Advanced Strength Of Materials
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Lecture – 24

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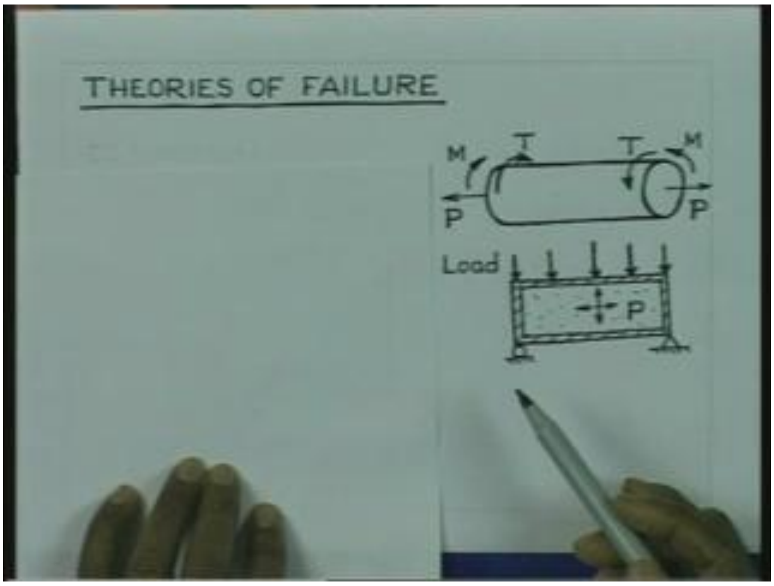

Theories of failures keep the condition to be satisfied by the material for leading to failure. That is to say it gives the condition for the initiation of failure of material at a point. We generally get components of machine made out of some material. It could be a shaft made out of some material or it could be a pressure vessel made out of some of the same material.

The shaft may be loaded by axial force. It may be also loaded by bending moment torsion. And when these loads are acting there will be some point in the material, which is most critically loaded. The condition at which the material will fails is given by the theory of failure.

Similarly in the second case, the pressure vessel is loaded by some internal pressure and it is also supported like a simply supported beam and there are some transverse loading. Under the action of this, there is going to be a point in the module which is going to be most critically loaded. And that critical condition will lead to failure of this component.

These theories help you to find out the load or the combination of load that we will call failure of the component. So therefore, it is very important from the point of view of design. In the design of a component like these you would be interested in finding out the combination of the axial load bending moment and torsion which we will call failure of this component, so that you can guard against the failure by keeping the load level below that critical state.

Similarly in this case, you can also find out the condition of the distributed load and also the pressure which we will call failure. And then to guard against the failure you can keep these values below the critical level. So, these critical conditions are determined by the theories of failures. Now, I would like to like you to consider one analogy.

In the case of the human beings, the suffering of pain or the psycho physical condition that arise in the human body. Due to let us say a boil or a cut or a swelling. It gives rise to a state of pain and irrespective of the condition doctor will give a medicine to reduce the pain. So therefore, in all the state whether it is due to a boil due to a cut or due to a swelling or due to a hurt. The conditions that arise that can that is common. And therefore, one can decide about it common remedy.

Similarly in the case of material, whether it is subjected to or a component made out of a material, whether it is subjected to an axial load or a bending moment or a torsion or a combination of all these. In each of these cases a condition arises which leads to the failure of the component. So, this commonness of the condition is what is the concern of the theory of failures?

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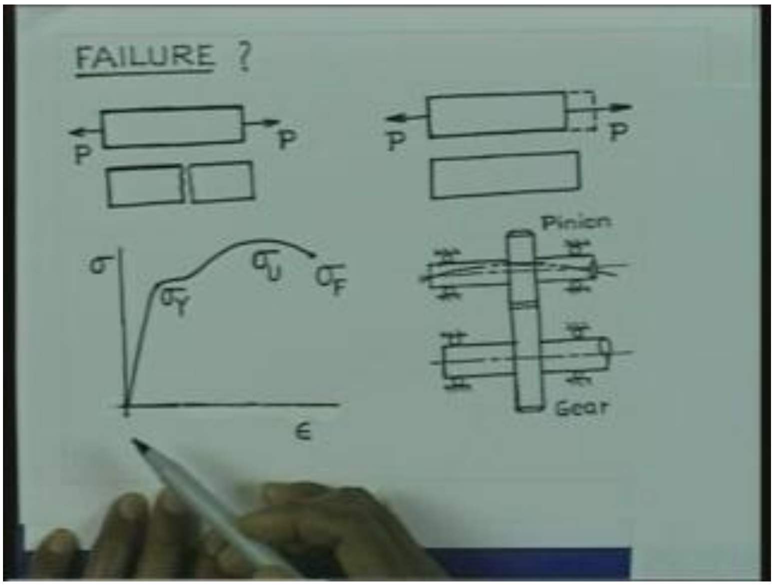

Now, let us try to look into what is a failure. Think of a component which is loaded in tension. If the load is applied gradually it may lead to a state, where it will break into two parts. This could be state of failure. Think of another situation herein, we have a component which is loaded. And under the action of the loading or as we increase the loading gradually there is going to be more deformation. And suppose you have loaded it.

After the plastic deformation occurs on unloading, what you are going to find is that, there is some residual deformation. So, look at this diagram again. You have this original length. So, after the removal of the load the component has now. Reached has got a length which is more than the original length. So therefore, the component got permanently deformed. This may be this may not be acceptable for some situations.

Let me give one example here. Think of this case. Herein we have two shafts which are coupled together by means of a pair of gears. This is the pinion and this is the gear while the shaft or the gears are transmitting power. There could be a situation that the loading which comes up the radial loading which tries to separate the two mating elements. If it is very high there is a possibility that the axis of the shaft can get deformed.

For example, we will find that after the load is removed the axis of the shaft got deformed from the straight geometry to geometry like this. So therefore, now the making of the pinion and the gear will not be perfect. And hence, the subsequent operation will be noisy. So, this is not acceptable. So therefore, we cannot allow in this case permanent deformation to take place.

In general, for the design of machines you would like to guard against this condition. So therefore, you would not permit any permanent deformation to take place in the component is you look into the tensile stress strain diagram of a material. Typically, we can think of plotting the engineering stress versus engineering strain as you load gradually the stress will vary. Linearly the strain and then after some time non-linearity will set in.

Subsequently we will have the a point here which is yield point at which the material will try to deform permanently or after which the material will try to deform permanently and it will load will increase due to deformation up to the ultimate limit and then it will finally, lead to fracture at this point which is the fracture stress. So, the separation will take place corresponding to this stress.

And in fact, there is this maximum stress which is the ultimate stress. That is the maximum stress that the component has withstood. And at this point itself the failure has initiated. And finally, it fails into two components at the stress level. So therefore, we can take this to be the critical load for failure. So, this is the failure corresponds to this case, but when we are trying to think of permanent deformation to be prevented.

Like the shaft for example, here we are trying to resort to this point that you will not allow the stress or the strain at this point to go beyond this point. So for our discussion here, we would like to say that the failure of the material here will correspond to permanent deformation setting in or the yielding of the material setting in.

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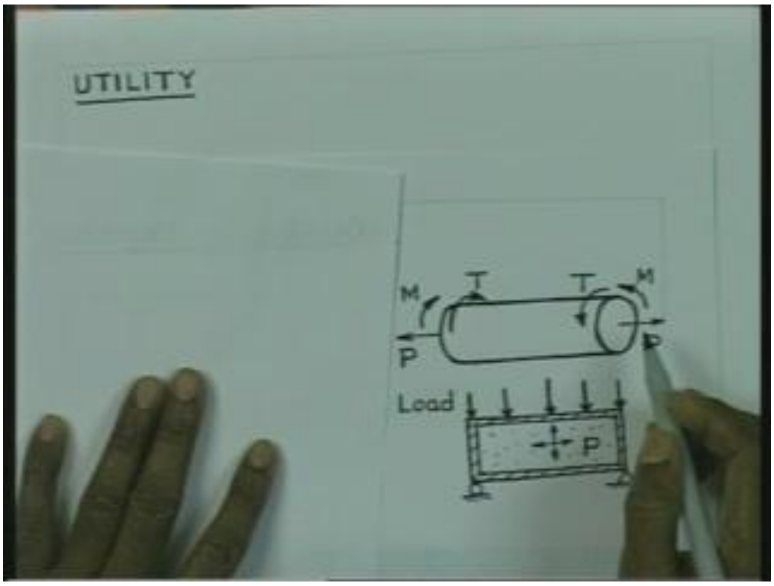

Now, what is the utility of the theory of failures? Let us try to look into this point. The component that, we were considering earlier. Think of this component, herein it is subjected to axial load bending moment and torsion. If it is subjected to axial load one would like to know, what is the load that the component will withstand given the cross sectional area and its dimension.

Or one may be interested in finding out the bending moment leaving the dimensions. Or what is the torque that can be withstood by the element given the dimensions. Or it can be a situation where in all the three are acting together. And we would be interested in what combination of the three will call failure of the component or will initiate permanent deformation at the most critical point.

So therefore, this theory of failure they help in finding out the load capacity of a component. That load capacity can be referred to a single load or it can be referred to a combination of loading. There is also another point which is resolved by the theories of failures. I would like to show you this.

When you try to pool a specimen in tensile testing machine particularly you have the experience when we test the mild steel specimen or aluminum specimen at the point of failure; that means, when the component has got broken down into two pieces. There is a typical cup and cone fracture.

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So, I would like you to show this. Think of this specimen. This has been this was the specimen which was tested in the tensile testing machine by gauge length of the specimen between this point and this point and as you pulled it. It met and finally felt into typical cup and cone fracture. You can see here that this side you have the formation of the cup and this side you have the formation of the cone. In this case, the central portion has filled perpendicular to the direction of the loading. And the outer portion has filled at an angle 45 degrees to the direction of. The question is why does it happen that way?

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Similarly, lets us consider the example of a chalk. If you take a piece of chalk and if you pull it this has filled. Perpendicular to the loading direction and the cross section is flat after the failure. You can see that you can join the two sides together to get almost our original geometry. So therefore, in this case you see that the chalk has filled in the cross sectional direction. This is the observation when you pull the chalk in tension.

Now, let us consider another loading. We take the same chalk piece. And we would now like to twist it. So, if I just apply the twisting that it is filled at a helix angle. It has filled in the helix direction and this angle is 45 degrees. Now, the chalk is made up of the same material. Herein under the action of the loading, we see it failure which is different from this failure which is resulted which has resulted from application of the torque.

How can we explain this? Does it mean that the two loading give rise to different failures? Of course physically they are different. But then is there commonness in the condition that existed in the two cases and that has led to the failure. I would like to repeat this. So, we have pulled the chalk in tension and in torsion. Or rather, we have tried to apply the tension loading on the chalk and torsion loading on the chalk.

And the failure in the two cases are different, but then does that mean that the condition that has been at the root of the failure is different that is the question. And in fact, we would see that although the two loadings are different. The two failures are seemingly different. There is a condition which is same in both cases which has lead to the failure. So therefore, this theory of failure helps us to find out the unity in the condition of failure in the two cases like this. And in fact, if there is multiplicity of loading, then also these theories of failure help us to find out the commonness of the condition of failure. Before, we go talk about the theories of failures.

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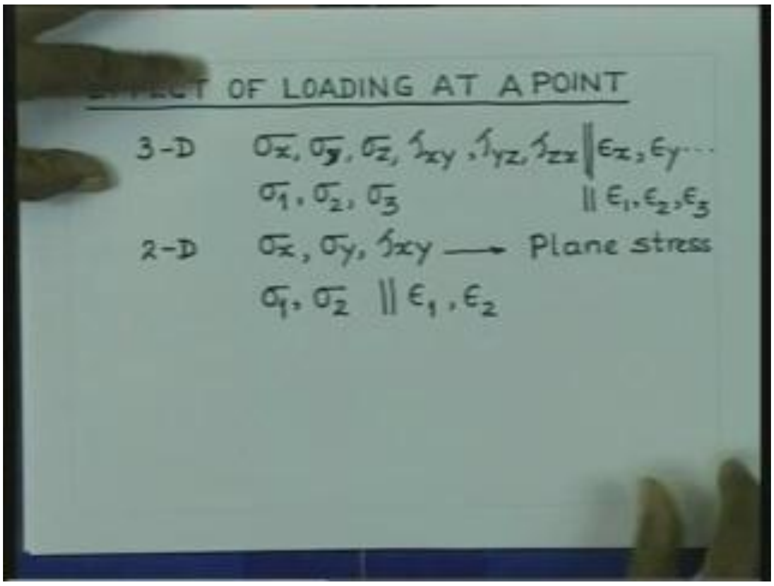

Let us try to look into the loading on components and the effect of loading at a point in the component. So, if you consider three Dimensional situations. In the 3-D, we can express the condition of state of loading at a point by the six components of stresses or six components of strain. So, we can certainly express the condition of loading at a point by the six components of stresses.

Alternatively, we can also express it by the six components of strain epsilon x epsilon y and all the other components. The same state of stress or strain we can express in terms of the three principle stresses. And here we can give it by the three principle strains epsilon 1 epsilon 2and epsilon 3.

In two dimensions, we can give it in terms of sigma x sigma y and tau x y. I would rather confine my discussion here to plane stress. So therefore for plane stress, we will have these three components. Alternatively, we can express the condition of loading at a point by the two stresses or the strains epsilon 1 epsilon 2.

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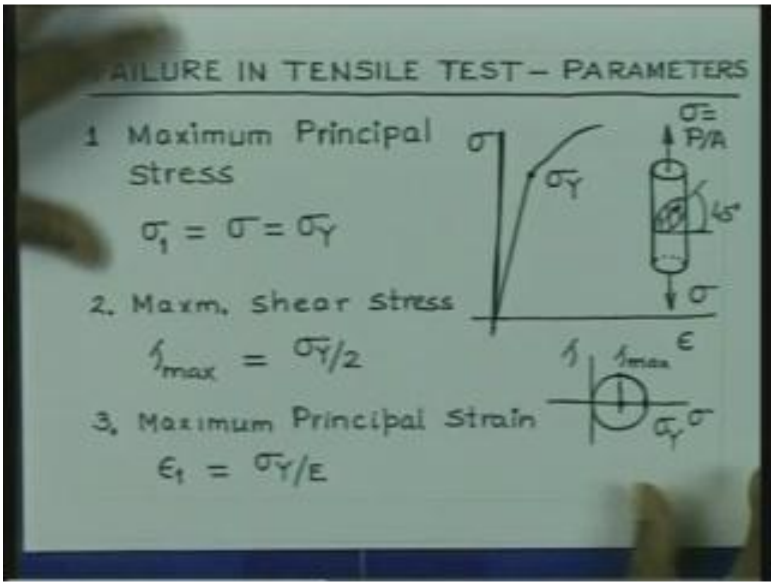

Let us look into the failure of materials in tensile testing. If we consider the specimen to be modeled like this and we plot the stress strain. As I have mentioned little, while ago that this is how the stress will be added with strain. And at some point, we will have yield stress demonstrated by the material.

And the condition of loading, which corresponds to this point is nothing but it is a case of one dimensional loading. So therefore, only one stress is acting here. So therefore, the condition of failure at the point of yielding is nothing but sigma 1stress sigma 1 is equal to sigma and that is equal to sigma y. So that is the condition of failure here.

If we try to consider now the maximum shear stress develops, because of this loading. You remember that in the case of the uniaxial loading, the maximum shear stress occurs on a particular degree plane. So, this is the plane which is at an angle of 45 degree. On this plane, there is a shear stress acting here that is the maximum shear stress.

This follows from the Mohr circle diagram. So, if you draw the Mohr circle diagram you have the shear stress in the y direction, normal stress in the x direction, your stress is here. Let us say, that is the yield stress. So, if we draw the Mohr circle, this is the Mohr circle. For this case and the maximum shear stress corresponds to this point.

And that maximum shear stress is nothing but sigma y by 2. And it acts at an angle of 45 degree to the uniaxial loading direction or that sigma direction. So therefore, it is at an angle of 45 degree. So, the maximum shear stress here, tau maximum is equal to sigma y by 2.

Now, what is the principal strain in this case? Since, there is only one directional loading. For one stress present here, so maximum principal strain here. Maximum principal strain corresponding to this loading is nothing but sigma y by E, where E is the modulus. So, approximately you consider that it is linear up to the yield point. So therefore, maximum principal strain in this case is nothing but sigma y by at the same point of failure, if you calculate the total energy density.

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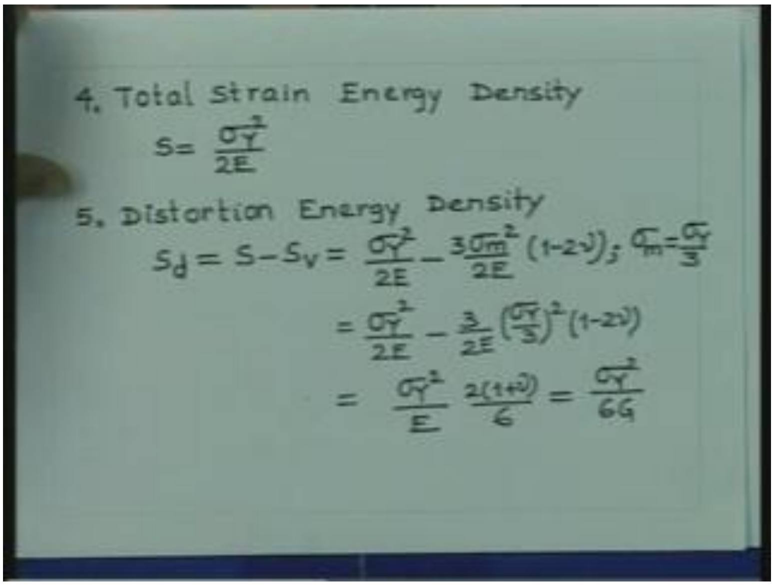

Total strain energy density that is represented by s is nothing but sigma y square by 2E. So that is the energy per unit volume in the material at the yield point. If you calculate that distortion energy density, distortion energy density is represented by s d. S d is equal to total strain energy density minus the volumetric strain energy density. And this is given by total strain energy density is nothing but sigma y square by 2E. And the volumetric energy density is nothing but 3 times sigma means stress square divided by 2E multiplied by 1 minus 2 nu square. The mean stress is given by since it is uniaxial loading. So, it will be some of these three stresses is nothing but sigma y and mean stress is average of that. So, it is sigma y by 3.

So, if you do the simplification, it gives us. And this is equal to sigma y square by E into 2 into 1 plus nu by 6. And that turns out to be sigma y square by 6G. So, the yielding or at the point of yield point, what we have is that? Material at the principal material at the stress state which is given by sigma 1 equal to sigma y maximum shear stress is sigma y by 2. Maximum strain principal strain is nothing but sigma y by E.

And the total strain energy density is given by sigma y square by 2E. And the distortion energy density is given by sigma y square by 6G. Here, one can say that the yielding of the material has been. Due to the attainment of the maximum principle stress, the level sigma y or the maximum shear stress reaching the value sigma y by 2 total to maximum principal strain reaching the value sigma y by E, total energy density reaching the value sigma y square by 2E. Distortion energy density reaching the value sigma y square by 6G.

So therefore, one can explain the yielding phenomenon to be associated with either a critical level of stress, critical level of strain or a critical level of energy density and this is the basis of different theories.

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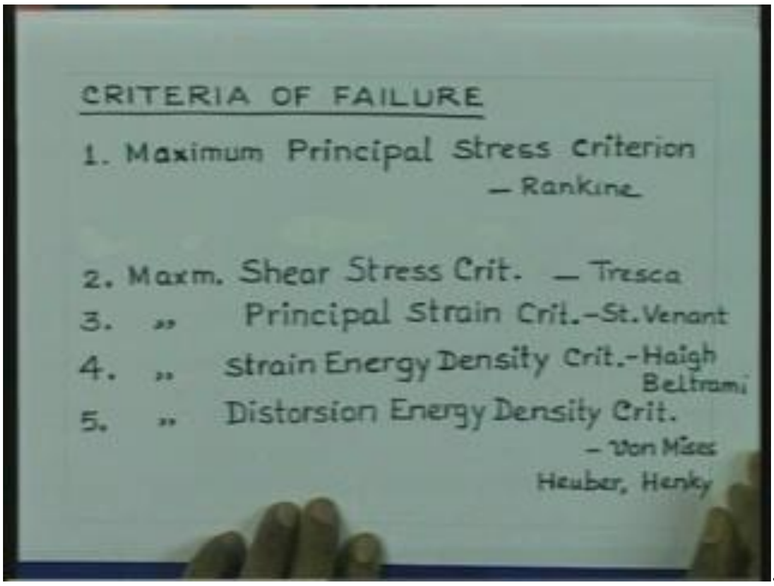

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And in fact, keeping this as the basis there are five important theories of failure or criteria of failure. They are maximum principal stress theory or criterion. And this criterion was proposed by Rankine. Similarly, maximum shear stress criterion and this is this has been proposed by Tresca. Maximum principal strain criterion has been proposed by St. Venant.

Maximum strain energy density criterion has been due to maximum strain energy density criterion is due to Haigh and it is also associated with the name of Beltrami. Maximum distortion energy density criterion is due to Von Mises. It is also associated with the name Heuber Henky. Herein, we would like to consider this criterion to be given by form mines.

Why is that we have to consider this theory? There is also another issue I must stress upon here. That if we have to find out the load capacity of a component under complex loading. For example, you would get back to our first example here. So in this case, if we have to find out the load capacity axial load capacity alone then we can do testing. Similarly if, we have to find out the bending capacity we can do the testing by applying bending load.

Similarly, if we are interested in finding out the torsion capacity you can apply the torsion load and find out the load capacity. This means that if we have a component subjected to varieties of loading. In each case it will be necessary to do the test and find out the loading. That is not really rational way of going about.

On the other hand, if we can do one test. For example, we do the tensile test. And we find out the capacity of the material. And the observation that, we have in the tensile test if that can be made use of in finding out the capacity of bending load capacity. If we can find out the torsion load capacity then in that case that is a rational way of going about it.

In fact, theories of failures do help us in achieving that. So, we try to just do single test collect the material data. And using those material data, we try to find out the load capacity in the new situations. And it is not necessary to go for testing in each and every case, before I go for taking up this example for these theories.

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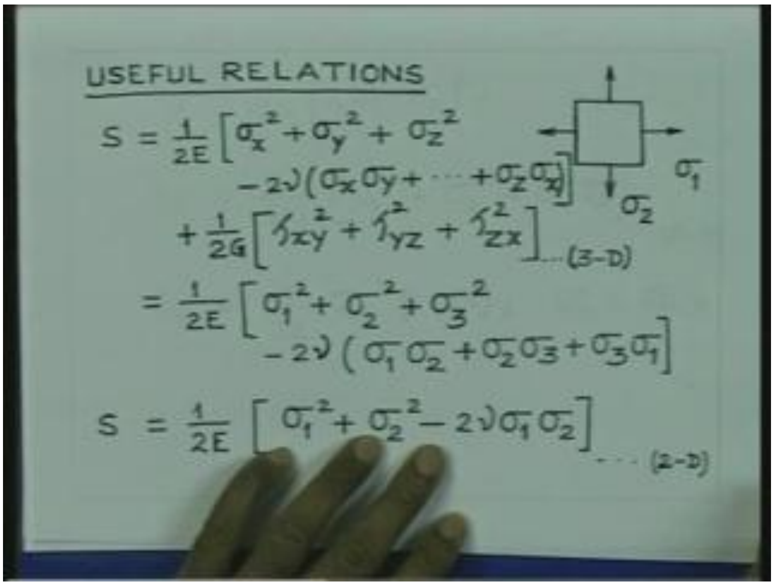

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Let us brush up some of the relationships we have derived in our initial stages. If you have a problem of three dimensions then the total strain energy density can be given in terms of the six stress tension components. This is nothing but 1 by 2E sigma x square plus sigma y square, plus sigma z square, minus twice mu sigma x sigma y, plus sigma y sigma z plus sigma z sigma x, plus 1 by 2G tau x y square plus tau y z square plus tau z x square. So, this is the relationship for three dimensions.

And if, we write the same relationship in terms of the principal stresses. Then in that case, it becomes little simpler expression which is nothing but 1 by 2E, sigma 1 square plus sigma 2 square plus sigma 3 square minus 2 nu sigma 1 sigma 2 plus sigma 2 sigma 3 plus sigma 3 sigma 1. In the case of two dimensions, we will have sigma 3 0. Then, we get the total strain energy density as 1 by 2E sigma 1 square plus sigma 2 square minus 2 times nu sigma 1 and sigma 2. So therefore, this is for two dimensions. I would again emphasize. That in my discussion I would like to confine to plane stress condition. We are talking about plane strain wherein third stress component can be present.

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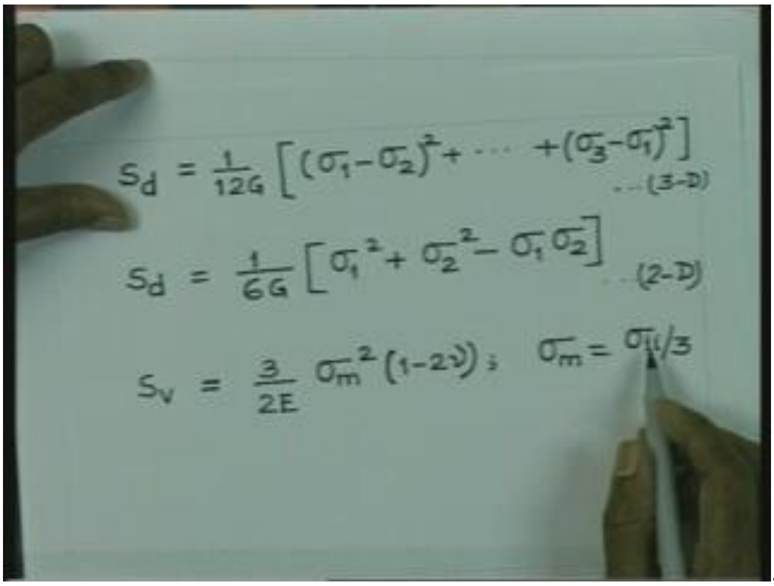

Similarly, the distortion energy density is given by 1 by 12G sigma 1 minus sigma 2 whole square plus sigma 2 minus sigma 3 whole square plus sigma 3 minus sigma 1 whole square. This is for three dimensions. And the same expression for two dimensions is given by 1 by 6G sigma 1 square plus sigma 2 square minus sigma 1 sigma 2. So, this is for two dimensions. The volumetric strain energy density is given by 3 by 2E sigma m square into 1 minus 2 nu, where sigma m is nothing but average of the three normal stress components.

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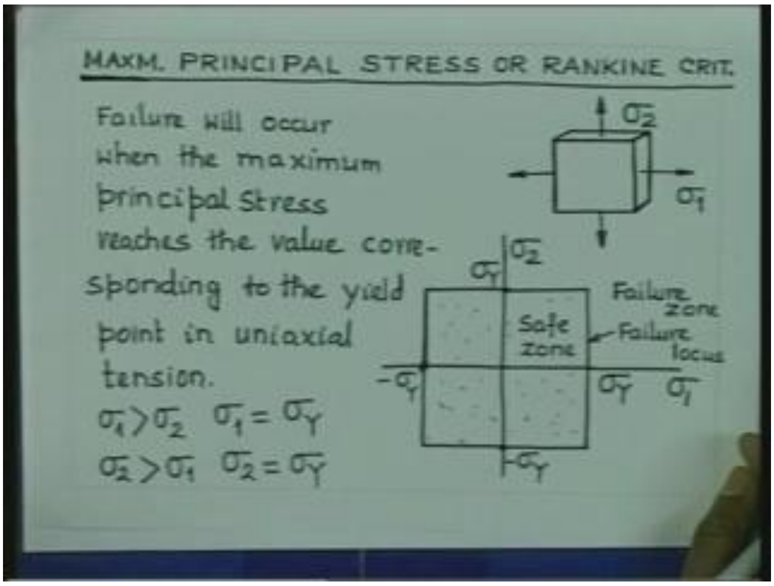

Let us consider the maximum principal stress or ranking criterion. So, we are considering the situation to be plane stress. If we consider, the most critically loaded point in the material in a component. Let us, represent the stress to be given by sigma 1 and sigma 2 acting in the two directions.

According to this criterion, failure will occur when the maximum principal stress reaches the corresponding value in uniaxial tension. So, I would like to write failure will occur when the maximum principal stress reaches the value corresponding to the yield point in uniaxial tension. Failure will occur, when the maximum principal stress reaches the value corresponding to the yield point in uniaxial tension.

Let us now, consider that the stress sigma 1 is greater than sigma 2. So, sigma 1 is greater than sigma 2. Then, this means that the failure will take place. When, sigma 1 is equal to the corresponding value in uniaxial test at yield point. So, we know that the corresponding stress at yield point in uniaxial tension. So therefore, this is the condition of failure.

So, if we try to plot the value of all the combination of sigma 1and sigma 2 which will cause failure. We can now show it graphically. Let us plot, sigma 1in this direction and sigma 2 in this direction.. So, if sigma 1 is greater, therefore this failure. So, going to occur, whenever the stress sigma 1 reaches the value sigma y, so therefore, this line will gives us the condition of failure.

Suppose, sigma 2 is the largest stress then sigma 1 then; obviously, failure will occur when sigma 2 is equal to sigma y. So, since sigma 2 is larger. Therefore again, we will have the failure here, when sigma 2 is equal to sigma y. So, this line gives the combination of again sigma 1 and sigma 2which will correspond to failure; that means the stresses which are lying within this boundary.

They are all set combinations. So, we can write that the stresses lying in this zone are all safe. So, this is the safe zone. And the stresses which are outside this boundary they are correspond to failure zone. So, you can write that this is failure zone. And the portion of the boundary which separates the safe zone from the failure zone is known as failure locus.

Generally, we consider the property of the material to be same in tension and compression. Therefore, we can now also find out the safe zone and the failure zone in the compressive region. So therefore, it is going to be without any difficulty we can find out this. If one of the stresses is positive and another stress is negative.

Again, we will have the zone given by these two lines and in the other quadrant also that zone is going to be given by this line. So therefore, the total space in two dimensions which is corresponding to safe zone is nothing but these. So these are stresses, which correspond to safety.

And the stresses outside this correspond to failure. So therefore, this is also sigma y. This is sigma y, and of course this is compressive sigma y. So therefore, this is how the failure zone can be found out making use of the maximum principal stress criterion or ranking criterion.

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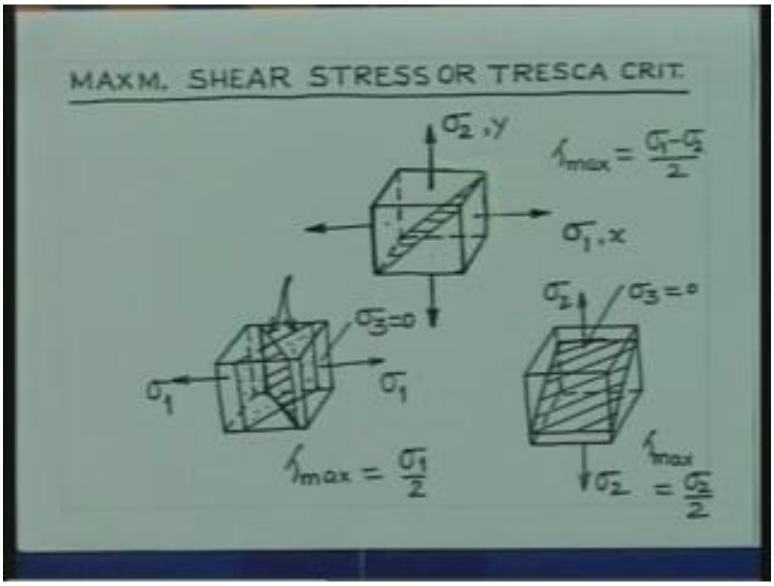

Now, we like to consider maximum shear stress or Tresca criterion. According to this criterion, the failure will occur when the maximum shear stress in the material reaches the value corresponding to the value at the yield point in uniaxial tension test.

Before we go into giving the theory. Let us, consider some points here. Think of this element which is loaded by biaxial loading sigma 1 and sigma 2. If we find out the maximum shear stress that shear stress is going to occur on this 45 degree plane. So, this is the 45 degree plane. This is one 45 degree plane.

The other 45 degree plane is going to be given by joining this with this edge. So therefore, the other 45 degree plane is going to be this one, if it is a perfect square. So, this is the maximum shear stress plane. And the magnitude of the shear stress is nothing but sigma 1 minus sigma 2 by 2 provided sigma 1 is greater than sigma 2. And it can be also given by sigma 2 minus sigma 1 by 2 provided sigma 2 is greater than sigma 1.

Let us, now consider a situation here. In this case, we have sigma 1 acting in this direction. Sigma 2 is acting in that direction. And in the curve direction, if you consider that this is your x direction. This is y direction. In the z direction, the stress is 0, what I mean is? That in this direction, we have the stress sigma 1 acting. And the stress in this direction which is sigma 3 is 0, because we are considering the plane stress condition.

Because of this stress sigma 1, we are going to get maximum shear stress. And that maximum shear stress is going to occur on a 45 degree plane which is inclined at an angle of 45 degree, 45 degree to these two axes sigma 1and sigma 3. So therefore, this 45 degree plane could be the plane like this. This is the 45 degree plane. So, let me draw this. So, the plane which is making an angle of 45 degree to both sigma 1 and sigma 3 direction is nothing but this one.

It could also mean the plane which is here. This could also mean this 45 degree plane. So, these are the planes. And this is one and this is the second plane on which the shear stress will be maximum. And the maximum shear stress, here is nothing but sigma 1 by 2. By the same type of considerations, if you consider the plane formed by direction sigma 2 and sigma 3, let us consider this direction sigma 3, again sigma 3 equal to 0.

Because of this sigma 2 which say uniaxial type of loading. We are going to get again shear stress acting on 45 degree plane which is going to be maximum. So, one such plane I will draw here. So, this is the 45 degree. And on this plane, we are going to get tau maximum is equal to sigma 2 by 2.

So, you find here, that in a case of two dimensional loading. We are likely to get three maximum shear stresses. One coming up, because of the loading in sigma 1 and sigma 2 plane and its value is sigma 1 minus sigma 2 by 2. And due to sigma 1 and sigma 3, we will have maximum stress sigma 1 by 2 and due to the loading in the plane sigma 2 sigma 3will have maximum space sigma 2 by 2.

Note that if sigma 2 is positive or both sigma 1 and sigma 2 s positive. Then in that case, we are going to get the higher stress to be given by sigma 1 by 2. If we also assume that both sigma 1 and sigma 2 are positive and sigma 1 is greater than sigma 2. Then in that case, this becomes a stress which is not the highest. This is the stress which is the highest.

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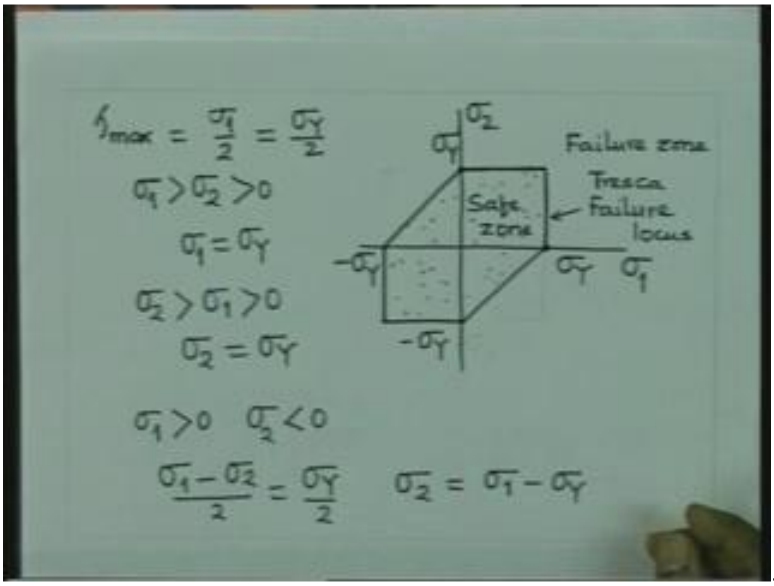

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So, according to this maximum principle stress theory, the failure is going to occur, when the maximum shear stress reaches the value corresponding to the value at the yield point in uniaxial tension. So, we can now find out the failure zone as for this theory. If we consider now, the maximum shear stress is given by sigma 1 by 2. Provided, we have sigma 1 greater than sigma 2 and it is greater than 0.

And the condition of failure is that this tau maximum reaching a value equal to sigma y by 2. And therefore, sigma 1 is equal to sigma y. That is the condition of failure. So, if you now try to plot it in this stress space sigma 1and sigma 2, what you find is that? We will have this level given by sigma y. So, whenever the sigma 1 stress reaches the value sigma y we are going to have the failure.

Is, we have sigma 2 greater than sigma 1 and both are positive. Then in that case, the failure is going to be given by sigma 2 equal to sigma y. Because, this maximum shear stress is sigma 2 by 2 and that is equal to sigma y by 2 and which simplifies to sigma 2equal to sigma y. So therefore, we will have the combination of stresses that will cause the failure is given by this line and this distance as usual is given by sigma y.

Similarly in the compressive region, if you have the sigma 1 stress greater than sigma 2. Then, the failure is going to be given by the line which is at a distance of minus sigma 1 sigma y from the origin. Similarly if, sigma 2 is dominating, then in that case the failure is going to be given by sigma 2 stress which is at a level of minus sigma y.

Now if, sigma 1 is positive and sigma 2 is negative. Then in that case, the maximum shear stress is going to be given by this expression sigma 1 minus sigma 2 by 2 equal to sigma y by 2. And therefore, this boils down to sigma 2 equal to sigma 1 minus sigma y. So, this is a straight line which is at an angle of 45 degree. So, we are talking of this zone wherein sigma 1 is positive and sigma 2 is negative and therefore, the straight line is going to be again given by this straight line.

So, all the combination of sigma 1and sigma 2 which will is given by this line. By the similar type of consideration, when sigma 2 is positive and sigma 1 is negative. You will find that the combination of stress which is going to cause failure is given by this line, so therefore this 0, which is a hexagon.

This hexagon separates the safe combination of stresses from the unsafe combination of stresses. So therefore, this is safe zone and outside is the failure zone. And this boundary is the failure locus. And this is due to Tresca, so it is Tresca failure locus. So by using this Tresca criterion, you can find out all the combinations of stresses that are going to be safe.