Advanced Strength of Materials
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Lecture – 23

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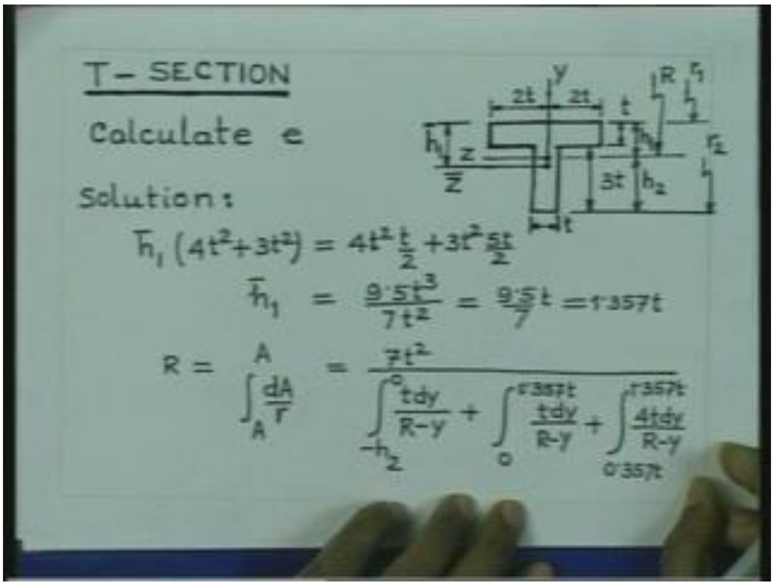

Last time we were considering the determination of eccentricity of a T section. The problem is shown here. We came up to the point, that the center of gravity of the cross section is at a distance of h 1 bar from the top fiber. And that is of magnitude 1.357 t. In order to calculate this eccentricity, it is necessary to calculate the radius of curvature r of the cross section. The formula for that is here R is equal to cross sectional area divided by the integral d A by r.

The cross sectional area is 7 t square. Now, to calculate d A we have to go in stages. We can consider now d A for the portion from the bottom fiber up to the origin. From origin to the junction of the flange in the web. And then again we can consider from the bottom of the flange to the top of the flange.

So, our integration would look like this minus h 2 when you consider this distance is h 2 minus h 2 to origin is at 0 and this d A is nothing but, t d y and small r is nothing but, capital R minus y plus from origin to the junction of the wave and the flange. So, which is at a distance of 0.357.

Since this height is equal to 1.357 t and this is t, this portion is 0.357 t. And this d A is nothing but, t d y divided by R minus y. Plus the integration from the height 357 t to 1.357 t. And in this case d y is going to be given by, for this portion the width is equal to 4 t. So, therefore, 4 t into d y divided by R minus y. So, note this that when you have the width of the section it changing. Then, your integration has got to be done in stages, noting those changes.

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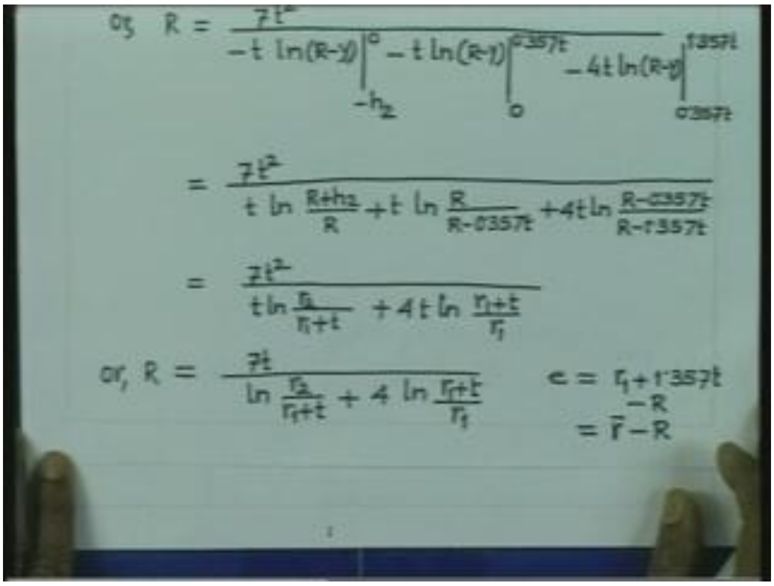

So, if you continue further R is equal to 7 t square and the integration is very straight forward here. So, it is minus t l n R minus y and the limits are minus h by 2 to 0. Second integral is minus t l n again R minus y 0 to 0.357 t minus 4 t l n R minus y and the limits are 0.357 t to 1.357 t. In fact, these two can be combined we can write really that this is 0, they will cancel out.

So, finally, we can write this thing as 7 t square or if would like to write. Let us write the intermediate steps also 7 t square t l n R plus h 2 by R plus t l n R by R minus 0.357 t plus 4 t l n R minus 0.357 t divided by R minus 1.357 t. And this is equal to l n r 2 by r 1 plus t plus 4 t l n r 1 plus t by r 1. So, we are substituting the value there in terms of r 1 and r 2.

Finally, this gives us t cancels out. So, we will have 7 t l n r 2 by r 1 plus t plus 4 l n r 1 plus t divided by r 1. So, that is the value of the radius of curvature of the cross section. And since, we are interested in calculating the value of eccentricity. So, eccentricity is going to be given by r bar nothing but r 1 plus 1.357 t minus R and R is already here. So, if you are given explicitly the value of r bar. Then, this is nothing but, r bar minus R.

So, this is how you can tackle the problem of t sections. You will find that, these types of sections are going to be utilized in the c frames of stresses. They are also used in the crane hooks. Another section, which is quite widely used in the case of crane hook, is trapezoidal section.

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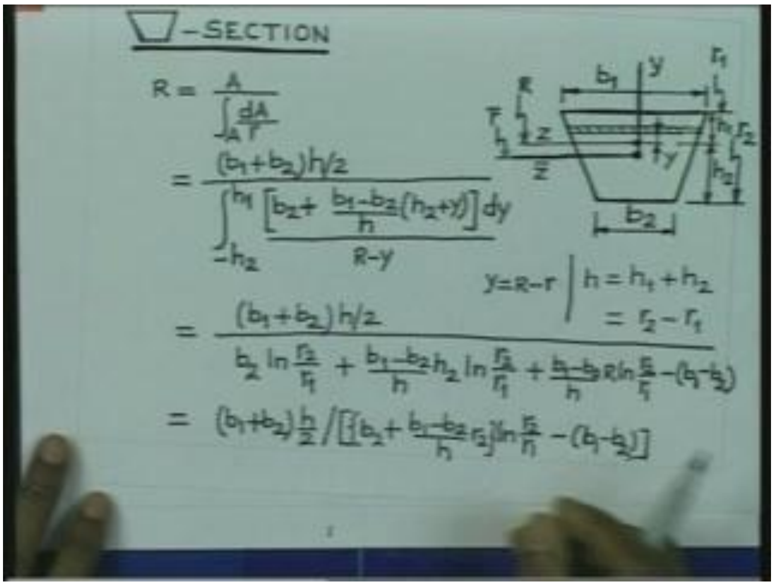

So, we will now look into how to calculate the eccentricity of a trapezoidal section. Consider the typical section to be as shown here. The center of gravity of the cross section is located here. As usual we consider that the radius of curvature of the centroidal axis is r bar. And the radius of curvature of the neutral axis is R. And we will show the extreme fiber ((Refer Time: 09:19)) i to be r 1 and r 2. The neutral axis is passing through this point.

Let us concentrate on a fiber, which is at a distance of y from the neutral axis. So, therefore, this distance from here to here is y. We also would like to indicate, that the fiber at the bottom is at a distance of h 2 from the neutral axis. This height is h 1. So, h is nothing but, h 1 plus h 2 which is same thing as r 2 minus r 1.

In this case, first thing that you have to do is locate the centroid of the cross section, which is very easy. I am not going to do that, but let us calculate the radius of curvature r of this cross section. We start with the formula R equal to A by integral d A by r over the whole section. Now, this is equal to the cross sectional area is nothing but, b 1 plus b 2 by 2 into h or b 1 plus b 2 into h by 2. That is the cross sectional area.

Now, we can write the area d a which is given by this width, let us say that is b multiplied by d y. We can consider that this height is equal to d y. Then, this b is given by b 2 plus b 1 minus b 2 by h into the height h 2 plus y. So, we have got this area given by b 2 plus b 1 minus b 2 divided by h into h 2 plus y into d y. So, that is really d a and the denominator is R minus y and our limits of integration is going to be minus h 2 to h 1.

Noting that, y you can write this thing as R minus r. We can now, write this thing as b 1 plus b 2 h by 2. We can integrate this portion very easily and that will give us b 2 l n r 2 by r 1. Similarly, this portion will be also straight forward b 1 minus b 2 by h and this is h 2 l n r 2 by r 1. This we can add r subtract r, thereby we will have two expressions. and this is going to be plus b 1 minus b 2 by h into R l n r 2 by r 1. And the last quantity is going to be b 1 minus b 2.

So, finally, the radius of curvature for this section is going to be b 1 plus b 2 h by 2 divided by b 2 plus b 1 minus b 2 by h r 2 l n r 2 by r 1 minus b 1 minus b 2. So, for the trapezoidal section, you can calculate r by making use of this formula. As I have indicated, these trapezoidal sections are also very common for crane hooks. And also sometimes the stress frames are also made with this cross section.

Let us now try to solve some problem. And see, what is going to be the stresses in a rectangular cross section, under the action of bending. When the member is curved and how these stresses are different from the stresses in a straight beam.

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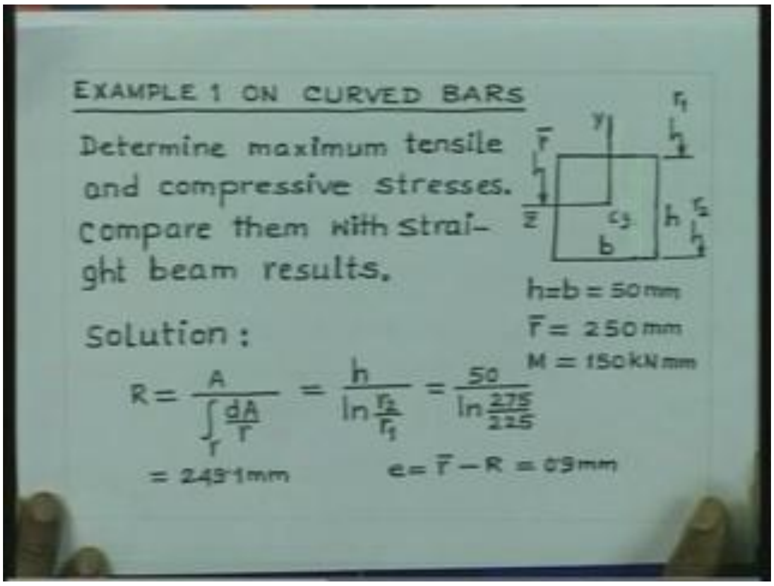

So, we will consider one example on curved bars. Here, the cross section is given with the width b equal to height. So, you say that square cross section and the dimensions are h equal to b equal to 50 millimeter, and the radius of curvature of the centroidal axis. So, the centroidal axis has a curvature r bar. That r bar is given as 250 millimeter, the bending moment applied on the cross section is 150 kilo Newton millimeter.

So, determine the maximum tensile and compressive stresses in the cross section. Compare them with straight beam results. So, we will proceed with this solution. Let us consider first this, radius of curvature R this. And we have already derived that, that is nothing but, l n r 2 by r 1. Here, the height of the cross section is 50 and r 2. R 2 is nothing but, the radius of curvature of the fiber at the bottom.

And that should be equal to this 250 plus the half of the depth which is 25. So, therefore, it will become 275. And the radius of curvature of the top fiber, which is r 1 which is nothing but, 250 minus half the height. So, it is again 250 minus 25 therefore, it will be 225. So, this gives us radius of curvature as 249.1 millimeter. Therefore, the eccentricity e is nothing but, r bar minus R which is 250 minus 249.1. Therefore this is 0.9 millimeter.

Let us now calculate the stresses in the cross section. We will make use of the formulas directly, that the maximum stress at the inner fiber was given by minus M into h 1 divided by A e r 1. So and the stress at the bottom fiber was M into h 2 by A into r 2.

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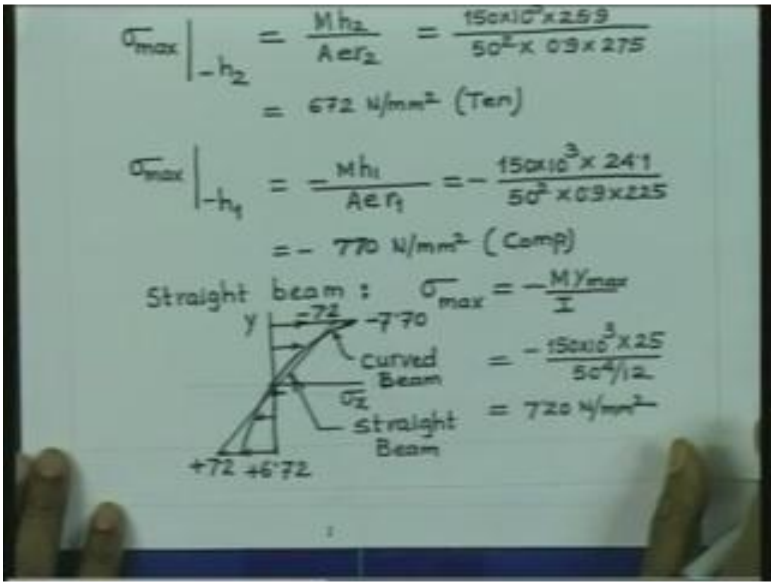

So, you can write now the maximum stress at the bottom fiber, which is this is given by M h 2 by A e r 2. That is equal to 150 10 to the power 3, that is the bending moment and this distance is 25.9 and cross sectional area is 50 square, eccentricity is 0.9 and the radius of curvature of the bottom fiber is 275. So, if you calculate this, this comes out to be 6.72 Newton per millimeter square and this stress is tensile.

Similarly, the stress at the top fiber is equal to minus M h 1 by A e r 1 150 10 to the power 3 24.1 divided by 50 square 0.9 into 225. And if we simplify this gives us 7.70 Newton per millimeter square. And this stress is compressive stress. If you compute the stress when the beam is straight, then it is simply going to be 6 mm by b h square or you can write that maximum stress is nothing but, m into y maximum divided by I.

So, therefore, for straight beam sigma maximum is equal to minus M y maximum divided by I. So, in this case you have the I is nothing but, 50 to the power 4 by 12. So, you can substitute now 150 into 10 to the power 3 for the moment. And this phi bar distance is 25 for both the top and the bottom fiber. And the moment of inertia is nothing but, 50 raised to the power 4 divided by 12. And this gives us 7.20 Newton per millimeter square.

So, you can see that the maximum stress that we are calculating it is underestimated. This value is 7.7, whereas the straight beam formula gives us 7.2. Similarly, the minimum stress is going to be overestimated, if the making of the straight beam formula.

Just to compare the two variations we can plot it. Let us say, that this is the direction we will plot the sigma x stress. And let us have our depth plotted in this direction.

Then, curved beam formula will give a variation of this type. We have the compressive stress this side and tensile stresses this side. The magnitudes are 7.70 Newton per millimeter square and this one is plus 6.72. So, this is the curved beam formula. On the other hand, if we show the stresses given by the straight beam formula, it has a variation which is straight.

So, this will pass through the centroid of the cross section and the variation is straight. So, we will have variation of this type. So, here this is the straight beam formula. And here the magnitude of this stress both at the top and bottom phi bar is plus 7.2. That is the difference between the straight beam and the curved beam formula.

Please note that, the zero of the straight beam formula is occurring here, whereas zero of the curved beam formula is occurring at a point, which is closer to the center. You will also have cross sections, which are going to be circular in shape. So, let us now see how we can calculate the eccentricity for such sections. We will consider now circular section.

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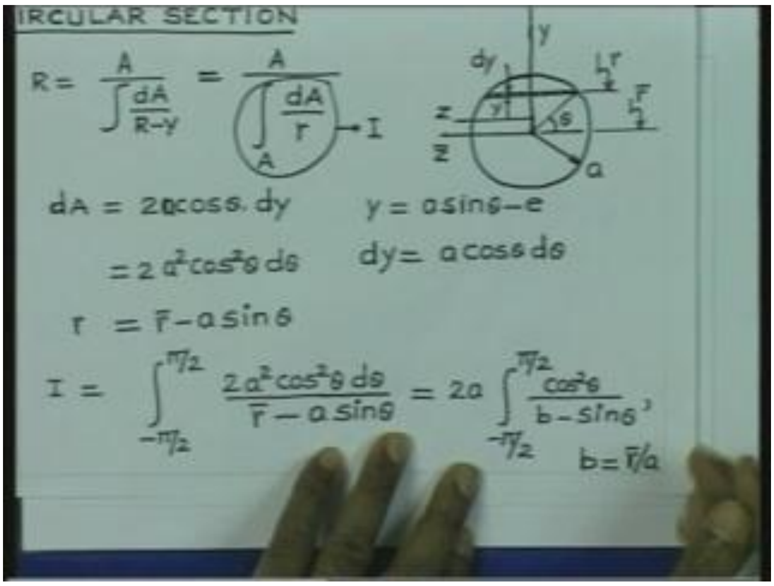

Let the section be of radius it is a solid circular section of radius a. And the c g is of course, at the center and the neutral axis is passing throughput the point here, which is the center of curvature is located somewhere at this side. Let us now consider a fiber here, which is at a distance y let us say. They radius of this fiber is equal to r. We can now directly go in for the formula here in d a if consider this distance it is going to be 2 a cos theta.

And if this is d y, then the area is going to be given by 2 a square cos square theta d theta. And here, d y is equal to a cosine theta d theta. We try to have the origin or reference of theta to be the for a little axis. And therefore, this is theta and it is increasing in the anti clockwise direction. Now, r bar or r is related to r bar by this relationship r bar minus a sin theta.

Now, if we just indicate this integral, that integral by I, this I is equal to limits are minus pi by 2 to plus pi by 2 2 a square cos square theta d theta divided by r bar minus a sin theta. And that is equal to 2 a minus pi by 2 to plus pi by 2 cos square theta by b minus sin theta, where we indicate that this b is nothing but r bar by a.

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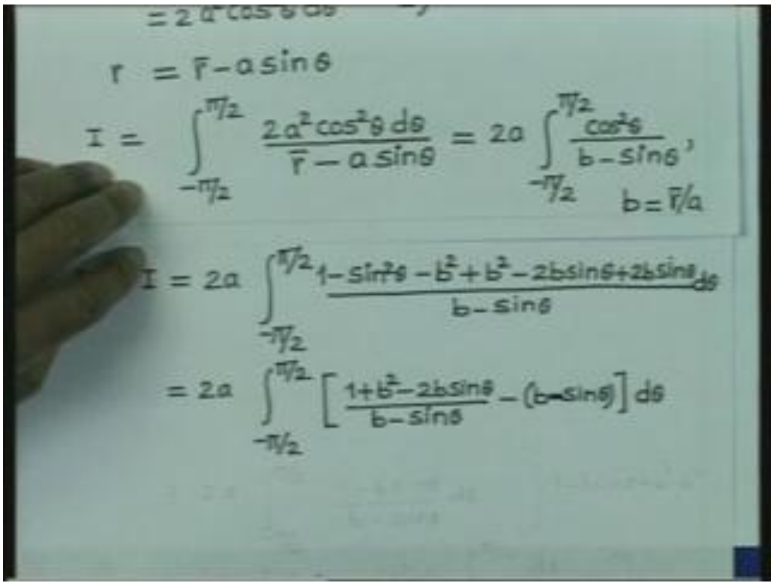

If we do little manipulation we get I is equal to 2 a minus pi by 2 pi by 2 1 minus sin square theta minus b square plus b square minus twice b sin theta plus twice b sin theta d theta. And the denominator is b minus sin theta. This will become 2 a minus pi by 2 to plus pi by 2 1 plus b square minus twice b sin theta by b minus sin theta minus b minus sin theta d theta.

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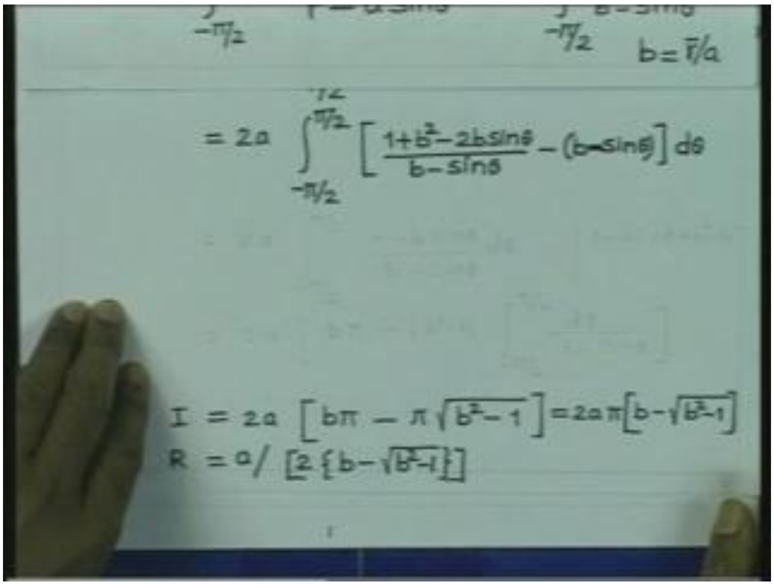

So, we have some to this point that I is given by this integration. If you carry on further finally, we can solve this. And it becomes equal to I is equal to 2 b pi minus pi b square minus 1 which is something as 2 a pi b b square minus 1. And hence, R is equal to area is pi a square divided by this I. So, pi will cancel and therefore, we will have this expression 2 b b square minus 1. That is the value of the radius of curvature. This circular cross section is again used in making links rings. And therefore, you will find the usefulness of this derivation. Let us now consider one example, which is given here.

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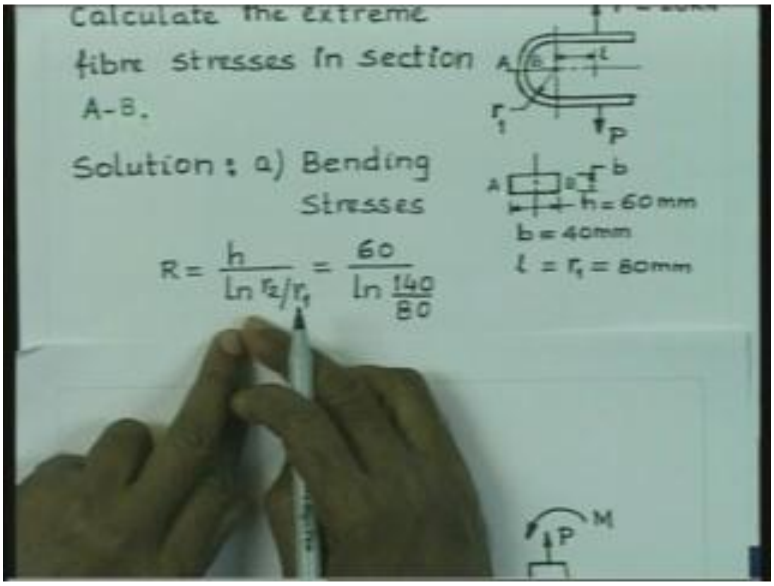

You have a frame like this, which is of the section here at this location A B is rectangular. It has height equal to 60 millimeter and this width here B, this dimension b is equal to 40 millimeter. The frame is loaded like this, the center of curvature of this segment is r 1 and the distance l is given as 80 millimeter. What is needed is that calculate the extreme fiber says in the cross section. So, therefore you have a u frame, which is loaded at some distance from the center of curvature of the portion, which is curved and you have to find out the stresses at point A and B.

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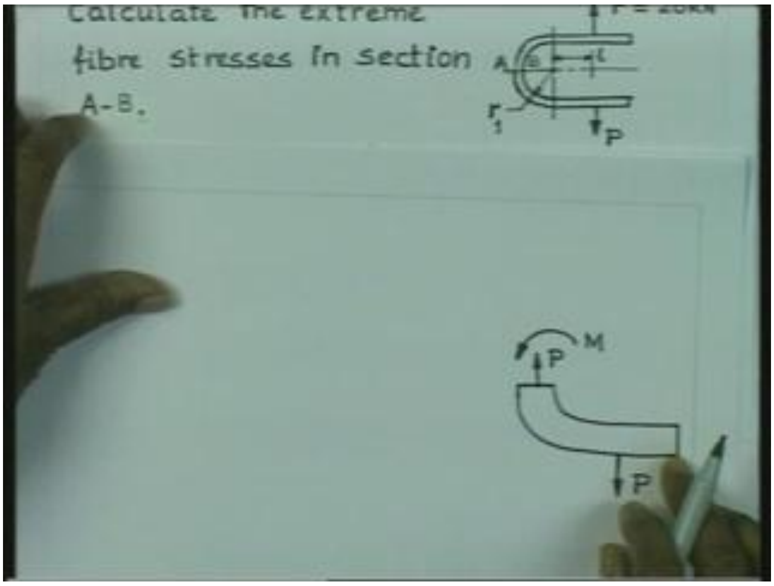

You can consider this problem, if I take this section A B. So, if I just consider a section like this, here in you have the force P is acting externally. And therefore, you are going to have a reaction force here P, which is passing through the centroid of the cross section. Then, you are also going to get a bending moment M.

So, here you have two forces unlike the problem, that we have considered earlier. That you have a direct force acting on the cross section and at the same time bending moment. So, in order to calculate the stresses we have to go into two stages, first we have to calculate the stresses in the cross section, due to the bending moment M. And then we have to calculate the stresses due to the force P.

So, we will go in stages ((Refer Time: 36:50)). We will now calculate the bending stresses or the stresses due to M. In order to do that, we have to calculate the radius of curvature of the neutral axis. If we make use of these formula, that we have derived R is going to be nothing but, h by l n r 2 by r 1 which is 60 by l n or the inner radius is 80. So, that is r 1 and therefore, r 2 is going to be 80 plus 60 which is nothing but, 140.

So, therefore, you have r is 60 by l n 140 by 80. This gives us r equal to 107.2 millimeter. And hence, the eccentricity of the cross section is r bar minus R. R bar is nothing but, the radius of curvature of the central axis. And which is going to be 80 plus 30 is 110 and r bar is 1 I beg your pardon. R bar is 110 and R is 107.2. So, therefore we get eccentricity as 2.8 millimeter.

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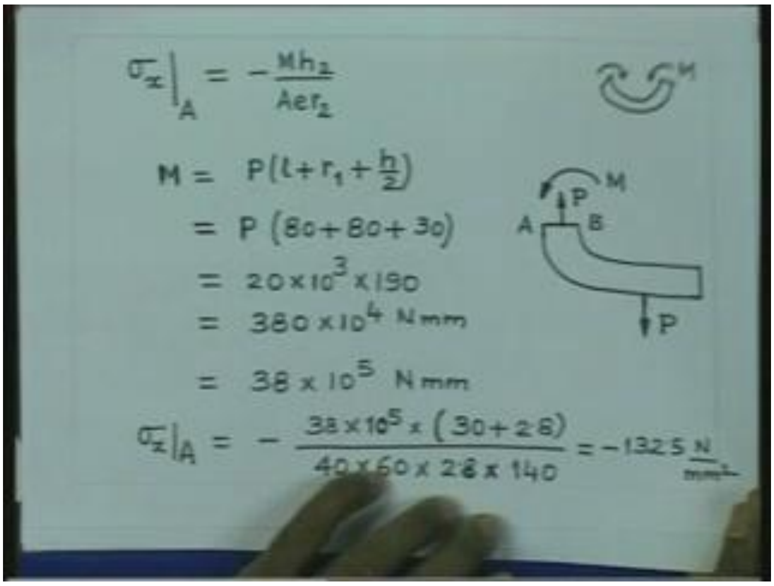

Now, we have to calculate this trace at the location A and B. We can write the formula sigma x at the location A. Note that this bending moment is trying to reduce the curvature or increase the radius of curvature. So, therefore, it is a negative bending moment. The bending moment that we consider to be positive was like this.

That was our curved member, we applied the bending moment like this. This bending moment is considered positive and it increases. The curvature of the beam or it reduces the radius of curvature of the beam. But, in this case you find that this moment is going to increase the radius of curvature. And hence, it is negative bending moment.

And therefore, the stress here is going to be minus M h 2 divided by A e r 2. This is you y axis, this is z axis. So, we will have this and M in this case, you have to take the bending moment produced by this force at this centroid. So, therefore, it is going to be P into l plus r 1 plus h by 2. So, if you do that P l is 80 plus r 1 is 80 and the height is equal to 60. So, therefore, it is 30.

So, this is nothing but, 20 the force is given as 20 kilo Newton's. So, therefore, it is 20 into 3 and this is 190. So, therefore, it is 380 into 10 to the power 4 Newton millimeter. Same thing as 30 into 10 to the power 5 Newton millimeter. So, we have got the bending moment as 38 into 10 to the power 5 Newton millimeter. So, we can substitute the value now. Sigma x at A is equal to minus 38 10 to the power 5 and this h 2 30 plus 2.8 millimeter is the eccentricity. So, we had that area is 40 into 60, eccentricity is 2.8 and r 2 is going to be 80 plus 60 140. So, if we simplify this, it gives us 132.5 Newton per millimeter square.

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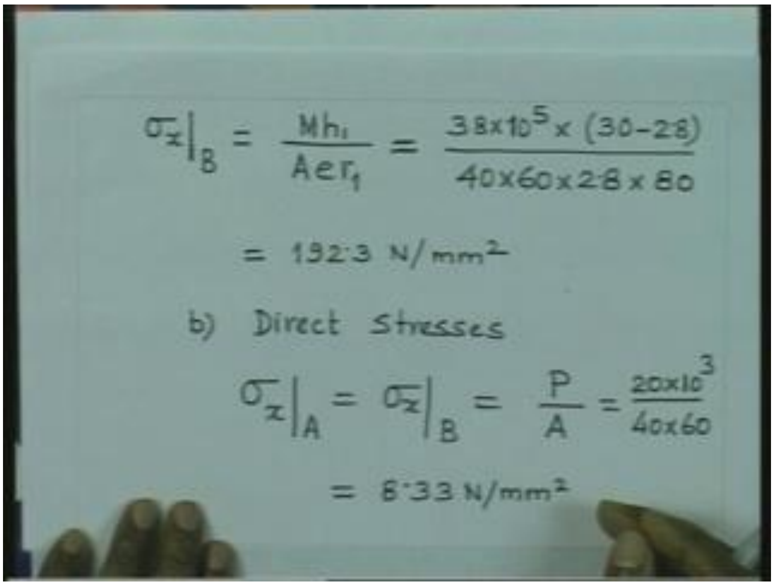

Similarly, the stress at point B is going to be tensile. So, it is M h 1 A e r 1. So, substitute the value now 38 10 to the power 5 h 1 is nothing but, 30 minus 2.8. This is 40 into 60 2.8 and r 1 is nothing but, 80. So, to calculate this it is 192.3 Newton per millimeter square. We have to calculate the direct stresses due to p, direct stresses are going to be given by it is nothing but, P by A.

Here, the stress will be same at both the locations and this is nothing but, 20 into 10 to the power 3 Newton divided by 40 into 60. And this is equal to 8.33 Newton per millimeter square. And this is a tensile stress. So, note that the stress at A due to bending moment is compressive at B it is tensile and due to the direct force P it is tensile everywhere. So, now we can get the total stresses.

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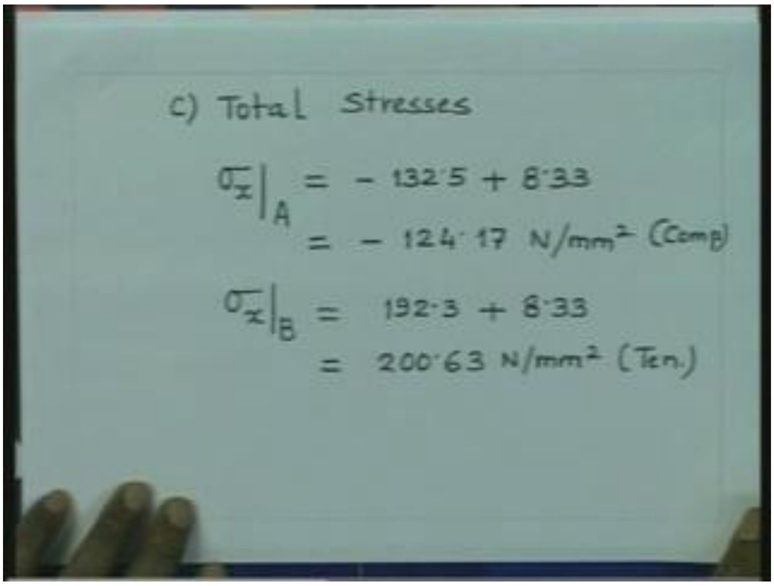

So, at A we have the bending stress is given by 132.5 Newton per millimeter square. That is compressive plus the direct stress 8.33. So, this gives us finally, 124.17 Newton per millimeter square compressive. Similarly, sigma x at B is equal to the bending stress is 192.3, direct stress is 8.33. The two will give us finally, 200.63 Newton per millimeter square and this is tensile stress. It is important to note the distribution of the stresses in the cross section.

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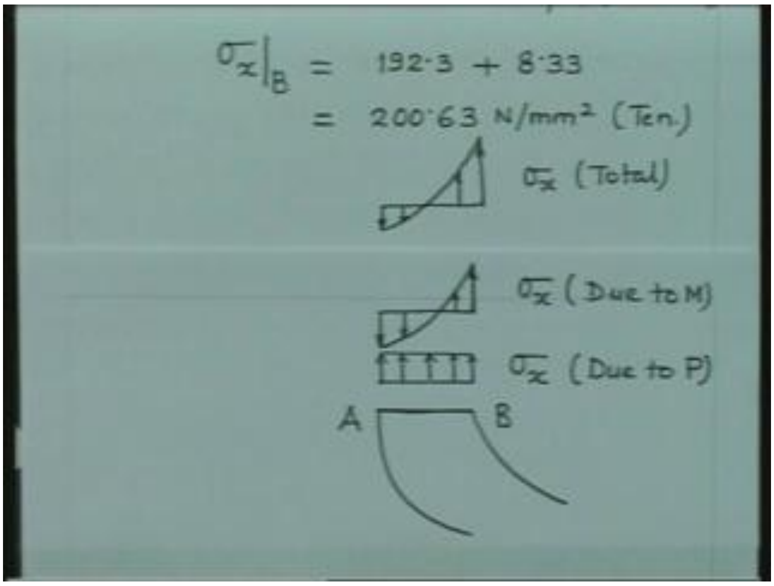

Let us draw this cross section to a larger scale. This is the section A B. Now, due to the direct force we are going to get the stresses, which is going to be uniform stress all over. So, this is the stress sigma x due to direct P. And the bending stress is, we are going to get the tensile stresses at the inner fiber and compressive stress at the outer fiber and the variation of the stress. This is the variation of the stress.

So, this is tensile, here it is compressive. Finally, the two are added and we get a stress variation, which will be like this ((Refer Time: 48:26)). We have a variation finally, which is going to be the total variation. So, this is sigma x due to M and this is the sigma x total. I would like you to consider certain problems for solving. We can look into the problem of this type.

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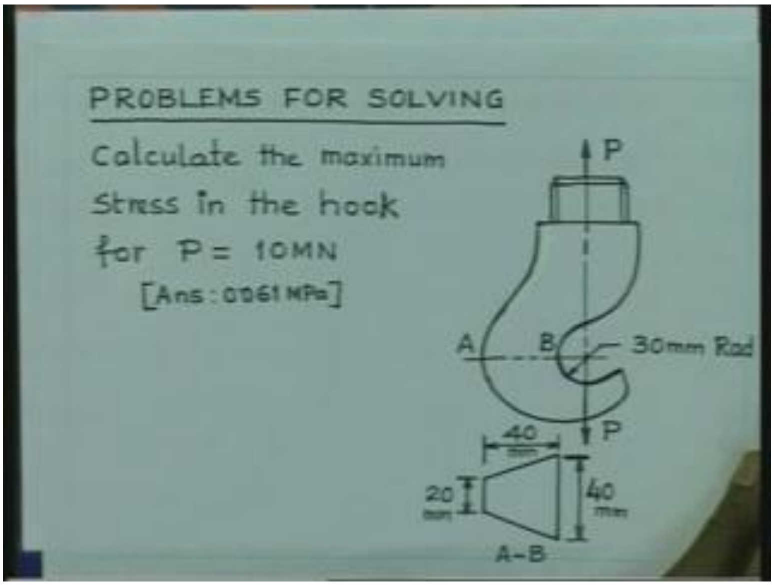

You have a crane hook, the dimensions of the crane hook. There is a radius here, which is just 30 millimeter. And this cross sectional area at A B is trapezoidal. Generally in this construction you will find that, the larger width is going to be at the inner radius. So, therefore, you have 40 here and 20 at the side.

The load applied on the hook is 10 mega neutron, you have to calculate the maximum stress in the hook, in the hook for this external load p. You can solve this problem for yourself. The answer is going to be 0.061 mega Pascal.

Finally let us try to look into what we have done today. We have considered the problem of calculation of stresses in curved box. We brushed up the calculations that you were acquainted with in connection with the straight beam. Then, we went into finding out the stresses in curved beam. And it came out, that in the case of curved beam, the stress distribution is going to be non-linear, unlike the case of straight beam where it is linear.

If you consider a rectangular cross section of the bar. In the case of straight beam, you are going to get extreme stresses to be equal. And that is given by 6 M by b h square, where m is the bending moment, b is the width of the cross section, h is the height. On the other hand, if you consider the case of the curved beam, it is going to have value which is higher at the inner fiber, lower at the outer fiber.

And the magnitude of this stress, if you apply the bending moment to reduce the radius of curvature, which you have considered to be positive. Then, in that case the space at the inner fiber is given by minus m into h 1 divided by area of cross section, into eccentricity into the radius of curvature of the fiber r 1. So, it was minus M h 1 by A e r 1.

If you consider, the outer fiber the stress is given by M h 2 by A e r 2, where h 2 is the height of the fiber or the distance of the fiber from the neutral axis. And r 2 is the radius of curvature of the fiber. It is important to understand, why is that the eccentricity is such that, the neutral axis always shifts towards the center of curvature.

In the case of curved bar, you will find that the neutral axis is always going to shift, towards the center of curvature. And it is not going to be otherwise. This will be clear from the fact that, if you consider two fibers, which are equidistant from the neutral axis. The stress in the fiber, which is closer to the center is higher, than this fiber which is away from the center.

And therefore, you are going to have higher stresses, towards the inner fibers or in the inner fibers. And since, the centroidal axis divides the cross section into two half's. Such that, sum total of force acting in the top half is exactly equal to the sum total of the force at the bottom half. And this is what you are going to find in the case of straight beam?

In the case of straight beam you will have the, linear distribution. And you will have the total force in the top half and the bottom half, exactly matching and this sum total is 0. But, in the case of curved bar, since the stresses are higher at the inner fibers to have the equality of the compressive force. And the tensile force, you must have the neutral axis moved towards the center of curvature.

Since, the higher stresses are acting at the inner fibers, than the outer fibers to have the equality of the forces, you cannot have the neutral axis coinciding with the centroidal axis. That is one of the reason, why you get the neutral axis shifting towards the center of curvature. In order to solve the problem of this type, the step should be that you calculate the radius of curvature of the neutral axis, thereby you get the eccentricity. Then, you make use of the formula straight away to calculate the stresses.