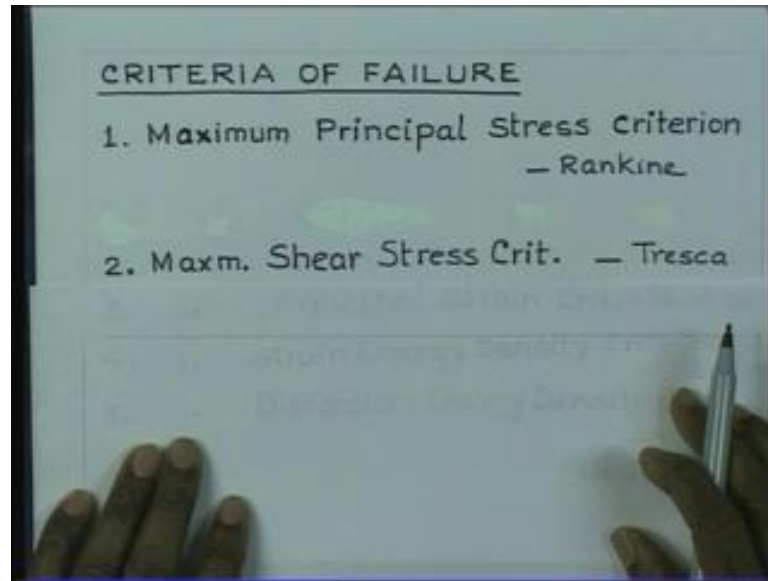


**Advanced Strength of Materials**  
**Prof. S. K. Maiti**  
**Department of Mechanical Engineering**  
**Indian Institution of Technology, Bombay**

**Lecture – 25**

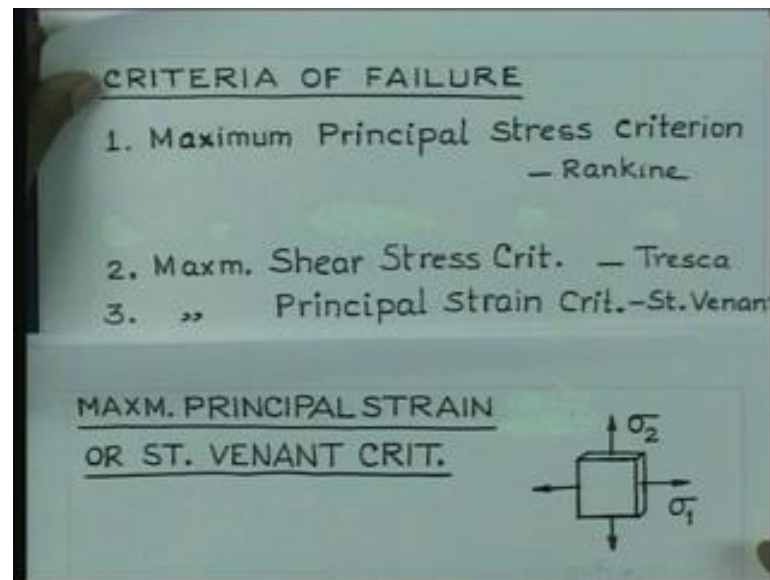
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Last time we were talking about the two stresses based criteria. The first one was maximum principle stress criterion or Rankine criterion. And the second one maximum shear stress criterion which is due to Tresca. It must be emphasized that in both these cases, what is considered is that? When a component made up of a material is loaded. The stress at the most critically loaded point, whenever it reaches a critical level there is going to be failure.

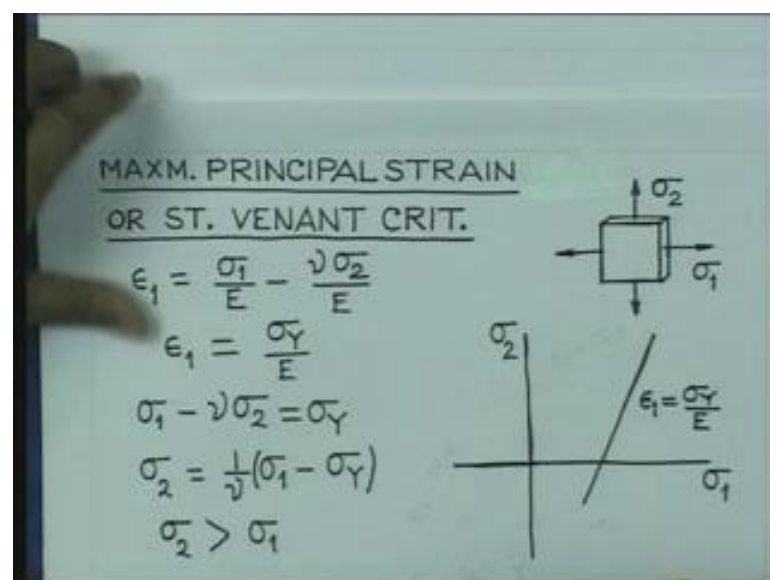
So, it is a question of stress reaching the critical level. And this critical level is determined from the tensile test. In the first case, it is considered that the maximum normal stress that is going to be the limiting factor for the material failure. And in second case, it is the maximum shear stress, which is the limiting factor for the failure of the material. So in both the cases, it is the stress which provides the basis for the failure of the material.

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Now, we would like to consider other three criteria. In the criteria which was proposed by St. Venant which is considered to be maximum principle stress strain criterion. In this what is considered is that? There is a limit in the normal strain that the material can withstand. And if the strain exceeds the value there is going to be failure. This is known as maximum principal strain criteria criterion.

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Now, we would like to consider its details. Consider the point in a material which is loaded like this. So, there is going to be strain in the two principal directions. And

according to this criterion, the material will fail whenever the maximum principle strain reaches the critical level. And this critical level is given by the strain at the yield point in uniaxial tensile test.

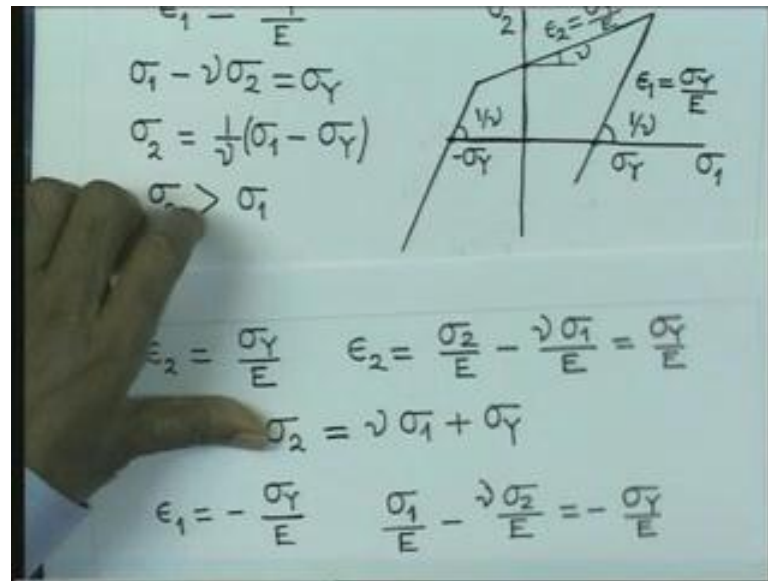
Let us now, consider the material points, wherein the stresses are  $\sigma_1$  and  $\sigma_2$ . Then, the maximum principle strain is  $\sigma_1$  is the larger stress. We can write that to be  $\sigma_1$  by modulus of elasticity  $E$  minus poisson's ratio multiplied by the stress  $\sigma_2$  divided by  $E$ . So, the condition of failure is that whenever this strain reaches the level corresponding to the yield point in uniaxial tensile test. So, therefore, this strain at yield point is nothing but  $\sigma_y$  by  $E$ . So, therefore, this is the condition of failure.

So, if we now simplify it means that  $\sigma_1$  minus  $\nu$  times  $\sigma_2$  should be equal to  $\sigma_y$ . That is the condition of failure and we can write this thing as  $\sigma_2$  is equal to  $\frac{1}{\nu}(\sigma_1 - \sigma_y)$ . So therefore, this is the straight line and this straight line is going to make an angle  $\frac{1}{\nu}$  with the stress direction  $\sigma_1$ .

So, if you consider the two dimensional stress spaces. Let this is the direction  $\sigma_1$ . Let us, consider the other direction is  $\sigma_2$ . Therefore, this is a straight line which makes an angle of  $\frac{1}{\nu}$  with the  $\sigma_1$  axis. So therefore, this is the locus in the quadrant in the right hand side of the plane. This one this straight line equation is  $\sigma_1$  is equal to  $\sigma_y$  by  $E$ .

So, all the points which are lying on the left side, there is strain less than this critical level  $\sigma_y$  by  $E$ . And if, it is on the right hand side it is going to be more than the strain  $\sigma_y$  by  $E$  and hence it will lead to failure. Now, if you consider the other case let us say that  $\sigma_2$  is the stress which is dominant.

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Then in that case, you will get this equation to be epsilon 2 is equal to sigma y by E is the condition of failure. And the epsilon 2 can be written as sigma 2 by E minus nu sigma 1 by e and this is equal to sigma y by E. Therefore we will have a case now. This is sigma 2 is equal to nu sigma 1 plus sigma y.

So, this is again a straight line which makes an angle of nu with the x axis sigma 1 axis. So, therefore this, when sigma 1 equal to 0. Sigma 2 is sigma y. So therefore, this straight line will pass through sigma y on the sigma 2 axis. So therefore, this point is sigma y. And think, you also should note that in this case sigma 2 is 0, then sigma 1 equal to sigma y. So, therefore, this is the point which is sigma y. So, this point is also sigma y.

And this straight line is going to make an angle of nu with the sigma 1 axis. So therefore, this angle tangent of this angle is nothing but poisson's ratio, whereas tangent of this angle is equal to 1 by nu. So, we can write that this equation is sigma epsilon 1 is equal to sigma y by E. And this straight line it is equation is sigma epsilon 2 is equal to sigma y by E. So, this is the other straight line.

So, therefore, this is the space which gives the combination of stress or rather combination of stresses which will cause no failure. And if, the stresses are lying outside this zone, that will lead to failure. We can also consider a case the strain to be loading is consisting of both positive and negative stresses are tension and compression.

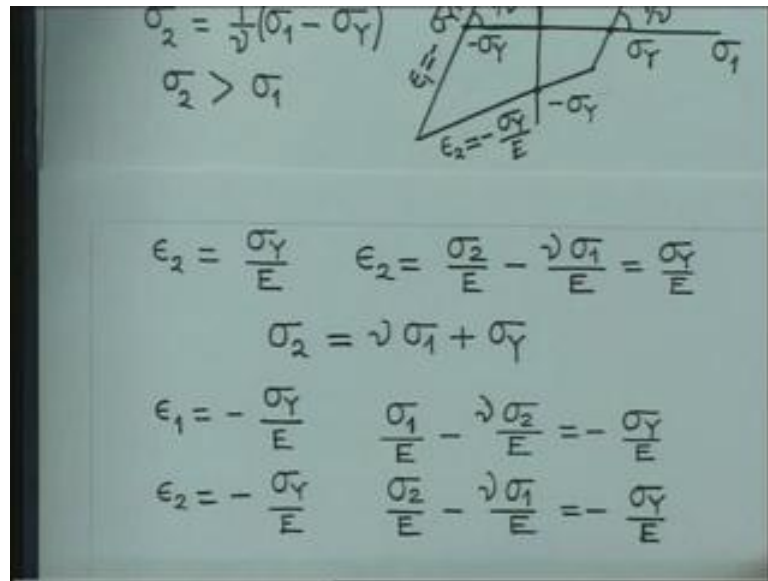
So if that is the case, then we can write that of the loading is compression. Then, we can have a condition that whenever this strain reaches the value minus  $\sigma_y$  by  $E$  that is going to lead to failure. And therefore, it will give us finally, that  $\sigma_1$  by  $E$  minus  $\nu \sigma_2$  by  $E$  is equal to  $\sigma_y$  by  $E$ . And this straight line is again going to have a slope of  $1$  by  $\nu$ .

With the  $\sigma_1$  axis and this straight line can be drawn and it is going to again pass through the point if let us say that  $\sigma_2$  is  $0$ . Then,  $\sigma_1$  is equal to minus  $\sigma_y$ . So therefore, it is going to pass through a point which is at a distance of  $\sigma_y$  from the origin. So, this is minus  $\sigma_y$  and the straight line that we are going to get out of this equation is nothing but this straight line which is parallel to this one. So therefore, this is also making an angle of  $1$  by  $\nu$ .

By similar consideration if we have the strain  $\epsilon_2$  is equal to minus  $\sigma_y$  by  $E$  that will give us  $\sigma_2$  by  $E$  minus  $\nu \sigma_1$  by  $E$  is equal to minus  $\sigma_y$  by  $E$ . So herein again, if you have  $\sigma_1$  equal to  $0$ , you will have  $\sigma_2$  equal to minus  $\sigma_y$ . So therefore, this is also a straight line which makes an angle of  $\nu$  with the  $\sigma_1$  direction.

And it passes through the point  $\sigma_2$  equal to minus  $\sigma_y$ . So therefore, this is the point. This straight line will pass and this straight line is parallel to the other straight line here. So therefore, this gives you the straight line corresponding to  $\epsilon_2$  equal to minus  $\sigma_y$  by  $E$ . So therefore, this is  $\epsilon_2$  is equal to minus  $\sigma_y$  by  $E$  and this straight line we have  $\epsilon_1$  is equal to minus  $\sigma_y$  by  $E$ .

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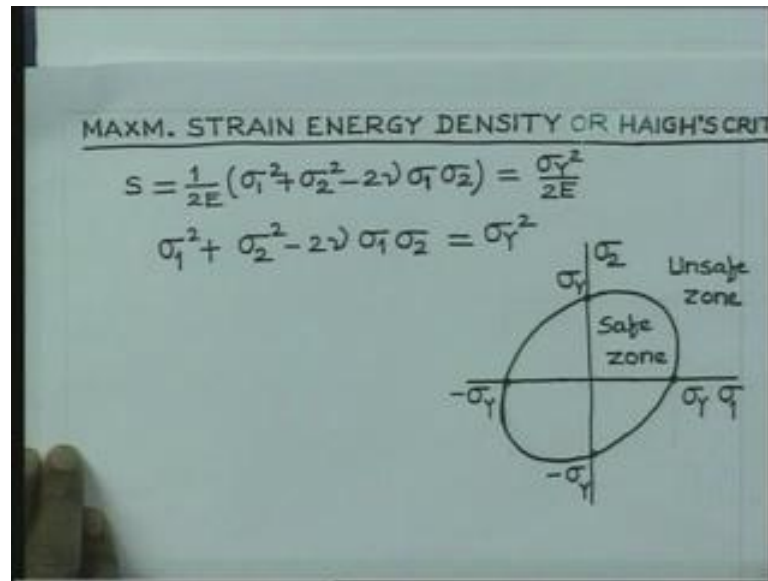


So, therefore, this finally, gives us the space as per the St. Venant criterion it is a rhombus. And all these points are at a distance of  $\sigma_Y$  from the origin. And therefore, the zone enclosed by this rhombus. It gives you the safe zone and the combination of stresses which lie outside this zone is unsafe zone.

So, you find the difference with the other two criteria. Are that herein, the failure is best on the strain rather than stress. So, if you like to call it that strain is nothing but a pain in the material there is a limiting plane that the material can withstand before the elastic failure.

And considering that situation, we find the combination of stresses that is going to lead to failure is given by this rhombus. Note that these two points are going to lie on a line which makes an angle of 45 degree with the x with the  $\sigma_1$  direction. So, rather  $\sigma_1$  axis, so this is the these are two stresses which are going to make an angle of these are going to lie on a straight line which is going to make an angle of 45 degree with the axis.

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So far the failure of the material has been based on the consideration of some level of stress and strain. But, some people were of the view that the failure of the material is guided by not by the stress not even by strain. But, it is guided by the amount of energy that it put into the material for deformation.

So, this was the consideration which was first proposed by Haigh and it is known as Haigh's criterion. And he proposed that the total strain energy of deformation in a material has a critical value. And therefore, the failure of the material will occur whenever the strain energy density at the most critically stressed point reaches the critical level.

And that critical level corresponds to the failure point in the tensile test at yield point. I have already given to you the strain energy density. The strain energy density in the case of uniaxial loading at the yield point is nothing but  $\sigma_Y^2$  by  $2E$ . And therefore, the amount of energy that the material can withstand before failure is nothing but  $\sigma_Y^2$  by  $2E$ .

So, this failure criteria can therefore be expressed again considering that the loading at a point can be given in terms of two stresses  $\sigma_1$  and  $\sigma_2$ . We can write the condition of failure that the strain energy density is which is related to the two stress components by  $\frac{1}{2E} (\sigma_1^2 + \sigma_2^2 - 2\nu \sigma_1 \sigma_2)$  is equal to  $\frac{\sigma_Y^2}{2E}$ . That is the strain energy density expressed in terms of the two principal stresses

at a point. And therefore, whenever this strain energy density reaches the level in uniaxial tensile test at yield point. That is  $\sigma_y^2 / 2E$  there is going to be failure.

So, this means that the combination of stresses that are going to lead to failure of a component made of a particular material is given this expression  $\sigma_1^2 + \sigma_2^2 - 2\nu\sigma_1\sigma_2 = \sigma_y^2$ . Note that in this case, the energy is always positive. And therefore, we will find that this is a quadratic sign of the stresses do not matter much.

And the surface, that are the curve which is represented by this equation can be drawn. We can consider now our two principal directions as the coordinates. This is  $\sigma_1$  direction, this is  $\sigma_2$  direction. Note here, when  $\sigma_2 = 0$ . The failure stress is going to be  $\sigma_1 = \sigma_y$ . So therefore, the curve is going to pass through the point which is at a distance of  $\sigma_y$  from the origin on the  $\sigma_1$  axis.

So, also if we have negative stress then also we are going to have  $\sigma_1 = -\sigma_y$ . So therefore, it is going to pass through this point which is  $-\sigma_y$ . By the similar consideration it is going to pass through this point which is at a distance of  $\sigma_y$  from the origin. And it is also going to pass through this point which is going to be at a distance of  $-\sigma_y$  from the origin.

So therefore, this curve it can be drawn now. These are the four points through which it is going to pass and it is an equation of an ellipse. So, this is the failure space given by maximum strain energy density criterion. And as usual the spaces which lie within this curve represent the safe combination. And therefore, this is the safe zone. And if the combination of stresses gives rise to the energy per unit volume more than this there is going to be failure. And therefore, outside this space we have the unsafe stress combinations. So, this is nothing but unsafe zone.



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MAX. DISTORSION ENERGY DENSITY OR  
VON MISES CRIT.

$$S_d = \frac{\sigma_Y^2}{6G}$$
$$S = S_v + S_d$$
$$= \frac{1}{6G} [\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2]$$

or,  $\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2 = \sigma_Y^2$

$$\sigma_1 = \sigma_2 = \pm \sigma_Y$$

The next criterion that was proposed it is also based on energy. But then, the energy that was considered to be responsible for failure is different. You remember the total strain energy density at a point can be split into two components; one is the volumetric component and the other one is distortion component. So, the total strain energy density is can be split into two components; one is volumetric part and the other one is distortion part.

So, this part is responsible for the change in volume of the element. And this part is responsible for the distortion of the element. According to this maximum distortion energy density criterion or von mises criterion, it is considered that this distortion energy density is the responsible energy for the failure of the material; that means it is the distortion energy density which has a limit and at the failure point the distortion energy reaches a critical level.

So we can state it, that the failure of a material point of a component will occur whenever the distortion energy density reaches a critical level. And this critical level is given by the strain energy density at the yield point of the material in uniaxial tension. So, we can write that the distortion energy density  $s_d$  should be equal to  $\sigma_y^2$  by  $6G$ , where  $G$  is the modulus of rigidity. So, that is the limiting value and this is the condition of failure.

Now, if you again try to represent the distortion energy density in terms of the principal stresses at a point. Then, this distortion energy density is given by  $\frac{1}{6G} (\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2)$ ; that means, the combination of stresses that are going to cause failure is given by  $\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2 = \sigma_y^2$ .

Here again, you find that the sign of the stresses is not important. And at the same time you find that this curve is going to pass through the four points on the stress axes which are at a distance of  $\sigma_y$ ; that means, when  $\sigma_2 = 0$  you have  $\sigma_1 = \pm \sigma_y$ . Similarly, when  $\sigma_1 = 0$   $\sigma_2 = \pm \sigma_y$ , so therefore if we now, consider the coordinates as  $\sigma_1$  and  $\sigma_2$ .

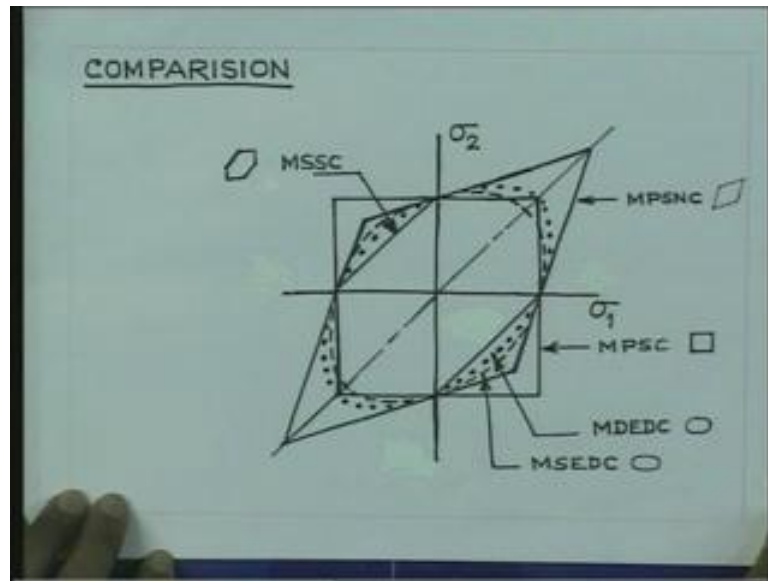
Then, this curve is going to pass through the four points which are at a distance of  $\sigma_y$  from the origin. And this is the equation of an ellipse, whose axes are making an angle of 45 degree with the two stress axes. So, we can now draw the ellipse here and this ellipse is also going to pass through  $\sigma_1 = \sigma_2$ . You will find that this is going to pass this I mean  $\sigma_1 = \sigma_2$ .

We are going to have again the stress the curve is going to pass through a point at a distance of  $\sigma_y$  from the two axes. So therefore, we can now draw the ellipse, what I meant is that? When  $\sigma_1 = \sigma_2$  then we get that this is nothing but  $\pm \sigma_y$ . And therefore, this ellipse is also going to pass through the point whose coordinates are  $\sigma_y, \sigma_y$  or  $\sigma_y, -\sigma_y$  or  $-\sigma_y, \sigma_y$  or  $-\sigma_y, -\sigma_y$ . So therefore, these are the other two points on the ellipse.

Hence, the combinations of stresses which lie within this curve represent the safe zone. And the combinations which are outside they indicate the unsafe combinations of stresses and the boundary representing the all safe zone. Thus we have got five criteria to explain the failure of a material. First one was based on the maximum principal stress. So; that means, it is the normal stress acting on a plane that guides the failure.

The second one is maximum shear stress theory. So, they are it means they are shearing stress is the guiding factor. Then, we had the strain criteria and in the case of strain criterion the maximum strain acting on the plane is the guiding factor. Lastly, we have talked about two energy criteria. One is based on the total energy density and the last one is based on the distortion energy density rather than the total strain energy density.

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Now, you would be interested to see, how they are different and where they are applicable. Is it that all the materials are going to obey all the criteria or it is some of the criteria are going to be suitable for certain materials? So now, let us try to see how these criteria are different in the stress space. So, we just consider the comparison of the five criteria here.

So, we have drawn the two principal stress axes  $\sigma_1$  and  $\sigma_2$ . You remember that the maximum principal stress criterion, which was due to ranking that represents a square of dimension  $2\sigma_y$ . So that is the ranking criteria criterion. And then, we have considered the maximum shear stress criterion. Maximum shear stress criterion it is nothing but this hexagon. And the maximum principal stress strain criterion is given by this rhombus.

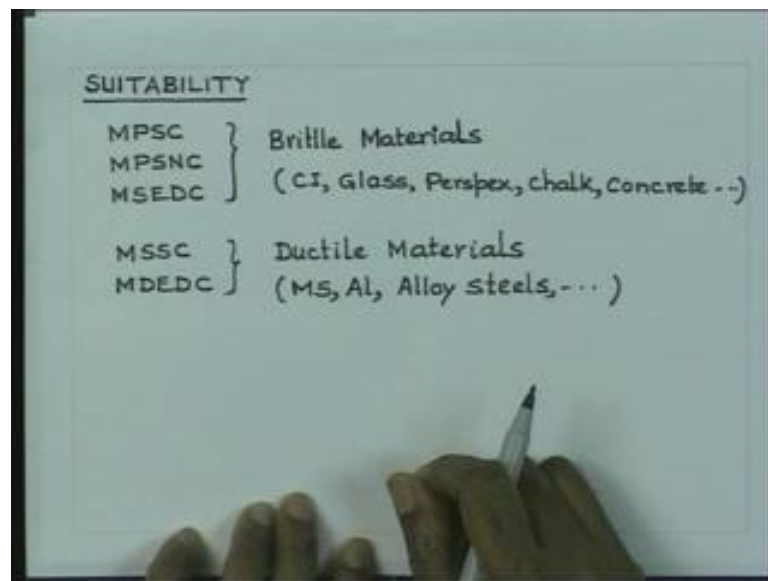
Then, we had talked about the two energy criterion. The first one this maximum strain energy density criterion is this. This is the ellipse here, this ellipse. Lastly, we have the maximum distortion energy density criterion which indicates this ellipse given by points dots here, so you of all the criteria at one place.

The maximum difference between maximum difference amongst the criteria are shown in this region, wherein the two stresses are equal or in this region, where the stresses are equal or even it is going to have. So; that means, it is in this region, where both the stresses are tensile or both the stresses are compressive. And we also find that, if one of

the stresses is compressive, another stress is tensile. There is also large difference. So also, this zone, but more or less the criteria are going to have not much of differences.

Then what is the material behavior in fact? Which criteria is going to govern the failure of what material; hat means, the suitability of the criteria. So, if you look into the issue of suitability.

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<u>SUITABILITY</u>	
MPSC MPSCC MSEDC	} Brittle Materials (CI, Glass, Perspex, chalk, concrete...)
MSSC MDEDC	} Ductile Materials (MS, Al, Alloy steels, ...)

Then material can be broadly classified into two categories. One set of materials are called brittle materials and the other set of materials are ductile materials. The brittle materials are those which fail without undergoing much of plastic deformation. And the ductile materials are those materials which undergo substantial plastic deformation before failure.

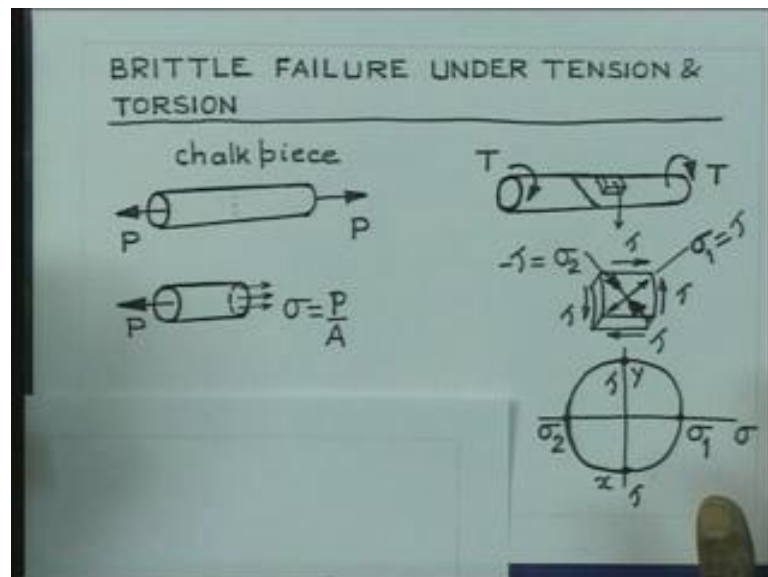
Now, the brittle materials or rather let me put it this way that the suitability of the criteria is dependent on the type of material. The loading, the temperature conditions the thermal field, so or the speed of loading. So, all these are responsible for the applicability of criteria, but when we are trying to think about quasi static loading for quasi static loading.

It is generally found that the criteria of maximum principal stress, maximum principal strain, maximum strain energy density. These criteria are found to be suitable for brittle materials. So, they are more governed by the three criteria, what are the examples. You

can consider cast iron, then glass, Perspex, chalk and concrete. So, you can go on adding. Just we would like to just mention these materials. These are the materials whose failure is governed by any of the three criteria.

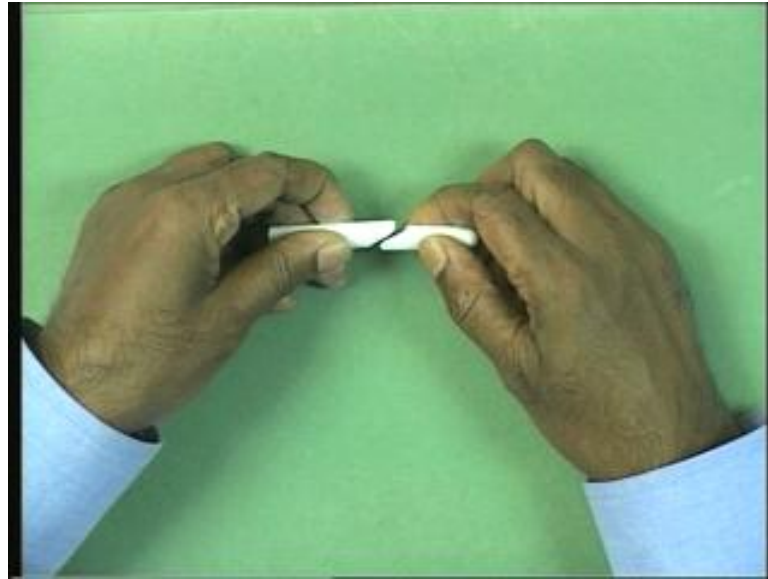
On the other hand ductile material failure is governed by the criteria of maximum shear stress and maximum strain energy density. So for ductile material, these two criteria are more suitable, what are the examples here? Majority of the steels that is mild steel, aluminum, alloy steels etcetera are the examples belonging to this category, before we go into considering applications of the theory.

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Let us get back to the two examples that we were considering in the last presentation. You remember that we considered the failure of chalk piece under tension and torsion. Chalk is a brittle material. And therefore let us now look into the failure of that brittle chalk piece under tension and torsion. Just schematically I have shown it here the same chalk piece. It is subjected to tension here and torsion, if you remember the failure that was demonstrated.

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We had the chalk piece here. And we pulled it and we pulled it, it showed a failure which is just along the cross section. So, it fails perpendicular to the loading direction. On the other hand when you consider the failure of the same chalk piece under the action of torsion then it failed in a helical direction. That it failed at an angle of about 45 degree with the axial direction. So here, that that angle is almost 45 degree. So, we would like to see this difference. Does it mean that the failure in both the cases occur under a common condition.

(Refer Slide Time: 34:44) If you now look into the loading of the chalk piece a typical section here is shown there. The stresses are all uniform and this stress is equal to  $P$  by area of cross section. So the failure took place, when these stresses reach the critical level of the material. Therefore, it failed by purely normal stress reaching the critical level. It is a principal stress and it reaches the critical value. In this case, whatever the value is we can measure it by tensile testing.

Now, in the case of torsion, the failure was observed at an angle of. So, it was observed at angle like this 45 degree. Now let us see, why it is so? Just consider this chalk piece to be a circular shaft like. And now if you concentrate your attention on an element on the surface of the chalk let us say this is the element.

I am trying to a larger scale. So, this is the element on the surface and this element if you draw it to a larger scale. So, we will draw this to a larger scale here. Under the action of

the twisting moment, we are going to get the shear stresses acting here, which will be directed in the same sense as the torque. Similarly, the stress on this space is going to be  $\tau$ . This you can find out that it is nothing but  $16 T \text{ by } \pi D^3$ , where  $D$  is the diameter of the chalk piece at that cross section.

Similarly, we will have the complementary shear stresses acting on this space which is also  $\tau$ . And on this space it is going to be again  $\tau$ . So, this is the stress state on the surface of the chalk. If we try to draw the Mohr circle for this case let us consider that this is our  $x$  direction. So,  $x$  plane is subjected to stress no normal stress. There is only shear stress.

Similarly, the  $y$  direction is subjected to no normal stress. Only there is shear stress. So, if you try to draw the Mohr circle for this point. This is our normal stress axis and the vertical axis is let us say shear stress axis. Now the stress on the  $x$  plane will be represented by there is 0 normal stresses and the shear stress here. For the Mohr circle it is negative shear stress, because it produces anti clockwise moment.

So, therefore, the shear stress will be on the  $x$  plane. So, therefore, this is the  $x$  plane. This direction is the  $x$  plane. Similarly, the stress which is producing clockwise moment it is nothing but positive stress. So therefore, this will be the point which indicates the  $y$  axis. So, this is the  $y$  axis. So, the circle radius is this, this is the origin. And therefore, if you draw the Mohr circle this is the circle for this case.

So, the maximum principal stress is occurring at this point, minimum principal stress which is  $\sigma_2$  which is here. Look at this the magnitude of the maximum principal stress is nothing but the shear stress  $\tau$ . And the minimum principal stress is compressive stress which is nothing but again  $\tau$ .

Now, what is the direction of this maximum principal stress? It is at an angle of 45 degree in the anti clockwise strains with respect to the  $x$  direction. So therefore, you will find that the maximum principal stress is going to occur in this direction, which is this  $\sigma_1$  direction. And this  $\sigma_1$  stress is positive and it is magnitude equal to  $\tau$ .

Similarly, the other direction, which is at angle of 90 degree with this direction, so therefore this, is the other octagonal direction. So, this is the direction of the other principal stress and this is  $\sigma_2$ . And herein of course, this  $\sigma_2$  is equal to minus

tau. So, this is minus tau. So, it is really a compressive stress. So, therefore, better to show it this way that this is the stress acting in this direction.

So, this is how the compression is acting in this direction, tension is acting in that direction. So therefore, the maximum tensile stress is acting in this direction. And therefore, the failure if it is to occur by the maximum principal stress then this is the plane in which the failure will occur. So therefore, you will find the failure to be occurring on this plane which is going to make an angle of forty five degree with the axial direction.

So therefore, you will find that in both these cases the failure is governed by the maximum principal stress. And therefore, unity is there in the failure. That arises in the two situations. So, herein the failure of this material is fully explained by the maximum principal stress criterion.

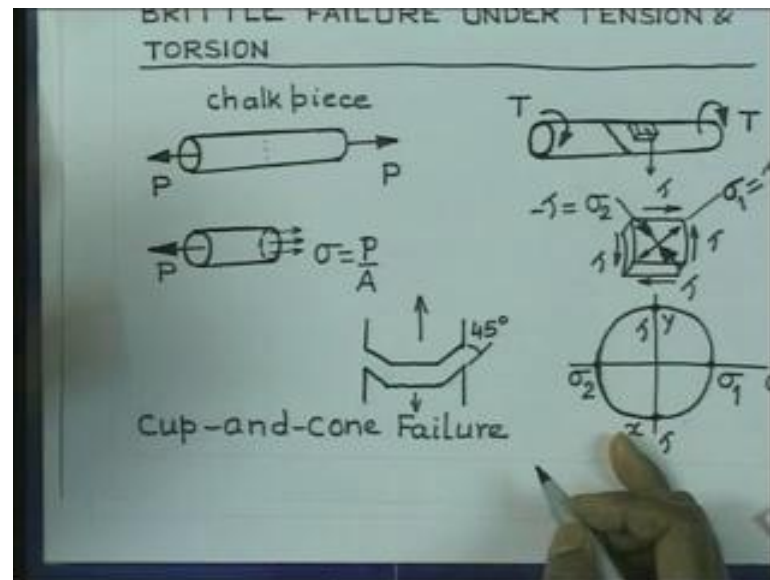
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The other example that we were considering the other day is the failure of the aluminum specimen in the tensile test. And in this case, we saw that the failure was at an angle of 45 degree. We had the cup and cone failure. This is the cone and this is cup and this angle is about 45 degree with the loading direction.

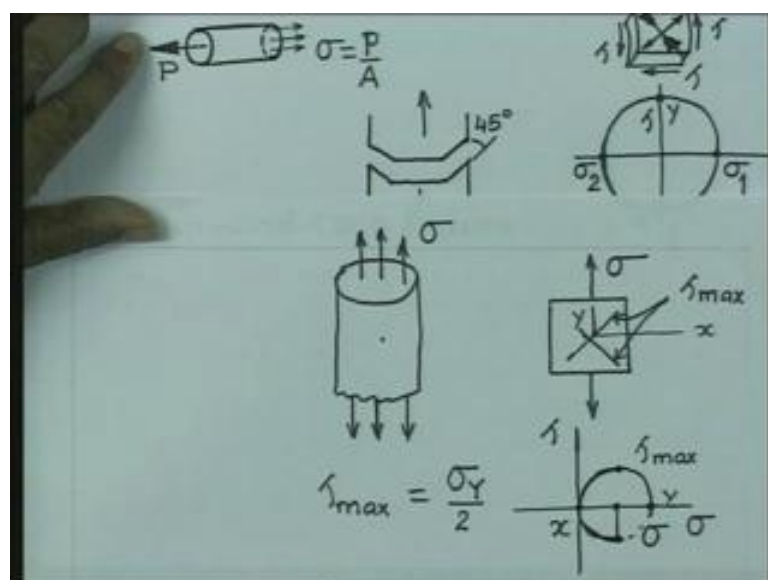


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So, let us try to see, why this failure occurs at an angle of 45 degree to the loading direction. So in the case of tensile specimen, if we schematically show the failure has taken place like this. The central portion has failed perpendicular to the loading direction. And this circumferential portion has failed at an angle of 45 degree. So, this angle is 45 degree. This is the typical cup and cone failure.

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Now, if you try to look into a point on the surface of the tensile specimen. So, we have the specimen like this. And it is loaded by uniform tension. Let us, say that that is equal

to  $\sigma$ . So, a point on the surface of the specimen is subjected to loading purely uniaxial loading.

So, if we now consider the Mohr circle diagram for this case. We have the x plane, this is the say x direction and this is y direction, x plane does not have any stress. So therefore, we can represent the state of stress given by this point and the. So, this is the location of the x plane and the stress on the y plane. We have the normal stress which is of magnitude equal to  $\sigma$ . So therefore, this is the diameter of the Mohr circle.

So, if we draw the Mohr circle just the one half, this is the Mohr circle. I have just drawn the top half, what we find is that? Maximum shear stress is going to occur at this point. This is the center of the Mohr circle. So therefore, this is your x direction, this is y direction. Now, this direction which makes an angle of 45 degree see the angle between this direction and this y direction is 45 degree. So therefore, if you move 45 degree with respect to the y, so this is y direction.

If you move 45 degree anti clockwise, you get to the plane where the maximum shear stress is going to act. So therefore, we have the maximum shear stresses acting on the plane of 45 degree. Similarly, the other plane also the other plane this plane which is exactly the diametrical plane. So, if you consider the other plane that is nothing but this plane. So, these are the two 45 degree planes, where maximum shear stresses are acting. So therefore, these are the planes of  $\tau$  maximum.

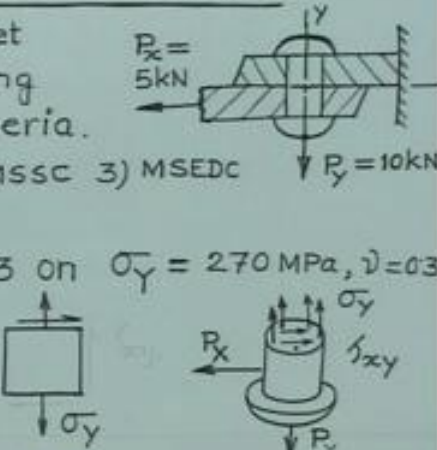
The material aluminum is a ductile material. And its failure can be guided by maximum shear stress criterion. And herein, what you find is that? Maximum shear stress is acting at an angle of 45 degree with the loading direction. Therefore, these are the planes on which the maximum shear stresses are acting. And therefore, when the loads reach a syllable such that the maximum shear stress reaches the value equal to  $\sigma_y$  by 2, there is failure of the material.

That is why you find that, there is a 45 degree fracture on the surface. And after some failure has taken place at the surface, the remaining portion of the cross section will fail by the normal stress. So therefore, it is generally considered that the circumferential portion of the cross section fails by shear and the central portion has failed by normal stress.

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**EXAMPLE 1 ON APPLICATION**  
Estimate rivet diameter using various criteria.  
1) MPSC 2) MSSC 3) MSEDG & 4) MDEDG  
Use FOS = 3 on  $\sigma_Y = 270 \text{ MPa}$ ,  $\nu = 0.3$

**Solution:**  
 $\sigma_y = R_y / A$   
 $\tau_{xy} = R_x / A$



Now, we would like to consider one example on the application of the theory that we have studied. This is an example wherein, we have a riveted joint. The riveted two plates are riveted by a single rivet. And the loading is like this that we have load in this direction.

Let us consider that this is our x direction. And this is our y direction. The loading in the x direction is 5 Kilo Newton. And there is also a loading at this point acting at the tip of the rivet, which is in the y direction it is of magnitude equal to 10 Kilo Newton. This is what the loading I am trying to consider. Now, the problem is estimate the rivet diameter.

Using the various diameter, using the various criteria and herein it is specified that we will make use of the maximum principle stress criteria, maximum shear stress criteria maximum strain energy density criteria and maximum distortion energy density criteria. And the material property of the rivet is given here. That yield point is 270 Mega Pascal poisson's ratio is 0.3.

And it is stated that, you make use of a factor of safety of three on the yield point. I would like to emphasize here that we do not want the component to fail. And in order to guard against the possibility of any failure. We do not allow the material to reach the critical level. So, we try to just see that the loading is at a level much lower than the

critical level. That is why; we always try to see that there is a load level which is some proportion of the critical level.

So, if we yield point is the critical load level. Then, we would like to apply some factor of safety some factor which is generally greater than 1. So in this case, the factor of safety is 3. And hence, you get the allowable stress in this case to be given by  $\sigma_y$  by 3.

Now coming to the calculation of stresses, if we consider the section of the rivet let us consider the section here around this joint of the rivet. So, the loading on the rivet is shown here. You will have  $P_y$  acting vertically downward.  $P_x$  is acting there. This  $P_y$  is going to give rise to normal stresses on this cross section which is of let us represent by  $\sigma_y$ . And this  $\sigma_y$  is nothing but  $P_y$  by  $A$ .

And this stress, we can consider that this is going to give rise to shear stresses uniformly on this cross section which is of magnitude  $\tau_x$ . And it is magnitude is; obviously, we can write it is  $P_x$  by  $A$  directly. Since the diameter can be taken to be small, we can consider uniform shear stress.

So therefore, a typical point on this cross section is subjected to loading which is shown here that you have the  $\sigma_y$  stress. And then we have this shear stresses acting. Look here, that this shear stress is this one. And this is of magnitude equal to  $\tau_x$ . So, these are the stresses.

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Handwritten calculations on a whiteboard:

$$\sigma_1 = \frac{\sigma_y}{2} + \sqrt{\left(\frac{\sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{12.05}{A}$$

$$\sigma_2 = \sigma_y - \sigma_1 = -\frac{2.05}{A}$$

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{7.05}{A}$$

$$\sigma_{all} = \sigma_y / FOS = 270/3 = 90 \text{ N/mm}^2 = 0.09 \frac{\text{KN}}{\text{mm}^2}$$

1) MPSC:  $\sigma_1 = \sigma_{all}$       $\frac{12.05}{A} = 0.09$       $A = 138.89 \text{ mm}^2$   
 $A = \frac{\pi d^2}{4}$ ;  $d = 13 \text{ mm}$

2) MSSC:  $\tau_{max} = \frac{\sigma_{all}}{2}$       $\frac{7.05}{A} = 0.045$       $d = 14.1 \text{ mm}$

Now, we would like to apply the criteria. So, we have sigma y given by 10 by A Kilo Newton per millimeter square, tau x y is 5 by A Kilo Newton per millimeter square. With these stresses, we can calculate the principal stresses which is given by sigma 1 equal to sigma y by 2 plus square root sigma y by 2 square plus tau x y square which comes out to be 12.05 by A.

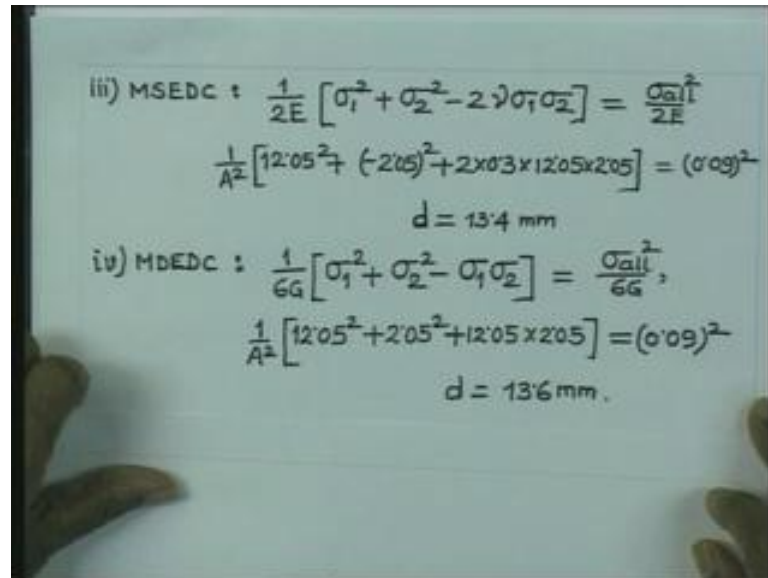
Similarly sigma 2, we can write sigma x plus sigma y minus sigma 1, sigma x is 0. Therefore, it is sigma y minus sigma 1 which comes out to be minus 2.05 A. Maximum shear stress we can calculate from sigma 1 minus sigma 2 by 2 which comes out to be 7.05 by A. Of course, note that A is the cross sectional area.

Now the allowable stress, generally we would have taken the limiting stress to be sigma y. But now, we are trying to have the allowable stress which is sigma y by factor of safety. So, therefore, it is 270 by 3, which is nothing but 90 Newton per millimeter square equal to 0.09 Kilo Newton per millimeter square.

If I apply the maximum principal stress theory wherein we write sigma 1 equal to sigma allowable, and therefore sigma 1 is 12.05 by A is equal to 0.09 which gives equal to 138.89 millimeter square. And if you consider the cross section to be circular then d comes out to be 13 millimeter.

Similarly, from the maximum shear stress criterion  $\tau_{\max}$  is now going to be  $\sigma_{\text{allowable}}/2$  and  $\tau_{\max}$  is 7.05 by A. That is equal to 0.045. And therefore, we simplify and we get  $d$  equal to 14.1 millimeter.

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Handwritten mathematical derivations for the Maximum Strain Energy Density Criterion (MSEDC) and Maximum Distortion Energy Density Criterion (MDEDC).

iii) MSEDC :  $\frac{1}{2E} [\sigma_1^2 + \sigma_2^2 - 2\nu\sigma_1\sigma_2] = \frac{\sigma_{\text{all}}^2}{2E}$   
 $\frac{1}{A^2} [12.05^2 + (-2.05)^2 + 2 \times 0.3 \times 12.05 \times 2.05] = (0.09)^2$   
 $d = 13.4 \text{ mm}$

iv) MDEDC :  $\frac{1}{6G} [\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2] = \frac{\sigma_{\text{all}}^2}{6G}$   
 $\frac{1}{A^2} [12.05^2 + 2.05^2 + 12.05 \times 2.05] = (0.09)^2$   
 $d = 13.6 \text{ mm}$

Now, if we consider the strain energy density criterion. Here, in the relationship that we have is going to be given by  $\frac{1}{2E} [\sigma_1^2 + \sigma_2^2 - 2\nu\sigma_1\sigma_2] = \frac{\sigma_{\text{allowable}}^2}{2E}$ . And if we substitute the value, then we get  $\frac{1}{A^2} [12.05^2 + (-2.05)^2 + 2 \times 0.3 \times 12.05 \times 2.05] = 0.09^2$ . And once, you simplify you get the diameter as 13.4 millimeter.

Similarly, if you consider the maximum distortion energy density criterion then the energy density is given by  $\frac{1}{6G} [\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2] = \frac{\sigma_{\text{allowable}}^2}{6G}$ . And that should have a limiting value of  $\sigma_{\text{allowable}}^2$  by  $6G$ . And on substitution, you find that this is equal to  $\frac{1}{A^2} [12.05^2 + 2.05^2 + 12.05 \times 2.05] = 0.09^2$ . And this gives you finally a diameter of 13.6.

So therefore, you see that applying the different criteria. You are getting diameter to be different though not very much. You find that by applying the maximum principal stress criteria. We have criterion we have got diameter to be 13.13 millimeter and by applying the shear stress criterion we have got diameter as 14.1. By applying the strain energy density criterion, we have got it as 13.4 and by applying the distortion energy density

criterion we have got 13.6. Now, the highest value is obtained by applying the maximum shear stress criterion. It is good idea for a designer to pick out the value which is the highest.