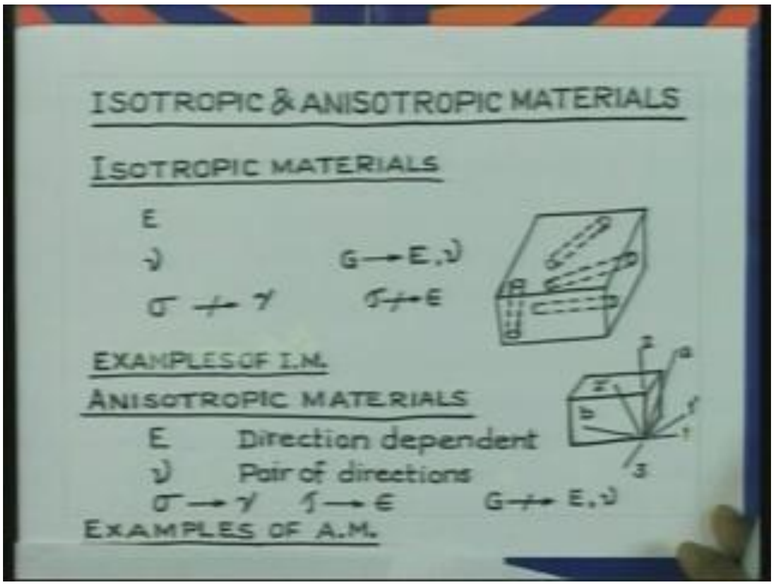
Advanced Strength of Materials
Prof. S. K. Maiti
Department of Mechanical Engineering
Indian Institution of Technology, Bombay

Lecture – 11

Today, we are going to look into isotropic and anisotropic materials. Most materials which are used in missile building they can fall into category of isotropic and anisotropic materials.

(Refer Slide Time: 01: 10)



What are isotropic materials? Isotropic materials are the once, whose elastic property are independent of directions. Let us say the modulus of elasticity. Modulus of elasticity of certain materials is the independent of the direction of testing. Think of a block of materials like these. If you considerate specimen oriented in this direction.

Test it you will get some modulus of elasticity. If you take analog specimen oriented in that direction or in this direction. You will find the same modulus of elasticity. Or when if you test in this direction, it will going to have the same modulus of elasticity. Therefore, the modulus of elasticity is independent of the direction.

Similarly, the Poisson's ratio indicates the influence between or interaction between two directions. So, if you consider a situation like these, that you have direction 1 and orthogonal direction 2. The interactions between the strains in these two directions are going to be expressed by Poisson’s ratio nu. And this Poisson's ratio between two directions is again independent of the orientations of the direction 1 and 2.

Suppose, you change it to one dash and two dash or a and b. You will find the same Poisson's ratio. Now, in such material you find that the modulus of rigidity. It is related to modulus of elasticity and Poisson's ratio. It is not an independent elastic constant. Another important feature of isotropic material is that. You will find that the normal stress applied in any direction. It is not going to induce any deformation or shear strain.

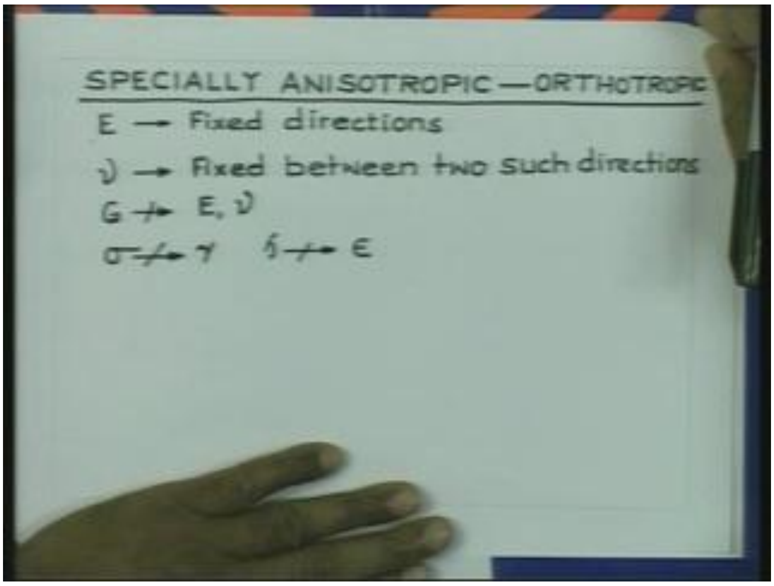
Similarly, if you apply shear stress in a plane, it is not going to produce any normal strain. So, the shear stress and normal strain or normal stress and shear strains they are uncoupled. Examples of isotropic materials, most of the materials particularly metallic materials they are really isotropic materials except in few cases which we will look into later.

Now, let us try to look into, what are anisotropic materials. So, indeed materials the elastic properties are direction dependent. So to say, that modulus of elasticity will depend on the direction of testing. So, you can understand now. Similarly, the Poisson's ratio which gives the effect between two directions forms interactions between two directions. This is going to be dependent on the pair of direction that one select.

And you find that, the modulus of rigidity for such materials is not related to modulus of elasticity and Poisson's ratio. So, unlike the case of isotropic materials, this G is not related to E and mu. It is an independent elastic constant. Another point which is also important for such material that the normal stress does not does produce a strain. So, shear stress produces normal strain. So, that is coupling effect.

Now, examples of anisotropic materials composites are important examples of anisotropic material. If you consider rolls sheet of steel, it is going to have some modulus of elasticity in the rolling direction. And it will have different modulus of elasticity in the perpendicular directions. So, this rolls steels they are really anisotropic materials.

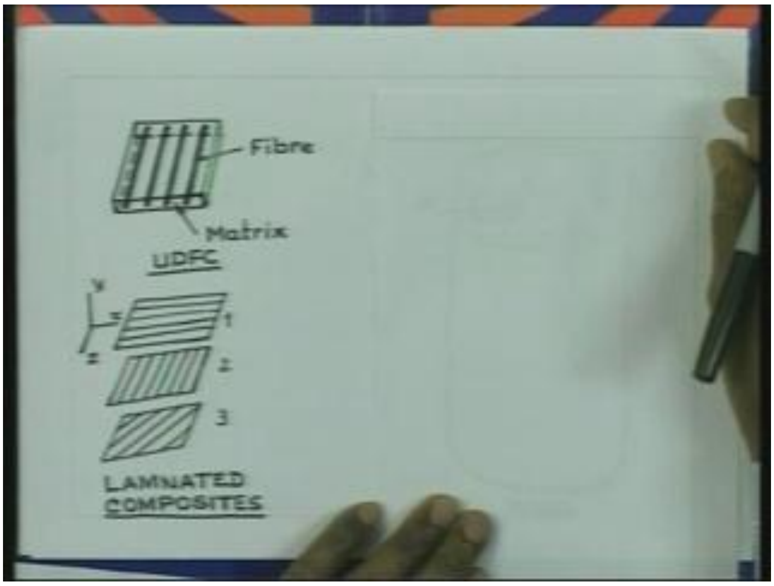
(Refer Slide Time: 06:11)



Now, there are certain timber is another example of anisotropic materials. Now, we will see that these anisotropic materials there are certain special type of anisotropic. And they are some are typically known as orthotropic material. So, I would like to look into orthotropic materials. So, there are sixth material directions for such material. You take that direction anywhere in the bulk of the material it will have the same modulus.

And in some cases, they have three such directions orthogonal directions, which have three independent moduli. And the Poisson's ratio between the two such directions, they are also constant. However, the modulus of rigidity is not related to E and nu. They are independent elastic constants. And for such materials, we will find that normal stress does not produce and shear strain. And shear stress does not produce any normal strain.

(Refer Slide Time: 07:17)



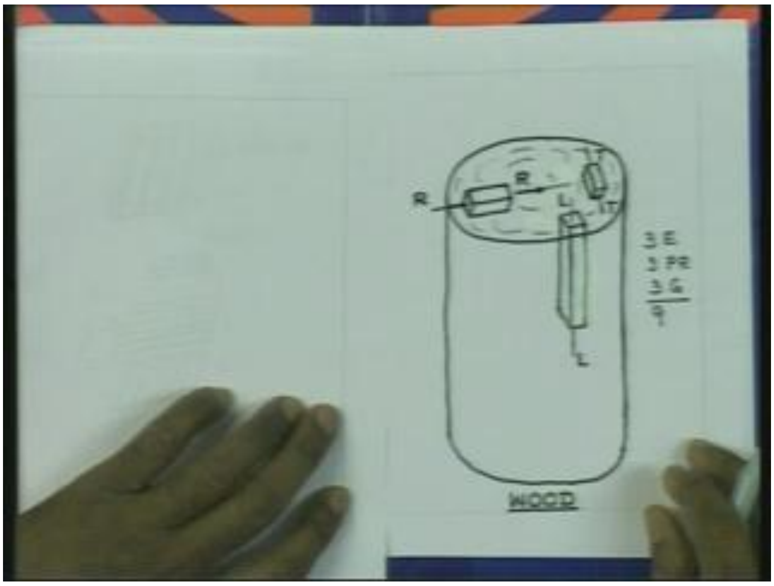
Now, we will look into some examples. First of all let us see, the composite materials you may not have occasion to come across these materials. Let us talk about them, if you consider some bulk material in the form of plate. And if you reinforce them by fibers located along the matrix you get. And this, fibers are generally made up of stronger material.

For example, the matrix could be made up of epoxy phosphates. And the fibers could be glass fibers, nylon or it could be carbon fibers. And you such materials are known as unidirectional fiber composites. They will have strong modulus in the direction of the fiber and lower modulus in the direction perpendicular to the fiber fibers. These materials are varying on orthotropic material.

They have material constants like one modulus in this direction, another modulus in the perpendicular direction. They have one Poisson's ratio constant relating that two directions. And also they have one modulus of rigidity in the plane of the material. So therefore, there are four independent elastic constants. If this system are all isotropic, then you would you would have had only two material constants.

Now, you think of a situation that we have unidirectional fiber composites in different layers of material. And then, if we put one over the other and make a combination that is going to give us, what is known as laminated composites. And these laminated composites are going to have anisotropic property. So, you can understand now how laminated composites are made. They are really constituted of orthotropic layers put one over the other.

(Refer Slide Time: 09:21)



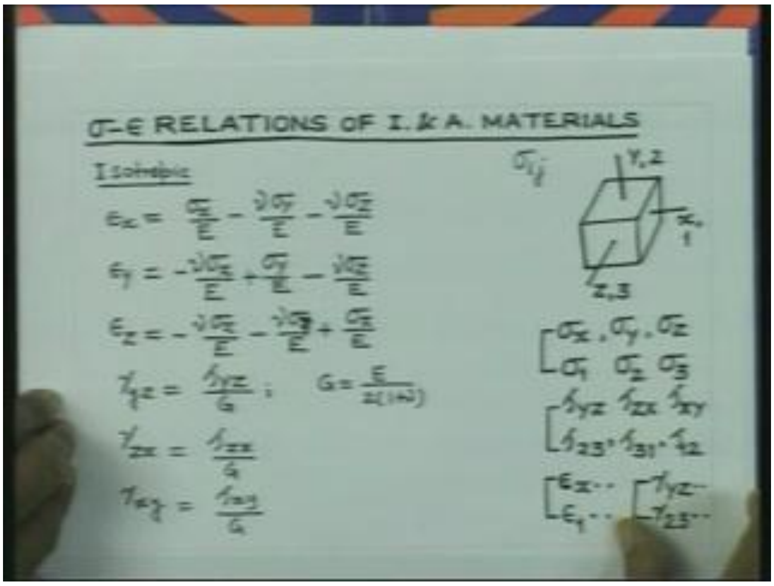
Now, one naturally you occurring anisotropic material is timber. We should look in the trunk of a tree. We will find that there are three distinct directions. One is in the longitudinal direction, another in the tangential direction and the other direction is radial. So, if you take specimen in the longitudinal direction it will show some modulus. Similarly, if you take the other two directions they will have two different moduli.

And you find that, modulus of elasticity in the longitudinal direction is the maximum. And that is why; the beams of the timber are made such that they are oriented in the longitudinal direction. Now in this case, you have three independent moduli. Then, you have the Poisson's ratio the effect between the direction L and T T and R R and L these three pairs. So, you will have three constants indicating the influence of the Poisson's ratio.

Then, you are going to get modulus of rigidity in the plane L T T R and R L 3 modulus of rigidity. So therefore, you are going to get 3 modulus of elasticity three Poisson's ratio constants and 3 modulus of rigidity. So therefore, you will find that for such materials you are going to have 9 independent elastic constants. But in the case of purely anisotropic material, the number of elastic constants would be 21 which we will saw in little later.

Now in the case of timber, you take these three directions anywhere in the whole trunk of the tree, they will show similar property. So, these are known as actually material symmetry directions. So, that leads to some advantage.

(Refer Slide Time: 11:40)



Now, let us try to look into the stress strain relations of isotropic anisotropic materials. In fact, we are going to talk about generalized Hooke's law. Let us, consider the direction xyz or in this ways 1 2 and 3. And we would like to have these symbols that sigma x sigma y sigma z they will be either indicated like this or sigma 1 sigma 2 sigma 3.

And shear stress is tau yz tau zx tau xy will be indicated as tau 23, tau 31, tau 12. Similarly, we will adopt also the symbols for strains epsilon x epsilon y epsilon z or epsilon 1, epsilon 2, epsilon 3 respectively. And the shear strains gamma yz gamma zx gamma xy as gamma 23, gamma 31 and gamma 12.

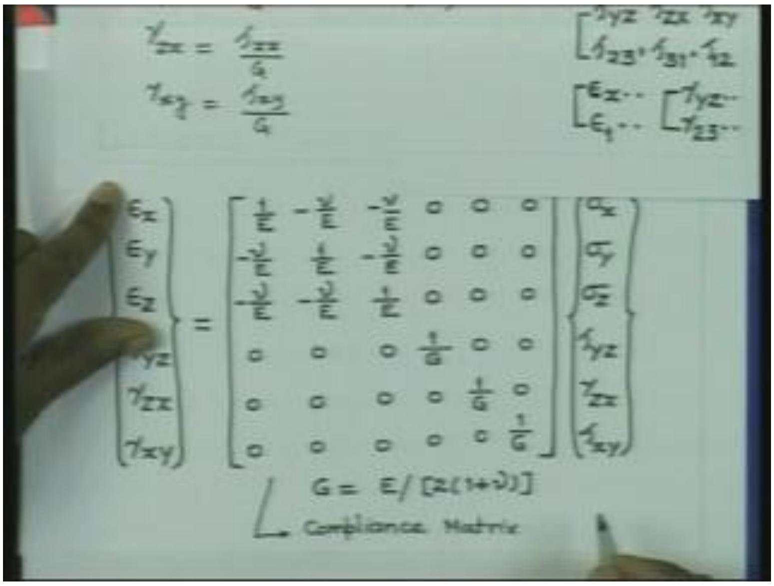
Now, if I consider first isotropic material, if we consider the strain in the direction x. Let us say that, we have all these stresses present here. There by meaning that we have all the six components of stresses. Now, if I consider epsilon x for the material. Then, under the action of the stress sigma x I will have stress strain coming up like sigma x by E.

Now due to sigma y, we will have the Poisson's ratios effect and shear stress upward the strain in the x direction will be like these. Similarly, the z stress will give rise to the strain in the x direction given by this one. Similarly, epsilon y is going to be given the strain sigma y by E in the y direction. And sigma x is going to give contribution nu times sigma x by E. Similarly, we will have z giving contribution nu times sigma z by E.

And epsilon z by similar consideration will have nu sigma x by E minus nu sigma, sigma y by E minus sigma I am sorry sigma z by E. So, these are the normal strain. And you see that there is no influence of this shear stress is. And now gamma yz is going to be tau yz by G. G is modulus of rigidity. And we will see that G is nothing but E by 2 into 1 plus nu.

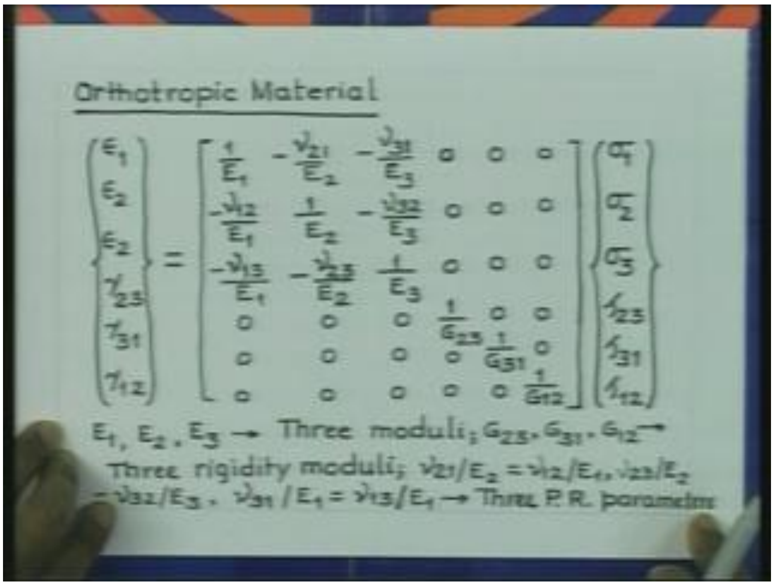
Similarly, gamma zx is going to be gamma tau zx by G and gamma xy is nothing but tau xy by G. So, these are the six stress strain relation. These are nothing but generalized Hooke's law. And if we write this thing in a matrix form, it looks all this relationship it will be it can be written in a compact form like this.

(Refer Slide Time: 15:19)



That the strains are related to the stresses by this matrix and these constants are 0. So, also these constants are 0 and all the rigidity moduli are coming in the diagonal. And you see here, that this matrix which is known as compliance matrix. So, this matrix is known as compliance matrix. It is a matrix of dimension 6 by 6 is connects. It connects the six strains to six stresses and it is a symmetric matrix.

(Refer Slide Time: 16:17)



Now, if we consider the material to be anisotropic. Then, we will have difference. So, we will now consider let us say we will consider material orthotropic. Then in it that case, we will find that if we now consider the six stresses to be indicated like this. Then, all the six stress if I now consider together.

Then, strain in the one direction or x direction is going to be nothing but sigma 1 by E minus nu sigma 2 into nu 2 1 into E 2. So, the strain in the direction to this sigma 2 by E 2 and it is Poisson's ratio. In fact, it is expressed by nu 2 1 and this is going to be negative, so therefore nu 2 1 by E 2. Similarly, this sigma three is going to give contribution to one direction like this.

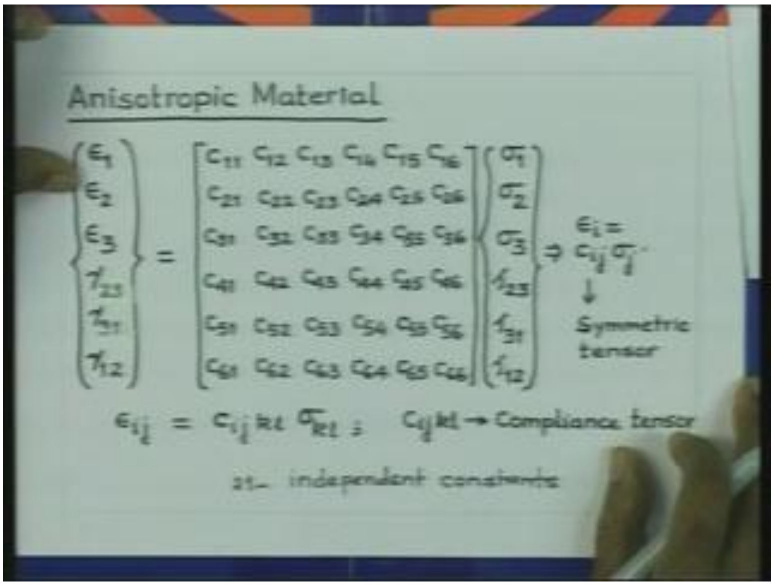
So, you can go on writing in the two direction 2 will have strain due to sigma 2 is sigma 2 by E 2. And strain due to sigma 1 it is sigma 1 by E 1 multiplied by nu 1 2. You see that this nu 2 1. This is nu 1 2. Similarly, this sigma 3 stress is going to give strain in the direction 2 given by sigma 3 by E 3 into nu 3 2 and it is negative.

So, if that strain in the direction 3 is tensile we are going to have contraction in the direction 2. Similarly, we can write this epsilon 2 also considering all the three stresses. And they are going to have constants like minus nu 1 3 by E 1 minus nu 2 3 by E 2 1 by E 3. And then, this shear strain gamma 2 3 is going to be given by tau 2 3 by G 2 3, where G 2 3 is the modulus of rigidity in the plane 2 3.

Similarly, the next strain is going too given by tau 3 1 by G 3 1 and gamma 1 2 is equal to tau 1 2 by G 1 2. So, you see here, we have three moduli of rigidity. Then, we have three I beg your pardon we have three moduli of elasticity. And then, we have three moduli of rigidity.

And now, you will find that this ratio nu 2 1 by E 2 is equal to nu 1 2 by E 1, so therefore in effective of only one elasticity constant. Similarly, this ratio nu 3 1 by E 3 is equal to nu 1 3 by E 1. So therefore, again another elasticity constant and this ratio is again same as this one. So, we have three Poisson's ratio related constant. So therefore, these are the equalities and therefore, we have three Poisson's ratio parameter in totally we have nine parameters. Again, this matrix is symmetric. And therefore, this is the relationship of orthotropic material.

(Refer Slide Time: 19:30)



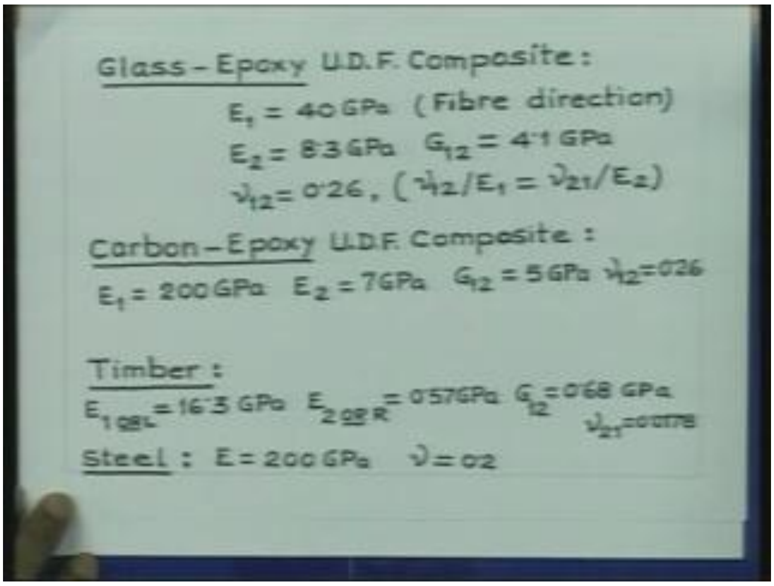
Now, if you consider purely anisotropic, then in that case you will find that the relationship is going to be little different. Now first of all, if you just concentrate on this part you have the six strains and you have the six stresses. Now, the effect of the stresses will be obtainable by considering epsilon 1 equal to C 1 1 into sigma 1 C 1 2 into sigma 2 c 1 3 into sigma 3 C 1 4 into tau 2 3 C 1 5 into tau 3 1 and C 1 6 into tau 1 2. So that is how you get the influence of all the six components in the direction 1.

Note that here, these C 1 4 C 1 5 Cc 1 6 constants are not zero. And therefore, you find that the shear stresses are going to contribute to the normal strains. So, you can write the expression for all the six strains. And in these there are it is a 6 by 6 matrix. So, this compliance matrix for tensor is a 6 by 6 matrix.

But then, it is a symmetric tensor. There by meaning that the 15 terms above the diagonal. And 15 terms below the diagonal. They are equal. And therefore, you have 15 of diagonal terms plus. These 6 diagonal terms total 21 constants. So therefore, there are 21 independent elastic constants 21 independent. So purely, anisotropic material would have 21 independent constants.

Sometimes this form can be simply written in this form by using the tensor notation epsilon i equal to c ij into sigma j varying i is varying from 1 to 6 j is varying from 1 to 6. But in general, we can write this in this form that this is the strain tensor. And this is nothing but compliance tensor into the stress tensor. So therefore, this is the generalized Hooke's law for anisotropic material.

(Refer Slide Time: 21:59).



Now, you must be interested in knowing some idea about the, what are of magnitude of the properties of the materials which are anisotropic. So, let us consider one composite you need some fiber composite which is made up of epoxy matrix and glass fiber. Then in that case, you find that if you test the material in the fiber direction. Then you have modulus of elasticity as 40 GPa.

And if you test it in the direction perpendicular then the modulus of elasticity is 8.3 GPa. And the modulus of rigidity in the plane is nothing but 4.1 GPa. And then the Poisson's ratio constant nu 1 2 is 0.26. And you can find out nu 2 1 by considering these equality. So therefore, you see there are four constants.

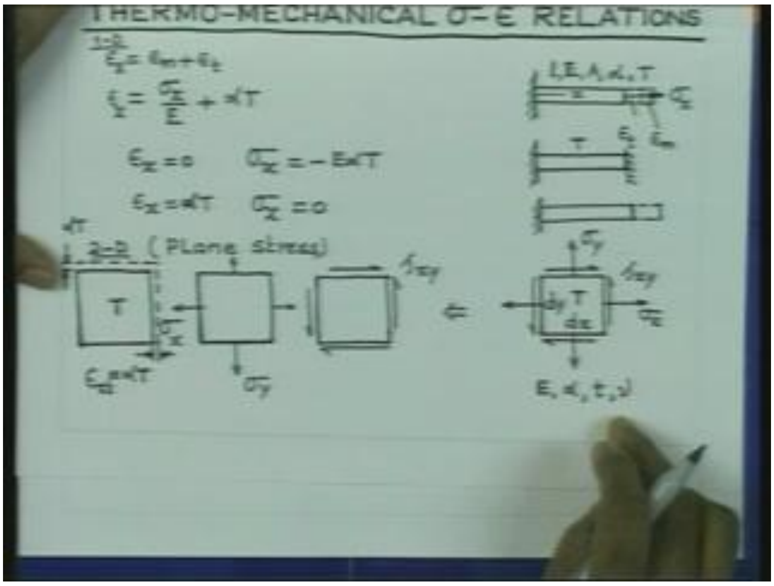
Now, if the reinforcement is done by carbon fibers. Then, you find that the modulus in the direction of the fiber is very high 200 GPa. And modulus in the perpendicular direction is 7. And the modulus of rigidity in the plane is 5 GPa and nu 1 2 is 0.26. And in the case of timber, so if you consider timber in the case of timber in the longitudinal direction; that means, in the direction of the trunk. You have 16.3 GPa in the direction radial which is 0.57 GPa.

And in the direction 1 2; that means, longitudinal radial the modulus of rigidity 0.68 GPa and nu 2 1 is 0.10178 that is the Poisson's ratio constant. You will find generally this modulus of elasticity in the tangential direction very close to this value. So therefore, you see the order of magnitude there is a ratio about 30. Now, compare this with the magnitude of steel which is an isotropic material. The modulus of elasticity is 200 GPa and the Poisson's ratio 0.2.

Now, I must mention that you can have composite material made up of metal matrix and some other metal as its reinforcement. We make now titanium aluminum composite whereas, the matrix is of aluminum. And the fibers are of titanium this gives very good strength and that is why, they are used for high strength applications. And this carbon composites, they can be also you can have matrix of carbon and the fibers of carbon.

That is why, they are known as carbon car carbon composites. And these materials are very high temperature resistant. And therefore, they are made and they are made use of in a making of the nodes of the reentering vehicles. The vehicles, which goes for space exploration while they are entering into the atmosphere. There is very high temperature which is as high as 3000 degree centigrade to stand these high temperature they make use of carbon composites.

(Refer Slide Time: 25:32)



Now, I would like to consider the stress strain relations, where in you have both thermal and mechanical loading. So therefore, we will now consider thermo-mechanical stress strain relations. That in most power plants in the chemical plants in the boilers or turbines. You find that the components are going to operate at high temperature. They are not only subjected to mechanical loading.

They are also temperature, they are also subjected to varying temperature and which are very high order of which are also having high magnitude. So, in the presence of both temperature field in a component and also mechanical loading you are going to have both thermal and mechanicals deformation or deformation due to both mechanical loading and thermal loading. So, we would like to consider how these calculations can be done. In order to do that, we must have relationship between stress and mechanical strain and also temperature.

Now consider the simple case, you take the example of a simple rod. And if you try to increase the temperature of this rod it is going to expand, if it will go on expanding depending on the thermal expansion. And if this expansion is prevented, then we are going to get some stresses developed in the component which is going to be compressing in nature. But if, we allow the free expansion of the component to take place, then we are not going to have any stress developed in it.

Now, let us try to see this little more elaborately. Consider one component of uniform cross section it is held like this. Let us consider that, it is length is l its modulus of elasticity material has modulus elasticity E. The cross section is A, and let us consider that the thermal coefficient of expansion for this material is alpha.

And we now raise the temperature of this material about the room temperature by T. And also this material is subjected to some mechanical loading due to this. There is a stress of magnitude let us say sigma x. So, let us consider that this is oriented in the direction x. Now, under the action of the mechanical loading, It will have strain and that strain is nothing but sigma x by E.

Now, under the action of the temperature field, it is going to have expansion of magnitude l into alpha into T. Now, if I want to calculate the strain, then the change in length is l alpha T original length is l. Therefore, this strain is nothing but l alpha T divided by l. Therefore, it is alpha T.

So therefore, the total strain, you are going to get or the deformation. You are going to get due to part of that is due to thermal. Let us say, this is the expansion due to the thermal expansion. And this, the expansion further up is due to the mechanical loading. So, therefore, we have strain due to thermal and strain due to mechanical loading.

So therefore, if I want to calculate the total strength epsilon it is nothing but epsilon mechanical plus epsilon thermal. So now, I can write the value mechanical strain is nothing but sigma x by E. And this thermal strain is nothing but alpha into T. So therefore, the relation if you would like to write this strain in this x direction, total strain in the x direction epsilon x is in equal to sigma x by E plus alpha into T. So, you find that this is the relationship between the strain stress and temperature.

Let us see that, we have a case wherein epsilon x equal to 0. So, the total strain equal to 0. Then, you find that sigma x is equal to minus E alpha T. Therefore, it means that if the component is not allowed to, what I mean is that? This that if I consider this component. And if I increase its temperature by T, and we do not allow this to expand. Then in that case, it is going to develop a stress of magnitude E alpha T which is compressing in nature. So therefore, this is the magnitude of the strain when there is no this is magnitude of the stress when there is no strain.

Now consider the other extreme, suppose we now say that epsilon x is equal to alpha T. Ten in that case, sigma x is equal to 0, what it means here? That if you now consider the same component and now let us say that this is originally here. Now, if I consider it to have expansion free expansion up to this marks. And; that means, we are not constraining the free expansion, then in that case stress will be equal to 0.

So therefore, you see whenever there is constant on the thermal expansion. We are going to have stress in the material. And this stress therefore, the thermal stress, we will find that it is going to lie all the time in one dimension in the range 0 to minus E alpha T. So, this is really the picture in the case of 1-Dimension.

Now, let us try to consider the case in 2-Dimension. So, if I now consider an element of a component in two dimension. And let us say that, this is two dimensional plane stress condition. So in the case of two dimensional plane stress, let us try to assume that the stresses on the element are sigma x sigma y and the shear stress is tau xy.

And at the same time, let us also assume that the temperature of the body is raised by T above the ambient. Now, the material properties will try to make use of E alpha and thickness is equal to T. Let us consider that, this height is equal to dy and this height is equal to dx. This loading I can split into two parts. Let us consider that, we have first is let us say that, this is the shear loading normally.

And then, we can also consider that there is mechanical loading. So, this is sigma x sigma y, this is tau xy. So, this is the mechanical constituent of the loading. And then, we can also consider that there is thermal effect. So this thermal effect, we can consider it to be again sep separate. So therefore, in this case we have rise in temperature by T. So, this problem can be now considered to be made up of three.

Now in this case, you can calculate the strains in both x and y direction. And if the material is isotropic, you will get only the normal stresses producing. Normal strain, there is no shear strain produced by them. And then, this is going to produce only the shear strain and it will not produce any normal strain.

And temperature rise is going to produce strains, which is going to be increase in dimensions of the body both in x and y direction. So therefore, in this case, the size of the body is going to increase both in x and y direction. And that increase in dimensions will depend on the magnitude of dx and dy.

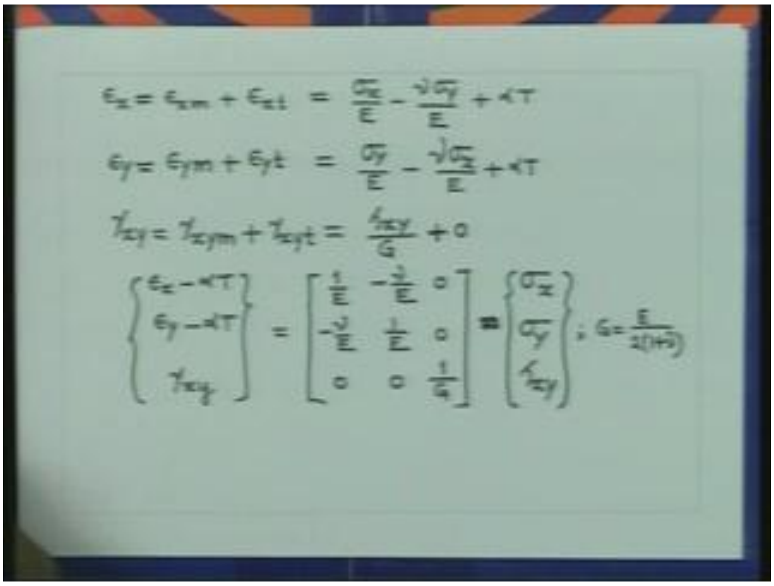
Note that this thermals rise or temperature rise is not going to give rise to any distortion of the body. So, this a very important point that in the case of temperature rise. You will find that the form of the body or the shape of the body is not going to undergo any change. So, if you had two orthogonal directions initially. They will remain orthogonal and there is no shear strain produced. So, we will can now calculate the strains under the action of all these loadings very easily. So, we will now calculate the strain due to the mechanical part and also thermal part.

Before I go into considering the mechanical part look at the thermal see the thermal if this side is equal to dx under the action of temperature rise. This is going to expand by dx into alpha into T. And therefore, this strain is going to be dx alpha T divided by dx therefore, it is alpha T. So therefore, the strain in this direction, this strain is going to be of magnitude equal to alpha T.

So, if you would like to consider epsilon x T is equal to alpha T. Similarly, if you consider this direction, the height is dy under the action of temperature rise. It is going to be expanding by dy into alpha into T. And therefore, strain is going to be dy alpha T divided by dy. Again this strain in this direction, it is also going to be of magnitude equal to alpha T. So therefore, the strain in the two orthogonal directions, are going to be alpha T and alpha T.

And due to these two stress in the x and y direction, we are going to get strain in the x direction to be given by sigma x by E minus nu. We will have of course, the Poisson's ratio constant nu. So, nu times sigma y by E. So therefore, I can now write that the total strain.

(Refer Slide Time: 38:46)

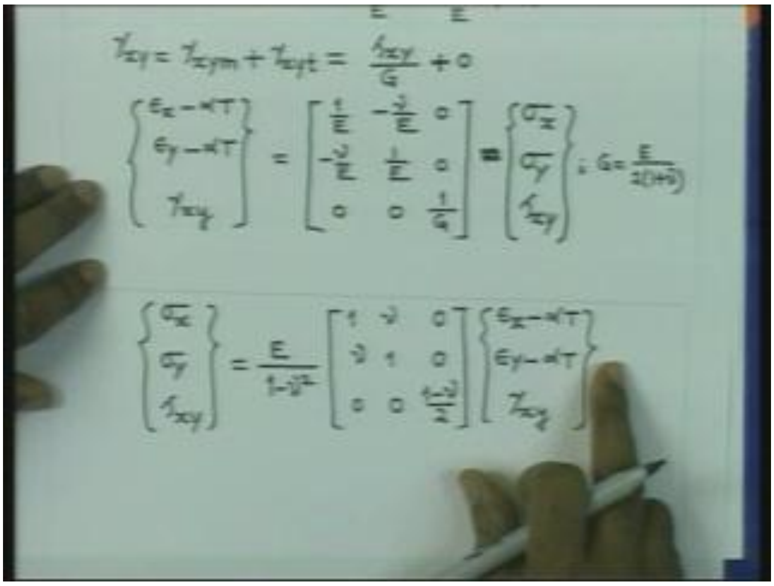


Epsilon x is equal to epsilon x mechanical plus epsilon x thermal. And now epsilon x mechanical is nothing but sigma x by E which is due to sigma x and due to sigma y. We are going to get nu times sigma y by E. And shear stress is not going to produce any normal strain. So therefore, we have no contribution plus, we are going to get this thermal strain which is of magnitude equal to alpha T. So therefore, this is alpha T.

Similarly, epsilon y is equal to epsilon y mechanical plus epsilon y thermal. So, this is going to be again sigma y by E minus nu times sigma x by E. And the mechanical strain is going to be you have already seen here. This is equal to alpha T. So therefore, this is alpha T. Now, the shear strain gamma xy. Again, we will have gamma xy due to mechanical loading gamma xy due to thermal loading. The strain due to mechanical loading is straight away given by tau xy by G and this part is equal to 0. So, please note this that under the action of the temperature rises. You are not going to have any shared information.

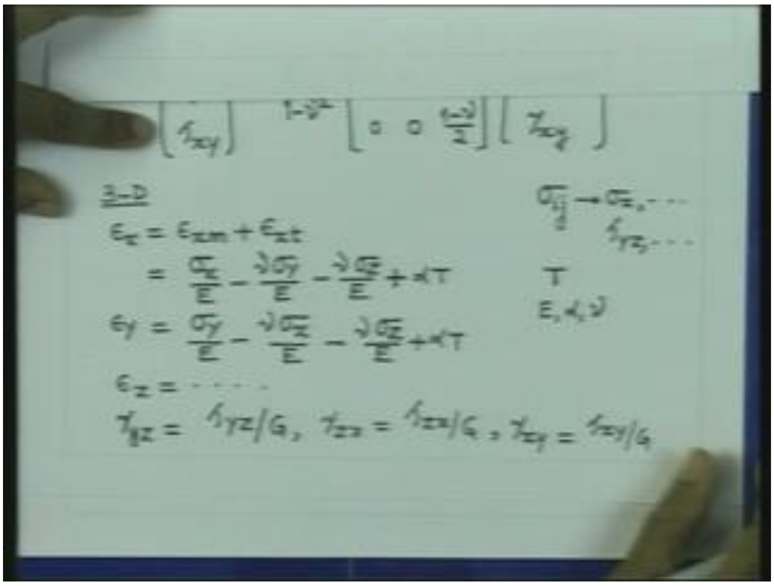
That is why you see that when you heat a rectangular plate it will remain rectangular if you heat a circular plate it will remain circular. So, therefore, two initially orthogonal directions will remain orthogonal even after deformation. So, this is the stress strain temperature relationship in the case of plane stress. And we can write now this thing in the form epsilon x minus alpha T epsilon y minus alpha T. And gamma xy is equal to 1 by E minus nu by E 0 minus nu by E 1 by E 0 0 0 1 by G. I am sorry this multiplied by we have sigma x sigma y tau xy. So, this is the strain temperature and stress relation. You can see that the elastic constant are involved here nu and E and G and since G is equal to E by 2 into 1 plus nu. We can invert this relationship.

(Refer Slide Time: 42:56)



And once, we invert this relationship we will get a form like this sigma x sigma y is equal to E by 1 minus nu square 1 nu 0 0 nu 0 0 1 minus nu by 2 multiplied by epsilon x minus alpha T epsilon y minus alpha T into and then last term is gamma xy. So, therefore, this gives you the relationship between the stress and strain and temperature. So, this is the relationship for plane stress.

(Refer Slide Time: 44:13)



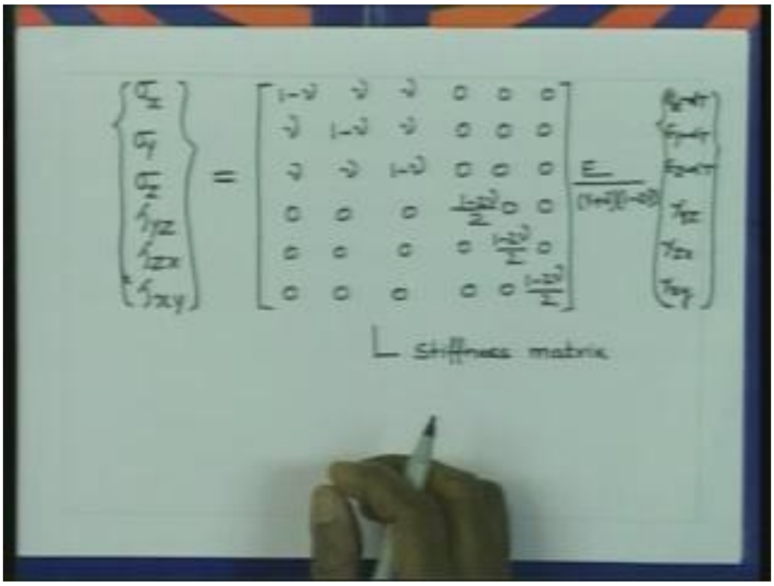
You can extend this relationship to 3-Dimensions very easily. So, if you consider 3-Dimensions, we will have all the components of stresses sigma x sigma y sigma z and then we will have tau yz tau zx and tau xy. And then, we are also going to get thermal field which is characterized by temperature rise T and if you assume the material constants again to be E alpha Poisson's ratio nu.

Then, we can now write that the strain in the x direction is going to be strain due to mechanical loading plus the strain in the x direction due to temperature. Now, if I consider the material to be isotropic. Then in that case, under the action of the three normal stress, we are going to get strained to be given by epsilon x by E minus nu times sigma y by E minus nu times sigma z by E and the thermal strain is going to be alpha T.

So, you can write the expression for sigma y sigma z along similar lines. So, I will just write for you sigma epsilon y. So, epsilon y equal to sigma y by E nu times sigma x by E minus nu times sigma z by E and the thermal strain will be alpha T. So, you can write very easily epsilon z. So, I will not write that. So, please fill up.

Now, you can consider the shear strain gamma yz. That is going to be nothing but tau yz by G and the thermal strain will be 0. So therefore, the shear strain is only due to mechanical loading. And you can write now, gamma zx along similar line zx by G. And of course, you can also write gamma xy to be given by tau xy by G. So, these are the six relations involving mechanical loading and thermal loading.

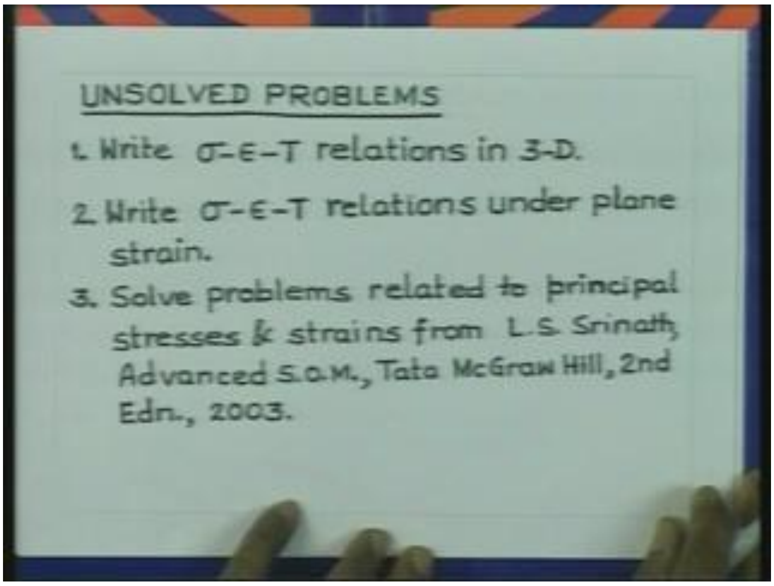
(Refer Slide Time: 47:33)



Now, if you try to invert this relation. Then, you will get the connection between the stresses on one hand and the strain and the temperature on the other. So therefore, if you invert this relationship which is going to give you like this stresses sigma x sigma y sigma z tau yz tau zx and tau xy. So, these are the six stresses.

And then, we will get the constants like this. It is 1 minus nu nu nu 0 0 0 nu 1 minus nu nu 0 0 0 nu nu 1 minus nu 0 0 0 1 minus 2 nu by 2 0 0 0 0 0 0 1 minus 2 nu by 2 0 0 0 0 0 0 1 minus 2 nu by 2. Then, we have the common multiplier E by 1 plus nu into 1 minus 2nu multiplied by epsilon x minus alpha T, epsilon y minus alpha T, epsilon z minus alpha T and this is gamma yz gamma zx gamma xy. So, this is epsilon x minus alpha T, epsilon y minus alpha T, epsilon z minus alpha T, gamma yz gamma zx gamma xy. So, therefore, this is the strain matrix. So, again it is a 6 by 6 matrix it is symmetric matrix. And this matrix is known as stiffness matrix.

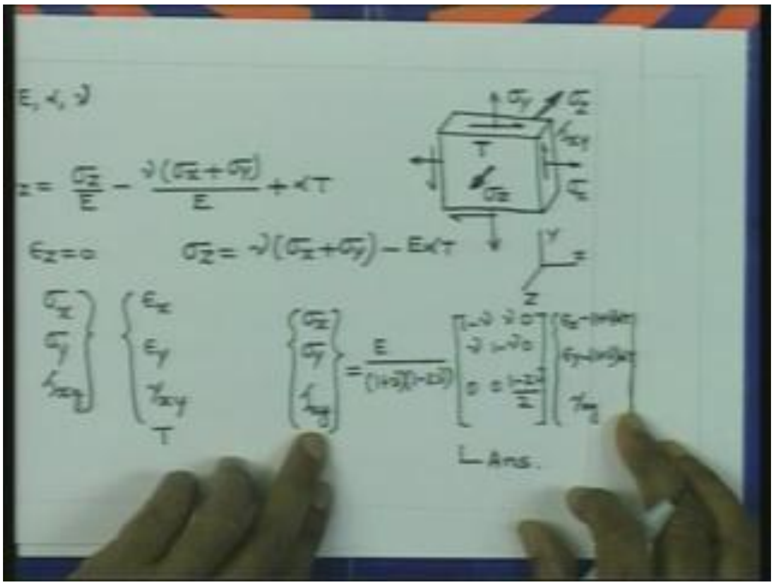
(Refer Slide Time: 51:06)



Now, this will be able to consider some derivations with whatever we have talked so far. Think of the problems like deriving of course, these relations in 3-Dimension. Just now I have derived for you. Now, this is something which you must consider write the relations involving sigma epsilon and temperature under plane strain condition. So, this is something you must derive and that will make all the derivations clear.

The third example which you can consider solve problems related to determination of principal stresses and strains from L. S. Srinath advanced strength of materials it is published by Tata McGraw Hill second edition in 2003. So this second problem, I would like you to think over.

(Refer Slide Time: 52:13)



So in this case, the picture is like this. You have a component. So, let us say the directions are x y and then we have z direction. So, you have the stresses acting in the x direction, sigma x sigma y acting in the y direction. Then, you have shear stresses acting these are in plane shear stresses. Let us, simply represent by tau xy and at the same time we are going to have some stress acting in the z direction. So therefore, we have sigma z stress also acting on the body. On top of this there is a temperature rise by T.

So, the material constants E alpha nu, you can find out the strain in the x direction due to mechanical. It will be due to normal stress as sigma x sigma y and sigma z. And the thermal strain is going to be simply alpha into T. Similarly in the y direction, you are going to get strains due to all the three component. And in the z direction, you are also going to get similar picture.

So, epsilon z is now going to be sigma z by E minus nu times sigma x plus sigma y by E plus the thermal strain is going to be alpha T. So therefore, epsilon z is due to mechanical is this much and thermal. Now, since we are considering plane strain condition, there is no expansion in the z direction. So therefore, epsilon z equal to 0 for plane strength. Therefore, you find that sigma z is equal to nu times sigma x plus sigma y minus E alpha T.

So therefore, in this case you get that sigma z is to be dependent on both the normal stresses sigma x and sigma y and also the temperature rise and modulus of elasticity thermal coefficient of expansion. So, you can now write the expression for sigma x sigma y in terms of only sigma x and sigma y eliminating sigma z. Thereby you are going to get relationship involving sigma x sigma y and tau xy on one hand.

And on the other side, you are going to get epsilon x epsilon y and gamma xy plus the temperature. So, you are going to also get temperature. So therefore, you will find that these stresses are now going to be related to the strains and temperature. So that was the relationship which you are to derive.

And this relation finally, it will turn out to be that sigma x sigma y tau xy is equal to E by 1 plus nu into 1 minus 2 nu into 1 minus nu nu 0 nu 1 minus nu 0 0 0 1 minus 2 nu by 2 that is the material constant. And on the other side, you are going to get epsilon x minus 1 plus nu into alpha T epsilon y 1 plus nu into alpha T and gamma xy. So therefore, this is what is going to be the relationship. So, this please note it carefully that this is going to be epsilon x minus 1 plus nu into alpha T epsilon y minus 1 plus nu into alpha T gamma xy. So, this is the relationship. So, your answer is going to be in this form. Finally the plane strain relationship will look like this.