Advanced Strength of Materials
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Lecture - 14

You are now in a position to calculate the stresses in a thick cylinder. And you can also decide where the maximum stress is going to occur, and what is it is magnitude? We will, now consider some applications of this analysis to advantage. You can make cylinder consisting of one material spreading all over the thickness. We can also have the same cylinder made by fitting, one cylinder on to the other.

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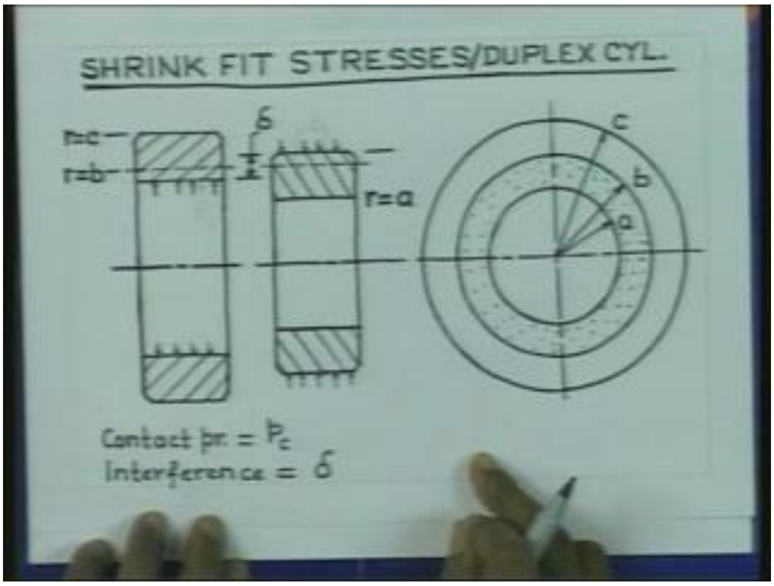

So, if we have such construction they are sometime called as duplex cylinder. And the assembly is done by shrink fitting. So, if you consider a duplex cylinder to be made up of two cylinders. Like this one of radii a and b, another of radii b and c. In this case, the inner cylinder has been fitted into the external one, to get this composite cylinder or duplex cylinder. Now in order to do that, we go for shrink fitting.

In shrink fitting, what is done is that, we start with the two cylinders which are like this. This is the outer cylinder. The inner radius of the outer cylinder is less than the outer radius of the inner cylinder. So, therefore these two radii they have the initial difference delta, which is known as interference. And the assembly is done. It can be done in two ways. Either, you go for cooling the internal cylinder and cool it.

So, that it can be easily inserted into the external cylinder. And then they have allowed the assembly to come to the normal temperature and you get the assembly. And while, this inner cylinder is coming to the room temperature it is beginning to expand. Therefore, it will try to induce some pressure at the interface. And that will try to expand the outer cylinder. And hence, this inner radius of the outer cylinder will get expanded.

And since, the expansion of the outer radius of the inner cylinder cannot take place freely. The pressure which developed at interface is going to also compress the inner cylinder. And in the process, the inner cylinder is going to get contracted by some distance. And they will finally, attain a common internal radius. So, there will be some radial pressure developed at the interface.

Let us say that this is how the radial pressure is going to develop at the interface. So, under the action of the radial pressure, this is going to expand. And the same radial pressure is going to cause, the contraction of the inner cylinder. Finally, they will attain a common radius from here to there, and the assembly will be made. Now, although I have shown the difference between the two radii to be substantially large, it is not so… It is very small, it is exaggerated here.

And the common radius that they attain, we can just represent to be. We can represent that dimension by b. Now, the stresses that develop in this assembly, it is very important to find out them. And after, the assembly is made, if the cylinder is made use of to store some to be at high pressure. Then, in that case the stresses are going to develop due to fluid pressure. And hence, the total stress that is going to come up in the assembly.

It is going to be the sum of the stresses due to fluid pressure and also the stresses due to shrink fitting. Now, I said that the assembly can be done in two ways. The other way is that, you can allow the external cylinder to expand by heating. And after, it has got expanded. It can be pushed over the inner cylinder. So, that on cooling they will come to room temperature and again the assembly can be made.

In either case, you are going to have contact pressure developing at the interface. And that contact pressure is going to expand the inner radius of the outer cylinder. And it is going to contract the outer radius of the inner cylinder. So, it is important to calculate these stresses. So, first we like to calculate these shrink fit stresses. Now, let us consider that the contact pressure that develops is equal to p c. And the final radius that the common interface picks up is going to be given by b. The initial interference is given by delta. So, let us say that this is the contact pressure interference is delta. In fact, this is radial interference. Now, we can consider the loading of the inner cylinder first.

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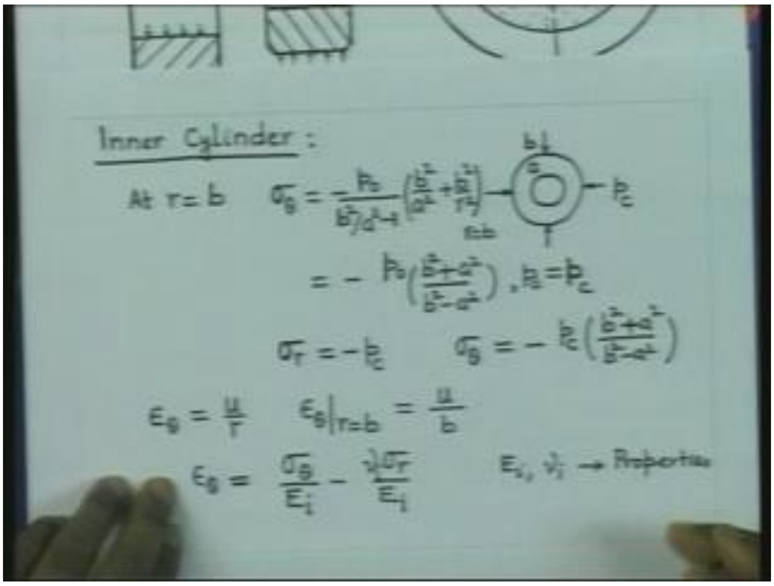

So, let us consider inner cylinder. So, the inner cylinder is subjected to pressure at its outer boundary. So, we have the inner cylinder with radii a and b. Now, in the assembly the inner cylinder is subjected to contact pressure p c. Therefore, we can find out the stress in the inner cylinder. Particularly, at r equal to b. We have this stresses sigma theta and that is given by the formula. Just now or the formula that, we have derived in the earlier lecture.

It is nothing but p 0 minus p 0 by b square by a square minus 1 b square by a square plus b square by r square. So, this of course we have to calculate for r equal to b. So, that gives us p 0 by b square by a square minus 1 into b square by a square plus 1. So, if you simplify this comes out to be b square p 0 into b square plus a square by b square minus a square. So, that is the value of the hoop stress.

Similarly, if we calculate the hoop stress I beg your pardon. Radial stress at the outer wall that is straight away minus p c, this would be pc. So, therefore this p 0 is equal to pc. So, p 0 is nothing but, p c here. So, at the end what we have is that sigma r is p c. And sigma theta is equal to p c b square plus a square divided by b square minus a square. Now, let us try to see what is the contraction of the inner contraction of the outer radius of the inner cylinder?

So, therefore to calculate that we can now consider the hoop strain, that is given by u theta equal to u by r. So, if epsilon theta is to be calculated at r equal to b. Then, that is going to be given by u divided by b. And by using the Hooke's law, we can write that epsilon theta is nothing but sigma theta by E minus nu times sigma r by E. Now, let us consider that in this case, the properties of the materials are different. So, therefore let us consider that this is modulus is given by E i. Here Poisson's ratio is given by nu i for the internal cylinder. So, E i nu i are the properties. Now, let us substitute the value.

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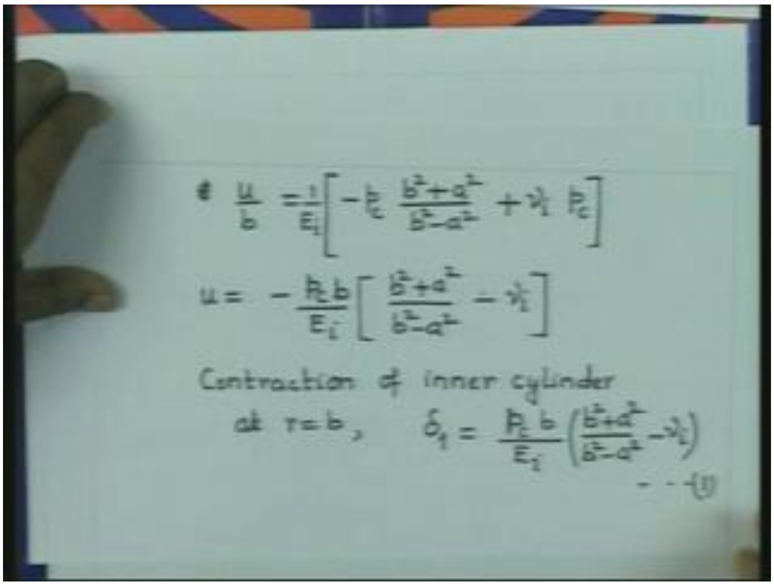

Therefore, u by b is equal to sigma theta already we have the value as r equal to b. So, we can substitute the value. So, we will have 1 by E i common here. And then in that substitution of sigma theta will give us minus p c b square plus a square by b square minus a square minus nu i into sigma r, sigma r is minus p c. So therefore, it will be nu i multiplied by p c. So, that gives us the value of the strain at r equal to b.

And if we calculate u, u is going to be given by minus p c by E i into b square plus a square by b square minus a square minus nu i. So, that multiplied by b. So, if we multiply by b that gives us the value of the displacement of the outer wall of the inner cylinder. Now, in this case you see that nu is always less than 1. And this ratio is greater than one. So, therefore, this value is going to be positive.

And here in all the quantities are positive. So, therefore we have a displacement which is negative. That means, it indicate that the outer wall is going to move inwards. So, there will be contraction. So, therefore the contraction of inner cylinder at outer radius r equal to b. Let us say that, that is delta 1 is equal to p c b by E i into b square plus a square by b square minus a square minus nu i. So, this is let us says 1. Now, we can calculate the deformation of the outer cylinder in the similar manner.

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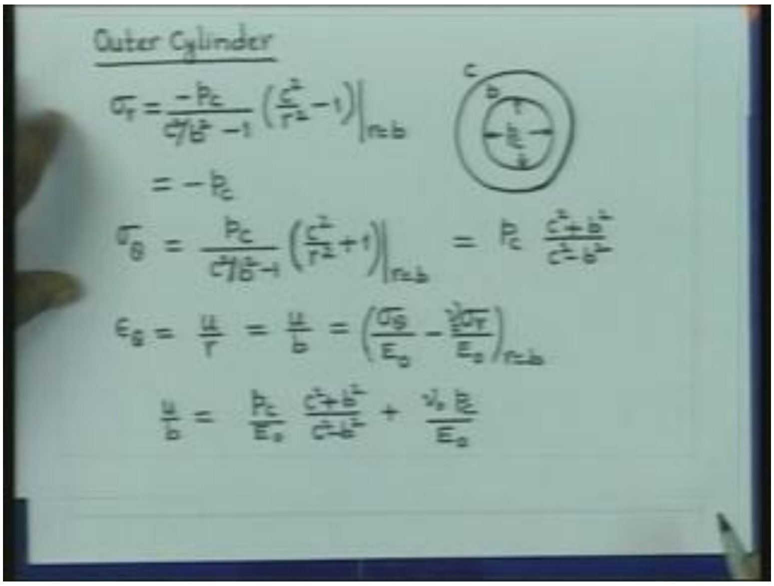

So, if we consider the outer cylinder now. So, you can now represent the outer cylinder to be loaded by a pressure loading at its inner boundary. So, therefore we have the outer cylinder. So, this is the outer cylinder. It is inner radius let us represent approximately by b and its outer radius by c. And this is loaded by p c the contact pressure at it is inner boundary. So, therefore this is again pc. Now, under the action of this pressure p c, the inner radius of the cylinder is going to expand. So, we will like to calculate that expansion.

First of all, let us calculate the stresses at the inner boundary of the outer cylinder. We have sigma r stress. Sigma r in this case, it is going to be given by this relationship. So, P c divided by c square by b square minus 1 c square by r square minus 1. And this is at r equal to b. So, if I do that there is a negative sign here. So, therefore sigma r if I put r equal to b, that gives us minus p c.

Obviously, we see directly that sigma r at r equal to b is nothing but, minus p c. Similarly, sigma theta is equal to p c divided by c square by b square minus 1 c square by r square plus 1. And if we put r equal to b, this will give us p c c square plus b square divided by c square minus b square. So, that is the stress at the inner wall of the outer cylinder.

If we want to calculate the change in inner radius then again we calculate from hoop strain which is given by u by r. And therefore, at r equal to b will have u by b is the strain. In term this is also given by the hoop strain in terms of the stresses, which is nothing but sigma theta by modulus of elasticity of the material. So, if we consider the modulus of elasticity of this outer cylinder to be given by E 0.

So, sigma theta by E 0 minus nu times sigma r by E 0. So, this nu again we will like to have suffix 0, to indicate the outer cylinder. That is to be calculated for r equal to b. Since, we have the stresses at r equal to b. Then, we can directly get u by b is equal to p c E 0 c square plus b square by c square minus b square minus nu 0 p c. Since the radial stress is negative. So, this will become positive. So, therefore it is nu 0 p c by E 0.

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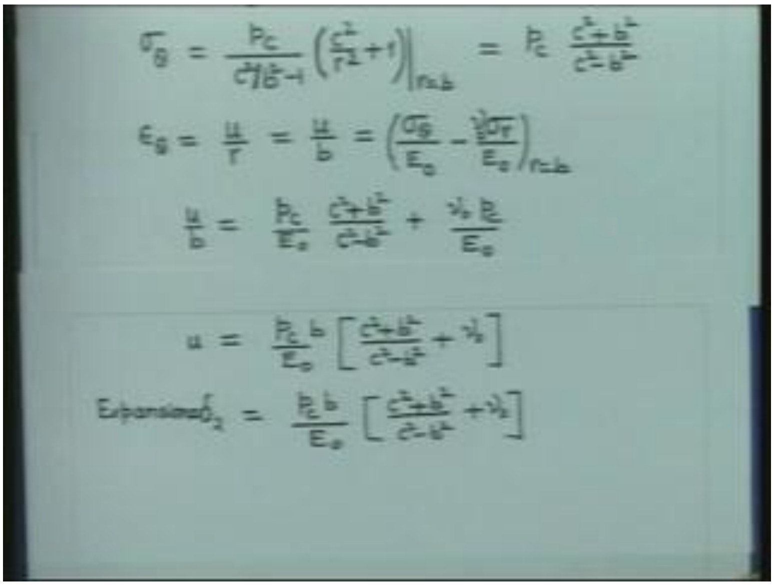

Finally, we get the radial displacement u at r equal to b given by p c by E 0 into b multiplied by c square plus b square divided by c square minus b square, nu 0 plus nu 0. So, that is the value of the displacement. Here in, you find that this is positive quantity greater than 1, this is positive quantity and these are all positive. So, therefore u is positive.

Thereby, indicating that the inner radius of the outer cylinder is going to get expanded and if we represent this expansion by delta 2. So, this expansion is equal to let us say delta 2. And that is given by p c b by E 0 into c square plus b square c square minus b square plus nu 0. So, we will like to write this equation as equation number 2. So, we have got now the expression for the contraction of the inner cylinder and the expansion for the outer cylinder.

So, in this problem while you are trying to do the shrink fitting it is necessary to find out the contact pressure or common pressure for a given value of this interference. So, how do we get it? So, let us now look into how we can get this value of the pressure given a particular value of this interference.

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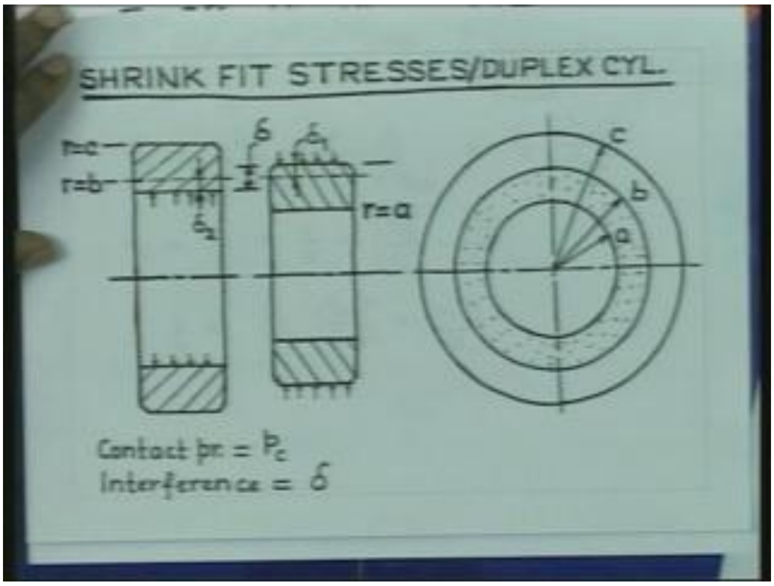

So, this diagram it shows that under the action of the contact pressure, the outer wall of the inner cylinder is going to get contracted. And it is going to move by a distance delta 1. This is delta 1. Similarly, under the action of the same contact pressure, the inner wall of the outer cylinder is going to get expanded by this amount delta 2. So, therefore the initial interference if the two cylinders attain a common radius likes this.

Then obviously, the sum of the contraction and expansion of the two cylinders must be equal to the interference. So, this is the compatibility condition here, that the contraction of the inner cylinder plus the expansion of the outer cylinder is nothing but, the initial interference. So, that gives the relationship to calculate the contact pressure. So, we can now substitute the values. First of all, let us write the compatibility condition.

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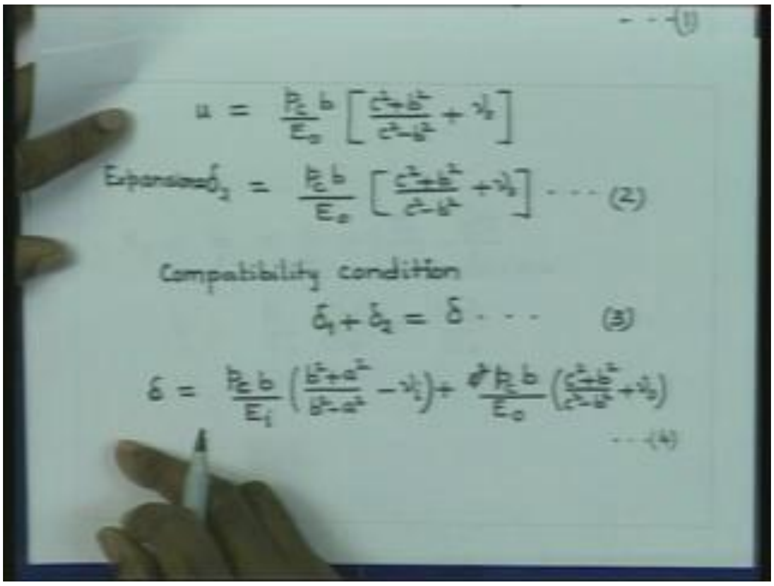

Compatibility condition delta 1 plus delta 2 is equal to delta. We have already seen that delta 1 and delta 2. They are functions of contact pressure and the dimensions of the cylinder plus the material constants. And we know, all the quantities accepting this contact pressure. So, therefore by substituting in this relationship we get an equation involving the interference and the contact pressures. So, therefore this gives us.

So, the final equation is delta equal to p c into b by E i into b square plus a square divided by b square minus a square minus nu i plus p c into b by E 0 into c i square plus b square divided by c i square minus b square plus nu 0. So, this is the equation wherein only unknown is p c. Therefore, given the value of this interference one can calculate the contact pressure. And then it is possible to calculate the stresses developed in the duplex cylinder.

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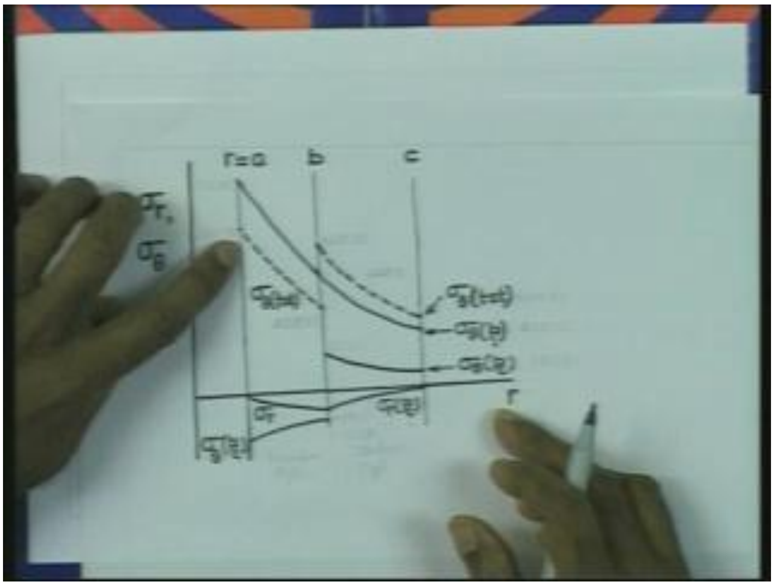

So, once this contact pressure is calculated, it is very easy to calculate the distribution of stresses in the duplex cylinder in the assembly. So, we would like to show those stresses. The radii, let us say that a is the internal radius b is the common radius c is the external radius as we have considered the symbol earlier. The internal cylinder is going to be subjected to external pressure loading.

So, therefore we are going to get the stresses developed due to the interference. They are going to be same as in the case of a externally pressurized cylinder. So, therefore we will have the variation of the stresses. This is the variation of sigma r and the variation of hoop stress is going to look like this. Therefore, this is sigma theta due to contact pressure. This is sigma r. Similarly, the outer cylinder is going to be subjected to internal pressure.

Therefore, we are going to get the hoop stresses given by this variation. So, therefore this is the variation of sigma theta due to contact pressure. And the radial stress is going to have a variation of this sort. So, this is sigma r due to contact pressure p c. So, this is how the stresses are going to develop in the inner and outer cylinder. Please note that, the stresses that develop at the internal radius of the duplex cylinder is going to be compressive.

And it is going to be tensile at the common radius, when you consider the outer cylinder. So, therefore this compressive stress is very important. You must note this point that, you are going to have compressive stress at the inner radius of a duplex cylinder. So, when you have a shrink fitting you are going to get compressive stress developed in this position. And that is quiet understandable, since you are developing some external loading of the inner cylinder and it is compressive.

Therefore, you are going to have compressive stress in the circumferential direction here. On the other hand, since this same contact pressure is trying to give internal pressure to the outer cylinder. We are going to have tensile stresses in the inner radius of the outer cylinder. Now, these stresses can be, it can be designed to lead to some advantages. Particularly, when you have this duplex construction you can now have the internal pressure internal fluid stored in the cylinder.

And that internal fluid is under pressure. Internal fluid under pressure is going to give tensile stresses in the inner wall of the whole cylinder. And since, there is initially some compressive stress. The net stress in the inner wall is going to reduce. So, therefore by duplexing this cylinder you can reduce the total stress acting at the inner wall of the duplex cylinder due to external loading. And hence the pressure capacity can be increased.

I would like to repeat, that if you have this duplex construction you are in a position to develop some initial stresses due to interference. And that initial stress, particularly in the circumferential direction is going to be compressive in nature. And when this particular assembly is made use of, to store some fluid under internal pressure, that internal fluid pressure will give rise to tensile stress at the inner wall.

Because of the initial stresses, you are going to get some reduction in the stress due to internal fluid pressure. And thereby, the pressure capacity of the cylinder increases. You would like to illustrate this point by considering one example. Let these cylinders be subjected to some internal pressure. In order to calculate the stresses due to internal pressure, it becomes little simple.

If we consider that these two cylinders are made up of the same material. Then, in that case if the internal pressure is given. Then, I can calculate the stresses considering the two radii to span from a to c. And you can calculate the stress distribution particularly hoop stress by considering a internally pressurized cylinder. And the variation of the stress would be sigma r will vary minus p i divided by c square by b square minus 1 into c square by r square minus 1.

And sigma theta is going to be p i divided by c square by b square minus 1 into c square by r square plus 1. And if you substitute the value of the two radii and plot the variation of stresses, it can give you a typical variation of this type. We will have the highest hoop stress at this point. And it is going to have a continuous variation like this. So, therefore this is nothing but sigma theta due to internal pressure p i.

And now, if we find the resultant stresses sigma theta due to interference is given by this. Since these are compressive stresses, you will find that this maximum is going to get reduced by this one. So, therefore we will have it reduced there from this magnitude. Similarly, there is a reduction by this magnitude at the outer wall. So, therefore it will come down to this value.

And hence, the variation of the sigma theta in the inner cylinder is going to be given in this form. So, this is the sigma theta total. Similarly, if you consider the outer cylinder the interference stresses are given by this. They are positive stresses. So, therefore the magnitude of the stress at this inner radius is this much. So, the stress here is going to increase. Similarly, here also the stress is going to add up and it will be increasing.

So, finally you will find that the variation of the hoop stress in the outer cylinder is going to look like this. So, this is sigma theta total for the outer cylinder. So, therefore the final hoop stress variation will be going through is given by this dotted line. See that, the magnitude of the hoop stress in the cylinder has been reduced from this value to this value. And therefore, we have reduction in the maximum stress and it is to advantage.

And hence, it will increase the capacity of the cylinder. And we have also increased the level of stresses in the outer cylinder. So, therefore here the stressing of the material has become little more uniform. Now, I would like to illustrate this by considering a specific example.

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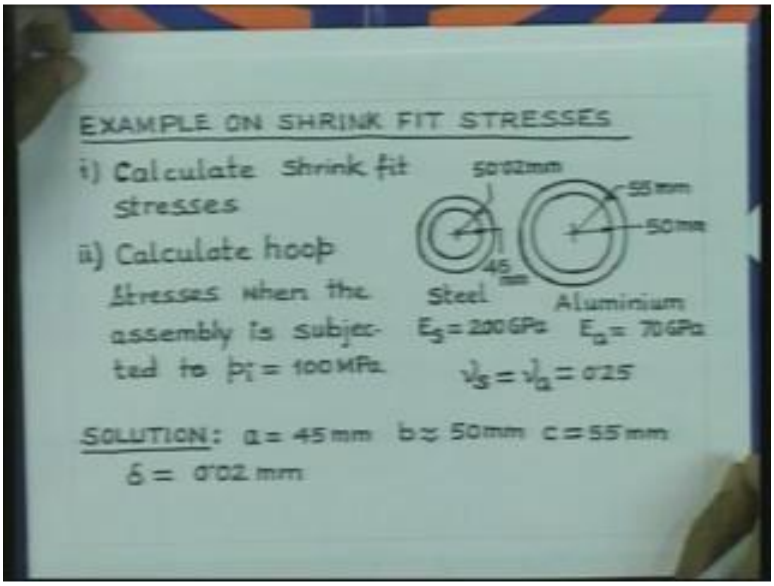

Let us consider this example. We have a steel cylinder of radii 45 millimeter and 50.02 millimeter. And we have aluminum cylinder whose radii are 50 millimeter and 55 millimeter. Now, these two cylinders are assembled together to make a duplex construction. The steel properties are given by modulus 200 GPa and its Poisson's ratio is 0.25. So, we would like to represent the modulus by E s and the Poisson's ratio by nu s.

Similarly, for aluminium the modulus is E a and that has a magnitude of 70 GPa. And in this case, the Poisson's ratio of aluminium is same as that of steel and it is given by 0.25. Now, the problem is stated to us calculate the shrink fit stresses. Calculate the hoop stresses, when the assembly is subjected to a pressure p i. In solving this problem, we proceed like this. We just assume that the common radius after assembly is going to have a value approximately equal to 50 millimeter.

And a according to our symbol, that we allocated earlier a is equal to 45 millimeter, c is equal to 55 millimeter. And the radial interference, you see that radial interference is nothing but this 50.02 minus 50. So, therefore radial interference is equal to 0.02 millimeter. So, if we recall the relationship that we have obtained, relating the interference contact pressure module and the dimensions.

So, we have this relationship. Here in we can substitute the values. Noting that this E i is nothing but E s nu i is nothing but, nu s. And E 0 is nothing but, E a nu 0 is nothing but nu a. So, if you substitute the value. So, we have after symbols that are shown here.

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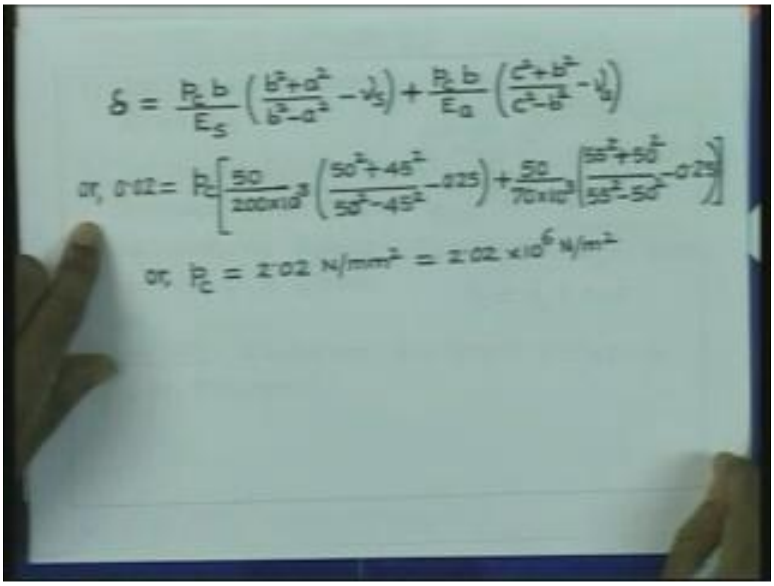

So, if we substitute now the value we will keep it all in Newton millimeter units. So, therefore it is 0.02 is equal to p c into b is 50 divided by modulus 200 into 10 to the power 3. It is in Newton per millimeter square. The dimension b here is 50, a here is 45. The Poisson's ratio is 0.25. Similarly, in this case b is 50. Modulus is 70 into 10 to the power 3.

And here, c is 55 b is 50 and the Poisson's ratio is 0.25. So, we can solve for now the contact pressure and that contact pressure is given by 2.02 Newton per millimeter square which is nothing but 2.02 into 10 to the power of 6 Newton per meter square. So, that gives you the answer for the first part. Now, it is also needed as for the problem statement. Calculate the stresses. You have to also calculate the stresses due to the shrink fitting. So, therefore if we now go back to our relationship involving the pressure and the stress distribution and substitute the value. So, we will calculate now the string fit stresses p c [FL] that p c.

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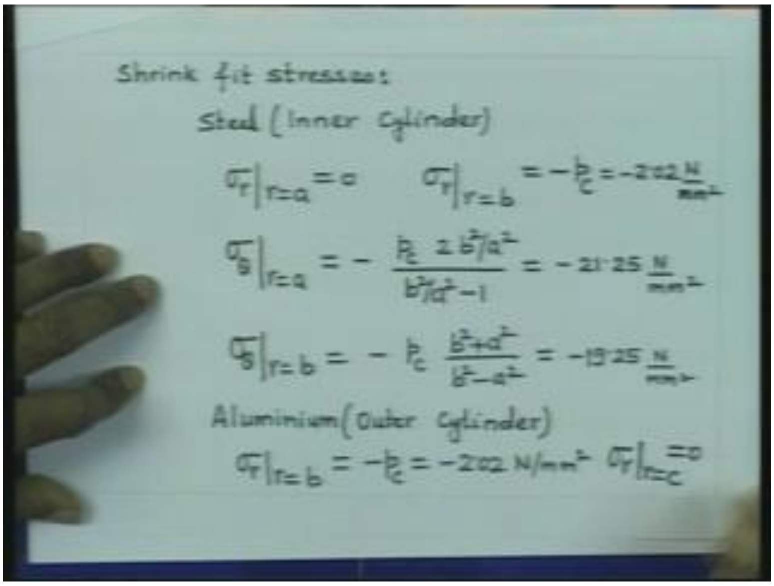

So, we will now calculate shrink fit stresses. Let us first consider the steel cylinder which is nothing but the inner cylinder. It is subjected to radial pressure from outside. So, therefore sigma r at r equal to a is 0 sigma r at r equal to b equal to minus p c, which is nothing but minus 2.02 Newton per millimeter square.

Similarly, hoop stress sigma theta at r equal to a is going to be given by p c 2 times b square by a square divided by b square by a square minus 1. So, if you substitute the values this gives you 21.25 Newton per millimeter square. Similarly, sigma theta at the outer radius this is given by p c b square plus a square by b square minus a square. And once you substitute the values, it gives you magnitude equal to minus 19.25 Newton per millimeter square.

Next, we will consider the aluminium cylinder which is the outer cylinder. Again it is subjected to internal pressure. So, therefore sigma r at r equal to common radius is going to be minus p c which is minus 2.02 Newton per millimeter square. And sigma r at r equal to c is 0.

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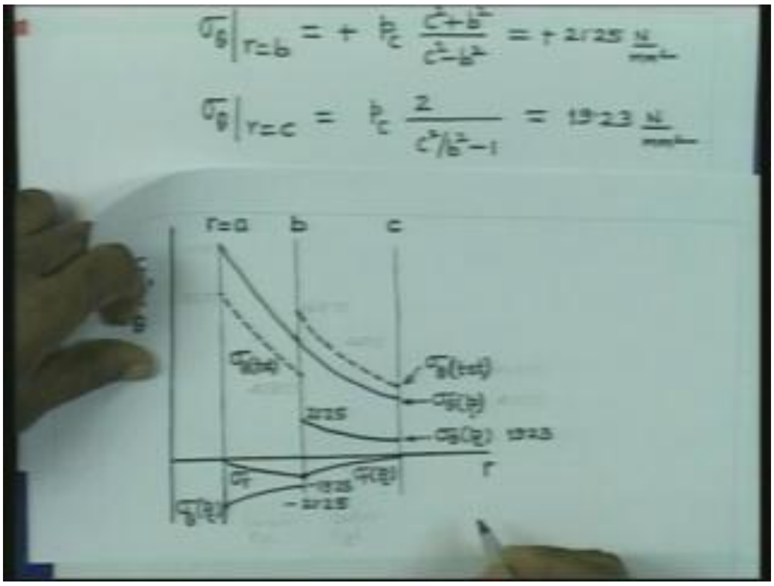

Then we have sigma theta at r equal to b. This is given by p c c square plus b square divided by c square minus b square. And this is of magnitude mi this is of course, positive. So, this is 21.25 Newton per millimeter square. And sigma theta at r equal to c is going to given by 2 times p c divided by c square by b square minus 1. And once you simplify this, this comes out to be 19.23 Newton per millimeter square.

So, with all the stresses if we now get back to our plot we get the following. We have the stress in the inner cylinder. The radial stress it has magnitude equal to 2.02 Newton per millimeter square, which is compressive. And the inner wall here, going to have a stress of magnitude equal to 21.25 Newton per millimeter square. And this stress here, it is 19.25 Newton per millimeter square.

Similarly, here the inner wall of the steel cylinder has a stress of magnitude 21.25 Newton per millimeter square. And the external radius has a stress of 19.23 Newton per millimeter square. So, the stresses are given with magnitude. We will now see how the stresses due to internal pressure can be calculated. If the two materials were of the same material, I mean the inner cylinder and outer cylinder were made up of the same material. Then in that case you would have calculated the stress variation in the whole cylinder. Considering a single material with internal radius a and external radius c, using the formula direct.

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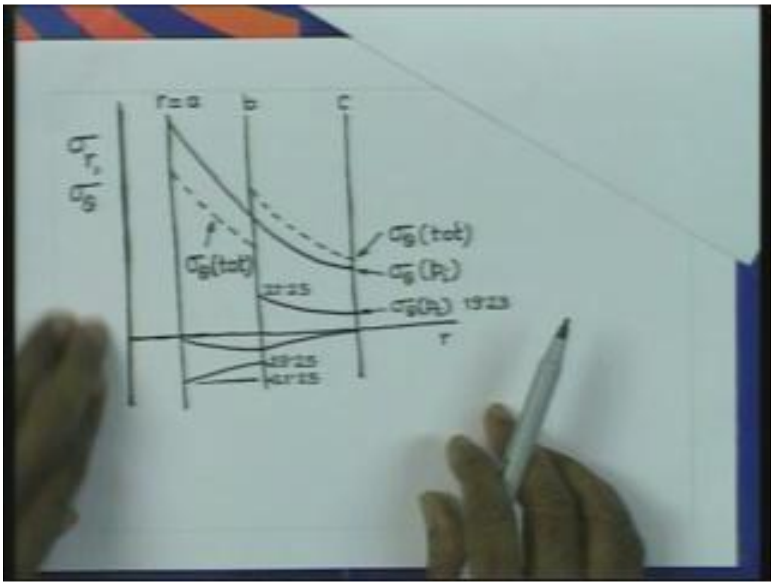

But then in this case, since the material of the inner cylinder is of steel and that of the outer cylinder is of aluminium, we have to have a different approach. So, let us now see how we can calculate these stresses.

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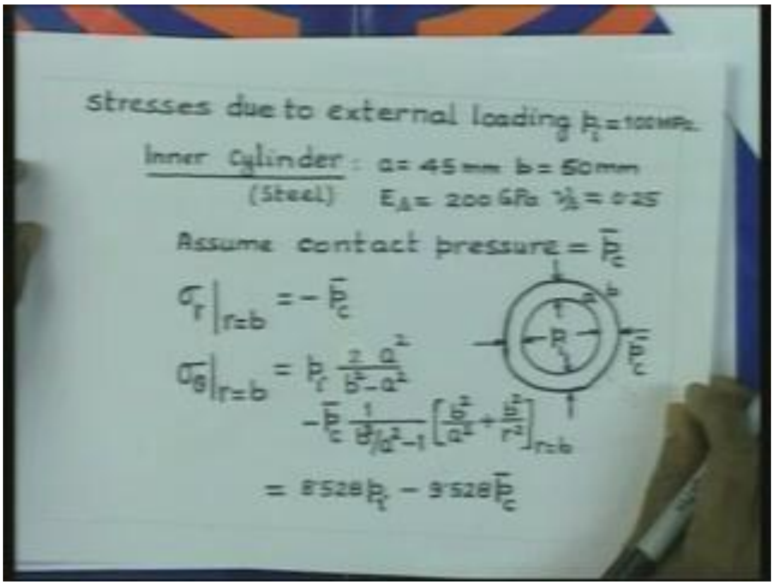

What you can consider here? Just think of the existence of the inner cylinder if the internal pressure p i is acting. Then, the cylinder will expand both a and b will increase in value. And since, after this radius the aluminium cylinder is existing the free expansion of the dimension b cannot take place. So, there will be some restraint on the expansion.

And let us say, that the restraint given by the aluminium cylinder is p c bar. And therefore, this p c bar is going to have some restrain imposed on the inner cylinder. And the same pressure will help to expand the aluminium cylinder.

So that both will, now come to a common position after expansion. So, if we consider now the contact pressure p c then we have to first of all find out the contact pressure p c bar by considering the condition that the radius b of the steel cylinder. And that of the aluminium cylinder will expand by the same magnitude. So, let us see how we can go about it? So, for this type of loading on this cylinder we have at r equal to b, the radial pressure is minus p c bar.

And since, it is a cylinder with both internal and external pressure, the tangential stress at r equal to b is given by this formula p i into 2 a square by b square minus a square minus p c bar 1 by b square minus a square minus 1 into b square by a square plus b square by r square r equal to b. And this on simplification, making use of the fact that a is 45 millimeter and b is 50 millimeter, we get these values. Now, if you consider that the expansion of the steel cylinder at r equal to b. Let us say it is, it has got expanded by a magnitude equal to delta i. So, that is the expansion.

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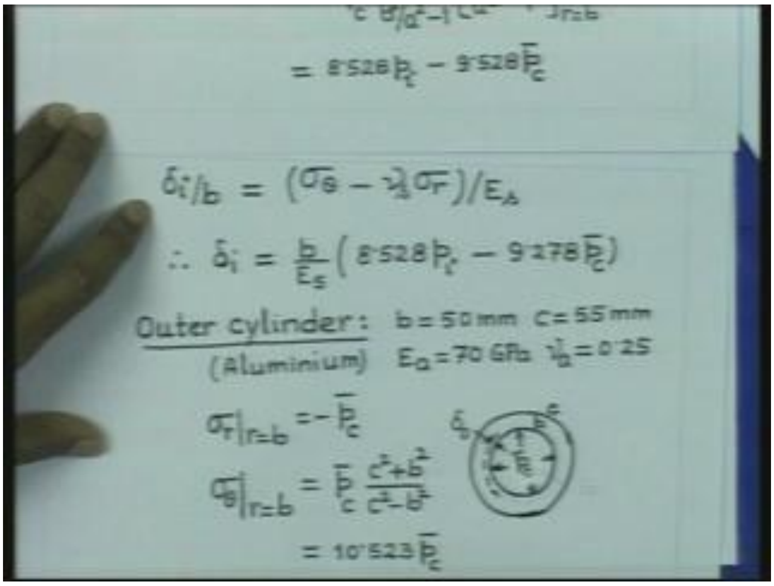

So, the strain that is going to develop in the circumferential direction is nothing but delta i by b. And that should be given by the relationship sigma theta minus nu s into sigma r divided by E s. Again, if you substitute if you just actually write now. Delta i is equal to b by E s, Es is the modulus of the steel cylinder or steel. And this is the pressure and sigma r is minus p c has been made use of. So, we get this simplified value.

Similarly, the outer cylinder is going to be subjected to this pressure internal pressure p c bar. And under the action of this p c bar, it is going to have some expansion all over in the circumferential or in the radial direction. And if we consider, that expansion to be given by, let us say that expansion is equal to delta 0 or delta o. Then we can calculate this by, first of all calculating the stresses at the inner radius of the steels aluminium cylinder. Sigma r at r equal to b equal to minus p c bar. And sigma theta r equal to b this is given by the lame's formula. And then we substitute the value. It comes out to be 10.523 p c bar.

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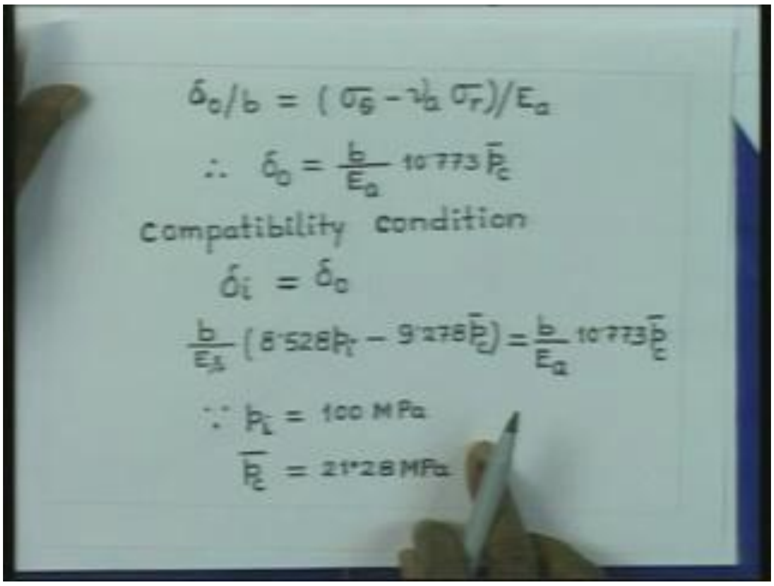

Obviously, this strain which is in the circumferential direction, it is given by delta 0 by b equal to sigma theta minus nu a into sigma r by E. And if we again substitute these values, here we get this much. So, the compatibility condition here is that. The expansion of the outer radius of the inner cylinder delta i must be equal to the expansion of the inner radius of the outer cylinder.

So, this if we make use of the values delta i and delta o that we have derived little while ago, it gives us this relationship see. And by simplifying, we get the common pressure and the internal pressure is 100 MPa. And therefore, this contact pressure p c bar is equal to 21.28 MPa. So, after having done this, having got this it is very easy to calculate the stresses due to p i. Now, it is going to be for inner cylinder sigma theta and sigma r is going to be due to both p i and p c.

And if we make use of the formula, it gives us this value. And sigma r is equal to minus p i, which is 100 MPa. Similarly, at the common radius r equal to b sigma theta already we had this formula earlier. So, we substitute the value we get this value and sigma r is given by this value. Similarly, you calculate for the outer cylinder the stresses, already we had this formula earlier. So, we applying the value we get this sigma r is this one. And for the external radius sigma theta is this one and sigma r is this one. So, all these values if we now superpose on the diagram.

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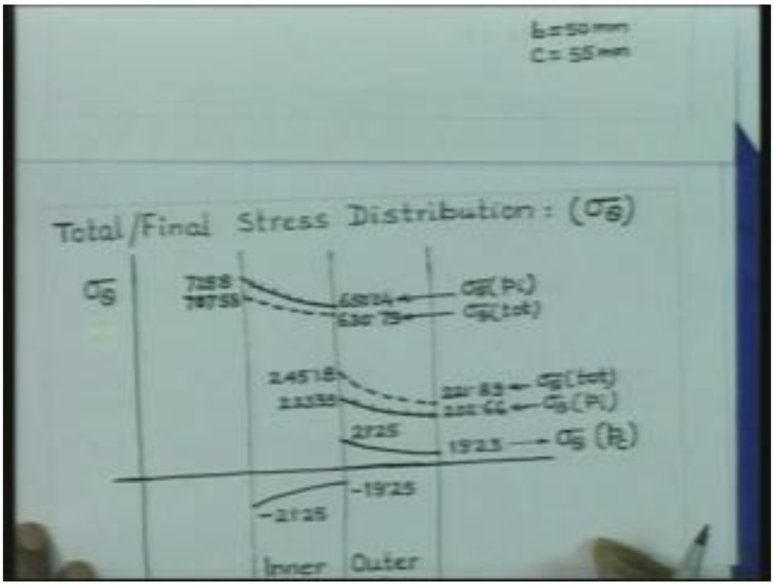

We have a picture like this. The stresses due to the internal pressure, is going to vary like this in the inner cylinder. And the stresses due to the internal pressure, is going to vary like this in the outer cylinder. So, this is the variation of the stresses in the inner and outer cylinder. So, the values are 728.8 and this is 650.04. And this is 223.93 and this is 202.66. So, these are the stresses due to sigma theta due to p internal.

Now, if we superpose the stresses due to shrink fitting, these are given here. So, only I am saying the variation of sigma theta. So, if you add up this compressive stress to this one, the stress magnitude will reduce. Finally, we are getting a variation of the total stress to be shown by this. And this one is now going to be additive. So, therefore it will increase to this level. So, if you write the value this sum total becomes 707.55 that is the value here.

And this is 630.79 and this one is 245.18 and this is 221.89. So, this is nothing but the sigma theta total due to shrink fitting and internal pressure. This is again the sigma theta total due to shrink fitting and internal pressure. And this is nothing but, sigma theta due to internal pressure. So, this gives you finally the variation of the stresses you can see that, higher stresses exist in the steel cylinder. Similarly, the stress varies and in the aluminium cylinder is going to be lower. And this difference is due to the fact that, modulus of steel is much higher than aluminium. So, this is how you can get finally, the stress distribution due to shrink fitting and internal pressure.