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Lecture No. #08 Gas Liquefaction and Refrigeration Systems

So, welcome to the eighth lecture of cryogenic engineering under the NPTEL program. During the earlier topics, we had introduction to cryogenic engineering, where we talked about introductory knowledge of cryogenic engineering. And the second topic was properties of cryogenic fluids wherein, we talked about properties of various cryogens. We talked about direct temperature entropy diagram.

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Also, we talked in detail about hydrogen, and helium. In helium, we talk about lambda transition, and super fluidity. The third topic which we talked about was properties of materials at low temperature. And here, we studied how the mechanical, electrical, and thermal properties change at low temperature for a various materials. Also we talked about super conductivity, which is nothing but super conductivity properties of materials of various materials, there are transition temperatures, and thing like that. This was a kind of introductory knowledge in order to understand, what cryogenic engineering is all about.

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Now, based on these topics, we are now entering the real Cryogenic engineering field. And that is to deal with the current topic, which is gas liquefaction, and refrigeration system. As you understand that Cryogenic engineering is basically the production of low temperature utilization of low temperature etcetera, the gas liquefaction at low temperature and refrigeration at low temperature is the most important topic in Cryogenic engineering's. This topic is a very important topic and under this topic, I will touch up on the basics of refrigeration and liquefaction, which possibly a few of may know. Talk about various techniques of production of low temperature and we will talk about each of those ways. And we will try to understand what is the plus points and the negative points about each of those techniques. The naturally every system has got some ideal system and then we sort of go on adding the non ideality the system.

So, once we understand how we produce the low temperature, we will talk about the ideal thermodynamic cycle for refrigeration as well as for liquefaction of any gas. The next topic to be dealt with will be, the various liquefaction cycles. Now, here in we talk about how to liquefy various gases? As you understand in order to liquefy the gases, first of all we should reach it is boiling point temperature. And all these gases have got their boiling point at a very low temperature. So, first of all one should reach the lowest possible temperature, with regards to the gas we are talking about to liquefy. And then we should talk about, how to liquefy these gases? So, first and foremost is how to attain low temperature? That is first thing we should to understand.

And second thing is now, how to liquefy those gases? Having attained those low temperatures which are close to the boiling point of these gases, all these four, five subjects under this topic will be covered in seven to eight lectures. And we see as we go around we will see how this topics develop. At a same time, each of this lectures will be having It is own tutorials and assignments. And they are all included at the end of each lecture. So, why do we require to study the liquefaction and refrigeration? What is it is usage?

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Possibly, we have covered that in the first introductory lecture. Just to cover up what exactly is the need for having gases in liquid form or refrigeration at very low temperature.

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So, first of all liquid nitrogen is used as precoolant in most of the cryogenic systems and also it is used to provide an inert atmosphere in welding industries. So, one can find the usage of liquid nitrogen in various manufacturing techniques as well as a precoolant for liquefying other gases. The Cryogens like lox, which is liquid oxygen, liquid hydrogen are used in rocket propulsion. And in the recent past liquid hydrogen is being considered for automobile. Hydrogen gas becomes liquid at 20 Kelvin.

The transportation of most of these gases, across the world is done in liquid state. So, here is the reason why one should liquefy the gases because they enable us transporting these gases more safely in a liquid form. At the same time the gas refrigeration at a very low temperature, when I talking about gas refrigeration. It is mostly less than 120 Kelvin can also be used as precoolant. So, if you refrigeration available, this refrigeration effect can be used to precool the other gases. That can be used as precoolant for liquefiers and also in applications where low temperature gases are required.

Now, whereas, superconductivity for example, requires low temperature it requires gas refrigeration in those cases. So, these are basically the reasons just to give you some usages of gas liquefaction and gas refrigeration there are plenty otherwise. Now, before I go into the real refrigeration system, as to how we produce the low temperature? We will study the basics of refrigeration; many of you again must have covered this in thermodynamics or in applications of thermodynamics. I will just touch upon the important points here and then we can go ahead.

So, what is refrigeration? The technique of preserving food and perishable goods is an idea of prehistoric time. We know this $((\))$ quite some time now. And a system which produces cold or maintains such low temperature is called as refrigerating system. So, basically refrigeration system will produce the cold, at the same time it will maintain this cold or low temperature, this process is called refrigeration. Normally, a refrigeration system operates in a closed cycle manner, just as what we see in a domestic refrigerator, it is a closed cycle system mostly.

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Now, we will apply, we will understand the first and second law of thermodynamics because we are going to use these laws later on to study different cycles. So, if you want to understand the basics of refrigeration, we just go through the statements of the first law and second law of thermodynamics.

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So, first law of Thermodynamics states, it is just the manifestation of law of conservation of energy. So, whatever goes in comes out. That is what we say as conservation of energy. Here, you can understand that if in this system, we transfer some heat. So, whatever change of heat happens in the system, this change of heat is utilized to increase the internal energy U of the system plus some of the Q is utilized in producing a work. This is what we call as first law of thermodynamics. It reads like this. The change in heat in a system is equal to sum of changes in the internal energy U and the work W, it produces. Basically, whatever goes in is equal to whatever gets stored and whatever comes out.

Mathematically, we can write d Q is equal to d U plus d W which is understandable from this figure. The second law of Thermodynamics also is stated over here. Normally, we know that heat flows from the high temperature to low temperature. The second law states that, if you want to transfer heat from low temperature to high temperature we have to do some work and this is what is shown in this system. The second law states that, the work is required to pump the heat from low temperature to high temperature. So, if you want to pump in heat from low temperature which is Q L then you have to supply W in that case.

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Hence, work input W is required to generate and maintain low temperatures T L in this particular case. Now, C O P or the coefficient of performance is defined as a ratio of heat extracted that is a Q L which is nothing but the refrigeration effect to the work input. So, whatever amount of work is required. So, that we generate this effect of Q L, the ratio of this Q L by w is what is called as C O P. And mathematically, we know that W is nothing but Q H minus Q L.

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So, C O P nothing but Q L upon Q H minus Q L, the best performance is delivered by a system when it adapts a reverse Carnot cycle as the working cycle. I think this all of you know because you have studied the same in Thermodynamics. So, this is being the most ideal cycle for refrigeration, a reverse Carnot cycle is what we use to get ideal C O P of a system. So, C O P of such a system is called as Carnot C O P which is nothing but ideal C O P and is given by T L upon T H minus T L. The Carnot C O P is often used as a benchmark to compare the performance of any refrigeration system.

So, once we know that, what is the ideal C O P of any machine? Which is operating between T H and T L, real system can be compared to the ideal C O P that is obtainable using a reverse Carnot cycle. The C O P represents Watt of cooling effect obtained per Watt of power input at a particular temperature, this is what we understand from Q L upon W. For example, if we avoid a system over here, wherein T H is equal to 300 Kelvin and T L is equal to 100 Kelvin. And if it is desired to maintain T L as 100 Kelvin with one Watt as cooling power and T H is 300 Kelvin, the Carnot C O P is given by T L upon T H minus T L. If we put the values, we will get 100 upon 200 which is 1 by 2.

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What does it mean? It means that two Watt of input power is required to deliver one Watt of cooling effect at 100 Kelvin. So, if I extend the same logic for different temperatures and if I want to produce low temperatures of 270 100 20 Kelvin or 4 Kelvin, you can understand from this table, that the Carnot C O P goes on reducing. So, the C O P is 9 first and 0.5 at 100 Kelvin and 0.07 at 20 Kelvin and 0.01 at 4 Kelvin. At the same time actual C O P is much less than what you get over here. W P by W C is nothing but one by Carnot C O P.

Basically, W P is a work input and W C is the cooling effect. And it talks about how much work input is required in order to produce one Watt of cooling effect at 270 Kelvin? So, if I want to produce 100 Kelvin, I require two Watts to produce one Watt of cooling effect at 100 Kelvin and if I want to produce four Kelvin then I require 74 Watts as power input to produce one Watt of cooling effect at 4 Kelvin. This basically gives you an idea, that as T L decreases as you want lower and lower temperature Carnot C O P decreases, which also means, that your power input will go on increasing to produce one Watt of cooling effect at lower and lower temperatures.

The point to be understood here is, if you want to produce low temperature, the work input is going to be higher and higher and your C O P is going to be less and less. Now, in this particular topic we are going to cover various cycles and all these cycles are shows in some kind of schematics.

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And in this schematics, we are using some symbols which are normally standard symbols and I am just going to explain to you those symbols are. So, that it will be beneficial for you to understand these cycles in future. So, symbols used in Liquefaction cycle schematics, these symbols are used for refrigeration cycle also. The symbols used in different cycle schematic of refrigeration liquefaction systems are as given below. Compressor is a very important part of this cycle. And a usual sign of compressor is given as this. A compressor increases the pressure of the gas interacts with surrounding in the following ways.

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It gives heat of compression Q R to the surrounding and W C denotes here the work required for compression. Then there is various connecting flow line. All this are denoted by this particular schematic, the flow of liquid assumed to be frictionless and there is no pressure drop during this flow. That is what the assumption is the direction of the arrow indicates the gas flow direction. Now, once you liquefy the gas, it is stored in liquid container and it is assume that the container is perfectly insulated from the surrounding.

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The schematic for the Liquid Container is as shown over here. So, what you can see from here? Whenever the liquefaction of the gas happens, the liquid is stored in this container and this is what we call as liquid container. This is what it represents liquid container. Now, in these cycles what we also required is an Expander. The schematic for Expander is as shown over here. The expansion is isentropic in these cases whenever the schematics like that, during expansion and it produces work done as W e. Also one of the very important components of this liquefaction refrigeration cycle is the Heat Exchanger. The heat exchanger could be a two fluid type or a three fluid type. And that is what a two fluid type heat exchanger is shown like that.

It has got one inlet from hot side may be and one inlet from cold side, if they are counter flow like arrangement. Or at the same time, there could be three fluids based on the liquefaction cycle, we will see that we can have three fluids. And it shows two inlets, one outlet here. Two outlets and one inlet showing that there are three fluids interacting at one time.

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With this background, I will take you to now, how do we produce low temperature? What are different methods by which we can produce low temperature? The first is very simple process, and which is followed in our domestic refrigerator called Throttling process. How does it take place? The high pressure gas flows through a small constriction which a valve could be there or a capillary tube could be there. Basically, it kind of offers resistance to the flow of gas, and because of which the gas gets expanded from higher pressure to low pressure. And it will result in lower of temperature.

So, lowering of temperature can be achieved when the gas expands from higher pressure to low pressure. When it passes through a constriction, produced by a valve or let us say capillary tube. The other method for production of low temperature is heat exchanger. In which you have got a one hot fluid going over here and one cold fluid entering from this side. And because of the heat exchange temperature of the hot fluid will come down to low temperature, producing cold. The cold is basically given by the cold fluid which is entering from other side. So, fluid A in, fluid A out. Fluid B in, which is possibly the cold fluid and the fluid B is coming out at this point.

Now, the bigger systems or the larger systems may be formed not only by one of these techniques, but it may have both these things in the circuit or in the schematic. So, one can have combination of throttling method or heat exchanger or expansion devices as we see in the next slide.

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This system or this will enable us to increase the capacity of the system or to reach very low temperatures. The third system is nothing but the compression and expansion of gases. So, we have a compressor in this, the gas gets compressed to high pressure here, and then the gas gets expanded in a expansion device and you can get lowering of temperature. And this lowered temperature could be transferred by using heat exchanger to the outside, gas to be cooled or outside material to be cooled and the gas again goes back. So, by compressor here and expansion over here, we can produce lower and lower temperatures. So, arrangements like precooking Joule Thompson expansion, expansion devices like reciprocating or turbo expanders may be used in these systems.

So, a system now will comprise of all these three possibilities A J T expansion or a Joule Thompson expansion valve or a heat exchanger or expansion devices like, turbo expanders or reciprocating expanders and important the compressors. These will what make a compression expansion device and this is what a complete system would be. C O P and capacity of these machines can be improved by proper choice of compressor expansion device, heat exchanger etcetera. One can improve it is C O P as well as capacity and one can really reach down lower in temperature. Now, if I were to operate a particular machine in the form of a refrigerator, this is what a cycle would be.

A refrigerator operates in a closed Thermodynamic cycle, the gas always moves in a closed cycle manner and therefore, we call it a closed Thermodynamic cycle.

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The rate of mass flow gram per second, which is flowing through the cycle, is going to be the same at all points. The heat is exchanged between the cold end and the object to be cooled. So, if I want to cool this object, this object is coupled with this heat exchanger and this is how the heat exchange happens in a refrigerator. At the same time, this heat exchanger also can be used to basically liquefy any other gases whose boiling point is above the lowest temperature produced by this circuit.

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So, this cold and heat exchanger can also be used to liquefy gases, depending upon the boiling point of a particular gas over here. So, infect a refrigerator can also be used as liquefier in this manner or it can also be used as a refrigerator in this manner, but in both the cases we can see that the cycle is a closed cycle. Now, when a particular circuit operates in a liquefier mode, what you can see here? The gas is compressed and the gas is expanded. And after the expansion, the gases got turned into liquid because of the lowering of temperature and because it is going to be used as liquefier. I will take some liquid out from the liquefier which means that, I have taken some of the mass which is flowing in the system I have taken it out.

As a result of which I have to add some makeup gas in the system. This is the difference between a liquefier and a refrigerator, that this is not anymore a closed cycle it is an open cycle. Because the working fluid is getting liquefied and I am removing some of the liquid which is generated in this cycle. So, liquefier often produces cold liquid that is drawn off from the system. For example, a nitrogen liquefier produces liquid nitrogen. If I got nitrogen flowing in this system or if nitrogen works as a working fluid, then some of the liquid nitrogen which is produced at this point, I will take it out.

And whatever amount I take out, I am going to add it as makeup gas. So, some of the nitrogen gas I would add in the circuit, as makeup gas in this particular circuit. Since, the mass is drawn out from the circuit; it operates in an open Thermodynamic cycle. The mass deficit occurring, due to loss of working fluid is replenished by a makeup gas connection and this is what we just talked about. So, this is basically a cycle or a schematic showing a circuit is working as a liquefier, which is different than what we saw earlier which was a refrigerator case.

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Now, if I got a case wherein I want to have refrigerator and liquefier both working in a circuit so, systems can also be used to liquefy gases as well as to cool one object as refrigerator. So, I want to produce a system, I want to have such a system which will work as also as liquefier and also as refrigerator and this is what we see here. So, here some of the working fluid is liquefied and it is transferred outside. At the same time liquefied working fluid is used to cool some object using this heat exchanger. So, in this case, if you take this off this is nothing but a refrigerator. At the same time if you add this, this is nothing but a liquefier. But as soon as you use it is liquefier, what you need to know is a open cycle.

And therefore, a makeup gas has to be added in order to overcome the deficit of mass which is taken out from this particular system. So, one such arrangement is shown over here and a cold heat exchanger you use to transfer cold from the liquid container to the object to be cooled over here. If I want to compare all these three systems, what you can see is, this is a refrigerator, at the same time this refrigerator is working like a liquefier also. But ultimately this is a closed cycle refrigerator or a liquefier. Now, this is a liquefier which is a open cycle and whatever gas is taken off a makeup gas is added to the system.

So, this is a liquefier. At the same time what I have got third system as a refrigerator plus liquefier. The difference between this system and this system, this is a open loop liquefier plus refrigerator. And the working fluid whatever is liquefied; some of the working fluid in the liquid form is taken out. And this liquefied working fluid also acting like a refrigerator in order to cool this object to be cooled so, herein the whole cycle is working as a refrigerator, at the same time it is working like liquefier. This is an open cycle and therefore, makeup gas has to be admitted to the system so, that the deficit can be overcome. This are three comparative cycles which are working like a refrigerator liquefier and refrigerators plus liquefier.

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What is to be understood is, a refrigerator is a closed cycle, while the liquefier is a open cycle. This is the most important thing to be understood from this comparison. Now, we will come to very important expansion which is Joule Thompson expansion of which we talked about earlier. Whenever high pressure gas passes through a constriction produced by let us say valve or a capillary tube. The gas expands from higher pressure to low pressure and this may result in lowering of temperature. So this initial condition and this is my final condition. The gas is entering at particular pressure and temperature. And it is leaving this valve at a particular low pressure, at the same time temperature here could be different than what it was at the initial condition.

If I want to compare various parameters at the initial state over here and at the final state over here, there are various parameters. And these parameters are the mass flow rate the enthalpy at the inlet, the velocity at the inlet, the datum level at the inlet. And in this process of expansion from high pressure to low pressure, how much heat is generated or supplied? And how much work input is given or taken off?

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So, all these parameters need to be related in order to understand what this Joule Thompson process likes. So, every time whenever we want to study certain system, what is to be done is, to apply the first law of Thermodynamics which is the algebraic sum of; whatever is going in is equal to whatever coming out plus whatever is stored in a system. So, as you know that, the statement reads as d Q minus d W is equal to d U. So, whatever heat goes in minus, whatever work it produces. The difference between two is basically is going to be utilized in increase or decrease of internal energy of this particular system. See, if I want to give a general form to this particular equation, we can say Q net minus W net is equal to sigma U.

That means final form of energy which is leaving the system minus sigma U initial. So, Q net minus W net is equal to sigma U out minus sigma U in that is energy leaving the system minus energy entering the system.

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So, if I apply the first law, the Q net minus W net is equal to final energy of the system which is m dot f into enthalpy of the system. The kinetic energy of the gas which is leaving the system, and the potential energy or the datum level of the gas which is leaving the system minus whatever is entering the system, that is m dot i which is the mass entering the system. It is heat content that is, it is enthalpy at the entrance. It is kinetic energy at the entrance and it is datum level of the potential energy at the entrance. So, Q net minus W net is equal to m dot f h f plus v f square by two g plus g z f minus m dot i h i plus v i square by two g plus g z i. As we know in a Joule Thompson expansion, there have been no Q and no W involved in this particular expansion device.

Because there are no moving components no heat is given or produced in this system and therefore, Q net and W net both these quantities are equal to 0 in this case.

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So, having done Q net and W net equal to 0, at the same time if we say that the velocity changes are hardly anything similarly, the datum levels are not changed, then this two quantities also get cancelled out. And therefore, what remains ultimately is m dot f h f is equal to m dot i h i. And if we say that the mass flows are equal at inlet and outlet which is, what the case should be, then what you ultimately get is h f is equal to h i what does it mean. It means that a Joule Thompson expansion is an isenthalpic expansion which also means that, the enthalpy during the Joule Thompson expansions remains constant. This is a very important derivation and this will be used because the Joule Thompson expansion is always used in liquefier and refrigerator.

And whenever we apply such equations, we have to apply these equations in order to find out the efficiency of the C O P of liquefier or refrigerator. This will be very, very important for us when we will analyze different liquefaction and refrigeration cycles.

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So, let us see now, what is joule Thompson effect? If you see at this particular graph, what you can see is, a temperature pressure plot for any gas. And what you can see on this temperature pressure plot is, the line of constant enthalpy which is h is equal to constant and you can see all these lines. And what you can see from this particular graph is that the constant enthalpy line shows maxima at a particular temperature. So, each of this line you can see that it goes through a maxima, while above this it is not showing any maxima. And this is very important to understand from Joule Thompson expansion point of view, there after expansion, pressure reduces and what we expect from it that, the temperature also should reduce.

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So, here if I join all the maxima of different curves, what you get is this red line. And if we join all this maximum of the isenthalpic line We can divide this region into region one and region, the region one is on the right side of this red line and region two is on the left side of this red line. So, we can say region two is basically enclosed by this red line. Now, if we consider this region one, consider point A over here, which is at this particular point. And correspondingly this will have this pressure and this temperature as denoted by these two horizontal and vertical lines.

Now, if I expand a gas from A to B, whenever I do expansion the pressure would reduce. So, whenever I expand the gas from A point to B point, what you can see from this particular figure? If I expand from this gas by a Joule Thompson expansion process, the enthalpy would remain constant and therefore, the point B would lie on this isenthalpic line. So, both A and B would lie on the same isenthalpic line and corresponding state at point B would be this pressure. Which is less than the pressure at point A, while what you can see from this figure is the temperature at pointy B is more than the temperature at point A.

That means, a isenthalpic expansion has resulted in increase in temperature. Whenever the gas is expanded from state A to state B in this region number one, what you found it? Pressure reduces. But the temperature has increased. So, it can be seen from this result, that there is increase in temperature of the gas and naturally would not like to have this because what we want to have is reduction temperature after expansion. So, what would I do now? I will come to region number two, which is enclosed by this red line. And now, if I take one more point called C over here on the same isenthalpic curve, but this is inside this red line. And now, I have got at C some pressure and temperature associated with this point C.

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And if I expand this gas at point C to point D, which is this state now. Again, point C and point D the expansion has happened due to Joule Thompson expansion process. The enthalpy would remain the same; that means, C and D is an isenthalpic expansion process. And if I see the state of D, I can understand that there is a decrease in pressure has resulted in drop in temperature at. What is to be understood therefore is if I expand the gas in region number one, I get heating. My temperature increases after expansion. But if I expand the gas inside this region number two after isenthalpic expansion, I get cooling or the temperature of the gas gets reduced after expansion. And this is a very important difference.

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So, ultimately, what I want to understand is, what is my del T by del P? What is my temperature change with change in pressure, if the enthalpy is kept constant? So, the parameter del T by del P at constant enthalpy is the most important parameter. If I reduce the pressure, if it results in increase of temperature then delta T by delta P for the process A B would be negative while for the process C to D. Wherein temperature decreases when the pressure decreases delta T by delta P at constant enthalpy is going to be positive.

And whenever delta T by delta P at constant enthalpy is positive, it would result in cooling. While whenever delta T by delta P is negative, it will result in heating if the expansion of the gas happens. And this is what is most important thing. This ratio delta T by delta P at constant enthalpy is called as Joule Thompson coefficient while this effect is called as Joule Thompson effect. And many times now I will refer to it as J T expansion, J T effect or J T coefficient. So, delta T by delta P at constant enthalpy is what we call as Joule Thompson coefficient. Which shows J T effect which also shows that, if the J T effect is going to be resulting in lowering of temperature or it is going to be resulting in heating of the gases or increase of temperature.

Mathematically it is called as mu J T. So, mu J T is nothing but Joule Thompson coefficient or A J T coefficient, which is del T by del P at constant enthalpy.

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If mu J T is more than 0 or positive, it will result in cooling as it what happens in a process C to D. If it is less than 0 or if it is negative, that is what is happening in A to B process which will result in heating. And if it is equal to 0 which is going to happen at this maximum points, it is not going to produce both of these possibilities; that means, it will neither show heating nor it will show cooling because mu J T is equal to 0 at this particular points.

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So, if I want to produce cooling for my liquefier or refrigerator I should ensure that I am in this region number two and I should not be in the region number one. And this dividing line between region number one and region number two is called as inversion curve. So, this is what we call as inversion curve and if I want to produce cooling, I should be on the left side of this inversion curve. If I want to produce heating; however, I should be on the right side of this inversion curve. And this are all the temperatures on the inversion curve and at P is equal to 0, the temperature is maximum on this inversion curve. And this is called as maximum inversion temperature or T inversion temperature at this particular point.

So, what we understand from this is? This is called as inversion curve. And the temperature at P is equal to 0 on the inversion curve is called as maximum inversion temperature can be identified as represented as T inversion also. It is clear that the initial state of the gas should be inside the region number two or below T inversion, if you want to produce cooling effect. You can see if it is above the T inversion temperature, it is not going to produce cooling, because every time the constant enthalpy line is just to going up over here. So, one should ensure that, if you want to produce cooling you should be below T inversion temperature. The starting temperature of the gas before expansion should be below T inversion or one should ensure that the state always lies inside this inversion curve.

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Now, let us see the mathematical form in order to understand what exactly happens beyond the inversion temperature and below the inversion temperature. From the earlier plot the enthalpy H is a function of both pressure and temperature. And mathematically, I can write H is a function of P and T. Using the calculus, the following can be derived that is if I differentiate H with respect to P keeping the temperature as a third parameter constant. So, if I do in a cyclic way del H by del P keeping temperature as same, del P by del T keeping enthalpy as same, del T by del H keeping P as same. Then the product of all these three parameter is equal to minus 1. This is what we understand from calculus.

So, del H by del P at constant temperature into del P by del T at constant enthalpy into del T by del H at constant pressure is equal to minus 1. What we understand is from this arrangement, if I want to understand mu J T mu J T is nothing but del T by del P at constant enthalpy. See, if I take del T by del P on right side, I will get del T by del P at constant enthalpy which is nothing but mu J T is equal to del T by del H at P constant pressure into del H by del P at constant temperature with a negative sign. This is very important to understand that mu J T now depends on del T by del H and del H by del P.

Please understand this equation and now, my further calculations are basically oriented to understand what are these two parameters? What is delta T by del H when pressure is constant? And what is del H by del P when temperature is constant. In order to derive mathematical formulation for this again use calculus and here now, we use entropy. Entropy is a function of temperature and pressure. Which we all know from Thermodynamics, what is the temperature? What is the pressure? Correspondingly, these are the two states, depending on which I will get what is the entropy.

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See, if I write this entropy and if I differentiate d s being a function of T and P. And I want to write d s as a partial derivative of entropy with respect to temperature and with respect to pressure. Then the whole thing I can write mathematically in this form which is d s is equal to del s by del t at constant pressure into d T plus del s by del P at constant temperature into d P. If I multiply the whole thing by T, what I get is, T d s is equal to T into delta s by delta T at constant pressure into d T plus T into del s by del P T into d. So, first I got entropy expression which I represent in a form of d s, in partial differential form of the two other variables.

And then I multiplied by T because T d s has got something very significant rule. So, here I want to understand what is T into delta s by delta T? It is nothing but c P value. Delta s by delta T at constant pressure multiplied by T from Thermodynamics we understand it is nothing but specific capacity at constant pressure. And if I use Maxwell's equation in Thermodynamics, I can calculate del s by delta P, at constant temperature as minus del v by del T at constant pressure. These relations are given in all Thermodynamics books and therefore, I will not spend time in deriving these two equations.

What we understand from this if I put the value of c p and minus del v by del t into this, what I get ultimately is, T d s is equal to c P d T minus T into del v by del T at constant pressure into d P. So, what I understood from this? Basically, is replacing these values by c P and del v by del T in the equation which we obtained for T d s. Now, if I know enthalpy, the equation is absolutely straight forward which is, what we understand in Thermodynamics? d h is equal to T d s plus v d p. So, I got expression here for d h which is enthalpy change, and here comes the T d s, and here we get an expression for T d s. So, if I put the value of T d s expression in this equation, I will get a new expression for d h which is d h is equal to c p d T plus all these parameters.

So, if I put this here, and rearrange it, because of the minus sign I get minus T into del v by del T at constant pressure minus v into d P. And this is the expression I get finally, for the d h which is what I am going to use later.

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Again using the calculus now, in the similar fashion I got enthalpy as a function of pressure and temperature. So, if I again represent d h in a partial derivative format. So, I get d h is equal to del h by del T at constant pressure into d T plus del h by del P at constant temperature into d p. This is exactly what we did earlier for entropy values. So, here, I get an expression for d h in this format, using the partial derivative format, and earlier if you recollect I got an expression for d h in this format. So, if you compare these two equations, we can see that there is a d T over here, and there is a d T over here. And therefore, the coefficients of d T in both the cases should be equal.

Similarly, the d p over here, and d p over here and the coefficients of d p in this case should be same as del h by del p at constant temperature. So, what I understand from here is, this bracket is nothing but c p and this bracket is nothing but this bracket and this is what we understand from this. So, these two red brackets denote the same thing while these two red brackets also denote the same thing, and this is the whole exercise what we did is basically to get these values. So, if you go back to our earlier expression of mu J T which is nothing but del t by del p at constant enthalpy which is this I will put these values over here now.

And if I put those values over here; that means, delta T by delta h is nothing but one by c p and del h by del p is nothing but the whole of this bracket which is kept over here.

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So, this is the form of mu J T which I get. At the same time what I get for ideal gas is P V is equal R T. So, if I differentiate it with respect to P, I get del v by del T at constant pressure nothing but R by P which is nothing but V by T. So, del v by del T is nothing but v by T, and if I put this value of del v by del T in my expression, del v by del T is equal to v by T, what you understand from this is, T and T gets cancelled, v minus v is equal to 0 and in this case I get mu J T is equal to 0. Now, mu J T is equal to 0 is true for the ideal gas where we assume p v is equal to R T. If the ideal gas undergoes Joule Thompson expansion then mu J T is equal to 0.

What does it mean? It means that the ideal gas does not show any change in temperature when it undergoes J T expansion. So, if I want to expand the ideal gas, it will not show any temperature change, because mu J T is equal to 0 in this case. So, what will show the changes? Not the ideal gas, but the real gas which is completely away from ideality.

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So, whatever we have covered till now, I would like to summarize, and because this is a little more important chapter, beginning of liquefaction and refrigeration I would give this summary in little detail. So, what we covered today is, the basics of refrigeration, and liquefaction systems. If you remember we started talking about the first law of Thermodynamics and second law of Thermodynamics. And then when we came up with the definition of coefficient of performance or C O P which is nothing but refrigeration effect available at a particular temperature divided by power input. And then we understood the concept of Carnot C O P or the Carnot cycle based which is nothing but the ideal C O P.

And if we know the lowest temperature T L, then Carnot C O P is nothing but lowest temperature T L divided by the difference between two temperatures that is T h minus T L. Now, this Carnot C O P is basically the ideal cycle, and what it serves? It serves a purpose that an actual system C O P could be compared with Carnot C O P. So, C O P calculated by Carnot cycle or Carnot C O P is the maximum value of a C O P. And the actual performance of the system of refrigeration can be compared with maximum possible C O P, which is obtained using Carnot C O P definition. We also studied that as

a required low temperature decrease; that means, as you want to attain lower and lower temperatures T L the Carnot C O P decreases.

That means, if I want to achieve temperatures, let us say 270 Kelvin, your C O P is going to be 270 divided by 30 in that case, that is 9 in that case. However, as you go on reducing temperature and as the T L value goes on decreasing, and it come down to 80 Kelvin 50 Kelvin or 30 Kelvin. The Carnot C O P decreases, and here we understand how important it is to get higher and higher C O P, and basically this talks about the efficiency of the system start decreasing as T L decreases. Now, there are various methods by which we can have production of low temperature. There are various methods like, using Joule Thompson expansion, heat exchanger, and compression expansion systems.

These are the systems that could be used to produce low temperature in Joule Thompson expansion. We are having an high pressure gas, and this high pressure gas is made to go through a valve, or a constriction, or a capillary tube. Because of which Joule Thompson expansion happens, and as soon as the expansion happens, the expansion may result in lowering of temperature. Now, for this case, what we need to have is, a pressurized gas. And we expand the gas from high pressure to low pressure which results in lowering of temperature. We can use heat exchanger; that means, we can have two fluids and one of the fluids could be a cold fluid when the other fluid passes through the heat exchanger, or other gas. Whatever is passing through can liquefy or it is temperature can get lowered.

Now, in both these cases Joule Thompson expansion, or heat exchanger in joule Thompson expansion, what we need to have is a highly pressurized gas. And in heat exchanger, what you need to have is a cold fluid, or a fluid at lower temperature. If these two things are not available then what you have to do is, to have a compression expansion system in which we compress the gas in the same cycle we expand the gas. So, we do not need in this case the pressurized gas and a J T expansion can serve expansion device also in this case. So, there are three different ways of producing low temperature. And now, if I want to make a new system, a new system could be a combination of all these methods.

That means, it may have a J T expansion valve, it may have a heat exchanger, and it may have a compression expansion device. And therefore, depending on how much cooling effect you want, depending on what is the C O P that is in your mind, to attain one has to device a mechanism in order to reach lower and lower temperature.

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We also studied, what is the difference between the refrigerator and a liquefier? And also we talked about having a combination of refrigerator and liquefier. We understood that refrigerator is a closed cycle system. And the working fluid gets compressed and expanded, the mass never crosses the control volume, or the mass never leaves the system. Liquefier the working fluid, itself liquefies. So, the working fluid gets compressed, gets expanded and then comes liquid. And some of the liquid is taken out of the system, and the makeup gases therefore, added to compress it the deficit. So, liquefier we can call as open system, while refrigerator what we call as a closed cycle system.

At the same time, the refrigerator can perform as a liquefier, if the boiling point of the gas which enters the refrigerator, heat exchanger lies above the refrigerator temperature. Then the diet fluid can liquefy. And therefore, in this case refrigerator can also function as liquefier. In this case what we call the refrigerator functions, both as a refrigerator and as a liquefier. Then we talked about the ratio delta T by delta p at constant enthalpy this is nothing but J T expansion coefficient. This expansion coefficient should be positive, in order that we get cooling effect; that means, when I reduce this pressure or when the expansion happens. During expansion, the pressure decreases, and therefore, when delta p decreases delta T should also decrease.

This will result in cooling, and a both numerator and denominator decrease, then delta T by delta P is going to be positive which will result in cooling. However, in the other case delta p decreases, we may land up in a situation when delta T increases. In that case, delta T by delta p at constant enthalpy is going to be negative, and in that case what you get is not the cooling effect, but what you get is a heating effect. And if you want to attain lower and lower temperature, we should avoid a case when delta T by delta p is going to be negative. We also found by mathematical algebraic equations, that J T expansion is an isenthalpic process; that means, enthalpy remains constant and that why delta T by delta P is always denoted at constant enthalpy.

The J T coefficient is given by this equation, this mu J T is nothing but J T coefficient and as I just said that, this mu J T has to be positive in order that it results in cooling effect. Now, you can see that mu J T is equal to 1 by C P into this bracket, and this bracket decides whether mu J T is going to be positive, negative or 0. If this bracket is positive, then mu J T is always positive. And in that case it will result in cooling effect. If this bracket is negative, then the mu J T will be negative, and therefore, expansion would result in heating effect in this case.

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We have derived that the mu J T for ideal gas is equal to 0; that means, it does not show any change in temperature when it undergoes J T expansion. This is with regards to the kinetic theory of gases which assume that the gas is ideal gas. And when you apply that this particular gas obeys the p v is equal to N R T. p v is equal to R T. In that case, we found that mu J T is equal to 0 which means that ideal gas in principle when expanded should not result in cooling, at the same time should not result in heating also. So, a Joule Thompson expansion cannot be applied to ideal gas, what we want to have is a real gas in this case. The temperature on the inversion curve at P is equal to 0 is called maximum inversion temperature T inversion.

So, we found that when the pressure is P is equal to 0, the inversion temperature in this case is maximum, and this is denoted by maximum inversion temperature. And this is a characteristic temperature for every gas. So, every gas will have maximum inversion temperature, which has to be taken into consideration when you device this particular gas for expansion, using Joule Thompson expansion process, so, in order that it should result in cooling during expansion. The initial state of the gas should lie inside the inversion curve, or the initial temperature of the gas should be below the maximum inversion temperature. This is a very important requirement for J T expansion.

If it should result in cooling; that means, we should ensure that the state of the gas, that the pressure and temperature of the gas is lying inside the inversion curve. Or the temperature of the gas at least should be less than maximum inversion temperature and pressure also should be less than particular pressure requirement which is coming from the inversion curve. So, basic requirement that should be satisfied is the temperature of the gas should be less than maximum inversion temperature. Then there is a probability that the gas will result in cooling after subjected to J T expansion or isenthalpic expansion. With this summary I think you all should be in position to understand what we talk about in the next following lectures, because there we will talk about various liquefaction cycles.

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Based on this lecture, a self assessment exercise is been given here. Kindly assess yourself for this lecture, and ensure that you are in proper position to go in details of gas liquefaction, and gas refrigerations. So, these are self assessment exercise. Please go through there are some gaps which should be filled by you, and you could tally your answers against the answers given in the at the end here. Thank you very much.