Advanced Strength of Materials

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Lecture – 26

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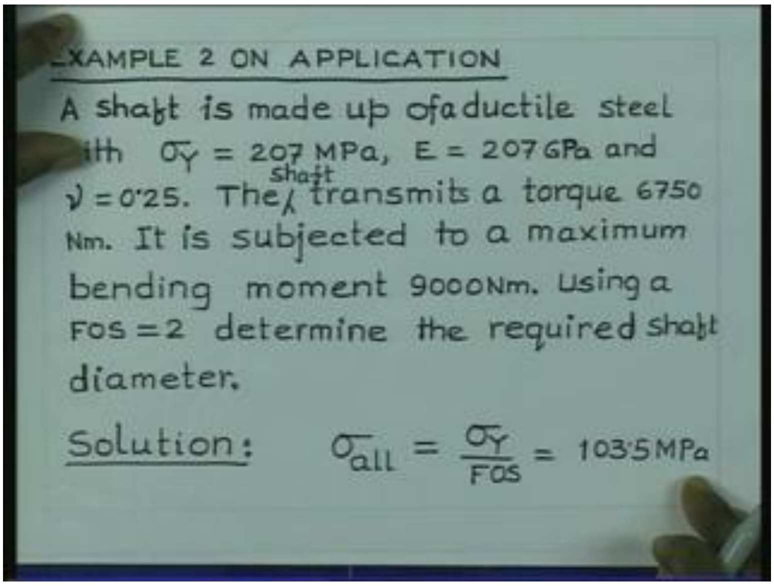

Last time we considered applications of criteria of failure. Let us consider second example on the application. It is given that a shaft is made up of a ductile material with sigma Y 207 MPa. Modulus of elasticity 207 GPa, and poisons ratio 0.25. The shaft transmits a torque 6750 Newton meter. It is subjected to a maximum bending moment 900 Newton meter. Using a factor of safety equal to 2, determine the required shaft diameter. So, the shaft is made up of a steel. And it is indicated, that we have to make use of a factor of safety equal to 2 on the yield point.

And therefore, if we have to go for a solution, we must make use of allowable stress equal to sigma Y by factor of safety. And this will give you 207 by 2, that is nothing but 103.5 MPa. Now, let us look into how we go about applying the criteria. Look into the component, if we draw the components schematically here.

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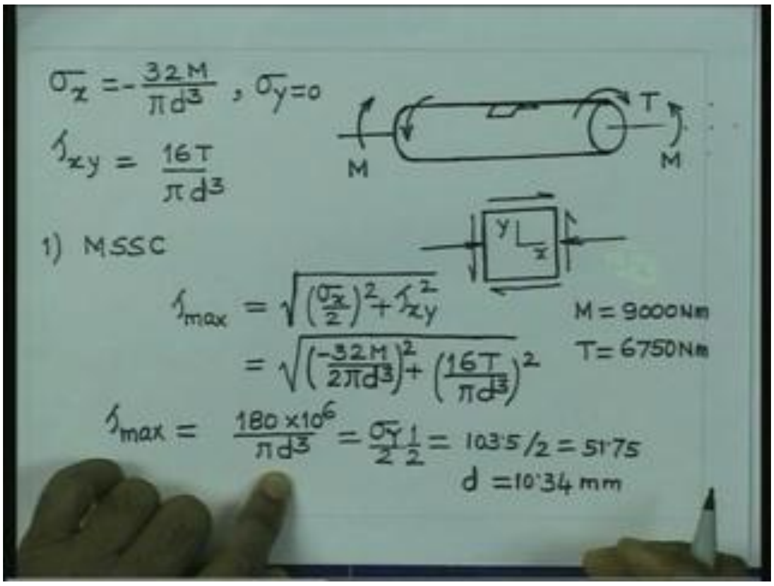

This is the shaft it is subjected to twisting moment. And also it is subjected to bending moment. You remember that when we apply the bending moment, with this type of bending moment applied on the shaft. You are going to have tensile stresses at the 6 o'clock position, and compressive stresses at the 12 o'clock position. And under the action of the twisting moment, you are going to get shear stress, which are going to be maximum around these outer circumference, or who are close to the outer circumference.

And therefore, if we consider an element at the 12 o'clock position or 6 o'clock position. That is these are the elements, which are going to be critically loaded. So, now let us consider an element, which is just around the close to the 6 o'clock position. So, I am trying to talk about this element, which is here. And it is just close to this position. And if I take this element and develop it, it is shown here. Let us consider, this is x direction and this is y direction.

Now, under the action of bending moment, you are going to have compressive stresses. And under the action of the twisting moment, you are going to get shear stresses. So, shear stresses on the right hand face, it will be directed like that in the same sense as the given torque. And therefore, the shear stresses, on this face is going to be like this. Similarly, the complementary shear stresses are going to come up on the top face. And also at the bottom face.

Now, the stresses therefore, are sigma x is equal to you can get the maximum stress, which is given by 32 M by pi d cube, where d is the diameter of the shaft. And M is the bending moment. This is obtained from the flexure formula. And in this particular case, you have clear cut specification, that sigma y equal to 0. Note that, I have written this sigma x to be negative, because it is subjected to compressive stress at the 12 o'clock position there.

Similarly, the shear stress, which you can obtain from the torsion formula of the circular shaft. And this is going to give you 16 T by pi d cube. If you want to consider this material for design,tThere are two criteria, which are applicable here. One is maximum shear stress criterion. And the other one is maximum distortion energy density criterion. This is a ductile material, therefore you have the possibility of considering these two criteria to be applicable.

And let us now try to see, if we apply the maximum shear stress criteria, criterion what is the diameter we get. So, first we consider the maximum shear stress criterion. This criterion states that the failure will take place, when the maximum shear stress reaches a critical value. Now, the maximum shear stress for this case, we have tau max. And it is given, you can calculate from the mode circle radius, which is nothing but square root sigma x by 2 whole square plus tau x y square.

And if we substitute the value here, then we get this is minus 32 M by pi d cube. And we have a division by 2. That is sigma x by 2 square plus 16 T by pi d cube square. We have been given that, bending moment is equal to 9000 Newton meter. And torque equal to 6750 Newton meter. So, we have M equal to 9000 Newton meter. And T equal to 6750 Newton meter.

So, if we make this substitution and we try to write that, this tau maximum can have the value at the most sigma y by 2 is the shear stress in uniaxial test tension. And you have a factor of safety of 2. Therefore, it is going to be sigma Y by 4. So, therefore, this can have a value equal to sigma Y by 2 is the shear stress in uniaxial tension. And the factor of safety of 2 comes here. So, therefore, it is sigma Y by 4.

And if you try to plug in the value of M and T. You get this thing at 180 into 10 to the power of 6 divided by pi d cube and these are all in millimeter units. And therefore, what you get from this is that diameter d as 10.34 millimeter. And you can write this value as, it is nothing but ((Refer Time: 09:22)) by 2, which is 51.75. So, therefore, you have opportunity of getting d, d as 10.34 millimeter.

We will now, consider application of the distortion energy density criterion, according to the distortion energy density criterion, the strain energy density. The distortion energy density at failure should have the value corresponding to the 1 at the uniaxial tensile at the yield point in the uniaxial tensile test.

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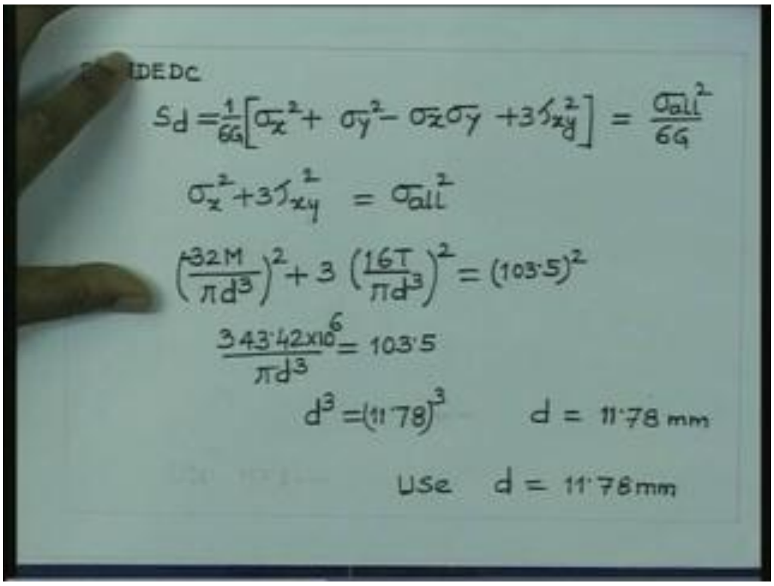

So, therefore, if we apply this criterion S d strain energy density of distortion. And it is given in terms of the Cartesian components of stresses by 1 by 6 G, sigma x square plus sigma y square minus sigma x sigma y plus 3 times tau x y square. And this strain energy density is to be limited by, sigma allowable square by 6 G. So, it gives us sigma x square plus 3 tau x y square is equal to sigma allowable square. And if you substitute the values, so we have minus 32 M by pi d cube whole square, plus 3 times 16 T by pi d cube whole square equal to 103.5 whole square.

And this gets simplified to 343.42 into 10 to power 6 by pi d cube is equal to 103.5. And this gives finally, d cube equal to and therefore, d equal to 11.78 millimeter. So, it means that, we may have we have to use, you see that the two value that we have obtained are ((Refer Time: 13:13)) 10.34 millimeter. And this has given 11.78 millimeter. Therefore, use the larger value it is good to keep the larger value for safety. And therefore, use d as 11.78 millimeter.

So, thus you see how the criteria of the failure can be applied for the design calculations. We have considered the theories of failure for materials, which are perfectly homogenous. There is no defect in the material. But in practice, you will find that materials are not defects, are not free of defects. You could have materials with cracks. These cracks can be inherent in the material. It can also come up during the service or the manufacturing.

Therefore, we have situations, where we may have to see the safety of a component, which is with defects. I would like to emphasize, that machine components are made up of materials. They are not all the time free of defects. Think of the casting, if we make a component out of casting, there you have a possibility that blow holes will remain in the material, which will act as a defect.

If we think of manufacturing of a component by welding, then in perfect welding can give rise to some disjoint, like a crack in the component and therefore, there is possibility of defect. So, these are the situation where, you find the defect in the material, which is coming up from the manufacturing process. Once, a machine has been made up of components, which are free of any defects.

And the machine is put into service, you will find that during the service, due to a loading on the component, there can be defect coming up. If a component is subjected to fatigue loading, it may give rise to some defect. If a component is subjected to high temperature, due to the degradation of the material or due to the creep of the material, there can be defect in the material.

So, therefore, you find that even if a machine has been put into service with components, which are made up of material, which are totally free of any defect. After some time, these components might develop defects, due to degradation. After some usage of the machine, when the machine is still under operation. The question that will be pertinent, under the action of the various loads, that act on the component. How do we guarantee the safety.

It boils down to asking a question, that if, there is a defect in the material. How do we ensure the safety of the component or how do we guarantee the safety, how do we assess the safety of the component. This was not possible till about 1950’s. But, it has been possible to assess the safety of components with defects. And the theory, which is useful in this connection, is known as theory of brittle failure.

I must mention that the brittleness or the ductility of the material is dependent on temperature at which the component is working. It will also depend on, if there is some crack like defect in the material. Say material like, mild steel can be very ductile. But, if it is put into use at sub 0 temperature, it may behave like a brittle material. Similarly, a ductile material normally, will show ductility. But, if a crack is introduced in the material, it will behave or it will fail like a brittle material.

So, therefore, the issue which is, which emerges out of all these discussion is that. If we have a crack like defect in a component. How do we assess the load capacity? How do we guarantee or how do we assess the safety of the component. So, this has been possible by considering the theory of brittle fracture.

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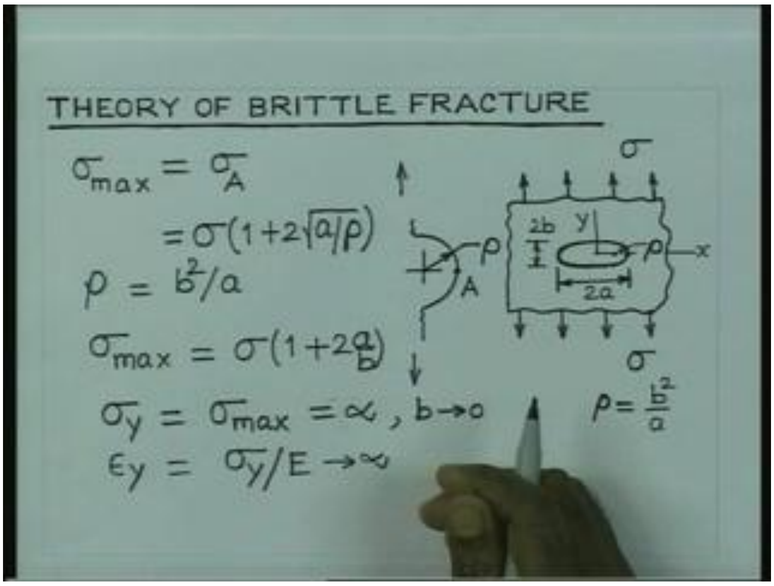

And this theory of brittle fracture it developed along some specific, it developed gradually. So, we would like to consider some of the point, which has led to the development of theory of brittle fracture. Suppose, you think of a component like a plate, have an elliptical crack here. The elliptical crack is oriented in the let us say, x y direction. And the plate is loaded with sigma. And it is major axis is 2 a. And the radius at the end of the major axis is let us say rho.

For a elliptical geometry this rho is nothing but it b the semi minor axis. Then it is nothing but b square by a. So, this b minor axis of the ellipse is 2 b. Then in that case, this radius of curvature at the end of the major axis is this curve by a, which is show through a larger scale here. So, this point, this point let us indicate by A. When, the component with this sort of cut out is subjected to loading sigma. It is well known, that there will be stress concentration at the point A.

So, if this sigma is the external stress, then maximum stress is going to occur at the point A. And therefore, this sigma maximum is equal to sigma A. And it has been calculated by ((Refer Time: 21:20)). And the value is given by sigma into 1 plus 2 square root a by rho, that is the value. And this rho as I have indicated, this rho is nothing but b square by a. Hence, if we now simplify by putting rho equal to b square by a. Then this maximum stress is given by 1 plus 2 a by b.

Look at these, that maximum stress is equal to 2 times sigma into 1 plus 2 a by b. Now, if the dimension b is reduced to 0 or the minor axis is reduced to 0. Then, what we find is that, the stress suites to infinity. So, therefore, what we have get now. That this stress sigma y, which is nothing but sigma max at the point A. So, at this point the maximum stress, which is the acting in the loading direction, it becomes infinity. When, b is tending to 0.

So, therefore, the stress is infinity. Similarly, the strain this point is subjected to purely tensile loading. And therefore, the strain is also equal to infinity, which is nothing but sigma y by E, that also suites to infinity. So, if we have to apply the classical theory of failure or classical criteria of failure, which could be based on the maximum principle stress or maximum principal strain, we find that it is not possible. Because, the magnitude of the maximum stress and the maximum strain suites to infinity.

So, that was the difficulty in applying the classical criteria of failure. And to aviate all these, there was a new way to look at the failure of brittle materials in the presence of crack. And it was pro founded by Griffith.

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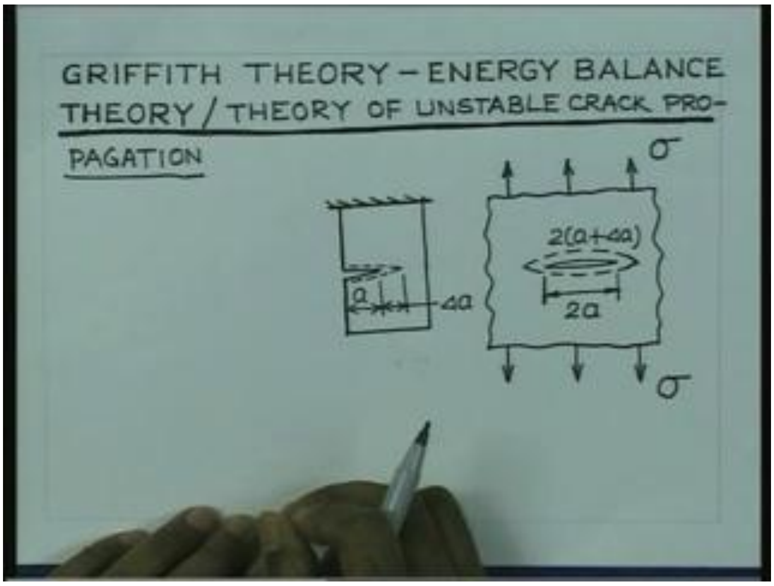

And it is known as Griffith theory or energy balance theory. It is also known as theory of unstable crack propagation. Now, let us look into schematically, a case of a plate again of uniform thickness and it is is consisting of a crack, sub crack of dimension 2a. What we are interested in looking for, what will be the condition. So that, this crack will extend unstably. Alternatively, think of a situation that you have a plate here, which is pulled at this end, it consist of a crack of dimension let us say a.

And what is the condition at which, it will extend unstably. Or if it extends to a enlarged length a plus delta a. What is the condition that is to be satisfied at the point of onset of extension. Similarly, in this case, if the crack is to extend through dimension let us say, 2 times a plus delta a. What is the condition to be satisfied for the extension of the crack. So, that is the subject, which was examined by Griffith. And he considered that this, extension phenomena of the crack, can be treated like a thermodynamic process. And it can be obtained the conditions for the extension can be obtained from the consideration of energy balance. So, let us consider the second example.

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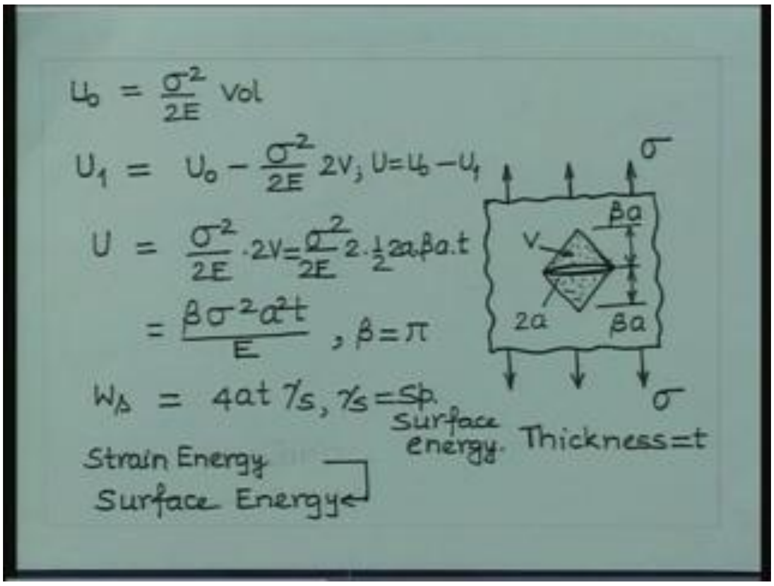

That we have the plate with this loading and the crack is of dimension 2a. If we consider for the moment that this particular component is not having any crack in it. So, if you think of situation, that there is no crack in it. And it is loaded with stress equal to sigma. Then the strain energy ((Refer Time: 27:05)) un cracked component. And therefore, the strain energy density at any point is going to be sigma square by 2 E. And therefore, the total strain energy stored in the body would be equal to sigma square by 2 E multiplied by volume of the body.

So, if you consider the un cracked component. Thus strain energy stored in the component is going to be nothing but sigma square by 2 E into the volume of the component. Now if we introduce a crack here, because of the crack, you will find that a portion of a material, which is just lying above the crack. And below the crack is going to be free of any loading, free of any space and strength.

And let us approximate that there is a zone here, which is like this, this is a zone it is height is let us say, it is a it is proportional to the crack dimension a. So, it is beta a let us say. Similarly, this side also, we have a length equal to beta a. So, that is the portion, which is going to be load free. If we just consider the thickness of the plate to be equal to t. Then in that case the difference in energy between the two cases. We have in the first case without the crack the whole body is available.

In the second case, we have the body with a portion, which is free of any stress and strain. And therefore, the energy in the second case, with the crack can be written as the strain energy of the earlier state minus the strain energy, which was stored in these two triangular region. So, if you consider the volume of the triangular region here is V. Then, you can write that the energy in the second case, is going to be equal to U 0 minus sigma square by 2 E.

That is the strain energy density multiplied by 2 times b that is the energy, which is going to be in the second case Now, we can write the value of this V. And therefore, this difference in energy, if you consider the energy in the second case is going to be reduced. So, therefore, the energy reduction, that reduction in strain energy if you represent by U. That is going to be nothing but U 0 minus U 0 plus sigma square by 2 E into 2 V.

And therefore, we can write now, let us write here that this U is nothing but U 0 minus U 1. And therefore, it is nothing but sigma square by 2 E multiplied by 2 times volume. And we can write this thing as sigma square by 2 E. That volume is nothing but half this length is 2 a. And this height is beta a. And the thickness is t. So, therefore, this is the volume of the triangular zone. And if you now, try to make simplifications, it comes out to be equal to beta sigma square a square into t divided by E.

So, therefore, the reduction in strain energy of the body is given by beta sigma square a square into t by E. And. In fact, Griffith did exhaustive calculations. And from the exhaustive calculations it was shown, that there is beta is nothing but equal to pi. You can also see another thing happening, that while the un cracked body is giving rise to a crack, there is breaking of material bonds. And this breaking of material bonds will require some energy.

And when you give energy as input to break those bonds, after the bonds are broken, that where does that energy go. That energy gets stored as the surface energy. That energy get stored at the surfaces. So, I would like to repeat this point again that when we try to pull the material to break the material bonds. We do give some input energy and as soon as the material bonds are broken. The energy that we have given as input, it gets converted into the surface energy.

And therefore, the energy gets stored at the surfaces. And if we consider, that the surface energy of the material is a constant. Then, if we have a crack of dimension 2 a in a material, then we have a surface, we have created a surface of area 2 a t at the top, 2 a t at the bottom, so therefore, total area is 4 a t. And therefore, the energy that has got converted into surface energy is given by 4 a t multiplied by these the specific energy, specific surface energy.

So, we can write now, that the surface energy of the cracked body is nothing but 4 a t multiplied by gamma s. Where, gamma s is the specific surface energy. Now, when you try to look, we try to consider the loading of a component like this, this component. ((Refer Time: 33:50)) Here in as you are deforming the component, you input strain energy in the body. And as soon as the crack extends, there is creation of new surfaces, so the breaking of the bonds.

And therefore, this strain energy is getting converted into the surface energy. So, therefore, while the crack propagation is taking place, you have really two forms of energy at plate. On the one hand, you have the strain energy. And we have the other form of energy is nothing but surface energy. So, these are the two forms of energy, which are going to be involved in the crack propagation process.

And as the crack extends, the strain energy of the body is getting reduced. At the same time the surface energy of the body is getting increased. Therefore, we can consider that there is a conversion of energy, from the strain energy, from the strain energy to surface energy during the crack propagation process. So, crack extension is a energy conversion process, where in the strain energy gets converted into surface energy. We can also think of that the strain energy, strain energy is the source is the source.

And surface is the sink. And therefore, energy getting transferred from the source to sink during the crack extension process. So, what is the condition for the unstable crack extension, if the energy is getting gradually, converted from the surface energy from the strain energy to surface energy. Then, what is the condition for the unstable crack extension. So, therefore, there is a rate, at which the strain energy is getting released from the body.

And at the same time there is a rate at which the energy is getting absorbed at the surfaces. If the energy availability or the rate of energy availability is more than the rate of absorption of energy at the surfaces, then the unstable crack extension will take place. So, Griffith stated, that the condition for the unstable crack extension is that. The rate of strain energy released is greater than equal to the rate of surface energy absorption.

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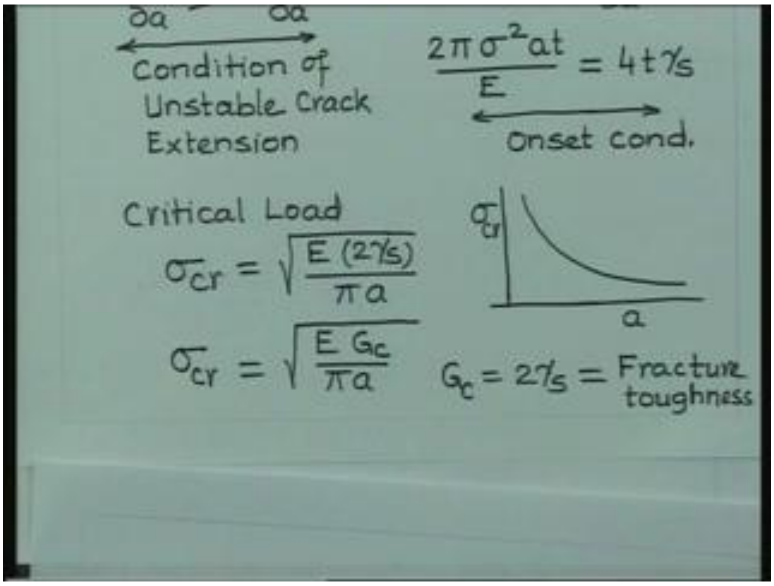

So, we can write now, the condition for the unstable crack extension, that delta u delta a. So, delta u delta a is the rate of release of strain energy. That should be greater than equal to rate of absorption of surface energy. So, that is delta W s delta a. So, rate of release of strain energy greater than equal to this. Rate of absorption of surface energy, this is the condition of unstable crack extension. So, the critical state is therefore, is reached when delta W delta a is equal to delta W s delta a.

So, critical condition when the crack is going to begin to extend is given by this condition delta W delta a is equal to delta W s delta a. So, the crack is going to extend, when delta W delta a is equal to delta W delta s. So, we can now, write the value of delta W delta a. Our u or pi sigma square ((Refer Time: 39:18)). U is given by pi sigma square a square into p by E.

So, therefore, if we take the derivative it is going to be twice pi sigma square a t by E. That is delta W delta a. And if, that is equal to delta W s delta a is nothing but 4 t times gamma s. So, this is the condition for the onset of extension. So, this is the onset condition. And therefore, it gives you the loading, which will trigger the extension, therefore, this critical load. So, critical load sigma critical is given by E into 2 gamma s by pi into a.

So, this is a very important relationship. It gives you the critical stress, which will trigger the extension of a crack in a material with surface energy capacity equal to gamma s. Where in you see that, E is the material property, gamma s is the material property. And therefore, the critical load capacity of a component is related to the crack dimension by this relation. So, this is the outcome of the Griffith theory that you can calculate the critical load, given the crack dimension in a material with material property gamma s.

In general, it is written as sigma critical is equal to E into G c by pi a. This is G c is equal to 2 gamma s. This is also a material property and it is known as fracture toughness or fracture resistance of the material. So, the critical stress is given by this one. In fact, Griffith did experiment with glass fibers. And he showed that the variation of this critical load with the crack dimension, it varies as expected it is going to vary like this. So, therefore, it was experimentally verified, that this rule is correct.

This particular relationship has lot of practical meaning. It gives you for the first time the capability of calculating the load capacity of a component with defect.

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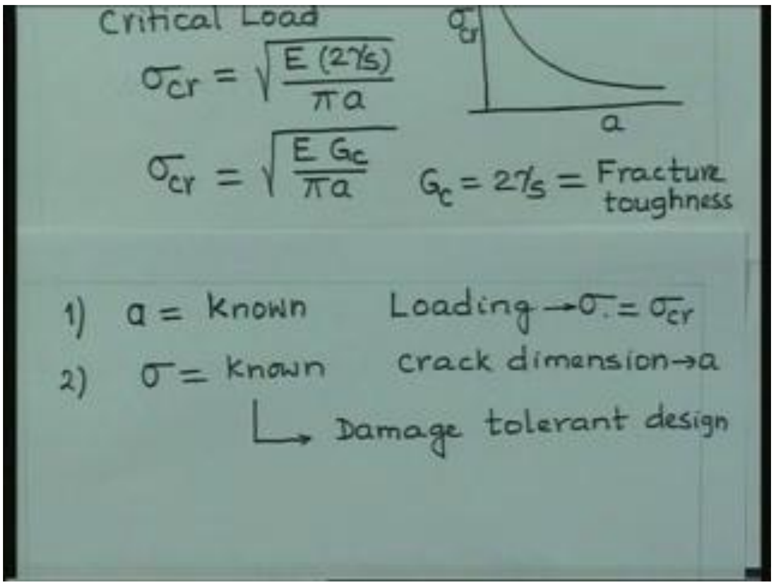

So, if you are giving the value of the loading. If you have given the value of let us say crack dimension a or it is known, if we consider a component, which is with a known crack dimension, then this relationship it gives you, what is the load capacity. Or what is the safe load, it gives you sigma. Let me put it this way, that if we know the crack dimension in the component. Then it is is possible to calculate the loading, which will trigger the crack extension.

And therefore, one can ensure safety, by keeping the load level below these level, alternatively if we know the loading on the component. Then, it is possible to calculate the crack dimension a, which will trigger crack extension or failure of the component. The second approach has given rise to a new consideration for design. It is known as damage tolerance design. What it means is that. We can calculate, how much damage the structure can tolerate. A component can tolerate given the loading and the material property.

Before, this theory was pro founded it was not possible to assess the safety of component with crack. But after, this was known and some more developments took place. The damage tolerant design has come into being. And it has been utilized for the aircraft structure design. What is done is that. Even if the critical component of a aircraft is having a crack in it. One can find out, whether the loading acting on the component is still going to lead to safe performance or not.

So, that is why, this all the airborne structures are examined for damage tolerant design. So, by considering the particular damage in the critical component. And taking the loading in the component one tries to ensure, whether the failure is going to take place or not by considering this relationship of the type given by this Griffith. One question, which can come up, how do we go about determining this fracture resistance of the material? So, let us try to consider, how do we get the factor toughness of the material evaluated experimentally.

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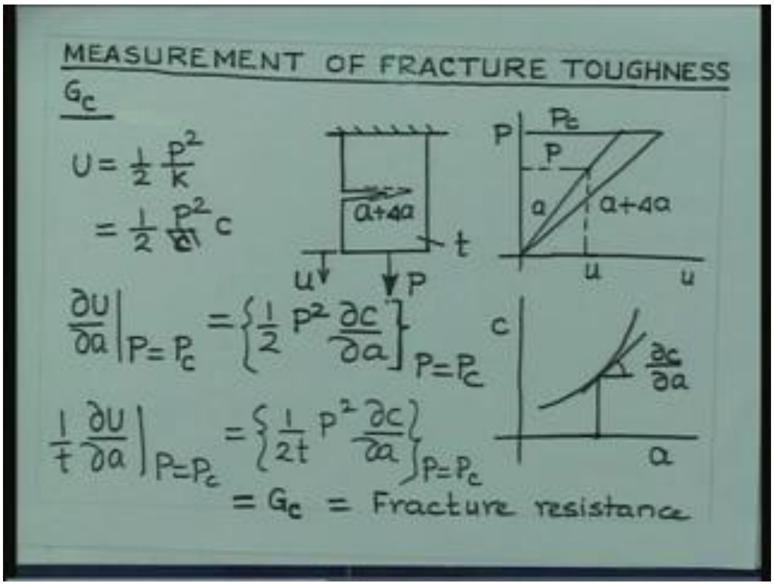

For that, you consider a specimen of this type and we apply the load here. And try to monitor the displacement. If we consider a particular crack size a. And if we try to plot the variation of the load with the displacement u. Then you find that, since it is elastic material. There is no plastic deformation or non-linear deformation. We are going to get the variation of load with deflection to be given by this. If you consider a crack of a larger size let us say, a plus delta a.

And if we try to plot the load versus reflection, you are going to see now. A variation of this type, where in for the same load you are expecting larger deflection, because the they are the crack of larger size. At the critical level of the load, when the P reaches critical level, you are going to see extension of the crack from length a to a plus delta a. If we calculate the strain energy at any level. Suppose, we just consider this level of the loading, let us say p and this is the deflection u.

Then the strain energy at this point is nothing but half P square by K Where K is the stiffness of this component. If we make use of the compliance then this is nothing but half P square by C. Where, C is known as compliance and it is reciprocal of stiffness. If we consider the rate of release of strain energy, delta U delta a, under a constant load P equal to P c. Then it is going to be given by half, I will take the part this should be mistake here it should be half P square into C.

And therefore, it is is going to be half P square delta c delta a. And this is we have to calculate the value for P equal to P c. So, this gives you the rate of strain energy release per unit crack extension. And when you have unit crack extension, if the thickness of this specimen is equal to t. Then, the energy release rate is given by delta U delta a 1 by t delta U, delta a. P equal to P c is given by 1 by 2 t P square delta c delta a, if we can draw a curve showing the variation of this compliance for different crackling. So, we can consider specimens of different cracklings here. And if we find out the slope that will give us stiffness and reciprocal of that will give us compliance. So, we can plot a curve. And this curve will this looking something like this. And if you are trying to do a experiment with a specific crack length. Then for that crack length a, you can find out what is going to be the slope of the curve delta c delta a.

And hence, this strain energy release rate at the points in the crack is extending is given by 1 by 2 t P square delta c delta a, where P equal to P c that is nothing but the critical strain energy release rate. And this is equal to G c fracture toughness of the material. At the point of crack extension, the strain energy release rate is critical and that is nothing but the fracture resistance of the material.

So, therefore, in order to determine the value of this fracture resistance of a material. We can do experiment like this. We can take a specimen of a particular geometry. And then we introduce a crack of certain dimension. And we do the testing by applying some load. And try to find out, what is the load at which crack is going to extend, and making use of this formula. We can find out, what is the critical strain energy relation.

So, before you can do that, you must have the information available about the variation of compliance of the specimen with crack length. And when you have got this P c, you take the crack length that is starting crack length. For that crack length, you find out from this curve delta c delta a. And you have already got from the experiment of the value of P c. And from this relationship, you can find out what is the critical strain energy release rate. And which is nothing but the fracture resistance of the material.

In fact, experiment is done along this line to find out the fracture resistance of the material. So, therefore, what we have done over the last couple of lecture. We have tried to consider the theories of failures of material, which are homogenous. And we have considering the theories, five theories. One is based on the maximum principal space.

Then the second one based on the shear space. Then we have consider the maximum principal strain criterion.

Then we have consider two criterion, which are based on energy density. The first one was based on the maximum strain energy density. And the second one was based on the maximum distortion energy density. We have shown that this criteria are not equally applicable to all the materials. The ductile materials are obeyed mostly by the maximum shear stress criterion, and the maximum distortion energy density criterion. And for the brittle materials the other 3 criterion, other 3 criteria are applicable.

We have also illustrated the usefulness of these criteria by considering design exercises. Then we have considered, if the material is consisting of a defect like a crack. Then how do we go about assessing the load capacity of such component or of such of component made up of such material. In that connection we have talked about the energy balance criterion that is due to Griffith.

And we have shown that the condition for the unstable extension is going to be rate of strain energy release rate, must be greater than equal to the rate of absorption of surface energy. Thereby, you have been able to calculate the load capacity of a component with a given crack size. And the same relationship can be useful for calculating the damage that can be tolerated for given level of loading on the component. Lastly we have tried to consider, how this fracture toughness of the material can be measured experimentally.