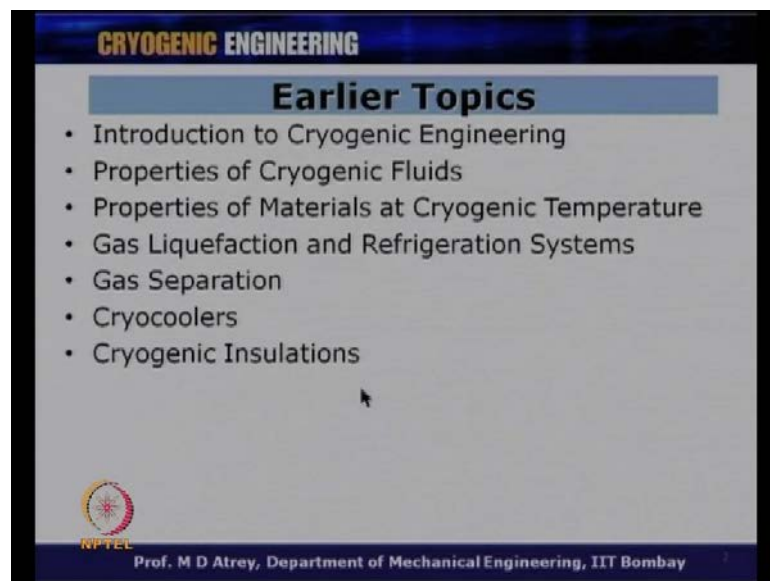


Cryogenic Engineering
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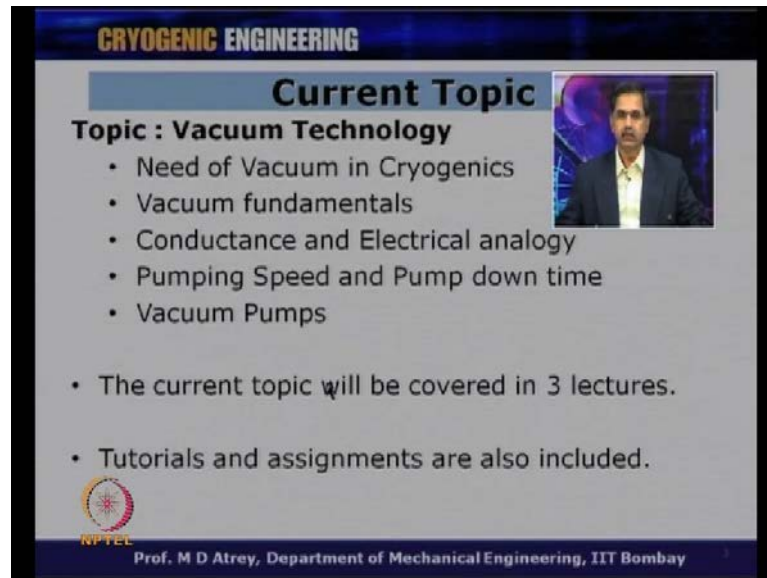
Lecture No. # 36
Vacuum Technology

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Welcome to the thirty six lecture of cryogenic engineering under the NPTEL program. We have covered various topics till now and they are as follows, and the last we finished was cryogenic insulations under cryogenic engineering.

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CRYOGENIC ENGINEERING


Current Topic

Topic : Vacuum Technology

- Need of Vacuum in Cryogenics
- Vacuum fundamentals
- Conductance and Electrical analogy
- Pumping Speed and Pump down time
- Vacuum Pumps

• The current topic will be covered in 3 lectures.

• Tutorials and assignments are also included.

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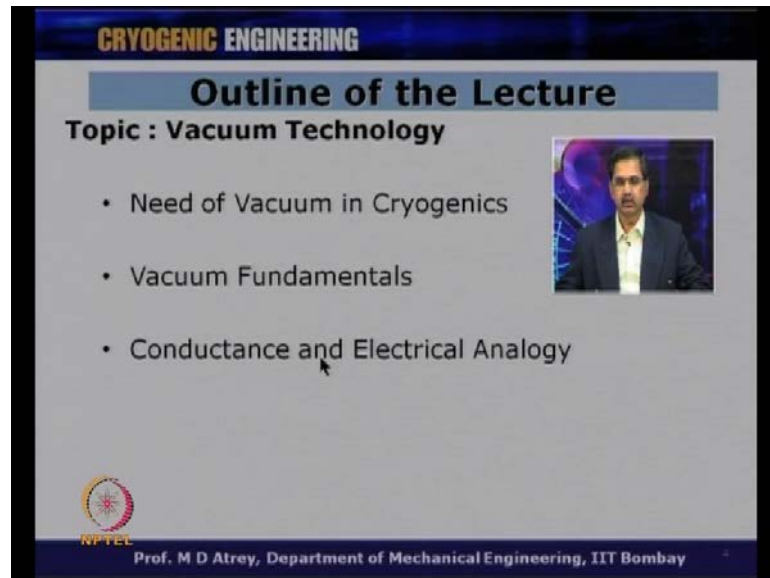
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So, now, the current topic which we are going to talk about is vacuum technology. A brief has already been mentioned about vacuum technology in the earlier lecture regarding cryogenic insulation. However, vacuum is a very important technology and it will be used everywhere in cryogenic engineering, and therefore, it is very important to understand what is this vacuum about, and if I want to get good vacuum, what is a good vacuum practice? What are different vacuum pumps that could be used and things like that?

What is most important to understand is what are vacuum fundamentals. So, under this topic, we will understand what is the need of vacuum in cryogenics although it is little bit clear in the earlier topic when we covered cryogenic insulation. We will know what are vacuum fundamentals. Very basic knowledge to understand how can good vacuum be obtained, what are different technologies, what are different nomenclatures that are used to get good vacuum. Then what is conductance and electrical analogy apply to vacuum technology?

Pumping speed, we use various pumps. How do we get different pumping speeds and what are pump down times for respective pumps, and then, we will talk about in brief about some vacuum pump that are normally used. This topic will be covered in around three lectures and we will have some tutorial assignment at the end of in various topics at various points and time.

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CRYOGENIC ENGINEERING

Outline of the Lecture

Topic : Vacuum Technology

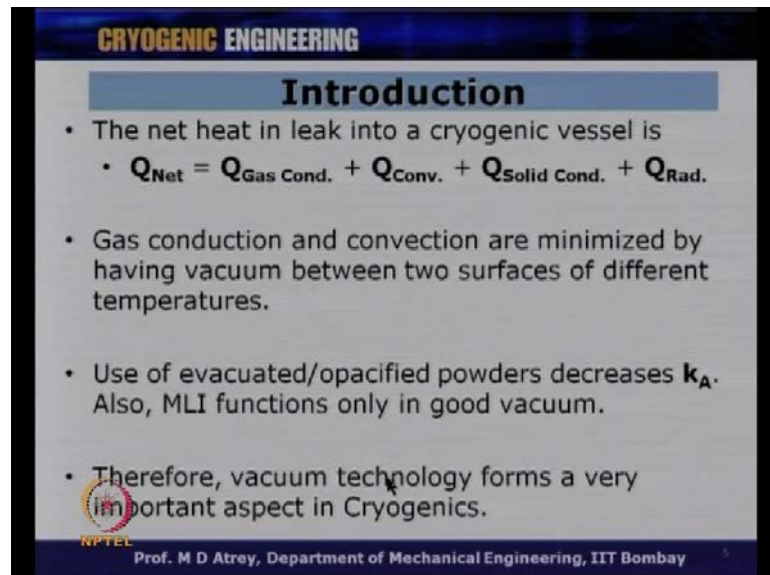
- Need of Vacuum in Cryogenics
- Vacuum Fundamentals
- Conductance and Electrical Analogy

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So, in today's lecture, we will understand what is the need of vacuum in cryogenics, some basics of vacuum that under vacuum fundamentals, and conductance and electrical analogy to understand. What is this conductance business that, is prevalently used in vacuum technology used in cryogenics.

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CRYOGENIC ENGINEERING

Introduction

- The net heat in leak into a cryogenic vessel is
 - $Q_{\text{Net}} = Q_{\text{Gas Cond.}} + Q_{\text{Conv.}} + Q_{\text{Solid Cond.}} + Q_{\text{Rad.}}$
- Gas conduction and convection are minimized by having vacuum between two surfaces of different temperatures.
- Use of evacuated/opacified powders decreases k_A . Also, MLI functions only in good vacuum.
- Therefore, vacuum technology forms a very important aspect in Cryogenics.

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We know that the net heat in leak into a cryogenic vessel is due to various modes of mechanism of heat transfer. So, I can write Q_{net} - the net heat in leak in a cryogenic vessel - will be because of $Q_{gas\ conduction}$, $Q_{gas\ convection}$, $Q_{solid\ conduction}$ and $Q_{radiation}$. This is also we have seen in cryogenic insulation.

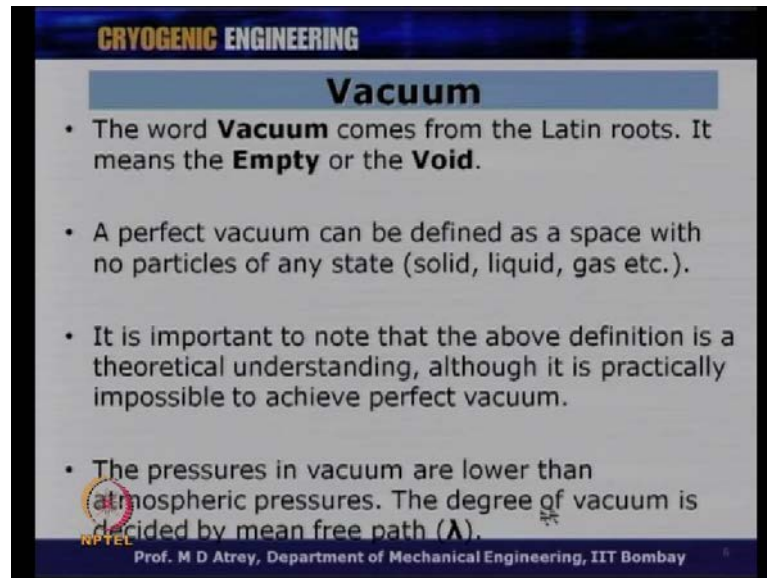
And if I produce good vacuum, the gas component can be completely ruled out; it will not be present over there. It will not be equal to 0, but we can say that the amount of gas available in a particular phase would be negligible or we can neglect $Q_{gas\ conduction}$, and therefore, we can also neglect $Q_{convection}$ as it is going to be caused presence of gas in that particular space.

So, if I got a good vacuum, the first two Q I can say get $Q_{gas\ conduction}$ and $Q_{convection}$ can be completely taken care of or they will get minimized. So, gas conduction and convection are minimized the losses occurring because of this or heat in leak occurring because of convection and conduction are minimized by having vacuum between two surfaces of different temperatures. See, if I have good vacuum, the first two losses or heat in leaks can be completely taken care of if I got good vacuum.

Second thing we also know that use of evacuated or opacified powder which basically uses vacuum over there. It will decrease k a apparent thermal conductivity and we also know that multilayer insulation which takes care of solid conduction as well as $Q_{radiation}$. Solid conduction is taken care by different aspects having, you know, very small size per lite powder or having spacers. We know that optimum number of m l i's could be used so that $Q_{solid\ conduction}$ also could be minimized.

So, m l I can definitely reduce $Q_{radiations}$ also it will minimized $Q_{solid\ conduction}$, and we know that all this powders and also multilayer insulation work only when good vacuum is there. So, that means that all this modes of heat transfer which bring about heat in leak into the system can be taken care of if we are having very good vacuum. So, vacuum technology forms a very important aspect in cryogenics. We have just seen that all this modes of heat transfer gets minimized when we have a very good vacuum, and therefore, vacuum becomes an integral part of cryogenic engineering or a very important technology to be use always with cryogenic.

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CRYOGENIC ENGINEERING

Vacuum

- The word **Vacuum** comes from the Latin roots. It means the **Empty** or the **Void**.
- A perfect vacuum can be defined as a space with no particles of any state (solid, liquid, gas etc.).
- It is important to note that the above definition is a theoretical understanding, although it is practically impossible to achieve perfect vacuum.
- The pressures in vacuum are lower than atmospheric pressures. The degree of vacuum is decided by mean free path (λ).

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So, what do we mean by vacuum? The word vacuum comes from Latin roots. It means empty or the void, so actually nothing. A perfect vacuum can be defined as a space with no particles of any states. So, no presence of solid, liquid, gas particles in a space. If that is so, then we will say it is perfect vacuum. It is important to note that the above definition is a theoretical understanding; we cannot have a perfect vacuum, and therefore, practically it is impossible to achieve perfect vacuum. You can have a very low pressure, very **very** low pressure but, you cannot get perfect vacuum; that means nothing is present over there; no particles are present over there. This is something which is very idealistic situation, and therefore, practically it is impossible to achieve perfect vacuum.


The pressures in vacuum are lower than atmospheric pressures. That is why we call it vacuum. The degree of vacuum is decided by the mean free path. We have touched upon lambda mean free path in the earlier lecture also, and depending on the value of this mean free path, we have got different degrees of vacuum, and before that, let us understand what is this mean free path although it has already when touched upon in cryogenic insulation.


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CRYOGENIC ENGINEERING

Mean Free Path

- Mean free path (λ) is defined as the average distance travelled by the molecules between the subsequent collisions.
- λ is given as
$$\lambda = \frac{\mu}{p} \left(\frac{\pi RT}{2} \right)^{0.5}$$
- Where,
 - μ – Viscosity of gas
 - p – Pressure of gas
 - T – Temperature of gas
 - R – Specific gas constant



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So, what is mean free path? Mean free path is defined as the average distance travelled by the molecules between the subsequent collisions; that means from one collision to second collision, whatever path it travels, whatever distance it travels, it is called as mean free path, and as you know that, as you go on reducing the pressure, the number of molecules will be getting reduced and reduced, and therefore, the path will increase; the path between two collisions will start increasing, and as the pressure gets low down to a very low level, their molecular collisions may stop and the molecule may heat the walls directly. In that case, that is the maximum mean free path possible in that case.

So, lambda is given by, a mean free path is given by this particular expression lambda is equal to mu by p into pi R T by 0.2 to the power 0.5 - where mu is a viscosity of the gas; P is the pressure of the gas and T is the temperature of the gas. So, there different parameters and one can calculate the value of lambda or mean free path accordingly and R is a specific gas constant. So, for every gas now, we can calculate the value of lambda.

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CRYOGENIC ENGINEERING

Mean Free Path

$$\lambda = \frac{\mu}{p} \left(\frac{\pi k T}{2} \right)^{0.5}$$

- It is clear that λ ,
 - Increases with decrease in pressure
 - Increases with increase in temperature
- The value of mean free path (λ) plays an important role in deciding the flow regimes in vacuum.

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Also we understand from these that lambda increases with decrease in pressure. So, as the pressure gets reduced, the value of lambda increases; that means the distance between two collisions are the path starts increasing between the two collisions which is understandable, because as the pressure gets reduced down, the number of particle will gets reduced. So, the number of collisions also gets reduced and molecule to molecule collision will take more path to be achieved by one molecules.

So, as the pressure gets reduced, the value of lambda increases; it also increases with increase in temperature. That can be understood from this because temperature happens to be in the numerator. The value of mean free path plays an important role in deciding the flow regimes in vacuum. So, this lambda is a very important characteristic or very important parameter which decides what is this flow regimes in vacuum. Similar to the fact that we have got different flow regimes in continuum region. We have got different flow regimes in vacuum also.

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The slide is titled "CRYOGENIC ENGINEERING" and "Flow Regimes". It contains two diagrams of a closed system. The top diagram, labeled "High Pressure", shows a rectangular box filled with many small red and green circles representing molecules. The bottom diagram, labeled "Low Pressure", shows the same box with only a few scattered red and green circles. To the right of the diagrams is a bulleted list: "Consider a closed system as shown in the figure.", "With the lowering of pressure", "The number of molecules are reduced", "The residual molecules are pulled apart.", and "As a result, mean free path (λ) of residual molecules becomes larger than the dimensions of the system." At the bottom left is the NPTEL logo, and at the bottom center is the text "Prof. M D Atrey, Department of Mechanical Engineering, IIT Bombay".

Let us see, what is this flow regime. Consider a closed system as shown in the figure and there are different molecules let say at high pressure; that means in compared to atmospheric pressures, we have got different molecules gram together in a given space or given volume, but as you go down lowering the pressure, the number of molecules will get reduced and you may land up in a situation like this where the number of molecules are reduced as the pressure is reduced and the residual molecules are now pulled apart; that means the distance between the two molecules will start increasing, and therefore, we say that here the molecules will have collisions at will, I mean the time they move, the the time they start to having movement, they will have a collision over here.


Why? Here it is a probabilistic model. The depending on what direction the molecules move, it will have a collision, and therefore, we say here the length between the two collision or the lambda value. The mean free path in this case will be more as compared to what it is over here. That is obvious from here.

As a result, mean free path of residual molecules becomes larger than the dimensions of the system. So, sometimes this means free path will become more than the dimension of the... If it travels over here, it will possibly never collide with any other molecule. In that case, the mean free path may be more than the diameter which is the characteristic dimension of this particular enclosure and mean free path may be more than this or it will be comparable to that.

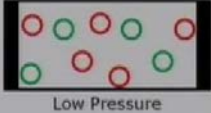
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CRYOGENIC ENGINEERING


Flow Regimes



- In such systems, the molecules collide only with the walls of the container.
- Such a flow of fluid is called as Free Molecular Flow.



Low Pressure



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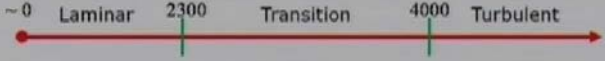
In such system, molecules collide only when, only with the walls of the container and they will never collide with the molecules themselves. So, molecule to molecule collision will be rear as the pressure goes down or, or, as we go down to lower pressures. Such a flow of fluid is called free molecular flow. Now, we are talking about having a free molecular zone over here and this is where vacuum would come into picture basically because normally we will deal with such flows in vacuum.

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CRYOGENIC ENGINEERING

Flow Regimes

- If λ is much smaller than the characteristic lengths, such flows are called as continuum flows.
- In fluid mechanics, Reynold's Number (**Re**) is used to categorize the pipe flow regimes as shown above.



- In these flows, molecules collide with each other as well as physical boundaries, if any.
- Pressures are in the range of atmospheric values.



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If lambda is much smaller than the characteristic lengths, such flows are called continuum flow. If lambda is very **very** small, that means the length between the collisions or the time between the collisions see very **very** small or as soon as the molecules start moving, it collides with other molecules. See, in this case, lambda is going to be very small and this will not normally happen in continuum flow, which is what normally we will deal above atmosphere pressures.

In fluid mechanics, Reynolds number is used to categorize the pipe flow regime as shown in the figure. So, we got a, normally we deal with Reynolds number in continuum flow and we know that Reynolds number is between 0 to 2300. What we call the flow is laminar. While if the Reynolds number is between 2300 and 4000, we say that the flow is transition flow. While above 4000, we say the flow is turbulent flow and this is very much known to all of you.

In these flows, molecules collide with each other as well as with physical boundaries if any; that means the molecules are in constant collision conditions; they are constantly colliding with each other and they also may collide with the walls or the physical boundaries of this pipe. Now, such thing may not exist at low pressure. So, in this case, pressures are in the range of atmospheric values. Why?

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CRYOGENIC ENGINEERING

Flow Regimes in Vacuum

- Knudsen Number (N_{Kn}) is used to categorize the flow regimes in vacuum.
- This concept is analogous to Reynold's Number (Re) in fluid mechanics.
- Knudsen Number (N_{Kn}) is given as
$$N_{Kn} = \frac{\lambda}{D}$$
- Where,
 - λ – Mean free path
 - D – Characteristic diameter

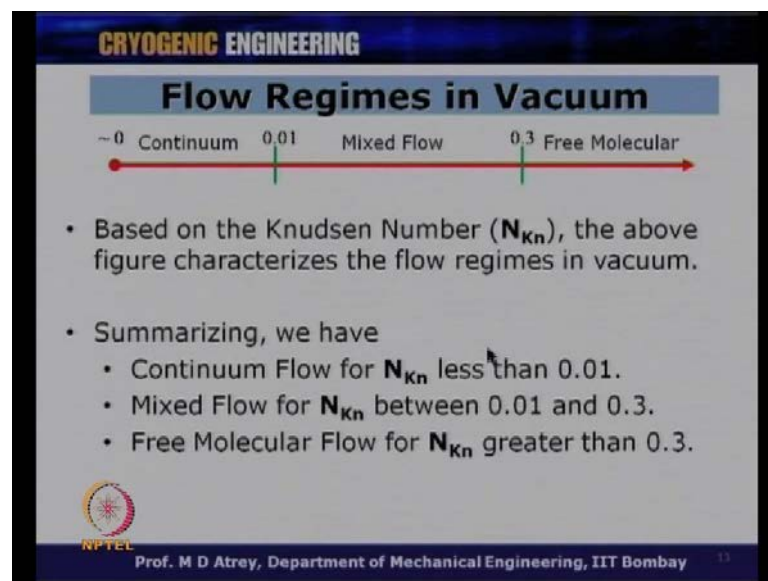
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If you go to the vacuum now, we have got a different flow regime now and we do not have Reynolds number over in vacuum now. What we have now here is calls Knudsen

Number. Normally written as N_{kn} is used to categorize the flow regimes in vacuum. So, here, we always deal with a different numbers and not, not, some verb and the moment we specify Knudsen Number, we know that we are talking about flow regimes in vacuum now. And what is this Knudsen Number now? This concept is analogous to Reynolds number in fluid mechanics, and the Knudsen Number N_{kn} is given as N_{kn} is equal to λ / D - where λ is mean free path and D is the characteristic diameter or dimension.

So, if we know that as we have lower and lower pressure, the value of λ will go on increasing; that means the Knudsen Number will go on increasing as we have got better and better and better vacuum, and according to the value of this Knudsen Number now, we will have different flow regimes in vacuum.

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So, I will have different Knudsen Number now. So, based on the Knudsen Number N_{kn} , the above figure characterizes the flow regimes in vacuum. So, when I say my Knudsen Number is less than 0.01, I will say I am in continuum region. That λ is much smaller than D in this case, but as λ starts increasing, Knudsen Number starts increasing. So, if I am between 0.01 to 0.3, I have something called as mixed flow over here.

So, my lambda is now increasing because of the pressure is decreasing, and if my Knudsen Number is between 0.01 to 0.3, I will have something called as mixed flow, and if my Knudsen Number is more than 0.3, that means lambda is 0.3 times dimension of the characteristic dimension. I am in something called as free molecular region. So, my molecules are in a free molecular flow region here there in the mixed flow regime. In this case when the lambda, when the Knudsen Number is between 0.01 and 0.3, and if the Knudsen Number is less than 0.01, I am actually calling I am still in continuum region. Here, the lambda value is going to be very **very** small as compared to the dimensions or the characteristic dimension of the container.

Summarizing, we, **we**, say that we have got a continuum flow for Knudsen Number less than 0.01 mixed flow between Knudsen Number having between 0.01 and 0.3, and free molecular flow for Knudsen Number greater than 0.3. The lambda value will go on increasing; Knudsen Number will go on increasing and I will reach to a free molecular regime when the Knudsen Number is more than 0.3.

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CRYOGENIC ENGINEERING

Units of Pressure

- In an S. I. system, pressure is measured in **Pascal** or **N/m²**. Very often, **Bar** is also used for pressure measurement.
- For example, the standard atmospheric pressure can be expressed as
 - 1.013×10^5 **Pa**
 - 1.013×10^5 **N/m²**
 - **1 bar**
 - **760 mm** of Hg column at standard sea level.

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So, let us call now, because as we go on lowering the pressures, we normally would call the pressures by different unit in this case. So, let us understand what are normal units of pressure, and then, we will define how do we refer to different vacuum levels. So, in SI unit, pressure is measured in Pascal or Newton per meter square; very often bar is also used for pressure measurement. You know this.

For example, the standard atmospheric pressure can be expressed as 1.013 into 10 to the power 5 Pascal, 1.013 into 10 to the power 5 Newton per meter square. This is what normally we used in SI or in thermodynamics also. We have got a 1 bar or we have got a 760 millimeter of mercury column at standard sea level. This is what, our standard units of pressure. This is what we used normally in thermodynamics and heat transfer and fluid mechanics and thing like that.

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The slide is titled "CRYOGENIC ENGINEERING" and "Units of Pressure". It contains the following text:

- In vacuum, normally unit for pressure is **Torr** or **milli bar**.
- This unit is named after **Evangelista Torricelli**, an Italian physicist, in the year 1644.
- **1 Torr** is defined as **1 mm** of Hg column at standard sea level.
- Therefore, **1 Torr = 133.28 Pa = 133.28 N/m²**.

At the bottom, there is an NPTEL logo and the text "Prof. M D Atrey, Department of Mechanical Engineering, IIT Bombay".

Now, in vacuum, normally unit of pressure is Torr - **t o double r Torr** - or sometimes also milli bar. This unit is named after Evangelista Torricelli. You must have heard about Torricelli - an Italian physicist in the year 1644. So, to honor this scientist Torricelli, we have got a unit of pressure specifically used in vacuum. Vacuum therefore will be called as so many Torrs or sometimes in milli bar also and this unit is named after this scientist Torricelli in the year 1644.

So, 1 Torr, what is this 1 Torr? 1 Torr is defined as one millimeter of mercury column at standard sea level. So, imagine that 760 millimeter of mercury column makes one atmosphere, 1 bar while 1 millimeter makes 1 Torr here or we can say one by 760 bar is nothing but 1 Torr or 1 by 760 atmosphere is nothing but equal to 1 Torr. So, actually the pressures in this range are much below atmosphere, much below 1 bar. Therefore, 1 Torr is equal to 133.28 Pascal or 1 Pascal is nothing but 1 Newton per meter square, so, 133.28 Newton per meter square.

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CRYOGENIC ENGINEERING

Units of Pressure

- The conversion table for pressure is as shown below.

	Pa	Bar	atm	Torr
1 Pa	1	10^{-5}	9.8×10^{-6}	7.5×10^{-3}
1 Bar	10^5	1	0.98	750.06
1 atm	1.013×10^5	1.013	1	760
1 Torr	133.3	1.33×10^{-3}	1.31×10^{-3}	1

- 1 milli = 10^{-3} .**
- 1 Kilo = 10^3 .**

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So, there is a conversion table for your reference which is go on over here. So, we can have different units; we can have vacuum could be presented sometime in Pascal bar atmosphere or Torr. Normally, it will be referred in Torr, and here, we have got different conversions from Pascal to atmosphere Pascal to Torr. So, I can say that 1 Pascal is equal to 10 to the power minus 5 bar 1 Pascal is equal to 7.5 into 10 to the power minus 3 Torr. Similarly, 1 bar is equal to 10 to the power 5 Pascal; 1 bar is equal to 750.06 Torr or 1 Torr which is what we vacuum is referred to as is equal to 133.3 Pascal or 1.31 into 10 to the power minus 3 atmosphere.

So, one can understand from these if I were to convert different units of vacuum or pressures into other units. This is kind of a conversion table for your usage. Similarly, we know that 1 milli is nothing but 10 to the power minus 3 and 1 kilo is 10 to the power 3. So, sometimes kilo Pascal could be written over here. So, I can say 1.33 kilo Pascal is equal to 1 Torr, because sometimes we can understand like this also.

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CRYOGENIC ENGINEERING

Degree of Vacuum

- As mentioned earlier, pressures are lower than atmospheric pressures in vacuum spaces.
- Depending upon the pressure in the system, the degree of the vacuum is categorized.
- The table on the next slide correlates the pressure and degree of the vacuum.

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Now, there are different degrees of vacuum depending on the levels of different pressures. So, as mentioned earlier, pressures are lower than atmospheric pressures in vacuum spaces, and depending on how low they are, depending on their values, we got something called as degrees of vacuum.

So, depending upon the pressure in the system, the degree of the vacuum is categorized. The table on the next slide correlates the pressure and degree of vacuum. So, normally we call I should have low vacuum; I should have very low vacuum. I should have ultra-low vacuum and things like that. There all classified based on what is the level of pressure over there, and therefore, let us have a look at this degree of vacuum.

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Degree of Vacuum	Pressure
Rough Vacuum	25 torr < p < 760 torr 3 kPa < p < 103 kPa
Medium Vacuum	0.001 torr < p < 25 torr 0.1 Pa < p < 3000 Pa
High Vacuum	10^{-6} torr < p < 10^{-3} torr 0.1 mPa < p < 100 mPa
Very High Vacuum	10^{-9} Torr < p < 10^{-6} Torr 0.1 μ Pa < p < 100 μ Pa
Ultra High Vacuum	p < 10^{-9} torr

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So, degree of vacuum normally called as rough vacuum. In most of the industrial applications, we have got rough vacuum, wherein the pressure is between 760 Torr which is nothing but 1 atmosphere approximately and 25 Torr. See, when your pressure is between 25 Torr and 760 Torr, we have got rough vacuum, or in k p a, we have got three k p a and, 10, 103 k p a.

So, if my pressure is between 103 k p a and 3 k p a, we got rough vacuum. Then we have got medium vacuum between 25 Torr to 0.001 Torr 10 to the power minus 3 Torr or also given in Pascals over here. Then you have got a high vacuum. So, you got a rough vacuum, medium vacuum, high vacuum. The high vacuum is between 10 to the power minus 3 to 10 to the power minus 6 Torr.

So, again you can understand that I am come down to 10 to the power minus 6 Torr, which is between high vacuum when the pressure is between minus 3 and minus 6 Torr. Sometimes it referred as when one would not say 10 to the power minus 3 to 10 to the power minus 6, one can always say that vacuum is of the order of minus 3 to minus 6 or it between minus 3 and minus 6.

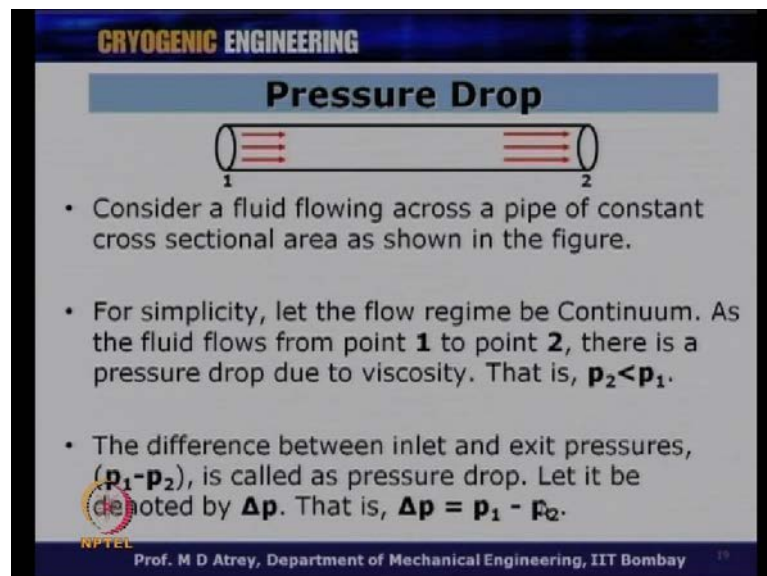
Normally, it is understood that this is 10 to the power minus 3 and 10 to the power minus 6. There are different levels of vacuum, and then we got a very high vacuum between minus 6 to minus nine levels. When we got a pressure of the order of 10 to the power minus 8 minus 9, we say we are in very high vacuum region and then we got a ultra-high

vacuum which is use for very special purposes and this is now less than pressure less than 10 to the power minus 9 Torr.

So, you can understand normally most of our operation happens between medium vacuum to high vacuum in this region at least in cryogenic unless very high levels of vacuum are expected. Then, you know very special applications would expect ultra-high vacuum to be there. In that case, what kind of material do you use; what processes do you use all these aspects become very important, because you need a very high level of vacuum in this case, and therefore, kind of process you use here are very **very** important.

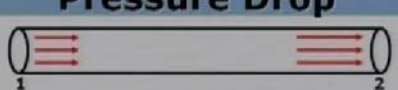
So, slide number 16 correction so we can say that from here one Torr is equal to 1.133 kilo Pascal. So, we have understood what are various degrees of vacuum now, and therefore, now we will go to little practical applications or how to get good vacuum, but before that, we will have to understand the flow rates, the pressure drops, because if I want to vacuum particular space, I will connect that space to a vacuum pump and I will connect that space to vacuum pump through a tube or a pipe, and to begin with now, we will have a pressure drops across this pipe, and therefore, we will have a molecular flow region.

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CRYOGENIC ENGINEERING

Pressure Drop



• Consider a fluid flowing across a pipe of constant cross sectional area as shown in the figure.

• For simplicity, let the flow regime be Continuum. As the fluid flows from point **1** to point **2**, there is a pressure drop due to viscosity. That is, $p_2 < p_1$.

• The difference between inlet and exit pressures, $(p_1 - p_2)$, is called as pressure drop. Let it be denoted by Δp . That is, $\Delta p = p_1 - p_2$.

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We will be now talking about not a continuum regime, but maybe we will talking about molecular flow regimes, and therefore, we will have to understand what is the pressure drop that happens what are the mass flow rate that happens when we are talking about molecular flow. And therefore, let us understand the pressure drop or the mass flow relationships as we go down from continuum flow to mixed flow and mixed flow to molecular flow region, and this is very important to understand, to calculate the mass flow rate and the pressure drop Δp across a tube of let us say length l , and therefore, let us understand what is this pressure drop and mass flow rate relationships for continuum region, mixed flow region and molecular flow region.

So, we know that in a pipe, if the fluid is traveling across the pipe of constant cross section area as shown in the figure will have pressure drop across this region from one to two will have a pressure drop with passage of time. For simplicity, let the flow regime be continuum. Let us assume its continuum regime now. As the fluid flows from 0.1 to 0.2, there is the pressure drop due to viscosity, and therefore, what we will have? Pressure at this point two will be less than pressure at 0.1. So, p_2 is going to be less than p_1 because of the friction, because of the surface conditions, because of the viscosity of the fluid.

The difference between the inlet and exit pressures which is p_1 minus p_2 is what we call as pressure drop. Let this, this, be denoted as Δp , and therefore, we can say Δp is equal to p_1 minus p_2 and it is a very standard fluid mechanics practice to calculate Δp across the region when the flow rate is around, let say \dot{m} or whatever it is.

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CRYOGENIC ENGINEERING

Pressure Drop

• Pressure drop for a laminar continuum flow is

$$\Delta p = \frac{128 \mu L \dot{m}}{\pi D^4 \rho}$$

• It is called as Poiseuille's equation, which correlates pressure drop (Δp) and mass flow rate (\dot{m}).

• Here,

- μ, ρ – Viscosity and Density of fluid
- L, D – Length and diameter of tube
- \dot{m} – Mass flow rate

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So, pressure drop for a laminar continuum flow, and when I say laminar flow, we are talking about Reynolds number less than around 2300, less than 2300 and this is the very standard equation Δp is equal to $128 \mu l \dot{m}$ upon π into d to the power 4 into ρ , and this is called as Poiseuille's equation in fluid mechanics which correlates pressure drop Δp to mass flow rate \dot{m} .

So, if the length is l and the mass flow rate is \dot{m} and a diameter is d , then during this travel, I will have a Δp amounting to this and one can see from here that Δp is directly proportional to the length it travels; Δp is directly proportional to the \dot{m} mass flow rate. If the mass flow rate is high, Δp is going to be high. The length is high, Δp is going to be high.

Why? If the d is small, Δp is going to be very **very** large; Δp is dependent on d to the power 4 which is the very important parameter. So, in continuum flow, we can see that Δp is inversely proportional to the fourth power of d . It is a very important parameter, please note that.

So, here, μ and ρ are viscosity and density. Here μ and ρ are viscosity and density of the fluid while l and d are the length and a diameter of the tube, and Δp is directly proportional to the length and Δp is inversely proportional to the fourth power of d as far as we are talking about laminar continuum flow.

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CRYOGENIC ENGINEERING

Pressure Drop

- For a rough vacuum ($N_{Kn} < 0.01$), as mentioned earlier, the operating pressures are in between **25** to **760** Torr.
- An ideal gas behaviour is assumed and hence, the correlation between average pressure and density is
- Here, $\rho = \frac{\bar{p}M}{RT}$
 - ρ, T - Density and Temperature of gas
 - M - Molecular weight of gas
 - R - Universal Gas constant
 - \bar{p} - Average pressure

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For rough vacuum now, when the Knudsen Number is less than point one as mentioned earlier, the operating pressure are in between 25 to 760 Torr which we know now. I am talking about Knudsen Number less than 0.01. An ideal gas behavior assumed here, and hence, the correlation between average pressure and density is rho is equal to p m by r t. So, we have got a ideal gas law here. So, here, rho and t are density and temperature of gas and we are talking about now rough vacuum Knudsen Number less than 0.01 or operating pressure between 25 to 760 Torr. We are having that ideal gas relationship is valid; m is the molecular weight of the gas; r is a universal gas constant while p is the average pressure.

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CRYOGENIC ENGINEERING

Pressure Drop

$$\Delta p = \frac{128 \mu L \dot{m}}{\pi D^4 \rho}$$
$$\rho = \frac{\bar{p} M}{RT}$$

- Combining the above two equations, the pressure drop (Δp) for a continuum laminar flow is

$$\Delta p = \frac{128 \mu L \dot{m} RT}{\pi D^4 \bar{p} M} \rightarrow \dot{m} = \frac{\pi D^4 \bar{p} M \Delta p}{128 \mu L RT}$$

- From the above equation, it is clear that the mass flow rate (\dot{m}) is
 - Directly proportional to pressure drop (Δp).
 - Directly proportional to 4th power of diameter (D).

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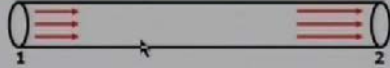
So, we have found that for this continuum regime, Knudsen Number less than 0.01. We have got a Δp is $128 \mu l \dot{m}$ upon π into D to the power 4 into ρ . We know also ρ is equal to $p m$ by $r t$, and if I put the value of ρ over here combining the above two equations, the pressure drop Δp for continuum laminar flow is this, and from here, I get relationship for \dot{m} having \dot{m} over here. I will get \dot{m} and Δp related. So, \dot{m} is equal to π into D to the power 4 $p m$ into Δp divided by $128 \mu l r t$.

So, basically Δp and \dot{m} directly proportional and you got a relationship between Δp and l and D . Therefore, you got a relationship between \dot{m} and d and $p D$ and Δp . From the above equation, it is clear that the mass flow rate \dot{m} is directly proportional to the pressure drop. So, if the pressure drop is higher, mass flow rate going to be higher and its directly proportional to the forth power of diameter. Here, Δp we were talking about when Δp had inverse relation with the forth power of D , while mass flow rate has now direct relationship between the, to the forth power of diameter.

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CRYOGENIC ENGINEERING

Pressure Drop



- With lowering of pressure in tube, ($0.01 < N_{Kn} < 0.3$), an intermediate flow regime between the continuum and the free molecular flows exists.
- This regime is called as Mixed Flow or Slip Flow.
- In such conditions, the gas molecules close to the wall appear to slip past the wall with a finite velocity parallel to axis of tube, and hence the name **slip flow**.

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Now, that was with the continuum flow and let us talk about now mixed flow or with the lowering of pressure in the tube when the Knudsen Number is between 0.01 and 0.3 and intermediate flow regime between the continuum and the free molecular flow exists now as we have seen different flow regimes in vacuum now. This regime is called as mixed flow or slip flow. So, what is happening now? We are somewhere in between molecular flow regime and continuum regime, and that is why it is called as mixed flow regime, and in such condition, the gas molecules close to the wall appear to slip past the wall with a finite velocity parallel to the axis of the tube, and hence, it is called as slip flow.

In the earlier case, in the continuum regime, we know that the velocity of the particles at the wall is equal to 0. We know that the pressure is, the flow rate is maximum in the center and is 0 at the walls of this pipe, but as the number of molecules go on lessening, as the Knudsen Number is getting increased or as λ starts getting increased, this molecule at the wall will also have some finite velocity, and therefore, it will not be velocity equal to 0 over in this case. It will have some finite velocity, and therefore, we say that the molecules appear to slip past the wall. It is going; it is not stopping over there. It is going; it is slipping past the wall with the finite velocity, and therefore, this velocity will also be showing up in the calculation of Δp or mass flow rate.

And therefore, in slip flow, we will have something what was happening in continuum flow plus because of the change in velocity or because of the velocity existing at the finite velocity, at the walls of this pipe, we will have one more parameter appearing in the pressure or in the mass flow rate calculations.

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CRYOGENIC ENGINEERING

Pressure Drop

- From the kinetic theory of gases, mass flow rate (\dot{m}) and pressure drop (Δp) for slip flow in a circular tube is given by

$$\dot{m} = \frac{\pi D^4 \bar{p} \Delta p}{128 \mu L \mathcal{R} T} \left(1 + \frac{8 \mu}{\bar{p} D} \left(\frac{\pi \mathcal{R} T}{2 M} \right)^{0.5} \right) \quad \dot{m} = \frac{\pi D^4 \bar{p} M \Delta p}{128 \mu L \mathcal{R} T}$$

- On comparison of above equation with mass flow rate (\dot{m}) for a continuum laminar flow,
 - The first term accounts for the internal laminar flow (away from walls).
 - The second term accounts for the finite velocity correction near the tube walls.

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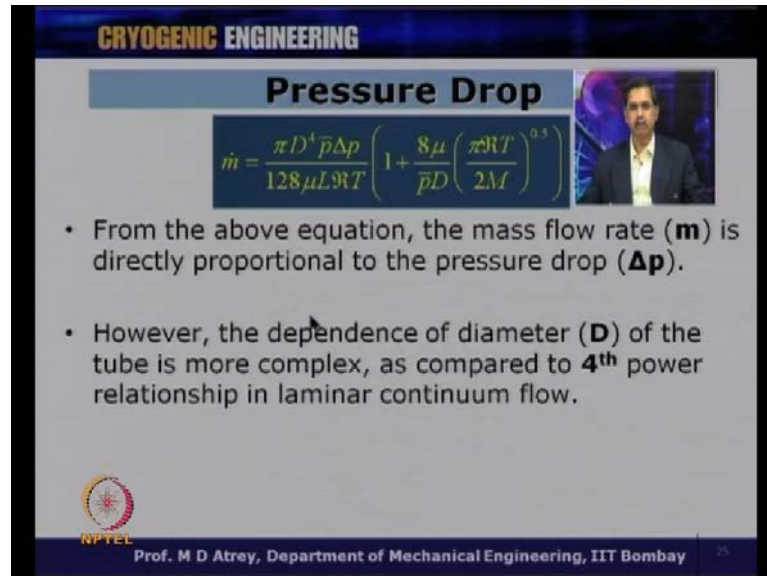
So, from the kinetic theory of gases, mass flow rate and pressure drop for slip flow in a circular tube is given by this formula. So, you can see now \dot{m} is equal to something into one plus something. So, this is same as what it was in a continuum flow and this is what we had in a continuum flow. This is the expression for \dot{m} in continuum flow and this is nothing but same.

On comparison of above equation with mass flow rate for continuum laminar flow, we know that the first term accounts for the internal laminar flow away from the walls; that means at the center, near the center of the tube, this will account for that but the second term is now accounting for the slip or the finite velocity the gas molecules will have near the walls.

The second term accounts for the finite velocity correction near the tube walls. This will make the matter little complicated, because in earlier case mass, flow rate was directly proportional to D to the power 4. In this case, however we got a d to the power 4 here and we got a d in the denominator over here, and therefore, the relationship between the

mass flow rate and a diameter is not so much clear. So, we can understand; however, \dot{m} is directly proportional to still Δp .

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CRYOGENIC ENGINEERING

Pressure Drop

$$\dot{m} = \frac{\pi D^4 \bar{p} \Delta p}{128 \mu L \mathcal{R} T} \left(1 + \frac{8 \mu}{\bar{p} D} \left(\frac{\pi \mathcal{R} T}{2M} \right)^{0.5} \right)$$

- From the above equation, the mass flow rate (\dot{m}) is directly proportional to the pressure drop (Δp).
- However, the dependence of diameter (D) of the tube is more complex, as compared to 4th power relationship in laminar continuum flow.

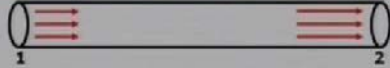
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So, from the above equation, the mass flow rate \dot{m} is directly proportional to the pressure drop which is Δp ; however, the dependence of diameter d of the tube is more complex as compared to the fourth power relationship in laminar continuum flow. So, earlier, we had \dot{m} directly proportional to d to the power 4. While in this case, we got a \dot{m} which is a complex relationship with diameter because diameter appears in the finite velocity here in the denominator. While in the continuum region, it was appearing in the numerator only. So, you can see the change that is occurred because we shifted from a continuum region to a mixed flow regime now or the Knudsen number between 0.01 and 0.3.

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CRYOGENIC ENGINEERING

Pressure Drop



- With further lowering of pressure ($N_{Kn} > 0.3$), the number of molecules are reduced as well as the residual gas molecules are pulled apart.
- This flow regime is called as Free Molecular Flow.
- In such conditions, mean free path (λ) of the molecules is larger than the diameter of the tube. The flow is limited due to collisions of molecules with the walls.

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Let us, if we go down the pressure earlier, Knudsen Number will be more than 0.3. With further lowering of pressure, when Knudsen number is more than 0.3, the numbers of molecules are reduced as well as the residual gas molecules are pulled apart. So, we have seen earlier the lambda value has increased now, and therefore, the collision path, the, the, path between having two collision has become comparable with the characteristic dimension. The Knudsen Number is more than 0.3; the pressures are less; the distance between the molecules will increase. This flow regime is called as free molecular flow. In such conditions, mean free path lambda of the molecules is larger or comparable with the diameter of the tube. The flow is limited due to collisions of molecules with the walls. This is what we had talked about earlier.

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CRYOGENIC ENGINEERING

Pressure Drop

- The mass flow rate (**m**) and the pressure drop (**Δp**) in a free molecular flow are related by

$$\dot{m} = \frac{D^3 \Delta p}{L} \left(\frac{\pi M}{18RT} \right)^{0.5}$$

- From the above equation, it is clear that the mass flow rate (**m**) is
 - Directly proportional to pressure drop (**Δp**).
 - Directly proportional to **3rd** power of diameter (**D**).

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In this case now, the mass flow rate \dot{m} and the pressure drop in a free molecular flow are related by this formula. So, we got a \dot{m} now directly proportional to D^3 over here, while \dot{m} is still directly proportional Δp . However, its dependence on diameter has changed. So, if you recollect in the continuum region, it was dependent directly on D to the power 4. In the slip flow, it was a complex relationship. While here, in the molecular region, \dot{m} is now directly dependent on power 3 of the diameter D^3 basically. So, things are changing as we shift from continuum region to free molecular region.

And this is very important because we will have to calculate or we will have to understand what is diameter should I use; what is the length should I use if I were to connect my vacuum pump with the space vacuum, or if I know that region to be vacuum has so much diameter and length, I will have to understand the dynamics of molecular flow in that particular space. From the above equation, it is clear that the mass flow rate \dot{m} is directly proportional to the pressure drop Δp and directly proportional to the third power of diameter. This is what we have understood.

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The slide is titled "CRYOGENIC ENGINEERING" and "Throughput (Q)". It contains the following text:

- Apart from mass flow rate (**m**), the rate of fluid flow is often measured by a quantity called as Throughput (**Q**).
- Throughput is defined as a product of volumetric flow rate (**V**) and pressure (**p**), measured at the point where **V** is measured.
- Mathematically, we have $Q = p \dot{V}$
- The S. I. unit for Throughput is **Pa-m³/s**. Very often at low pressures, it is also expressed in **Torr-Lit/s** or **bar-Lit/s**.

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Now, let us come to the next nomenclature which is prevalently used in vacuum technology, which is nothing but throughput and normally designated as Q. So, apart from mass flow rate m, the rate of fluid flow is often measured by a quantity called as throughput Q. So, we, we can call something as mass flow rate which is k g per second or something like that, but then, we have got a one more parameter called Q and it is a very important because it talks about pressure and volume flow rates also.

So, what is this Q? The throughput is defined as a product of volumetric flow rate v and pressure measured at the point where v is measured. Wherever we are measuring mass flow rate or volumetric flow rates, at the same place, we will measure pressure also and pressure into multiplied that volumetric flow rate will give me what is called as throughput, and therefore, it will have units of pressure as well as volumetric flow rates.

So, mathematically we have Q is equal to p into v dot. So, volumetric flow rates, let say liter per second, liter per minute, liter per hour or any other dimensions we can have and pressures we can have pressure dimensions as Torr or bar or mill bar, kilo Pascals or whatever.

So, the, the, SI units of throughput therefore are Pascal meter cube per second, Pascal for pressure meter cube per second for volumetric flow rate very often at low pressure. It is also expressed in Torr liter per second. Mostly I have seen that various places we got a Torr liter per second or milli bar liter per second, so, Torr liter per second. Torr is for

pressure liter per second or miter per second will be for the volumetric flow rate and this is the way normally the throughput is mentioned.

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CRYOGENIC ENGINEERING

Throughput (Q)

- Assuming an ideal gas behavior, the volumetric flow rate (\dot{V}) is expressed using an ideal gas law as
$$\dot{V} = \frac{\dot{m}RT}{pM}$$
- From the definition of Throughput, we have $Q = \rho \dot{V}$
- Combining the above two equations, we get
$$Q = \frac{\dot{m}RT}{M}$$
- Here,
 - \dot{m} – Mass flow rate, M – Molecular weight of gas
 - T – Temperature, R – Universal Gas constant

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Assuming an ideal gas behavior, the volumetric flow rate v is expressed using ideal gas law which is nothing but $V \text{ dot}$ is equal to $m \text{ dot} r t$ divided by p and r by m is nothing but specific gas constant. So, therefore, we have got a this m as molecular weight of the respective gases. From the above definition and this is come from basically $p v$ is equal to $m r t$.

From the definition of throughput now, can we put the value of $v \text{ dot}$ in that equation, and therefore, what we have is Q is equal to $p v \text{ dot}$, and if you have put the value of $v \text{ dot}$ in this equation, this pressure and this pressure will get cancelled out, and therefore, I will get Q is equal to $m \text{ dot} r t$ upon capital m . So, I have got a relationship between now mass flow rate and specific gas constant and temperature to value of Q , and here, m is a mass flow rate; m , capital M is molecular weight of the gas; t is the temperature; r is the universal gas constant.

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CRYOGENIC ENGINEERING

Electrical Analogy

- It is important to note that vacuum systems involve complex piping arrangements.
- In order to analyze these systems, a mathematical theory is developed based on an analogy between electrical circuits and piping systems.
- Linear transport laws like **Ohm's law** and **Fourier's law** are used in formulating the problem.

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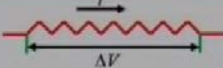
Now, it is important now to note that vacuum system involves complex piping arrangements. If I want to vacuum a particular space, I will connect that space by different, by having different piping arrangement and this pipes could be in parallel to each other; this pipes could be in series with each other, and to analyze if I connect pipe of a particular dimensions d and l to another pipe of d and l , it will give me different throughputs; it will give me different something called as resistance the flow or Δp , and therefore, it is very important to understand what happens if I connect this pipes of a given dimension in series or in parallel, and therefore, this series connection and parallel connection could be understood, could be well understood by having electrical analogy. We know Ohm's law basically; we know the resistances in series, resistances in parallel.

And, therefore, understanding this piping in connection, in series and parallel based on the electrical analogies always very simple. So, let us try to understand that parameter. So, in order to analyze these systems, a mathematical theory is developed based on an analogy between electrical circuits and piping systems. Linear transport laws like Ohm's law and Fourier laws are used in formulating the problem. So, let us understand electrical analogy.

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CRYOGENIC ENGINEERING

Electrical Analogy



- Consider a small electric conductor as shown.
- When a current (**i**) flows across this conductor, there is a voltage drop (**ΔV**) due to the resistance (**R**) offered by the conductor.
- These quantities are mathematically related by **Ohm's Law** as given below.

$$\Delta V = iR$$

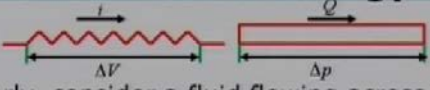
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We know that when a small current of I is passed through a resistance which has got a potential difference of Δv . We got a certain resistance which is automatically connected by ohms, ohms law. So, consider a small electrical conductor as shown. When a current I flows across this conductor, there is the voltage drop ΔV due to the resistance R offered by the conductor and this is very simple and they are related to each other by Ohm's law. These quantities are mathematically related by ohm's law as given below. So, ohm's law says ΔV is equal to I into R .

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CRYOGENIC ENGINEERING

Electrical Analogy



- Similarly, consider a fluid flowing across a small pipe as shown above.
- For a throughput (**Q**), there is a pressure drop (**Δp**) due to conductance (**C**) offered by this pipe.
- Comparing the above figures, we have

ΔV analogues to **Δp**
i analogues to **Q**
R analogues to **1/C**

$$\Delta V = iR \rightarrow \Delta p = \frac{Q}{C}$$

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Now, can we compare this electrical circuits of having voltage difference across a given resistance of length l through which a current I flows. So, now, let me compare this with a pipe and this I say that a pipe flow occurs Q which is comparable with I and this Q occurs only when the Δp occurs across the length of the pipe and the length of the pipe is l and Δp is the pressure drop across the length of the pipe l .

So, similarly, consider a fluid flowing across a small pipe as shown above. So, I can now compare electrical circuit with a pipe. So, I can see that I gets compared with Q and Δv can get compared with Δp , while the resistance is nothing but may be some kind of conductance I can get. You know have a different parameters which will be analogous to, which will be related to the resistance in the electrical circuit.

So, for a throughput Q , there is a pressure drop Δp due to conductance c offered by this pipe. So, in pipe, normally we do not call resistance but we referred to this as conductance, and therefore, what is conductance here? Reciprocal of r is nothing but, conductance. So, if I have got a parameter r in electrical analogy, I will say $1/r$ is nothing but conductance. So, from electrical circuit, I can compare this with the conductance of the pipe and conductance of the pipe c is related to one by r of electrical circuit.

So, I have got three different comparisons I with Q , Δv with Δp and c in the conductance of pipe to the $1/r$ value in the electrical circuits. So, comparing the above figures, we have Δv analogues to ΔP , i analogues to Q and r is analogues to $1/c$. Now, therefore, we can write Δv is equal to i into r , and therefore, I can write Δp is equal to Q into $1/c$; Q is nothing but similar to $r i$ and $1/c$ is nothing but similar to r , and therefore, I will get Δp is equal to Q by c in our equation for fluid flow in a small pipe and this is what basically a comparison of fluid mechanics or a flow regime in vacuum will be with the electrical analogy when, **when**, the current flows through a conductor.

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CRYOGENIC ENGINEERING

Conductance in Vacuum

$$\Delta p = \frac{Q}{C} \rightarrow Q = C(\Delta p)$$

- It is clear that for a given pressure drop (Δp) across a pipe, Throughput (Q) is directly proportional to conductance (C).
- For an ideal gas, the following equations hold true.

$$Q = \frac{\dot{m}RT}{M} \quad \Delta p = \frac{\Delta \rho RT}{M}$$

- Substituting, we have

$$\frac{\dot{m}RT}{M} = C \frac{\Delta \rho RT}{M} \quad C = \frac{\dot{m}}{\Delta \rho}$$

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So, let us we have just plotted term called C but we have not yet defined, and therefore, we will try to understand what this conductance in vacuum defined as. So, conductance is related to delta p. By this equation, delta p is equal to Q by c. Therefore, I can say Q is equal to C into delta P. It is clear that for a given pressure drop delta p across a pipe throughput Q is directly proportional to conductance c.

So, for, if I got a some value of Q given over here, then I will have throughput Q directly connected or directly related to delta p or directly connected to conductance C. So, if C increases, Q increases. If my delta p is given to me, so Q will vary in accordance to the variation in c directly; they are directly related to each other.

So, if I want to have more and more Q, I should increase the conductance of that particular pipe. So, increases c, increase the value of Q and this will good for creating vacuum in a particular space. For an ideal gas, following equation is holding true. Q is equal to m dot R T upon m. This is what we have talked about earlier, and we know from here, delta p is equal to delta rho into R T by m. This is true with the gas which follows p is equal to m R T, and from where, we could conclude that Q is equal to m dot R T upon m dot and delta p is equal to Q upon C or delta p is directly getting related to the gas law by this equation delta rho R T by m.

Now, substituting this values over in this equation Q, and this Q, we can put this Q is equal to this Q and we can have delta p as this over here putting these values, we will get now m dot R T upon m is equal to C into delta rho R T upon m; delta rho is nothing but density difference. Now, from here, we can cancel out R T and R T; we can cancel out m and m and we get therefore relationship between m dot and conductance and delta rho. From here, we can see that therefore this will get cancel and I get C is equal to m dot upon delta rho. So, now, my conductance is nothing but mass flow rate divided by delta rho. The density difference which is occurring because of the pressure difference p 1 and p 2 or the pressure drop occurring across the length l.

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CRYOGENIC ENGINEERING

Conductance in Vacuum

- Conductance for a pipe for different flow regimes can be derived by rearranging the pressure drop – mass flow rate equations derived earlier.
- Continuum Flow ($N_{Kn} < 0.01$) – $C = \frac{\pi D^4 \bar{p}}{128 \mu l}$
- Mixed Flow – ($0.01 < N_{Kn} < 0.3$) $C = \frac{\pi D^4 \bar{p}}{128 \mu l} \left[1 + \frac{8 \mu}{\bar{p} D} \left(\frac{\pi R T}{2 M} \right)^{0.5} \right]$
- Free Molecular Flow ($N_{Kn} > 0.3$) – $C = \frac{D^3}{L} \left(\frac{\pi R T}{18 M} \right)^{0.5}$

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So, conductance for a pipe for different flow regimes can be now derived by this formula, can be derived by rearranging the pressure drop mass flow rate equations derived earlier. So, let us see all the derivations earlier. For continuum flow now, we, we, will get conductance C is equal to pi into d to the power 4 p, because now we know that from the earlier calculations, we know that c is equal to m dot upon delta rho. I will put this into all other equation, we have, which we have calculated for m dot for various regimes, just divided by delta rho. It will cancel out delta rho in that particular equation.

And therefore, what we will get is - for continuum flow, we will get C is equal to this conductance is directly proportional to the d to the power 4. For mixed flow, for the Knudsen Number between 0.01 and 0.3, we will get c is equal to this formula. Similar to

what we had done earlier, and similarly, for free molecular region now, we will get conductance is equal to this. What does it mean? It means that, if I know the length of a pipe, if I know the diameter of the pipe, if I know the average pressure in the system, if I know the temperature, if I know the gas to be vacuum, I can calculate conductance for that pipe, for that gas, for a given Δp , for a given pressure for a given length and given diameter of the pipe, I can calculate conductance of different pipes.

Many times now we can have standard equations to calculate conductance for a bend, for a conductance for a straight length, conductance for diametral changes in the pipe lengths and these are the formula which will help us to calculate conductance for different pipes of different length and diameters for different gases, and therefore, it is very important to know such relationships which are normally standardized.

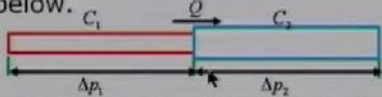
To understand what happens to the conductance, when there is a bend in a pipe, when the pipe is straight, when pipe has got some you know curvature for example, all this equations help us to calculate conductance of pipe, which ultimately would help us to calculate the what will the pressure drop that will happen if I have this vacuum pump over there or how much time it will take for me to reach down to the lower and lower vacuum levels for a given pump of the, when I connect the vacuum pump to a given space through such pipes. So, very important to understand what is this conductance and how it is related to the operating parameters and the dimensions of the pipe.

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Conductance in Vacuum

- Consider a series combination of two pipes with C_1 and C_2 as individual conductances respectively as shown below.



- Let Q be the Throughput for this system. It is clear that for a series combination, Q is same for each of the pipe.
- The pressure drops in each of the pipes are Δp_1 and Δp_2 respectively. That is,

$$\Delta p_1 = \frac{Q}{C_1} \quad \Delta p_2 = \frac{Q}{C_2}$$

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So, conductance is vacuum now. We can have conductance when the two pipes are connected in series and the pipes could be connected in parallel the way we have in resistances connecting in series and parallel. So, consider a series combination of two pipes with C_1 and C_2 as individual conductances respectively with Δp_1 and Δp_2 as the pressure drop happening. Let Q be the throughput for this pipe. It is clear that a series combination of Q is a same for each pipe. Q is going to be the same, because whatever Q happens to these pipe, the same Q is going through this pipe, is similar to I. I is the same when it is entering at this point ends going through this particular resistance.

The pressure drop in each pipe are going to be Δp_1 and Δp_2 respectively, and therefore, we can say Δp_1 is equal to Q by C_1 . C_1 and C_2 are conductances of this pipe of different dimensions and length. Accordingly we will have Δp_1 is equal to Q by C_1 ; Δp_2 is equal to Q by C_2 , and what we know let us have a overall conductances, conductance as C_0 and let us try to relate that to this. And we also know that the overall pressure drop Δp is equal to Δp_1 plus Δp_2 .

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Conductance in Vacuum

Diagram showing two pipes in series with conductances C_1 and C_2 , and pressure drops Δp_1 and Δp_2 . Throughput Q is indicated.

- Let the overall conductance and the total pressure drop of the system be C_0 and Δp respectively.
- Therefore, we have $\Delta p = \frac{Q}{C_0}$, $\Delta p_1 = \frac{Q}{C_1}$, and $\Delta p_2 = \frac{Q}{C_2}$.
- Using $\Delta p = \Delta p_1 + \Delta p_2$, we get $\frac{1}{C_0} = \frac{1}{C_1} + \frac{1}{C_2}$.
- Extending to N pipes in series, we have $\frac{1}{C_0} = \sum \frac{1}{C_i}$.

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So, let the overall conductance and the total pressure drop in the system be C_0 and Δp respectively, and what we therefore is total for the equation for these is a Δp is equal to Q upon C_0 . And we know also Δp_1 is equal to Q by C_1 ; Δp_2 is equal to Q by C_2 . Now, I know Δp is equal to Δp_1 plus Δp_2 , and therefore, I will

get an equation as $\frac{1}{C_0}$ is equal to $\frac{1}{C_1} + \frac{1}{C_2}$, where Q and Q will get cancel if they are put in this particular equation.

What does it mean? It means that when the two pipes are connected in series, the effective or overall conductance of this pipes when they got a C_1 and C_2 of respective conductances will get $\frac{1}{C_0}$ is equal to $\frac{1}{C_1} + \frac{1}{C_2}$, and this is what happens in the resistance in series $\frac{1}{C_2}$ nothing but overall resistance r_0 is equal to $r_1 + r_2$. Same thing happen over here when the conductances are getting connected to each other in series.

So, if I have got this, we did only for two pipes. I can have now n number of pipes. So, extending to n pipes in series, I will get one upon c_0 is equal to sigma $\frac{1}{C_i}$, i ranging from 1 to n in that case. So, if I know different pipes connected in series, what will I do? I will calculate their respective conductance and get overall conductance calculated by this formulas. So, $\frac{1}{C_0}$ is equal to $\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$ etcetera would give me the overall conductance of a given series pipe connection.

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Conductance in Vacuum

- Similarly, consider a parallel combination of two pipes with C_1 , C_2 and Δp as conductance and pressure drop respectively.

Diagram showing two parallel pipes with conductances C_1 and C_2 and pressure drop Δp . Flow rates Q_1 and Q_2 are shown in each pipe, and the total flow rate Q is shown entering the parallel combination.

- Let C_o and Q be given, we have

$$Q = C_o(\Delta p) \quad Q_1 = C_1(\Delta p) \quad Q_2 = C_2(\Delta p)$$
- Using $Q = Q_1 + Q_2$, we get

$$C_o = C_1 + C_2$$
- Extending to N pipes in parallel, we have

$$C_o = \sum_i C_i$$

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Now, let us see what happens if the pipes are connected in parallel. So, similarly, consider a parallel combination of two pipes. Let say C_1 and C_2 are the conductance of these two pipes. While as you can say that some Q_1 will go through this pipe some Q_2 will go through this pipe depending on the diameters of this pipe.

Let us see that there having same length, and therefore, we will have the delta p happening across them is going to be the same, because the same gas will gets split up or the flow will gets split up here and the flow will come back here. As a result of which, the pressure at the entrance and the pressure at the exit will be same, and therefore, both in both this pipes which are connected in parallel, the pressure drop is going to be the same. This was not the case when their connected in series.

So, let C_0 and Q be connect be given and then we have delta p be the same. We know that Q is equal to $C_0 \Delta p$ which is overall Q and we know that Q is equal to Q_1 plus Q_2 also, because Q is coming from here; Q is coming out from here. When they are travelling through in this parallel pipes, they getting split up into Q_1 and Q_2 , and Q_1 is nothing but $C_1 \Delta p$. Δp is same all over. Q_2 is equal to $C_2 \Delta p$.

And therefore, what we know is Q is equal to Q_1 plus Q_2 , and if I write them together $\Delta p \Delta p$, it will get this common basically, and therefore, I will get C_0 is equal to C_1 plus C_2 . So, when the pipes of C_1 and C_2 conductances are connected in parallel to each other, the overall conductances will get added as C_1 plus C_2 directly, and therefore, this is the overall conductance for pipes which are connected in parallel with each other. So, extending this to n pipes, we will have 1, C_0 is equal to $\sum C_i$ when i is extending from 1 to n pipes. So, we saw what is conductance in their connected n series; what is overall conductance when they are connected in parallel to each other.

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Summary

- Heat in leak is minimized by having vacuum between two surfaces of different temperatures.
- λ is defined as the average distance travelled by the molecules between the subsequent collisions.
- Based on Knudsen Number (N_{Kn}), we have Continuum Flow ($N_{Kn} < 0.01$), Mixed Flow ($0.01 < N_{Kn} < 0.3$), Free Molecular Flow ($N_{Kn} > 0.3$).
- Conductance

Series :	$\frac{1}{C_o} = \sum_i \frac{1}{C_i}$	Parallel :	$C_o = \sum_i C_i$
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Summarizing this lecture - heat in leak is minimized by having vacuum between two surfaces of different temperatures. Lambda which is nothing but, mean free path is defined as the average distance travelled by the molecules between the subsequent collisions. Based on Knudsen Number which is λ/d , we have continuum flow when Knudsen Number is less than 0.01. We have got a mixed flow when Knudsen Number is between 0.01 and 0.3, and we got a free molecular flow when the Knudsen Number is more than 0.3 when the lambda is increasing from continuum flow to free molecular flow.

And we know that conductances of pipe which are connected in series is $1/C_0$ is equal to $1/\sum C_i$, and when they are connected in parallel, we know that C_0 is equal to $\sum C_i$. They will can added C_1 plus C_2 plus C_3 etcetera. Thank you very much.