


Cryogenic Engineering
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Lecture No. # 30
Cryocoolers
Ideal Stirling cycle

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CRYOGENIC ENGINEERING

Earlier Lecture

- We have seen the schematic and the working of a Gifford – McMahon Cryocooler (**W. E. Gifford** and **H. O. Mc Mahon**, 1950).
- It has a valve mechanism to generate the pressure pulse. The relation between the pressure pulse and the expander – displacer motion is vital.
- The basic components are Compressor, Flex lines, Regenerator(s), Displacer(s), Valve mechanism.
-  GM system can reach much lower temperatures as compared to a Stirling system.

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So, welcome to the thirtieth lecture of cryogenic engineering under the NPTEL program. In the earlier lecture, we talked about GM Cryocooler or differed Mc Mahon Cryocooler, we have seen the schematic and the working of the GM Cryocooler which was invented by Gifford and McMahon, in 1950. It is high let was it has a valve mechanism to generate the pressure pulse, the relation between the pressure pulse and the expander displacer motion is vital and this is what we saw in detail during the last lecture. The basic components of the GM Cryocooler are compressor flex lines regenerator displacer valve mechanism etcetera. A GM system can reach much lower temperatures as compared to a Stirling system.

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Earlier Lecture

- Multistaging is done to reach lower temperatures (4.2 K to 10 K).
- Single stage (~ 30 K), **SS** mesh.
- 2 – stage (~ 10 K), 1st stage: **SS** mesh, 2nd stage: **Lead** balls.
- 2 – stage (~ 4.2 K), 1st stage: **SS** mesh + **Lead** balls, 2nd stage: **Lead** balls + **Er₃Ni** balls.
- Commercially available cryocoolers have rotary valves to control/regulate the flow of working fluid.

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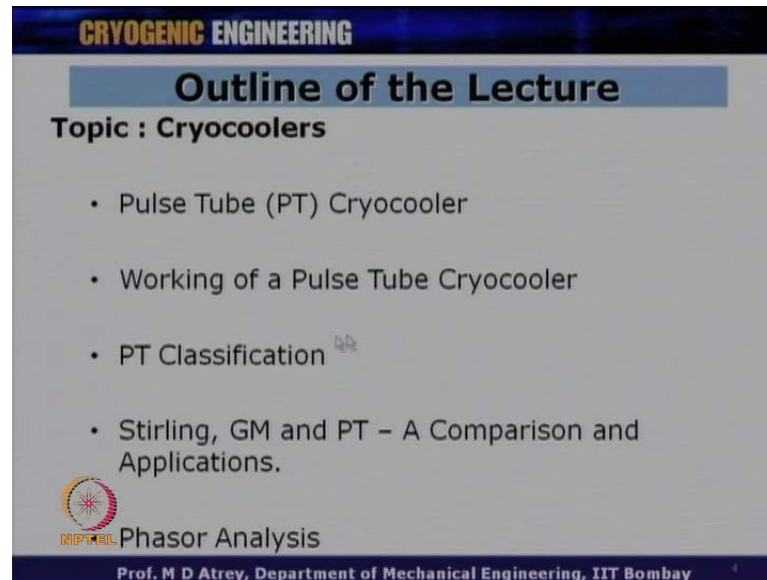
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Basically, the difference between the GM and the sterling is GM has a valve and trans at very low frequency of around 1 to 2 hertz well Stirling operates without a wall and normally it trans at very high frequency as compared to GM Cryocooler. Now, in GM Cryocooler multistaging is done to reach very lower temperature and of the order of around of 4.2 k which is helium boiling point and up to 10 Kelvin. Now, these are temperature which are normally use for basically various scientific experiments for example, 10 k while 4.2 k is normally use to Liquify or re-condense helium gas. A typical regenerator material in a single stage GM Cryocooler which would possibly bring the temperature down to 30 Kelvin is stain less steel mesh and if you want to come to by using a two stage Cryocooler to reach around 10 Kelvin there in the first stage we will have stainless steel mesh material and in the second stage we will have around lead balls.

Similarly, if I want to reach using a two stage Cryocooler to 4.2 Kelvin temperature the first stage may have the stainless steel mesh plus lead ball as originator material. And in the second stage we may have lead balls plus erbrianethical holmium copper neodymium etcetera the all spears made out of normally these materials at around 0.2 millimeter diameter. Now, when I am telling these a very broad reginator configuration SS mesh plus lead ball it may have 100 percent SS mesh and second stage may have lead balls plus erbrianethical, but this is what we generally, would at the reginator material for a four point two Kelvin Cryocooler.

Commercially available Cryocoolers have normally rotary wall to control regulate the flow of the work in fluid the work in fluid normally will be they helium gas this is what we saw in detail during in the last lecture.

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Outline of the Lecture

Topic : Cryocoolers

- Pulse Tube (PT) Cryocooler
- Working of a Pulse Tube Cryocooler
- PT Classification
- Stirling, GM and PT – A Comparison and Applications.

 **Phasor Analysis**

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Now, this lecture we will now completed to dedicated to pulse tube Cryocooler which is very important to Cryocooler right now, amongs all the Cryocooler that are use and you will understand why it is so. Let us see the working of a pulse tube Cryocooler let us see how the pulse tube Cryocooler are classified as and normally I will refer pulse to be as now PT Cryocooler or PT PTC sometimes. So, pulse to classification and let us compare this Stirling, a Gifford McMahon and pulse tube Cryocooler and let see different applications. And let us try to understand the pulse to Cryocooler using a freezer analysis, which you very important concept to understand why it different phase shift mechanism are employed in pulse tube Cryocooler.

And will have the next lecture also, dedicated to pulse tube Cryocooler where we can see more hardware that has been generated here in our laboratory.

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CRYOGENIC ENGINEERING

Introduction

- In the earlier lectures, we have seen a regenerative type Stirling and GM systems.
- These systems have a mechanical expander – displacer to displace the working gas.
- The displacers are either free moving or driven by an external mechanism.

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So, in the earlier lecture we have seen a regenerative Cryocooler and of different types let us Stirling type and GM type. So, this is Stirling type this is just to recap to understand what we have done in the last lectures. So, we have got Stirling type machines Stirling Cryocooler where this now wall in between the compressor and the expander and therefore, the displacer moves at a same frequency as that are compressor piston. And there are the fix phase difference between the motion of the compressor piston and the motion of the displacer while, in a GM Cryocooler you have got a pressures of wall mechanism between the compressor piston and expander. And there is the phase difference not between the displacer and the piston, but between the wall mechanism and the displacer which is moving at (()) low frequency in a GM Cryocooler.

These systems have a mechanical expander displacer to displace the working gas in both the systems we have got one component moving in a expander we have got a displacer moving here, at higher frequency we are going to displacer moving here at around low frequency of around 1 to 2 hertz in a GM Cryocooler. The displacers are either free moving or driven by an external mechanism. So, in order to drive this piston or displacer we may have to have a different driving mechanism for driving this displacer up and down or sometimes it could be free displacer, which moves because of the pressure drop across the displacer. There could be small pressure drop across the displacer the gas

across the displacer, because of which the piston moves up and down the displacer the normally very light weight moving component.

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CRYOGENIC ENGINEERING

Introduction

- The cold end displacers pose few problems as given below.
- The rubbing seal on displacer is difficult to maintain.
- The motion of the displacer induces unnecessary vibrations at the cold end.
- PT cooler overcomes these problems as this does not have a mechanical displacer.

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The cold and displacer pose few problems as given below sometimes, the in GM Cryocooler especially as you can see there is a seal which works at a low temperature, because this said is a low temperature. And therefore, will have a rubbing seal at this point and these seals which have rubbing in this can as can pose some problem from working point of view. The rubbing seal on displacer is difficult to maintain and therefore, when I was to really to take care of theses seal and the seal definition is very important how this seal is composed of.

So, at low temperature there could be shrinkages across and seal, which is perfect and room temperature may not work very well at low temperature and sometime this is a trouble for lot of displacers moving in this cylinder. The motion of displacer induces unnecessary vibration at the cold end so, whatever, object I want to cool the whatever the load comes over here in case of Stirling at this point in case of GM and at this point. Because the mechanical component moves up and down this creates lot of vibration for the component to be cooled and sometimes this component may not function whatever, scientific action you want take over here may get hamper because the vibrations. That means, this experimental will not tolerant the kind of vibration generated by this moving component which is displacer in this case.

To overcome this problem pulse tube cooler comes to rescue and therefore, what happens in pulse tube cooler? Pulse tube cooler overcomes these problems as pulse tube cooler does not have a mechanical displacer their nothing moving at in the in the cylinder like here you got a displacer the nothing is moving in case pulse tube. And therefore, one complete mechanical component absent in case of pulse tube Cryocooler and this is very difference between Stirling GM and the pulse tube Cryocooler. The moment I do not have something moving over here, I do not have vibration at this point at the same time I do not have any need to have drive mechanism. So, as in to move this mechanical component this is a very important difference between the pulse tube cooler and Stirling and GM coolers.

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Pulse Tube Cryocooler

- Consider the schematic of a Stirling system as shown in the figure.
- In a Pulse Tube cryocooler, the mechanical displacer is removed and an oscillating gas flow in the thin walled tube produces cooling.
- This gas tube is called as **Pulse Tube** and this phenomenon is called as **Pulse Tube action**.

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So, now what happens? Considers schematic of a Stirling system as shown in this figure this is a normal Stirling type Cryocooler and if I want to convert this to pulse to Cryocooler what will I do I will just replace as you just saw I will just replace this displacer by a gas column cylinder or tube filled with gas column. So, in a pulse tube Cryocooler the mechanical displacer is removed and then oscillating gas flow in the thin walled tube produces cooling. So, what is going to happen is I will just have a empty tube filled up with the gas, which is then subjected to oscillating pressurization and depressurization and this generates cooling and this is simple definition of a pulse to Cryocooler.

So, in effect we have got regenerative Cryocooler, when the pulses oscillating pulses enter a tube which is subjected to pressurization and depressurization the cold generator, there is something called PSM which is what I will talk about this gas tube is called pulse tube and this phenomenon is called as pulse tube action.

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Working of PT Cryocooler

The diagram shows a schematic of a pulse tube system. At the top, a compressor is shown with a downward arrow labeled w . Below it, a heat exchanger is shown with a downward arrow labeled Q_c, T_0 . This is connected to a Phase Shift Mechanism (PSM). Below the PSM, a regenerator is shown with a downward arrow labeled Q_c, T_1 . This is connected to a pulse tube. The pulse tube is shown with a downward arrow labeled Q_c, T_2 . The pulse tube is connected to a cold generator. The cold generator is shown with a downward arrow labeled Q_c, T_3 . The cold generator is connected to a regenerator. The regenerator is shown with a downward arrow labeled Q_c, T_4 . The regenerator is connected to a pulse tube. The pulse tube is shown with a downward arrow labeled Q_c, T_5 . The pulse tube is connected to a compressor. The compressor is shown with a downward arrow labeled w .

- The components of a PT system are Compressor, Heat exchangers, Regenerator, Pulse tube and Phase Shift Mechanism (**PSM**).
- The details and the requirement of the Phase Shift Mechanism (**PSM**) is explained at the later part of the lecture.
- During pressurization, the high pressure gas flows across the regenerator and into the pulse tube.

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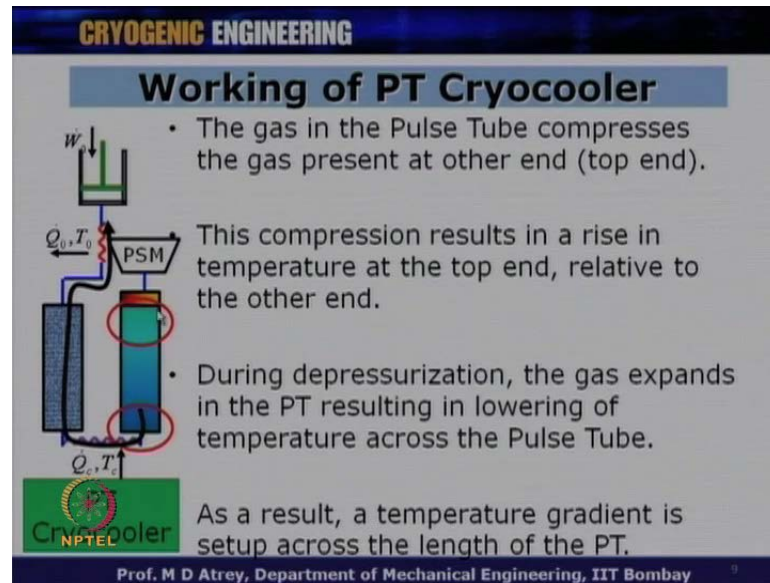
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The components of a pulse tube system therefore, are compressor heat exchanger regenerator, pulse tube and phase shift mechanism. So, earlier we had a displacer and as I said every time the displacer and the compressor have got some relationship in there motion. Now, we do not have any moving component as such here mechanical moving component here and therefore, whatever gases moving at this point I have to see that the some kind of a the gas moves in a cordons with what I want which would generate cooling. So, would ensure that the oscillation setup in this tube the move in a fix manner the move in a manner, which generates cooling and this manner in which they should oscillate in this tube is basically due to the pay shift mechanism PSM that is what is return over here.

We will talk about this PSM in detail in the this lecture and also in the next lecture. So, details and the requirement of the phase shift mechanism which is very important this is a very important and this is present only pulse tube cooler not in the Stirling in the GM type machines. So, therefore, these PSM has to be understood and this will be explained at the later part of the lecture or in the next lecture also. So, what happens how do we get

cooling? So, the piston comes down and the pressurization happens the piston goes back and the depressurization happens during pressurization now. The high pressure gas flows across the regenerator and in to the pulse tube as shown here, by this arrow the gas comes down the after cooler takes the compression the gas enters the regenerator and then the gas enters the pulse tube.

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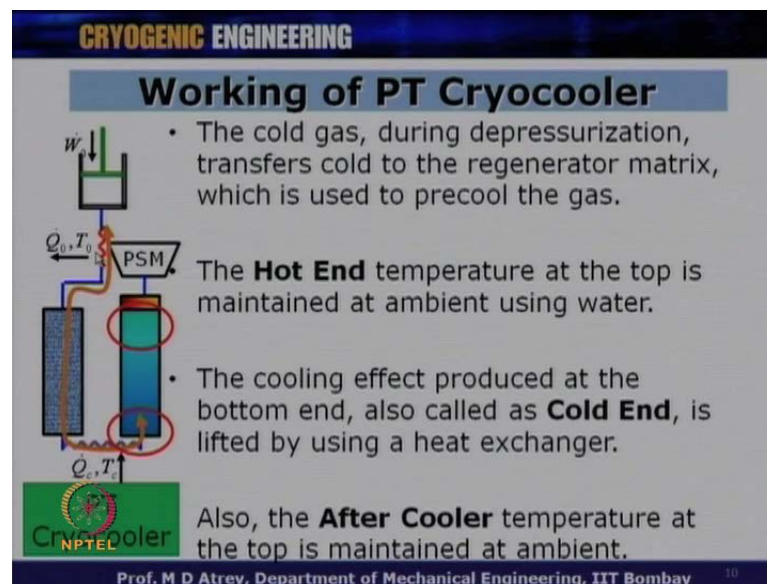


The gas in the pulse tube compresses the gas present at the other ends. So, everything was field up earlier at an average pressure or a charging pressure as soon as the gas gets charge of as soon as the gas gets compress the depending on the amplitude of the pressurization the gas enters and these gas will compress the gas present already in the tube which initially, is present at the charging pressure. So, gas in the pulse tube compresses the gas present at the other end let us called as the top end let us call as the bottom end of the pulse tube.

Now, these compression this compression of a earlier gas by the gas which is coming, because of pressurization this compression result in a rise of temperature at the top end. Infect the temperature rises at every point, but the temperature rise at the hot end at the top end will be much more as compare to the gas in this tube below this top end. So, during pressurization the temperature at this point increases, what happens after that the piston goes back and the depressurization happens. So, during depressurization the gas expands in the pulse tube resulting in lowering of temperature across the pulse tube.

So, as soon as the piston goes back the depressurization happens and therefore, the gas in the pulse tube expands, which will result in the lowering of temperature. So, temperature at the top end also reduces temperature at the bottom also reduces, but previously the top end was at higher temperature than the temperature at the bottom end. So, what does happen? First during pressurization we got increase temperature at top end as compare to the cold end and during depressurization the gas expands and temperature lows down. As a result of which a temperature gradient will gets setup over here across the length the temperature start decreasing down. So, gas pressurization happen temperature at the top end increases depressurization happen temperature decreases as a result a temperature gradient is setup across the length of the pulse tube.

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Now, the cold gas, this temperature will be less than the temperature at the top end temperature of the bottom end is going to be less than the temperature of the top end and slowly and steadily the temperature at the bottom end will start reducing. While depressurization when the gas goes back this gas gives the heat to the regenerator matrix and during pressurization this gas will get precooled, because of the heat transfer from the regenerator matrix to the pulse tubes. The cold gas during depressurization transfer cold to the regenerator matrix and this cold is again given which is use this cold is again use to precool the incoming gas, during pressurization.

The hot end temperature at the top end is maintained at a ambient temperature now, has I said earlier the temperature increases, but what I am going to do is the temperature at this top end is going to the maintain at ambient temperature by running water over here. As the result of which this will be ambient temperature and the temperature of the bottom end we slowly start coming down below the ambient and therefore, you will get cooling effect as soon as the temperature comes below ambient you will get you will generate cooling effect, which is transferred by the gas we need goes back during depressurization. So, this is how the pulse tube cooler works.

The pulse tube cooler will compress the gas and setup a temperature gradient across the length pulse tube during the depressurization the gas goes back the hot end of. The pulse tube is always maintained at ambient temperature, there by the lower end of the pulse tube temperature starts going down and how much it should go down will depend on what is the capacity of regenerator matrix material to store the heat, this is true for Stirling cooler this is true for even GM cooler. So, the basically this are all regenerator Cryocooler, but the major difference is now, there is no displacer and therefore, the oscillating gas flow sets of a temperature gradient across the length of the pulse tube.

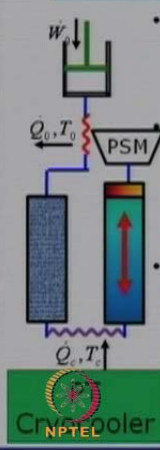
This is a very simple way of you know understanding the pulse tube action the cooling effect produced at the bottom end also called as cold end is lifted by using a heat exchanger. So, the heat exchanger here with basically, exchanges the cold generated because of pulse tube action. So, normally a pulse tube cooler will be identify the hot end of the pulse tube which is maintained at the room temperature and cold end the pulse tube were the cooling effect gets generated, this is very simple terminology the hot end and pulse tube and the cold end of the pulse tube. Also, the after cooler temperature at the top is maintained at ambient.

So, we have got a after cooler heat exchanger here which maintains the temperature gas to be ambient temperature. So, when the gas gets compressed the gas enters the regenerator at ambient temperature, because of the after cooling.

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Working of PT Cryocooler



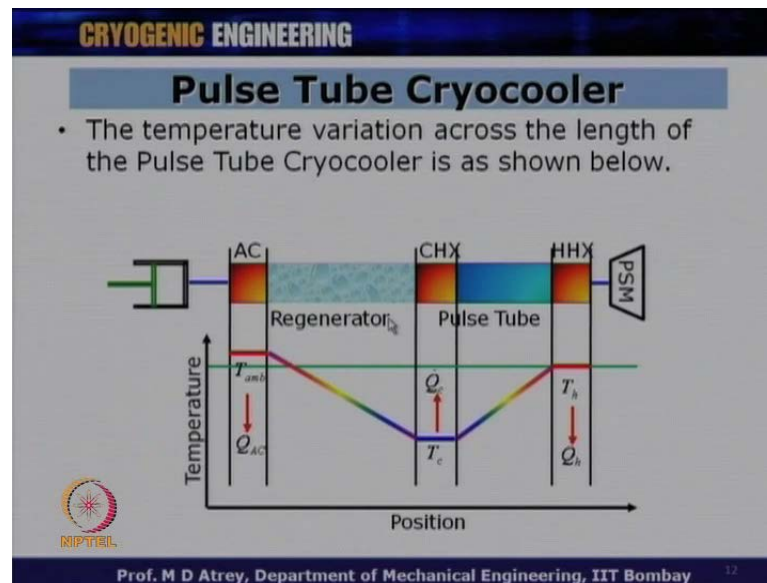
- The gas movement in the PT does not need any mechanical drive.
- Hence, the vibrations in the PT Cryocooler are less as compared to Stirling and GM Cryocoolers.
- The schematic of the PT Cryocooler, with three heat exchangers, namely, After cooler (**AC**), Cold end heat exchanger (**CHX**), Hot end heat exchanger (**HHX**) is as shown in the next slide.

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The gas movement in the pulse tube does not need any mechanical drive as we understood that is only oscillating flow and it does not require any mechanical drive. Hence, the vibration in the pulse tube Cryocooler are less as compared to Stirling and GM Cryocoolers obvious there is no moving component and therefore, the vibrations are megnititude a megnititude less then as compared to Stirling and GM Cryocooler. The schematic of the pulse tube Cryocooler with three heat exchangers namely, after cooler which is called AC, cold end heat exchanger which can be called as CHX and hot end heat exchanger HHX is as shown in the next slide. So, now, I will be formatting the entire figure so, as we understand the pulse tube action in better way vary we call AC, CHX and HHX will be shown on this figure.

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So, if I want to show now the temperature variation across the length the pulse tube Cryocooler it will be like this is a compressor there is the after cooler regenerator cold and heat exchanger pulse tube and hot and heat exchanger and the PSM. So, this is pulse tube Cryocooler shown in line and this is called as inline configuration the pulse tube, were the gas travels in straight line and comes back in a straight line during pressurization gas goes like this during depressurization gas comes back. Now, if I well to compute the temperatures across from this point up to this point from after cooler up to the end of the pulse tube up to the hot end across temperature across position graph would look like this and this is my ambient temperature.

So, during the after cooling the temperature will be gas is going to be little above the ambient temperature, where Q is given to the after cooler. In the regenerator at study state we have temperature gradient develop, because of in coming because of the precooling the gas going back and the gas coming in and the study state temperature distribution will get generated. In the CHX that is cold and heat exchanger the temperature normally will be maintained constant because there is load on a system that is constant load on a system Q_c at temperature T_c . And across the pulse to also reliable temperature gradient generated and therefore, temperature gradient would look like these in the hot and heat exchanger now, again the temperature would remain constant normally.

So, is very important diagram to understand that the cold is getting generated at this point the lowest temperature is generated in the cold and heat exchanger or the near the pulse tube end, which is closer to the regenerator. So, here we generate cooling effect so, whatever, object I want to cool should be kept attach to the cold and heat exchanger across which the cold is transferred.

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Pulse Tube Cryocooler

Advantages

- No moving parts in the expander, hence, less vibrations.
- No sealing requirement at low temperature.
- High reliability.

Disadvantages

- No reliability data due to less history.
- Orientation effects.

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Now, let us see the pulse tube Cryocooler in again a different perspective what are the advantages and disadvantages what are the uses of pulse tube Cryocooler in general? So, advantage of pulse tube cooler are obvious now, no moving part in the expander hence less vibrations, which is very important advantage pulse tube cooler as compare to other Cryocoolers, there is no sealing requirement the moment you do not have a mechanical drive there is no sealing requirement at low temperature. So, therefore, problem of rubbing seals do not arise high reliability the moment we do not have in a drive mechanism moment we do not have any moving component it will high reliability.

So, pulse tube cooler does have high reliability as compared to other expander in which one of the components is moving the disadvantages; however, are no reliability data due to less history. So, there is no failure data over period of time and therefore, we do not have any reliable data as such to come to conclusion that this pulse tube cooler will never failed. And the same time you can understand the pulse tube cooler being a gas dominant phenomena smallest angle here and there to the pulse tube cooler, because the gravity

driven also. Because it will have some convective current the hot and on the top the cold and the bottom we can have some gas heat transfer in the gas column itself and therefore, the pulse tube cooler is subjected to some kind of orientation effect.

So, as soon as the pulse tube cooler is not vertical and is inclined at some angle the cooling effect generated by the pulse tube cooler will be different so, pulse tube cooler normally is operated in a vertical mood only. So, will have orientation effect, which is also a negative characteristic of a pulse tube Cryocooler.

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Pulse Tube Cryocooler

Uses

- Cooling of infrared sensors.
- Space applications.
- Re-condensing of **LHe** and **LN₂**.
- Gas liquefaction.

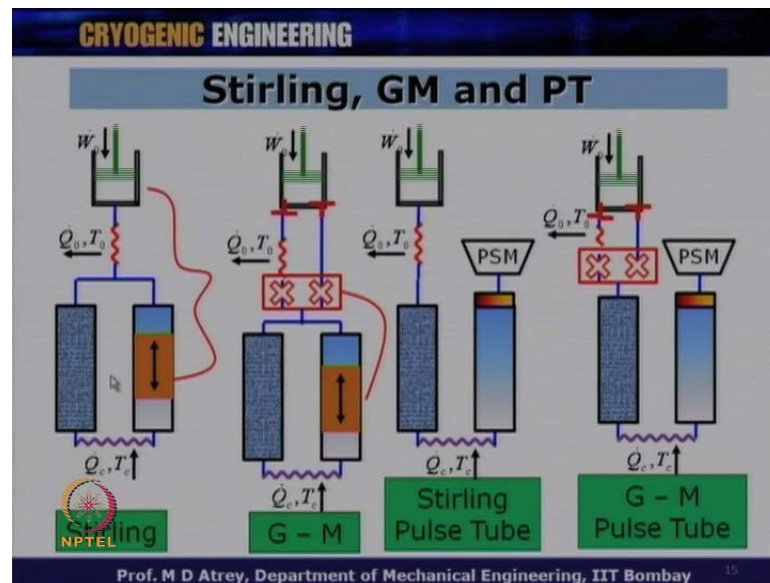
Recent Developments

- Reached below 4 K.
- Miniaturization.

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The uses of pulse tube cooler it is used to cool the infinite sensors for space application re condensing liquid helium and liquid nitrogen which is very important function it can be use for re-condensation. And therefore, it is normally club to the object to be cooled where the helium boil of happens where the nitrogen boil of happens and the pulse tube can be kept coupled to that particular equipment. And sometimes the pulse tube cooler can be use for nitrogen liquefaction or even helium liquefaction depending on it will be single stage or two stages for this respective operations. The recent developments one can obtain less than 4 Kelvin temperature using a two stage pulse tube Cryocooler and miniaturization is absolutely possible, if you go for Stirling type pulse to Cryocooler and therefore, pulse tube cooler the very important candidate in Cryocoolers.

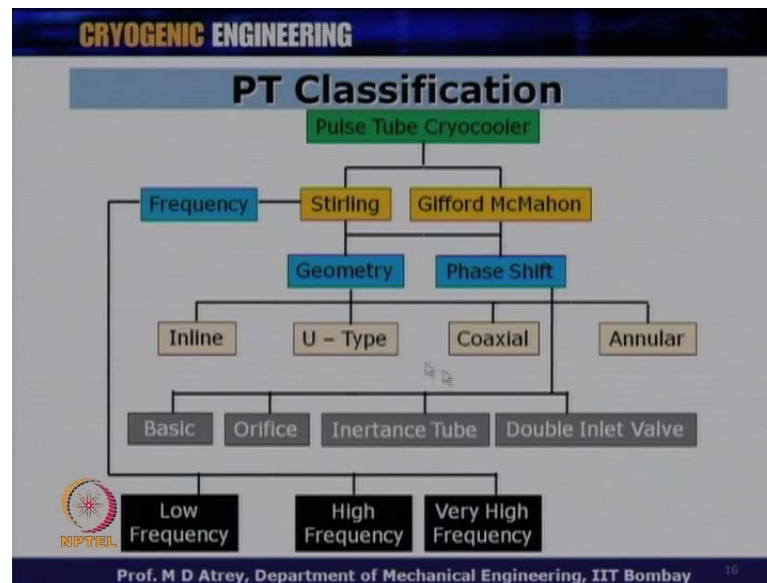
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So, if I want to see all three together the Stirling the GM and the pulse tube cooler they would look like these. So, this Stirling Cryocooler the GM type Cryocoolers these the Stirling type pulse tube cooler. Now, in pulse tube cooler we will have Stirling type pulse tube cooler where they will not be any wall and therefore, this is a very compact and miniature unit Stirling type pulse tube cooler. And we can also have a GM type pulse tube cooler the moment we have a got wall between the compress have a expander this will be called as GM type pulse tube cooler. So, very important to understand Stirling cooler as different GM cooler is a different Stirling type pulse tube cooler different and GM type pulse tube cooler is different.

So, one has to really understand whenever we say particular Cryocooler what Cryocooler we have in talking about this to be fix on a mind the moment I say GM cooler I got a wall between the compressor and expander, moment as a pulse tube I do not have a displacer moment as a Stirling I do not have a wall between the piston and the expander. So, this classification of Stirling type pulse tube GM type pulse tube cooler has to be completely understood by all of you.

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Now, let us come to very important classification under the pulse tube categories. So, pulse tube Cryocoolers can be classified based on as we know Stirling type or gifford mcmahon type. So, moment we got a wall Gifford McMahan type moment we do not have a wall is Stirling type. Under both of this type now, we can have various other classification based on the geometry of the pulse tube cooler and depending on the phase shift mechanism that we use in the pulse tube Cryocooler. So, let us come to the geometry first now, based on the geometry we have inline configuration we have got U type configuration we got a coaxial configuration and we got a angular configuration will see each of this in detail this will basically, refer to the position of a regenerator and pulse tube.


If the regenerator and the pulse tube are inline as I shown earlier in the long horizontal pulse tube Cryocooler this will be called inline configuration if they are parallel to each other this called U type configuration, if their having a same axis coaxial and angular will see that. Other classification based on the kind of phase shift being used we have not at come to understand why this phase shift mechanism is being use in pulse tube Cryocooler, but what is use for the phase shift mechanism also will decide what kind of pulse tube Cryocooler it is. So, we can have will understand this in detail later when we understand the phase shift mechanism and that time I will show this classification to you again.

So, basic pulse tube Cryocooler or orifice pulse tube Cryocooler inertance type pulse tube Cryocooler and double inlet valve pulse tube Cryocooler. So, will understand about this later in the lecture and under Stirling Cryocooler now, we know the Stirling cooler is high frequency as come to the GM cooler, but when I say high frequency how height is? And therefore, based on the kind of frequency we use in a Stirling type pulse tube cooler based on the frequency I can again identify the low frequency high frequency and very high frequency.

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PT Classification

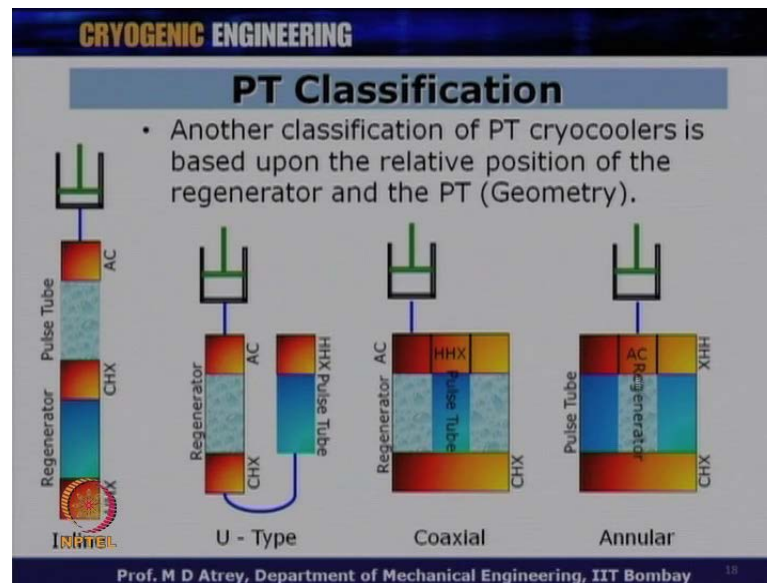


- Depending upon the usage of valves, Pulse Tube cryocooler can either be
 - Stirling type PT Cryocooler
 - GM type PT Cryocooler
- Stirling systems are high frequency machines where as, GM systems are of low frequency.
- Each of the systems has its own advantages and disadvantages.

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Now, let us see this classification in little more detail. So, as you know that depending on the usage of valve the pulse tube Cryocooler can either be Stirling type pulse tube cooler or a GM type pulse tube cooler. The moment as a GM type pulse tube cooler I got a wall over here Stirling systems are high frequency machines whereas, GM systems are low frequency machine I hope this is now, entirely clear to all of you each of the system as know earlier is at got own advantages and disadvantages.

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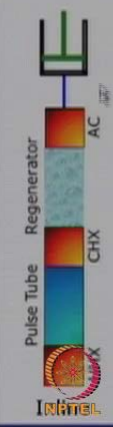
Another classification of pulse tube cooler is based on the relative position of the regenerator and the pulse tube as we had just seen. So, this is my inline configuration where the pulse tube cooler and the regenerator are inline. Second classification is U type classification, where the pulse tube cooler and the regenerator are parallel to each other and the U type connection given for the gas flows the gas comes over here gas goes to these and comes order. Let us the advantages the merits and demerits each of this type in the next slides and then a coaxial Cryocooler coaxial pulse tube Cryocooler; that means, the pulse tube Cryocooler and the regenerator have the same coaxial their basically coaxial they got a same axis around, which they are basically assembled the gas come in the angular regenerator from outside and enters in to the pulse tube like this.

So, you could a hot and heat exchanger here, while this are angular regenerator over here this is what we call as coaxial, when the regenerator is in angular position and we have got a annular pulse tube when the pulse tube now, is an annular position while the regenerator is at the center. So, these are four different types inline U type coaxial and annular now, let us see the advantages and disadvantages of each of this type.

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PT Classification



- The gas does not change the direction of flow. Hence, the pressure losses are minimum.
- This arrangement delivers best performance as compared to others.
- The cold end is at the center of the system.
- The system is less compact since it occupies huge space (length wise).

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Let us come top first the inline configuration the gas does not change the direction of the flow hence, the pressure losses are minimum. We can understand that during pressurization the gas will travel up to entire link and during depressurization the gas will go back. So, basically during pressurization the gas comes trades there is no change of direction and therefore, there is no pressure drop losses across this. So, thermodynamically sign, that this will have minimum pressure drop and therefore, this pulse tube cooler will be most efficient. So, what is the problem this will take a long length the space occupied this by this pulse tube cooler in this length dimension will be pretty high as compared to other units.

Also, this arrangement delivers the best performance as compared to others this is what we just saw thermodynamically, the cold end is at the center of the system this is an important, if I want to cool something I have to access the middle portion of this pulse tube Cryocooler which sometimes is not accessible. I have to enter from this side or this side, but it is not at the sides it is at the center of this thing and therefore, whatever object I want to cool I will not have a really good accessibility to it and therefore, sometime this is may not be acceptable for user from uses point of view. The third disadvantage of the system goes to say the system is less compact since it occupies huge space length wise.

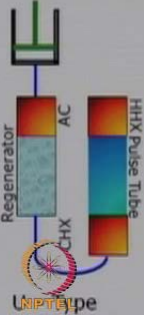
All the thermodynamically it is advantages is efficient you may have loose on the non-accessibility of the cold and heat exchanger and the huge length wise dimension required in the in line pulse tube Cryocooler.

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PT Classification

- The gas flow undergoes a 180 degree change in flow direction. Due to which the system exhibits pressure drop.
- The cold end is exposed and it is easily accessible.
- The system is more compact and it occupies less space.
- The performance is dependent upon the sharpness of the bend.



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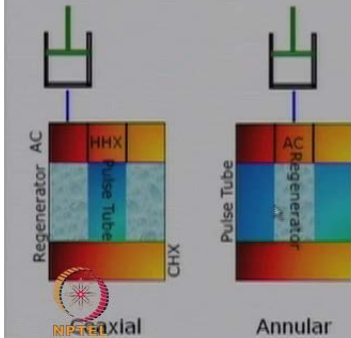
The next as you see is a U type Cryocooler now, suddenly you can see that the length wise dimension has decrease, but what you can see now? The gas flow undergoes a 180 degree change in flow direction so, gas during pressurization will come to regenerator and then take U turn and then enter the pulse tube cooler. So, it will take a 180 degree change in the flow direction due to which the system exhibits pressure drop. The cold end is now; however, exposed I can access the cold and from the bottom so whatever, I want to cool can easily be attach to the cold and heat exchanger and this is very important and it is easily accessible.

The system becomes more compact now, and it occupies less space as again is what it was doing earlier the performance is dependent upon the sharpness of the bend. So, how much pressure drop could happened depend on how sharp this spend is made basically is fabricated and therefore, if we made a very gradual bend the pressure drop in the askets will be minimized. So, this is the U type pulse tube Cryocooler where the regenerator and the pulse the cooler are parallel to each other.

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CRYOGENIC ENGINEERING

PT Classification



- The system exhibits maximum pressure drop due to change in flow direction.
- The cold end is exposed and it is easily accessible.
- The system is more compact but there is a possible heat transfer between the PT and regenerator.

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Let us come to now coaxial and annular Cryocoolers and you can see that, the system exhibits maximum pressure drop due to change in flow direction. So, here the system the gas will come like that and it will have a sharp bend direction in both this cases and therefore, the pressure drop in both the coaxial and the pulse tube Cryocooler will be very high. In this case the gas will come from the center and then go to the pulse tube again having a it has to turn in a sharp way basically is it not it, because there all assembled together.

So, in the both this cases the pressure drop process could be tremendous the cold end is exposed and it is easily accept the best part of about this two configuration is whatever, I want to cool is accessible from this bottom end. And therefore, from users point of view this is the most important or wanted system this a very compact system also, because there all you know assembled together is very compact system the cold and heat exchanger availabilities accessibilities perfect in this case. So, other than the thermodynamic component that is high pressure drop losses, we have got the both the advantages system is very compact and system is the cold end is accessible for the user. The system is more compact, but is a possibility of a heat transfer now the only disadvantage also other than the pressure drop it as their coupled up together you can have a heat transfer between the gas in the pulse tube and the gas in the regenerator. In both this case we can make this system little bit in efficient there are ways to overcome this; however, normally there can be heat transfer between the gas in the regenerator and

gas in the pulse tube and therefore, we can lose some cooling effect over here in this particular configuration.

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CRYOGENIC ENGINEERING

PT Classification

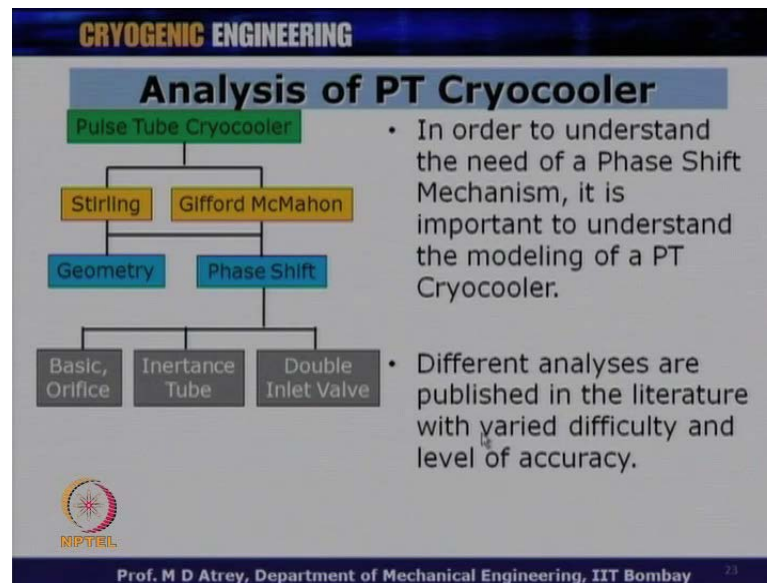
- Depending upon the operating frequency, the Stirling PT systems can be classified as listed below.
- Low Frequency (< 30 Hz)
- High Frequency (30 Hz - 80 Hz)
- Very High Frequency (> 80 Hz)

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So, this is the four configurations which we just saw. Now, pulse tube Cryocoolers can also be classified depending on the frequency under the Stirling cooler (()) which I had seen. So, depending upon the operating frequency the Stirling type pulse tube can be classified as listed below, I can call this as low frequency if the frequency less than 30 hertz be clear I am talking about Stirling type pulse tube cooler. Now, the GM type pulse tube cooler will operate at 1 to 2 hertz only while Stirling type pulse tube cooler will be called a low frequency a Stirling type Cryocooler, if is frequency less than 30 hertz high frequency if it operate between 32 hertz 80 hertz and very high frequency when it is frequency is going to be more than 80 hertz. So, we can call low frequency machine high frequency or very high frequency depending on the frequency at with Stirling type pulse tube cooler operates.

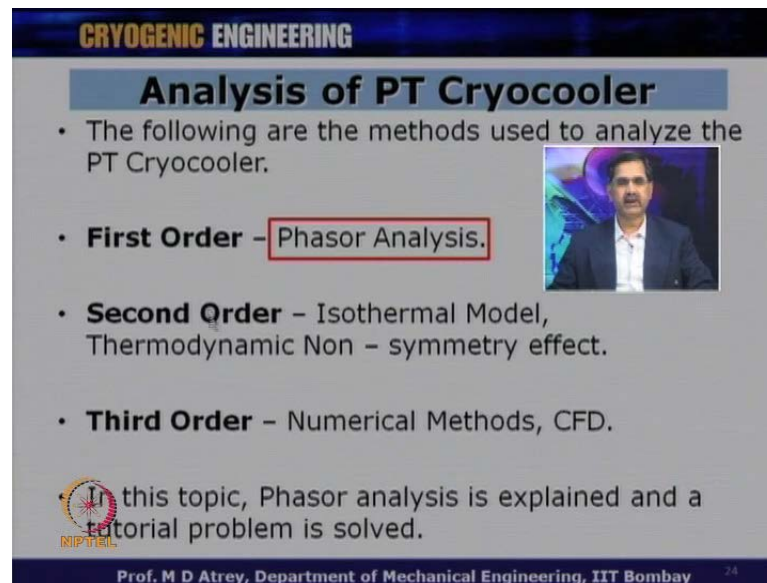
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So, after understanding this pulse tube cooler classification let us see now, the analysis of the pulse tube Cryocooler. So, just saw that the pulse tube cooler we got a Stirling type and Gifford McMahon type and we just saw the classification based on the geometry now, let us understand this phase shift mechanism. We have shown here that based on the phase shift mechanism we can further classify the pulse tube cooler as Basic type pulse tube cooler orifice type pulse tube cooler Inertance tube pulse tube cooler and Double inlet valve type pulse tube cooler. So, basically these are the nothing but the phase shift mechanism, which are important to bring cooling effect to be get more and more cooling effect in the pulse tube Cryocooler.

So, phase shift mechanism very important part of the pulse tube Cryocooler and also it is very important to understand what is the requirement of this phase shift mechanism and how do the induce cooling in case of pulse tube Cryocooler. So, in order to understand the need of phase shift mechanism it is important to understand the modeling of a pulse tube Cryocooler. So, modeling can be very you know modeling as the simulation business can be very difficult in case of pulse tube Cryocooler very simple to a very a complicated mathematical modeling, that can be done for pulse tube Cryocooler it is a very important research topic. But in this particular lecture I would to show a simple modeling in order to understand what is this Basic or orifice Inertance and Double inlet requirement and how do they create cooling effect in case of pulse tube Cryocooler.

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CRYOGENIC ENGINEERING

Analysis of PT Cryocooler

- The following are the methods used to analyze the PT Cryocooler.
- **First Order** – Phasor Analysis.
- **Second Order** – Isothermal Model, Thermodynamic Non – symmetry effect.
- **Third Order** – Numerical Methods, CFD.

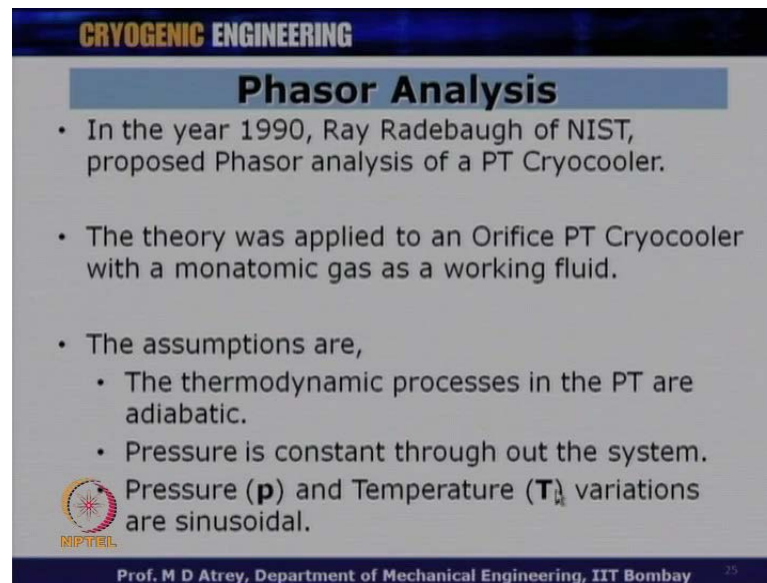
• In this topic, Phasor analysis is explained and a tutorial problem is solved.

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So, with a very simple modeling exercise it is simple analysis we can understand that. Different analyses are published in the literature with varied difficulty and level of accuracy. And this analysis could be classified we had use this earlier also first Stirling type Cryocoolers, the following are the methods used to analyze the pulse tube Cryocooler we have got a first order analysis in this pulse tube cooler phasor analysis. We have got a second order analysis where we can have isothermal model, thermodynamic non symmetry model and thing like that and we can have third order analysis where we can numerical methods computational flow dynamics and etcetera.

Depending on the kind of difficulties or computer time requirement and more and more realistic analysis as you (()) up to top to bottom it becomes very complicated it requires lot of time to solve these equations. While in order to understand the phase shift mechanism the first analysis very important and therefore, in this particular lecture, I am going to take Phasor analysis is going to be explained and I will solve is small tutorial problem also may be in the next lecture to understand this Phasor analysis. So, this Phasor analysis make is understand what is the requirement of phase shift mechanism, while in order to calculate the cooling effect and to get dimensions more complicated models like second order model third order model could be utilized.


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CRYOGENIC ENGINEERING

Phasor Analysis

- In the year 1990, Ray Radebaugh of NIST, proposed Phasor analysis of a PT Cryocooler.
- The theory was applied to an Orifice PT Cryocooler with a monatomic gas as a working fluid.
- The assumptions are,
 - The thermodynamic processes in the PT are adiabatic.
 - Pressure is constant through out the system.

 Pressure (**p**) and Temperature (**T**) variations are sinusoidal.

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So, let us come to Phasor analysis in the year 1990, Ray Radebaugh from NIST, proposed Phasor analysis of a pulse tube Cryocooler. The theory was applied to simple orifice pulse tube Cryocooler with a monatomic gas like helium gas use as a working fluid. So, let us consider orifice type pulse tube Cryocooler and use helium as a working fluid. The simple assumption made are, the thermodynamic processes in the pulse tube are adiabatic, in the pulse tube not in the heat exchanger only in the pulse tube that the processes are adiabatic; that means, there is no loss of heat energy from the pulse tube cooler which is realistic assumption.

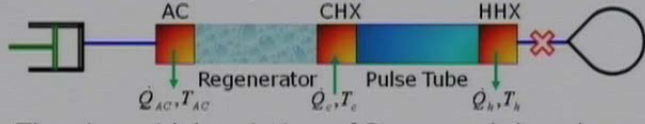
At the same time the pressure is constant throughout the system; that means, there is no pressure drop in the system both this assumptions are very realistic. The pressure P and temperature T at any location where is sinusoidal the variations also sinusoidal. So, because of oscillating pressure flows the temperature also at every point will vary and let us assume that these variations are sinusoidal.

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CRYOGENIC ENGINEERING

Phasor Analysis

- Consider an Orifice Pulse Tube Cryocooler (OPTC) as shown in the figure.



- The sinusoidal variation of Pressure (**p**) and Temperature (**T**) are as given below.

$$p = p_0 + p_1 \cos(\omega t) \quad T = T_0 + T_1 \cos(\omega t)$$

- In the above equation, **p₀** and **T₀** are the average pressure and ambient temperature respectively. **p₁** and **T₁** are the variations respectively.

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So, let us see orifice type pulse tube cooler what does it mean? Between the hot end and the reserve wire this is the reserve wire and this hot end of the pulse tube cooler as you can see this is a inline kind of pulse tube Cryocooler the compressor after cooler regenerator cold and heat exchanger pulse tube cooler and hot end. And this hot end is attach to reserve wire is through is small orifice and therefore, this is called as orifice pulse tube Cryocooler. Now, what is requirement of this orifice why it is kept and all that we understand in this analysis? The sinusoidal variation of pressure and temperature are as given below as we have just given assumption that the pressure and temperature variations are sinusoidal.

So, let us assume pressure **P** is equal to **P₀** plus **P₁ cos omega T**, while temperature **T** is equal to **T₀** plus **T₁ cos omega T** this are the sinusoidal variation of pressure and temperature. In the above equation, **P₀** and **T₀** are the average pressure and ambient temperature respectively there average value basically around which the variations happen where we can cos **T₀** as ambient temperature and **T₀** as a pressure variation. While we can assume the **P₁** and **T₁** are the variations basically there are amplitudes **P₁** and **T₁** are the amplitudes of the pressure and temperature variation.

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CRYOGENIC ENGINEERING

Phasor Analysis

The diagram shows a pulse tube cryocooler system. It starts with an AC (Alternating Current) source on the left, followed by a Regenerator, a CHX (Cold Heat Exchanger), a Pulse Tube, and an HHX (Hot Heat Exchanger). Mass flow rates are indicated by arrows: \dot{m}_c at the cold end, \dot{m}_{pt} in the pulse tube, \dot{m}_h at the hot end, and \dot{m}_o at an orifice. Heat transfer rates and temperatures are labeled as Q_{AC}, T_{AC} , Q_c, T_c , and Q_h, T_h .

- Let \dot{m}_c , \dot{m}_{pt} , \dot{m}_h and \dot{m}_o be the mass flow rates in the cold end, PT, hot end and orifice respectively.
- Using the law of conservation of mass, we have

$$\dot{m}_{pt} = \dot{m}_h - \dot{m}_c$$
- In an OPTC, the following holds true.

$$\dot{m}_h = \dot{m}_o$$

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Now, let us see the let us come the mass flow rate let \dot{m}_c and \dot{m}_{pt} , \dot{m}_h and \dot{m}_o be the mass flow rates in the cold end \dot{m}_c as cold end heat exchanger here at this point. So, as we can see \dot{m}_c as cold and heat exchanger mass flow rate \dot{m}_{pt} as the mass flow rate let say center of the pulse tube \dot{m}_h is the mass flow rate in the hot end heat exchanger while \dot{m}_o is the mass flow rate to the orifice at this point. Using the law of conservation of mass, we have \dot{m}_{pt} is equal to \dot{m}_h , if I want to compute the \dot{m}_{pt} we can assume that by conservation of mass \dot{m}_{pt} is equal to whatever, is living minus whatever is incoming.

So, \dot{m}_{pt} is equal to \dot{m}_h minus \dot{m}_c in the OPTC, that means, in the orifice pulse to Cryocooler the following holds true where we can say that \dot{m}_h is equal to \dot{m}_o ; that means, whatever is the mass flow rate living the hot and heat exchanger is the mass flow rate the \dot{m}_o , which is very again valid because is orifice is very close to hot and heat exchanger. And also is perfect assumption that \dot{m}_h which is living at this point is the mass flow rate through the orifice, orifice is the very small opening it opens into reserve wire and the pressuring the reserve wire is going to be the average pressure is going to be average pressure in the system, this is not subjected to oscillating pressure rearranging the above mass equation.

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CRYOGENIC ENGINEERING

Phasor Analysis

The diagram shows a pulse tube refrigerator with three main components: an AC (Acoustic Compressor), a CHX (Cold Heat Exchanger), and an HHX (Hot Heat Exchanger). Mass flow rates are indicated as \dot{m}_c (cold), \dot{m}_{pt} (pulse tube), \dot{m}_h (hot), and \dot{m}_o (output). Heat transfer parameters are \dot{Q}_{AC}, T_{AC} for the AC, \dot{Q}_c, T_c for the CHX, and \dot{Q}_h, T_h for the HHX.

- Rearranging the above mass equation, in terms of volume, we have

$$dV'_{pt} = dV'_h - dV'_c$$
- Upon multiplying with pressure **p**, we get

$$pdV'_{pt} = pdV'_h - pdV'_c$$

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We just computed \dot{m}_{pt} is equal to \dot{m}_h minus \dot{m}_c and if I want to replace the same thing in terms of volume we have got the dV_{pt} is equal to dV_h minus dV_c . That means, volumetric variations a small variations in volume in the pulse tube is equal to variation in the hot and heat exchanger minus the variation in the cold and heat exchanger this can be obtained by multiplying the earlier mass equation by then $c t$ and area that is why we can calculate this. Upon multiplying this equation by pressure p if you multiply entire thing by pressure p we get pdV_{pt} is equal to pdV_h minus pdV_c .

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CRYOGENIC ENGINEERING

Phasor Analysis

- Consider an Ideal Gas equation.

$$pV = mRT$$
- Differentiating the Ideal Gas Equation, we have

$$\frac{pdV}{dt} = RT \left(\frac{dm}{dt} \right) \quad pdV = RT (dm)$$

$$pdV'_{pt} = pdV'_h - pdV'_c$$
- Combining the above two equations, we get

$$pdV'_{pt} = RT_h dm_h - RT_c dm_c$$

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Now, let us come to the ideal gas equation the earlier equation can be use later, we know pV is equal to $m RT$ or ideal gas differentiating the ideal gas equation we get $p dV$ by $d t$ is equal to RT and $d m$ by $d t$ RT is constant at any temperature. So, we can get $p dV$ by $d t$ is equal to RT in to $d m$ by $d t$ where, if I cancel the $d t$ what we get is $p dV$ is equal to RT in to $d m$ and we have already describe we have already got an expression for $p dV$ earlier. So, if I want to have expression for $p dV$ $p t$ which is what we are calculated $p dV$ $p t$ is equal to $p dV$ h minus $p dV$ c this is what we had calculated and I want to replace these by these now. So, I got a $p dV$ h is equal to RT in to $d m$ h now here. So, I can replace this $p dV$ h by this term I can have $p dV$ c by RT c in to $d m$ c .

So, combinem, if I put the values from here into this equation I will get replacing these by these replacing $p dV$ c by again this I will get $p dV$ $p t$ is equal to RT h in to $d m$ h , $p dV$ h is equal to RT h in to $d m$ h minus RT c in to $d m$ c . So, basically now I am replacing this $p dV$ by RT so, and I will have a mass term introduce over here.

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CRYOGENIC ENGINEERING

Phasor Analysis

- The Pressure (**p**) and Temperature (**T**) variations are sinusoidal. They can be written as

$$p = p_0 + p_1 \cos(\omega t) \quad T = T_0 + T_1 \cos(\omega t)$$
- At any cross section of the PT, let there be a phase α between pressure and temperature variations.

$$p = p_0 + p_1 \cos(\omega t) \quad T = T_0 + T_1 \cos(\omega t + \alpha)$$
- For an adiabatic process, we have

$$\frac{T_0 + T_1 \cos(\omega t + \alpha)}{T_0} = \left(\frac{p_0 + p_1 \cos(\omega t)}{p_0} \right)^{\frac{2}{5}}$$

$$\frac{T_1}{T_0} = \frac{2}{5} \left(\frac{p_1}{p_0} \right)$$

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Now, let us come to the pressure variation and temperature variations and because there is sinusoidal they could be return as P is equal to $P_0 + P_1 \cos \omega t$, T is equal to $T_0 + T_1 \cos \omega t$. At any cross section in the pulse tube let there be a phase α between the pressure and temperature variation they need not be in face and therefore, we can introduce the face difference between this pressure and temperature variations in the pulse tube. So, P is equal to $P_0 + P_1 \cos \omega t$ and T is equal to $T_0 + T_1$

$\cos(\omega T + \alpha)$. So, we can just introduce phase difference between the pressure and temperature variation at any cross section in the pulse tube, at any cross section in the pulse tube will have this variation and we assume that the entire pulse tube cooler works in an adiabatic way there is an adiabatic.

So, for an adiabatic process we have T/T_0 is equal to $(P/P_0)^{1/\gamma}$, this is the temperature and the pressure relationship connected together by the γ of particular gas, if I am talking about helium as work in fluid the γ for a helium is 1.67. So, putting that value of γ in this equation and putting the expression for P and temperature as shown over here, we can see that T/T_0 expressed using the keeping the value of T in this expression so, T/T_0 is equal to putting the value of P/P_0 to the power $\gamma - 1$ by γ is equal to $2/5$.

So, once we put γ is equal to 1.67 we will get the value of $\gamma - 1$ by γ has to be 2.5, expanding this further by binomial theorem and neglecting the second order terms what we get is therefore, T_1/T_0 is equal to $(2/5)^{1/2.5} P_1/P_0$. So, you can see there if I expand T_1/T_0 get canceled T_1/T_0 P_1/P_0 get canceled P_1/P_0 will come and then I expand further and neglect the further terms, I will get a relationship between the temperature T_1/T_0 this is my amplitude to average value relationship related to the pressure amplitude the average pressure relationship. This we will use again later connecting temperatures and pressure variation in the pulse tube cooler.

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CRYOGENIC ENGINEERING

Phasor Analysis

- For a Pulse Tube, an adiabatic law between the pressure and the volume is as given below.

$$pV_{pt}^\gamma = k$$
- Differentiating, we have

$$V_{pt}dp + \gamma p dV_{pt} = 0$$
- The mass equation is

$$p dV_{pt} = RT_h dm_h - RT_c dm_c$$

$$V_{pt} dp = -\gamma (RT_h dm_h - RT_c dm_c)$$

$$\frac{V_{pt} dp}{\gamma} = -RT_c dm_c + RT_h dm_h$$

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So, pulse tube cooler an adiabatic law between the pressure and the volume is given below, we know that pV to the power γ is equal to constant this is what we say the adiabatic law. So, let us apply this law for the gas in the pulse tube, because we know that the gas in the pulse tube behave you know adiabatic manner. So, let us say $p dV$ to the power γ is equal to constant differentiating now, differentiating this we have v_{pt} into dp plus γ types $p dV_{pt}$ is equal to 0. Just differentiate this and rearrange the terms what we get here is $v_{pt} dp$ plus γ type $p dV_{pt}$ is equal to 0 differentiating constant what you get is 0. Now, what we have derived earlier as mass equation we have already got it here.

So, $p dV_{pt}$ is equal to $RT_h dm_h - RT_c dm_c$ which we have derived earlier now, what I would like to do is replace this term $p dV_{pt}$ by this. So, putting this term over here I will replace this by this so, what I get is $v_{pt} dp$ is equal to and putting this on the right side now, minus γ taking γ common here in to bracket $RT_h dm_h - RT_c dm_c$. Please understand this steps we may have to take more time to understand I am just goings step by step, I have try to give as many steps as possible that you understand this derivations.

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CRYOGENIC ENGINEERING

Phasor Analysis

- Rearranging the above equation, we have

$$-\frac{V_{pt} dp}{\gamma} = -RT_c dm_c + RT_h dm_h$$
$$dm_c = \frac{V_{pt}}{\gamma RT_c} dp + \frac{T_h}{T_c} dm_h$$

- Dividing the above equation with an infinitesimal time step **dt**, we have

$$\frac{dm_c}{dt} = \frac{V_{pt}}{\gamma RT_c} \left(\frac{dp}{dt} \right) + \frac{T_h}{T_c} \left(\frac{dm_h}{dt} \right)$$


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So, here $v_{pt} dp$ by γ if, I do I just put γ on this side I got this expression further also, taking the minus side and putting this on this said you got a dm_c and you got a dm_h plus term over here. Rearranging the above equation, what I get now $-\frac{v_{pt} dp}{\gamma}$ is equal to $-RT_c dm_c + RT_h dm_h$. So, what I get is dm_c basically now I am finding the relationship between the mass flow rate at the cold end and the mass flow rate at the hot end it is my entire objective, relating entire thing to the pressure variation in a system.

So, dm_c now is equal to $\frac{v_{pt}}{\gamma}$ upon γ taking RT_c in the denominator if I put RT_c over here R and R will get cancel transferring on this said I get plus term $\frac{T_h}{T_c}$ into dm_h . Dividing entire theme by dt , now, so that I will get $\frac{dm_c}{dt}$ by dt $\frac{dm_h}{dt}$ by dt by infinitesimal time step dt we have $\frac{dm_c}{dt}$ which nothing but the mass flow rate at the cold end heat exchanger or the cold end of the pulse tube is equal to $\frac{v_{pt}}{\gamma RT_c} \frac{dp}{dt}$ this is my pressure variation with time $\frac{T_h}{T_c} \frac{dm_h}{dt}$ this my mass flow rate variation at the hot end the pulse tube. So, now, I am relating the mass flow rate at the cold end the pulse tube to, the pressure variation and the mass flow variation at the hot end to the pulse tube this is a very important derivation now, basically relating the mass flow rate at the cold end mass flow rate at the hot end and the pressure variation.

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CRYOGENIC ENGINEERING

Phasor Analysis

- The mass flow rate through the orifice is directly proportional to the pressure drop.

$$\dot{m}_h \propto \Delta p$$

$$\dot{m}_h \propto p_0 + p_1 \cos(\omega t) - p_0$$

$$\dot{m}_h = C_1 p_1 \cos(\omega t)$$

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Now, let us understand this in this inline pulse tube, the mass flow rate at the orifice is directly proportional to the mass flow rate to the pressure drop. Now, I am coming to \dot{m}_h or \dot{m}_o we have said that let us find the relationship of \dot{m}_h \dot{m}_h is equal to \dot{m}_o as far as the orifice type pulse tube cooler is considered. So, what is \dot{m}_h proportional to? The \dot{m}_h proportional to the pressure drop across this orifice correct, because the mass flow rate orifice will depend on what is the pressure drop at this point and this point. Now, the pressuring the reserve wire is always average pressure which is p_0 while the pressure on this said of the orifice is always oscillating flow which is p_0 plus $p_1 \cos \omega t$ which is what we have earlier seen.

So, \dot{m}_h is always proportional to Δp across the orifice. So, what is \dot{m}_h proportional to $p_0 + p_1 \cos \omega t$ which is the pressure variation on the left said of the orifice while, on the right said of the orifice is just the p_0 . So, if I want to calculate to Δp it is $p_0 + p_1 \cos \omega t$ that is the pressure on the left side orifice and minus p_0 which is pressure on the right side the orifice. So, if you see this p_0 and p_0 will get cancelled and \dot{m}_h directly proportional to $p_1 \cos \omega t$. So, p_0 p_0 will get cancelled and I will remove this constant of this proportional at this signed and introduce a constant of proportionality therefore, I can right \dot{m}_h is equal to C_1 which is constant $C_1 p_1 \cos \omega t$ this a very important.

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CRYOGENIC ENGINEERING


Phasor Analysis

- Combining the following equations, we have

$$p = p_0 + p_1 \cos(\omega t) \quad \dot{m}_c = C_1 p_1 \cos(\omega t)$$

$$\frac{dm_c}{dt} = \frac{V_{pl}}{\gamma RT_c} \left(\frac{dp}{dt} \right) + \frac{T_h}{T_c} \left(\frac{dm_h}{dt} \right)$$

$$\dot{m}_c = -\frac{\omega p_1 V_{pl}}{\gamma RT_c} \sin(\omega t) + \frac{T_h}{T_c} C_1 p_1 \cos(\omega t)$$

$$\dot{m}_c = \frac{\omega p_1 V_{pl}}{\gamma RT_c} \cos\left(\omega t + \frac{\pi}{2}\right) + \frac{T_h}{T_c} C_1 p_1 \cos(\omega t)$$


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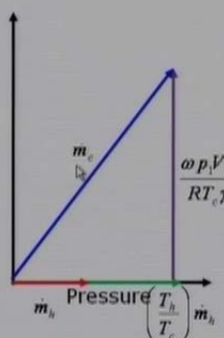
Combining the following equations we have p is equal to $p_0 + p_1 \cos \omega t$ we got \dot{m}_h is equal to $C_1 p_1 \cos \omega t$ now. And now we have got a relationship between what we have derived earlier which is basically, mass flow rate at the cold end mass flow rate at hot end and dp by dt , can I get dp by dt from this term I will get dp by dt from this term I will get \dot{m}_h is nothing but dm_h by dt . So, I can now put this term over here I can get dp by dt and now, I can get relationship between of this parameters in a better way yes. So, get a dp by dt from this get \dot{m}_h as a dm_h by dt over here and if I right this things what I get is \dot{m}_c is equal to the differential of $\cos \omega t$ will be minus $\sin \omega t$. So, will get minus term $\sin \omega t$ and I will put $C_1 p_1 \cos \omega t$ as this point over here.

So, I got \dot{m}_c as a some of two components one component which is this $\cos \omega t$ the other component is $\sin \omega t$, if I want to write this $\sin \omega t$ instead of that if I write in terms of \cos than, I will get minus $\sin \omega t$ as $\cos \omega t + \pi/2$ this is clear from the trigonometry. So, \dot{m}_c now can be return as one \cos term $\cos \omega t$ and other term is $\cos \omega t + \pi/2$; that means, it is sure that \dot{m}_c is a vectorial addition of one term which is in one direction. And the other term is going to be at $\pi/2$ phase difference with this term is it not it so, \dot{m}_c is equal to this plus this.

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CRYOGENIC ENGINEERING

Phasor Analysis



$$\dot{m}_c = \frac{\omega p_1 V_{pl}}{\gamma R T_c} \cos\left(\omega t + \frac{\pi}{2}\right) + \frac{T_h}{T_c} C_1 p_1 \cos(\omega t)$$

$$\dot{m}_c = \frac{\omega p_1 V_{pl}}{\gamma R T_c} \cos\left(\omega t + \frac{\pi}{2}\right) + \frac{T_h}{T_c} \dot{m}_h$$

From the above equation, it is clear that vector \mathbf{m}_c is a sum of two vectors which are at 90° to each other.

- Plotting these two vectors, we have the figure as shown.

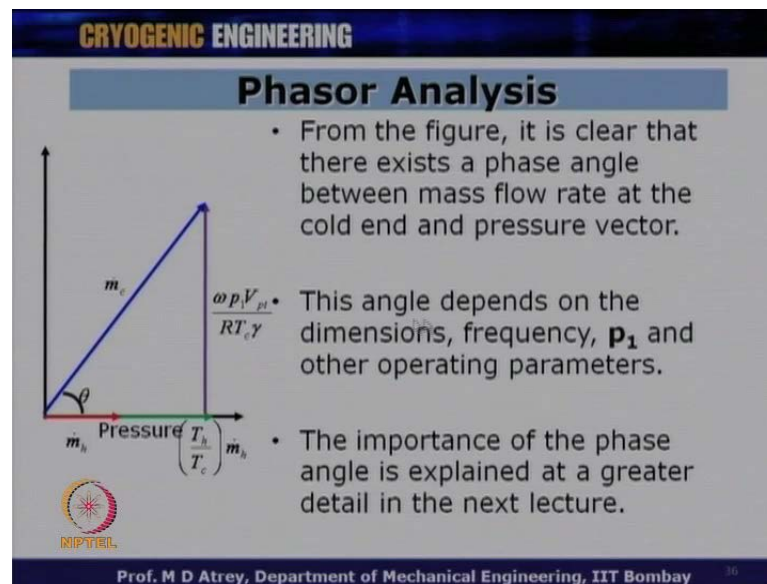
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So, I can right now \dot{m}_c as we just wrote is this term and what is this $C_1 p_1 \cos \omega t$ is nothing but \dot{m}_h and as we know that \dot{m}_h is directly proportional to the pressure \dot{m}_h is always going to be in line with pressure, in other way \dot{m}_h is always in phase with the pressure. The mass flow rate at the hot end is always in phase with pressure axis, if I want to plot it will be always pressure axis while the second term or this term is going to be at 90 degree angle to the pressure axis. So, pressure axis and \dot{m}_h are in the same axis while the second term is going to be at a phase angle of $\pi/2$ or 90 degree to this. So, now, in order to calculate \dot{m}_c I got two terms, one term which is always in the direction of phase a pressure which is \dot{m}_h and second term is going to be 90 degree angle to that particular term.

So, from the above equation it is clear that vector, if I want to have a vectorial addition of this to get to calculate \dot{m}_c vector \dot{m}_c is a sum of two vectors which are at 90 degrees two each other is it clear. So, this term is going to be in line with pressure axis and this term is going to be 90 degree to this term. So, if I want to have if suppose I pressure is in this action I know that, plotting these two vectors we have figure this is my \dot{m}_h which is in line with pressure, but what is amplitude of this T_h/T_c into p_1 this is my axis over here. So, I will get T_h/T_c into \dot{m}_h this is my pressure axis on which \dot{m}_h matches where in phase, the amplitude of this vector is T_h/T_c into \dot{m}_h .

So, T_h being larger than T_c this axis will be T_h by T_c $m \cdot h$ larger than the $m \cdot h$ to which now, I got a vertical axis vertical Phasor which is having a magnitude of this is having a amplitude of this and which is going to the 90 degree angle. So, if I want to calculate from here my $m \cdot c$ is equal to these vector plus vertical component which is at 90 degree, which has got a magnitude of ωp_1 in to v_{pt} divided by $RT_c \gamma$ and if I add them together vectorally I will get the value of $m \cdot c$.

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What does this mean? This means that from this figure it is clear that there exists a phase angle between the mass flow rate at the cold end which is my $m \cdot c$ and the pressure vector. So, we can see in pulse tube cooler the mass flow rate at the cold end and the pressure axis they are not in phase, but they have got a some theta angle between them. While $m \cdot h$ has got a same phase with the pressure at the hot end the pressure and the $m \cdot h$ R is same phase basically they are parallel to each other while $m \cdot c$ is making an angle of theta. This is very important and in the next lecture will talk more about the value of this theta, but what does this theta determine this theta is a phase difference between the mass flow rate at the cold end and the pressure.

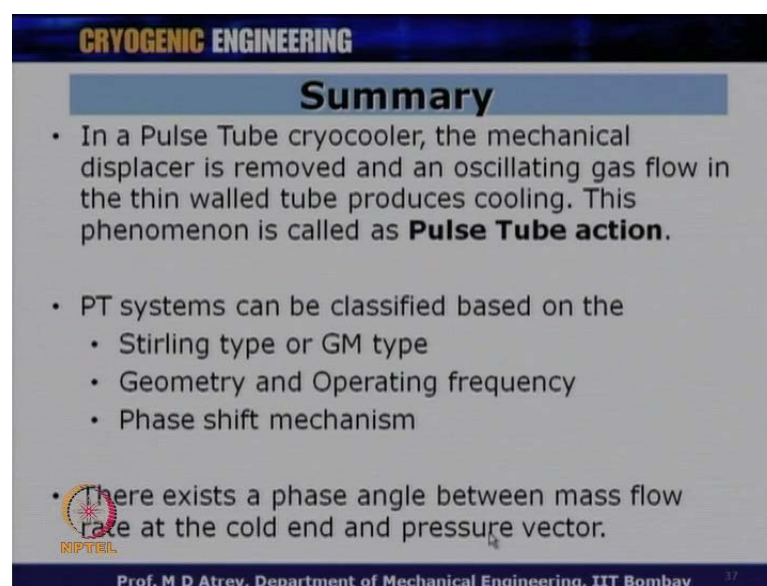
This angle theta depends on what parameter it will depend basically on this is it not it will depend on this quantity this quantity small we can have, this vertical vector could be of very small like and therefore, theta in that case will be very small. These parameters are very large in that case $m \cdot c$ can be very very large and therefore, the theta also can

be very large quantity. So, this theta angle depends on $\omega p_1 V_{pt}$ what is ω frequency what is p_1 is a amplitude of pressure what is V_{pt} is the volume of the pulse tube. So, if got a very large pulse tube we can have very large theta, if I got a higher frequency we can have a very high theta if the amplitude is the pressure variation is very large we can have very high theta similarly, if we got a low temperatures very very low temperature this theta could be larger.

So, the theta basically the angle theta depends on the dimensions that mean V_{pt} volume of pulse tube cooler the frequency which is ωp_1 which is amplitude of pressure variations and other operating parameters like temperature etcetera. This theta is very important component which will be basically determined by all this parameters. The importance of the phase angle is explained at a greater detail in this next lecture, because I cannot explain everything in this today's lecture. And all this phase shift mechanism what we have talked about will depend on through this theta is minimized, we will understand this we want for would pulse tube action theta to be as minimum as possible and therefore, all this phase shift mechanism would ensure that theta is getting reduced.

So, that the pulse tube cooling action is better and better and therefore, all this orifice double inlet inheritance to are employed to basically change this angle of theta for over benefit this will what will be explained in the next lecture in the Phasor analysis continued during that lecture.

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Summary

- In a Pulse Tube cryocooler, the mechanical displacer is removed and an oscillating gas flow in the thin walled tube produces cooling. This phenomenon is called as **Pulse Tube action**.
- PT systems can be classified based on the
 - Stirling type or GM type
 - Geometry and Operating frequency
 - Phase shift mechanism
- There exists a phase angle between mass flow rate at the cold end and pressure vector.

RIPITEL

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So, I will stop here now, the summary of this lecture therefore, is in a pulse tube Cryocooler the mechanical displacer is removed and an oscillating gas flow in the thin walled tube produces cooling this phenomenon is called as pulse tube action. The pulse tube systems can be classified based on the Stirling type or GM type; that means, presence of wall or low wall high frequency or low frequency. The geometry and operating frequency geometry will decide with the inline U type coaxial angular etcetera.

The operating frequency in the Stirling also will be determined whether the low frequency, high frequency, very high frequency etcetera. Also, they can be classified based on the kind phase shift mechanism employed in the pulse tube cooler, and they can be call as basic pulse tube cooler orifice pulse tube cooler double inlet or inertuf pulse tube cooler. This part we have not seen, and this is what we are basically understanding now, what we also understand there exist phase angle between the mass flow rate at the cold end and the pressure vector. And this is what we derived in (()) details to understand why this theta appears, between $\dot{m} \cdot c$ and the pressure vector, because of what parameter this theta appears; thank you very much.