Cryogenic Engineering

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Lecture No. #06
Material Properties at Low Temperature

Welcome to the sixth lecture of cryogenic engineering for NPTEL program. The title of this is continuing from the earlier lecture which is properties of materials at low temperature.

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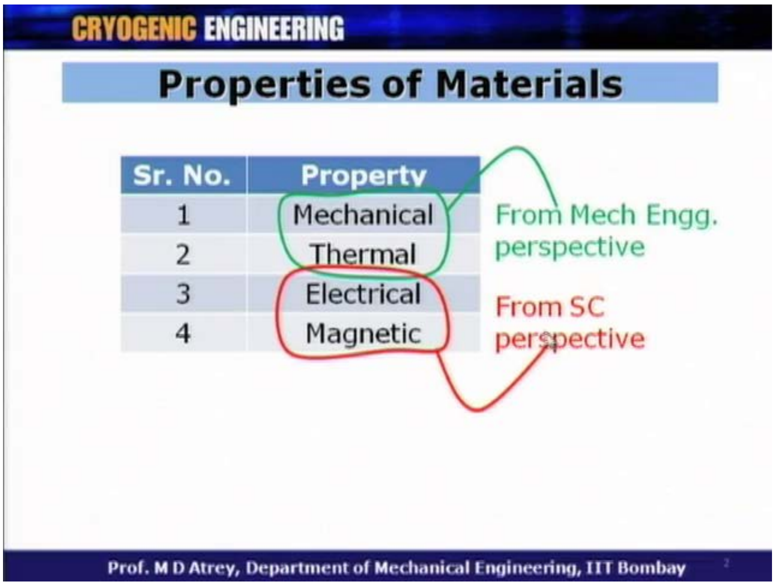

Here, we are considering various properties, there are mechanical properties, thermal properties, electrical properties, and magnetic properties. We have talked about this in the last lecture, out of which the mechanical, and the thermal properties are basically made for mechanic engineers, and this is what the course is meant for mechanical engineers. And therefore, we will talk about this properties, mechanical properties and thermal properties at low temperature in detail. At the same time, I will talk about electrical and magnetic property I will touch up on this properties, and I will not go in to the details of these properties.

As per as electrical and magnetic properties are considered at low temperature, we will talk more about super conductivity, this S C basically short from super conductivity. So, I will talk more about mechanical and thermal in detail for mechanical engineers. While, I will just give some information about these two properties, giving more stress on super conductivity as such.

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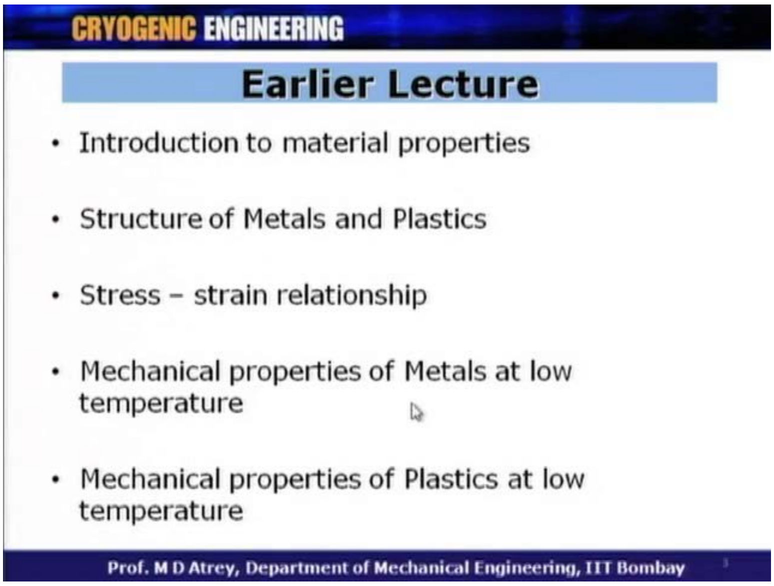

My earlier lecture introduce to you different material properties. We talked about structure of metals and plastics. We talk about BCC, HCC, FCC etcetera, all those materials and the relevance to the mechanical properties. We also talked about the stress-strain relationship, which are very very important in order to understand mechanical behavior of a material at any temperature. We talked about in detail, about the mechanical properties of metals at low temperature. At the same time, I touched upon mechanical properties of plastics at low temperature, because plastics also play an important role in cryogenics.

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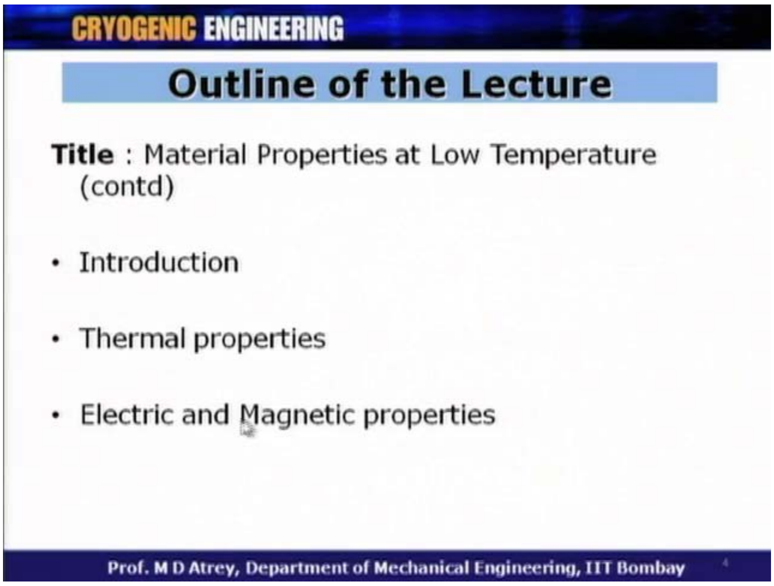

In today lecture, I will continue with the material properties of low temperature, I will give a small introduction as to what exactly I am going to talk about in today lecture, then the thermal properties. The essence of today lecture of the main focus of today lecture will be on thermal properties and thermal property variation at low temperature also, I will talk in brief or the electric and magnetic properties.

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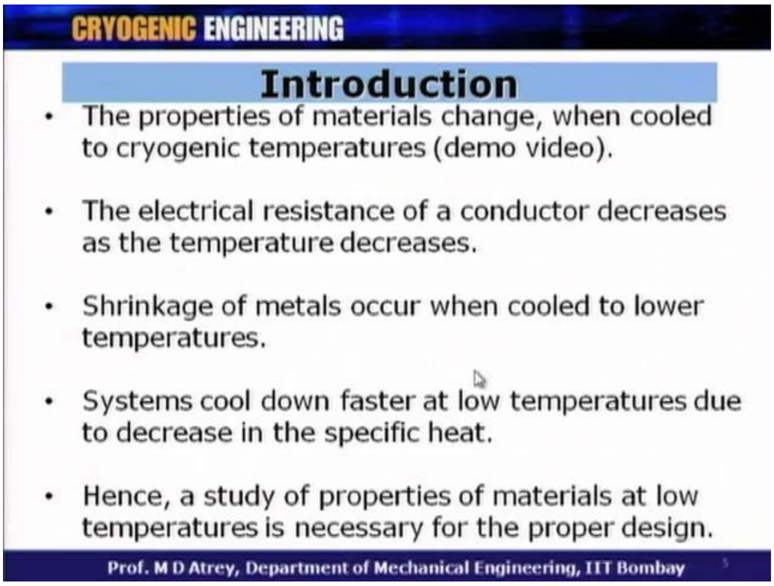

Now, as for the properties of the materials at low temperature are considered, the properties of materials change when cooled to cryogenic temperature and we have seen a small video. We showed to you, how the properties of the material change at low temperature. We know that, the electrical resistance of a conductor decreases as the temperature decreases; in fact that is why the electrical conductivity increases, which lead to super conductivity. So, as you go on reducing the temperature, the conductivity increases in other word, electrical resistance decreases. At the same time we know that, if the materials temperature is increased it expands.

Similarly, in cryogenics if the materials temperature is decreased, the material will shrink and therefore, the shrinkage of material occurs, when cooled to lower temperature and this is a very important consideration and this is what we will see in this particular lecture. At the same time, we understand that the systems cool down faster at low temperatures, due to decrease in the specific heat. If I want to cool down the system at room temperature, the system will take some more time as compare to, what it would take at lower and low temperature essentially, because they will be decrease in the specific heat at low temperature.

This is an aspect which has to consider while cooling different materials at low temperature. With this small examples, we are very general examples we have talked about, we know that the study of properties of materials at low temperatures is necessary for the proper design, because every cryogenic design, every cryostat design, every cryogenic device has to understand, how the materials behave at low temperature? And their behavior has to taken in to consideration while designing that particular device.

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Now, what are different material properties, we are going to talk about under the thermal properties? So, under thermal properties, I will talk about essentially three properties. One is thermal expansion or contraction in cryogenics will come across more contraction if we are reducing temperature while, if we are increasing temperature will get expansion. The second properties specific heats of solids because the specific heat is the very important function of temperature and this is with try to understand in this particular lecture, important thing is thermal conductivity. So, we will study these three properties and the variations of these properties, at low temperature in detail.

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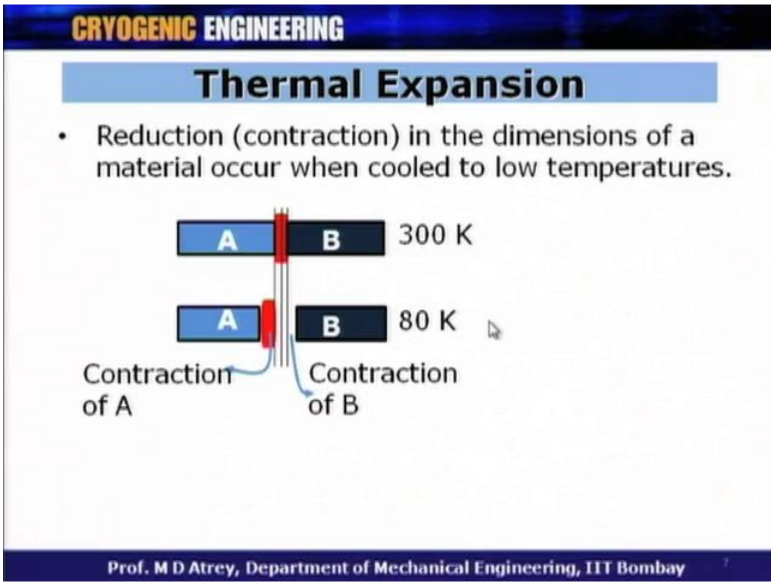

So, let us first come to thermal expansion. What is thermal expansion? Thermal expansion or contraction is nothing, but reduction in the dimensions of a material occurring, when cooled to low temperature. Simple, we know this that; if we heat the material its dimension increases. Similarly, if the material is subjected to very very low temperature, its shrinks and that has to be considered in the design of cryostat or a cryogenic device. Now, I just show a small schematic or small animation here, where, we got a material A and material B and this two materials are joint together by brazing or welding whatever, poses of joining we use over here and this joining naturally is done at 300 Kelvin, I mean room temperature. 300 Kelvin is always treated as room temperature.

Now, this joint when subjected to low temperature will look like this, now why does it look like this? It shows that material A has shrunk, also material B has shrunk, we can also understand that the joint which was perfect at room temperature. Now, that joint is not perfect here and the joint will start leaking and why does it leak? It leak because both the material shrunk and the joint therefore, gave way to whatever cryogen or whatever gas flowing beneath this; that means, at 300 Kelvin the joint was perfect, at 80 k the joint has become imperfect, the joint has started leaking. Now, this is the very important thing, this is where cryogen comes into picture, this is where thermal engineer comes into picture, this is the contraction of A and this is the contraction of B.

Now, any designer who knows this will have to consider the shrinkage of material A and the shrinkage of material B before he starts joining mechanism. This requires some of the calculations to be done. Some properties of material have to be known the shrinkage is a function of temperature. So, one should know, how much shrinkage it comes across? How much material A shrinks? Or how much material B shrinks? Does material A shrink more than A or more than B? That you have to understand and those things have to be taken in to consideration while designing a cryostat or any machine.

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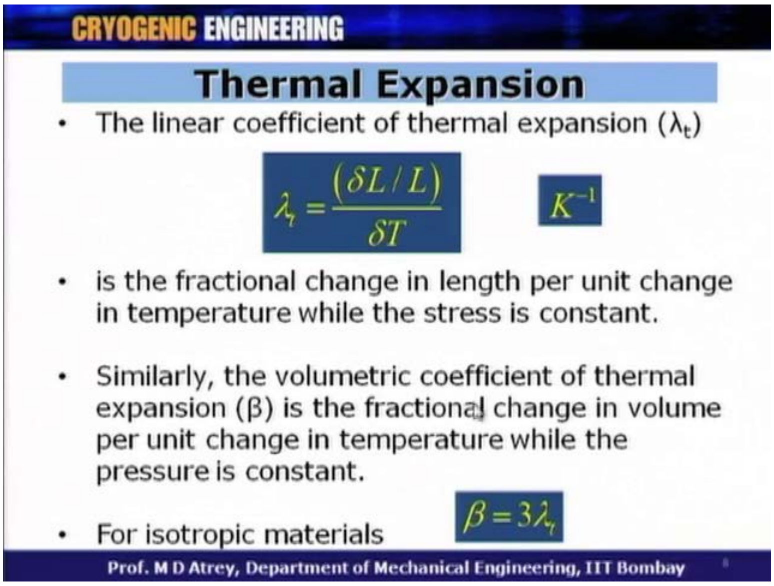

So, let us see what is the tarameter? Which defines the shrinkage, the shrinkage is now defined by the linear coefficient of thermal expansion, when we are talking about one dimension or let us say length, width or height etcetera. We can define the shrinkage by lambda T and this lambda T is defined as del L by L upon delta T. What does it mean? It means that, the fractional change in length; that means, if L is length then del L is a contraction or expansion per unit change in temperature, while the stress is constant. So, what is del L by L for a given material per unit change in temperature or per degree centigrade or per Kelvin?

That is very inherent property or a characteristic property of any material. So, lambda t is specific property of material and it depends on, what is lambda t at a particular temperature because lambda t at room temperature could be different. That means, the fractional change in length per unit change in temperature could be different at 300 Kelvin at room temperature, it could be different at 150 Kelvin, it could be different at 80 Kelvin. So, this behavior, this variation of the lambda t or what is lambda t at room temperature? What is lambda t has 200 Kelvin temperature? Have to be known, before we go for further design of any joint.

This is very important thing because; some additional stresses can get generated, because of contraction and expansion of the joints. The unit of lambda t from this definition is per Kelvin, which is K inverse or 1 by k, whatever way you want to write it could be written, sometimes it is written as per Kelvin per k. Similarly, now this was linear coefficient of thermal expansion. Similarly, we can have volumetric coefficient of thermal expansion beta and is the fractional change in volume, per unit change in temperature, while the pressure is constant. So, we can have one in length, breadth and height, which will give you volumetric change in thermal expansion, volumetric change occurring in all the three dimensions and for isotropic materials while lambda t in all the three direction is same, we can have beta is equal to 3 lambda t that is understandable.

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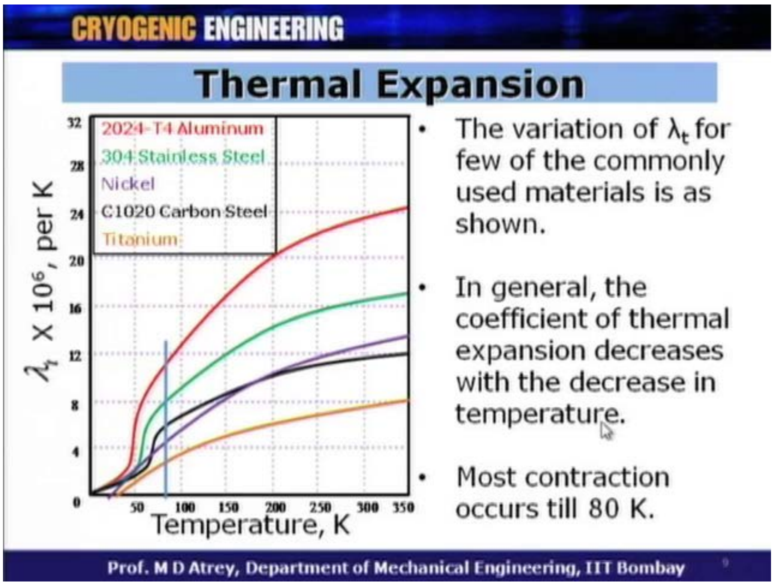

Now, this variation, this particular graph gives the variation of lambda t with respect to temperature. So, we can understand now, that lambda t is basically decreasing trends for every curve, this curve are meant for different materials. This is for aluminum, stainless steel, nickel, carbon steel, and titanium as shown over here. And what you can see from here, that lambda t is basically decreasing with a decrease of temperature. So, one thing is sure that, the variation of lambda t for the few commonly used material in all the materials. We see that the coefficient of thermal expansion is decreasing with the decrease in temperature.

So, this has to be understood. Also, what you can see is, at 300 Kelvin lambda t is values are much higher as combined to let say 80 Kelvin temperature; that means, at room temperature the shrinkages are much higher, delta L by L per degree centigrade are much higher as compare to very low temperature. This aspect also has to be considered while designing the machine. So, we can see below 80 Kelvin, the lambda t value is just 10, 8 or 4, while at 300 it is of the order of 24, 25 etcetera. So, most of the contraction we can see that, it happens from 300 to 80 Kelvin, while below 80 Kelvin the contractions are that way negligible.

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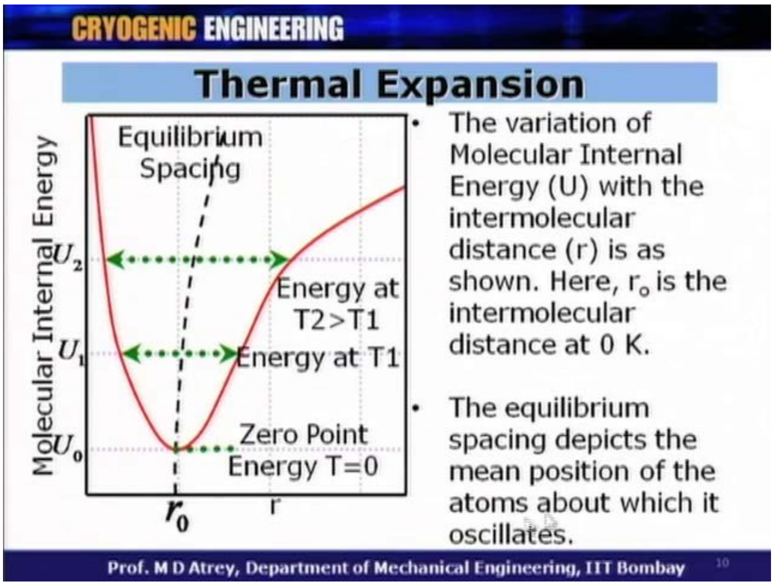

Now, why does this contraction happen? In order to understand that, we can understand that, we can see from this figure, that in the molecular internal energy variations verses the inter molecule distance which is (( )) here. This figure gives the variation of molecular internal energy verses intermolecular distance, which is given by r. What you can see here is at zero point that is at temperature equal to 0, r 0 has got particular value what you can see from here, this particular curve shows that, as the temperature increases this is the oscillation of the items in a particular lattice and this oscillation amplitude has increased.

As the temperature increases, this oscillation amplitude increases and you can see here, as the temperature increases further, this oscillation amplitude has further increased. Also, what you can see from here, that, the equilibrium spacing curve which basically denotes the intermolecular distance, the curve is tilting towards flight, which shows that this is not a very symmetric curve, this is what we see from here. The equilibrium spacing basically depicts the mean position of the atoms about which it oscillates.

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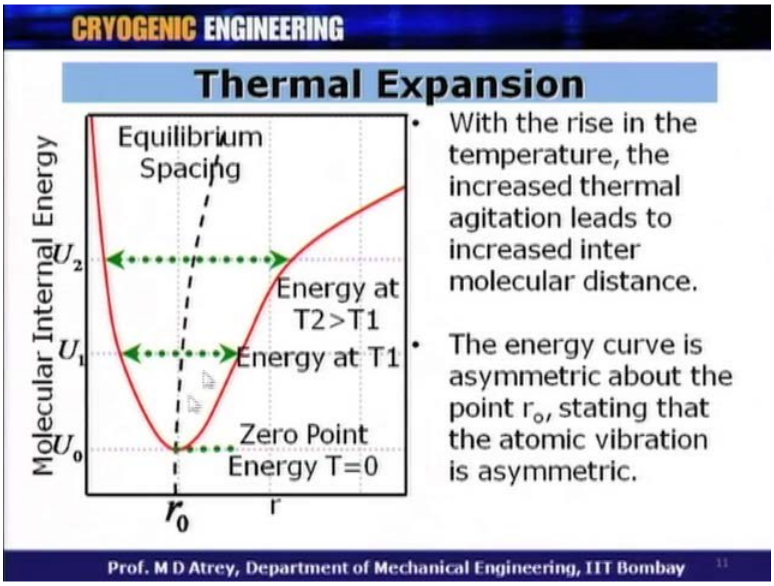

With the rise in temperature, the increased thermal agitation leads to increased intermolecular distance, which is what we just saw and the energy curve is asymmetric about the point r 0 stating that, the atomic vibration is asymmetric.

So, we can understand that, as the temperature increased, the curve is leading towards straight it means that, this are not symmetric. While, when the temperature increased from T1 to T2 the oscillation increased. While, if you goes from T2 to some additional T3 value, the oscillation will further increased, under increased will be much more than the ratio of this temperature T3 to T2 or T2 T1. As soon as you understand this, we can understand that the lambda value are changing and the lambda values are more at higher temperature then at lower temperature.

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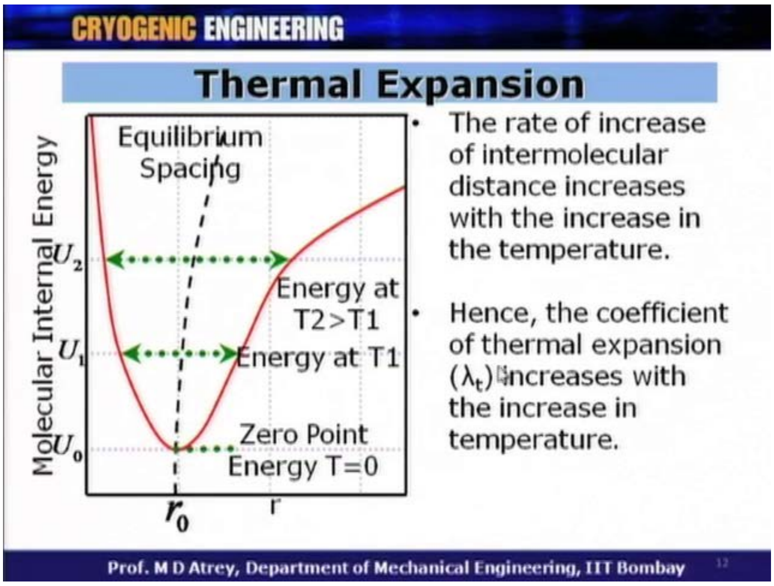

This is what the conclusion could be drawn from this. The rate of increase of intermolecular distance increases, with the increasing temperature. So, as you increase from T1 to T2 or from T2 to T3, smallest increase at T2 to T3 can lead to much more increase in the inter molecular distance which basically means that, lambda t value is much more here as compared to what it is over here and this basically offers the explanation for the curve which we earlier saw that lambda values at 300 Kelvin are much higher as complete toward their at least 80 Kelvin or 50 Kelvin.

Hence, the coefficient of thermal expansion lambda t increases, with the increase in temperature and this is basically the reason, why materials expand when heated and why do they contract when they are cooled.

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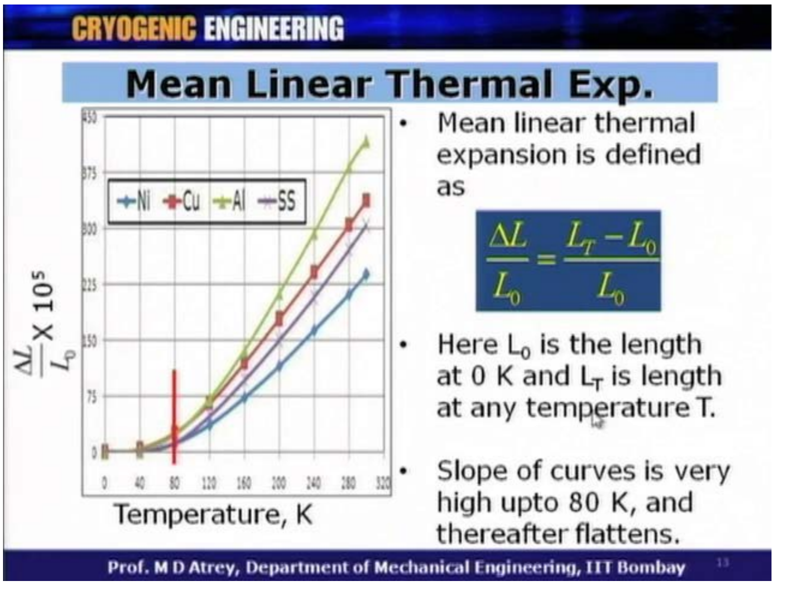

There is one more parameter which has to be understood called mean linear thermal expansion, which is delta L by L0. Now, this parameter also gives you understanding as towards exactly happens main the temperature changes happens from let say from 0 Kelvin to a particular temperature T. So, if I define mean linear thermal expansion as Del L by L0 which is LT minus L0 divided by L0; that means length at any temperature T minus L0 that is length at 0 Kelvin divided by L0. So, this curve basically derived from the values of lambda t for different material. So, we can see from here that, aluminum has got more delta L by L, as compared to copper as compared to stainless steel as compared to nickel.

So, here, we can understand that let say for temperature equal to 280 Kelvin delta L by L at 280 Kelvin will be around 375; that means, if we change the temperature from 0 Kelvin to 280 Kelvin, the length increases around 375 or delta L by L into L0 into 10 to the power 5 is equal to 375, which means around 3.75 millimeter, if we got 1 meter length at 0 Kelvin. So, this curve gives you the change in length for a given change of temperature from 0 Kelvin. Again, from this curve you can see that below 80 Kelvin the delta L by L value are much smaller as compare toward they are at higher and higher temperature. The slope of the curve is very high up to 80 Kelvin and thereafter it flattens.

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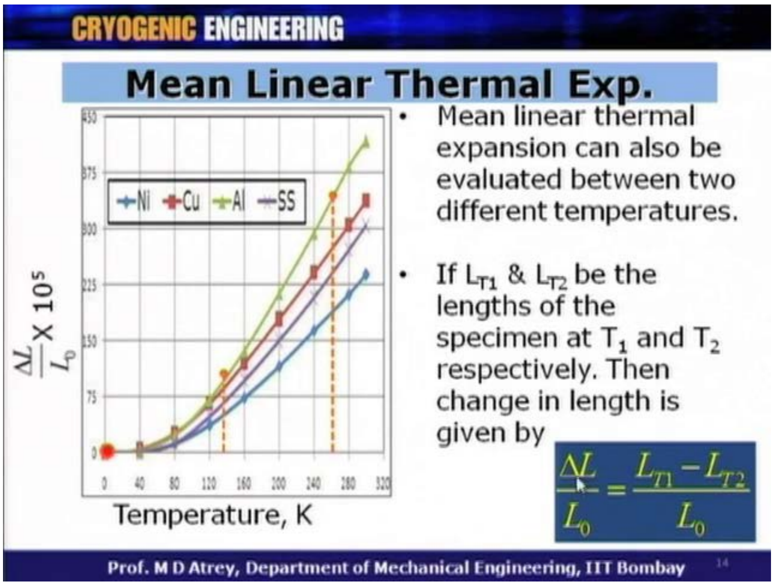

The mean linear thermal expansion can also be evaluated between two different temperatures. Earlier, we saw that the mainly thermal expansion was with respect to 0 Kelvin, but now if I got two temperature in to consideration like L T1 and L T2 are the lengths of the temperature at T1 and T2 respectively, then the change in length, if we change the temperature from 280 to 80 Kelvin or 80 to 280 Kelvin whatever, then we can compute those two temperatures, by this formula. So, in this case now, delta L by L0 will be L T1 minus L T2 upon L0. So, if I get a value of L T1 by L0 which is let say this and L T2 by L0 which is value this and if I get difference between this two values, then what I get is a delta L value if I change the temperature from T1 to T2.

Now, this is a very important calculation, which one has to do. If one uses liquid nitrogen at 77 Kelvin from room temperature, how much that particular part will shrink can be computed over here and this is very important to understand, what will be the stresses generated? How the joint will behave at lower and lower temperature? This is very important and with this formula one can calculate the delta l value between two temperature changes.

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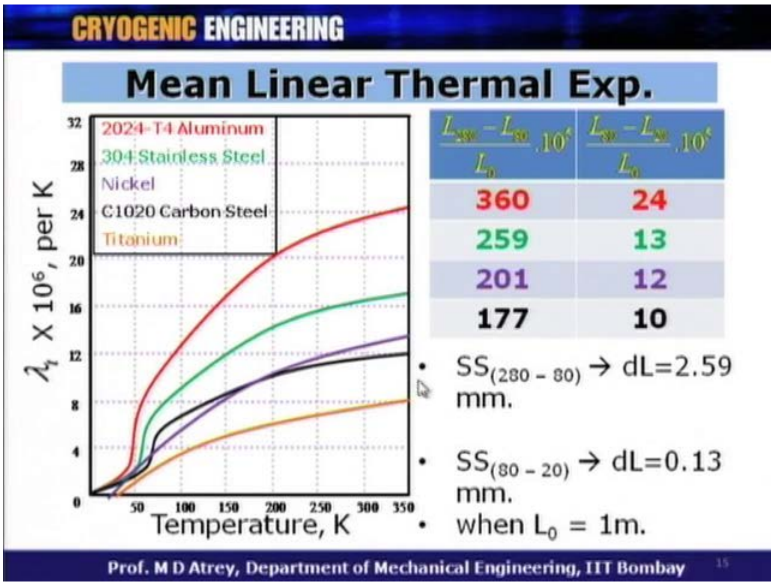

So, here we can see that for different materials, we have calculated, if the temperature of the stain less steel road decrease from 280 Kelvin to 80 Kelvin, we find from here this is stain less steel over here, that d L equal to 2.59 millimeter if L equal to 1 meter at 0 Kelvin and from 80 Kelvin to 20 Kelvin d L is only 0.13 millimeter telling you that, this indicates you that, maximum shrinkage can again has occurred up to 80 Kelvin while below 80 Kelvin it is only 0.13. One can compare these with the value of lambda t which we just saw, from here you can understand that for aluminum, the shrinkage is much higher which is clear from this particular portion, that values from aluminum are much higher then comes stainless steel, then comes nickel and then comes carbon steel.

So, basically the shrinkages go in order of the lambda t values and if one wants to compute the exact shrinkage, one can compute by this formulae and this are very very important in order to understand, how much shrinkage would occur? And therefore, what kind of jointing mechanism we should device? Or how much stresses will get generated if this particular (( )) is subjected to low temperature.

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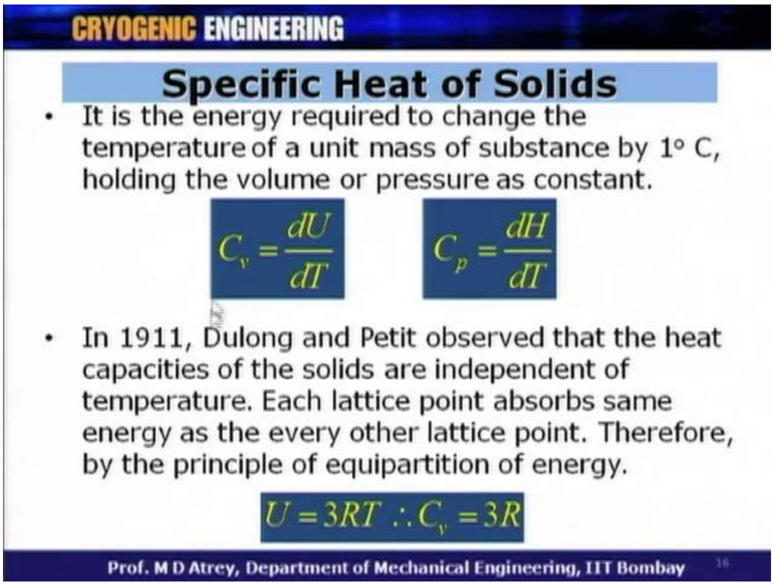

With this now, I will switch to the next property which is specific heat of solids. What is specific heat? This is also very important property. It is the energy required to change the temperature of a unit mass of substance by 1 degree centigrade, holding the volume or pressure as constant. So, basically how much amount of energy is required to change, let say if a 1 k g substance by 1 degree centigrade which talks about it C p value or C v value. By thermo dynamics you know that, C v is nothing, but d U by d T, that means, the change of internal energy per degree centigrade temperature.

Similarly, the C p value or the specific heat at constant pressure is nothing, but variation of enthalpy d H by d T change of enthalpy per degree temperature change. Now, in order to understand how to calculate c V and c P or how the C v and C p change at low temperature, I am going to consider only solids over here, because that is a material normally we come across in calculations. Now, lot of things evolved from right earlier from 1911 and in 1911, Dulong and petit observed that the heat capacities of the solids are independent of temperature, see that time the load temperature availability was so much.

Dulong and petit came to the conclusion, that the C p values or C v values for solids are independent of temperature, why? Because each lattice point absorbs same energy as the every other lattice point. So, that means, all the lattice point behave in the same way. They observe the same amount of energy and therefore, there is no dependence of amount of energy absorbs over temperature and by equipartition of energy. They calculated that U is equal to 3RT and therefore, d U by d T is equal 3R, which is C v is equal to 3R and therefore, they found that d U by d T nothing but a constant figure. So, whatever happens, even if the temperature decreases or increases, C v is equal to 3R values, this is what we observed by dulong petit's rule.

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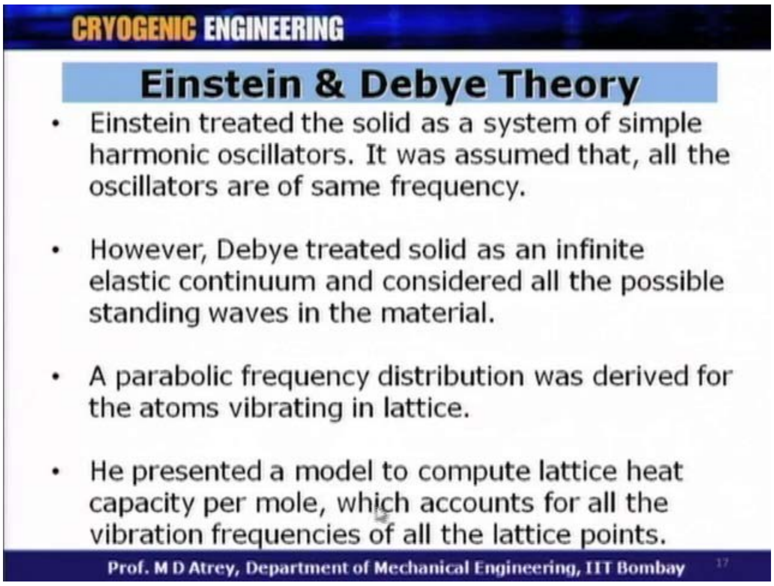

The next observation which went to Einstein, where the quantization came in to picture and Einstein treated the solid as a system of simple harmonic oscillators. Now you know that, all the atoms in molecule oscillate, What Einstein observed was, that all this oscillations are of the same frequency. So, all the lattice moment that is happening in the lattice, all the atoms they are which are vibrating, they are all vibrating at the same frequency which is not very realistic observation then, but again it was one of the evaluation that happens over Dulong petit's observation. However, the next scientist who came in to picture was Debye. Now, Debye treated the solid as an infinite elastic continuum and considered all the possible standing waves in the material.

Because there are lots of different vibrations, that are coming to picture. What are different frequencies coming to picture, different oscillating frequencies all the molecules are not oscillating really at the same frequency, but they got different frequencies and what Debye concluded is, a parabolic frequency distribution exists a derived parabolic frequency distribution was derived for the atoms vibrating in lattice and he consider all this frequency distribution in order to calculate C p and C v. So, he presented a module to compute lattice heat capacity per mole, which accounts for all the vibration frequencies of all the lattice points and this is very important thing because this comes very close to the realistic picture was lattice.

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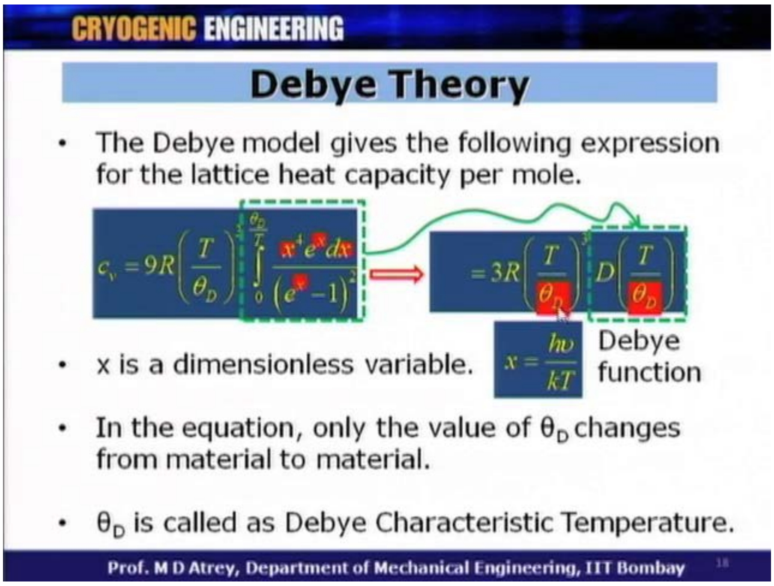

So, what he did was, the Debye model gives the following expression, with lot of calculation it came out with the expression which gives lattice heat capacity per mole and expression is like this, 9R T upon theta D and theta D is nothing, but characteristic Debye temperature and then, this expression takes care of the all the lattice vibration frequencies, all the oscillating frequencies in the lattice and he again, simplified this by having this expression as D of T by theta D which is called as Debye function. So, this is the most important thing that, whatever complicated part was there in order to considered all the parabolic frequency distribution, he indicated or he calculated that has Debye function and one can get the value of Debye function

And therefore, we know the Debye function at a particular temperature; we can compute the C v at those temperatures. In this particular expression, x is a dimensionless variable and this x basically nothing, but h U upon k T which is Planck's constant in to frequency Boltzmann's constant k, a temperature. So, from here you can understand that, x considered all the frequencies of vibration. In this equation, in the equation which is over here the only value of theta D is going to change and what is that theta D is basically nothing, but Debye characteristic temperature. So, for different material now, it will have a characteristic Debye temperature and if I know this Debye temperature, I can complete the value of C v provided, I know the Debye function values. So, the whole thing became a very simple when can complete the effect of temperature variation on the values of C v.

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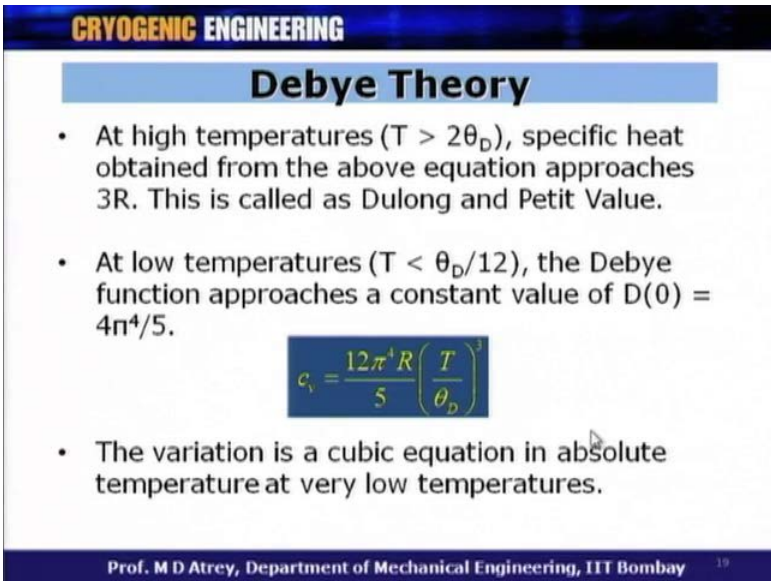

From this expression, we can understand that at very high temperature; that means, T greater than 2 times theta D, the specific heat obtained from the above equation approaches to 3R values and this is nothing, but what we got earlier by dulong petit's value. So, at higher temperature, at very high temperatures, at room temperatures and above, what will have is, of the C v is equal to constant values which is equal to 3R and at low temperatures, where T is less than theta D by 12, this will come in to very low temperature category.

The Debye function here, in this case approaches a constant value and at D0, what we have is 4 pi to the power 4 by 5 constant values. If I put a Debye value over here, particular C v value will be function of only temperature. Now, in this case, while all other things are constant, you can see here now at lower temperature the C v variation is T cube directly proposed to T cube and this is very important conclusion Debye it came up with that, the variation of a cubic equation in absolute temperature at very low temperature C v, is directly proposal to absolute temperature power to 3. So, at lower and lower temperature, we can imagine the behavior of C v will be.

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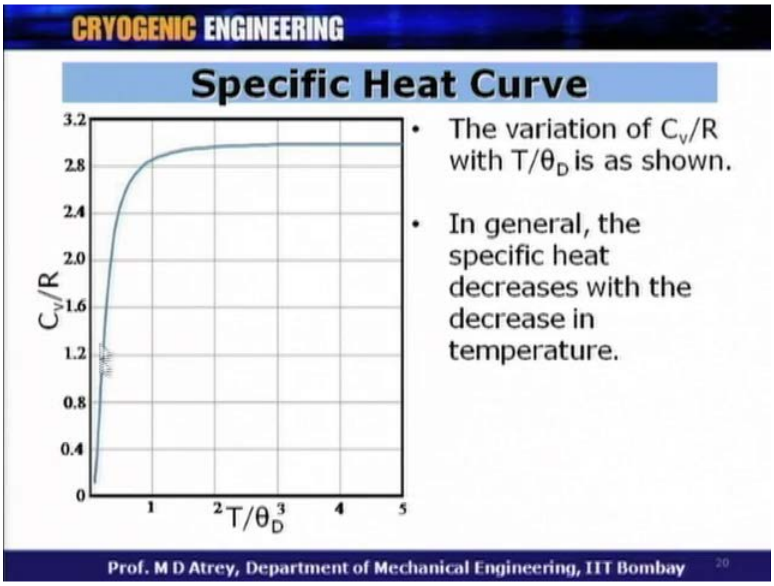

This is the curve, which basically gives the value of C v by R with T by theta D. So, if I know the T and if I know theta D, theta D is a characteristic temperature for any material. So, what you can see for higher temperature, where t is more than two times theta D or 3 times theta D, the C v by R is almost constant which is actually equal to 3 times R. While, at lower and lower temperature, what you see is C v by R is coming down with the power of 3 temperature coming down and therefore, you can see the steep decrease at all lower temperature and this is a very important curve in a order to calculate the value of C v. So, in general what you can see from here, that the specific heat decrease with the decreasing temperature, this is obvious from this particular figure.

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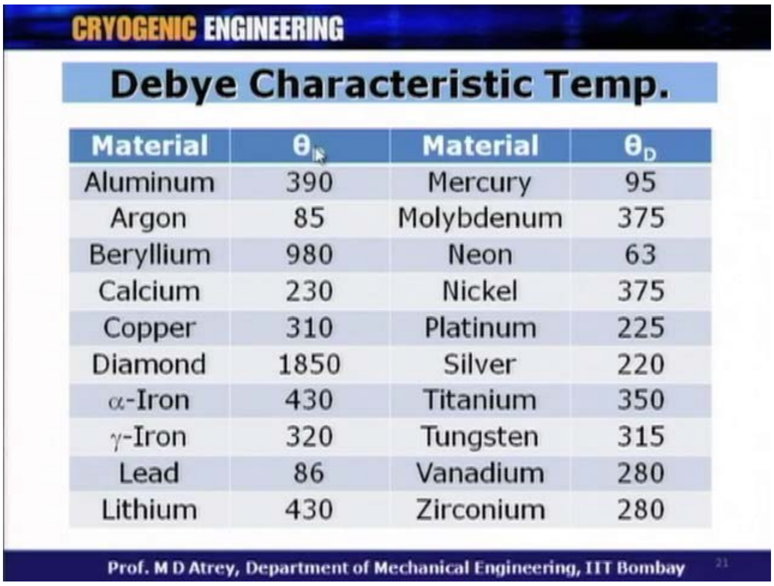

Material	θ_D	Material	θ_D
Aluminum	390	Mercury	95
Argon	85	Molybdenum	375
Beryllium	980	Neon	63
Calcium	230	Nickel	375
Copper	310	Platinum	225
Diamond	1850	Silver	220
α-Iron	430	Titanium	350
γ-Iron	320	Tungsten	315
Lead	86	Vanadium	280
Lithium	430	Zirconium	280

Now, this table gives you different values of the theta D for the different material. So, you got an aluminum 390, you got a beryllium, you got a calcium copper etcetera. If I know the theta D value for a different temperature, for different materials, then, I can calculate T by theta D and I can go back to this curve I can complete the T by theta D and calculate C v of that particular material it is as simple of that. So, if I know theta D, I can complete T by theta D and then I can correspondingly complete the value of C v in that case.

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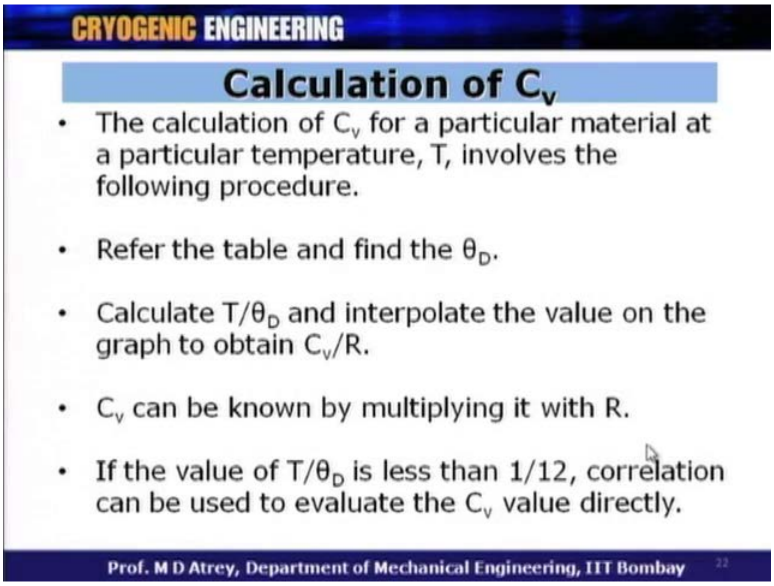

So, if I want to calculate the value of C v, what I am saying, repeating the same thing. The calculation of C v for a particular material at a particular temperature T involves following procedure, refer to the table and find out the value of theta D, calculate T by theta D and interpolate the value on the graph to obtain C v by R. C v by R is known just multiplied by R value, universal constant and there what you get is value of C v. If the value T by theta D is less than 1 by 12 then, we know that we are got one more expression which has been given earlier, you can use that co relation, use the value of T by theta D and compute the value of C v directly in that case. So, this is how you can calculate the value of specific heat at lower and lower temperature, if one knows the value of theta D for that particular material.

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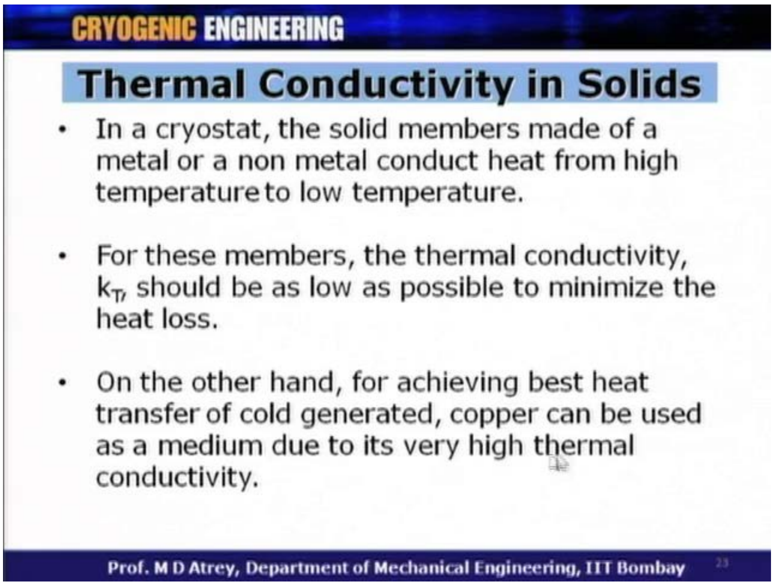

The next property, which I am going to talk about now is the very important property again thermal conductivity in solids. Now, why do we require the knowledge of the thermal conductivity in solids at low temperature, because of some examples I am trying to give you. So, in a cryostat, the solid members would be metal or non metal, this stainless steel could be copper, could be plastic, nylon etcetera and these material conduct heat from high temperature to low temperature. Sometimes, we have got some suspension member in order to support a cryostat, which will definitely take heat from room temperature and take it to the low temperature.

So, for these members the thermal conductivity k T should be as low as possible otherwise, the carry lot of heat from room temperature to the cold temperature and therefore, the cryogen, the more loss of cryogen or some additional cooling effect has to be used for this. Therefore, we have to see that the conductivity of this particular material is very very low; we have to see that thing. On the other hand, I will give other extreme example; where in, you want to know transferred cold because you want to cool something because lot of cold has been generated in a system, let say in cryo cooler and we want to cool a sample or we want to cool a gas.

Therefore, we want to have a material, which has got very high conductivity at low temperature and in this case, I would like to achieve base heat transfer of the cold generated and therefore, copper can be used as a medium, due its high thermal conductivity. So, I have to select material in such a way that, where I want to minimum losses to be coming on a system then I sure use low thermal conductivity material like nylon, stainless steel, but wherever I want that the heat transfer should be as highest possible therefore, I should use the material which has got high thermal conductivity like copper, aluminum etcetera.

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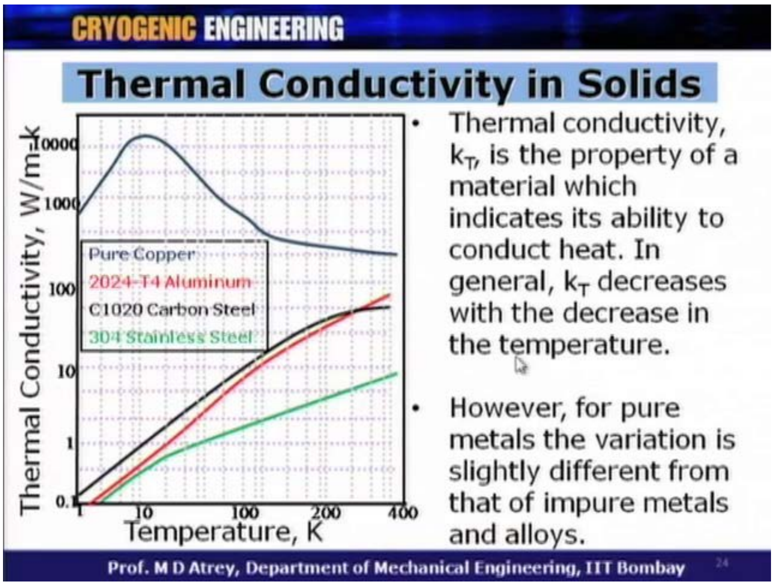

The thermal conductivity in solid, let say k T is the property of a material which indicates its ability to conduct heat. Now, this particular curve, this particular graph shows the thermal conductivity variations at lower and lower temperature. You can see two types of curves, one is curve which is going up and then coming down, which is for pure material or pure copper in this case. While, you can see other materials like aluminum, carbon steel stain less steel, we show that, as the temperature is coming down the temperature below, let say 200 Kelvin is coming down continuously like this.

So, these are basically alloys or impure material we can call them, while, this is pure material, but in general you can understand for these materials k T decreases with the decrease in the temperature. However, as I just indicated to you earlier, for pure materials, the variation is slightly different or one can say quite different from as compare to the impure materials or alloys. So, this is very important to understand, what exactly happens over here and what exactly happens over here, I will just try to give you some explanation for this two difference curve.

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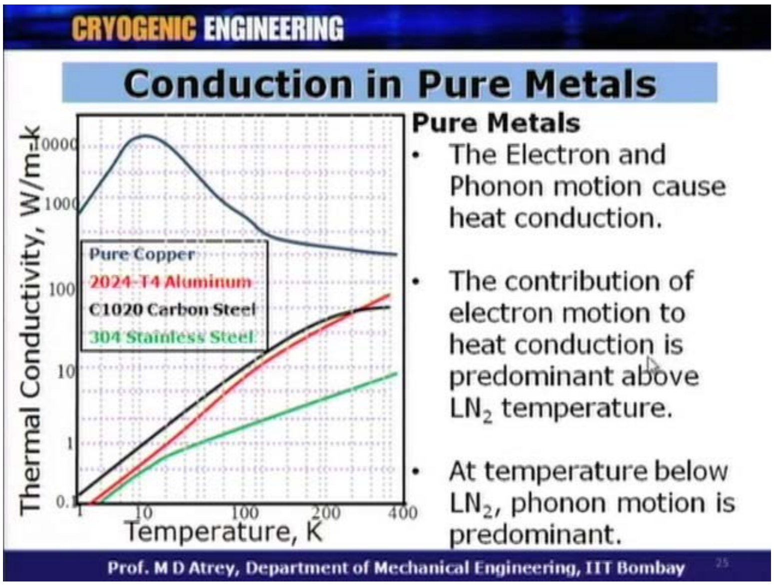

For pure materials, the for any materials for that matter the electron and phonon motion cause heat conduction. Basically, thermal conductivity is because of the motion of electrons and phonon. Phonon is basically the lattice vibration and this two contribute the heat conduction. Now, the contribution of the electron motion to heat conduction is predominant above liquid nitrogen temperature. As you know that, liquid nitrogen temperature dividing line in cryogenics and above liquid nitrogen temperature all, let say 77 to 80 Kelvin above this, the electron motion of heat conduction is predominant as compared to phonon motion.

So, at temperatures below LN 2, below liquid nitrogen temperature phonon motion is dominant. So, you got two variations, above liquid nitrogen temperature, what you have is electron motion contribution is more while, below it is not electron motion, but phonon motion is predominant at lower temperature and this is basically, what makes difference in the behavior as you see in pure material thermal conduction.

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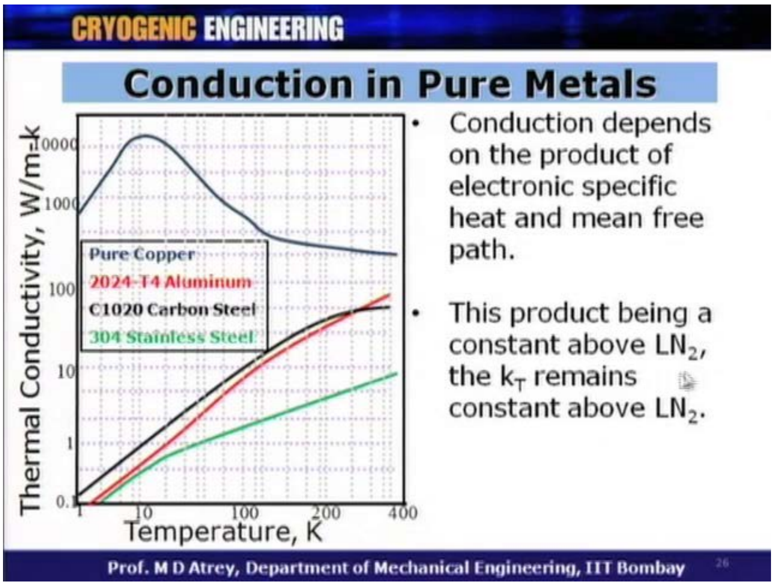

Now, in the thermal conduction, depends on the product of electronic specific heat and mean free path. So, if I say Q which is conduction is basically directly proportional to two parameters that is electronic specific heat and mean free path. Now, this product is constant above LN. So, what is happening above LN 2 is, as you come down, the temperature up to let say 80 Kelvin from room temperature, the electron specific heat is decreasing while, the mean free path is increasing, the electron mean free path is increasing and therefore, somehow the product of this specific heat, electron specific heat and mean free path is remaining constant above liquid nitrogen temperature, which makes that the thermal conductivity k T is remaining constant above liquid nitrogen temperature.

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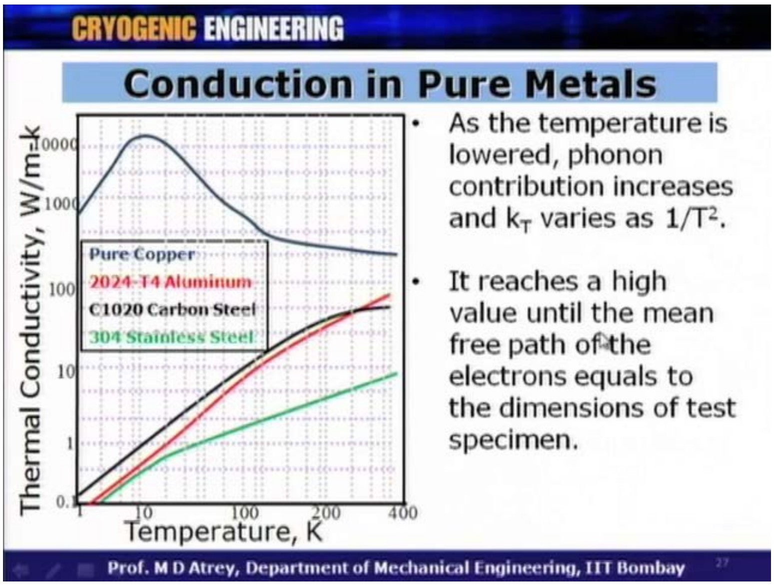

What is happening below? As I said, below liquid nitrogen temperature it is not electronic specific heat which is predominant, but the phonon specific heat is predominant. So, as the temperature is lowered, the phonon contribution increases and the k T are varying as 1 by T square. So, we can see that the, if the temperature is decreasing the phonon contribution is increasing in that case which was not so, as per as the electronic contribution was considered. Also, as we know earlier, that as temperature is coming down; the mean free path is increasing.

So, here, what you see now at below LN 2 temperature, the phonon contribution mean free path also increasing the value of k T. So, at lower temperature from, let say below LN 2, the phonon contribution increase plus the mean free path also has increase which leads to increase in thermal conductivity for pure material, but this will happen only up to the particular temperature. What happens at this temperature? At this temperature, the thermal conductivity reaches a very high value until, the mean free path of the electrons equals to the dimensions of test specimen.

So, naturally the mean free path has got some limitation and it increases to maximum to a value, which is equal to the maximum dimension of the test specimen. So, beyond which now, it cannot increase; that means, it has reached a constant value and therefore, below this temperature. Now, below this temperature, the conductivity starts coming down at this point or it reaches maximum over here.

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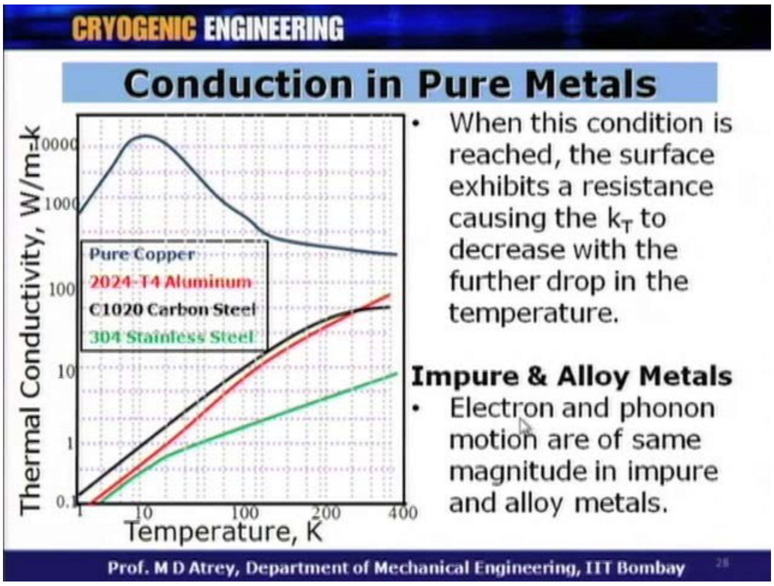

Because it has reach maximum value, when this condition is reached, that it has to reach the maximum value, the surface of the material exhibits a resistance to the motion of the electrons. So, basically the resistance offered by this material to the flow of electrons and flow of phonons and therefore, it causes the value of k T 2 come down below low temperature, below 10 Kelvin region etcetera. So, this explains, why you got a constant kind of a curve up to let say 80 Kelvin and below 80 Kelvin it goes very high up to this point because of the phonon contribution and because of the electronic mean free path increase and then its starts coming down.

Because now, electron mean path has reached to maximum value and the surfaces started offering resistances to the flow of electrons. This is the very funny behavior, this very typical behavior as far as the pure materials are considered. As far as the impure and alloy metals are considered, we can say that electron and phonon motion in this are of same magnitude. So, the contribution which is coming from electron and phonon both is almost of the same magnitude in impure and alloy metals. So, below a particular temperature phonon is predominate, above a particular temperature electron is predominant that aspect does not exist in this case. Both of them contribute equally, as far as the impure material or the alloy materials are considered.

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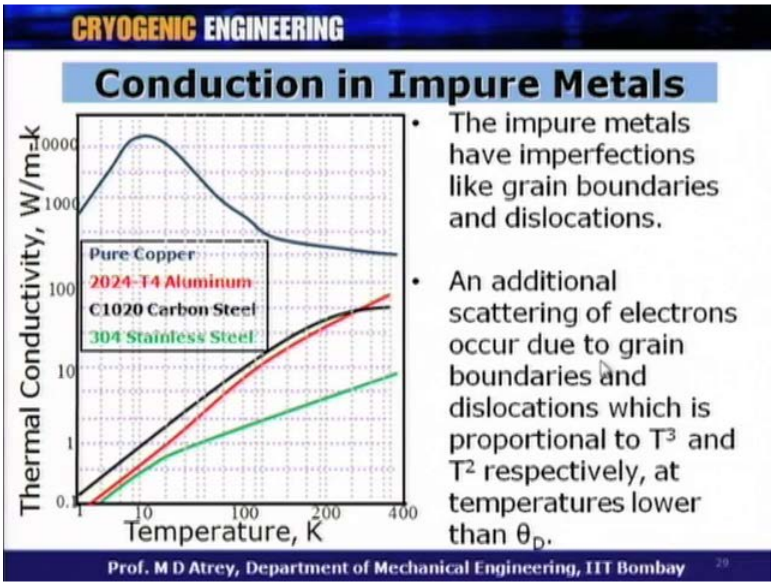

What happen for the decrease in the thermal conductivity in these materials? Then, the impure metals have got imperfection and that is why they are called as impure materials and they got imperfection like grain boundaries and dislocation, you know about this we talked about this in an earlier lecture. So, we got imperfection like grain boundaries and dislocation and this basically are kind of hindrances to the motion of electrons. What happens the electrons strike this imperfection and they get scattered? So, an additional scattering of electrons occur due to grain boundaries and dislocations.

Now, this scattering is proportional to T cube for grain boundaries and T square for dislocation, at very low temperature. What it means is, as the temperature comes down, the scattering become less and less and the less is T cube proportional, T square proportional, if we talk about a particular dislocation of grain boundary or dislocation respectively, which means that, we are getting an additional advantage of scattering because of each electrons where conducting heat from heat from one point to another.

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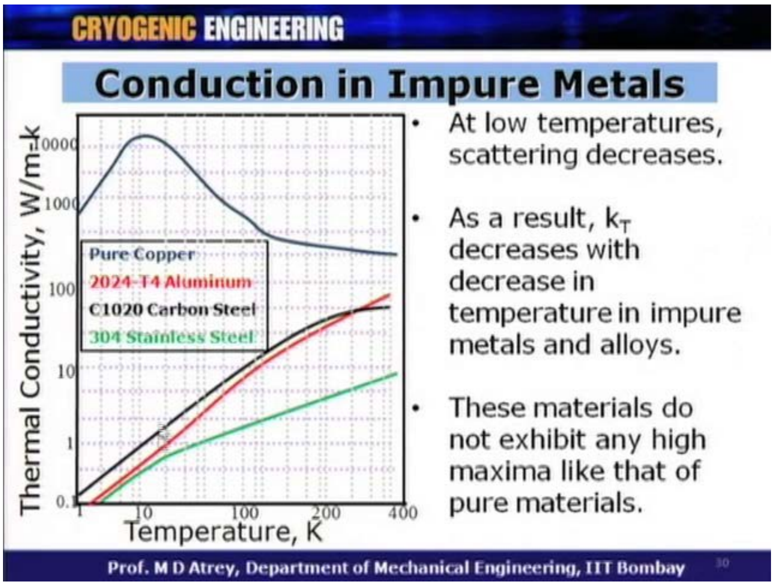

But this is not happening at low temperature, because the scattering has reduced drastically at low temperature. So, at low temperature scattering decreases, as a result k T decreases with decrease in temperature in impure metals and alloys. I am sure it is clear to you that in these particular cases, the scattering phenomena decreases at low temperature. Therefore, the thermal conductivity in this case decreases at low temperature, these materials do not exhibit any high maxima like that opera material. So, what we saw in a pure material, we do not see in this particular in this case.

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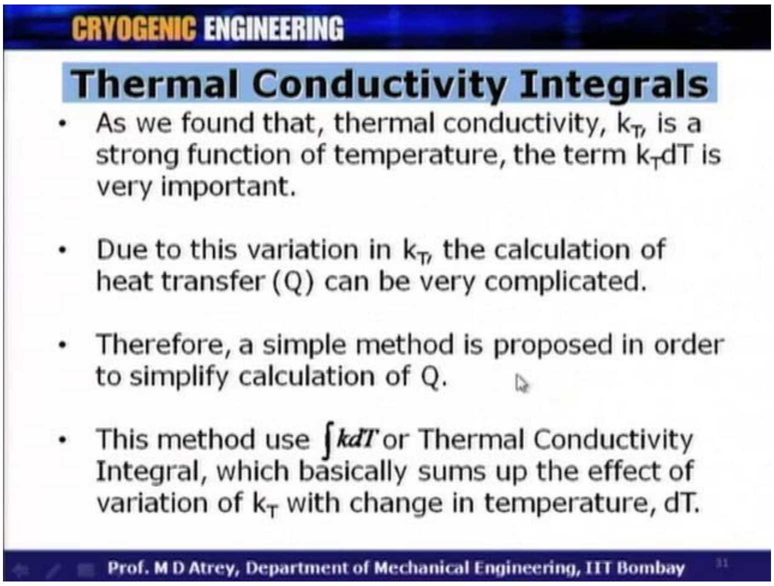

Thermal conductivity integrals, this is an important parameter which has to be understood while calculating the thermal conductivity loss at low temperature. As we found that, thermal conductivity k T is strong function of temperature, the term k T d T is very important. So, we have just seen that, the k thermal conductivity is a strong function of temperature. Due to this variation in k T, the calculation of heat transfer cube can be very complicated. If I want to calculate the conduction loss between 300 Kelvin to 80 Kelvin I have to consider the k variation temperature and then calculate for each temperature, in order to know how much, what is the loss of the conductivity between these two temperatures.

Therefore, a simple method is proposed in order to simplify the calculation of Q. This method uses integral k d T as a parameter, is called as thermal conductivity integral, which basically sums up the effect of variation of k T with respect to temperature change. So, k T d T is a parameter, which is directly offered, for a particular material and therefore, one can gate the, what is the k d T parameter in a particular given temperature in trouble. This benefits a lot while calculating the loss due to thermal conduction.

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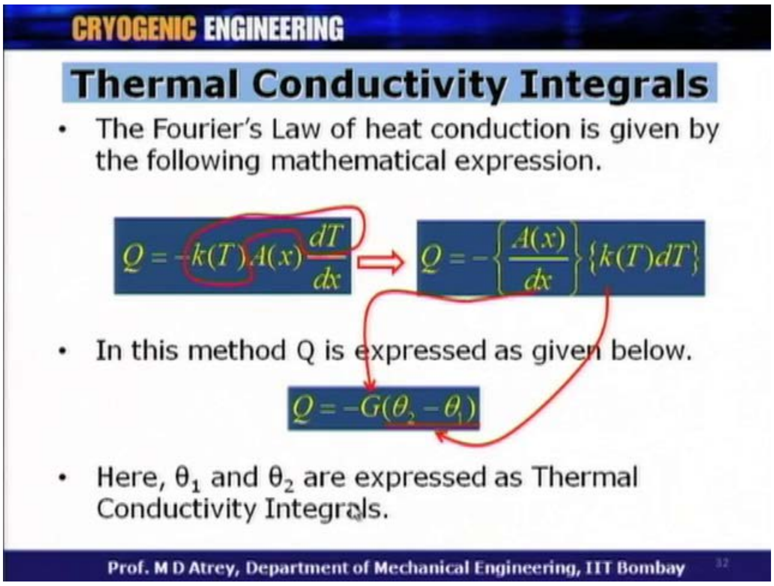

How? We can see now. The Fourier's law of heat conduction is given by the following mathematical expression which you know; Q is equal to minus k into A into d T by d x. Now, this we can club as, k T and d T can be club together, while A x upon d x can be club together and if we do that thing what we get is this. So, Q is equal to now here minus A x upon d x into k T by d T. In this method, Q is expressed as below; now, I am just representing this Q in this simple format, minus G into theta 2 by theta 1.

Now, this G is nothing but this parameter A x upon d x which almost remains constant for regular physical material and k T d T is nothing, but theta 2 minus theta 1, that is temperature, let say T 2 and T 1 talks about this two parameters. So, A x and d x is basically represented by G, while k T d T is given by theta 2 minus theta 1. Here theta 1 and theta 2are expressed as thermal conductivity integrals. So, theta 2 is thermal conductivity integral at temperature T 2, while theta 1 is a thermal conductivity integral at T 1.

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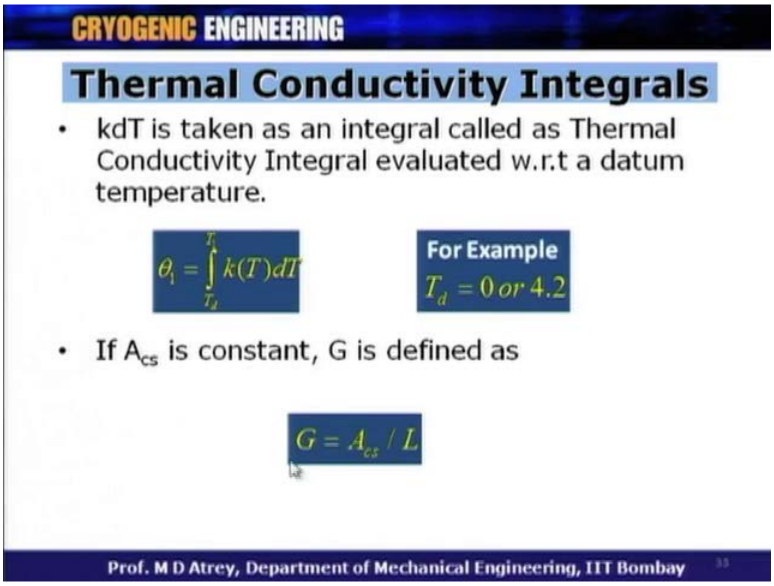

So, k d T now, is taken as integrals called thermal conductivity integrals evaluated with respect to datum temperature. So, we found that, theta 1 is nothing, but what is the k T d T from T d or any datum temperature two temperature of (( )) let say T 1 and similarly, theta 2 will be from T d to T 2, the datum line should be same in both the cases. At this datum line, various books gives various datum lines, could be 0 Kelvin or 4.2 Kelvin.

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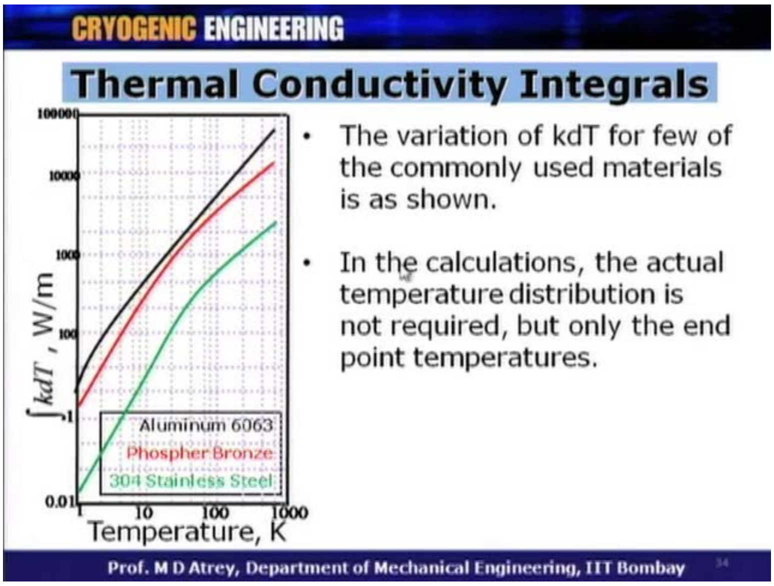

So, if A c s that area of is constant, G is defined as A c s by L which is what we saw. At from this now, what we see is a curve which gives variation of k d T versus temperature. So, if I talk about 100 Kelvin, I got k d T value for a particular material as this, which means that from 0 Kelvin to 100 Kelvin I got k d T value up to this point and let say at 10 Kelvin, I got k d T value at this and if I take difference of this two, I can calculate the loss of conduction, how much loss we have because of the thermal conductivity from 100 Kelvin to 10 Kelvin.

In the calculations, the actual temperature distribution is not required, but what you require only the end point temperature, I do not have to worry what happens from hundred to 10 Kelvin, I do not have to worry about 90, 95, 85 exactly. I just know the value of k d T of 100 Kelvin, I need to know the value of 10 Kelvin. If I know these two values, I can state we calculate the amount of Q that will be loss because of the thermal conduction.

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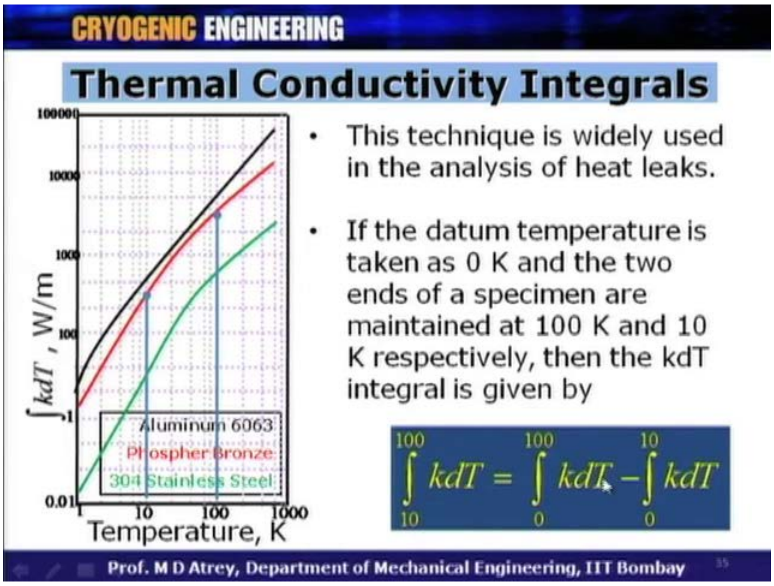

So, here this technique is widely used in the analysis of heat leaks. If the datum temperature is as 0 Kelvin and the two ends of a specimen are maintained at 100 Kelvin and 10 Kelvin respectively, then the k d T integrals are given by, if I want to calculate now amount of loss is occurring because of temperature change from 100 Kelvin to 2 Kelvin for particular material, if I am talking about aluminum, phosphor bronze or stain less steel.

Whatever, then what I am going to do is find a value of k d T for a particular material for phosphor bronze at 100 Kelvin which could be around this, minus at value of k d T at 10 Kelvin and difference between the two is nothing, but the heat loss due to conduction in a temperature region of 100 to 10 Kelvin temperature. What I need to know? Only one value here, one value here and difference between the two will give me k d T multiplied by G is basically, going to give me the amount of heat loss because of thermal conduction.

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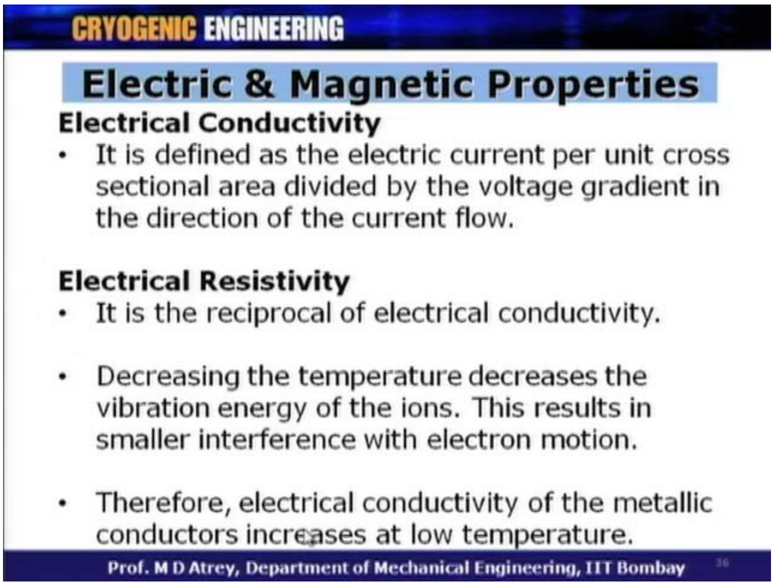

With these three properties, what we just talked about is thermal expansion, we talk about specific heat capacity, and we talked about the thermal conductivity. With these three thermal properties, I will come to now electric and magnetic property; I will touch in very brief in this case. The electrical conductivity as you know, it is defined as the electric current per unit cross selection area, divided by the voltage gradient in the direction of the current flow. So, we know that V upon I is nothing, but resistance. I am talking about the reciprocal of that that is electric current per unit cross sectional area divided by the voltage gradient in the direction of the current flow.

This is what electrical conductivity is given as the electrical resistivity is oppose to that. So, electrical resistivity is, it is the reciprocal of electrical conductivity. As you know that decreasing the temperature decreases the vibration energy of the ions. As you know that as the temperature decreases the energy decreases and therefore, the vibration energy of the ions also decrease the, this result in smaller interference with electron motion this ions. This positive charge actually ions, which are basically are the resistance that is offered to the motion of electron motion, electron motion creates the correct.

So, at lower and lower temperature the resistance offered by the ions is less and less. Therefore, the electrically resistivity in this case decreases. Therefore, electrical conductivity of the metallic conductors increases at low temperature and this is a very simple physical explanation to understand, why at low temperature electrical conductivity of the metallic conductor increases at low temperature?

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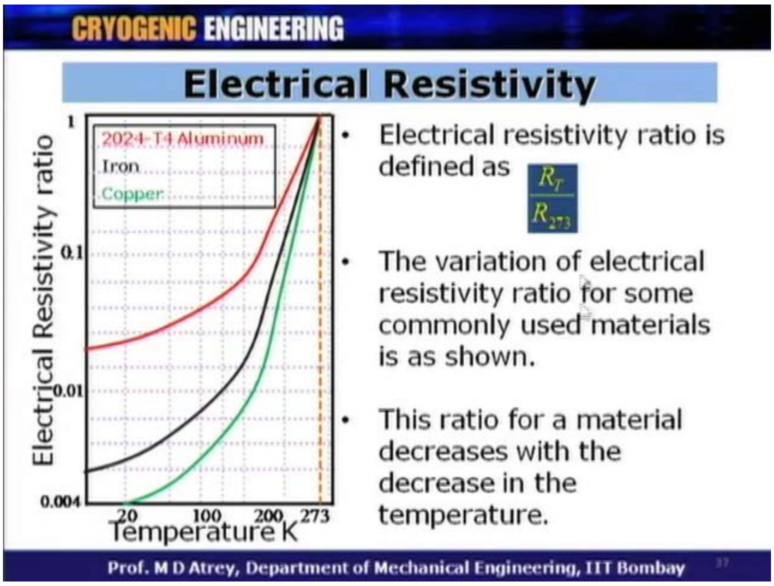

Now this particular curve gives the electrical resistivity ratio with temperature for different material like aluminum ion and copper and what you define electrical resistivity ratio as is defined over here the ERR defined as R T upon R 273; that means, R T is at any temperature T and R 273 is nothing, but 273 Kelvin at that 0 degree centigrade. So, you can understand that R T by R 273 is equal to 1 for all the materials and it is this point the variation of electrical resistivity ratio for some commonly used materials is as shown over here. And, what you can see now? R T upon R 273 decreases for all the materials how are the decreases are different with different temperatures with different materials.

So, what you can see for copper R T at any temperature? Let say 100 Kelvin R T upon R 273 for copper is much smaller as compared to what it is for aluminum. What does it mean? That, R T at 100 Kelvin are 100 is much smaller a value as compared to R 100 for aluminum which means conductivity of copper at 100 Kelvin is much higher as compared to what it is for iron or it is for aluminum which is what we know. This is very important to understand that how this resistivity varies at low temperature? Or how this ratio varies at low temperature?

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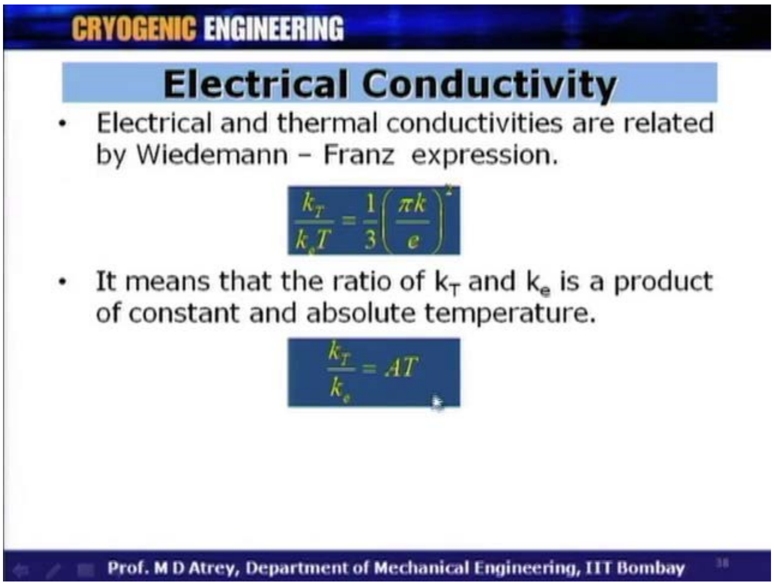

Also important, understand that the electrical and thermal conductivity are related by wiedemann-Franz expression what does it do wiedemann and Franz expression basically finds a relationship between thermal conductivity and electrical conductivity. So, k T and k e are nothing, but thermal conductivity and electrical conductivity. So, k T upon k e into T is equal to 1 by 3 pi k upon e to the power 2. You can see the this parameter is all most constant because k is Boltzmann constant is the charge for electron and we can see that this is completely constant value which will not change with temperature. If suppose the T comes on this side, then you can understand that k T upon k e is function of temperature only, it means that the ratio of k T upon k e is a product of constant which is this and absolute temperature and is represented like this.

So, k T upon k e is equal to AT. It means that, as the temperature gets lower if you reducing the value of T, the k T upon k e also get reduced. It means that k T gets reduced or k e increases. The two possibilities we know that the thermal conductivity decreases at lower temperature, we also know that electrical conductivity increases at low temperature because of decrease in thermal conductivity and because of increase in k e at lower temperature. k T by k e they move in a such way that at a particular temperature k T by k e remain constant or if the variation of temperature is given k T by k e will move according to this expression. This finishes whatever I want to talk about with regard to the electrical conductivity just to some up what we have done in this particular lecture.

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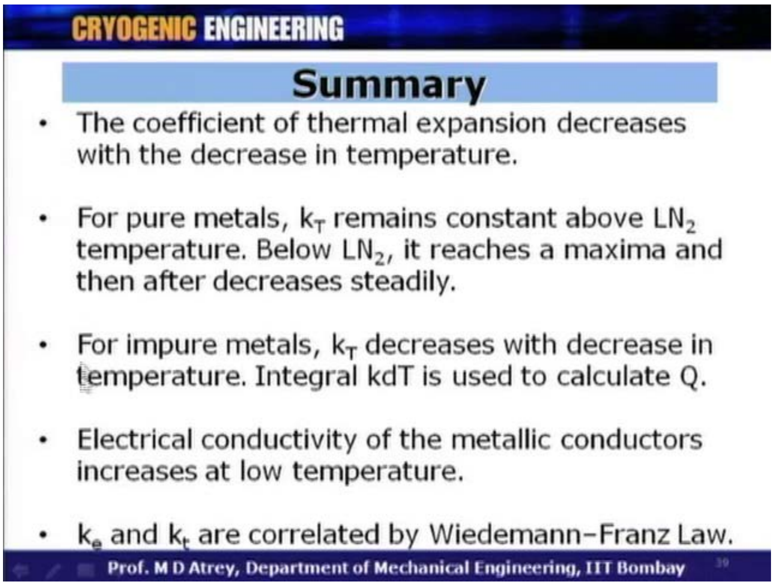

The coefficient of thermal expansion decreases with decrease in temperature this is we talked about lambda t lambda t was nothing, but equal to del L by L divided by delta t.

So, what we understand from here is for different materials you have got specific lambda t value and in all those materials as the temperature decreases, the lambda t value decreases which mean that the material whatever it is, it shrinks when temperature gets lower or it expands, when the temperature increases. For pure material like copper, we saw k T remains constant above liquid nitrogen temperature, below liquid nitrogen temperature it reaches a maxima lets around up to 10 Kelvin.

And, after that below this particular temperature it decreases steadily and we have understood the reasons why does it remain constant above liquid nitrogen temperature why does it go from LN 2 temperature below up to 10 Kelvin and why does it basically electron and decreases below 10 Kelvin to the value 0 impure materials k T decreases with decrease in temperature and also we understood that the integral k d T is used to calculate Q and we also saw how this k d T value is calculated now from the curve the electrical conductivity of the metallic conductor increases at low temperature. This what we understood, at the same time the electrical conductivity and thermal conductivity are correlated by wiedemann fraz law.

So, here we summarize whatever we have discussed in today lecture. Now, we must every less that there are lot of equation, lot of curves, lot of tables where in you should be able to calculate the specific heat at particular temperature for particular material. Or the shrinkage of the particular material at particular temperature or the loss due to thermal conduction from let us say a room temperature to 77 Kelvin or room temperature to 4.2 Kelvin or if the particular road is attached between 100 Kelvin to 10 Kelvin. Then how much load it gives on 10 Kelvin cooled that has to be calculated, that has to be understood.

And, all this problems in the form of tutorial, we will discuss in the next lecture which will be very important lecture to understand, how to calculate this losses? Where, the mathematical relationship between this particular parameters, and the tables will come into picture. At the end, I have got some a self -assessment exercise is given. Kindly asses yourself for this lecture, very honestly there all self assessment, please go through them try to answer, and at the end we have given the answer for those questions. So, that you know whether you answered them correctly or not? Thank you very much.