

Cryogenic Engineering
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Lecture No. # 09
Gas liquefaction and Refrigeration Systems

So, welcome to the ninth lecture of cryogenic engineering of this NPTEL program. My earlier lecture, I will just take a review of my earlier lecture; what we talked about in the earlier lecture, it was regarding gas refrigeration and gas liquefaction introduction.

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CRYOGENIC ENGINEERING

Earlier Lecture

- Basics of Refrigeration/Liquefaction, coefficient of performance and importance of Carnot COP.
- Throttling, heat exchanger, compression/expansion systems. Definition of a refrigerator, liquefier and a combination of these two systems.
- J - T expansion is an isenthalpic process and initial temperature of the gas should be less than T_{INV} to have a cooling effect.
- For an ideal gas $\left(\frac{\partial T}{\partial p}\right)_h = 0$, Hence, $\mu_J = 0$.

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So, what we talked about was basics of refrigeration, liquefaction. In order to liquefy particular gas, you have to low the temperature to boiling point of this particular gas. And therefore the importance or the procedure how to reach those lower and lower temperatures is very important. And the performance of the refrigerator could be calculated by COP or the coefficient of performance parameter, and importance of Carnot COP was also mentioned. It serves as an ideal COP or the maximum possible COP to reach a particular temperature.

We talked about various mechanisms to get lower and lower temperatures. These temperatures can be obtained by throttling mechanism, heat exchangers or compression expansion systems. We talked about all these three possibilities and we also talked that a complete system may come out as a combination of all these parameters. We talked

about a definition of a refrigerator, liquefier and a combination of these two systems. We talked that refrigeration is normally a close cycle process; that its working fluid never leaves the cycle. Well, a liquefier can be called as an open cycle process, because the working fluid itself gets liquefied. And whatever liquid we collect, the same amount of the gas has to be added to count off the deficit.

We talked that J - T expansion is an isenthalpic process; enthalpy gets conserved during this process. And the initial temperature of the gas should be less than the T inversion temperature to have a cooling effect. We talked about that, every gas has got a characteristic inversion temperature. And in order that after expansion by J - T expansion after isenthalpic expansion if it has to result in lowering of temperature, then the initial state of the gas should have a temperature less than the inversion temperature of that particular gas. We also saw that for an ideal gas μ_{J-T} or ΔT by Δp at constant enthalpy is equal to 0. That means if the gas is subjected to isenthalpic expansion, it should not result in any lowering of temperature or even in heating of the gas, because μ_{J-T} is equal to 0 for an ideal gas.

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Outline of the Lecture

Topic : Gas Liquefaction and Refrigeration Systems (contd)

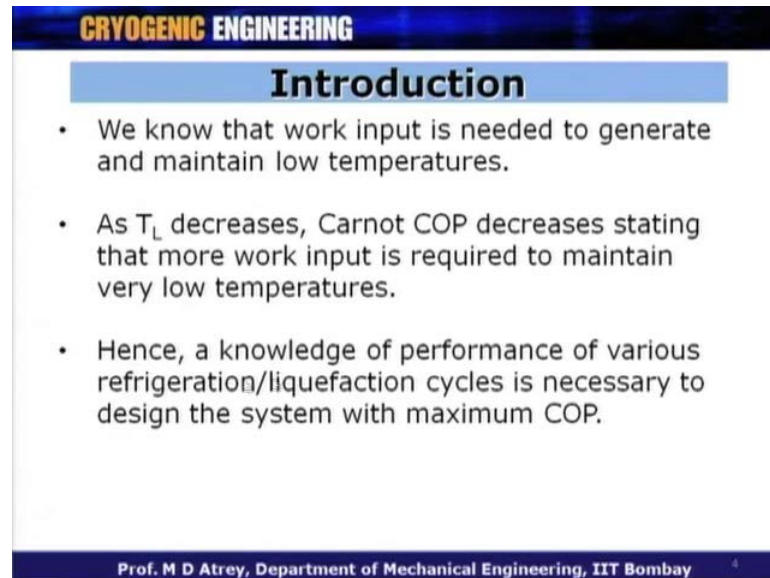
- J - T expansion of a real gas
- Isentropic expansion
- Comparison of J - T and Isentropic expansion
- Ideal Thermodynamic Cycle

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In this particular lecture, I am continuing with gas liquefaction and refrigeration systems. What we are going to talk about now is J - T expansion of real gas; what we talked in the earlier lecture was J - T expansion of ideal gas. Let's see how the real gas behaves when it is subjected to J - T expansion. Then we will talk about isentropic expansion; then we

will compare the J - T expansion and isentropic expansion. And then we will see the first cycle which is ideal thermodynamic cycle which is use for gas liquefaction **all right**, this is what we will try to cover in this particular lecture.

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CRYOGENIC ENGINEERING

Introduction

- We know that work input is needed to generate and maintain low temperatures.
- As T_L decreases, Carnot COP decreases stating that more work input is required to maintain very low temperatures.
- Hence, a knowledge of performance of various refrigeration/liquefaction cycles is necessary to design the system with maximum COP.

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We have seen that work input is needed to generate and maintain low temperatures. We have seen that the first law and the second law tells us that we have to put in some work, we have to put in some efforts in order to generate low temperature and to maintain the low temperature. We have seen that as T_L decreases as lower and lower temperature if we want to reach, the Carnot COP decreases stating that more work input is required to maintain very low temperature.

If you remember the last lecture, we had talked about watt of power input per watt of refrigeration, and we found that the watt of power required per watt of cooling effect required increases as we go down lower and lower temperatures; if you want to reach lower and lower temperature your work input increases. Hence, a knowledge of performance of various refrigeration liquefaction cycle is necessary to design the system with maximum COP. See ultimately important thing is to our maximum COP or maximum efficiency, and therefore one has to understand what cycle one should use in order to reach particular temperature or in order to liquefier particular gas. This is very, very important considering the fact that efficiency and COP are very important functions

of what cycle you choose in order to refrigerate in order to reach lower temperatures or to liquefy particular gas.

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CRYOGENIC ENGINEERING

Joule – Thompson Coefficient

- Enthalpy (h) is the sum of the internal energy (u) and pv work.
$$h = u + pv$$
- $$\mu_{JT} = \left(\frac{\partial T}{\partial p} \right)_h = - \left(\frac{\partial T}{\partial h} \right)_p \left(\frac{\partial h}{\partial p} \right)_T$$
- Substitution of Enthalpy (h) in above expression of J – T coefficient, we get
$$\mu_{JT} = - \frac{1}{c_p} \left[\left(\frac{\partial u}{\partial p} \right)_T + \left(\frac{\partial (pv)}{\partial p} \right)_T \right]$$
- In order that the J – T expansion results in cooling, the bracket should be negative.

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Now, let us understand what is Joule - Thompson coefficient is for the real gas, and let us start with the definition of enthalpy which is nothing but some of internal energy u plus pv work, and mathematically I can write h is equal to u plus pv. So, by definition mu J - T or J - T expansion coefficient is nothing but del T by del p at constant enthalpy. And if I split that it becomes del T by del h into del h by del p with a negative sign over here. This is what we have seen earlier and therefore, I will not go in the details of all this things.

So, when I know now del h by del p I have to find at constant temperature and del T by del h at constant pressure. Substituting enthalpy h, we know h is equal to u plus pv and if I substitute this h over here in the above expression of J - T coefficient what we will get is this mu J - T is equal to 1 by c p, because delta T by delta h is nothing but 1 by c p which we have seen earlier. And now I am putting the value of h as u plus pv, and therefore this expression becomes del u by del p at constant temperature plus del by del p of p v at constant temperature here.

Now, this is the bracket will **will** decide whether mu J - T is positive which is required for cooling or negative which result in heating or is equal to 0 which will not result in cooling or even heating. See, in order that J - T expansion results in cooling, the bracket

should always be negative. So, net result of this bracket should be negative. So that taking care of this negative, the whole thing becomes positive and in that case μ_{J-T} is going to be positive.

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Joule – Thompson Coefficient

$$\mu_{JT} = -\frac{1}{c_p} \left[\left(\frac{\partial u}{\partial p} \right)_T + \left(\frac{\partial(pv)}{\partial p} \right)_T \right]$$

- The first term represents the departure from Joule's law.
- At low pressures, the molecules are pulled apart. This results in increase in the potential energy. As a result, the kinetic energy decreases to keep the total energy constant and hence the temperature.
- Therefore, the first term is always negative for a real gas.

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So, this is the expression I am talking about, and what it has? It has got two parameters; ∂u by ∂p at constant temperature and ∂pv by ∂p at constant temperature. Now, let us talk about the first parameter that is ∂u by ∂p , because essentially the values what we get from ∂u by ∂p at constant temperature and $\partial(pv)$ by ∂p of $p v$ will decide what is the net sign of this bracket. The first term is the variation of internal energy which pressure at constant temperature. Now, this is nothing but this term represents departure from joules law. The joules law states that the internal energy is constant for a given temperature. And here we are talking about changes in internal energy with pressure when the temperature is maintained constant. So, this basically talks about how you are going away from the ideal gas **right**. The ideal gas will not accept these. That means, we are going away from ideality which means that we are taking into consideration the real effects of the gas and we are talking about real gas now.

At lower pressures, when you expand the gas from higher pressure to low pressure and when we are talking about low pressures, the molecules are pulled apart. This is what happens in expansion process. This results in increase in the potential energy of the gas.

So, whenever the molecules go apart, the potential energy increases, as a result the kinetic energy decreases, and as you know the kinetic energy decreases, because the total energy remains constant **alright**.

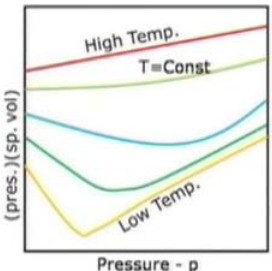
So, when the kinetic energy decreases, kinetic energy directly proportional to temperature. So, as soon as the kinetic energy decreases, the temperature of the system is also decreases. That means, Δu by Δp when the pressure decreases, the Δu by Δp should result in increase of u value, because the potential energy of the system increases, and therefore this term Δu by Δp always becomes negative which helps in the reducing the temperature gas when the gas expands. Please understand that Δu by Δp therefore will always be negative and will always help in reduction of temperature when the gas is subjected to expansion, when the pressure of the gas reduces. This is true for the real gas also.

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Joule - Thompson Coefficient

$$\mu_J = -\frac{1}{c_p} \left[\left(\frac{\partial u}{\partial p} \right)_T + \left(\frac{\partial (pv)}{\partial p} \right)_T \right]$$



- The second term can either be positive, negative or zero. It represents the departure from the Boyle's law.
- At high pressures, the molecules are squeezed together and hence, are less compressible than the Boyle's law prediction.

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Now, we will talk about the second term. So now, if we talk about the second term, this second term is the most important thing and this term will decide if this bracket is going to be positive, negative or 0. The second term now it can be positive, negative or 0 and this represents now departure from Boyle's law. Again you can see now, this is Δ of Δp of pv at constant temperature. The Boyle's law says that the product pv remains constant for a given temperature. Now, we say that we are talking about changes in the value of pv although the temperature remains constant with pressure changes. So, this is

basically departure from the Boyle's law where the non-ideality or the real gas comes into picture.

Please look at this graph and here it talks about the pV variations versus pressure. And as you can see here, the pV variation for higher temperature, pV value decreases when the pressure decreases, and therefore, $\frac{d(pV)}{dp}$ in this case is going to be positive. Well you can see as the temperature goes on lowering down, you can see that there is the variation in the curve and for some part you can find pV decreases, for some part the pV increases, and here comes the pV variation with pressure the sign changes. So, here in this case, the value pV decreases with a decrease in pressure saying that $\frac{d(pV)}{dp}$ in this case is positive.

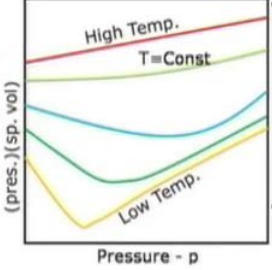
Well if you come in this region, we understand that pV value increases when pressure decreases and in this region therefore, $\frac{d(pV)}{dp}$ in this case is going to be negative, because there is a pV increase happens although the pressure decreases. So, in this case is going to be negative. And if we want to have cooling in this case, because you want $\mu_J - T$ to be positive, we expect that $\frac{d(pV)}{dp}$ should be negative which should help us it having $\mu_J - T$ has a positive quantity. So, if we see at this variation, we can have two parts; one at the higher pressure and one is at a lower pressure. We can see at a higher pressure, the molecules are very, very squeezed. The **molecule is already subjected** different molecules are already subjected to very higher pressure, and therefore, they cannot be compress further.

And what we can see now, hence, they are less compressible than Boyle's law prediction in this case. When the compressibility of the gas is very, very less in this case, when they cannot be compressed further, the pV should not show an increase in this case. Because the pressure decreases, but the volume will not increase in this case in a proportional manner, because the gases are less compressible in the higher pressure region; that means even if the pressure is decrease, the value of the pV also shows decrease in this case, and therefore, in this region, if you expand the gas, this $\frac{d(pV)}{dp}$ term is going to be positive in this case. That in pressure reduction happens $\frac{d(pV)}{dp}$ also reduces, and this is not going to help you in reduction of temperature or $\mu_J - T$ cooling effect.

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Joule - Thompson Coefficient

$$\mu_{JT} = -\frac{1}{c_p} \left[\left(\frac{\partial u}{\partial p} \right)_T + \left(\frac{\partial (pv)}{\partial p} \right)_T \right]$$


The graph shows the Joule-Thompson coefficient, μ_{JT} , on the y-axis (labeled '(pres.) (sp. vol)') versus pressure, p , on the x-axis. Three curves are shown for different temperatures: 'High Temp.' (top curve, red), 'T=Const' (middle curve, green), and 'Low Temp.' (bottom curve, yellow). The 'High Temp.' curve is a straight line with a positive slope. The 'T=Const' curve is a horizontal line. The 'Low Temp.' curve is a downward-opening parabola that crosses the x-axis at two points, indicating regions where the coefficient is positive and negative.

- The second term is negative at low pressures and low temperatures, where the gases are more compressible than the Boyle's law.
- For a real gas, J - T coefficient depends upon the relative magnitude of both the terms.

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However, if you see the other half that is the second term is negative at lower pressures and lower temperatures. So, if we talk about this region now, we find in this region which is low pressure and low temperature; the gases are more compressible than Boyle's law predictions **right**. Why see here, because now we are talking about lower pressure value. This is not a higher pressure value as it was in earlier case. We are talking about lower pressure values, and when I expand the gas from this pressure let us do up to this pressure. The pv product increases, because the volume increase will be there, and volume increase will be more, because the gases are more compressible in this region. And therefore, as soon as I expand the gas from higher pressure to low pressure in this case, the pv value increases, and therefore, $\frac{\partial (pv)}{\partial p}$ in this case would be negative which will help in making μ_{JT} as positive. And therefore, if you expand the gas in lower temperature and low pressure region, it is always beneficial or it will always result in cooling when subjected to isenthalpic expansion.

And this is where we found there are the temperature or the pressure above which it will not result in cooling when expanded; while in this case, the gas will result in cooling when expanded in a J - T expansion manner. So, for a real gas Joule - Thompson coefficient depends upon the relative magnitude of both this terms. So, what is this magnitude positive, negative or 0. This is always going to be negative, but what is the net result of this bracket will decide, if μ_{JT} is going to be positive or negative or when

subjected to Joule - Thompson expansion, is it going to result in cooling or is it going to result in heating will be decided by this parameters.

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The slide is titled "CRYOGENIC ENGINEERING" and "Equation of State". It contains the following text and equations:

- van der Waals equation of state for a real gas is as given below.
$$\left(p + \frac{a}{v^2}\right)(v - b) = RT$$
- where, **a** and **b** are constants, which gives the measure of intermolecular forces and size of the particles respectively. For an ideal gas both **a** and **b** are 0.
- Rearranging the terms, we get
$$p = \frac{RT}{(v - b)} - \frac{a}{v^2}$$

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Now, this was true when we talked about real gas and we talked about the enthalpy equation. Now, same thing could be understood when I am using a real gas equation or van der Waals equation. So, van der Waals equation of state for a real gas is given as this, which is $p + \frac{a}{v^2}$ into $v - b$ is equal to RT where a and b are constants, and this a and b gives the measure of intermolecular forces which is what a is while, and size of the particle respectively the value of b denotes basically the size of the particles, which we say that the finite volume of value to this b and a value in this case. For ideal gas, we assume that both a and b is equal to 0, and therefore what we get is p is equal to RT for ideal gas. If we rearrange this term, what you get is p is equal to RT upon $v - b$ minus $\frac{a}{v^2}$. You can do this exercise by yourself to derive this particular term.

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Equation of State

- Upon differentiating the following equation at constant pressure (only T and v are variables), we get

$$p = \frac{RT}{(v-b)} - \frac{a}{v^2}$$
$$0 = \frac{R}{(v-b)} \left(\frac{\partial T}{\partial v} \right)_p - \frac{RT}{(v-b)^2} + \frac{2a}{v^3}$$

- Rearranging the terms, we have

$$\left(\frac{\partial v}{\partial T} \right)_p = \frac{\frac{R}{(v-b)}}{\frac{RT}{(v-b)^2} - \frac{2a}{v^3}}$$

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Now, if I want to differentiate this term, real gas equation, van der Waals equation; upon differentiating the following equation at constant pressure, this is my equation, because the pressure remains constant in this case. I will get p is equal to RT v minus b and minus a by v square. If I differentiate I will get 0 here, because I am keeping pressure as constant and then I get this particular term. From where if I rearrange the term what I ultimately want to get is a value of del v by del T at constant pressure. This is the most important thing, because I would like to put this value in my earlier equation. See if I get del v by del T at constant pressure at this value, I will substitute this del v by del T at constant pressure in the J - T coefficient equation, this is the value and this my J - T coefficient equation.

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Equation of State

- Substituting $\left(\frac{\partial v}{\partial T}\right)_p$, in the J - T Coefficient equation, we have

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{\frac{R}{(v-b)}}{RT - \frac{2a}{(v-b)^2} - \frac{2a}{v^3}}$$

$$\mu_{JT} = \frac{1}{c_p} \left[T \left(\frac{\partial v}{\partial T}\right)_p - v \right]$$

- For real gas, we get
- For large specific volumes

$$\mu_{JT} = \frac{\frac{2a}{RT} \left(1 - \frac{b}{v}\right)^2 - b}{c_p \left[\left(1 - \frac{2a}{vRT}\right) \left(1 - \frac{b}{v}\right)^2 \right]}$$

$$v \gg a, b$$

$$\therefore \mu_{JT} = \frac{1}{c_p} \left(\frac{2a}{RT} - b \right)$$

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So, here this $\frac{\partial v}{\partial T}$ at constant pressure, I will put from here into this equation. And if I do that for real gas, what I get is this. Now, I will try to approximate these, if v is specific volume is very, very large as compare to the values of a and b . If I assume this and this is a very realistic assumption, if I do that then $\frac{b}{v}$ will tend to be equal to 0, well this also tend to be equal to 0, and therefore, ultimately my expression for μ_{JT} comes like this - $\frac{1}{c_p} \left(\frac{2a}{RT} - b \right)$. So, here the value of μ_{JT} is now determined by this particular equation, and the sign of μ_{JT} will be decided by this bracket - $\frac{2a}{RT} - b$. As we know $\frac{1}{c_p}$ is always going to be positive, because c_p is always a positive term.

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Joule – Thompson Coefficient

- For a real gas with large specific volumes

$$\mu_{JT} = \frac{1}{c_p} \left(\frac{2a}{RT} - b \right)$$

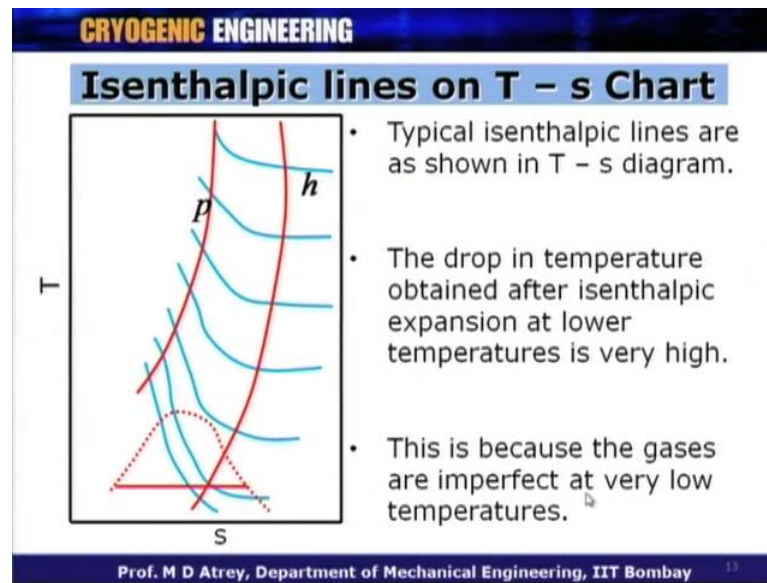
μ_{JT}	Cond.	T
>0	$\left(\frac{2a}{RT} - b \right) > 0$	$T < \frac{2a}{Rb}$
<0	$\left(\frac{2a}{RT} - b \right) < 0$	$T > \frac{2a}{Rb}$
$=0$	$\left(\frac{2a}{RT} - b \right) = 0$	$T = \frac{2a}{Rb}$
	$a = b = 0$	Ideal

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So, what I understand now? For a real gas with large specific volume, I will have this equation, and if I want to have μ_{JT} to be more than 0, this bracket should be more than 0, and therefore, temperature should be less than $2a$ by Rb . If I want to be μ_{JT} to be less than 0 which means it is going to be result in heating in this case, I would like the temperature of the gas - initial temperature of the gas to be more than $2a$ upon Rb . And if I want μ_{JT} to be equal to 0 in that case, my temperature is going to be $2a$ upon Rb . In this sense, this is nothing but the inversion temperature for the gas at a given pressure. See, if my temperature is less than $2a$ upon Rb , my μ_{JT} is going to be positive and when expanded it going to be result in lowering of temperature.

Well here, if the temperature is more than $2a$ upon Rb which is nothing but the inversion temperature for a particular gas; my expansion is going to result in heating in this case. Well this is going to be point where nothing will happen. And if a is equal to b is equal to 0, ultimately everything gets reduced to the ideal gas, again in which case μ_{JT} is equal to 0. So, this essentially talks about what the values of inversion temperature would be for a particular gas depending on the values of a , b and r for those gases. This is the van der Waals equation basically would decide, what are the values of the inversion temperature for a particular gas at different pressures etcetera. One can based on this values, one can construct the inversion curve also.

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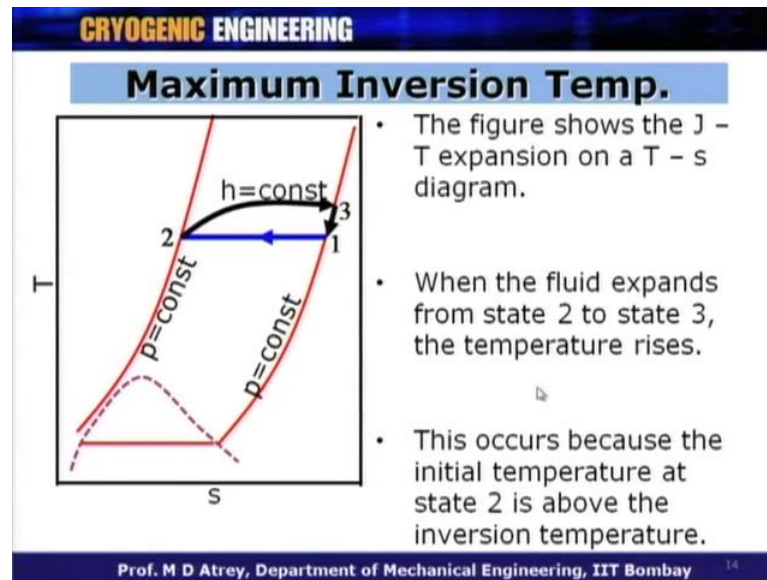
See, if I want to plot isenthalpic line on a T - S chart, you can see this is the dome in which the liquefaction happens, temperature and entropy here. And you can see, suppose this is a room temperature at this point, the isenthalpic lines are like this. See, if I expand the gas from this high pressure isenthalpically, I will get this as ΔT in this case, this minus this. But as you go down the temperature here, and if I expand the gas isenthalpically from this pressure to this pressure, my ΔT in this case is much larger than what I get near room temperature. What does it tell? It shows that the drop in temperature obtain after isenthalpic expansion at lower temperature is very, very high.

So, if I expand the gas at lower and lower temperatures, I am going to get bigger and bigger temperature drop if expanded isenthalpically. Why, because as I am going down and down I am reaching in a real gas state, I am going away from ideality. And therefore, my J - T expansion is going to be more and more prominent, I am going to get more and more real gas expansion, and in this case I am getting very large cooling when expanded isenthalpically **all right**. One should always have isenthalpic expansion when one goes away from ideal gas, and best way to go away from ideal gas is to have liquid; liquid is nothing but completely away from ideality.

So, normally you find that J - T expansions are mostly the J - T expansion would be for a two phase system; when you are completely away from ideal gas. So, you have got a mixture of let say gas plus liquid or complete liquid also which is what is normally

prepared in order to have more and more cooling effect, if the gas is **or the mixture** or the two phase mixture is subjected to J - T expansion. Why this happening? This is because the gases are imperfect at very low temperature, and therefore, they show the real gas behavior and they are completely away from the gas nature in these cases.

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So, if you want to see what happens near room temperature, you can see now. If I expand the gas from 2 to 3 which is let say at room temperature now, it is not result in **in** cooling, but it is result in heating in this case. Why, because this initial state of the gas is above the inversion temperature, and therefore, instead of getting into cooling zone it is going into heating way. And therefore, one should ensure that this state 2 is lying below the inversion temperature before it takes off for isenthalpic expansion. So, in the fluid expands from state 2 to 3, the temperature rises. This occurs because the initial temperature at state 2 is above the inversion temperature. This is the very important parameter one has to understand.

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Maximum Inversion Temp.

Gas	T_{inv} (K)
Helium	45
Hydrogen	205
Neon	250
Nitrogen	621
Air	603
Argon	794
Oxygen	761
Methane	939

- For the gases like He, Hydrogen and Neon, in order to experience J - T effect, they have to be precooled below T_{INV} .
- While the other gases show J - T cooling when expanded at room temperature.

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Now, there are various gases and there got the one inversion temperature. What you can see from here is the gases like Helium, Hydrogen, Neon have got their inversion temperature has 45, 205 and 250 kelvin which are below the room temperature, while all other gases over here having their inversion temperature above the room temperature; the room temperature being 300 kelvin. It means that if I want to have isenthalpic expansion to be resulting into cooling for these gases, their initial state has to be brought down below their inversion temperature. Then and then only when they are subjected to isenthalpic expansion or a J - T expansion, they would result in cooling **right**.

So, for gases like Helium, Hydrogen and Neon, in order to experience J - T cooling, they have to be precooled below T inversion temperature first and then subject to the J - T expansion. However, the other gases if they are subjected to isenthalpic expansion at room temperature only, they will result in cooling in this case, because their inversion temperature lie above the room temperature. This is very important for cryogenic temperature for these gases, we have to precool these gases first below their inversion temperature and then get cooling effect. Now, we talked about isenthalpic expansion.

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Isentropic Expansion

- Enthalpy (h) and Entropy (s) are the two thermodynamic state properties of matter which are functions of pressure and temperature.
- When the gases are expanded at constant enthalpy, as in a J - T expansion, it is called as an Isenthalpic expansion.
- On the similar lines, when the high pressure gases are expanded at constant entropy, it is called as an Isentropic expansion or a Reversible-Adiabatic expansion.

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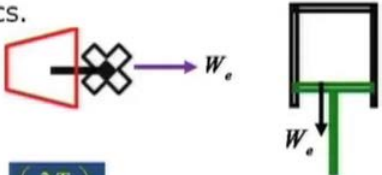
Now, we will talk about isentropic expansion, whether entropy remains constant. Enthalpy and entropy are two thermodynamics state properties of matter which are functions of pressure and temperature. When the gases are expanded at constant enthalpy as in J - T expansion it is called as isenthalpic expansion. On the similar line, when the high pressure gases are expanded at constant entropy it is called an isentropic expansion or a reversible adiabatic expansion. This is all what we know from thermodynamics.

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Isentropic Expansion

- The commonly used expansion devices are turbo expanders and reciprocating expanders.
- This is a work producing process as shown in the schematics.



- The ratio $\left(\frac{\partial T}{\partial p}\right)_s$ is called as an Isentropic Expansion Coefficient.

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The commonly used expansion devices are turbo expanders which is a turbine for isentropic expansion or reciprocating expanders where cylinder and piston arrangement could be used to cause isentropic expansion. This is a work producing device; now, this is a difference between a J - T expansion and an isentropic expansion. Here in this case, when expansion occurs, there is some kind of work gets produced and we will get some work from this expansion.

So, this is a turbo expander and you will get some kind of a W e or a shaft work done, when the gas expand from high pressure to low pressure. Or similarly, if we have got a reciprocating expanders, what you get ultimately is a work output W e when the expansion occurs. Now, similar as what you saw earlier, delta T by delta p at constant entropy is what we will consider now and this ratio is called as isentropic expansion coefficient.

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CRYOGENIC ENGINEERING

Isentropic Expansion

- The enthalpy (s) is a function of both pressure (p) and temperature (T).

$s = f(p, T)$
- Using the calculus, the following can be derived.

$\left(\frac{\partial s}{\partial p}\right)_T \left(\frac{\partial p}{\partial T}\right)_s \left(\frac{\partial T}{\partial s}\right)_p = -1$
- Rearranging the terms, we have

$\mu_s = \left(\frac{\partial T}{\partial p}\right)_s = -\left(\frac{\partial T}{\partial s}\right)_p \left(\frac{\partial s}{\partial p}\right)_T$

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Now, let see how we derive the value of delta T by delta p at constant entropy. The entropy is a function of both pressure p and temperature T, and therefore, what we can write is s is equal to function of pressure and temperature. Using the calculus as what we have done earlier, we derive following expression, where we say now, delta s by delta p at constant temperature into delta p at delta T at constant entropy into delta T by delta s at constant pressure is equal to minus 1. And therefore, if I rearrange this what ultimately I want to have is a mu s which is nothing but delta T by delta p here, delta T by delta p at

constant entropy which is what this value is. If I rearrange this term what I get is equal to minus sign delta T by delta s at constant pressure into delta s by delta p at constant temperature. So, here I have got an expression now for mu s; in a similar way, what I have earlier got expression for mu h or mu J – T.

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Isentropic Expansion

- For the same variables, entropy (s), temperature (T) and pressure (p), using the calculus, we can arrive at the following.

$$ds = \left(\frac{\partial s}{\partial T}\right)_p dT + \left(\frac{\partial s}{\partial p}\right)_T dp$$

$$Tds = T\left(\frac{\partial s}{\partial T}\right)_p dT + T\left(\frac{\partial s}{\partial p}\right)_T dp$$

c_p

$-\left(\frac{\partial v}{\partial T}\right)_p$

Maxwell's Equation

$$\mu_s = \left(\frac{\partial T}{\partial p}\right)_s = -\left(\frac{\partial T}{\partial s}\right)_p \left(\frac{\partial s}{\partial p}\right)_T$$

$$\mu_s = +\frac{T}{c_p} \left(\frac{\partial v}{\partial T}\right)_p$$

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Now, for the same variables entropy s, temperature T and pressure p, we use again calculus and now I can write ds as, basically I am writing this expression by using partial derivative in terms of T and p - temperature and pressure. So, delta s by delta T keeping pressure constant into dT plus delta s by delta p keeping temperature constant into dp. If I multiply by T and this is again in the same line as what we have done earlier. We find that Tds is nothing but multiplied by T to this term, multiplied by T to this term. Again using the same approach, this particular thing is nothing but c p. While this delta s by delta p gives minus del v by del T at constant pressure which is what we obtained from Maxwell's equation.

If I use these two terms now in my original equation over here, I will put the values and what I put the values as delta T by delta s as c p, because if I compare this with this, I get this is equal to this and I get this is equal to this **all right**. So, I put the value of delta T by delta s as 1 by c p, and I get delta s by delta p as nothing but delta v by delta T at constant pressure. This is a negative sign over here and del 1 negative sign here, over

here. If I put them together, I get μ_s is equal to T by c_p into $\frac{\partial v}{\partial T}$ at constant pressure.

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Isentropic Expansion

- For an ideal gas, the equation of state is $v = \frac{RT}{p}$
- Differentiating w.r.t **T** at constant **p**, we get
- On substitution, we get $\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p} = \frac{v}{T}$

$$\mu_s = \frac{T}{c_p} \left(\frac{\partial v}{\partial T}\right)_p = \frac{T}{c_p} \left(\frac{v}{T}\right) = \frac{v}{c_p}$$

- For an ideal gas $\mu_s \neq 0$, unlike the case in the J - T expansion $\mu_J = 0$. It means that the ideal gas does exhibit a cooling effect, when it undergoes an isentropic expansion.

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So, now I apply the ideal gas and I know for ideal gas pv is equal to RT , and therefore, v is equal to RT by p . If I differentiate that with respect to temperature keeping pressure p constant, I will get $\frac{\partial p}{\partial T}$ is equal to $\frac{R}{p}$, because a T will go. And $\frac{R}{p}$ by p is nothing but equal to $\frac{v}{T}$. See if I put these values in this equation, and if I replace $\frac{\partial v}{\partial T}$ by $\frac{v}{T}$ is equal to $\frac{v}{T}$, I will get T and temperature constant, and my μ_s ultimately gets reduce to the value of $\frac{v}{c_p}$. So, μ_s is equal to $\frac{v}{c_p}$ by using ideal gas law, what does it mean? It means that for an ideal gas μ_s is non-zero quantity.

In earlier case for ideal gas, μ_{J-T} was 0. Unlike the case, in the J - T expansion when the enthalpy was constant we found that μ_{J-T} was equal to 0. It means that the ideal gas does exhibit a cooling effect when it undergoes an isentropic expansion. This is the very important thing to understand that whatever happen when the gas is expanded isentropically, it would always result in cooling, and it is true for the gas also; which was not true in earlier case when the J - T expansion happened. Now, what we will see? We just found that for ideal gas μ_s is positive non-zero, and what we will see now is what happens μ_s when it is subjected to real gas.

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CRYOGENIC ENGINEERING

Isentropic Expansion

$$\mu_s = + \frac{T}{c_p} \left(\frac{\partial v}{\partial T} \right)_p$$

- The derivative term represents the variation of volume with temperature at constant pressure.
- This term is called as the volumetric coefficient and is always positive and hence the isentropic expansion coefficient.
- It is clear that the isentropic expansion results in cooling irrespective of its initial state, unlike the J - T expansion.

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So, here μ_s is equal to this particular quantity. The derivative term represents the variation of volume with temperature; $\partial v / \partial T$ is nothing but volume changes with temperature at constant pressure. This term is called as volumetric coefficient and is always positive, and hence isentropic expansion coefficient will always be positive in this case. Whatever happens μ_s is going to be always positive, because volumetric coefficient will always be positive in this case. It is clear that the isentropic expansion results in cooling irrespective of its initial state unlike J - T expansion, because this term is always going to be positive in this case **all right**.

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CRYOGENIC ENGINEERING

Equation of State

- As derived in the earlier slide, differentiating the van der Waals equation, we get

$$\left(\frac{\partial v}{\partial T} \right)_p = \frac{\frac{R}{(v-b)}}{\frac{RT}{(v-b)^2} - \frac{2a}{v^3}}$$

- Substituting in μ_s , we get

$$\mu_s = \frac{v \left(1 - \frac{b}{v} \right)}{c_p \left[\left(1 - \frac{2a}{vRT} \right) \left(1 - \frac{b}{v} \right)^2 \right]}$$

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Now, if I apply the same for real gas using van der Waals equation, we get the similar term as what it was in the earlier case. And if I substitute that in the same value as μ_s is equal to this term which is what we got from earlier state.

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CRYOGENIC ENGINEERING

Equation of State

- For real gas

$$\mu_t = \frac{v \left(1 - \frac{b}{v}\right)}{c_p \left[\left(1 - \frac{2a}{vRT}\right) \left(1 - \frac{b}{v}\right)^2 \right]}$$

- For large specific volumes

$$v \gg a, b$$

$$\therefore \mu_t = \frac{v}{c_p}$$

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What you can find, again with approximation, I get $\mu_J - T$ is equal to v by c_p which is means that for real gas also $\mu_J - T$ is always going to be positive here. We found that whether is a gas or real gas $\mu_J - T$ is always going to be positive; that means, when expanded isentropic, it will always result in lowering of temperature.

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CRYOGENIC ENGINEERING

Comparative Study

J - T Expansion	Isentropic Expansion
It has a condition of T_{INV} .	No such condition exists.
It produces no work. This is an Internal Work process.	It produces work. This is an External Work process.
The device is simple in construction.	The device involve complex mechanisms.
Normally used for a phase change of fluids.	Normally used for a single phase fluids.
The clogging of constriction is a disadvantage.	Regular maintenance and periodic checks are required.

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If I want to compare isenthalpic expansion with isentropic expansion; so, J - T expansion with isentropic expansion, I should get the J - T expansion as a condition of T inversion or inversion temperature. I do not have any such condition in this case. Similarly, when I have a J - T expansion, I get no work output and this is called therefore as internal work process. While here I get some work output and therefore it is called as external work process. The J - T expansion is a very simple device is a very simple in construction. While in this case, you will have a lot of rotary mechanism, reciprocating mechanism, lot of linkages, frictional losses, etcetera, and therefore the device is a complex device in this case. It is normally used for phase change of fluids that is what we talked about, because J - T expansion always bucks for real gas, and therefore, we always prepared to have phase change of fluid during the expansion. While here we should work with single phase fluid, because it have got two phase fluid, it will not be acceptable to the turbo expander or reciprocating expander.

And lastly, because the J - T expansion is subjected through small wall constriction, the constriction can get subjected to clogging with the dust or the particles coming with the gas. And the clogging of the constriction is a disadvantage, and many times you have to clean it. While here one has to do regular maintenance and periodic checks are required, some time the lubrication has to be taken care of anything like that. So, this is what we will compare a J - T expansion on isentropic expansion. And one has to prepare depending on what you want, whether one should go for J - T expansion or one should go for isentropic expansion.

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Gas Liquefaction Systems

System
Thermodynamically Ideal System
Linde Hampson System
Precooled Linde Hampson System
Linde Dual Pressure System
Claude System
Kapitza System
Heylandt System
Collins System

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So, let us talk about the gas liquefaction systems here. I will understand in what with the expansions happen. We talk about now various gas liquefaction systems. And there are various gas liquefaction systems now. And first one is Thermodynamically Ideal system, Linde Hampson system, Precooled Linde Hampson system, Linde Dual Pressure system, Claude system, Kapitza system, Heylandt system and Collins system. Let us talk first about the thermodynamically ideal system, because we will have to compare all other systems with this ideal system as normally happens in engineering. So, **this we** this system serves as a comparison basis and let see how does this system work.

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Thermodynamic Ideal System

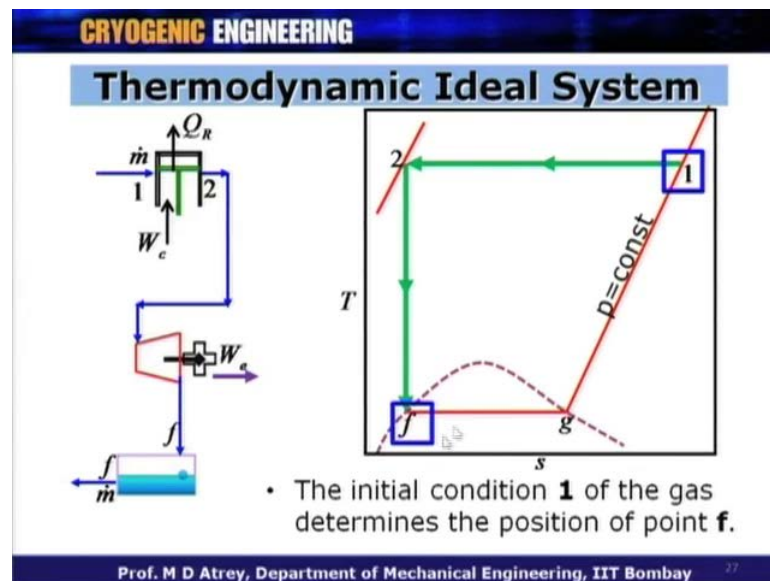
- The salient features of this system are as follows.
- All the gas that is compressed, gets liquefied.
- All the processes are ideal in nature and there are no irreversible pressure drops.
- Process of compression and expansion are isothermal and isentropic respectively.

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So, thermodynamically ideal system will have following salient features. Here this is a compressor, and this is a expander, and this is a container whether liquid after liquefaction is stored and the liquid is transferred from this. So, you can see that process 1 to 2 is a compression process, process 2 to f is a expansion process, and some work output W_e is obtained over here. While in compression process W_c is supplied, this is the work of compression and this is heat which is generated Q_r during compression. So, 1 to 2 process is a compression process, while 2 to f is a expansion process **all right**.

So, here what we can see is whatever gas is compressed is expanded, and it is converted to liquid and this gas is collected as a liquid form from this container. So, there is no return path for this gas. So, in a thermodynamically ideal system, all the gas that is compressed gets liquefied. So, all the processes are ideal processes in this and there are no irreversible pressure drops in a system. The process of compression and expansions are isothermal and isentropic respectively.

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So, if I were to represent this system on a T - S diagram, this would look like this. These are two pressure lines and this is starting point at point 1 over here. So, 1 to 2 process is a isothermal compression process, temperature remains constant. And after pressurization at point 2, here the gas is expanded isentropically, therefore entropy remains constant. And this gas after expansion hits this end which is a such as a liquid curve at this point.

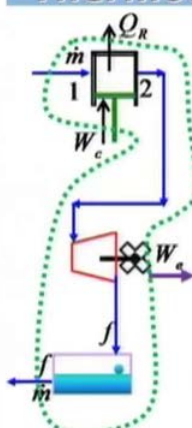
So, whatever gas is compressed completely gets liquefied at this point; this is what the liquid varying, we get at this particular point **all right**.

So, this point is shown as here and this is going to lie at a 1 bar pressure line, because this is my constant pressure, gas is expanded from point number 2 to f from very high pressure to 1 bar which is my starting point. So, this is ambient point 300 kelvin and 1 bar, and this is 1 bar line, and this is what my final state would be. What does it mean? The initial state 1 and final state f can be determined as soon as I know the pressure at this point. So, pressure is known, temperature is known, I know the f condition. So, initial condition 1 of the gas, this condition determines the final position f of the gas after liquefaction.

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CRYOGENIC ENGINEERING

Thermodynamic Ideal System



- It is an open thermodynamic system because the working fluid flows across the system.
- Consider a control volume for this system as shown in the figure.
- 1st Law of Thermodynamics is applied to analyse the system.
- The changes in the velocities and datum levels are assumed to be negligible.

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This is an open thermodynamic cycle and that is why we get the mass transferred gets transferred from the working fluid only, because the working fluid flows across the system. My **(())** consider this as a control volume, I can apply the 1 st law of thermodynamics to this system and I can analyze this system. And the changes in velocities and datum levels are assumed to be negligible and they are neglected while doing this analysis.

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Thermodynamic Ideal System

- The quantities entering and leaving the system are as given below.

IN	OUT
$m_1 @ 1$	$-Q_R$
$-W_c$	W_e
	$m_1 @ f$

- Using 1st Law , we get

$$E_{in} = E_{out}$$

$$\dot{m}_1 h_1 - W_c = Q_R + W_e + \dot{m}_f h_f$$

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The quantities entering and leaving the systems are given as below. So, if you see this table, I have got what is entering this control volume and I have to see what is leaving this control volume. So, what is coming in the control volume is the mass at point 1 which is m_1 at point 1, and W_c that is the work which is being supplied to the compressor W_c . So, what is coming out of the system is the heat of compression or Q_r , W_e is the work of expansion. And what again coming out is m_1 , because whatever compress is m_1 ; same thing is liquefied m_1 at point f .

See if I apply the 1st law, I will say energy coming in the control volume is equal to energy leaving the control volume. So therefore, $m_1 h_1$ is the energy associated with this gas which is entering at point 1, which is h_1 is nothing but the enthalpy of the gas at point 1, W_c which is work done is equal to Q_r which is leaving the system plus W_e plus m_f and h_f associated enthalpy at point f **all right**. So, m_1 dot and **m** m_f dot are nothing but the flow rates or the mass flow rate of the system. In fact as you can see in this case, m_1 dot is equal to m_f dot.

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Thermodynamic Ideal System

- The work W_e produced by the expander is negligible as compared to other terms.

$$\dot{m}_1 h_1 - W_c = Q_R + W_e + \dot{m}_2 h_2$$

- Rearranging the terms, we have

$$Q_R - W_c = \dot{m}_1 (h_2 - h_1)$$

- The compression process is assumed to be isothermal.

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If I further rearrange this system, the work W_e produced by the expander actually is negligible, and therefore can be neglected as compare to the systems. And therefore, if I neglect W_e , my equation gets reduced to $Q_R - W_c$ is equal to $\dot{m}_1 (h_2 - h_1)$, because \dot{m}_1 is equal to \dot{m}_2 . The compression process is assumed to be isothermal that means the temperature remains constant and this is what we talked about.

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Thermodynamic Ideal System

- Hence, from the Second Law of Thermodynamics, we can write

$$Q_R = \dot{m}_1 T_1 (s_2 - s_1)$$

- Also, the expansion process is an isentropic process. Therefore, $s_2 = s_1$.

- By substitution,

$$W_c = \dot{m}_1 T_1 (s_2 - s_1) - \dot{m}_1 (h_2 - h_1)$$

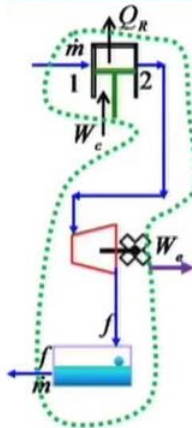
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Hence, from second law of thermodynamics, we can write that Q_r is equal to $m \dot{T}_1$ into difference between two entropy between the process 1 to 2 which is the isothermal compression process. So, heat generated during compression is nothing but $m \dot{T}_1$ into $d s$ that is s_2 minus s_1 . Also the expansion process is isentropic process, therefore s_2 is equal to s_f , and therefore, I can replace this s_2 by s_f . So, by substitution what you get is W_c is a work of compression is equal to $m \dot{T}_1$ into s_f minus s_1 minus $m \dot{T}_1$ into h_f minus h_1 . So, entropy difference and enthalpy difference between the point 1 and final point most important parameters to be use for calculation of W_c .

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Thermodynamic Ideal System



- This work of compression is done on the system. Hence, the value is expressed as a negative quantity.

$$-W_c = \dot{m}_1 T_1 (s_1 - s_f) - \dot{m}_1 (h_1 - h_f)$$

- Work required per unit mass of the **gas compressed** is given by

$$-\frac{W_c}{\dot{m}_1} = T_1 (s_1 - s_f) - (h_1 - h_f)$$

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This work of compression is done on the system and therefore, hence, the value is expressed as a negative quantity. So, work done on the system can be represent as negative quantity. So, minus W_c is the work of compression which is equal to now $m \dot{T}_1$ into s_1 minus s_f minus $m \dot{T}_1$ into h_1 minus h_f . So, work required per unit mass of gas compressed which is $m \dot{T}_1$ is equal to minus W_c upon $m \dot{T}_1$ is equal to T_1 into s_1 minus s_f minus h_1 minus h_f . This is the most important thing that if I know the point 1, I get T_1 into s_1 minus s_f and then I can know the enthalpy difference h_1 minus h_f also. And this determines the work required for compressing this gas and this is work required per unit mass of gas compressed this gas.

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Thermodynamic Ideal System

- Since in an ideal system, mass of gas compressed is same as mass of gas liquefied, $m_1 = m_f$.
- Work required per unit mass of the **gas liquefied** is given by

$$\frac{W_c}{\dot{m}_f} = T_i (s_i - s_f) - (h_i - h_f)$$
- Work required per unit mass of the gas is dependent on the initial condition of the gas.

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Since in an ideal system, the mass of gas compressed is same as mass of gas liquefied, and therefore we got m_1 is equal to m_f or \dot{m}_1 is equal to \dot{m}_f . So, work required per unit mass of gas that is liquefied, this is now W_c by \dot{m}_f . So, I am just replacing \dot{m}_1 by \dot{m}_f . So, W_c by \dot{m}_f with a negative sign here is equal to T_1 into s_1 minus s_f minus h_1 minus h_f . So, here again we can understand that - the work required per unit mass of gas liquefied depends on the initial condition of the gas, because the moment initial condition is T_1 and s_1 are obtained or h_f obtained, and the pressure remains the same, and therefore s_f at saturated liquid condition can be obtained at the same pressure, and h_f is the enthalpy **at the** of the saturated liquid condition obtained at the same pressure. Therefore, moment your initial condition gets decided, this value gets decided immediately. Now, based on this we can have a simple tutorial to understand how do we calculate the work of compression to liquefier gas.

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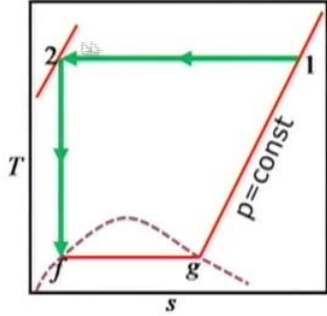
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Tutorial - 1

- Determine the ideal work requirement for liquefaction of nitrogen beginning at 1 bar pressure and 300 K.

• **Step 1**

- The T - s diagram for an ideal thermodynamic cycle is as shown



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So, here there is a problem. Determine the ideal work requirement for liquefaction of nitrogen beginning at 1 bar pressure and 300 kelvin. So, what I know is my initial condition is at 1 bar and 300 kelvin. So, my first step is basically get a T - s diagram for ideal thermodynamic cycle, I shown. This is my T-s diagram and this is what is available in various books; this is available in net also. For nitrogen, we should get this diagram. And find out a condition, as soon as I know the point 1, I will locate the point f. Because I will come down at constant pressure, go the constant pressure line and locate the point f. I draw a horizontal line from 1 to 2, I draw a vertical line from over here, and the intersection would otherwise give me the point 2 at this. So, this is 1, 2 and f is obtained or basically I should know what is my 1 point and what is my f point. This is the most basic requirement to calculate the **work** ideal work requirement for liquefaction of any gas.

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Tutorial - 1

- **Step 2**
- The state properties at different points are as given below.

	p (bar)	T (K)	h (J/g)	s (J/gK)
1	1	300	462	4.42
f	1	77	29	0.42

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Step 2 now; the state properties at different points are as given below. So, what are the state properties at 1 and f? At 1 I have got the pressure of 1 bar, at f also I have got the pressure of 1 bar. Here I got a temperature of 300 and 77 kelvin **300 and 77 kelvin** at 1 bar; 77 is the boiling point of nitrogen. Corresponding enthalpy values have to be taken from this chart now. So, a T-s chart has to be made available, you can see from different books. And the read enthalpy at point f and read enthalpy at point 1; these are the two values. Similarly, read entropy at point 1 and read entropy at point f.

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Tutorial - 1

- **Step 3**
- Substitution into the equation.

	p (bar)	T (K)	h (J/g)	s (J/gK)
1	1	300	462	4.42
f	1	77	29	0.42

$$-\frac{\dot{W}_l}{\dot{m}_f} = T_1(s_1 - s_f) - (h_1 - h_f)$$

$$= 300(4.42 - 0.42) - (462 - 29) = 767 \text{ J/g}$$

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And here put those values in the equation; substitute in the equation these values. So, this is my equation now. W_i we can call ideal work done per mass of the gas which is liquefied. So, W_i by $m \dot{f}$ dot with a minus sign into T_1 into s_1 minus s_f minus h_1 minus h_f , if I put these values what I get is a 767 joule per gram. So, my compressor requirement, per gram of mass which is liquefied, I will get I will have to put in 767 joules. So, my work requirement, therefore ideal work requirement for nitrogen gas to be liquefied is 767 joules per gram.

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CRYOGENIC ENGINEERING

Tutorial - 2

- Calculate the ideal work requirement for liquefaction of Helium and Hydrogen beginning at 1 bar pressure and 300 K. Compare the results.

- **Step 1**
- The T - s diagram for an ideal thermodynamic cycle is as shown

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If I go to the second tutorial now; calculate the ideal work requirement for liquefaction of helium and hydrogen beginning at 1 bar pressure and 300 K, and compare the results. I will follow the same technique as what I did earlier for nitrogen gas. My initial state at 1 bar and 300 K will give me the final position also. So, the T - s diagram for a thermodynamic cycle is as shown exactly same thing. Now, again I will go for helium and hydrogen T-s chart. Look at the point 1 at 1 bar and 300 kelvin, this is the 1 bar line. And for hydrogen, this will heat at 20 kelvin, for helium it will be at 4.2 kelvin. So, the temperature will remain actually 4.2 kelvin. One need not be bothered to locate this point 2, because this point 2 will be at a very high, high pressure - very high pressure. So, what I need to locate is point 1 only and point f only.

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Tutorial – 2

- **Step 2**
- The state properties for hydrogen and Helium at different points are as given below.

	p (bar)	T (K)	h (J/g)	s (J/gK)
Hydrogen				
1	1	300	4190	65
f	1	20	-75	18
Helium				
1	1	300	1575	31.5
f	1	4.2	9.5	3.45

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So, step 2 is the state property of hydrogen and helium at different points are as given below. So, here are my properties. This is point 1 and f for hydrogen, and this is point 1 and f for helium, all are at one bar pressure which is the room temperature is 300 kelvin in both the cases; while the point f is at the boiling point of respect to gases. So, point f is going to be at 20 kelvin for hydrogen and at 4.2 kelvin for helium. Read the corresponding enthalpy values and read the corresponding entropy values as what we have done earlier.

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CRYOGENIC ENGINEERING

Tutorial – 2

- **Step 3**
- Substitution into the equation.

	p (bar)	T (K)	h (J/g)	s (J/gK)
Hydrogen				
1	1	300	4190	65
f	1	20	-75	18
Helium				
1	1	300	1575	31.5
f	1	4.2	9.5	3.4

$$-\frac{\dot{W}_t}{\dot{m}_f} = T_1(s_1 - s_f) - (h_1 - h_f)$$

H₂

$$= 300(65 - 18) - (4190 - (-75))$$

$$= 9835 \text{ J/g}$$

He

$$= 300(31.5 - 3.4) - (1575 - 9.5)$$

$$= 6864.5 \text{ J/g}$$

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See, if I put this values in this equation, this values, what I get for hydrogen is 300 into s 1 minus s f which is this minus h 1 minus h f which is this, look at this minus point, because this is with respect to a particular initial condition or 0 point condition, and therefore, this is minus 75 is written over there, this will become positive in this case. What I get is 9835 joules per gram for hydrogen and if I do it for helium put the same values here, I get only 6864 joule per gram. What does it indicate? It indicates that the work done for liquefaction of hydrogen is more than work done requirements for liquefier helium gas.

So, it is not although the boiling point of helium is just 4.2 kelvin which is less than that of hydrogen. Still the work done to liquefier 1 gram of helium gas is less than that for hydrogen. So, basically it depends on what is my h 1 minus h f values or what is my latent heat in this case. This value is much higher for hydrogen as compared to helium. Similarly, this value also is much higher as compared to what it is for helium. So, it basically depends on the nature of the gas, what is the amount of work required to liquefier hydrogen and work required to liquefier helium.

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Ideal Work Requirement		
Gas	Normal Boiling Point (K)	Ideal Work (kJ/Kg)
Helium	4.21	6819
Hydrogen	20.27	12019
Nitrogen	77.36	768.1
Air	78.8	738.9
Argon	87.28	478.6
Oxygen	90.18	635.6
Ammonia	239.8	359.1

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So, this table give you ideal work requirement for various gases. And you can see the boiling point of helium is 4.21, 20.27; still the ideal work requirement for hydrogen in this case is more than that for helium. Similarly, nitrogen is only 768, but for air it is 738 argon, oxygen and ammonia. These are different values given. We will expect that you

can work out your own ways of calculating is ideal work requirement, and you can compare this values with this table or you can refer to any literature, and you can compare your findings with this ideal work requirement in terms of kilo joule per kg or joule per gram.

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Assignment

1. Calculate using the charts, ideal work of liquefaction for
 - Air
 - Oxygen
 - Helium
 - Ammonia
2. Compare the values obtained with the values given in the table

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With these tutorials you should be in position to calculate using charts, the ideal work of liquefaction for air, oxygen, helium, we have already done ammonia. Take the T - s diagram - T-s chart for these gases, find out the enthalpy values, find out the entropy values and calculate the ideal work requirement to liquefy this gas. Compare these values with the values given in the table. So, we have already given the table and whatever values you calculate, please compare, the values will be little different depending on how good or bad, your charts you read, sometimes you can read some wrong values and according to which your values can show some difference.

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Summary

- For an ideal gas $\mu_J = 0$, but for a real gas, J - T coefficient depends upon the relative magnitude of departure from Joule's Law and Boyle's Law.
- The gases like nitrogen, air show J - T cooling when expanded at room temperature because the T_{INV} is more than room temperature.
- While the gases like He, Hydrogen and Neon, are to be precooled in order to experience J - T effect.

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In order to summarize, whatever we have covered today, we found that for ideal gas μ_{JT} is equal to 0. But for a real gas J - T coefficient depends upon the relative magnitude of departure from joules law and Boyle's law. We saw the two parameters and we **we** found out that these two parameters in the bracket decide whether the μ_{JT} is going to be negative, positive or 0. The gases like nitrogen, air, show J - T cooling when expanded at room temperature. Because the inversion temperature for these gases is more than room temperature; so, one need not worry about temperature in this case, because the inversion temperature for these gases is way above the ambient temperature of 300 kelvin, and therefore, if they are subjected to J - T expansion, they would immediately result in cooling. However, the other gases like helium, hydrogen and neon they required to be precooled to bring down their initial temperature below the inversion temperature, and then only they will experience J - T effect or J - T cooling.

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Summary

- In expansion devices like turbo-expanders and expansion engines, the expansion process is isentropic or reversible adiabatic.
- Coefficient of an isentropic expansion is given by
$$\mu_s = + \frac{T}{c_p} \left(\frac{\partial v}{\partial T} \right)_p$$
- The isentropic expansion is always positive for both real and ideal gases. It results in cooling for any initial state, unlike the J - T expansion which is dependent on T_{INV} .

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In expansion devices like turbo expanders and expansion engines, the expansion process is isentropic or reversible adiabatic. The coefficient of isentropic expansion is given by μ_s value which is T by c_p into $\frac{\partial v}{\partial T}$ at constant pressure. And we found that the isentropic expansion is always positive for both real and ideal gases. Therefore, one need not worry about inversion temperature for search cases. It results in cooling for any initial state, unlike the J - T expansion which is dependent on what is the T inversion temperature for that particular gas.

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Summary

- J - T expansion is normally used where phase changes are required, where as isentropic expansion is used for single phase fluids.
- In a thermodynamic ideal system, all the gas that is compressed gets liquefied.
- The work required per unit mass of the **gas compressed** and **gas liquefied** are given by
$$-\frac{W_c}{\dot{m}_f} = T_i (s_i - s_f) - (h_i - h_f)$$

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The J - T expansion is normally used where phase changes are required, **where** whereas isentropic expansion is used for single phase fluids; this is what is met clear **right**. Because J - T expansion normally we want to have real behavior, we want to have non-ideal behavior, and therefore, we prepared to have a phase changes or we would prepare to have liquid in this case at initial condition also. While isentropic condition expansion is normally used for single phase fluid, because having liquid there can spoil can have problems for the turbo expanders or reciprocating expansions.

In a thermodynamic ideal system, this is what we saw all the gas that is compressed gets liquefied. And the work required per unit mass of gas compressed and gas liquefied are given by W_c by m_f dot minus sign is equal to T_1 into entropy difference between the initial state and the final state minus the enthalpy difference between the initial state and the final state. Thank you very much.