Advanced Strength of Materials
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Lecture – 22

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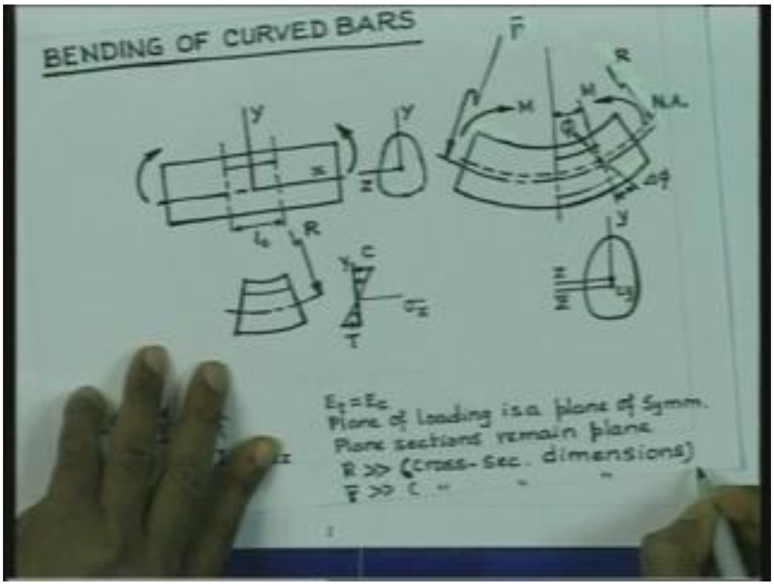

Today, we are going to study bending of curved bars. I am sure in your earlier course you have studied the bending of straight bars. You must have considered a component like this. It was subjected to bending moment, because of the bending the bar gets into the shape. I am just showing a portion of the bar here. And we select the coordinates that, this is the direction aligned with the centroidal axis of the bar. And this is y axis, which is pointed towards the center of curvature and z is out of the plane.

So, it is really trying to bend above the z x plane or rather the z x plane is trying to get curved. You have seen that, for such cases, if you just consider a portion of the bar. Let us say this portion. This portion after the application of the load takes a step like this. What you have considered? That this section rotates by some angle.

So, also this section rotates and all the fibers of the bar gets or becomes curved. And they are going to have common center of curvature. And you have considered, that the radius of curvature of the neutral axis, which pass through the center of gravity of the cross section is equal to R. In this case, you are going to have the bending stresses developed under the action of the bending moment.

And the distribution of the bending stress is going to be linear. It is going to give rise to compressive stress at the top fiber and tensile stress at the bottom fiber. You might recollect the flexure formula that was derived in connection with this. That sigma x divided by the distance of the fiber from the neutral axis is equal to M by I is equal to E by R, where sigma x is the stress. So, we are plotting the sigma x stress in this direction and y is in this direction.

So, therefore, this is the variation of this stress. And this sigma x by y, M is the bending moment, I is nothing but, the moment of inertia of this section of the beam about the z axis. So in fact, this I is nothing but, moment of inertia about the z axis I z z. But, it is simply written as I, E is the modulus of elasticity of the material. And R is the radius of curvature of the fiber, which passes through the centre of gravity of the cross section or that is the neutral fiber.

Now, in this case you must note that the fiber all the fibers initially are of length. If you consider that this distance is equal to l 0. Then, all the fibers initially are of the same length and after getting built, they are going to have different lengths. So, the strengths are going to be different. Just to refresh, what were the assumptions that you made in connection with the derivation of this formula.

You assumed, that the modulus of elasticity of the material in tension E t is equal to the modulus of elasticity of the material in comparison. Similarly, you also assume that the plane of loading is a plane of symmetry. In fact, the loading plane is nothing but, the y x plane and that plane is a plane of symmetry. So, plane of loading is a plane of symmetry.

Another important assumption that was made that after the bending the section does not get distorted. This section, which as initially plane it is also plane after it is bent. So, plane sections remain plane. The other assumption is that, the radius of curvature that we get out of this bending is going to be very large compared to the cross sectional dimensions. Like the dimensions in the horizontal direction or in the vertical directions.

So, R is very large compared to cross sectional dimensions. So, these are the major assumptions best of that, we derived this relationship. Now, let us consider the situation, that the bar is not straight initially. But, it is curved with some radius of curvature. Let us say R bar for the centroidal axis. And it is reasonable to assume, that this R bar is again very large compared to cross sectional dimensions.

Such that all the fibers can be considered to have the same center of curvature. So, if the beam is initially curved and it is subjected to bending, what will be the difference in the stress distributions? How can we calculate them. And you must also understand that this type of components do arise in practice. So, you must have come across the chain links, which are carved to apply the load it is going to have bending action developed. So, just I would like to show you certain components, which are going to have configurations that are initially curved. Look at some of the components which are shown here.

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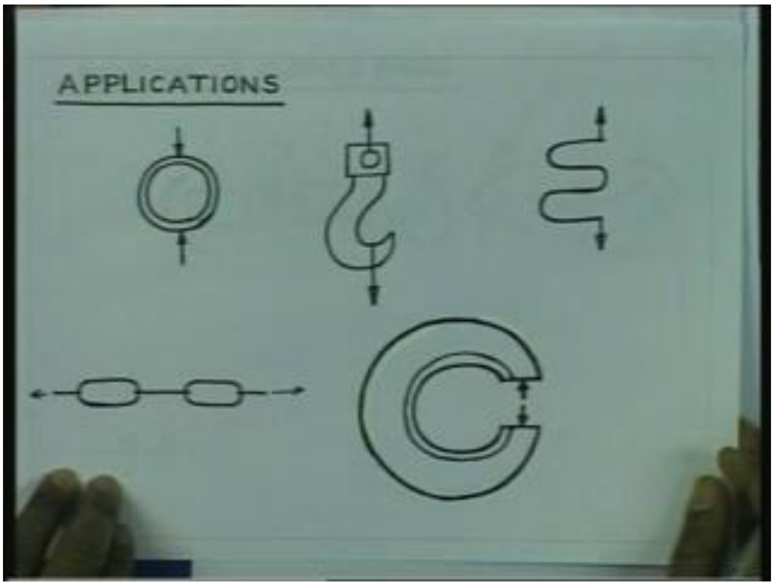

Think of a circular ring. It is subjected to diametrical compression. Think of the hook of a crane, which is curved here. And it is subjected to loading and under the action of the loading, there is going to be bending, developed. Similarly, you comes across pipelines like this, which are subjected to loading. You can also have the press frame c press frame, which is going to look like this. And it is subjected to loading and that is going to cause bending in the portion here.

Similarly, you also have come across chain links. Think of the chain links and it is subjected to loading in the axial direction. Under the action of this loading you are going to have bending in this portion. So, there are certainly applications and we would like to see, how the stresses in these components can be calculated.

You know that to derive the ((Refer Time: 11:06)) relationship for the bending stresses, in response to some bending moment applied on the component. Let us concentrate on a portion, which is uniform in cross section. Let us also assume that the cross section is like this. It does have a plane of symmetry and the loading is in the plane symmetry. So, we make the same assumption, as you deal in the case of straight beams.

Let us now consider that the c g of the cross section is here. And therefore, this is the y axis. And let us consider that this axis passing through c g is now z bar. Just I would like to differentiate with z I will come to why it is different little later. Let us concentrate on a portion of the beam, which is included between this section and a section, which is here.

So, let us just consider a portion which is included between these two sections. And let us say, that this portion is included it makes an angle at the center of curvature, which is equal to phi under no loading. As you apply the bending moment, this cross section is going to rotate. Let us say that it is going to rotate like this. And this rotation, that is very small compared to phi and let us represent this by delta phi.

Now, you can see that the fibers, if you consider this fiber it has some length initially. Or if you consider a fiber here, this is the length of the fiber initially. And it got changed by this distance, after the application of the bending moment. And you see here, that the fiber got compressed when it is located at this position. On the other hand, if you consider a fiber, which is somewhere located there this fiber is of length this much.

But, it got extended by this much after the application of the bending moment. So, we see the that phi bar here is getting stressed. And the phi bar here it is getting compressed. The same picture you have noted in the case of straight beam, the fibers which are located towards the center are getting compressed. And the fibers which are located they are getting extended.

Obviously, if the fibers are getting stretched, it is getting compressed here and stretched here. Obviously, there will be some location in between, where there is no change in the length of the fiber. And that is the fiber, which is considered to be neutral fiber. So, in this case let us we do not know, what is the location of that fiber? Let us consider that the neutral fiber is going to be located somewhere at this location. And let us show that this is the neutral phi bar.

So, this is the neutral axis of the beam after bending. The corresponding point on the cross section is here. And let us indicate the axis, which passes through this point by z. So, we are now trying to have the origin located at the point, where there is no strain. I would like to emphasize, that we will like to have the derivation again. Considering that the modulus of elasticity of the material in tension is equal to the modulus of elasticity in compression. Plane of the loading is a plane of symmetry.

In this case it is the y x plane, which is the plane of symmetry. And we also make assumptions, that there is no distortion of the cross section, after the load is applied. And we are also assuming, that the radius of curvature. This radius of curvature, now we would like to indicate that radius of curvature of the neutral axis or neutral fiber, that is the fiber which is neutral and let us indicate the radius of curvature of this fiber as R. So, this r is now very large compared to cross sectional dimensions.

In fact, in this case we can also consider that this R bar, which is the radius of the centroidal axis, this is also large compared to the cross sectional dimension. So, with these assumptions, we would now like to proceed for the derivation of the formula. So, I would like to now segregate that portion, which we considered which is making an angle of phi at the center of curvature.

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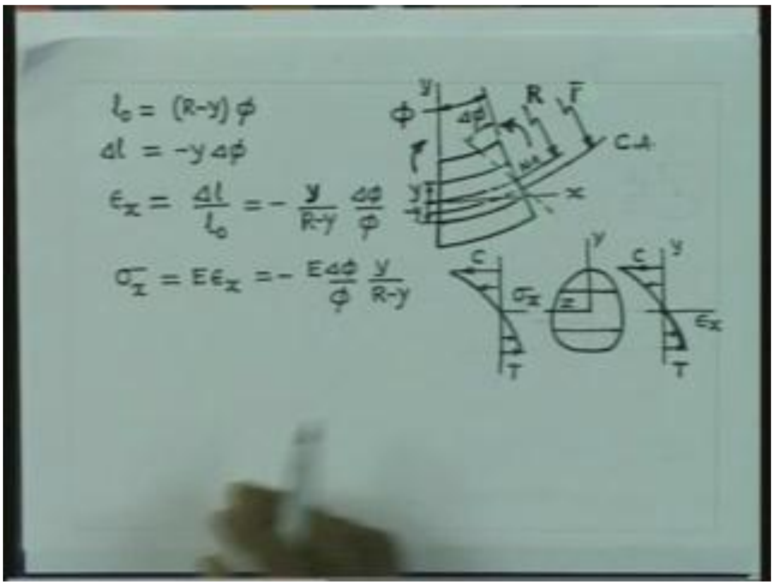

So, that is the portion and this section gets rotated by an angle, this much which we have indicated by delta phi. So, this is the this axis is centroidal axis and this is the neutral axis. So, this one is neutral axis. If we now concentrate on a fiber, which is located here. Please note that we have indicated y axis here and our x axis is located in this direction. So, therefore, this is really the direction of x axis.

Now, we are considering a fiber, which is at a distance y from the neutral axis. So, we are concentrating on a fiber, which is a distance of y from the neutral axis. If R is the radius of curvature of the neutral fiber, then the initial length of this fiber l 0 is going to be given by R minus y multiplied by phi, because phi was the angle made by the sector at the center of curvature.

After the loading has been applied, the section gets rotated. And therefore, there is a change in the length here. And we can approximate that length by this cord, which is delta l let us say. This delta l is equal to it is reducing in length, which is nothing but y multiplied by delta phi. So, the strain in the direction x is equal to delta l by the initial length l 0. And therefore, this is equal to y by R minus y into delta phi by phi.

So, the strain is going to have a non-linear variation. You remember that in the case of straight beam, we had the strain beam varying in the proportion y or the exact expression was minus y by R. But, in this case it is minus y by R minus y. So, therefore, it is a non-linear variation. Now, if you would like to plot this variation, let us consider that the section is here. This is our z axis, this is y axis.

Now, if I plot the strain in this direction Epsilon x and this is y. Then, this strain variation you will find that, it is going to have a variation like this. For y equal to 0, the strain is 0 and it is going to have comparison value here. And it is going to be opposite nature in the other direction. When, y is negative which is true in this direction, we are going to have positive strain.

So, therefore, this strain is tensile in the bottom fibers and it is compressive in the top fiber. So, this strain variation is now non-linear. In fact, you see that the length of this radius of curvature, the initial length of the fibers are different depending on this y. And therefore, you are going to have different starting length. And hence, the strains are going to be different and it is not going to vary linearly as in the case of straight beam.

You can now also consider this point. That is the fiber is here, which is what the center of curvature at a distance of y. Let us consider exactly the fiber, which is located on the other side of the neutral axis at a distance of y. Now, if you consider this fiber, which is located again at a distance of minus y from the neutral axis.

Now, the extension of this is exactly the same as this. But, since the length of this fiber is larger to begin with the strain is going to be less. So, therefore, the strain what we are saying is that, we have the fiber here which is a distance of y. Another fiber exactly located on the other side of the neutral axis at the same distance. The strain in this fiber is going to be more than the strain in this fiber. So, therefore, you will find that the strains are less in the tensile zone.

If we want to calculate the stresses, stresses are going to be given by directly sigma x is equal to modulus of elasticity E multiplied by strain e x. And therefore, this is equal to E delta phi by phi into y by R minus y. So, this is the stress variation and these are constants for the sector. And therefore, they are not varying the strain, the stresses are going to vary along similar line as in the case of strain.

So, if would like to plot the variation of space, you will get a variation of this type, same type or to a some scale, this variation gives the stress variation. So, we would like to draw it separately. So, we are plotting sigma x stress here and the variation is going to be given by again this y. So, here the stress is compressive and here it is tensile.

So note that, this is due to the fact, we have applied the bending moment in this fashion. Therefore, we get compression and tension in this direction. So, this is due to the direction of the moment, that we get compression there tension there. If you try to reverse this directions, you are going to get tension there and compression on the other side, again a phi bar which is located at the same distance from the neutral axis.

You will find that, stress in the phi bar which is located towards the center of curvature is going to be more than the stress, which is located away from the center of curvature. So, we have got the variation of stress. But, we are not yet in a position to say, how this stress is dependent on the applied bending moment. And what is this radius of curvature R, we may be given the value of the radius of curvature of the centroidal axis.

But, we are not so far clear about the value of this radius of curvature R. So, therefore, we really have to dissolve the two issues. How these constants delta phi by phi are related to the bending moment, and also how we can calculate this radius. So, we will proceed to determine these things in terms of the bending moment. How do you do that? You remember in the case of straight beam, what we did is that we considered a fiber at a distance y from the neutral axis.

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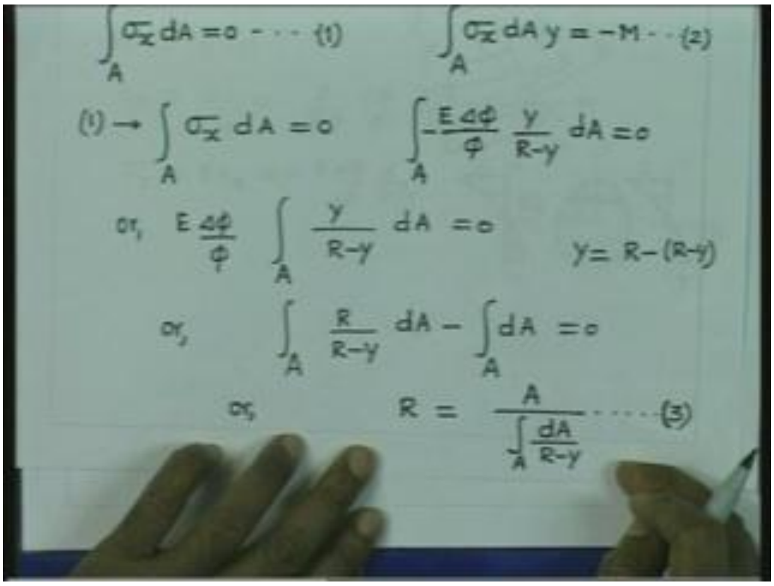

Therefore, the force acting on this fiber is nothing but, sigma x. If you consider that this area is nothing but, d A. Then, sigma x into d A is the force acting on that element. And now if we integrate the total force acting on the cross section. That is going to be sigma x into d A. This particular cross section is only subjected to bending moment in and there is no axial force acting on the cross section. And therefore, we must get the sum of the axial stresses to be equal to 0.

So, therefore, sigma x d A is equal to 0. So, that is the equilibrium equation, that is one of the equilibrium equations. Now, if we try to consider the stresses and their moment about the z axis. So, the moment of the force sigma d A, which is at a distance of y. So, that is the moment. Now, if we try to integrate this moment over the whole cross sectional area.

Obviously, this is the positive x direction. The stress which is considered to be positive is going to be directed like this. And it is moment is going to be clockwise, but the external moment is applied anti clockwise. So, therefore, we must have the equality like this, that this integral is equal to minus m, which is the applied bending moment in the anti clockwise direction.

So, this is the second equations of equilibrium. And from these two equilibrium equations, we can resolve the constants like R, and also this delta phi which is also unknown. Let us proceed with the first equation. So, starting from 1 we have sigma x d A is equal to 0 I think I would like to point out at this stage. That, this neutral axis which you have arbitrarily taken, we do not know it is location.

So, therefore, that is also an unknown. What is this y or what is the reference for this y is really unknown for us, so far. So, therefore, we have to in that and in fact, you are going to see that. This is going to be obtained from this relationship. So, if you now substitute the value of sigma x, we will have E delta phi by phi y by R minus y d A is equal to 0. Or we can take this for the element, that we have considered are all constants.

So, therefore, what we really have is integration of this quantity R minus y d A is equal to 0, we can these are constants. So, therefore, we can now consider this integration is nothing but, y by R minus y d A equal to 0. And if we do the manipulation, we can write now, we can add R subtract R. And therefore, will have R by R minus y d A minus integration of A over d area is 0. So, what we are doing is that, we have written y equal to R minus R minus y.

So, this gives us this R is a constant and this will be area of the cross section. So, therefore, we will have R is equal to A by integration d A by R minus... So, that is really the value of the radius of curvature. So, we are in a position to get this radius of curvature, which is going to come up after the application of the bending moment. So, this is a property of the cross section, note that this is a property of the cross section, it does not depend on the amount of load which is applied.

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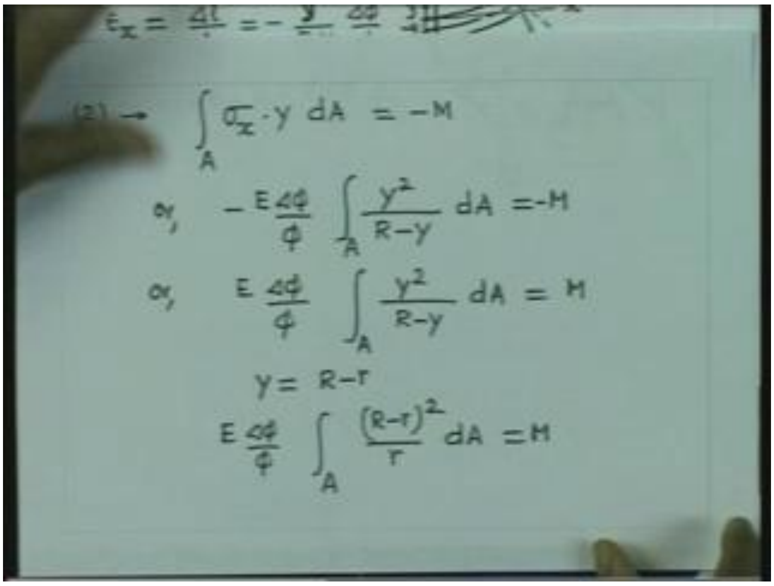

We will consider now the 2nd equation which is nothing but, sigma x multiplied by y d A over the whole cross sectional area is equal to minus M. Or you can write minus E delta phi by phi y square by R minus y d A is equal to minus M or E delta phi by phi y square R minus y d A is equal to M. We can now consider one simplification here or geometries shown ((Refer Time: 34:12)).

In this, we are considering the typical fiber, which is at a distance of y. And let us represent, that the radius of curvature of this fiber is molar. So, what I am saying, that the radius of curvature of this is equal to... So, if you make it represent it here, that this radius of curvature is equal to R. Therefore, that is nothing but, the radius of curvature is R here and this is the distance y. So, therefore, R minus y is nothing but, this molar. So, making use of that fact, we can write here that y is equal to R minus R. And therefore, we can write E delta phi by phi R minus R whole square divided by R d A is equal to M.

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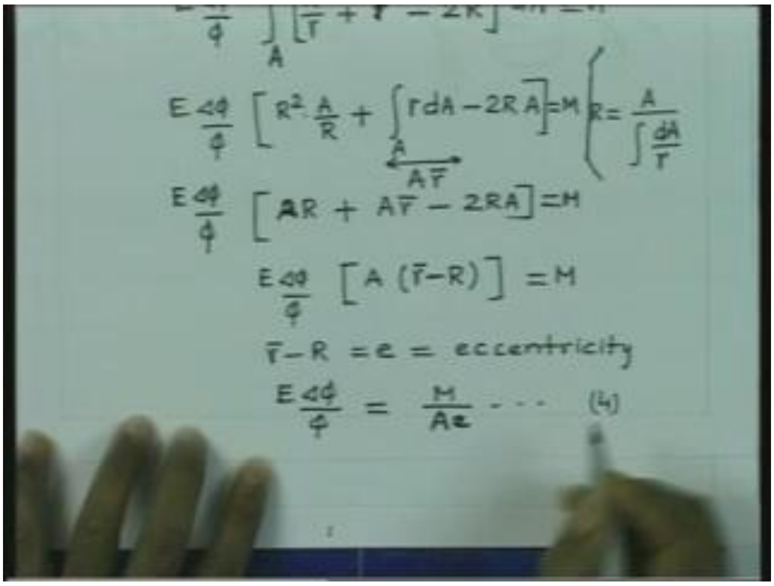

This will give us E delta phi by phi, you will find that these are going to have very simple form. So, it is A R square by R plus this is going to be R. Then, we are going to have minus 2 R. So, that is d A. So, R square by R plus R, this is equal to M E delta phi by phi. I would like you to consider, the point we had already derived that R is equal to A by integration d A by R minus y which is nothing but R. So, that was the relationship.

So, therefore, here what we can do, that we can write integration of d A by R is equal to A by R. So, we can now substitute here for this we can write R square into A by R. And this is nothing but, integration of R d A the whole cross section minus 2 R into A and that is equal to M. So, you can now simplify E delta phi by phi is equal to twice I beg your pardon, this is A into R plus what is this?

Please note this point ((Refer Time: 38:02)). This quantity, the radius of curvature of this fiber radius of curvature of this phi bar or it is at a distance of R from the center of curvature. So, therefore, R into d A is nothing but, the moment of this area about the center of curvature. And therefore, integration of this whole quantity would be nothing but, A into R bar.

So, this quantity is nothing but, whole cross sectional area multiplied by R bar. So, therefore, we will have A R bar minus twice R A is equal to M. Finally you will find that E delta phi by phi into A and we will have now R bar minus R equal to M. And if we represent this R bar minus R by E which is considered to be eccentricity.

Finally what we have is that E delta phi by phi is equal to M by A into E. So, the quantity delta phi is now related to the applied bending moment and the cross sectional dimension, and also the eccentricity of the cross section. So, this is relation number 4. So, we have obtained that if delta phi by phi is equal to M by A earlier we have already derived ((Refer Time: 39:54)) that the stress sigma x is equal to E delta phi by phi into R minus y.

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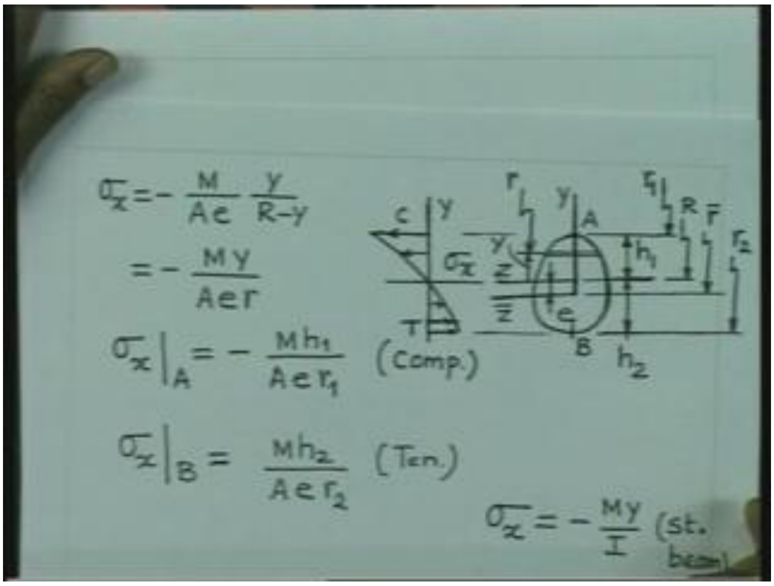

So, if we now substitute the value in that stress relationship, we now find that sigma x is equal to minus M by A e y by R minus y, which is same thing as M y A e into R. The radius of curvature of the fiber. So, what we are considering, that this is the fiber we are concentrating and this fiber has a radius R. So, the stress in this phi bar is given by M into y, where it is located at a distance of y from the neutral axis.

So, this distance is y M y A e into R, where eccentricity between the centroidal axis and the neutral axis is e. We can now see the value of this stresses, that is occurring in the cross section. If you consider the fiber which is at the extreme interior, it is at a distance of let us say h 1 from the neutral axis. Then, the stress is going to be the highest and that sigma x at A is equal to minus M h 1 A e.

If you consider that the radius of curvature of fiber is equal to r 1. Then, it is A e into r 1 and note that this stress is compressive in response to the bending moment, that we have applied. Similarly, if you consider the fiber B, which is at a distance of minus h 2 from the neutral axis this distance is h 2. So, it is minus h 2 y axis is positive towards the center of curvature. And therefore, sigma x at B is going to be... So, minus h 2 will cancel the negative sign.

And therefore, it is going to be M h 2 A e and if consider that the phi bar radius of this fiber is r 2, then it is a e into r 2. So, this is a tensile stress. So, we expected that the compressive should be there. And the tensile stress should come up at this point. And in fact, we have also got the values accordingly. So, these are the extreme stresses in this section. I would like you to again look back into the relationship for the straight beam.

So, if you consider the straight beam sigma x for straight beam, it was nothing but, M into y by I. That is for straight beam. So, the difference is quite evident, that here in it is non-linear, whereas, in this case it is linear relation. So, you not to solve the problem of curved beam, what is necessary? You will be given bending moment applied. You will probably given the value of the radius of curvature of the centroidal axis.

And you will be required to find out the stresses in the cross section to extreme stresses in the cross section and to solve the problem. Since, the relationship is sigma x is equal to minus M into y by A e into R. What is necessary is that, we have to first locate the neutral axis, which means that you have to find out the radius of curvature of the neutral axis, which is capital R. And the difference between the radius of curvature R bar and this capital R will give you the eccentricity of the cross section.

Then, you are in a position to find out exactly, the radius of curvature of the extreme fibers also. And then you get back to the relationship sigma x is equal to M into y by A e R. That will give you the stresses at any layer of the cross section. So, we will now look into what is the eccentricity, it is a property of the cross section. So, it will be nice idea to look into how we can calculate this eccentricity for number of cross sections.

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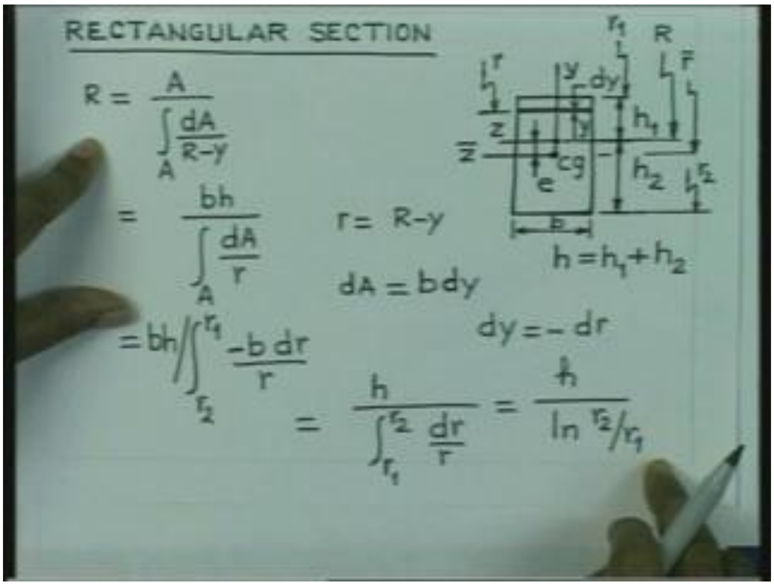

Let us consider the rectangular section. For the rectangular section, we have the radius of curvature of the neutral axes R. And the radius of curvature of the centroidal axis is r bar. Let us now represent that the height of the inner fiber, from the neutral axis is h 1 and the height of the extreme outer fiber is h 2 from the neutral axis. So, this it will be the convention that will make use of. And total height of the cross section is h 1 plus h 2, that is h.

What is r bar ((Refer Time: 46:02)) that is what we have to determine. And we have the formula that R is equal to cross sectional area divided by d A by R minus y integrated over the whole cross section. So, if you consider this section to have width equal to b. Let us say that width of the cross section is b. And for any fiber that is arbitrary fiber, we have this radius of curvature equal to r. And this fiber is at a distance of y from the neutral axis.

So, we can write this integration that is there is nothing but, write one more step d A by r. Since, r is equal to capital R minus y. We can write also d A given by, if you consider that thickness of this layer is equal to d y. Then, this d A is nothing but, b into d y. And these limits of integrations are going to be minus h 2 to h 1.

Alternatively we can write, that the radius of curvature in terms of the radius of curvature of the extreme fibers. If you consider the radius of curvature of this extreme fiber is equal to r 1. And the extreme bottom fiber radius of curvature equal to r 2. Then, we can certainly write from this relationship d y is equal to minus d r. So, we have now we will have the limits r 2 to r 1 b h divided by d A is nothing but, b minus d r divided by r.

So, that is the we are moving from this fiber to that fiber. So, d r is negative and now this gives us b is canceling, this will give us h divided by we can now make the limits to be interchanged. So, therefore, it will be d r by r and this gives us h divided by logarithmic r 2 by r 1. So, that is the radius of curvature of the rectangular cross section. So, once you have got this radius of curvature, it is given from the three dimensions. Once you know the radius of curvature r bar, you know what is the value of r 2 which is nothing but r bar plus h 2. And the value of r bar is nothing but r bar minus I beg your pardon r 2 is going to be equal to r bar plus h by 2. And r 1 is going to r bar minus h by 2. So, you do have this value known.

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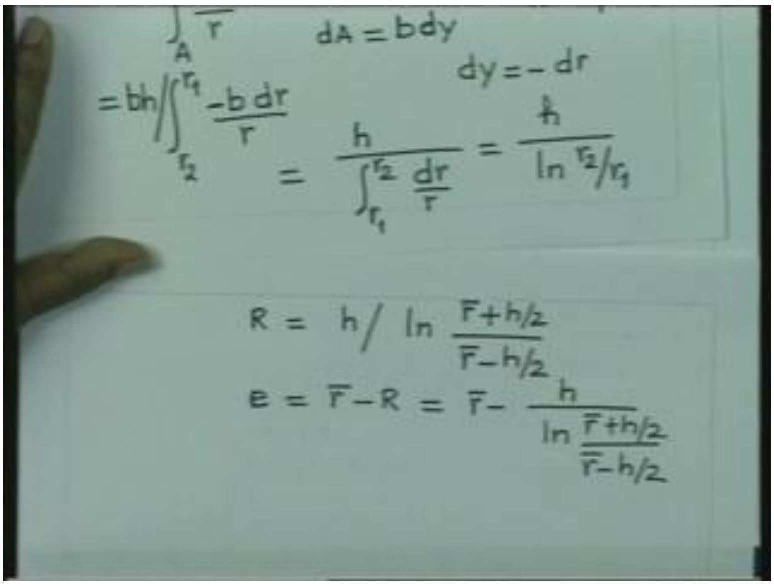

So, you can write in terms of the radius of curvature of this centroidal axis. So, this is nothing but, l n. This is r bar plus h by 2 divided by r bar minus h by 2. So, that is the value for the radius of curvature. And the eccentricity is nothing but, r bar minus R. So, we have the eccentricity now given by h divided by l n r bar plus h by 2 divided by r bar minus h by 2.

Let us now consider a section which is different. You will find that curved members are not always made up of rectangular cross section. They could be made up of t section, they could be made up of trapezoidal section for circular section. So, you would like to look into how to calculate the radius of curvature for such cross sections.

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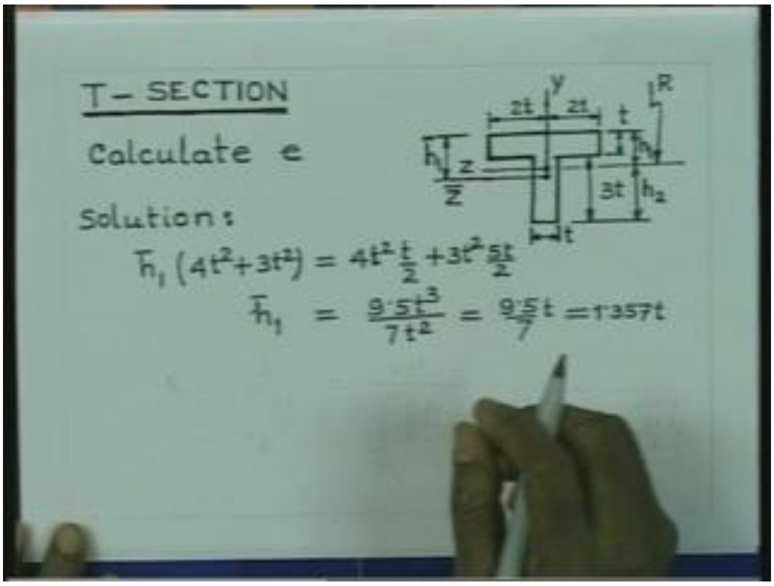

So, let us now consider a T section. So, the section is shown here, that we have 1 t formed by a rectangular portion of dimension 4 t by t and another portion. So, the flange is of dimension 4 t by t and the ((Refer Time: 52:19)) is of dimension 3 t by t. The centroidal axis will pass through the centroid of the cross section. Therefore, in this case it is not obvious, where the centroid is located. So, it has got to be located first.

And the problem is to find out, the eccentricity of the cross section. So, if you consider that this is the axis or this is the neutral axis or the z axis. Then, what is the difference between the locations of z and z bar. You would like to consider again the top fiber. The distance of the top fiber from the centroidal axis. Let us say, that is equal to h 1 bar which is unknown to us.

And you would also indicate, that the distance of the top fiber from the neutral axis, as usual given by h 1. And the distance of the bottom fiber from the neutral axis is h 2. So, to solve this problem, we will have to first of all find out the location of this centroid of cross section. And then we have to find out the radius of curvature of the cross section.

So, we will get into solving this problem. If we take the datum, datum to be this lin. Then, we can take the moment of this flange about that line. And moment of the ((Refer

Time: 54:25)) above the same datum. And the total moment would be equal to nothing but, total area multiplied by the distance of this c g that is h 1 bar.

So, we can write now h 1 bar into area of the flange is 4 t square and the area of the ((Refer Time: 54:48)) is 3 t square. So, therefore, that is the total area. So, moment of the total area about the datum is equal to the moment of the individual area about the same datum. So, the flange area is 4 t square and it is distance from the datum is t by 2.

Similarly, the ((Refer Time: 55:11)) area is equal to 3 t square. And it is distance is nothing but, this t plus half of 3 t. So, therefore, it is going to be 5 t by 2. This gives us h 1 bar equal to 9.5 t cube by 7 t square which is nothing but, 9.5 t by 7 which is nothing but 1.357 t. So, we have got the c g located with reference to the top fiber it is 1.357 t. We will consider the calculation of the radius of curvature in our next presentation.