Advanced Strength of Materials
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Lecture – 21

Last time we considered torsion of segmented box sections. Today, we would like to illustrate the formula that we have derived by considering one example.

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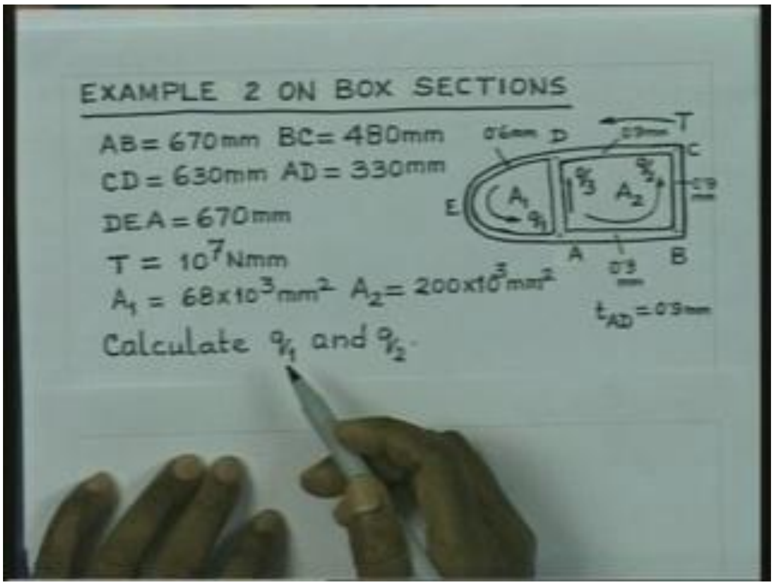

The problem is shown here, the segmented section is illustrated. We have one formed by A, B, C, D and another formed by A, D, E and the thickness of the wall, A to B, then B to C and C to D. So, also from D to A, there is same thickness, which is 0.9 millimeter and the portion D, E, A is of thickness 0.6 millimeter. The length of the portion, A B is 670 millimeter, B C is 480 millimeter, C D is 630 millimeter, A D is 330 millimeter and this length D E A is 670 millimeter.

This given that the torque of magnitude 10 to the power 7 Newton millimeter is applied. The areas enclosed by the cell 1, that is A 1 is 68 into 10 power 3, millimeter square. So, also the cell 2, it is area is 200 into 10 to the power 3, millimeter square. What is required is that; find out the flows in the cell 1 and 2. So; that means, you have to find out, q 1, which is in this loop and q 2 in this loop.

So, note that, this q 1 is flowing in the same sense as the applied torque; q 2 is also acting in the same sense as the applied torque. And the net flow in the segment A to D is q 3. It is directed in this direction. So, that is the direction of the flow q 3. So, we will look for the solution; first let us consider the point D.

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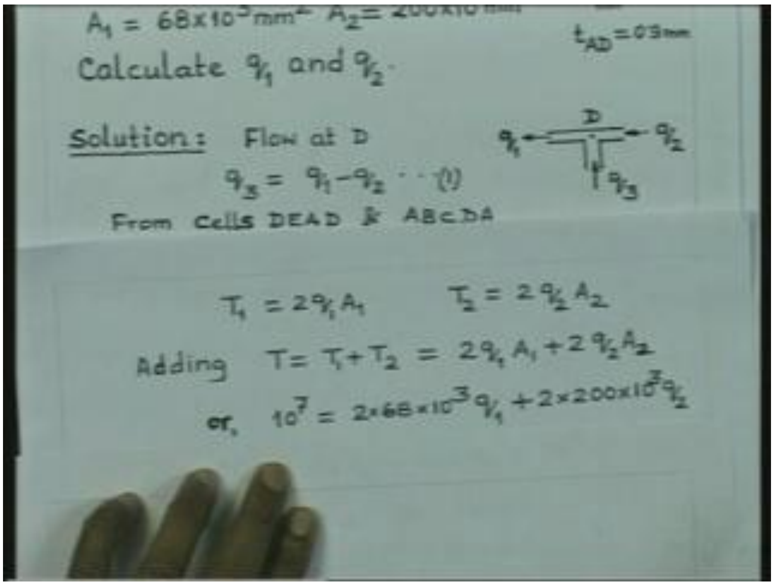

For the point D, flow leaving the point is q 1. Flow coming in towards the point is q 2 and q 3. So, from the flow of the quality, you can write q 3 is equal to q 1 minus q 2. So, this is, how the flows are related and this also can be obtained from the consideration of equilibrium. Now, let us consider the cells D E A D. Now, let us consider the cell, D E A D ((Refer Time: 01:00)) and ABCDA. So, these two cells separately and if you consider these cell separately. Then, for the first cell, we will obtain, the torque taken by the first cell, cell 1 is going to be given by the formula. That is the formula directly; T 1 is equal to 2 q 1 into A 1, where A 1 is the area, enclosed by the cell D A D.

Similarly, torque taken by the second cell, A B C D A is T 2 is given by 2 times q 2 into A 2, wherein q 2 is the flow in the cell and A 2 is the area enclosed by the cell. So, adding the two equations or two relations, we will get the total torque, which is T 1 plus T 2 and therefore, this is 2 q 1, A 1 plus 2 q 2, A 2. Now, if you substitute the values given, this is 10 to the power 7 and A 1 is 68 into 10 to the power 3, A 2 is 200 into 10 to the power 3. So, we will get this equation.

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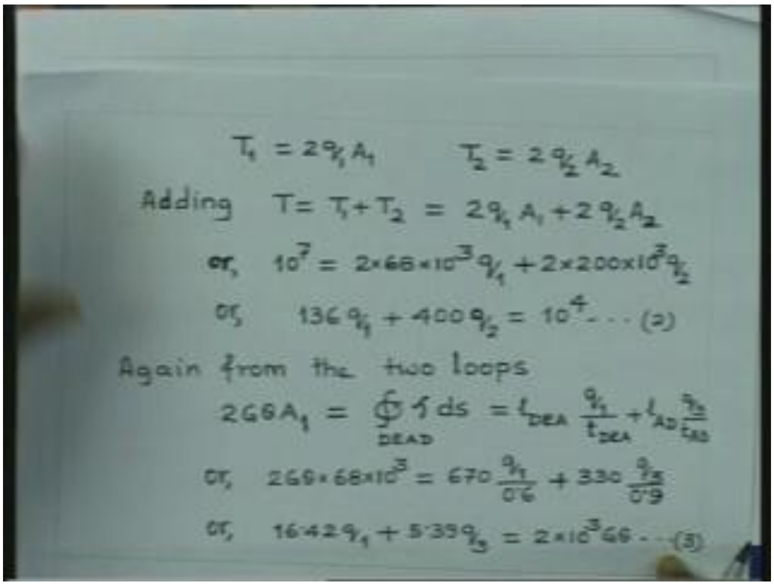

Finally, we can make some simplification and we get the relationship 136 q 1 plus 400 q 2 is equal to 10 to power 4. Let us consider that equation number 2 and already, we have got equation number 1, which is q 3 is equal to q 1 minus q 2. Now, if we consider, again the 2 loops separately. So, we will consider the loop D E A, D E A D. Then, we can write 2 G theta into A 1 is equal to integration of the product tau into d s over the whole loop.

And since, length of the portion D E A or rather a thickness of the portion D E A is constant. We can write this tau in terms of length of D E A. Flow of that portion is q 1 divided by thickness of D E A. Similarly, the portion A D, it is length is l A D and q 3 is constant. Therefore q 3 by t A D, it gives us the shear stress, which is constant. Now, we can substitute the value of the areas and also the thickness. Then, we get this relationship. On simplification, this turns out to be 16.42 q 1 plus 5.29 q 3 is equal to 2 into 10 to the power 3 z theta. That is equation number 3. So, we get another relationship, involving the two flows and the angle of twist per unit length theta.

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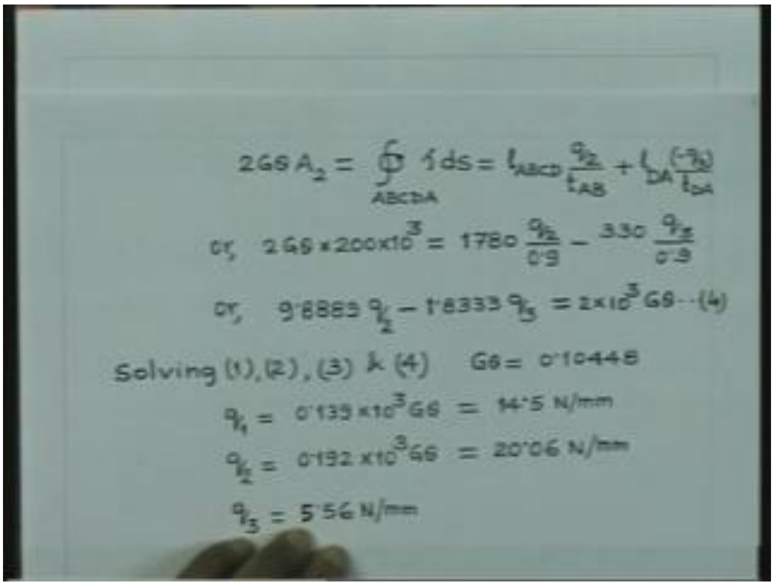

Let us consider applications of similar relationship for the second loop, which is nothing but, A B C D A. And we can write now, 2 G theta into A 2 is equal to over the whole loop A B C D A tau into d s. And since the thickness about the portion A B C D is constant. We can replace this thing by q by the thickness of the portion t A B and length of the portion A B C D.

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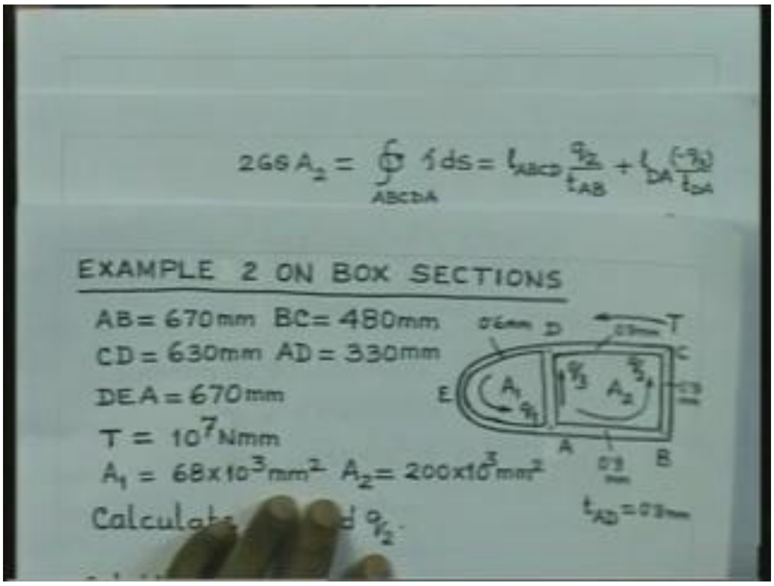

Similarly, for this portion D A, we are, this is something, we can note that, when we are trying to talk about this loop. We are trying to move in this direction. And since, q 3 is now in the opposite direction here; flow is directed in the opposite direction of the path. Therefore, we have to take negative sign. So, l D A minus q 3 divided by t D A. That is the value of the integral from d 2 a. So, we have constant thickness all over this.

Therefore, you could write straightaway length A B C D into q 2 t of A B and for the remaining portion, you have to write this. This negative sign comes up, because the moment is in this anti clockwise direction. But, the flow is taking place in the clockwise direction.

((Refer Time: 08:41)) So, if we substitute the value of the areas and the thicknesses, also the length. Then, we get this relationship 9.8889 q 2 minus 1.8333 q 3 is equal to 2 into 10 to the power 3 G theta. So, that is equation number 4. So, we have four equations. The first is q 3 equal to q 1 minus q 2. Second equation gives us 136 q 1 plus 400 q 2 is equal to 10 to the power 4. And then, we have got 3rd equation, which relates q 1 and q 3 to G theta and the 4th equation, it will q 2 and q 3 to G theta.

Now, in a problem, we have unknowns, what are the unknowns we have? We have four unknowns. They are nothing but, q 1, q 2 and q 3 and at the same time, angle of 2 is theta and we have four equations. We have sufficient number of equations to solve for the number of unknowns.

((Refer Time: 08:41)) Once you solve it, you obtain z theta as 0.10448 and q 1 equal to this, which is equivalent to 14.5 Newton per millimeter, q 2 is 0.192 into 10 to the power 3 G theta, which is nothing but, 20.06, Newton per millimeter. And also, you can have 10, q 3, which is 5.56 Newton per millimeter. So, this is how, you can solve the problems involving segmented box sections. The process can be repeated, even if you have more number of closed loops. The application can be done in a routine manner.

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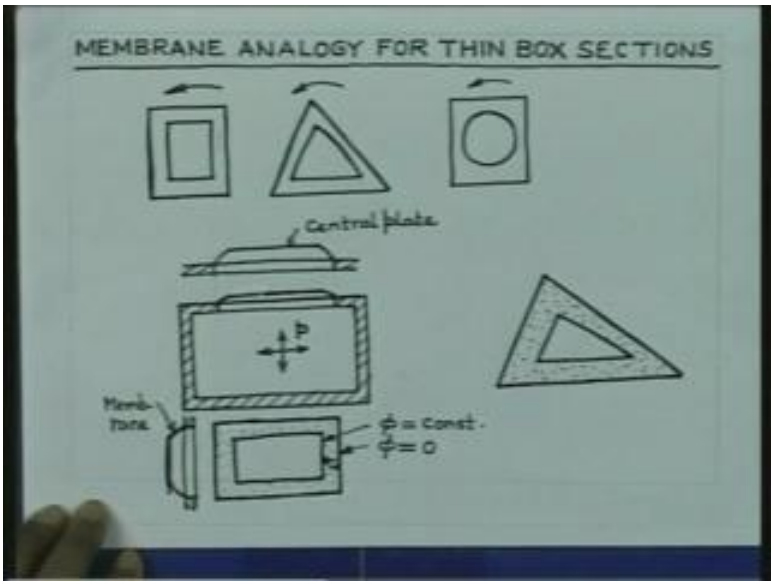

Now, to get some physical feel about the box sections, rather how does a space develop in the box section? I would like to get back to consideration of membrane analogy for thin box sections. What I am concerned really is the following sections. You have a section of this type or you have a section of the triangular shape or you could have a section of this type. They are subjected to angle of twist.

Now, if the membrane analogy is to be considered for such sections, how do you go about it? We are giving you the membrane analogy for solid rectangular sections, square sections, triangular sections, elliptical sections. But, if you have thin box sections, how do, we consider the membrane analogy? Just to get back to membrane analogy, if you are recollect that, if you have a membrane analogy for this type of section. Then, in that case, the box, sealed box was like this.

And we had an opening here, that opening should correspond to the outer boundary of the section. So, in this case, our outer boundary is like this. So, we have the outer boundary. Now, in this case, since the section is not solid, since it is hollow. We consider a metallic plate sort of plate like this. Now, your membrane is tied, between the outer boundary and this metallic plate with constant uniform tension for boundary.

What I mean is that, we try to tie the membrane at the outer boundary and also to the boundary of this metallic plate. And there is tension per unit length of this boundary, which is constant. Now, if you try to apply, the box is now closed by the wall of the box and the membrane and this metallic plate. If you now increase the pressure, the membrane is going to deflect like this.

So, if we carefully look into it, it will have that scale. So, this is the outer boundary and the metallic plate is located there and the membrane has now deformed like this. So, this is central plate. You remember that, the membrane, the stress function phi is proportional to the deflection of the membrane and since, phi is to be 0 here; 0 at the outer boundary. So, we have zero deflection, all along the boundary and this phi is constant along the boundary.

It also due to the fact, that there cannot be any acting perpendicular to this boundary. So, this phi is constant along the boundary, because if there is going to be a space acting perpendicular to this boundary. Then, the slope along this boundary cannot be 0, but since phi is constant along this boundary the slope is 0, in this direction. Therefore, shear stress in this direction is equal to 0.

By the same reasoning, there cannot be shear stress perpendicular to this boundary, because enough surface of the component is not subjected in any stress. And therefore, there cannot be any perpendicular to this boundary. And if the shear stress cannot be present perpendicular to this boundary, the membrane deflection has got to be constant along this boundary. So, therefore, all along this boundary, internal boundary also, the membrane deflection has got to be constant.

So, while, these membrane is deflecting, we must ensure that, the deflection of the central plate is constant. So, there is a constant elevation of this central portion. And hence, all along the boundary of the membrane, we have the deflection constant. Hence, panels stress function is also constant. So, I repeat to ensure the shear stress to be 0, perpendicular to this boundary.

The membrane deflection has to be constant, and therefore we must ensure that the central portion is going to have constant deflection is a particular direction. You can imagine the situation, that think of a case, that you have the soap bubble attached to this outer cut out, and then you apply, you inflate. Then, you try to bring in one plate like, component to truncate the membrane or that soap film. And that is, what is the type of situation that we have trying to talk about roughly. It is not exactly, so but it is roughly like that.

So, you can understand now, that this is the membrane, which is, if you try to take a section along the vertical plane here, this is the membrane. And that would be shear stress is going to be constant. You see that slope, you can approximate this thing by a straight line and the shear stress is approximately constant. And you find the shear stress to be parallel to the boundary and it is a constant intensity or shear flow is constant along this direction.

Similarly, if you consider this direction, you will also find that, the intersection of the membrane with the vertical plane, passing through this straight line. Will also, have you will observe the deflection of the membrane to be like this. That you have this, and then this is the central plate and then of course, we have the deflection here. So, therefore, this is the central plate and this is the deflection of the membrane and this is the wall of the box.

So, this is the membrane, here also, you can see that, the slope is more or less constant and therefore, the shear flow will remain constant along this segment. That is, if the thickness of the wall is constant, you are going to have constant, shear stress coming up at each point of the boundary and it is going to be directed parallel to the point. If that section is of this type, then your outer cut out in a box will be like this.

And the central plate will have a shape conforming to the internal boundary of the triangular box section. And you make sure that, the membrane is attached to the outer and inner boundary, with a constant tension per unit length. And then, try to deflect the membrane making sure that the central portion has a constant deflection. Then, you will get the shape of the membrane.

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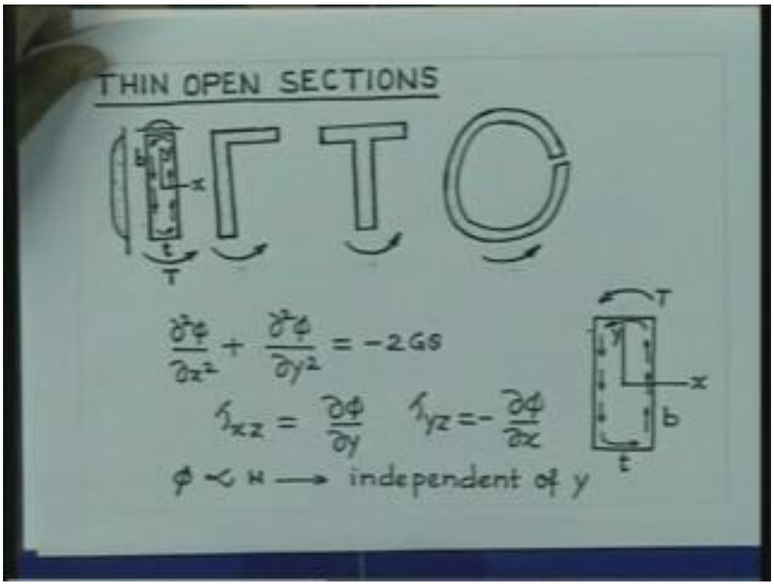

So, this is how, the membrane analogy can be applied to thin box sections. Now, after considering closed sections, let us look into thin open sections. What do you mean by this? You have sections like this, very slender rectangular sections like this or you have sections of the L shape or a T section of a circular section, but it is not closed. So, these are examples of open sections.

I would like you to consider, the membrane analogy for these sections. Think of the rectangular section. Let us consider that it is height is b and it is thickness is equal to t. The torque applied is equal to T. If we consider a membrane to be attached, then the membrane deflection will occur, if you look from this direction or if you consider the intersection of the vertical plane passing through the center.

Let us introduce the axis x and y. So, there are the axes. If you consider a vertical section containing the x axis, the intersection of the vertical plane with the membrane, will give us a curve like this. So, this is the membrane shape. Similarly, if you take a vertical plane passing thorough the y axis, then the intersection of that plane will be somewhat like this. So, this is the membrane. This is the shape of the membrane that you are going to get and here it is this.

It is also very clear that, these shear force or shear stress will be directed like this. Along this boundary, it will be directed like this, along this boundary, it will be directed like this. Similarly, along this boundary, it is going to be directed like this. So, it takes place in a current, in the same sense as the applied torque. If you see, if you forget about this portion here and portion there, since the deflection of the membrane in the center region is constant.

We can expect to get the same shape from this portion to somewhere this portion. So, approximately, if we neglect the two ages, the membrane is going to be constant and it is independent of the coordinate y. So, let us draw based to a larger scale. So, we have these directed like this. Looking into the membrane shape, what I was saying that, the shape of the membrane, over the portion, from here, somewhere there. If you look from this direction, it is going to look the same.

If you look into the equation, governing equation of the torsion, it was in this form delta 2 phi, delta x square plus delta 2 phi, delta y square is equal to minus 2 G theta. Wherein, we had already considered that tau x z is nothing but, delta phi, delta y and tau y z is equal to minus delta phi, delta x. If we just neglect the portion of the membrane, around top and bottom edges, what you find is that, this membrane shape is independent of y. So, phi, which is proportional to the membrane deflection and this is independent of y.

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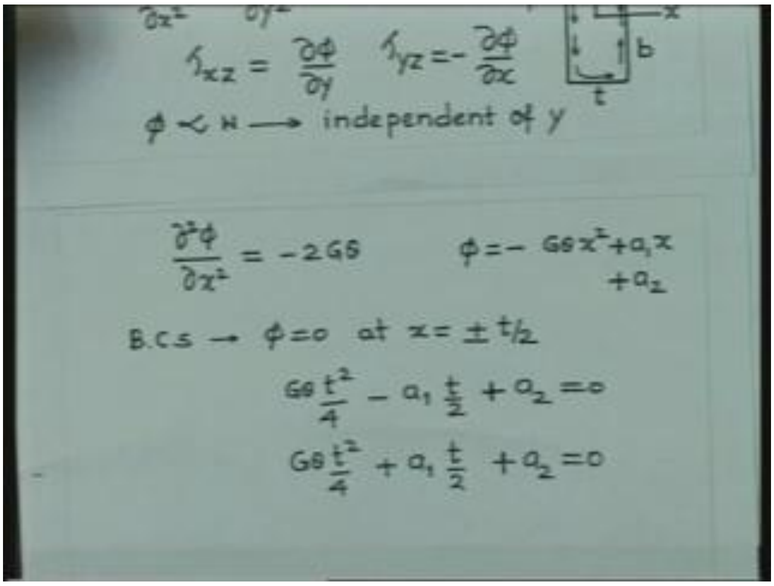

So, from this equation number 1, what you get is that delta 2 phi, delta x square is equal to minus 2 G theta. That is phi; it is just a function of x. So, you can now write, phi is equal to minus G theta x square plus some constant a 1 x plus another constant a 2. That is the form of phi. How to you get the two constants a 1 and a 2? We have the boundary conditions that phi is equal to 0 at this boundary, which is x is equal to minus t by 2 and it is also 0 at this boundary x is equal to plus t by 2.

So, therefore, what I am saying phi is equal to 0 here. So, also it is 0 at this boundary. So, you can introduce these conditions, boundary conditions are phi equal to 0 at x is equal to plus minus t by 2. So, if we substitute the values, what we have G theta t square by 4 minus a 1 t by 2 plus a 2 is equal to 0. Similarly, for x is equal to plus t by 2 plus a 1 t by 2 plus a 2 is equal to 0. So, we have two equations to solve for the two constants.

And once, you solve for it, you will find that, a 2 is equal to minus G theta, t square by 4 and a 1 is equal to 0. Substituting the values for the constants, what you find finally is that, phi is equal to G theta into t square by 4 minus x square. So, that is the variation of phi with x. We also know that, the toque capacity of the shaft is given by 2 times the integration of phi over the whole area.

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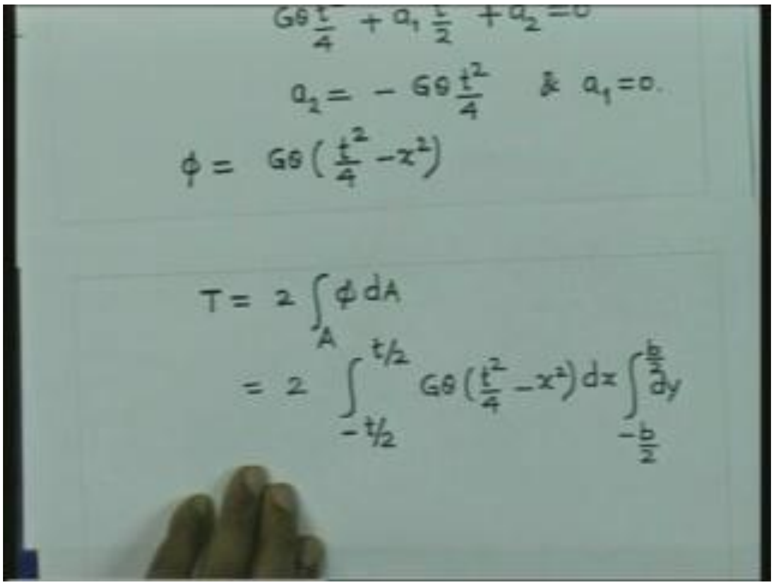

So, if we will consider that relationship, we have t equal to 2 times integration of phi over the area. And this therefore, means that, we have to do the integration from minus t by 2 to plus t by 2, G theta t square by 4 minus x square d x. And integration of y is minus b by 2 to plus b by 2, d y. Once you do this, we get this simplified form G theta b t cube by 3. That is t equal to G theta b t cube by 3.

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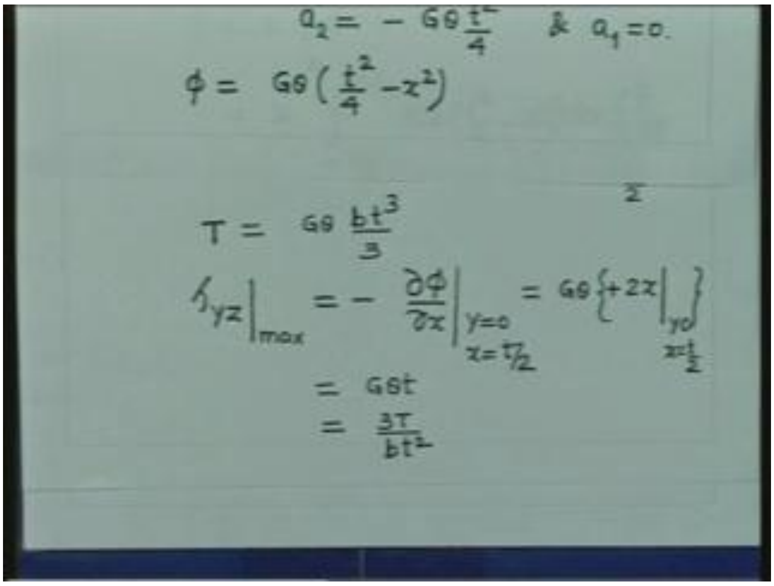

Now, if you calculate this stress. So, let us now calculate this stress at this point, at the intersection of x axis with the right boundary. So, that is nothing but, tau y z and it is going to be maximum stress. So, tau y z x is nothing but, delta phi delta x, at the point y equal to 0, x is equal to t by 2. So, this gives us since our form of phi is already known. So, if you do that, it is G theta minus 2 x. So, minus 2 x becomes plus and it is y equal to 0, x is equal to t by 2. So, if we do that, it gives us z theta into t.

So, the maximum shear stress is this much and since, we have z theta available in terms of torque, if we substitute value of G theta, you call to t 3 times b 2 cube. Then, it gives us 3 t by b t square. So, that is the maximum stress. So, this is how, you find that the problem of thin rectangular sections can be tackled. The angle of twist per unit length is related to the applied torque by the relationship t equal to G theta b t cube by 3. And the maximum shear stress which acts at the boundary is going to be 3 t by b t square. So, we have seen the rectangular section, that it is more or less. The space is constant along the longer, at the edge of the longer boundary.

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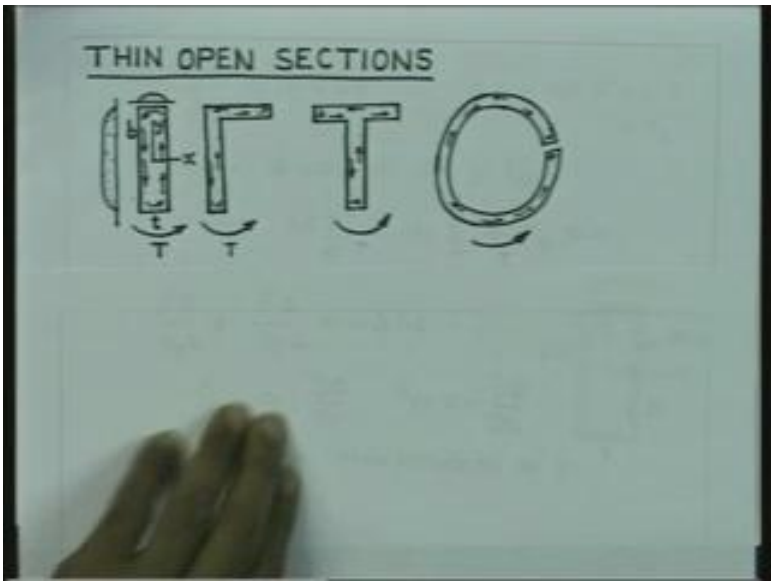

And it is given by G theta into t. And at the same time, the torque capacity is given by G theta into b t cube by 3. The same analysis can be applied to sections of this type or here or even these. If t is acting in a cross section like this, then the flows are going to be directed. This part of the boundary, it will be like this. Here, at the bottom, it will be directed in this manner. At the top edge, it will be directed here and on this edge, it will be directed like this and this will close the flow and it got to be directed like this. Similarly, here also, it is going to be directed like this. You can also consider similar picture here. That the flows are going to be directed at the edges like this.

For D section, if this is the torque, then at the outer boundary, it will revolve anti clockwise. And at the inner boundary, it is going to be clockwise. And of course, at the edges, it will try to close the loop. So, therefore, here it will be like this and at this edge, it will be like this. Neglecting this junction, the shape of the membrane over this portion, we can consider to be same as the shape of the membrane here.

Similarly, the shape of the membrane over this segment can be compared with the shape of the membrane here. And since, the stress is going to remain to remain constant at the edges, we can write the spaces in terms of the angel of twist per unit length and the thickness here. For this segment, also you can write the stress in terms of the angle of twist per unit length and the thickness here.

And the torque capacity for this portion can be written in terms of angle of twist per unit length and the length of the segment and the thickness. For this portion, also you can write in a similar fashion. Similarly, here, we can just consider this to be consisted of two portions. You can consider it to be consisted of this segment, rectangular segment and this rectangular segment and we can apply them in a membrane analogy.

And in this case, you can just develop it and you are going to have a strip of length approximately equal to the circumference's length. And it is a rectangular section like and hence, we can find out the capacity of this open section from the results that we have obtained there. So, we would like to illustrate this with examples.

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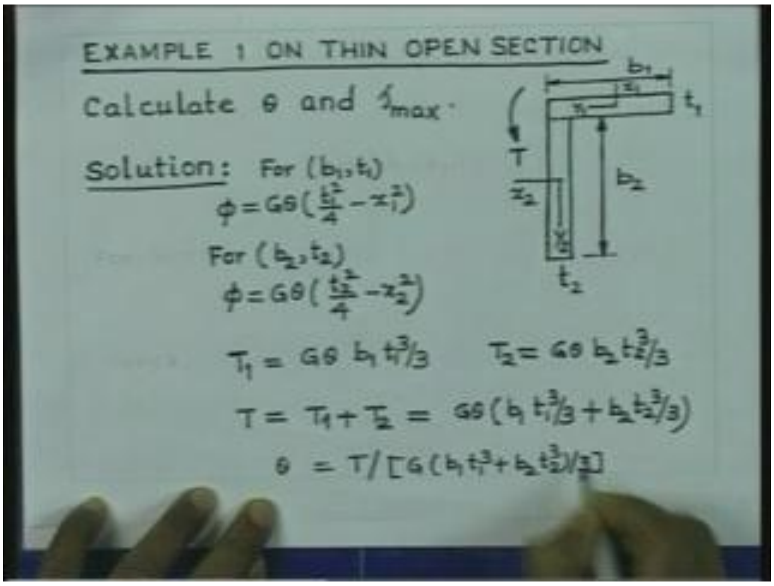

So, let us consider the examples. First, let us consider this example 1, on this type of section. So, here we have this portion of length equal to b 1, this is thickness t 1. And let us say, that the remaining portion is b 2 in height and t 2 is the thickness. If you try to consider the axis to be oriented for this segment 1, let us say that, x 1 is in a thickness direction, y 1 is in the height direction. Then, for this branch for b 1, t 1 we can write phi as G theta T 1 square by 4 minus x 1 square.

Similarly, for branch 2; b 2, t 2, we can write phi is equal to G theta t 2 square by 4 minus x 2 square, provided, you consider that the axes for the segment is like this. Now, if I apply the formula for torsion, for the portion b 1, t 1, torque T 1 is going to be equal to G theta, b 1, t 1 cube by 3. Similarly, for the portion 2; t 2 is going to be G theta b 2, t 2 cube by 3.

And hence, the total torque is equal to given by the sum of the 2 torques, T 1 and T 2 and that must be equal to G theta into b 1, t 1 cube by 3 plus b 2, t 2 cube by 3. So, that is the total torque. Now, in this case, what is asked for is calculate the theta and maximum, for the given torque T. So, we have got the relationship here, relating the angle of twist per unit length with torque. And therefore, your theta is given by t by G, b 1, t 1 cube plus b 2, t 2 cube by 3.

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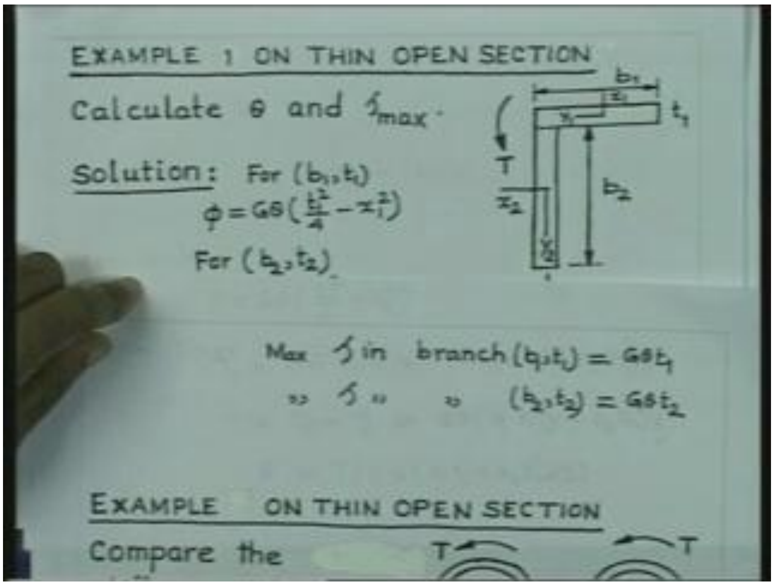

You can now calculate the maximum shear stress for the branch 1 is going to occur here and it will be simply z theta t 1. And for the branch 2, it is going to be shear, which is nothing but, G theta into t 2. So, therefore, for branch b 1, t 1, maximum shear stress, maximum tau. So, if you write maximum tau in branch b 1, t 1 it is nothing but, G theta into t 1.

Similarly, maximum tau in branch b 2, t 2, it is given by G theta into t 2. So, we have got both the angle of twist per unit and also the value of spaces in both the segments. And obviously, if t 2 is larger than t 1, you are going to have larger stress in branch number 2.

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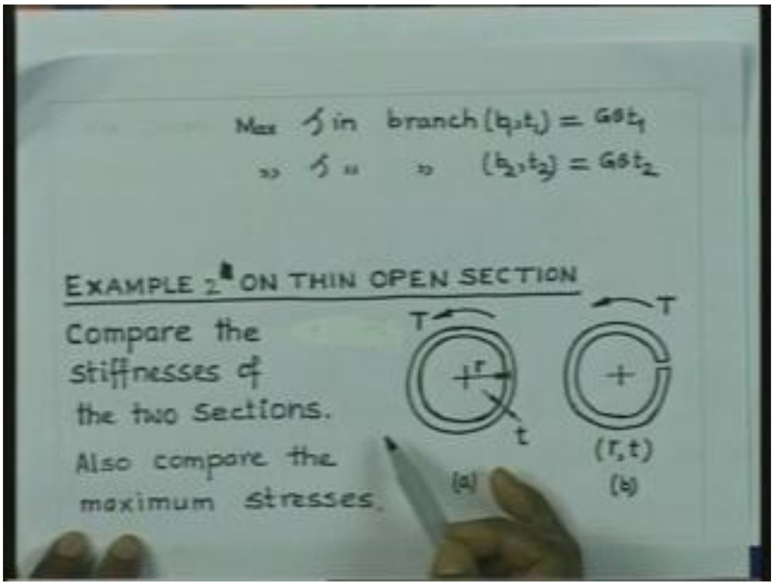

Let us consider another example. Let us now consider example number 2, wherein what we have? We have two sections. One is closed box sections of uniform thickness; it is circular in shape and another open section, the radius in both the cases are same. So, let us consider this case a and case b. So, we have thin circular sections here and in the other case, we have open circular section. So, what is needed is that, compare the stiffness's of the two sections and also compares the maximum stresses. So, we will try to solve this case. First of all, if you consider the closed section, we can apply the formula that you have derived in the strength of materials course, related to circular cross section.

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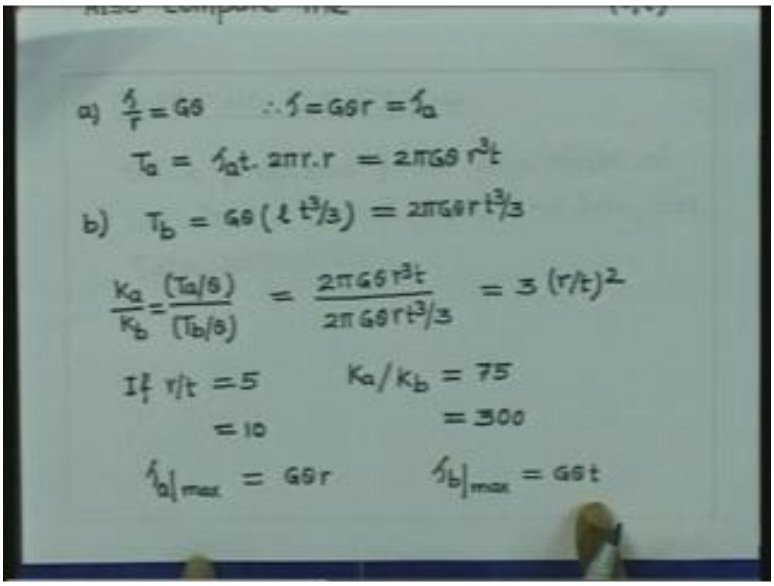

So, you can write for the section a tau by r is equal to G into theta, where theta is the angle of twist per unit length. And therefore, for this case tau is equal to G theta into r and let us say that, this is the shear stress maximum in the section or problem a. Now, T a, since it is a thin section, we can consider the shear stress to be uniform all along the thickness.

And therefore, shear flow is constant and shear flow is tau a into t and that is the force per unit of the boundary. And if you consider the whole boundary, which is 2 pi r. The total shear force is nothing but, 2 pi r into tau a into t. And if it take the moment of the forces about the center, we have to multiply by r. And therefore, this moment, that we get is nothing but, tau a into t. That is the shear flow multiplied by the total circumference l n that gives us total and it is acting at a distance of r.

So, therefore, that is the total moment and therefore, this is 2 pi G theta r cube into t. That is the torque in the case number 1. Now, let us go for the case number 2. For case number 2, we can calculate now T b. Now, we make use of the formula, that we have derived for the rectangular section that G theta b t q by 3. So, wherein, b is nothing but length of the whole circumference, which is nothing but 2 pi r.

So, you can write now G theta multiplied by l, l is the total length of the circumference. So, that is the total length of the circumference l t cube by 3. It is simple application of the formula that we have derived for the thin rectangular sections and that gives us 2 pi G theta r t cube by 3. What is stiffness? Stiffness is defined as torque per unit angle of twist. So, we can now write K a stiffness of the section a is given by T a by theta.

Similarly, K b is the stiffness in the case 2. It is given by T b by theta. So, that is the second case. So, if you now try the ratio of the 2, you will get these. And since, you know they value of T a by theta T b by theta, if you substitute that gives us 2 pi, G theta, r cube into t by 2 pi, G theta r t cube by 3. This gives us on simplification, 3 times r by t square.

Let us get the numbers. If r by t is equal to 5, then the stiffness ratio K a by K b is equal to 75 and if this ratio is 10, then this ratio is going to be 300. So, look at this, juts to, if we make that section open, the stiffness changes substantially. The stiffness of the closed section is much higher compared to the thickness of the open section. The second part, we have to calculate the shear stress in the two cases.

Now, if I consider the case 1, tau a, tau in case a maximum is equal to G theta into r and tau in the case b and the maximum value is equal to G theta into t. So, you can look into the magnitude order or magnitude of the stresses. If r is larger than t, you are going to get larger stress developed in the closed section compared to that in the hollow open section. So, higher spaces that we are going to develop and also you have seen that, the stiffness in the closed section is going to be much higher. So, the capability of the angle of twist or rather actually torque capability in the case of closed section is much higher. And also the stresses developed are going to be much higher.

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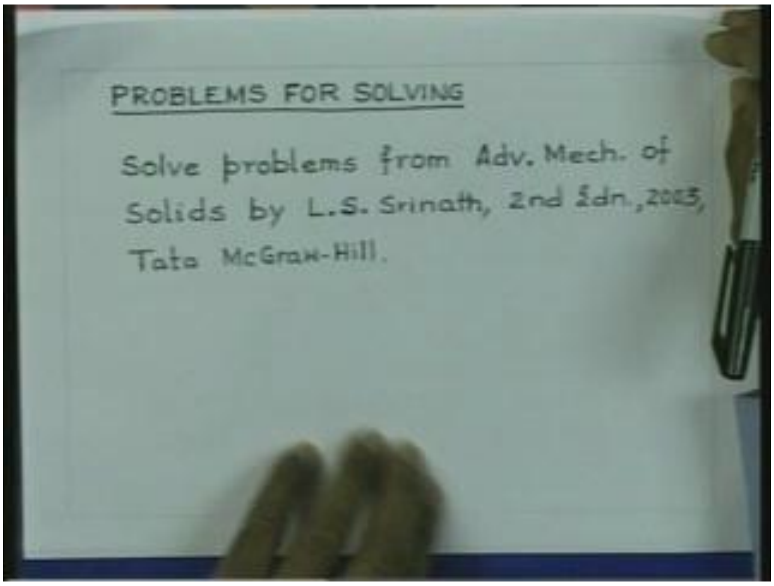

So, with this, we have illustrated, how the thin sections can be dealt with very easily. You can consider problems for solving to get clarity on the theory that we have dealt with in the torsion of non circular sections. You can look into the problems, which are not solved in the advance mechanics of solids by L. S. Srinath, second edition, which is published in 2003 by Tata McGraw-Hill.