Advanced Strength of Materials
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Lecture – 16

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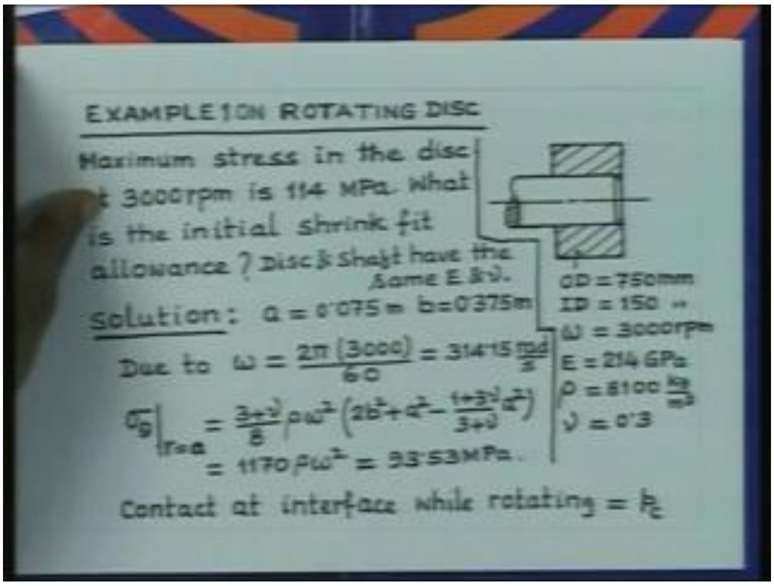

Now, we will consider examples on rotating disc. One problem is giving like this. One hub and shaft assembly is rotating at 3000 rpm for disc mounted on a shaft is rotating at 3000 rpm. And the maximum stress in the disc is 114 mega Pascal. So, the problem is to find out, what is the initial shrink fit allowance. The disc and the shaft have the same E and modulus of elasticity.

And these are the data we have to make use of outside diameter of the disc is 750. Inner diameter is 150 millimeter rpm is already I have given you 3000 rpm. The modulus of elasticity is 214 GPa. And specific weight is 8100 kg per meter cube Poisson’s ratio is 0.3. So, in this problem our first job is to find out, when the disc and the shaft assembly is rotating. We have to find out, what is the pressure at the interface?

This is something you must appreciate, that we mount the disc on the shaft with some initial interference. And as soon as we rotate the assembly, both the disc and the shaft are trying to expand in the radial direction. And the disc generally, would have more expansion than the shaft. And therefore, at the common interface you will find that.

Since, the expansion of the disc is more than that of the shaft the pressure is gradually relaxing. And therefore, at the particular rpm you will find that, the pressure that was there upper initial shrink fitting will not increase.

It will gradually reduce and if the speed of rotation of the unit is very high. It may happen, that day will become free from one another. So, therefore this sort of possibilities are there in this type of applications it is sometimes necessary also to find out. The critical speed at which, the disc will become loose of the shaft. But, in this particular problem, it is stated that the disc is still in touch with the shaft. And there is a contact pressure existing at the interface at the particular rpm.

So, the problem is to find out that pressure, which is existing at the rotational speed. Now, in order to find out the initial interference. We must be able to find out, the pressure which is existing at the rotational speed with 3000 rpm. So, let us get into doing this.

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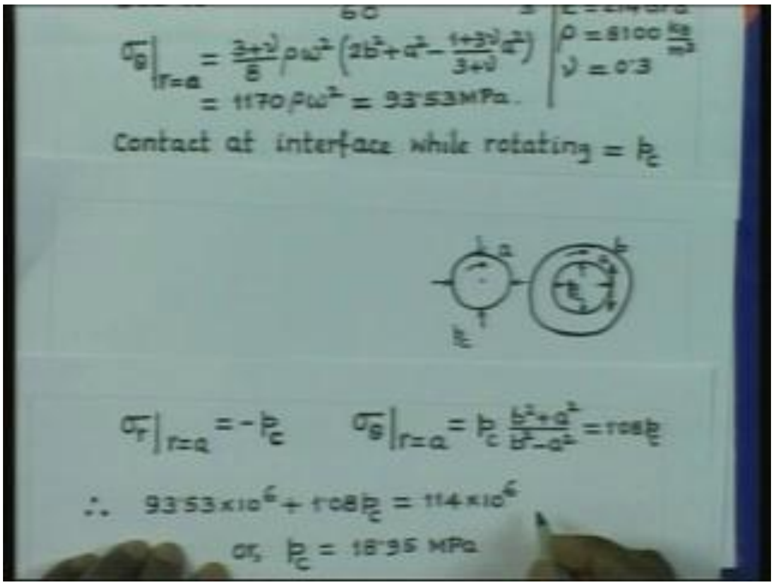

We can consider now, that the shaft let us consider the hub. So, at the rpm we have the picture like this. This is rotating and at the same time will have some contact pressure P acting on the, let us say P c acting on the shaft. Similarly, the disc is rotating plus at the same time, it is going to hub some contact pressure same P c acting at the inner surface. So, we can now find out the stress. Stress is going to be high in the case of disc only.

So therefore, we will calculate the stress at this point. And that will be due to the rotational speed plus also the internal pressure P c acting on the surface. So, let us calculate now. The solution, first of all let us find out the stresses due to rotation. So, if you ((Refer Time: 06:25)) see the angular speed it is going to be 2 pi n by 60, where n is the rpm and that is 3000, so it gives us 314.15 radian per second.

Now the radians says, the expression of the radians is 3 plus mu by 8 rho omega square into 2 b square plus a square minus 1 plus 3 nu by 3 plus nu a square. And that radial space is due to the rotation at the inner radius. So, this is a, this is also a. And this is b. So, therefore, ((Refer Time: 07:12)) we have the stress equal to r a. And it comes out to be 93.53 MPa.

We have made use of the value nu equal to 0.3, rho equal to 8100 and omega is equal to 314.15. And b is equal to 375 millimeter and a equal to 75 millimeters. Now, there is also the contact at P c the stresses due to contact pressure, we have also to calculate. So, if we calculate the stresses due to contact pressure. We will have this cylinder experiencing tensile stress in the circumferential direction. So therefore, you can have that from the Lami's formula. And therefore, it is given by this and it comes out to be 1.08 P c. And sigma r is obviously, minus P c.

Now, the sum total of the P c stress that is going to be in the tangential direction in the disc, is due to this pressure P c plus the stresses due to the rotation. And therefore, if you now consider the stress due to rotation is 93.53 MPa. Therefore, that is nothing but 93.53 into 10 to the power of 6 Pascal. And this, space due to the pressure is 1.08 P c and that is given as 114 into 10 to the power of 6 Pascal.

So, therefore, this is the equation to solve for the contact pressure that is there at the instant of rotation at 3000 rpm of the assembly. This is therefore, 18.95 is the contact pressure. So, we have now found that at 3000 rpm the contact pressure existing is equal to P c. And that P c is 18.95 mega Pascal. Now, we have to find out the initial interference. So, to consider this, let us consider the picture here.

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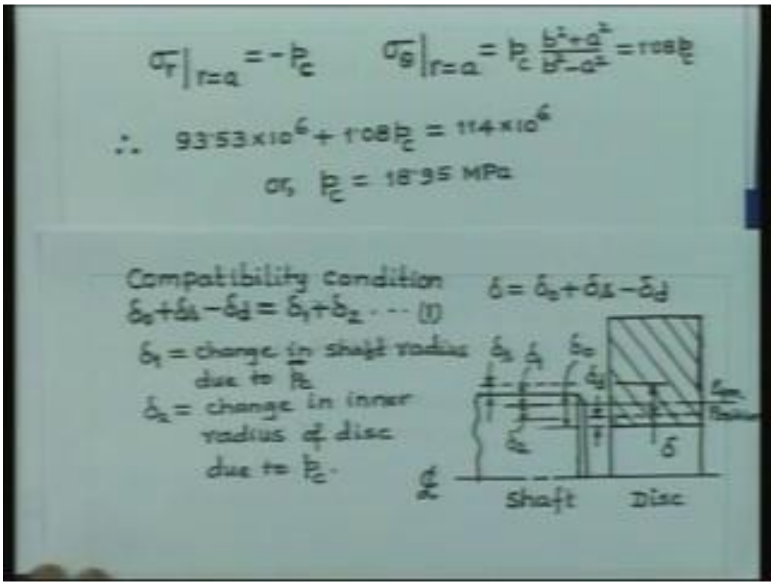

That will represent the disc to be shown like this and the shaft shown like this. Now, let us consider, that the common radius at which the shaft and the disc are going to be contacting that is giving by this distance. Therefore, this is the equilibrium position, this is the equilibrium position. That means, they will attain common radius of this value. I just want you to consider, that the initial interference. Let us, consider that this is the initial interference.

This is the outer radius of the shaft at no rotation. And no when they are free, they are not assembled together. And this is the inner radius of the hub when the two nodes are assembled together. So, therefore, we have the initial interference, giving by this value. So, therefore, this is the initial interference. Let us represent the thing by delta 0. Now, you just imagine that you are rotating, both the disc and the shaft at rpm 3000. So therefore, if you rotate the shaft at 3000 rpm.

It is going to have it is radius, outer radius increasing. So, therefore, let us say, that the outer radius will take up the position like this. So, this is the outer radius. And therefore, there is a change in the radius of the solid shaft, which is given by s delta. Similarly, the disc inner radius is also going to increase, the annexation of the rotation let us say and this is the movement of the inner radius. And hence, let us say that this is the increase in the radius of the disc.

So, what we find is that, when the two are rotating. Then, the gap between the two is given by this value. So, that is therefore, at the rotational speed that is giving here, that the interference that is going to be observed is nothing but this value. So therefore, the interference that we are going to see at the rotational speed is given by this magnitude. We can calculate this, that is nothing but it is delta 0 plus delta s minus delta d.

So, therefore, this delta is nothing but delta 0 plus delta s minus delta d. So, this gap between the two has been briefed by the contact pressure. So, therefore, we have a contact pressure acting at the rotational speed. On this surface of the shaft to decrease it is dimension and the same pressure is acting in the inner surface of the disc to increase its dimension.

So, let us now say, that the increase, that the decrease in the shaft radius equal to delta 1. Similarly, the increase in the inner radius of the disc is equal to delta 2. So therefore, this delta 1 plus delta 2 is nothing but this delta. So, you can now write the compatibility condition. So, we have the compatibility conditions delta 0 plus delta s minus delta d is equal to delta 1 plus delta 2. So, this is a very important equation, when the two components assembled together are rotating.

They would have the compatibility condition satisfied at the rotational speed like this, initial interference. Plus the expansion of the shaft on rotation, minus the expansion of the inner radius of the disc is nothing but the changes in the shaft radius under contact pressure plus the change in the inner radius of the disc under the same contact pressure. So therefore, this delta 1 is really, the change in shaft radius due to P c. And let us, consider it to be P c bar to distinguish between that. This is actually due to P c and delta 2 is equal to change in inner radius of disc due to P c. So, you can now calculate all these quantities to find out the initial interference delta 0. We will go into calculation of each of these slowly.

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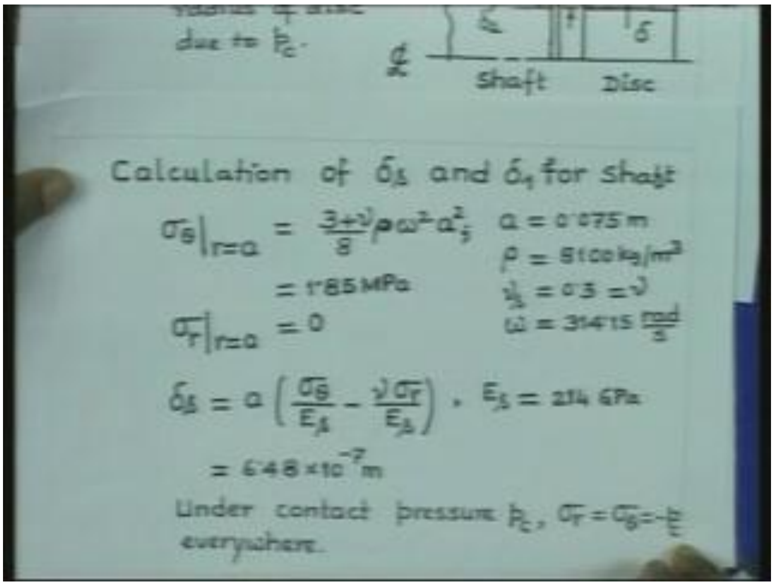

Let us, first calculate delta s and delta 1 for the shaft, delta s due to rotation. Now, shaft is a solid shaft and the maximum stress in the circumferential direction at it is radius a, is going to be 3 plus nu by 8 into rho omega square a square. Here a, is equal to 0.075 millimeter ((Refer Time: 18:16)) meter, rho equal to 8100 kg per meter cube. And the shaft per unit ratio is let us say, nu that is nu equal to 0.3 and rotation is 314.15 radian per second.

And the radian stays obviously, while it is rotating independently you can imagine, that this radial stress 0. So, now the delta s due to rotation is going to be simply given by a into sigma theta by E s minus rho into sigma r by E s. And since, sigma theta is equal to 1.85 MPa, we can calculate after substitution of E s as 214 GPa that delta s is equal to 6.48 into 10 to the power of minus 7 meter. So, that is the delta s.

Now, under contact pressure, under contact pressure P c you can now realize, we discussed in our earlier lecture, that the radial and the tangential stress everywhere is going to be equal to minus P c. So, it is really a state of hydrostatic compression, and therefore we know both these stresses.

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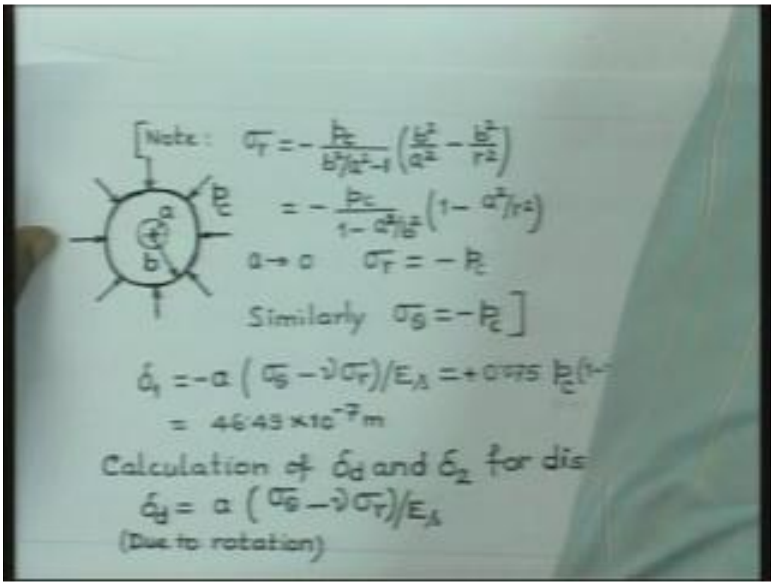

So, we can now calculate it is shown here. How this sigma r sigma n theta are going to be P c. Now, if you want to calculate the contraction of the radius of the shaft under the action of P c. We again try to go for calculation like this in the contraction is nothing but it is minus. You take the minus, because the contraction is negative therefore, it is minus a into sigma theta minus nu sigma r by E s. And P c we have obtained that, as 18.95 MPa.

And value of a is 0.075 meter and nu is 0.3 and E s is 214 GPa. So, if you substitute the value of all these, we get the change in the radius of the shaft as 46.49 into 10 to the power of minus 7 meter. You have to calculate now, the changes in the inner radius of the disc due to rotation, and also delta 2 of the disc under pressure P c. So, delta d due to rotation is nothing but a, into the strain in the circumferential direction, which is nothing but sigma theta minus nu by sigma r in whole divided by E s. So, the modulus is same as that of the shaft, that you have kept as it is…

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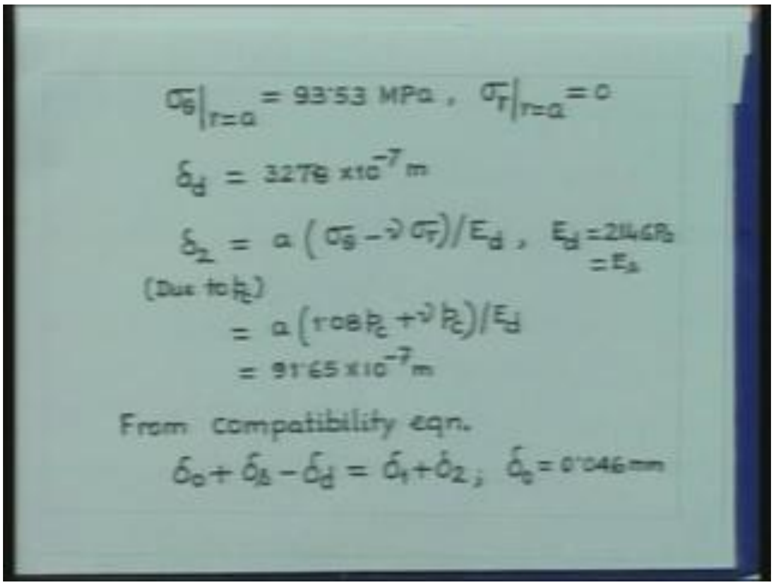

Once, you try to calculate we have already obtained the sigma theta space, due to rotation at r equal to 93.53 MPa earlier, sigma r at a equal to 0. Therefore, sigma d is equal to 327.8 into 10 to the power of minus 7 meter. And delta 2 due to P c is given by a into sigma theta minus nu into sigma r by E of the disc. And E of the disc is same as E of the shaft. So, that is equal to 214 GPa. And now, we have already seen that the ((Refer Time: 22:51)) is equal to 1.08 P c earlier and radial space is equal to minus P c.

So, therefore, we have substitution giving us like this. And this delta 2 on substitution on all the values it comes out to be 91.65 into 10 to the power of minus 7 meter. Now, we go back to the compatibility equation, that is delta 0 plus delta s minus delta d is equal to delta 1 plus delta 2. And after substitution of all the values, here delta 0 is unknown rest of the things are known. So, if you substitute the value, you get delta 0 at 0.46 millimeter. So, that is the radial interference at no rotation or even before assembly that was the interference. So, this is how, you have to solve these problems. Let us, consider another example.

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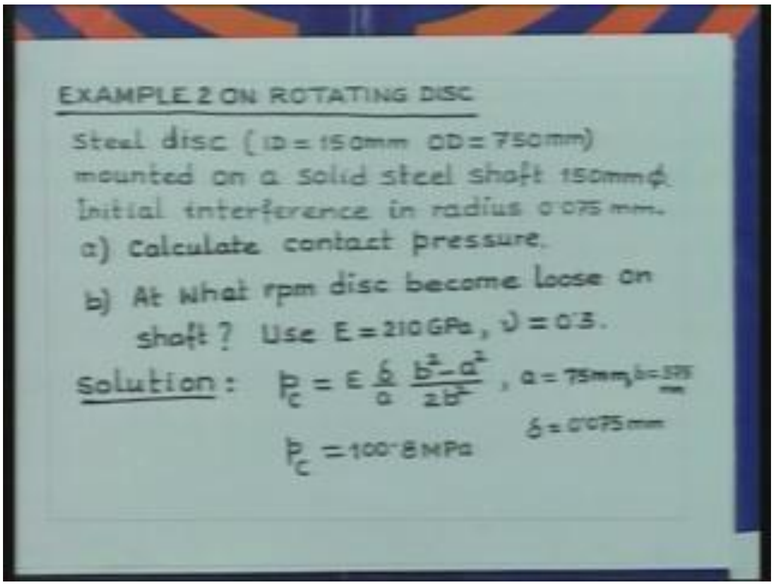

There is a steel disc of inner diameter 150 millimeter, outer diameter 750 millimeter mounted on a solid shaft of 150 millimeter. Initial interference in radius was 0.075 millimeter. Calculate the contact pressure. And it is required to find out the rpm, at which the disc become loose on the shaft. And the values of the material properties are given as E for both the shaft. And the disc as 210 GPa and nu equal to 0.3.

So, let us understand, what is really to be found out in this problem. So, you are bringing the initial interference. And you are required to find out the contact pressure that will develop after shrink fitting. And then you have to also find out the speed of the assembly at which the disc will become loose on the shaft. So, let us go into for solution.

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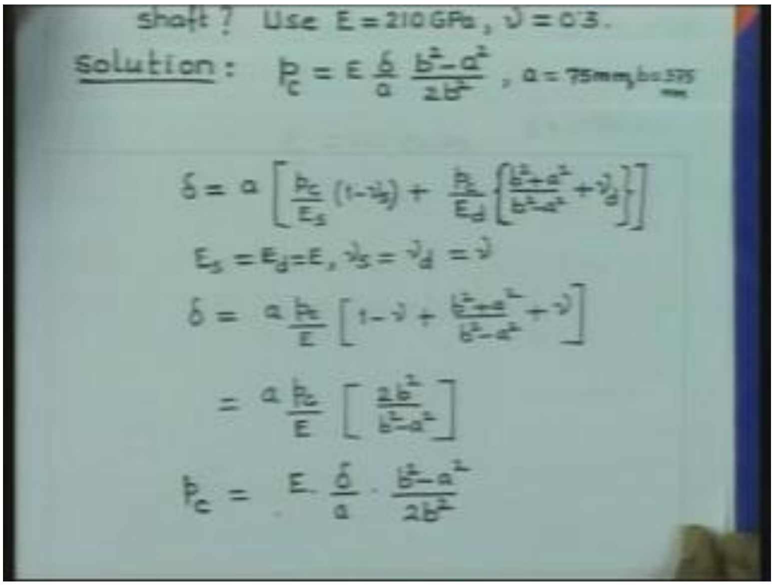

If you remember, we had earlier derived that interference delta was given by a P c by E s 1 minus nu s plus P c by E of the disc into b square plus a square minus divided by b square minus a square plus gamma into gamma of the disc, that was the relationship. So, the interference is given by delta is equal to common radius multiplied by the tangential strain in the outer radius of the shaft, where P c is the contact pressure. And this is the strain in the inner radius of the disc. Therefore, this is the expression that we get.

Now, this can be simplified we can now write, that when you have the same material. What we are giving here is that, E s is equal to E d and nu s is equal to nu d. Therefore, this delta is equal to and let us say that, these two moduli are equal to E. And this presence ratio are equal to let us say nu. Then we have a into P c by E 1 minus nu plus b square plus a square by b square minus a square plus nu, which is further reduces to 2 b square by b square a square.

Therefore, P c is equal to E delta by a, into b square minus a square 2 b square. So, that is the relationship for a situation of the type, that we have considered is nothing but P c is equal to E delta by a, into b square minus a square by 2 b square. So, that is the relationship we have made, use here giving a value of delta you can get the value of P c. So, we just ((Refer Slide Time: 30:05)) make use of this relationship here in a, is equal to 75 millimeter, b is equal to 375 millimeter and delta equal to 0.075 millimeter. So, once you substitute the value that P c is equal to 100.8 mega Pascal.

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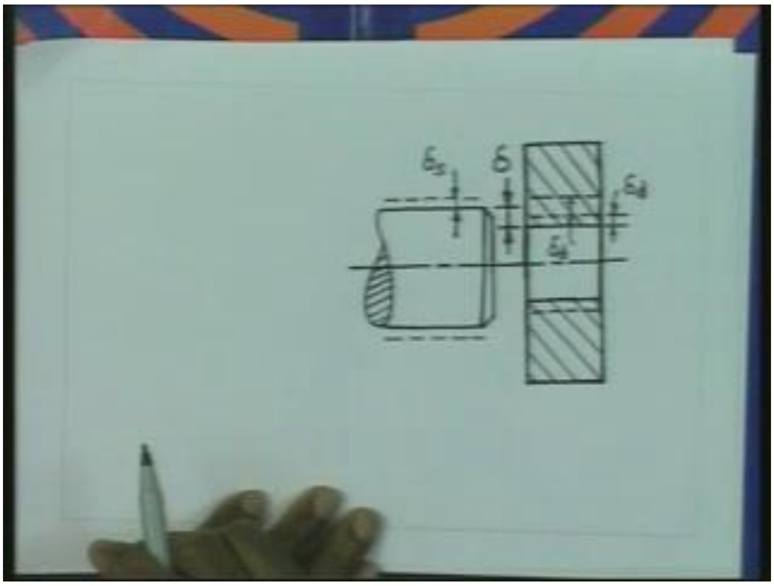

Now, we have to find out the speed, at which the shaft will become free from the disc or disc will become free ((Refer Slide Time: 30:41)) of the shaft. Now, you can recall the solution strategy of this earlier problem. So, here we have the initial interference between the disc and the shaft given by delta. When the two components are rotating independently, you can imagine that the inner radius of the disc is going to increase. And let us say, that increase in radius is equal to delta d.

Similarly, the shaft outer radius or the shaft radius will also undergo, going to undergo change. And that change let us say, that is equal to delta E s. So, how do you find out the rotation, which is going to make the disc free of the shaft. So, that is the ((Refer Slide Time: 32:22)) we have to look into. Imagine when the disc is going to be loose on the shaft. As the speed increases the shaft is also increasing in dimension. So, also the disc is increasing in dimension.

If the inner radius of the disc increases such that, it picks up the displacement equal to some of this interference plus the increase in the shaft radius. Then at that point, it is just above to be free on the shaft. So therefore, what I mean is that, if the extension of the inner radius of the disc has come to this level. That means, it has taken up the value of this magnitude delta d. Then you see, this two has picked up the same radial position. And therefore, the disc is about to be free on the shaft. So, this is the condition of disc becoming free.

So, what is needed is that, you are giving the initial interference. You calculate the increase in the radius of the shaft, due to rotation. Also calculate the increase in the inner radius of the disc due to rotation, wherein of course, the rotational speed is unknown. And once you see, that the increase in the inner radius of the disc is equal to initial interference plus the increase in the radius of the shaft that gives an equation involving the speed.

So, your speed is only the unknown and you have only one unknown. You have one equation, that delta d is equal to delta plus delta s. So, we would like to continue solving this problem. Let us now, see the stresses in shaft due to rotation.

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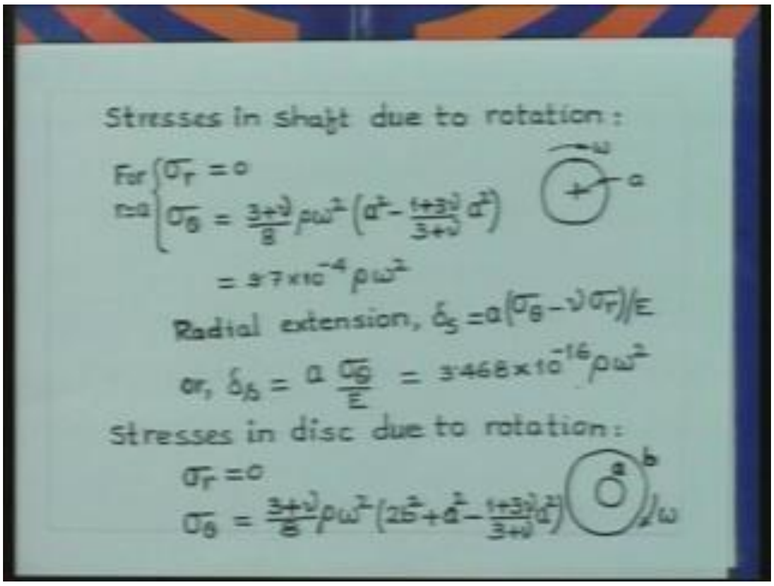

So, for r equal to a, we have sigma r due to rotation is 0, sigma theta due to rotation is going to be given by 3 plus nu by 8 rho omega square into a square minus 1 plus 3 nu a square. So, this is the solid disc formula, that we have made use of and this sigma theta. Once, you substitute the value of a and nu, rho we have tried to make use of rho equal to let us, we have not actually substituted rho here. So, we have kept the value of sigma theta here, sigma theta in terms of rho and omega. And that is 9.7 into 10 to the power minus 4 into rho omega square.

Now, the radial extension is going to be given by the delta s is nothing but a into the hook strain, which is sigma theta minus nu sigma r by E and since, sigma r E 0. Therefore, delta s is equal to a sigma theta by E. And this is 3.468 into minus 16 rho omega square. Similarly, the stresses in the disc due to rotation, if you consider the stresses for r equal to a again the radial stress is 0. And the tangential stress is going to be given by 3 plus nu by 8 rho omega square, 2 b square plus a square minus 1 plus 3 nu plus 3 plus nu a square.

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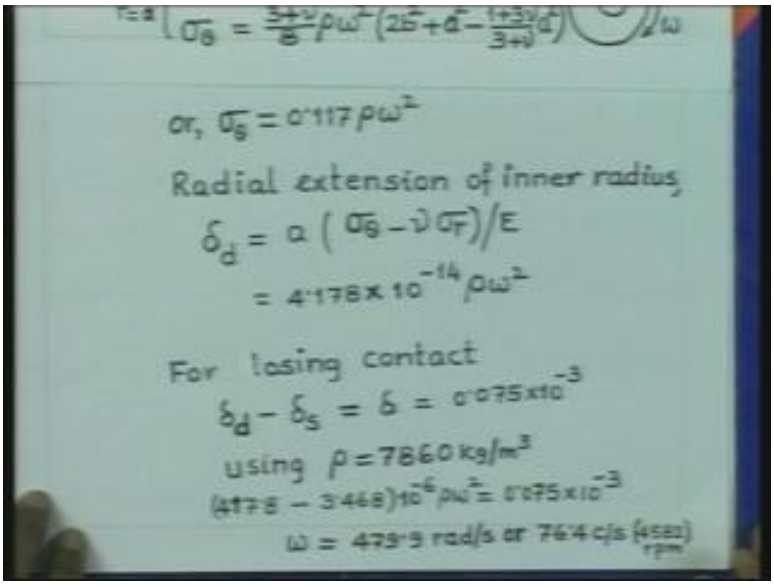

This gives us sigma theta is equal to 0.117 rho omega square. And therefore, the extension of the inner radius of the disc is given by delta d. And that is equal to a into the hook strain, which is sigma theta minus nu sigma r by E. And if you could see that the value of sigma theta. And then we also make use of this substitution then it is 4.178 into 10 to the power 14 rho omega square.

Now, condition for loosing contact is that the expansion of the disc minus the expansion of the shaft must be equal to the initial interference, which is given as 0.075 10 to the power minus 3 meter. If you make you use of the density equal to 7860 kg per meter cube. Then, we just substitute the value, which comes out to be this. And you can solve for omega from this relation it comes out to be 479.9 radian” per second, which is nothing but 76.4 cycle per second or 4582 revolutions per minute.

So, therefore, this is the speed, at which the disc is going to be loose on the shaft. So, this gives you ((Refer Time: 39:12)) good idea. How to solve the problem of rotating disc. Now, I want you to consider some problems for working on your own. First of all, I would like you to determine the thick solution under plane strain conditions. Just to give you some hints on it.

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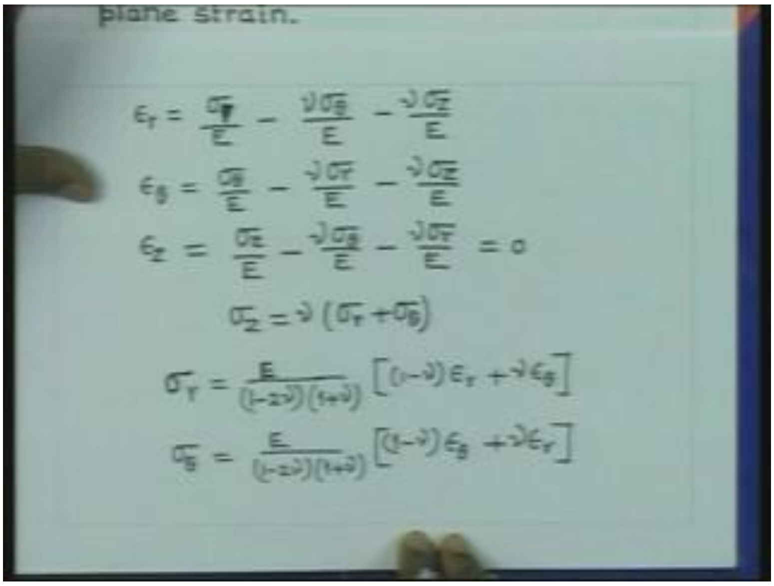

In the case, of the plane strain, we have the strain epsilon r is equal to sigma theta sigma r by modulus of the material minus nu times sigma theta material minus nu times sigma z by E of the material. Epsilon theta is equal to sigma theta by E minus nu times sigma r by E minus nu times sigma z by E. And epsilon z, epsilon z in this case is taken in the z is taken in the axial direction epsilon z is going to be sigma z by E minus nu times sigma theta by E minus nu times sigma r by E. So, these relations you can now put the constant that epsilon z is equal to 0.

In fact, when a long shaft is rotating, you can consider that each of the cross section remains plane. And it does not move or stay in the axial direction is 0. And therefore, you have this condition and hence we have sigma z is equal to nu times sigma r plus sigma theta. And once you make use of it. Then ((Refer Time: 41:57)) what you are going to find is that. You are going to have sigma r is equal to E by 1 minus 2 nu into 1 plus nu into 1 minus nu into epsilon r plus nu into epsilon theta.

Similarly, if you calculate sigma theta, which is going to be E by 1 minus 2 nu, 1 plus nu into 1 minus nu into epsilon theta plus nu times epsilon r. So, the stresses in terms of the strains are given here. Already in the case of cylinder, a thick cylinder we see that at the axis deformation is there. And epsilon r is given by d u d r. And epsilon theta is given by u by r. So, if you try to make use of those relationships in terms of displacements.

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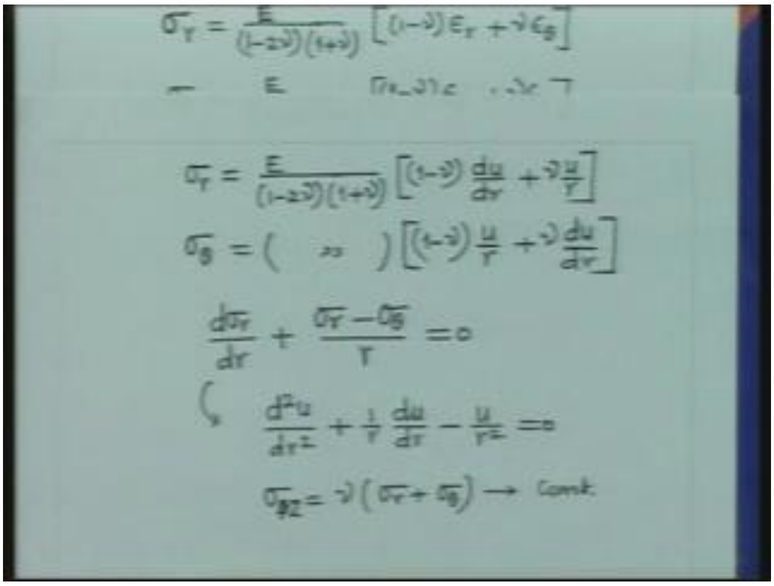

We have sigma r equal to E by 1 minus 2 nu into 1 plus nu, 1 minus nu epsilon r is d u d r plus nu times u by r, left sigma r. And sigma theta is equal to again this same constant. This is going to be 1 minus nu u by r plus nu times d u d r. So, these relationships are obtained for the stresses. And the governing equation for the thick cylinder is d sigma r d r plus sigma r minus sigma theta by r equal to 0.

So, you can now substitute these two stresses in terms of the displacement into this relationship. And you are going to get a governing equation, which will give rise to this will be converted. And you will find that this governing equation is finally, going to be of the form d 2 u d r square plus 1 by r d u by d r minus u by r square equal to 0. So, this will remain the same as in the case of thick cylinder on the plane stress and the solutions will remain unchanged.

And therefore, finally, by applying the boundary conditions, you can get the distribution of sigma theta and sigma r, which will remain the same as in the case, of plane stress. Only the difference you are going to see finally, that sigma theta is now going to be non zero. And it is given by sigma r plus sigma theta. And you have seen that, in the case of thick sigma r plus sigma theta remains same independent ((Refer Time: 45:51)).

And therefore, this sigma z is going to be related to some of the radial and tangential stresses like these. And this will remain a constant. And it does not vary along the radius. So, this problem you solve fully and you will have the better understanding of the whole derivation. Now, I would like you to consider similar problem. Let us, consider before that second problem.

What is the error in calculation of sigma theta max by thin cylinder formula for t by a less than 0.1. We have considered, that the thin cylinder formula, which is nothing but hook stress is equal to p r by t, where a is the radius.

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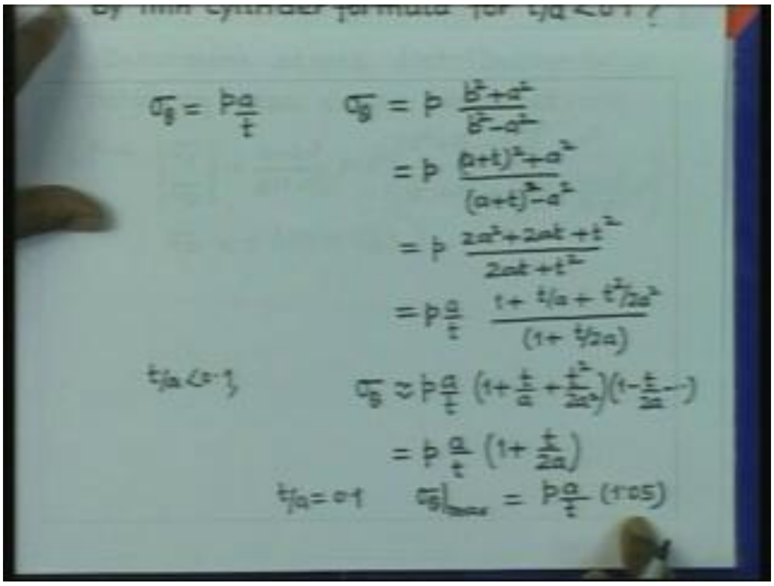

So, therefore, it should be hook stress is equal to, sigma theta is equal to p a by t, but if you consider it thick cylinder. The hook stress is maximum at the inner radius. And this is given by p into b square plus a square by b square minus a square. So, the maximum stress is going to be at the inner radius. And now, you can write this b as a plus t whole square. So, we will have this expression and now we have 2 a square plus 2 a t plus t square divided by 2 a t plus t square.

Let us take come on 2 a square from the numerator and 2 a t from the denominator. So, this will give us p a by t. And this one will turn out to be t by a plus t square by 2 a square. And the numerator is going to be 1 plus t by 2 a. Since, t by a, we are talking of t by a very small. We can expand these things binomially. And we can write now, this sigma theta is approximately equal to p a by t into 1 plus t by a plus t square by 2 a square into 1 minus t by 2 a. And the higher order terms must be there.

Now, if we just retain up to this much and then simplify this comes out to be equal to p a by t 1 plus t by 2 a. So, when you try to calculate the maximum stress for t by a equal to 0.1. Then we get this maximum stress, sigma theta maximum is equal to p a by t multiplied by 1.05. So, the maximum stress is first from the thin cylinder formula, which is this by just 5 percent.

So, if t by a, is less than 0.1, then in that case this error in the maximum stress would be less than 5 percent. That is why we make the restriction that the thin cylinder formula applies for t by a less than 0.1 thereby indicating that error is less than 5 percent. Now, the other problem I would like you to consider, determine the phase distribution in a rotating disc under plane strain. And the approach would be similar to that, the analysis of thick cylinder under plane strain. And you will find the answers for the plane strain problem for sigma r and sigma theta.

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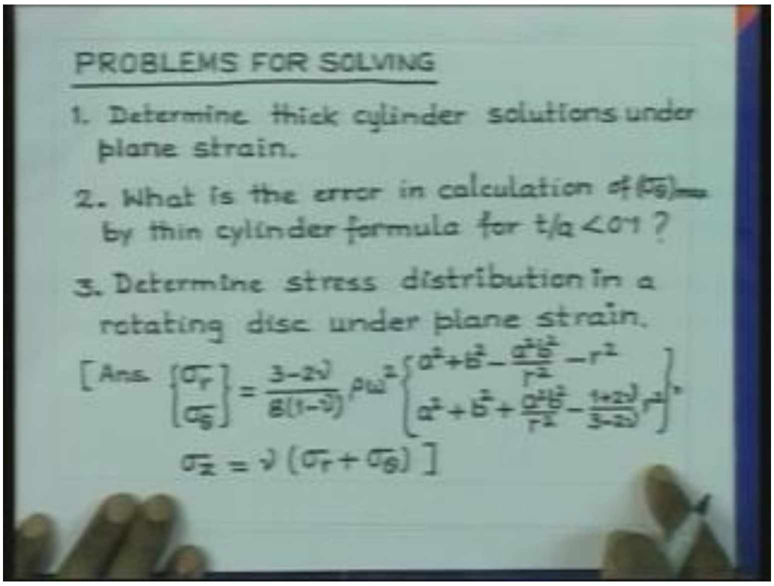

It is going to be 3 minus 2 nu by 8 into 1 minus nu rho omega square. And the expression for sigma r would be given by this. Expression sigma theta will be given by this one. There will be a difference here and the sigma z stress is going to be given by nu times sigma r plus sigma theta.