Cryogenic Engineering
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Lecture No. # 31
Cryocoolers Ideal Stirling Cycle

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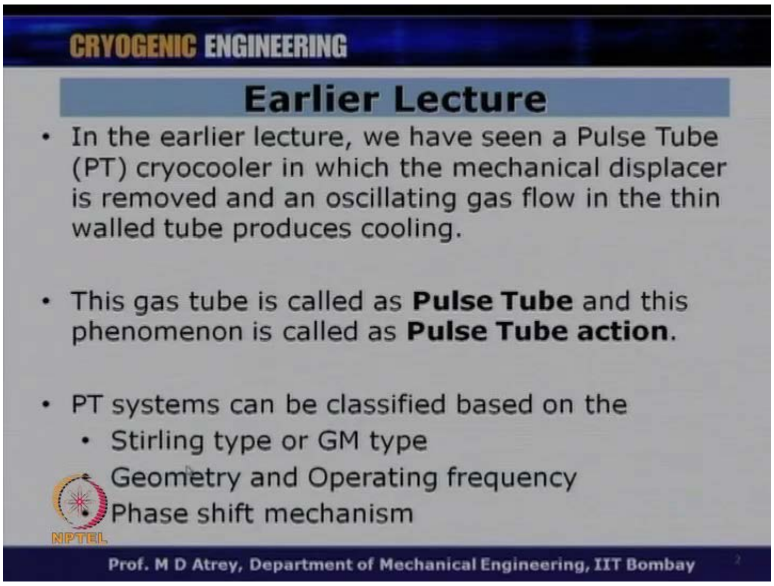

So, welcome to the thirty first lecture of cryogenic engineering under the NPTEL banner. In the earlier lecture, we have seen a Pulse Tube cryocooler in which as you know in the Pulse Tube cryocooler the mechanical displacer is removed, if we compare it with GM cooler as well as this Stirling cooler. And simply and oscillating gas flow in the thin walled tube produces cooling, and this is what we saw how the Pulse Tube cooler works, by what principle.

And this gas tube which is filled with gas with oscillating pressures is called as Pulse Tube and this phenomenon is called as Pulse Tube action. So, Pulse Tube action basically is responsible to produce cooling. We also saw that the PT system that the Pulse Tube systems can be classified based on the Stirling type or GM type, if they have valve or no valve; geometry and operating frequency. We have seen this detail classification earlier and also depending on what kind of phase shift mechanism they incorporate.

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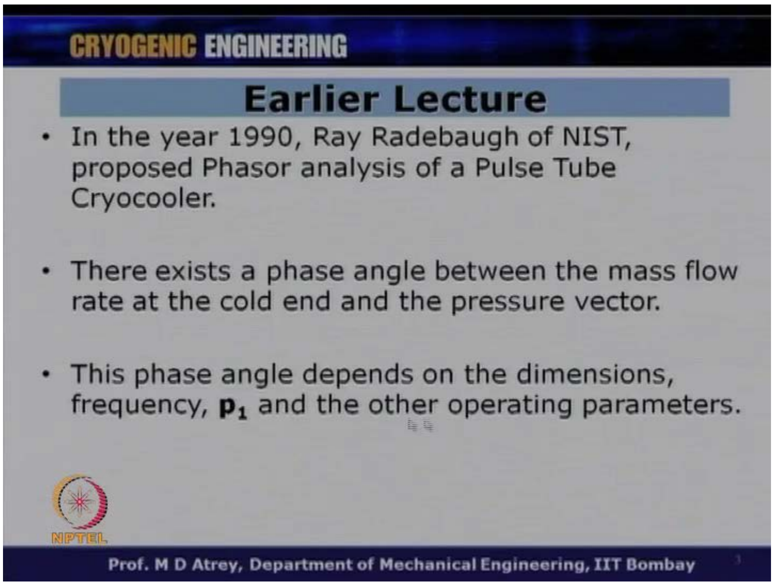

This phase shift mechanism is what we are going to concentrate on in this present lecture. We also saw that in the year 1990, Ray Radebaugh from NIST proposed a phasor analysis of a Pulse Tube cryocooler and have done some initial analysis to understand what is the effect of this phase angle business, which is the angle between the mass flow rate at the cold end and the pressure axis. There exists a phase angle and this is what we concluded, there exists a phase angle between the mass flow rate at the cold end of the Pulse Tube and the pressure vector. This phase angle depends on the dimensions of the Pulse Tube cooler, the length, the diameter, the frequency at which it operates, P 1 the amplitude of pressure pulse and various other operating parameters.

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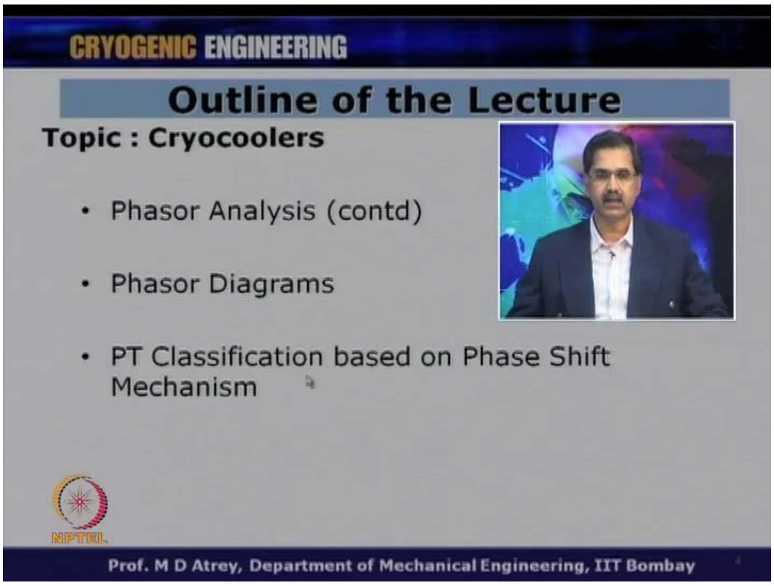

Now, in this lecture, we are going to take ahead the same analysis, we deal with the phase angle and what we call as phasor analysis will be explain. And this would basically, let us to draw phasor diagram for various Pulse Tube cryocooler. And from where we can understand the Pulse Tube classification based on phase shift mechanism. So, basically we can understand why this phase shift mechanism has to be employed in the Pulse Tube cryocooler. And I told you in the previous lecture that this will be dealt with when we go with the analysis or when we go with the phasor diagram.

So, this three aspects are very important aspect to understand different phase shift mechanism that are employed in the Pulse Tube cryocooler. And this lecture will be basically aimed at having this phasor diagrams and phasor analysis which is leading to different phase shift mechanisms.

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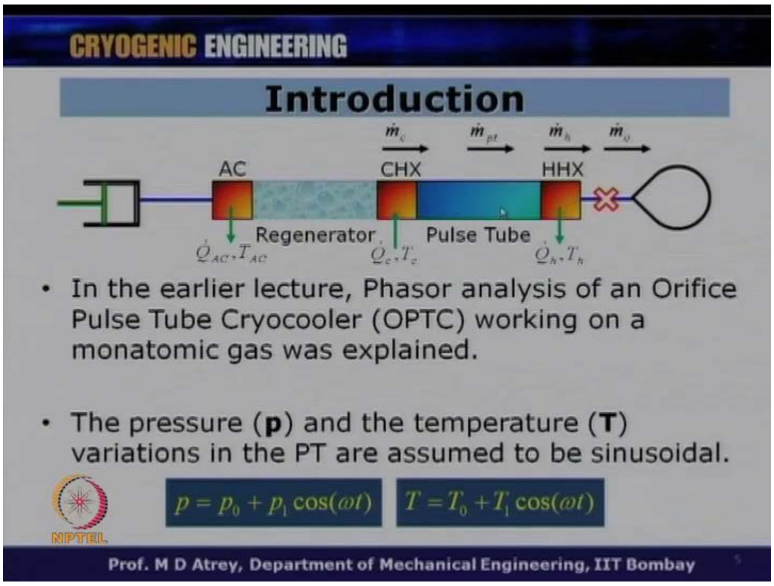

In brief repeat what we have done earlier and lead to your conclusion, what is this phasor diagram and how does it come. So, this is a simple OPTC - Orifice Pulse Tube Cooler where we can see a simple orifice opening at the hot end of the Pulse Tube cooler which opens in a reservoir. In the earlier lecture, phasor analysis of an OPTC working on a monatomic gas was explained. The pressure pulse P and a temperature variations in the Pulse Tube are assumed to be sinusoidal, and we had variations of pressure pulse like this, and we had variation of temperature like this. And this exists in the Pulse Tube where we assumed that the gas behaves in an adiabatic manner.

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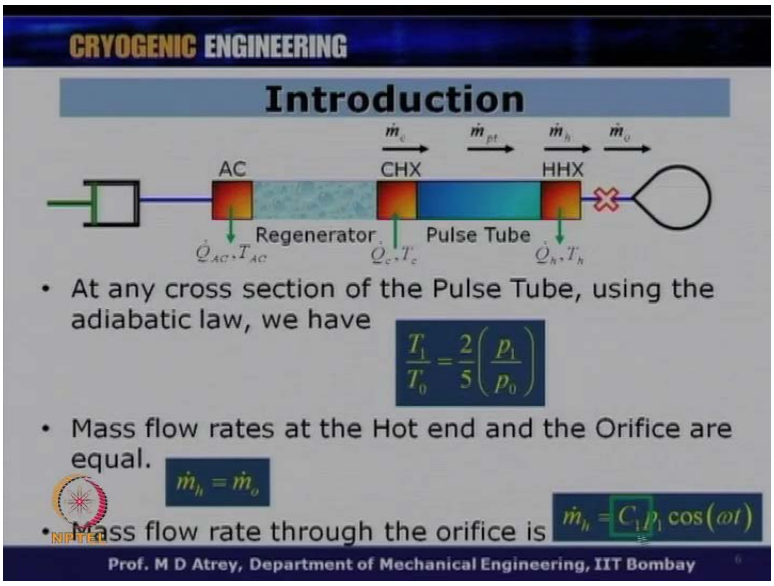

At any cross section of the pulse tube; so, you can take any cross section and we have got a pressure variation there and we have got a corresponding temperature variations over there. So, at any cross section of the Pulse Tube using the adiabatic law, we had derived earlier that T 1 upon T 0. That means the amplitude of temperature variation to its mean temperature value. And we will relating this temperature variations to the pressure variation at the same cross section. So, at any cross section, we can see the temperature variation, we can see a relationship between a temperature variation and pressure variation using an adiabatic law; we had derived this earlier.

The mass flow rates at the hot end and the orifice are equal. And we had seen that m dot h is equal to m dot o in a OPTC. Mass flow rate through orifice is and we had calculated this as this m dot h into C 1 p 1 cos omega t, C 1 p 1 is the amplitude of the mass flow rate at the hot end or the mass flow rate through the orifice.

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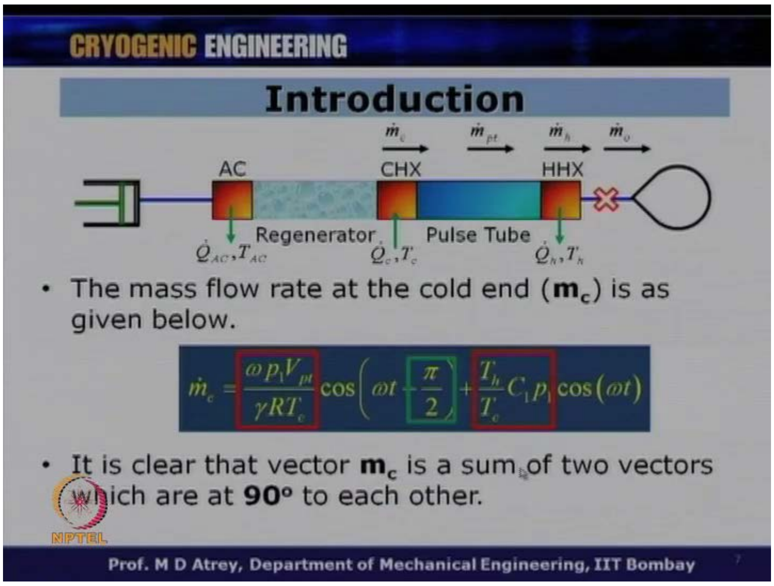

We are also seen that the mass flow rate at the cold end which is m dot c can be given by this vector formulation. We know that m dot c is equal to some quantity basically, this right hand side is nothing but the m dot h into T h by T c plus another vector we deals with the geometry of the Pulse Tube and which is an angle of pi by 2 between this two vectors. This will vectorially could be added. So, it is clear that vector m dot c is a sum of two vectors which are at 90 degree angle to each other.

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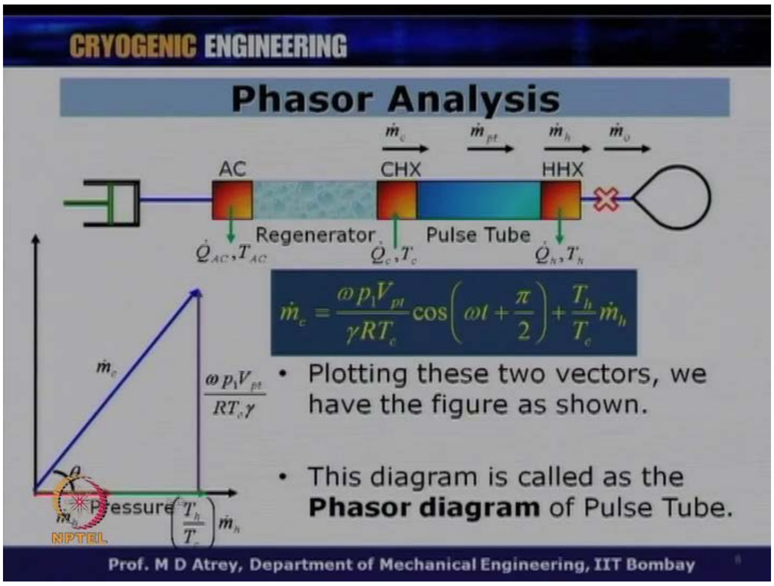

And I can draw a vector diagram m dot c is equal to T h by T c into m dot h which is this quantity plus the vertical quantity which is equal to this parameter. And this will give an angle theta between m dot c and the pressure axis or the pressure vector. So, plotting these two vectors we have the figure as shown here. This diagram is called as the phasor diagram of the Pulse Tube only not the Pulse Tube cryocooler. We are drawing a diagram only for this Pulse Tube hot end we can say, hot end and the cold end, because we are relating the mass flow rate at the cold end to the pressure vector. And we are relating basically the mass flow rate at the cold end to the mass flow rate at the hot end.

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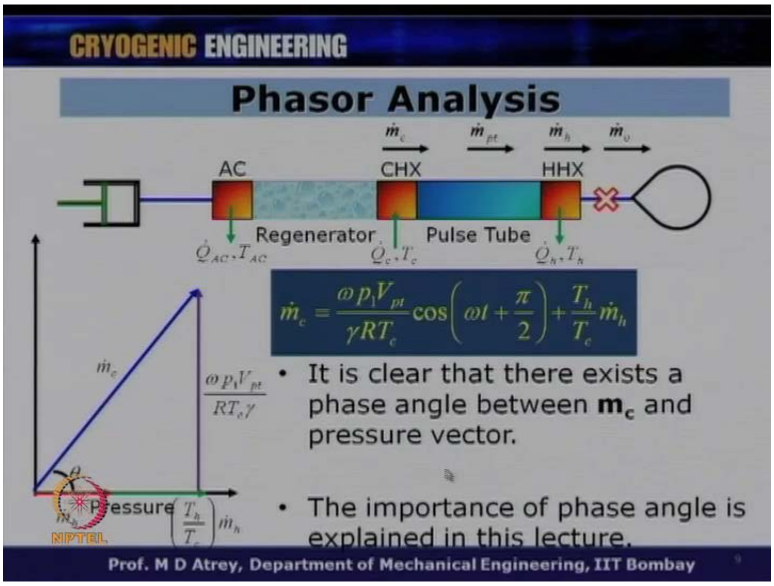

The hot end mass flow rate is in the same axis or is in phase with the pressure axis or pressure vector. It is clear that there exists a phase angle between m dot c and pressure vector. And the importance of phase angle is explained in this lecture. This is the point which we want to bring in over here.

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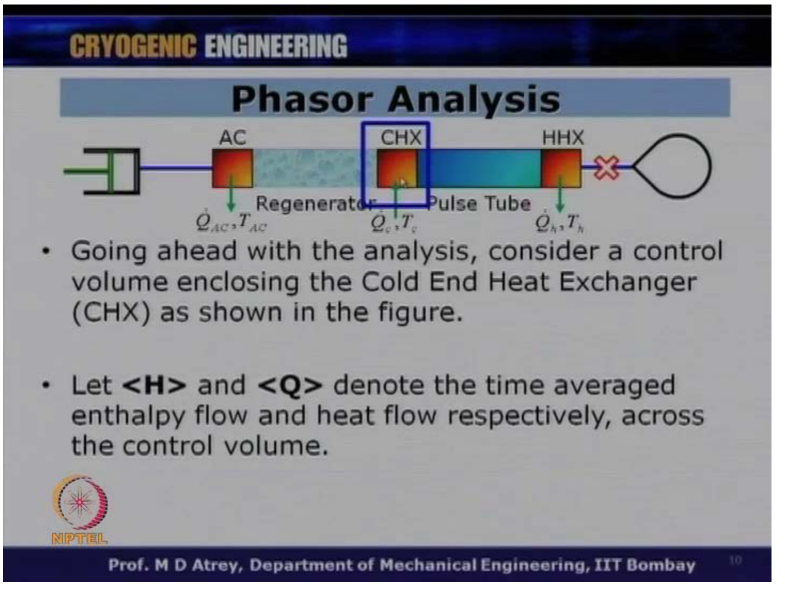

So, going ahead from this analysis; this is what we had done till the last lecture. We had brought out the importance of theta and going ahead from there, consider now a control volume as shown over here. So, let us have a control volume around the cold end heat exchanger as shown in the figure. Let H and Q denote the time average enthalpy flow and the heat flow respectively across the control volume. So, let us the enthalpy entering, enthalpy leaving this cold end heat exchanger; while Q denotes the heat flow - time average heat flow that means in one cycle how much heat inflow happens in the cold end heat exchanger.

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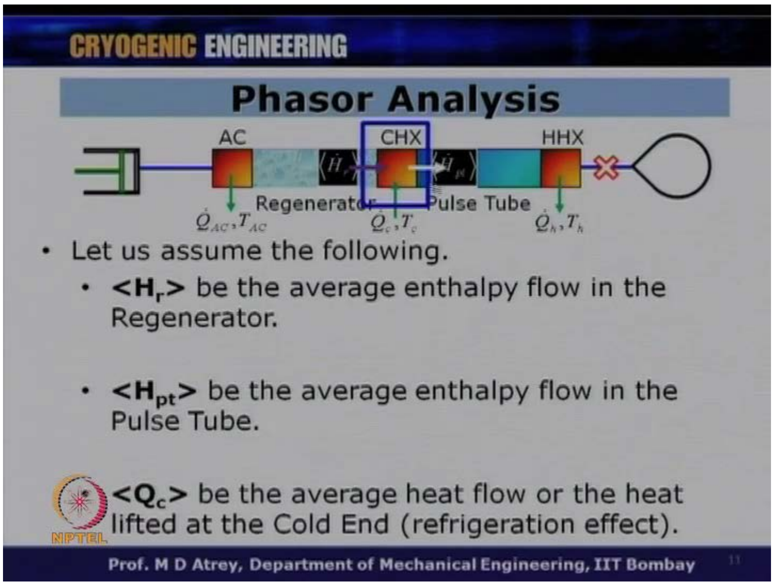

See if I do let us assume the following. Let H r with the average enthalpy flow in the regenerator from this side; let H pt with the average enthalpy flow in the pulse tube. So, the cold end heat exchanger connects the regenerator and the pulse tube. So, the enthalpy which we receive from the regenerator side let it be H r, and the enthalpy which with it leaves which with the gas leaves, the cold end heat exchanger and enters the Pulse Tube let it be H pt. And this is the average enthalpy flow in the one cycle.

And let Q c be the average heat flow or the heat lifted at the cold end. And this is nothing but the cooling effect or refrigeration effect. So, I have shown here Q c basically. So, if I do a energy balance, we have got H r coming in, H pt leaving and Q c entering the cold end heat exchanger.

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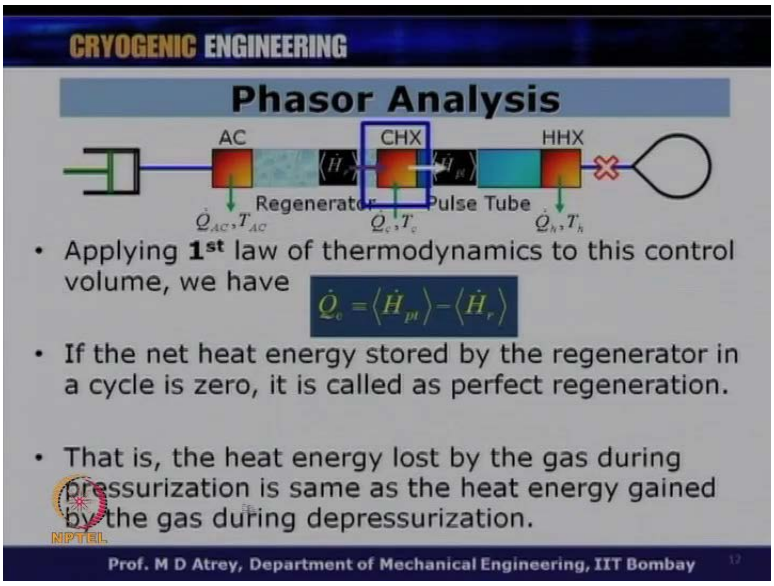

So, if we have apply the 1st law of thermodynamics to this control volume, we have got Q dot c which is the cooling effect is equal to whatever is leaving, whatever is entering that is H pt minus H dot r all right. So, this is the enthalpy difference across the cold end heat exchanger and Q c is nothing but the cooling effect. If the net heat energy stored by the regenerator in a cycle is 0, it is called as perfect regenerator.

Now, let us see what this H r value is. And let us define before that - if the net heat energy stored by the regenerator in a cycle is 0. That means when the gas traverse to regenerator, it gives the heat to the matrix, and when gas comes back from the Pulse Tube during depressurization, it takes away heat from the matrix and the gas goes back to the compressor. So, we can say that in one cycle, the gas gives the heat and gas takes the heat. And whatever it gives, if it takes back, exactly all the mode it will takes back then we can call it as perfect regeneration all right.

Therefore, we can say that - the heat energy lost by the gas during pressurization, if it is the same as the heat energy it gains during depressurization right. During pressurization the gas will go through the regenerator and it will give the energy to the regenerator matrix, and during depressurization the gas will go back to the compressor, and during this travel it will take the heat from the matrix. And if both these heat energies are same; that means we are having a perfect regenerator all right.

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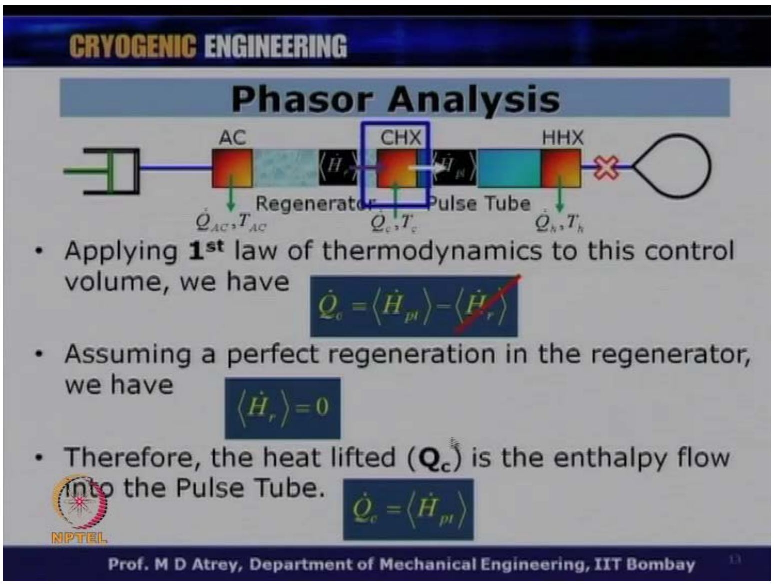

Let us come to this H r quantity. Assuming a perfect regeneration now in the regenerator; that means whatever is given to the matrix is taken by the gas in one cycle. That means H r will be equal to 0. That in a one cycle whatever H r it gives, it takes it back while going back. So, if we go and integrate H r over a cycle what we will get; over a cycle H r is will be equal to 0. The same will not be true with H pt, the same will not be true with Q c. Q c is of course, is a cooling effect which is given from outside.

But if we have a perfect regenerator, that means H r over a cycle will be equal to 0. And if we compute that we can get that we can cancel this H r equal to zero have H r is equal to 0. And therefore, the heat lifted or Q c or the cooling effect or the refrigeration effect is equal to the enthalpy flow into the pulse tube. That means H pt. So, we can say that Q c is equal to H pt all right. This is simply from the 1st law of thermodynamics and assuming that the heat exchanger or the regeneration is perfect in case of a pulse tube.

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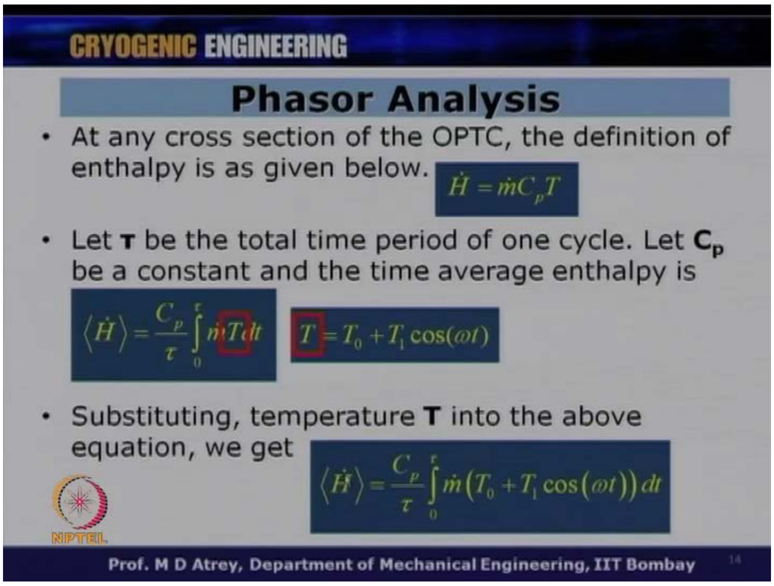

Now, let us take any cross section in the Pulse Tube and lets have our all the discussion right now only for the Orifice Pulse Tube Cooler. So, at any cross section of the OPTC, the definition of the enthalpy is as given. What is enthalpy basically - nothing but m dot C p d t - m dot C p into temperature. So, let tau be the total time period of one cycle. Let C p be a constant quantity, let us not assume the C p variation with temperature and pressure and let us assume to be simple constant quantity. And the time average enthalpy therefore, will be h dot will be equal to C p by tau integrating from 0 to tau m dot t into dt. So, what is varying in the Pulse Tube is m dot, what is varying in the Pulse Tube also is temperature. While C p is constant we have assumed and tau is a time for one cycle. So, basically we are trying to integrate and find out enthalpy during entire cycle or tau time.

Now, let us replace this T, we had assume that T has a variation of T 0 plus T 1 cos omega t and let us put this value over here. So, now we got a this quantity. Understand what I am doing is basically calculating the value of enthalpy, and for which I am doing all this algebraic calculation, in order to reach some expression which leads to this phase angle business between the mass flow rate at the cold end and the pressure axis. I am trying to do an exercise which will lead to us to calculate the cos theta or calculate a relationship between the mass flow rate at the cold end and the pressure axis, and put this in equation which will give me cooling effect. Because ultimately Pulse Tube cooler is basically producing cooling effect.

So, I want to find a relationship between what is this cooling effect and how it is connected to the cos theta business. Because for which all this phase shift mechanisms are incorporated. So, we got an expression for H dot which is nothing but equal to cooling effect also as we have found out.

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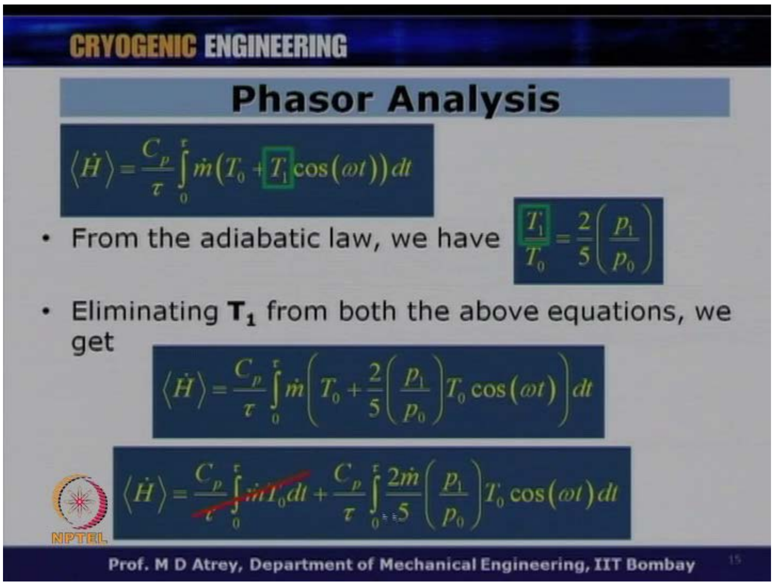

So, putting this values over here. We have already derived in the last lecture, a relationship between temperature and the pressure in the Pulse Tube where adiabatic law holds good. So, from the adiabatic law, we have got this variation. So, I can replace this T 1 value from based on this expression and put this value over here. So, I will replace this T 1 by 2 by 2 by 5 P 1 by P 0 into T 0 and have it over here. And expand it further that m dot T 0 and C p by tau, I will have this tau.

Now, understand that the first term is nothing but 0 to tau m dt, T 0 is the constant quantity, T 0 is a mean value of temperature at any location. So, if I integrate mass flow rate and if I assume that mass flow rate which is going in one cycle in this direction and opposite direction, we will find that if you integrate it will be 0. Unless and unless of mass gets stored over there which is not the case in this case. So, we can see that 0 to tau m dot dt is nothing but equal to 0. So, I will have to basically cancel out this term, because this term becomes equal to 0. Therefore, I will get an expression between as enthalpy is equal to the second term in this expression.

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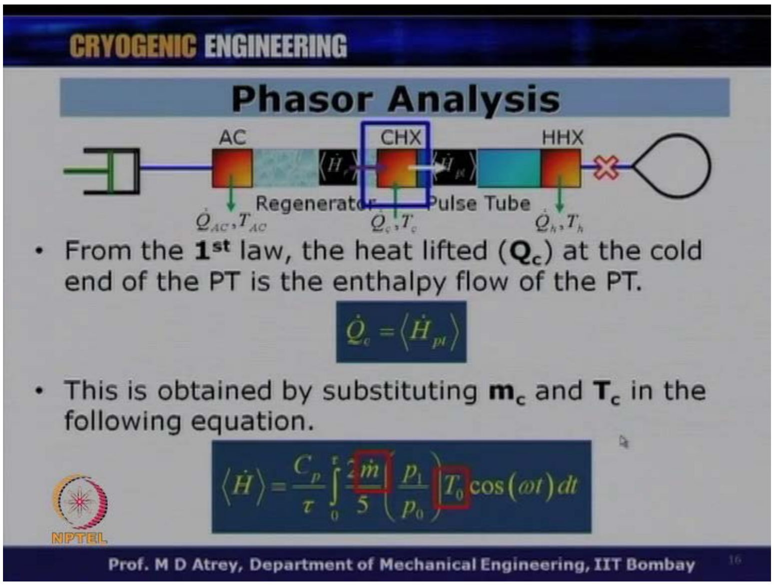

So, from the 1st law the heat lifted Q c at the cold end of the Pulse Tube is enthalpy flow of the Pulse Tube. We have found that Q c is equal to H pt; this is obtained now. So, we have got an expression of H pt as a function of some parameters. We know this H pt is equal to Q dot c and this Q dot c is the basically cold end of the pulse tube.

So, now whatever mass flow rate expressions we have had there, whatever temperature we have had, we should replace that mass flow rate by m dot c and temperature by T c in the following equation. So, this is the equation this is the equation which we have derived. In this now, we had taken m dot and T 0. I will now have m dot as m m dot c and T 0 as T c, because we are talking about the cold end of the Pulse Tube cooler where the cooling effect is derived right. This is the… We are talking about Pulse Tube at this cold end. So, therefore replacing this mass flow rate by m dot c and temperature as T c will get the following expression.

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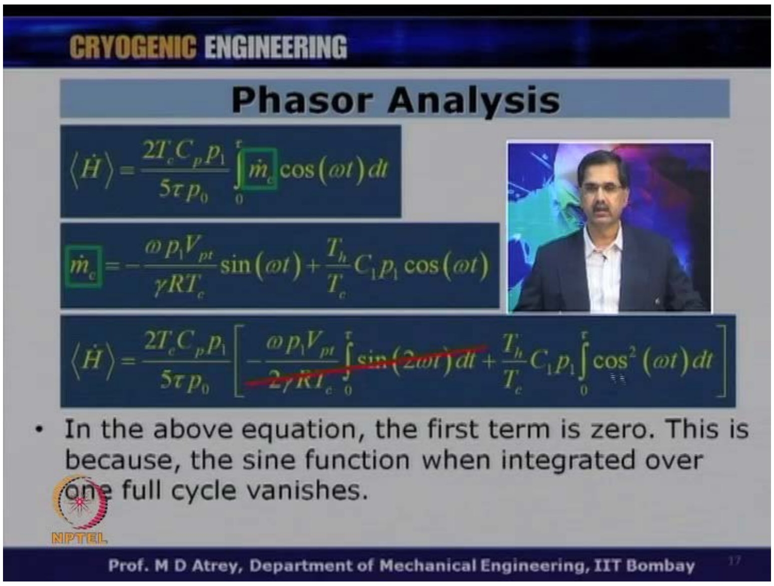

So, this is my expression now here. I got a m dot c m dot c here and I got temperature T c over here. Now, we have already calculated expression for m dot c. This is what we required in a… This is the expression which connects theta business. So, m dot c is equal to some quantity which is depicting T h by T c into m h plus a function which is this which is at 90 degree to this. So, putting this value of m dot c in this expression, I get a major (( )) expression as one 0 to tau 2 omega t and 0 to tau cos square omega t coming from multiplication of cos omega t and cos omega t over here.

So, again we can find now all this thing, if we take the first term here. All this terms are are basically constant terms. And now we have got a we have got a integration 0 to tau sin 2 omega t dt. And we know that in the above equation, the first term come equal to 0, why? This is because the sin function when integrated over one full cycle vanishes. So, basically you have got a sin pulse like that, and whatever area include in the top is equal to area which is below, and if you integrate therefore, in one cycle this sin 2 omega t will seeks to exist. It will become equal to 0. Therefore, h that is equal to this quantity a cos cos square omega t term will appear over there.

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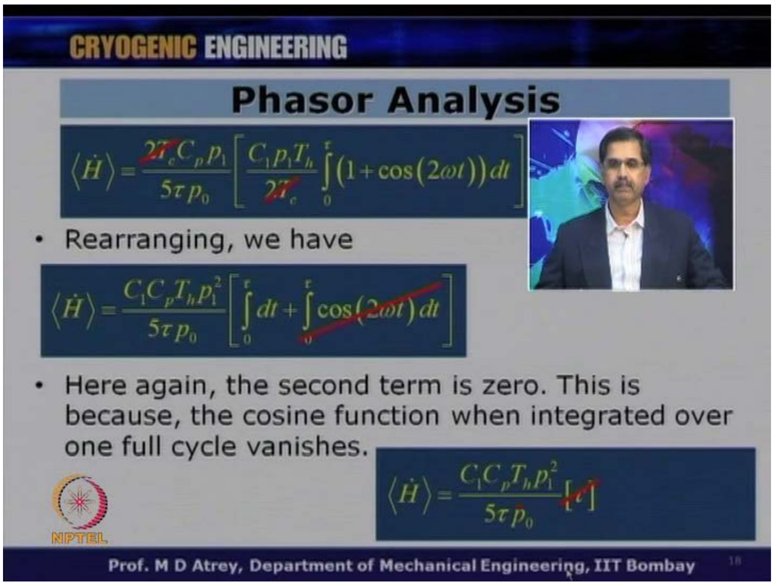

So, now I can write that cos square omega t is equal to 1 plus cos 2 omega 2 by simple trigonometry, I know that this is the parameter. And now I will again split this expression into two part; cancelling this T c over here I have got this term and then integrating. So, 0 to tau into 1 dt which is what I will get over here and 0 to tau cos 2 omega t dt.

Again by the same expression; if I have a cos to be integrated over a cycle, this expression again will vanish. In the similar way, we had sin 2 omega t as equal to 0 over integration over a cycle. So, we will have the same expression here, here again the second term is 0. This is because the cosine function when integrated over one full cycle vanishes. And if I have 0 to tau dt, what I will here it tau right. So, I get expression now H dot is equal to this into tau. And therefore, tau and tau will get cancelled over here and one simple expression will come from here which is equal to C 1 C p T h P 1 square into divided by 5 P 0.

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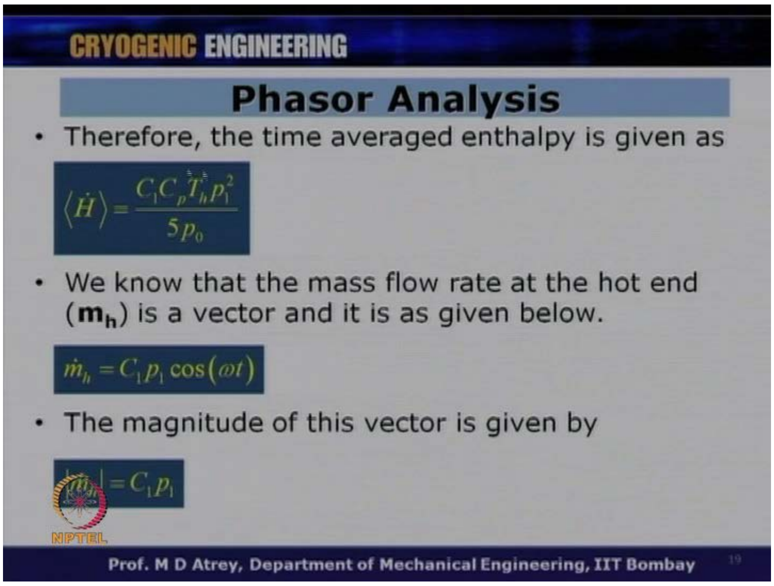

So, my time average enthalpy will come like this. Now, let us see what is this C 1 into P 1 is. We know that the mass flow rate at the hot end m dot h is a vector. We have calculated an expression for m dot h which is the mass flow rate at the hot end. And we know that m dot h is equal to C 1 P 1 cos omega t. So, can you find that the magnitude of m dot h is nothing but C 1 P 1 all right. So, magnitude of C 1 m m dot h is nothing but C 1 P 1. And here we got a term C 1 P 1. In fact, we have got a P 1 square and if we take 1P 1 from here, we got a C 1 P 1 and that can be replaced by m dot h over here.

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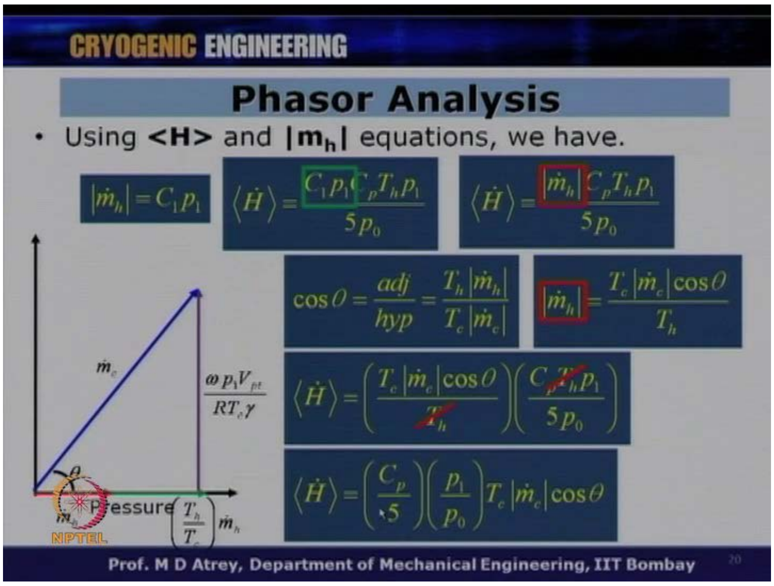

So, if I do that thing and if I put this value over here in this case, I will write this as m dot h over here. So, now I am basically trying to relate the cooling effect to the mass flow rates. And ultimately they will have a cos theta or a theta function over there. We have already derived this expression which leads me to get a cos theta value all right. So, you know m dot c is equal to some vector which has got a m dot h and some vertical vector basically which is at 90 degree to pressure vector. So, what does it give you? A cos theta is equal to adjacent side divided by hypotenuse. What it is adjacent side? T h by T c into m dot h divided by m dot c.

So now, this m dot h which we have derived can be written in terms of m dot c into cos theta; are you understanding? And therefore, now I can write m dot h is equal to T c by T h into m dot c cos the. And I will try to put this value over in this expression now. So now, my h dot expression becomes as a function of m dot c cos theta and all other parameters where this T h and this T h will get cancelled over here. And I have got a expression now H dot is equal to C p by 5 P 1 by P 0 T c m dot c cos theta. What is this C p? Specificate at constant pressure. Now, can I relate it to for a monatomic gas? Yes, I can. I can replace that C p by some universal gas quantity expression. And therefore, let us see what is this.

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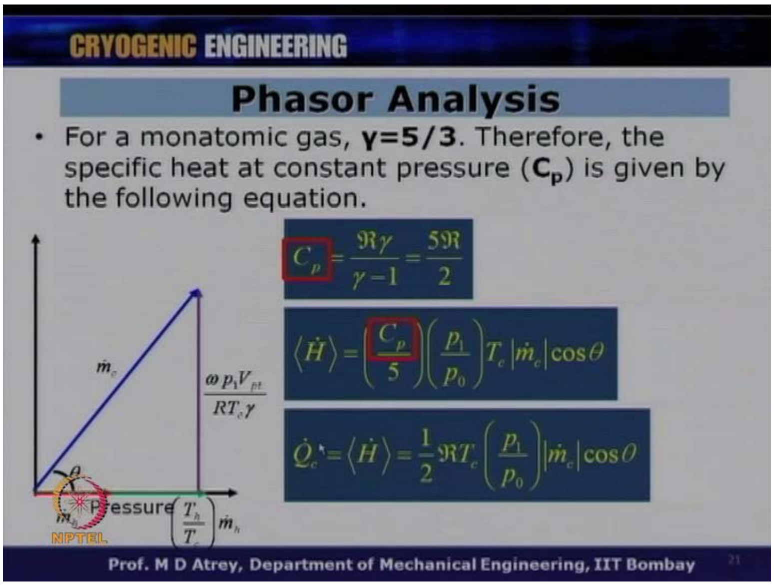

For a monatomic gas, like helium, we have got a gamma is equal to 5 by 3. Therefore, the specific heat at constant pressure C p is given by following expression which is C p is equal to universal gas constant R gamma divided by gamma minus 1. And if you put gamma is equal to 5 by 3 in this expression, what you get C p is equal to 5 by 2 R.

So, if I put this value over in this expression, put C p is equal to 5 by 2 R, I get an very good expression which is simple this. So now, we know that this enthalpy is nothing but cooling effect. This enthalpy is nothing but refrigeration effect. Q dot c is equal to H dot is equal to half RT c into P 1 by P 0 into m dot c cos theta. So, this leads to a very simple expression in order to calculate the cooling effect at in the Pulse Tube cooler which is taken at the cold end heat exchanger at the cold side of the Pulse Tube cooler. So, on what parameter this Q dot c will depend? Let us see this.

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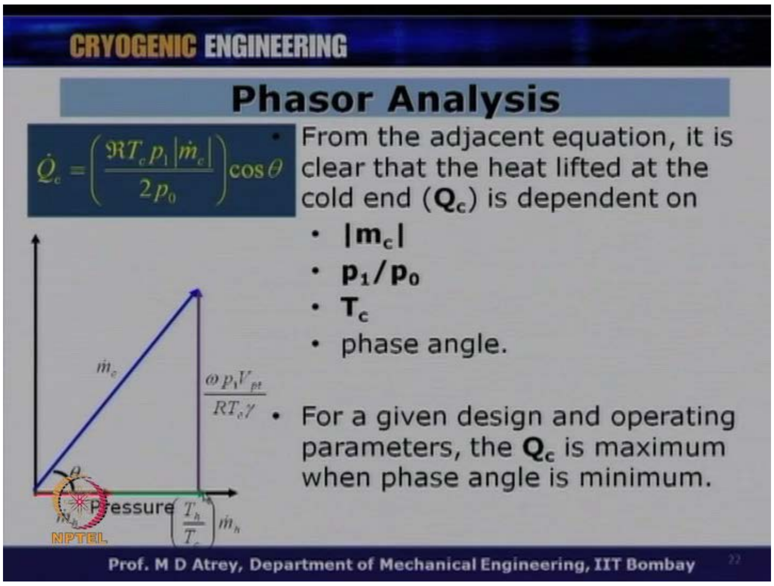

So, this is my expression. From the adjacent equation, it is clear that the heat lifted at the cold end Q c depends on m dot c, quite obvious Q dot c depends on m dot c, quite obvious it depends on P 1 by P 0, T c and phase angle which is cos theta. R is universal gas constant; so, no problem over this. Now, can you relate this; cooling effect definitely depends on mass flow rate that is m dot c. If you increase m dot c, you will get higher cooling effect which is absolutely to be in refrigerator.

Similarly, it depends on the P 1 by P 0. That means your amplitude of pressure should be very, very large as compared to the mean value. So, this is also clear, because this will increase the pressure ratio. And as soon as your pressure ratio is very large P max by P P minimum is very, very large, the cooling effect that means the p V area in the expansion phase is going to be more, and therefore cooling effect is going to be more.

So, I can understand that Q c is directly dependent on m dot c; Q c is directly dependent on P 1 by P 0. At the same time I know that if my cooling cold end temperature goes on increasing, I can get more and more cooling effect. So, if I have got a cooling effect at 80 kelvin I will definitely get more cooling effect at 100 kelvin or 150 kelvin. So, if T c increases I will get more and more cooling effect. What is the important is a last parameter cos theta. My Q c is going to be more when cos theta is going to be more and more. So, what is the maximum value of cos theta equal to 1. That means I should have cos theta equal to 1 which means I should have I should try the design in such a way that theta is equal to 0 or theta is tending towards 0.

So, I should have theta to be as small as quantity. So that my cos theta quantity goes on increasing for a given m dot c. So, Q c in order to get lot of cooling effect, the cos theta should be high or theta should be as small as possible, and what is this theta? Theta is the angle between the mass flow rate at the cold end of the Pulse Tube and the pressure axis all right. This is the general terminology; m dot h is happens to be in phase for OPTC all right for a Orifice Pulse Tube Cooler. But in general I should make a statement that theta is a angle between mass flow rate at the cold end of the Pulse Tube and the pressure axis or the pressure vector. This angle should be as small as possible. And in order that we want to have this angle as small as possible we have to incorporate various phase shift mechanisms.

So, the objective of phase shift mechanisms is basically aim at making this phase angle to be as small as possible. So that the cos theta is going to be maximum. So, that my Q dot c is going to be maximum. So, for any given design, if I got complete design available and I fix my operating parameter. So, I have got a diameter of Pulse Tube, length of Pulse Tube everything designed. So that this parameter is already given to me; V pt is known to, the volume of the Pulse Tube is known to me, frequency is known to me, P 1 is known to me, temperature I want to design for a particular cryocooler.

What should I do? I would like to maximize my Q c by making this phase angle as minimum or making this cos theta as maximum. So, this is you know external parameter attach to the Pulse Tube which called as phase shift mechanism, which should be design in such a way that the cos theta that is the angle in the between the Pulse Tube mass tube at the cold end and the pressure vector should be as small as possible.

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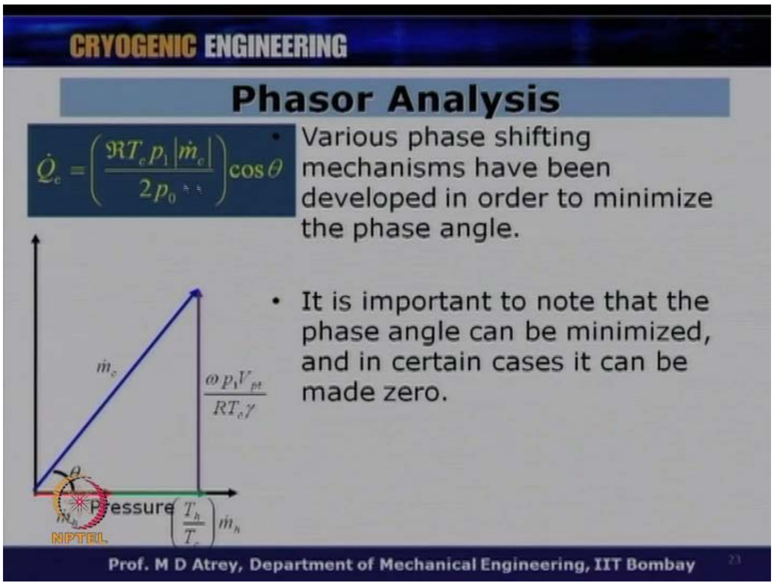

So, various phase shift mechanisms have been developed in order to minimize the phase angle. So, lot of research has happened in order to understand the importance of phase shift and what could this phase shift mechanism be all right. And depending on that we have got further classification of Pulse Tube which is what we will deal with later. So, it is important to note that the phase angle can be minimized. For example, in OPTC, I can minimize this phase angle, I cannot make equal to 0 for OPTC, because V pt is always going to be some finite quantity.

So, as long as this expression is correct that m dot c is equal to this vector plus this vector, and as long as this vector is going to be a finite quantity, and as long as m dot h is going to lie on the pressure axis on the pressure vector. As long as they are going to be in phase, I cannot make this theta to be equal to 0. So, what I can do? I can try to make this as small as possible by employing correct orifice opening. In OPTC, I will have a correct orifice opening. And therefore, which will make this theta to be as small as possible. So, it is important to note that the phase angle can be minimized. And in certain cases, now we will study different mechanisms of phase shift mechanisms and certain cases it can be made equal to 0 or also are very close to 0 also all right.

So, understand that Q c is a function of all this parameter which depends on cos theta which depends on m dot c P 1 by P 0 and T c also. Based on this cos theta - function of this cos theta based on, why do we have to reduce this phase angle and what are the phase shift mechanisms that have to be employed in order to get more and more cooling effect from pulse tube.

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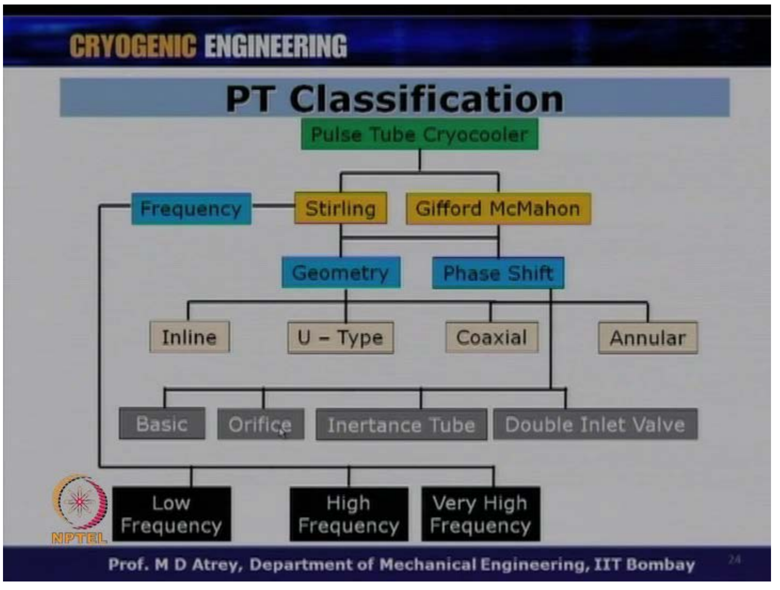

We have got further Pulse Tube classification. And we have seen this Pulse Tube classification earlier which can be done based on Stirling type, GM type, Geometry, Frequency. But one of the parameters for stirling type Pulse Tube cooler or GM type Pulse Tube cooler was also based on the phase shift mechanisms. And I had that time said that I will talk about this later in detail, although I had shown the names then. So, let us understand based on this phase shift mechanism what is the further classification. And you can see that they are further four parts; one is the basic type pulse tube cooler which we say basic phase shift mechanism or we can call it basic type pulse tube cooler, orifice type pulse tube cooler which we have already dealt with, then we have got a inertance tube pulse tube cooler and also we have got a double inlet valve pulse tube cooler all right.

So, these are four different types of phase shift mechanisms that are employed. In addition to all these (( )) classifications; so, this could be combination of U type and orifice, U type on inertance tube, U type and double inlet, and things like that. So, as far as the phase shift mechanism that has to be employed in the pulse tube cooler, we have got this four different possibilities; basic, orifice, inertance and double inlet. And now, depending on whatever diagram we had done, we have shown you the diagram of the pulse tube which was meant for only orifice, because that was simple to understand.

Now, let us draw the phasor diagram for Pulse Tube which are of basic type. Again orifice we can cover, inertance tube or a double inlet type, and let us try to understand what are this different mechanisms and how do this different mechanism help to minimize theta angle or to maximize cos theta. This is what we will concentrate on.

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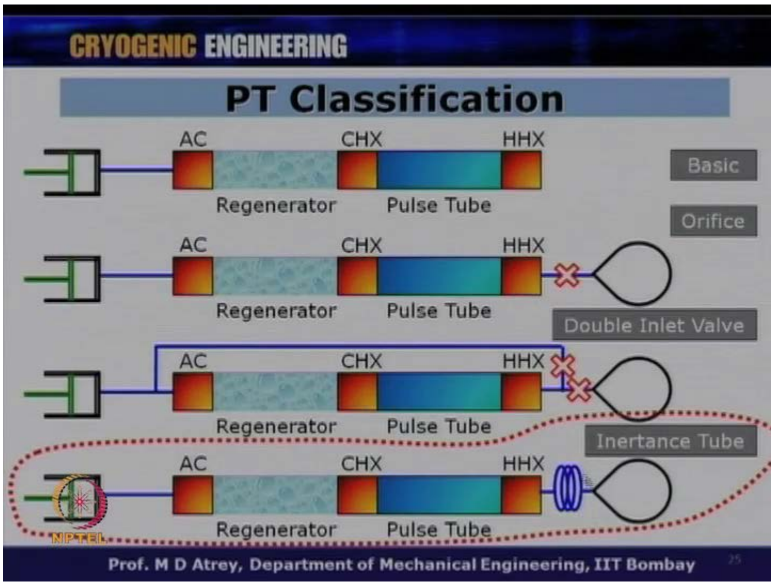

So, this is what we call as a basic Pulse Tube. What is this basic pulse tube? That means there is nothing at the hot end, there is the Pulse Tube cooler and there is a hot end heat exchanger, and after that there is no orifice and there is no reservoir, and this is what was pursued as Pulse Tube initially when Gifford came up with idea in 1963 and this is what we call as is basic pulse tube.

Now, let us see what happens to this basic Pulse Tube in detail in the next slide. But the second classification is orifice pulse tube. So, we have at got at the hot end now, we have got an orifice which is leading, we got a small inlet for the gas. So, gas does not stop at this point. This gas goes through an orifice and opens up in a reservoir. So, this reservoir is a very large volume as compared to the Pulse Tube volume. So, the gas during pressurization will go through this orifice depending on the orifice opening, it will just go through the orifice and go to the reservoir; and reservoir, because it has got a very large volume. It will hardly have any pressure fluctuations. So, we have got a pressure fluctuation on this side, while you do not have any pressure fluctuations normally or the pressure fluctuations are absolutely minimum in reservoir and this is called as OPTC. Some people called at OPTR - orifice Pulse Tube refrigerator. This is called as BPTR. So, we were calling as BPTC, OPTC then what we have is a double inlet valve.

In double inlet kind of phase shift mechanism, what is happening? We have got a same thing as OPTC. We have got orifice and reservoir. In addition to that we have got a one more valve and this is called as double inlet valve. So, we can see that gas after compression before going entering the after cooler some pressurize gas is just taken off and it is allowed to join in the hot end in Pulse Tube cooler. So, this is which joining at the hot end near the hot end after the Pulse Tube cooler after the hot end heat exchanger. This is called as double inlet valve. And now when the gas gets pressurized, some part of the gas or very small part of the gas actually goes in this double inlet circuit, and less gas would go through the regenerator and the Pulse Tube all right.

So, this is the mechanism which helps basically to reduce the phase angle theta, and we will see how. And lastly we have got one more addition; we have got a inertance tube. That means at the hot end instead of having orifices, what we have got a very small capillary kind of a tube of 1 or 2 millimeter diameter and a very large length, which can lead to inertance tube. And this could be one double inlet inertance tube also. So, this is called as inertance tube Pulse Tube cooler. And it one can have even the combination of double inlet with inertance tube. So, which will called as DIPTC, this is called as ITPC and we can call double inlet ITPC also.

So, these are the different four classifications based on the phase shift mechanism. What is the phase shift mechanism here in the basic Pulse Tube, there is no phase shift mechanism. Here you have got a orifice plus reservoir, here you got two orifices and that is why we call it as double inlet valve leading to an orifice again over here, we got a inertance tube Pulse Tube cooler, inertance tube leading to a reservoir, and we may have double inlet inertance tube PTC also. This is what Pulse Tube classification would look like when you got a classification based on various phase shift mechanism.

Now, let us try to see how do you get cooling, how does it affect theta in a basic Pulse Tube, orifice Pulse Tube and double inlet pulse tube. So, as far as this discussion goes for these, we will go basically for this three types; basic, orifice and double inlet type. While the inertance tube I will explained to you based on RIC circuit; the resistance, inductance, capacitance kind of mechanism which we will do in the next lecture.

So, let us try to understand these basic three mechanisms, because these are very important while inertance tube is nothing but combination of various parameters. So, let us come to the phasor diagram for BPTC. Understand I am drawing a phasor diagram only for the Pulse Tube and not the entire cryocooler. Once you understand the phasor diagram for Pulse Tube then we will draw the phasor diagram for the entire cryocooler.

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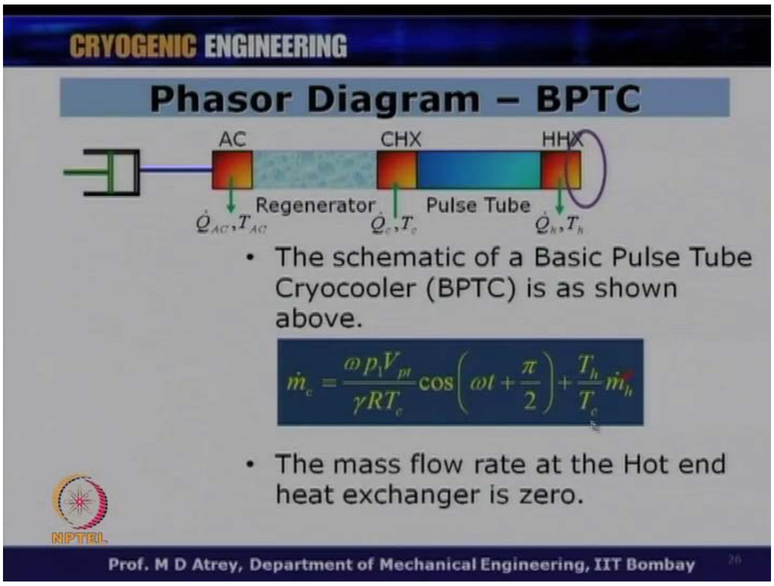

So, this is my BPTC, there is no orifice, there is no reservoir. The schematic of a basic Pulse Tube cryocooler BPTC is as given above. What do you see here? We have got m dot c, do you have m dot h leaving the hot end heat exchanger? No, because there is no orifice, we know that the gas ends up over here, when gas gets pressurized it hits the valve here and then gas goes back from here only. What does it mean? It has got no m dot h actually.

So, an expression which we have derived earlier was m dot c that is the mass flow rate at the cold end of the Pulse Tube, which is at this area is equal to T h by T c into m dot h, which is the mass flow rate at the hot end of the Pulse Tube plus a parameter depending on what is the volume of the Pulse Tube, what is the frequency of the Pulse Tube, what is the amplitude of pressure which is at a 90 degree vector addition to vector of m dot h. As you can see from BPTC m dot h is equal to 0. So, the mass flow rate at the hot end heat exchanger is 0 in this case, because this mass flow rate which is leaving the hot end heat exchanger is actually equal to 0. See if I want to draw a phase diagram I will say now m dot c is equal to only this quantity while the second part is equal to 0.

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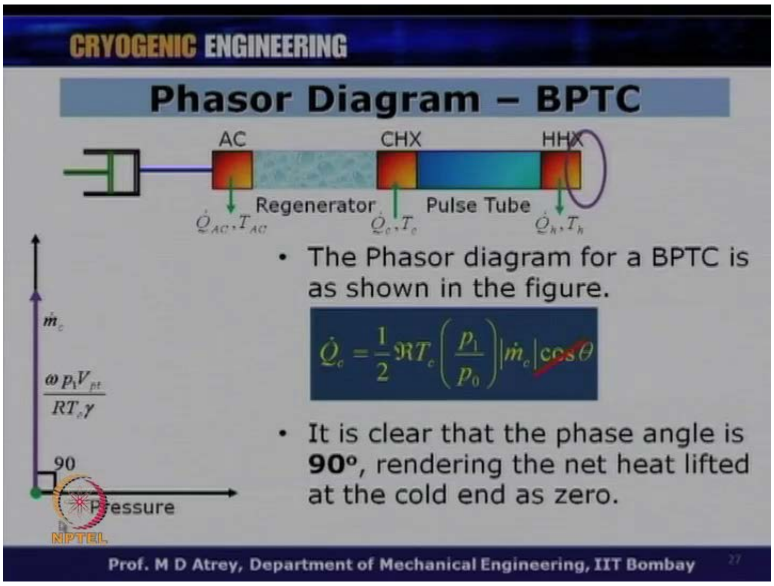

So, let us the second part vanishes and therefore, the phasor diagram now will look like this. So, pressure axis remains like that. There is no m dot h on the pressure axis, what we had seen earlier. And now m dot c is going to be at a 90 degree to the pressure axis. So, it will not start from here, because m dot h is equal to 0, m dot h is equal to 0 is going to be only at this point. And therefore, m dot c will start from origin only. And therefore, I have got m dot c value which is equal to the amplitude which is shown earlier.

And now what is theta therefore. The theta in this case is angle between as I defined earlier is an angle between m dot c that is the mass flow rate at the cold end of the Pulse Tube and the pressure. So, my cooling effect therefore which we had talked about is given by this expression. Theta is equal to 90 in this case. See if I put the value of theta is equal to 90 for BPTC what does it mean? Cos 90 is nothing but equal to 0 and therefore, it is clear that the phase angle is 90 degree rendering that the net heat lifted at the cold end is equal to 0. That means Q dot c for a BPTC is equal to 0.

So, as far as BPTC is considered I should not get any cooling and why should not I get any cooling, because the angle between m dot c and the pressure is equal to 0. This is the very important thing one understands from phasor diagram. In principle what happens, because the viscous flow of the gas with the valve, this angle becomes not equal to 90, but close to 90 and it will have some angle. Because of the resistive force, because some heat transfer, what we call as some surface pumping effect. This angle is not equal to 90, but close to 90 still, and because it is not cos theta is not equal to 0, you may have some cooling.

So, when Pulse Tube was invented - Pulse Tube cryocooler was invented it was only that BPTC was invented. And of course, it had got some cooling effect. However, temperature never came down below let us say 200 kelvin. And therefore, BPTC was what was invented. But we had not understood then what is the logic, because BPTC could not give you cooling in a cryogenic region. The reason was this that the theta was very, very close to 90 and therefore, it could not yield any cooling effect, and therefore, temperature could not come down.

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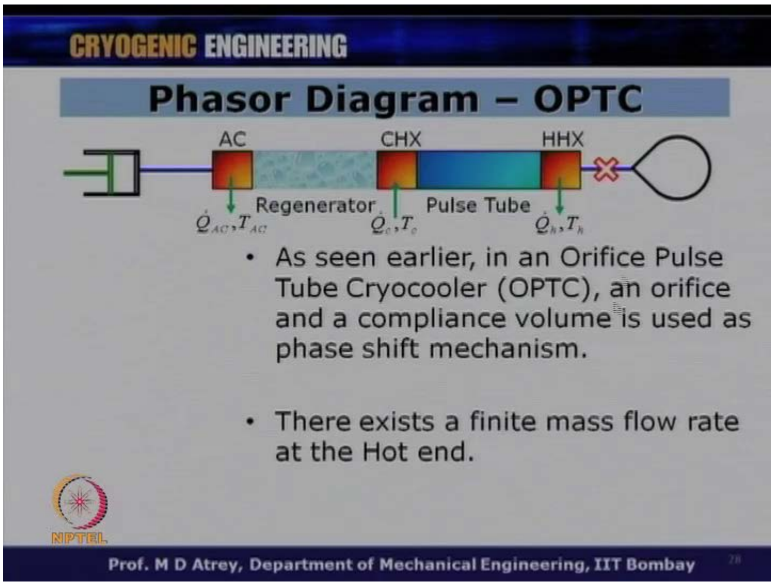

Now, let us see a phasor diagram for OPTC. When I am think OPTC actually I am drawing your attention only the Pulse Tube of the OPTC. So, in OPTC we know that your Pulse Tube is now connected to… Actually I am not taking into consideration the hot end heat exchanger; I am just calling this hot end of the pulse tube and cold end of the pulse tube. Later when I understand the Pulse Tube thing, I will add the hot end heat exchanger, cold end heat exchanger, after cooler, regenerator when I am drawing a phasor diagram for the entire OPTC.

So now, please understand I am talking about hot end of the Pulse Tube and cold end of the Pulse Tube, because we are drawing the diagram only for the Pulse Tube as of now. So, I have seen earlier in an Orifice Pulse Tube Cooler, in fact OPTC is clear, because entire derivation of theta we have done with OPTC in mind; so, OPTC and orifice and a compliance volume as we have done. There exists a finite mass flow rate at the hot end. We know that there is a finite mass flow rate. Therefore, there is the m dot h is equal to m dot o.

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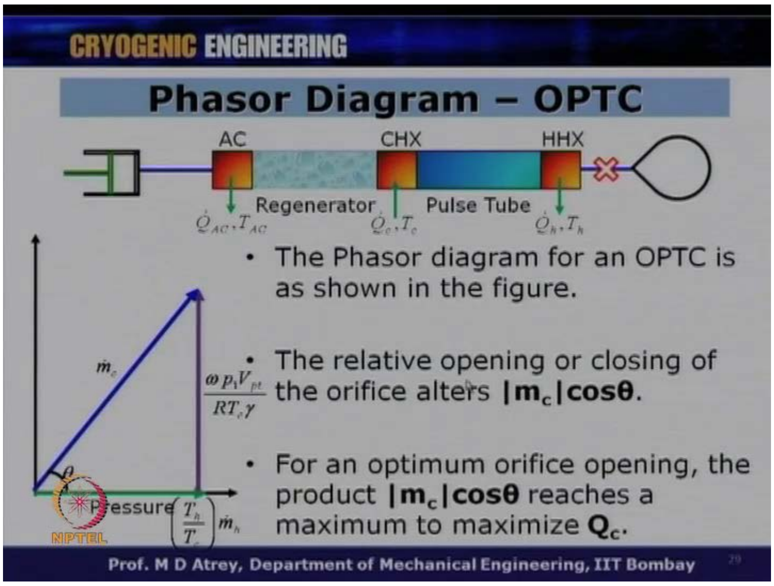

And therefore, our phasor diagram would look like what we had done earlier, we plot the pressure axis, pressure vector on which we have got T h by T c into m dot h, we have got a vertical height of a vector, which is at 90 degree to this and therefore, we have got m dot c and therefore, we know that there with theta and therefore, we know that there will m dot c cos theta, and depending on theta, the theta would basically depend on what is the design parameter which we have taken. We will try to keep it as small as possible by opening this orifice by optimizing this orifice opening; so that at a particular orifice opening I will get large m dot c cos theta. And therefore, large m dot c cos theta would determine what my cooling effect would be.

So, the relative opening or closing of the orifice will basically alter m dot c into cos theta and on which the cooling effect depends. So, we will not talk much about OPTC, because it is pretty clear and we have use this in earlier derivation also. For an optimum orifice opening, because I shall go on changing this orifice till the time I get more and more cooling effect or till the time I get lowest temperature. For an optimum orifice opening, the product m dot c cos theta reaches maximum and this maximizes our Q dot c or the cooling effect or the refrigeration effect obtained at temperature T c, which is obtained at this particular temperature.

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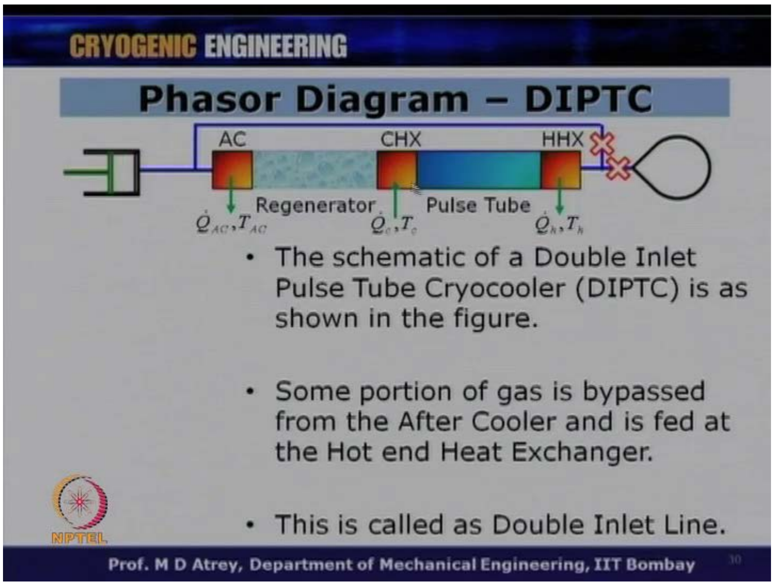

Now, let us come to DIPTC, this is very important. And now we are talking about double inlet Pulse Tube cooler. So, what is happening? As I had explained earlier; the schematic of double inlet Pulse Tube cooler is as shown over here. Some portion of the gas after compression and a very small quantity in fact, a very small quantity of gas after compression is bypassed from the after cooler, before the after cooler actually in fact, and is fed at the hot end heat exchanger. So, this goes from the top and it goes to hot end heat exchanger through a an orifice, through an orifice. So, how many orifice are there? You have got two orifices over here now. And this is called as double inlet line, this line is called as double inlet line. Otherwise, this is the OPTC and now it becomes double inlet PTC.

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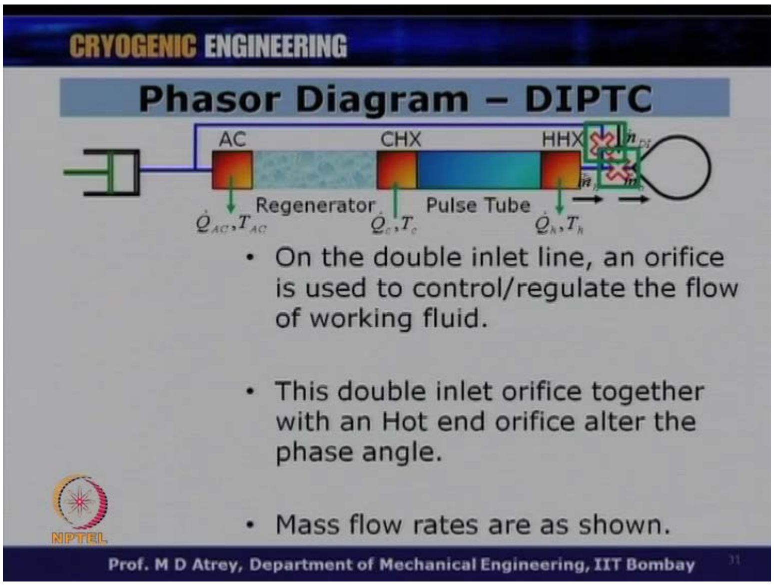

On the double inlet line, an orifice used to control or regulate the flow of working fluid. So, how much gas should go on this side which will control by this orifice opening. In the similar way, as OPTC orifice was doing. Now, you see I have got gases coming through orifice from one from double inlet side and therefore, I will have some mass flow rate form this side, which call as m dot D i. And we have got some mass flow rate coming for m dot h all right. So, I have got some mass flow rate coming from the hot end heat exchanger, I have got some mass flow rate coming from the m dot DI while I have got some m dot o over here depending on the orifice opening.

In earlier case we had seen in the OPTC, we had said that m dot h is equal to m dot o. Can I say that now? No, I cannot say that now. And what will be the expression, we can see in the next slides. So, double inlet orifice together with hot end orifice, alter the phase angle. So, we have got two possibilities now. Double inlet orifice and this orifice and this would result in the optimum phase angle basically. The mass flow rates are as shown over here. So, what are the mass flow rates now? I have got m dot DI coming over here which is the mass flow rate through this double inlet valve. I have got m dot h which is coming from the hot end heat exchanger, and I have got m dot o. I am not sure if you can see that thing, but what I want say is there are three mass flow rates now; m dot o is the mass flow rate through the orifice which depends on the pressure across its. That means m dot o is always going to be in phase with the pressure axis.

In earlier case, it was m dot h, because m dot h was nothing but equal to m dot o. While m dot DI is the mass flow rate through coming through the double inlet valve and m dot h is the mass flow rate coming from the Pulse Tube side or the hot end heat exchanger side.

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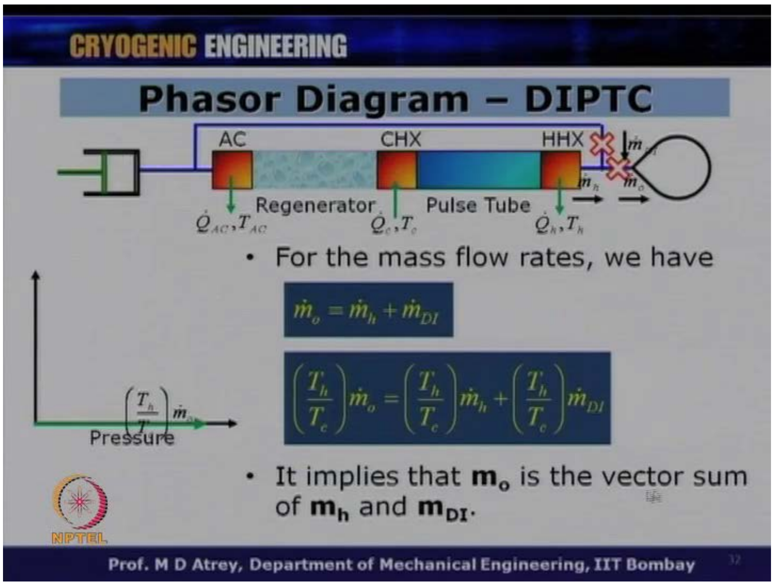

So, if I want to plot a phasor diagram, I have got a pressure axis and I know that m dot o or T h by T c into m dot o would be always be in phase with pressure, because the mass flow rate through orifice always depend on the pressure difference across it. So, for the mass flow rates we have if I do the mass balance m dot o, if I am doing during pressurization, I know that m dot o is equal to what is coming from the Pulse Tube plus, what is coming from double inlet. So, m dot o plus m dot h plus m dot D i. So, I am not to do vectorial addition of this in order to get m dot o.

And if I multiply all this three quantity by T h by T c, because we know that ultimately, I want T h by T c into m dot h in order to calculate m dot c. This is what expression we have already derived earlier. So, I have got T h by T c into m dot o is equal to T h by T c into m dot h plus T h by T c into m DI. It implies that I have just said that m dot o is a vectorial addition vector sum of m h plus m dot DI.

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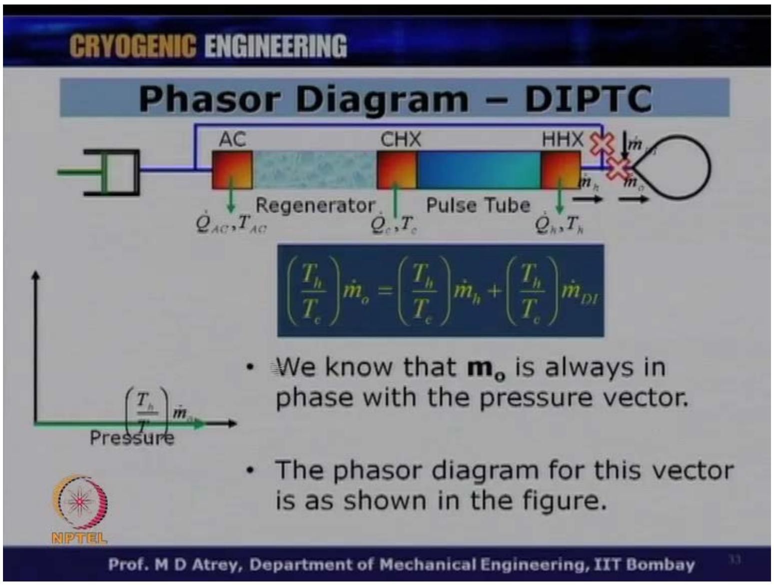

This is expression what we derived. We know that m dot o is always in phase with the pressure vector, because it is of (( )) orifice which depends on a pressure difference increases, the m dot o will increase in the same proportion. See if I want to draw now a phasor diagram, I will first draw a pressure axis and I will draw on this T h by T c into m dot o.

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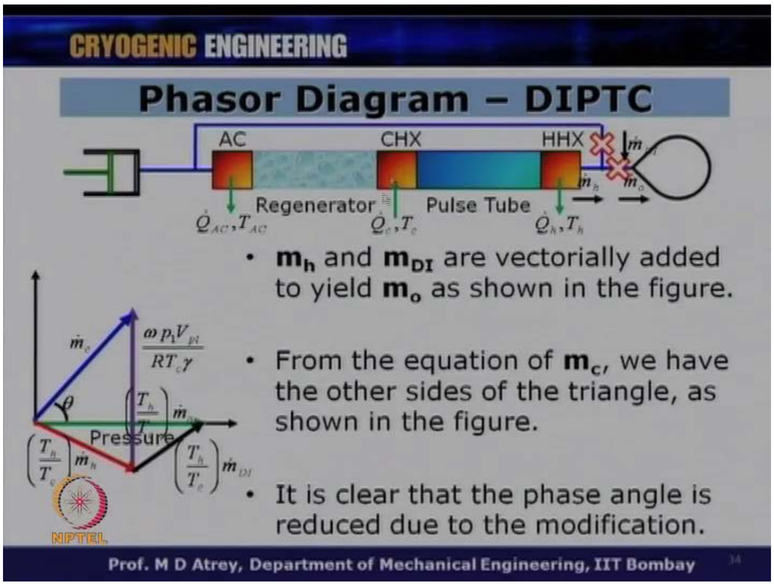

Now, I will know that m dot o is equal to m dot DI plus m dot DI plus m T h. And if I do the vectorial addition over here now, m dot h at and m dot DI are vectorially added yield m dot o. And now I can have the position in such a way depending on the orifice openings on either side that my m dot h would lie below m dot all right. So, I will have possibilities of having m dot h to lie below m dot o vector. And therefore, T h by T c into m dot h plus T h by T c into m DI. Do not go by this length right now. We are just showing that the relative positions of m dot h and m dot DI would look like this. And therefore, vectorially if I add this is m dot h into T h by T c plus m dot DI into T h by T c. This plus this is equal to T h by T c into m dot o.

Now, I know T h by T c into m dot h. So, we have calculated the mass flow rate at this plus the Pulse Tube cooler volume if I want to add then I will get m dot c. So, we have already developed a relationship between m dot c is equal to m dot h into T h by T c plus you know we have an expression depending on the which is at 90 degree to this. So, if I want to from the equation of m dot c now. We have the other side of the triangle as shown in the figure. So, from here I will have omega P 1 into V pt divide by RT c gamma which is representing the adiabatic action in the Pulse Tube, because the gamma figures over here. And this is my vector quantity depending on V pt pressure omega etcetera drawing at 90 degree to the pressure pulse, because there at 90 degree to the pressure vector. And this plus this vectorially added together will give me m dot c. So, now my m dot c will be like this. What you can see from here is earlier this m dot h was in phase with the pressure vector.

Now, I have pulled this m dot h below, because of the presence of m dot DI. As soon as I pull it pull it below, this vertical quantity equal to this gets started from the point which is below this pressure vector, because of which even m dot c gets pulled down. And therefore, my theta which is the angle between m dot c and pressure also gets reduced. So, what is happening; only because we have incorporated DI value, because of the double inlet because of the double inlet orifice presence. I have got this vectorial addition which gives me m dot o. Because of which my m dot h gets pulled down below the pressure vector. And therefore, as a result of which my theta which is this gets reduced as compared to if you see with OPTC. It is clear that the phase angle is reduced due to this modification.

And this is what basically the just why this DIPTC as employed. The DIPTC will pull this m dot h down below the pressure vector, because of which theta automatically will get reduced; theta being the angle between m dot c and the pressure pulse. Please understand this three things what why we do not get cooling in the BPTC theoretically; why the phase angle for a double inlet PTC is going to be less than as compared to the OPTC. And what is basic behind all that thing is basically this phasor diagram all right.

So, this is the phasor diagram for a double inlet PTC and as I just said we have never taken into consideration, the dead volume in the hot end heat exchanger, cold end heat exchanger, regenerator after cooler. And now I would like to take all this parameter into consideration. So, what we can do is now extend this and in this Pulse Tube now. This is the diagram for Pulse Tube we can just add a cold end heat exchanger to these and see a change, what happens, if I add cold end heat exchanger to my phasor diagram all right. So, till now we have done phasor diagram for BPTC, OPTC and DIPTC only for the Pulse Tube. I am adding one more component to this Pulse Tube to see how it alters the phase diagram and then we will derive the phasor diagram for the entire Pulse Tube cooler.

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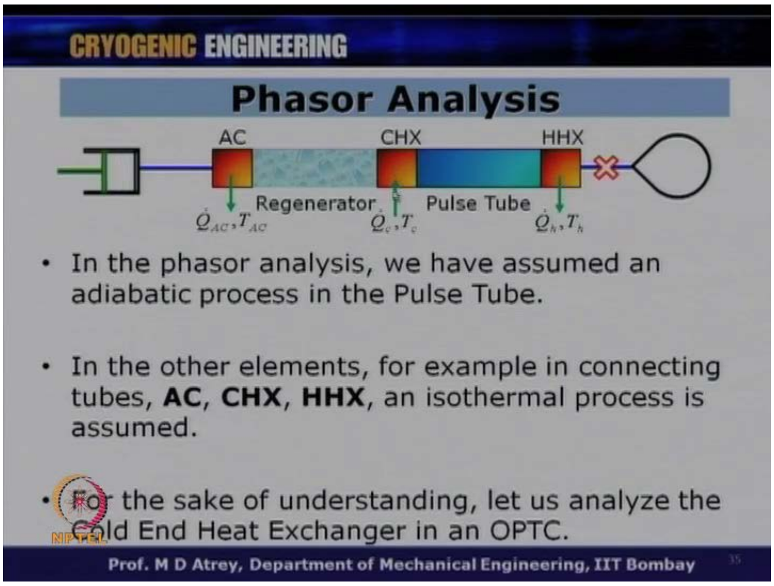

So, phasor analysis; in the phasor analysis we had assumed an adiabatic process in the pulse tube. So, as far as Pulse Tube was concerned we had adiabatic compression and expansion happening over here. While in the other elements for example, in the connecting tube this is my connecting tube after cooler cold end heat exchanger, regenerator, hot end heat exchanger, we are assuming the temperatures to be constant. So, we can assume the entire hot end heat exchanger to be at particular temperature regenerator, if I do slices I have got variation of regenerator temperature happening across the length like this or I can assume the entire regenerator to be at one fixed temperature also.

So, we can assume this to be isothermal temperatures over here and for the sake of understanding let us now analyze the cold end heat exchanger to the pulse tube. So, I am just taking this Pulse Tube, I am adding one cold end heat exchanger. Understand this Pulse Tube was happening, the process were happening at adiabatic law, while heater it will here in this case it will be isothermal at a fixed temperature. The regenerator will be isothermal at a temperature T r, let us say AC will be at particular temperature T r some temperature after cooler.

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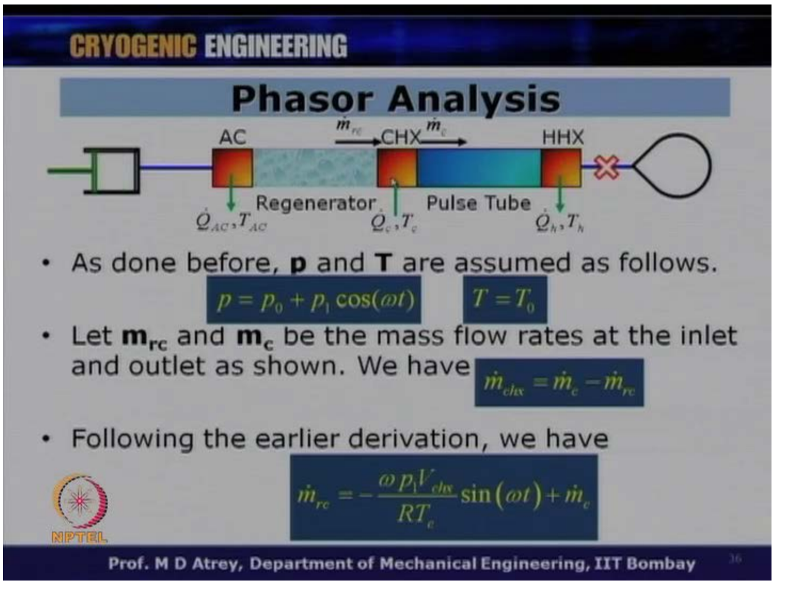

So, I has done earlier will have pressure variations, but will have temperature equal to T 0. So, for example, T 0 will be some some temperature over here; some temperature T r over here; and let us understand if I want to do a mass balance at this, I have got m dot r c entering this I have got m dot c leaving this and therefore, I have got some m dot CHX that is the mass flow rate in the cold end heat exchanger will be equal to m dot c minus m dot r c. From the earlier derivation now; if I go for earlier derivation, I can do all this algebraic calculation, which we have done and we can now see that m dot r c, which is the mass flow rate at this thing and it is related to m dot c by this formulation. So, m dot r c is equal to m dot c plus a quantity, which does not have a gamma here, because this is not adiabatic process.

We had derived the similar expression for m dot c, which is equal to m dot c, which is equal to m dot h plus depending on the volume of the pulse tube, but which had a gamma in the bottom, because it was having a adiabatic action; please understand this. So, I am now calculating m dot r c, if my m dot c is known to me, I have got a one more in a similar manner, I have got a same expression; however, because this process in the cold end heat exchanger is happening is isothermal process, I will not have a gamma figure, I am note here.

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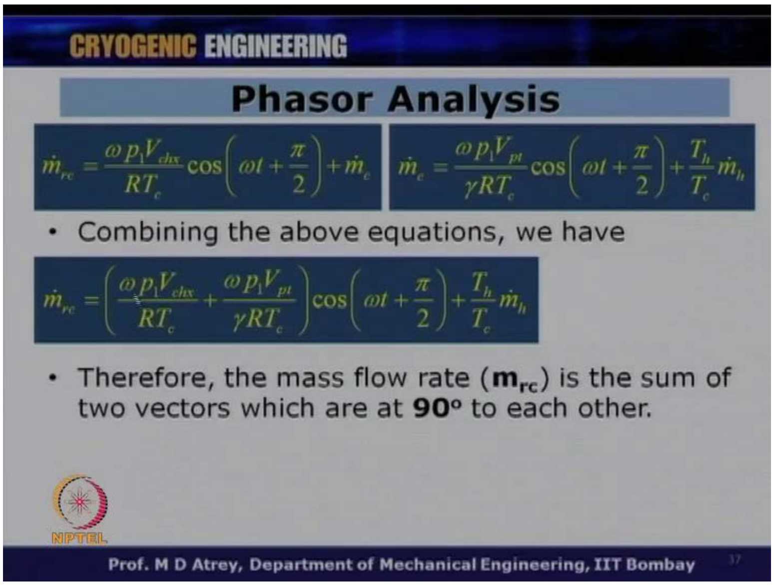

And if I do a phasor diagram for this, so m dot r c is equal to this plus this and I can put the value of m dot c, which we had derived earlier. So, m dot c is basically this as m dot h putting this value of m dot c in this, I will get m dot r c is equal to cos pi by 2 T c by T h by T c m dot h and this is the quantity. So, this quantity depicts the volume of pulse tube, this quantity depicts the volume of heat exchanger, which is at T c temperature; therefore, the mass flow rate m dot r c is the sum of two vectors, which has having a quantity of this plus this at a pi by 2 angle with m dot h.

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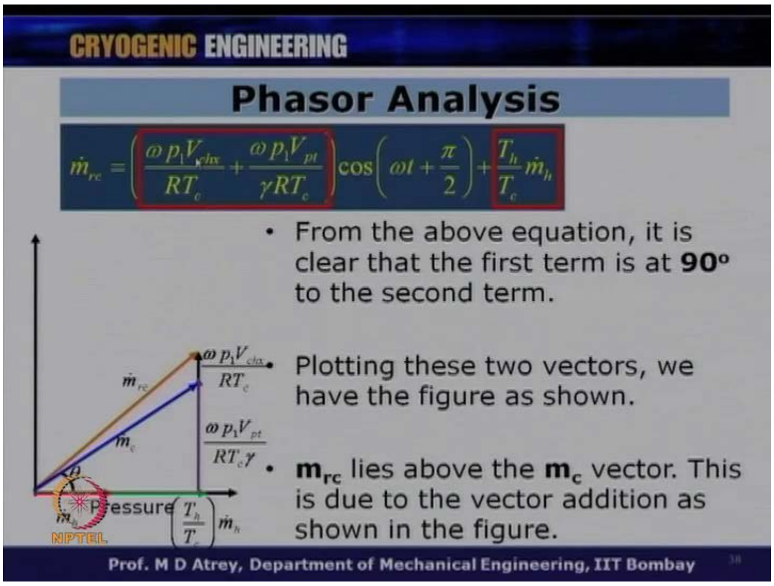

So, now if I want to do phasor analysis to this; this is my normal phasor diagram for m dot c, where we have taken this quantity has over here; I will add to this quantity, from the above equation it is clear that the first term is at 90 degree to the second term; plotting this two vectors, we have second quantity, which depends on what is the volume of heat exchanger over here; and now this will give me m dot r c, which is the mass flow rate of the regenerator at the cold end.

And similarly I will now have a regenerator volume, I will have a after cooler volume, I will have a compressor volume and thing like that. So, I got to add all this volumes over here, in order to get the mass flow rate near the compressor. So, basically this vertical heights are nothing but representative of different volumes and temperatures associated with those volumes. So, omega p 1 R gamma is remaining constant; what is this vertical height will represents; what is the volume of pulse tube divided by volume of pulse tube temperature; what is the volume of this heat exchanger divided by its temperature; then we comes the regenerator, then comes the after cooler, then comes the compressor. So, I have to basically derive this phase diagram for the entire pulse tube cooler in this way. And this is the way I have just shown an example how the cold end heat exchanger is added to our earlier derived OPTC phasor diagram. So, m dot r c lies above m dot c vector; this is due to the vector addition as shown in the figure; this is basically due to this vector addition as shown in the figure; understand that the Pulse Tube has a gamma factor, because the adiabatic action happening in the pulse tube, while the heat exchanger assumed to be a isothermal process and therefore, there will no gamma figuring out this. In fact, only in the Pulse Tube we will have a adiabatic gamma shown up over here.

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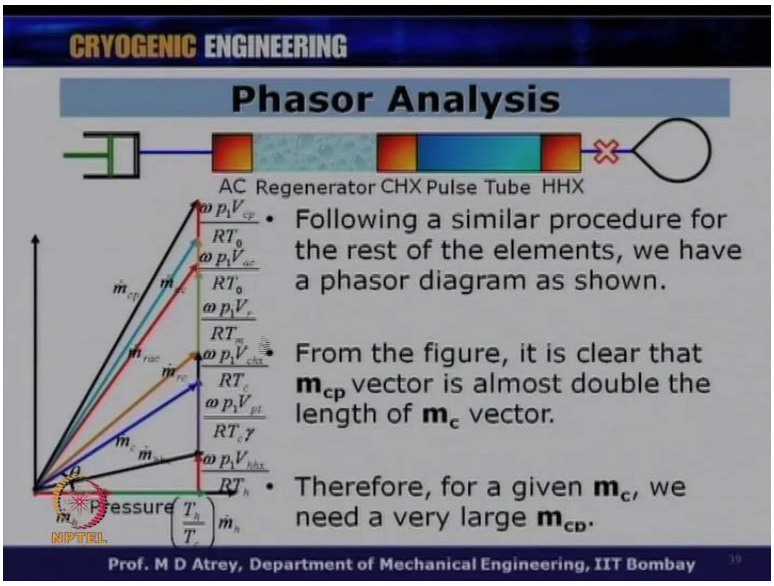

So, if I want to show now, as I said earlier I had not taken even the hot end heat exchanger, cold end heat exchanger, I have to take all this into consideration, I will have a same procedure to be followed out; I will incorporate the hot end heat exchanger and divided by T h over here, then I will add the regenerator volume. So, this is up to m dot r c, I will add entire regenerator volume depending on the temperature of T r and I will get m dot r at this point, let us call it r a c, then I will have after cooler volume added over here at its temperature, I will again have m dot r c at this point, and then I will have some connecting tube, if I add connecting tube also, I will have this, and then I will have a compressor at a top end; I have not shown the connecting tube volume over here, but that also would figure out here.

So, following a similar procedure for the rest of the elements, we have a phasor diagram for the entire cryocooler. Now can you imagine that for getting cooling effect, what I want is only this m dot c, but to get this m dot c, I am going to compress vectorially; if I see so much mass in the compressor. So, my compressor is going to be bigger and bigger and correspondingly, I have to give a lot of power to it. So, phasor diagram basically tells you how much gas needs to be compressed in the compressor, in order to get a relative m dot t c; and this is a phasor diagram for a OPTC.

So, from the figure it is clear that m dot C p vector is almost double the length of m dot c vector and therefore, my compressor power also accordingly will get decided. So, this is basically in nutshell; phasor diagram we have to construct and I will take some tutorial in the next lecture on development of this phasor diagram for some cryocooler, where we take some practical values and we can develop a phasor diagram, but understand you'll have to spend some more time to understand, but understand what is this physical understanding of this theta - cos theta and how does one derive this phasor diagram for OPTC, DIPTC etcetera; therefore, for a given m dot c, we have we need a very large m dot C p. So, I have to compress such a big gas, which will make my compressor to be a very high therefore, it will take lot of power input and therefore, my cop of the entire refrigerator could be as low as possible; this is nutshell, I would like to tell in this lecture, I will not complicate try to understand this physics behind this phase shift mechanisms.

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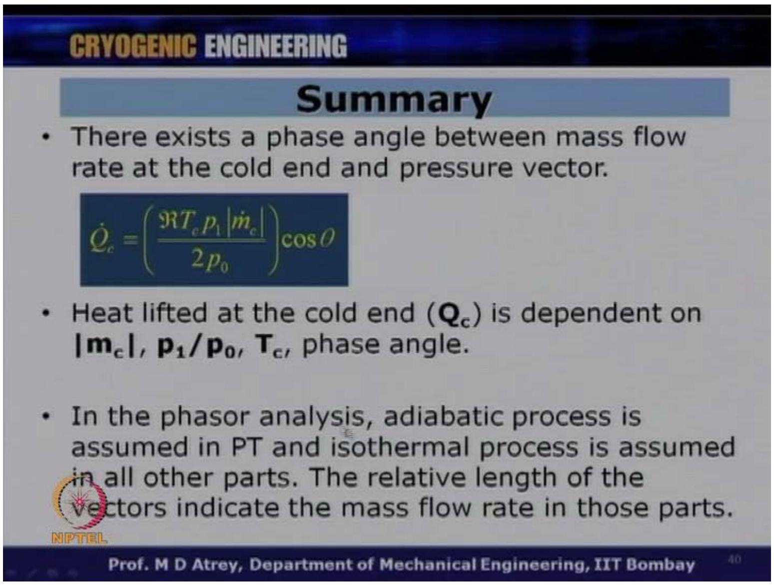

So, summary of the entire lecture; there exists a phase angle between mass flow rate at the cold end and the pressure vector and there is related on by this algebraic equation, the heat lifted at the cold end Q c depends on m dot c, P 1 by P 0, T c and phase angle. In the phasor analysis adiabatic process is assumed in pulse tube and isothermal process is assumed in all other parts, the relative length of various vectors, indicate the mass flow rate in those parts. So, we have got a mass flow rate to pulse tube, mass flow rate to after cooler mass flow rate to compressor and their related by different design or different volumes divided by the respective temperatures; a self-assessment exercise is given after this slide. Thank you very much.