Cryogenic Engineering
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Lecture No. # 27
Cryocoolers Ideal Stirling Cycle

So, welcome to the 27 th lecture of cryogenic engineering under the NPTEL banner. During the last lecture, we have initiated the study on Cryocoolers; and the first introductory lecture on Cryocooler was given; and if you see the highlights of earlier lecture, I mean only the introductory remarks of Cryocooler, we can just go through this following points.

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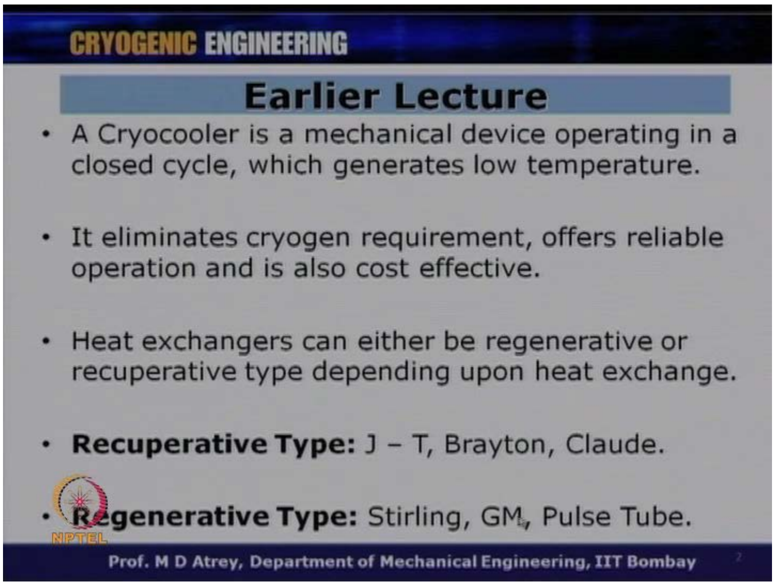

A Cryocooler is a mechanical device operating in a closed cycle manner, which generates low temperature; and I compared this to a domestic refrigerator, wherein you get low temperature you know, closed cycle manner. What do they do? It basically, eliminates cryogen requirement, offers reliable operation, and it is cost effective. There are different heat exchangers, and this heat exchangers can either be regenerative type or of recuperative type, depending on the type of heat exchange we want to have; and depending on this, we had classified the Cryocoolers.

If we have recuperative heat exchanger, then we can have Joule-Thomson Cryocooler, Brayton Cryocooler or Claude Cryocooler; and if you got regenerative type of heat exchanger, then we have got Stirling Cryocooler, GM or Gifford McMahon Cryocooler and Pulse Tube Cryocooler; and this was the broad classification of a close cycle Cryocooler, under heat exchange type, under the heat exchanger that is used in the Cryocoolers. Today in this lecture we will talk about Stirling Cryocoolers.

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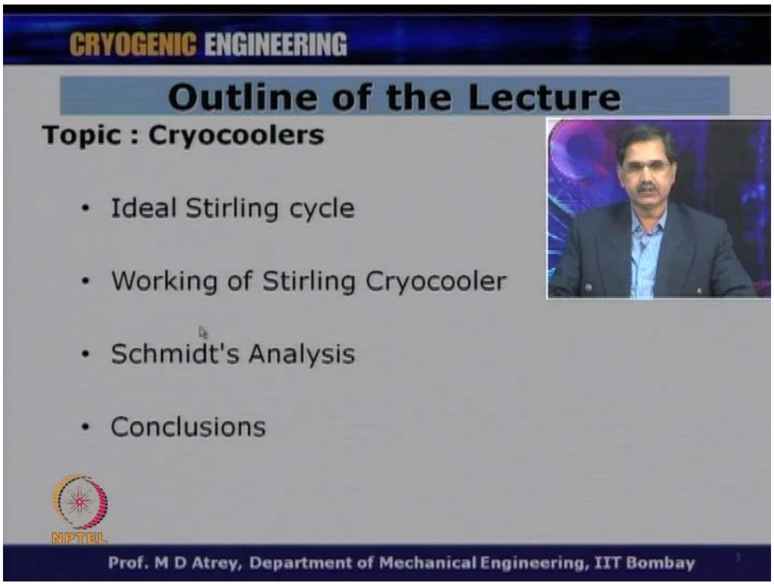

So, today's topic, we will talk about Ideal Stirling cycle. How is this cycle executed, how does this Stirling Cryocoolers work; that means, how is this Stirling cycle is executed in Stirling Cryocooler. And they will have a simple Schmidt's analysis, if you want to design a simple Stirling Cryocooler broadly; I mean a first guess of design basically not very accurate, but the first guess. And finally, I will have some conclusions on what we have done during this lecture. So, let us come to first ideal Stirling cycle; before that we will have a history of Stirling cycle, and how this Stirling cycle was brought into effect by having different machines.

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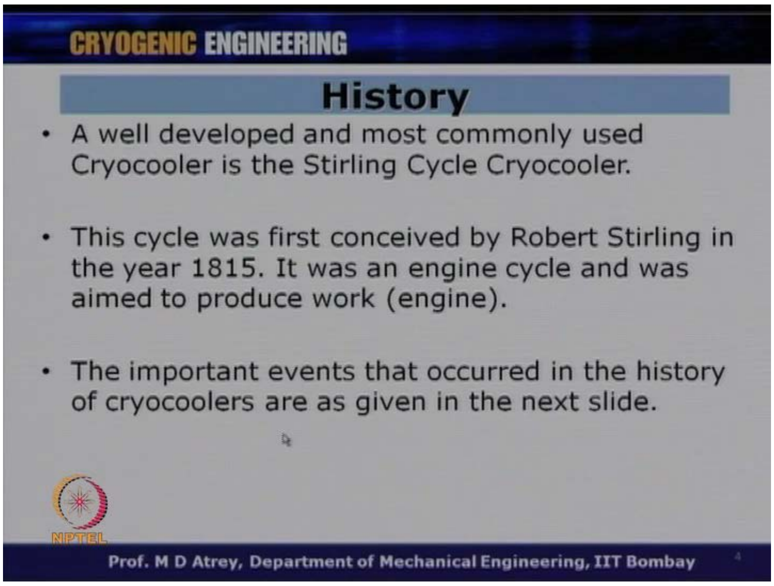

So, a brief history of Stirling cycle; a well developed and most commonly used Cryocooler is the Stirling cycle Cryocooler is very commonly used and has been used for space application for quite some time and therefore, lot of reliable data is available today; and therefore, the efficiency and reliability of Stirling cycle is considered to be very high. This cycle was first conceived by Robert Stirling in the year 1815. If you remember Stirling cycle, the Stirling Cryocooler works on Stirling cycle, and it is named after this inventor called Robert Stirling. When it was invented, it was basically made for engine cycle, and was aimed at producing work, being a engine cycle, it was producing work. And you know, the refrigerator cycle is reverse of engine cycle, so the same cycle what we call as reverse Stirling cycle is used for producing cold. The important events that occured in the history of Stirling Cryocoolers are given in the next slide.

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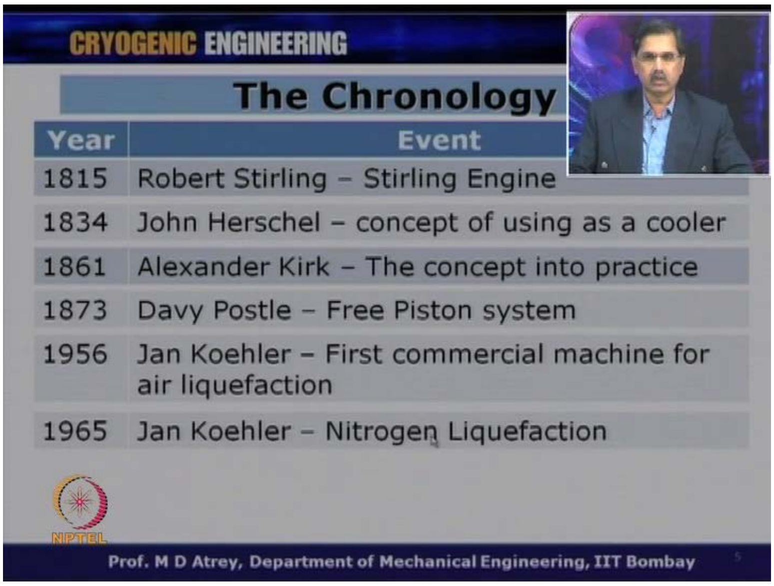

So, the chronology of events is if you see 1815, where Robert Stirling first talked about Stirling cycle, and talked about a possibility of Stirling engine. In 1934, John Herschel, for the first time talked of the concept of using this cycle as cooler; that means we talked about having a reverse Stirling cycle. In 1861, Alexander Kirk, he got this concept into practice, of using Stirling cycle as cooler. So, in 1834, the concept was given by Jan Herschel, while Alexander Kirk realized that concept in practice. Later on in 1873, Davy Postle came with a new idea called free piston system, so you have got free piston, free displacer kind of Stirling cycle also, which we will talk about later, and this idea was first proposed by Davy Postle in 1873.

And lately, after 1950 during 1956, John Koehler first time, he showed a commercial machine for air liquefaction. So, air liquefies around 78 kelvin temperature, and this Stirling cycle Koehler was for the first time used to demonstrate liquefaction of air in 1956. And further in 1965, Jan Koehler again use the same machine for nitrogen liquefaction, wherein nitrogen has a liquefaction point or a boiling point of 77 kelvin; and after that this machine become commercial machine and that is available everywhere in the world. So, looking at 1915 to 1965 was a real period, during which time Stirling cycle got its birth, and then it got evaluated over a period of time, and then Stirling cycle based commercial liquefiers are now available all over the world.

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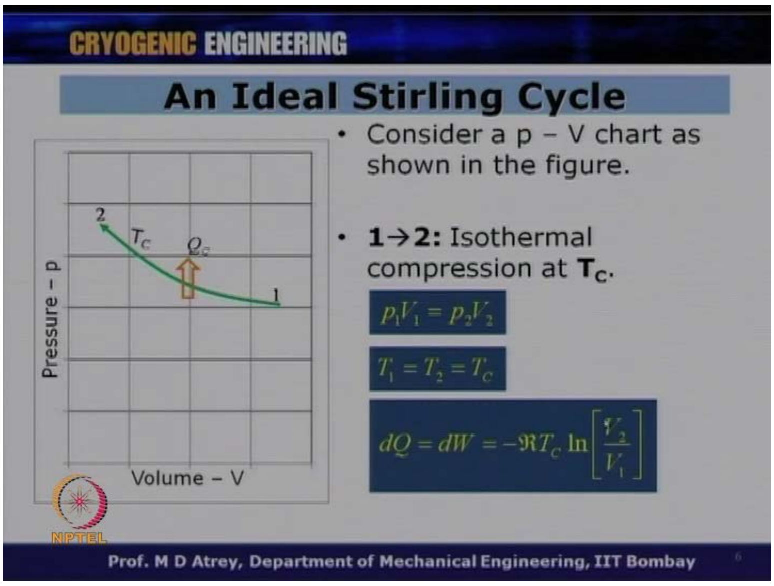

So, let us see how this Stirling cycle works. Working of Stirling cycle has been shown on this p-V diagram; so consider this p-V chart as shown in the figure. So, you got a 1 to 2 points here, which is isothermal compression at temperature T c. So, this is 1 to 2 process occurs at constant temperature that is why it is called as isothermal compression. So, if I were to write some equations for 1 and 2 process, we will have p 1 V 1 is equal to p 2 V 2, temperature remaining constant, this is very well known; T 1 is equal to T 2 is equal to T c, and the heat transfer is equal to work done, heat transfer is nothing but Q c during this time, and which is equal to minus RT c log V 2 upon V 1; this is what will happen in isothermal compression at temperature T c.

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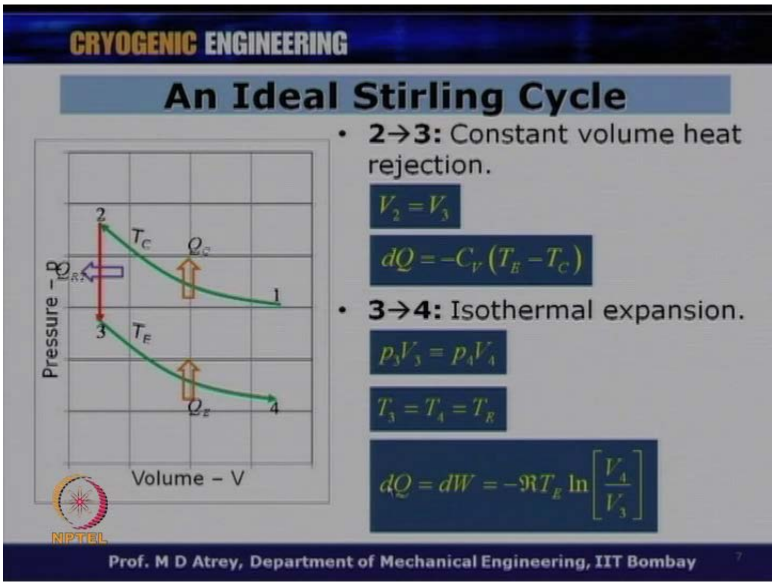

The second action is 2 - 3, which is constant volume heat rejection. As soon as the heat is rejected at constant volume, will come down and the pressure will get reduced here. So, 2 to 3 process is constant volume heat rejection; here in we have got V 2 is equal to V 3 and the amount of heat rejected during this time is equal to dQ is equal to minus C V T E minus T C; as the heat is rejected, we have got a negative sign, final temperature minus initial temperature, and this is the amount of heat rejected during the process 2 - 3, which is constant volume heat rejection.

So, here the pressure automatically got reduced, and then what we do is isothermal expansion. So, 3 to 4 process is again an isothermal expansion, wherein p 3 V 3 will be equal to p 4 V 4. So, point 3, point 4 and point 3 will have same temperature and therefore, we have p 3 V 3 is equal to p 4 V 4, T 3 is equal to T 4 is equal to T E. And during this time, the amount of cooling effect that one gets at during this isothermal expansion process is dQ is equal to RT E log V 4 upon V 3. So, here the heat is rejected Q C by here what you get is a cooling effect or Q E and this is what we get as a refrigerator.

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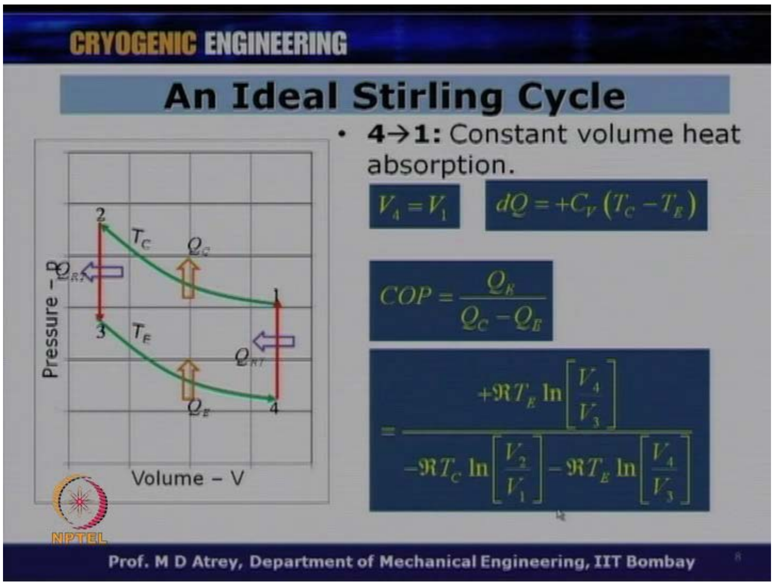

And again during the 4 - 1 process, which is constant volume heat absorption, this was constant volume heat rejection; 2 - 3 was constant volume heat rejection; and now in 4 - 1, we have constant volume heat absorption, the amount of heat absorbed is going to be at constant volume; therefore, V 4 is equal to V 1, dQ is equal to C V into T C minus T E; this is the amount of heat, which is absorbed during the process 4 - 1. Now, you know COP is given as Q E upon Q C minus Q E that is the refrigeration effect Q E divided by the work input, and this work input is going to be equal to Q C minus Q E, this is the COP or the coefficient of performance of any cycle. So, if I were to put to get the value of COP, and I put Q E value, which is obtained during the process 3 - 4 and I put respective value of Q C minus Q E my equation comes down to this.

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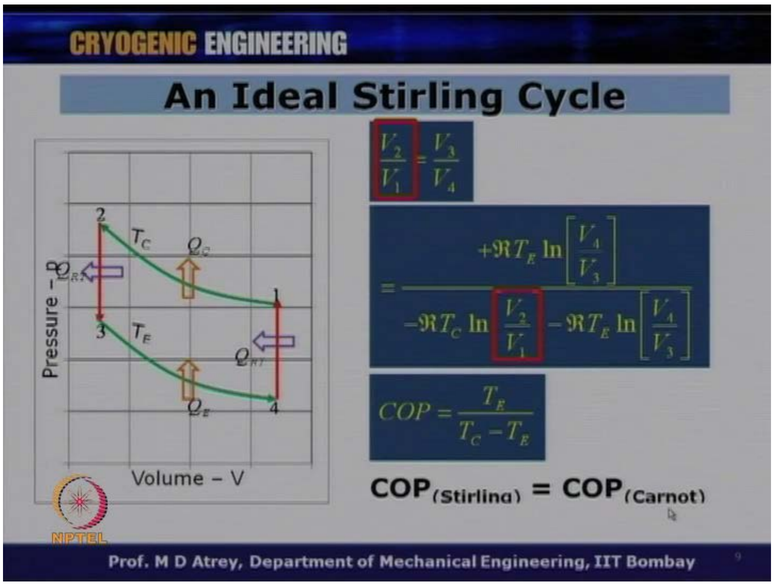

If I were to manipulate these values, we know V 2 upon V 1 is equal to V 3 upon V 4; V 2 upon V 1 is equal to V 3 upon V 4 for isothermal process. Putting up those values, I will replace this V 2 upon V 1 by V 3 upon V 4 and take this minus sign also into consideration, this will become V 4 upon V 3; and therefore, log of V 4 upon V 3 gets cancelled over all, R gets cancelled over all, and what you ultimately get is COP is equal to T E upon T C minus T E; that means, expansion space temperature T E, at which cooling effect is obtained divided by compression space temperature T C minus T E. So, T E upon T C minus T E is a COP of Ideal Stirling cycle; and if you remember, the same expression exists for COP of carnot cycle also; alright carnot cycle considered as the ideal cycle operation and therefore, we can conclude from here that COP is Stirling cycle is equal to COP carnot cycle right. So, we say the COP of ideal cycle Stirling cycle is equal to COP of carnot cycle.

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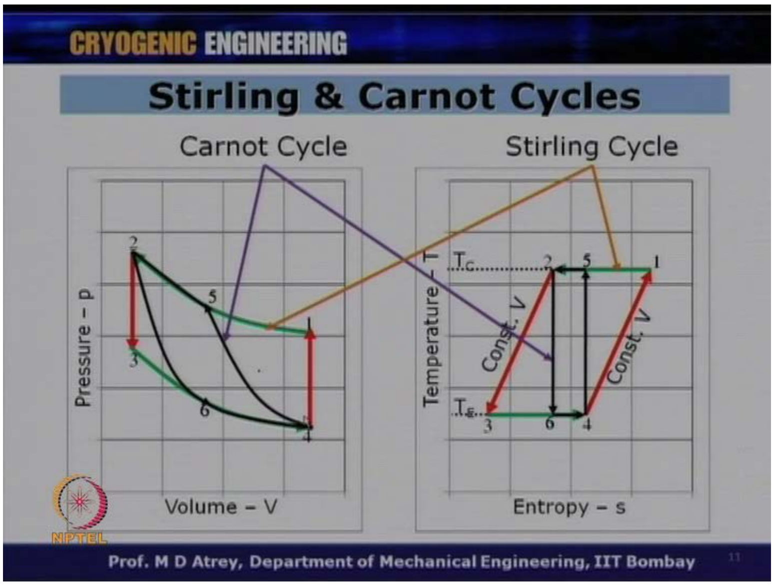

And now if I want to show both the diagrams, both these cycles on p V chart as well as temperature entropy chart. So, you can see 1, 2, 3, 4 as Stirling cycle, and the same cycle now is shown on temperature entropy diagram, which is normally what we refer in cryogenics. 1 - 2 is the isothermal compression; 2 to 3 is the constant volume process, heat rejection; 3 to 4 is a isothermal cooling effect obtained at this point, isomer expansion; and 4 to 1 is constant volume heat addition and the cycle continues. So, this is what a Stirling cycle would look like.

And if I were to plot a carnot cycle on the same diagram, under the same pressure and temperature limits, it would look like this. So, we have got now carnot cycle, which is put on the same maximum pressure and minimum pressure, maximum temperature and minimum temperature; and you can see that COP of carnot cycle will also be same as COP of Stirling cycle; ideal carnot cycle COP will be same as COP of ideal Stirling cycle; and you have got a different diagrams shown over here as carnot cycle. So, this is what it would look like, if I were to compare a Stirling cycle with the carnot cycle.

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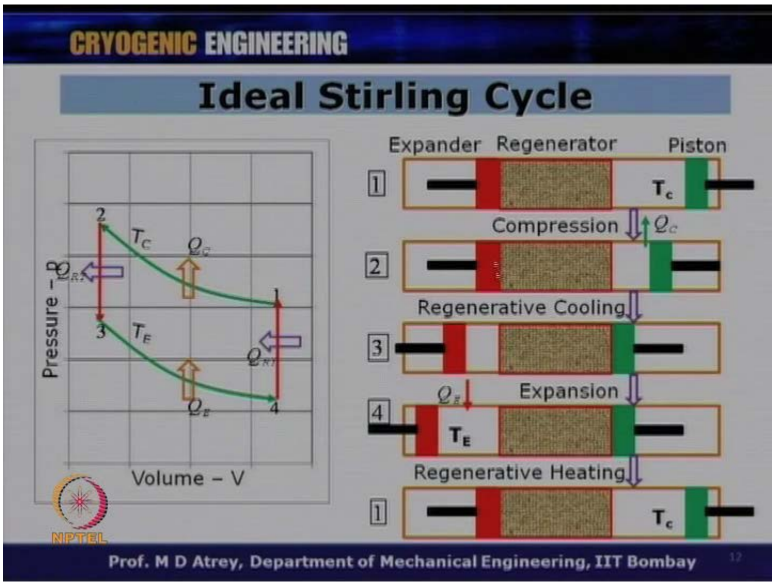

Now, ideal Stirling cycle, how can it be realized? If I were to have these constant volume processes, if I going to have constant isothermal process, isothermal compression, constant volume, heat rejection, isothermal expansion, constant volume heat addition. If I were to realize this process, I have to look first some kind of a process, which will be realized in practice. So, if I were to understand how to realize this ideal Stirling cycle into practice; I have to imagine a process like this, in which we have got a compression piston on the right side in green color and the left side, I have got some expander piston or it could be expanded displacer connected through a heat exchanger called regenerator. And this regenerator is the process through which heat is absorbed for some time, which we have seen last time is a regenerative heat exchanger, this heat exchangers towards heat during the heat rejection and gives back the heat during the heat absorption, when gas flows back and forth in this regenerative heat exchanger.

So, if (( )) want to plot this process on p - V chart and to understand actually what happens, we can see over here. So, this is my initial position to begin with, my piston it has the bottom dead center, while at this position the expander is at top dead center, and the process of compression now begins from 0.1 to 0.2. So, 1 to 2 is a compression process, during which Q C is released and a temperature remains constant over here. And here we can see that during this time, the expander piston remains at the same place, where it was initially; while the compressor piston has come up, during which time the Q C is released over here and this is 1 to 2 process is a isothermal compression process.

Now, I got next process, which is the regenerative cooling or constant volume heat rejection, and what will happen during this time? During this time, the boiling will remain constant; the total gas now, this is the volume, this piston will come forward here, and in order to keep the volume constant, corresponding to that thing, this expander will move back so that the volume of the gas remaining constant alright ; and that is why, that is the way, we can achieve constant volume process during ideal Stirling cycle. So, 2 to 3 process is a constant volume, during this process, the gas will give up its heat to the matrix, and this matrix will regenerator matrix; the matrix will store the heat during which time the pressure will come down.

Now, as you can see that this expander piston has come back, the volume has been kept constant, and this is what we can call at regenerative cooling. Further now, the gas is now in the regenerator as well as in the expander, this gas will not be expanded; and how will it be expanded, the piston has to - the expander piston has to move back. So, as soon as the expander piston moves back from this position and comes down over here, the gas will get expanded from process 3 to 4 and during which time, because the process is isothermal, we will have Q E as the cooling effect that will be realized during this process.

So, 3 to 4 process is isothermal expansion process; please understand again, during this time the piston is at a top date center; piston is at a top dead center, while the expander piston was top dead center during the process 1 – 2. But during this process 3 - 4, the compressor piston was top dead center; while the gas is expanded from 3 to 4 isothermally. Now during the 4 to 1 process, it is a regenerative heating or heat absorption process, the expander piston will come back to its original process - original place and all this gas which was held over here will be moved back during regenerative heating; during this travel, the gas will take back the heat from the regenerative matrix; during this time the piston, the compressor piston will move back to its original position over here one, in order to accumulate all the gas and the process will repeat.

So, you got a 1 to 2 is isothermal compression, which have occurs over here, 2 to 3 is constant volume process, which happens over here; 3 to 4 is a isothermal expansion and 4 to 1 is a regenerative heating or constant volume heat addition alright. This is the way, the piston and the displacer will have to move, in order to realize all this isothermal and constant volume processes in practice. Now what you can see from here is, how this pistons and how this displacers or compressor piston or expander piston move with respect to whether which is very important thing. So, in order to understand that let us see the next slide.

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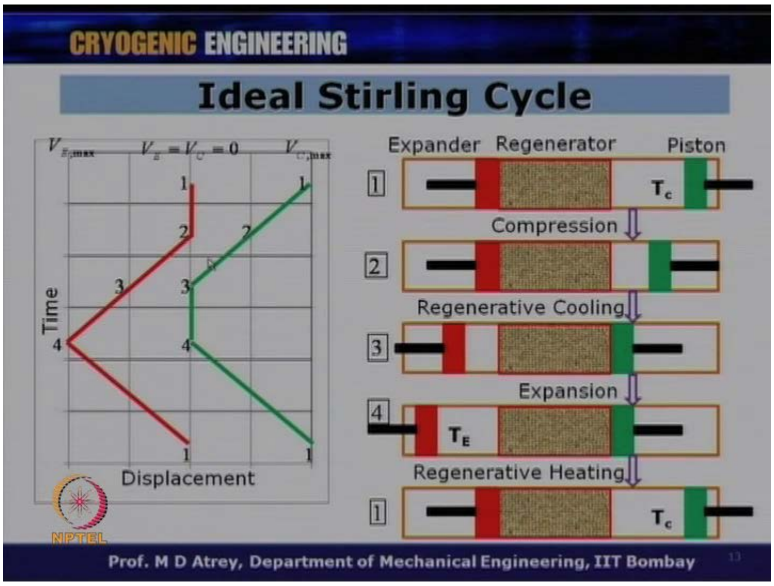

And here we can plot the locus of the top portion of the expander piston as well as the compressor piston. So, initially at point one, we had compressor piston at the bottom dead center, while the expander piston was at the top dead center. During the process 1 to 2, during which compression happened this green color line, which indicates the locus of the motion of the compressor piston, while the red color line gives the locus of the motion of the top version of the expander piston.

So, you can see that during this time, 1 to 2 shows that how this piston moved forward up to this point. However, during this time, the expander piston remained at top dead center only. So, this is moving piston is moving, but expander piston remain at the same place. Now during this process 2 and 3, which is regenerative cooling or constant volume heat rejection; now we can see that both the pistons are moving, so 2 is moving front, the compression piston moved front up to the top dead center, while the expander piston has started moving back, so that this process becomes a process at constant volume. So that is why you can see that, this volume between this two is always remaining constant during this process.

So, 2 to 3 is the motion of the compressor piston, this 2 to 3 is the motion of the expander piston, you can realize from the top, I have written here V E is equal to V C equal to 0 at this central rank. So, whenever the piston is at the top at this point, the at this point, the V C value or the compressor volume is 0 or V E volume, which is the volume above the compressor expander piston is also 0; while at the two extreme position, what we have shown is V C max, when the piston is at this point, the this is amount this distance amounts to V C max, while this distance amounts to V E max on the expander piston side.

If we go further from 3 to 4 now, the expansion process occurs isothermal expansion process, during which time, compressor distance remains at top dead center as shown over here, while the expander piston goes back up to the bottom dead center, this is what is shown. And during regenerating heating or constant volume process, again you can see 4 to 1 is a constant volume, heat addition process, it comes back to its original position, what we had earlier at point one and this where the cycle continues. So, what you can see here that this is the motion for some time, then there is no motion for some time, again there is the motion both for compressor piston and expander piston or expander displacer. This is what I want to show that the motion is not continuous, the motion is for some time, there is motion after sometime, there is no motion.

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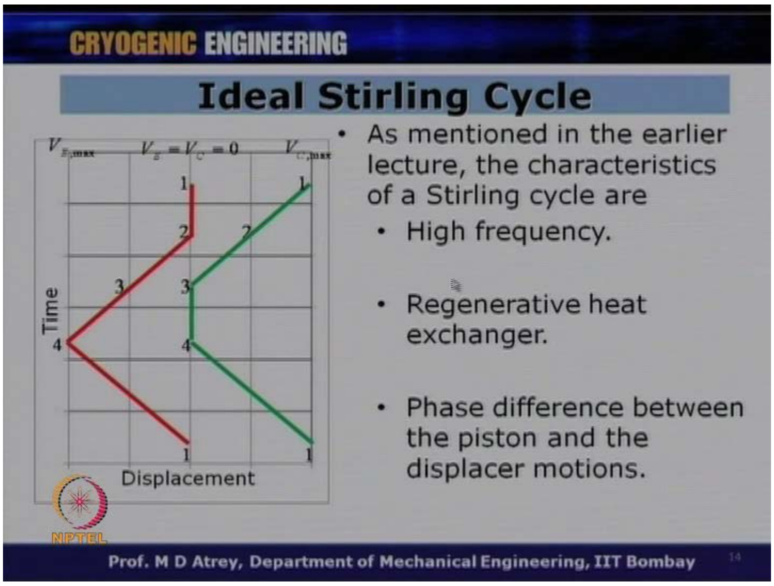

As mentioned in the earlier lecture, the characteristic of a Stirling cycle are high frequency; we remember that the Stirling cycle, there are no walls between the compressor and the expander and therefore, whatever is the frequency of the compressor piston, the same is the piston of the expander piston or displacer. So, they move at very high frequency between let us say 30 hertz to150 hertz or so; they were regenerative heat exchanger alright as well as there is the phase difference between the piston and the displacer motion.

So, both of them, do not go to the top dead center at same time or both of them do not reach the bottom dead center at the same time, which we just saw, which we can see from this motion also. This comes to the top data center much later and the expansion piston is already at the top dead center. So, this is what basically very important is to understand the importance of this phase difference between the piston and the displacer or expander piston also what is sometimes called as.

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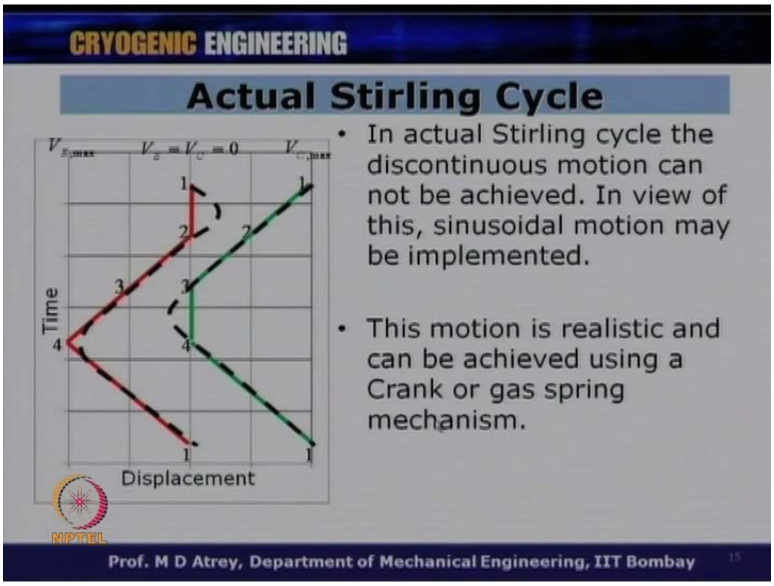

So, in actual case now, if I were to realize such a motion in actual case, the discontinuous motion what we just saw cannot be achieved. Can I have a motion, which is the motion for some time, then it stops abruptly, again there is a motion after some time. So, this is not possible. So, what is possible is normally as a simple harmonic motion or a sinusoidal motion. So, in order to realize this practice, in view of this a sinusoidal motion may be implemented, this is the very important aberration from ideal Stirling cycle. So, actual Stirling cycle may not have discontinuous motion, actual Stirling cycle may have sinusoidal motion, because that is possible to be given in actual practice.

This motion is realistic. So, whatever motion we just saw that motion was there is no motion, then there is a motion, and again there is a motion in the reverse direction. Instead of that, can we have a sinusoidal variation like that? So, we have a sinusoidal motion like that, which is a simple harmonic kind of a motion, which is possible to give to be given in actual practice and therefore, we called this motion is realistic, and can be given using the crank or a gas spring mechanism. So, this is something, which can be realized in practice.

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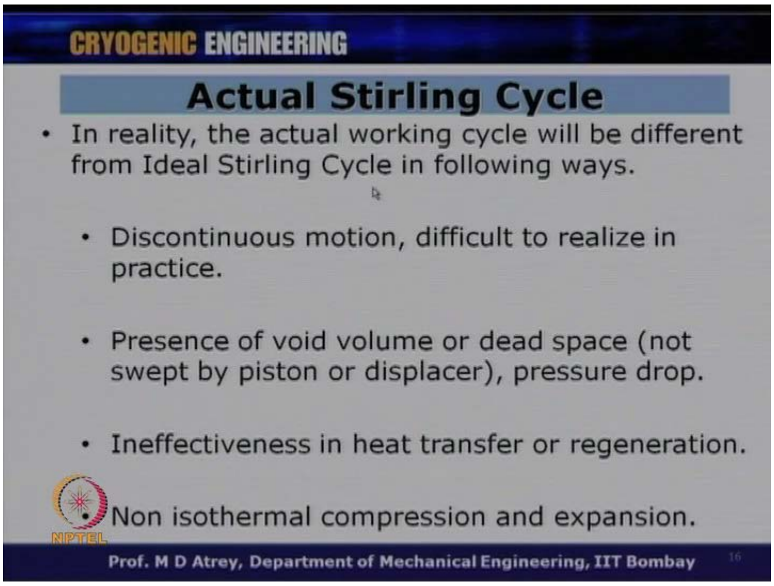

Actual Stirling cycle in reality the actual working cycle may be different from ideal Stirling cycle in following ways. So now, what we are doing we are going away from ideal Stirling cycle; and we are talking about in what ways, the actual Stirling cycle could be different than ideal Stirling cycle. What are different possibilities; the first possibility we just pointed out is a discontinuous motion, it is difficult to realize in practice.

So, in the actual case, we may have sinusoidal motion; we cannot possibly have discontinuous motion over there. Also the presence of void volume, what we just saw was we got a compression swept volume, we got expansion swept volume and we got a regenerative volume. But in order to realize this in practice, we may have some piping, we may have some cubes through which the gas traverse from compression space to the expander space; that means we got some more volume to what we just saw. So, this volume, which is not travelled by piston or displacer, is normally called as void volume or dead volume. In fact, the regenerator volume is also called as dead volume. So, presence of void volume or dead volume is a very, very realistically possible in case of a actual Stirling cycle, but having this, additional void volume or dead volume is going to kill the COP of the machine, we have to sacrifice COP of the Stirling cycle in that case.

Also, we will have pressure drop, because the gas is travelling through regenerator and therefore, gas will realize some resistance to the motion of the gas, depending on its viscosity, depending on the porosity of the regenerator etcetera. So, we will have actual, in the actual Stirling cycle, we will have some pressure drop, that also is taking the cycle away from the ideal Stirling cycle. Also, we talked about having heat exchange between the regenerator matrix and the gas, and this heat exchange may not be perfect; and therefore, we will have some ineffectiveness associated with this heat exchange. So, this is a very important thing, which has to be considered while designing actual Stirling cycle.

So, you will have ineffectiveness in heat transfer or regeneration, is the gas transferring all the heat to the heat generator, is the gas taking all the heat from the regenerator, it will all depend upon, how effective this regenerator is; and therefore, we will have to consider the effectiveness of heat exchange during this actual Stirling cycle. Also, the fourth possibility is non isothermal compression and expansion; now, in order to realize compression process isothermally it is very difficult as you know; this has to be otherwise a very slow process; however, we called Stirling cycle process is a speedy process, it is basically high frequency process. And therefore, to realize isothermal compression in actual case is not so simple, it is rather difficult; and therefore, we may not have isothermal compression actual properties or we may not have actual isothermal expansion in practice; and therefore, we will go away from ideality in this case.

So, these are different possibilities, because of which the actual Stirling cycle will go away from ideal Stirling cycle and therefore, the COP of actual Stirling cycle will be quite less than what you otherwise get from ideal Stirling cycle. So, ideal Stirling cycle will give a same COP as carnot cycle, but actual Stirling cycle will not be as efficient as the ideal Stirling cycle.

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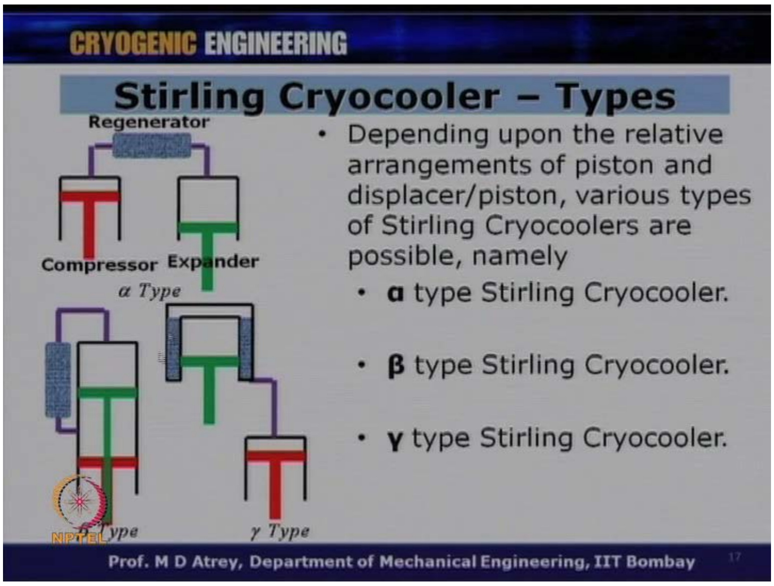

Now, there are different Stirling Cryocoolers types, and we will just briefly touch upon those types. So, depending on the relative arrangements of piston and displacer or this expander piston, we can have a displacer or we can have a piston, various types of Stirling Cryocoolers are possible namely, alpha type Stirling Cryocooler, beta type Stirling Cryocooler and gamma type Stirling Cryocooler. So, these figures show over here, this is alpha type Stirling Cryocooler, beta type Stirling Cryocooler and gamma type Stirling Cryocooler.

So, here you can see that we have got a compressor piston, the compress gas goes to the regenerator, and this is the expander piston again, this is the two piston kind of arrangement over here, and this is what we call as alpha type. Then here, we have got a beta type, here the compressor and the expander displacer or a piston is housed in one unit only; while they are in gamma type, there are two different housings here and this is called as gamma. So, alpha, beta and gamma are they are also called as different name which we will see now.

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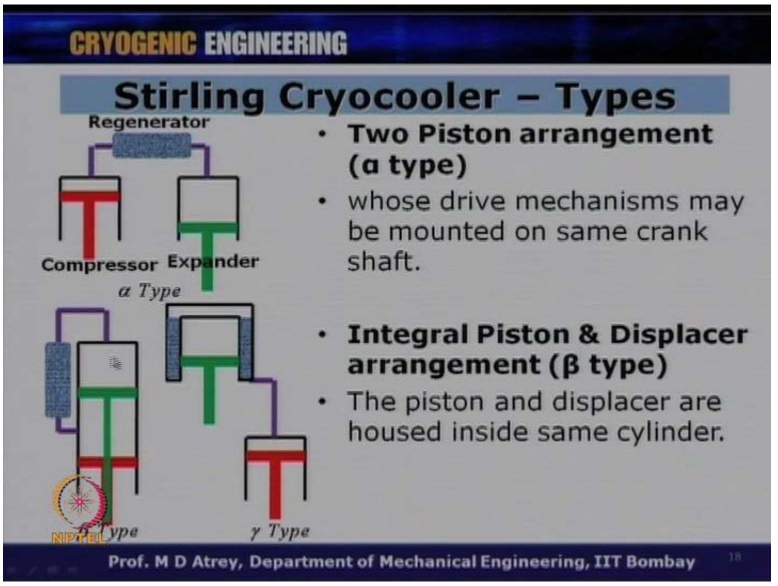

So, alpha is also called as two piston arrangements. So, here in this two piston arrangements, the drive mechanism may be mounted on the same crank shaft. So, we can have a same crank shaft here, and it may be having two cranks; one is driving the compression piston, one is driving the expander piston. So, here in this case, we can have drive mechanism may be mounted on the same crank shaft over here. The other arrangement it is beta type is also called as integral piston and displacer arrangement; that means, the piston and displacer are housed inside the same cylinder. So, here we can see that the piston is over here, and this is compression swept volume while above the expander displacer here, we have got a expansion volume and both of them are now they could be driven by the same crank shaft in this case.

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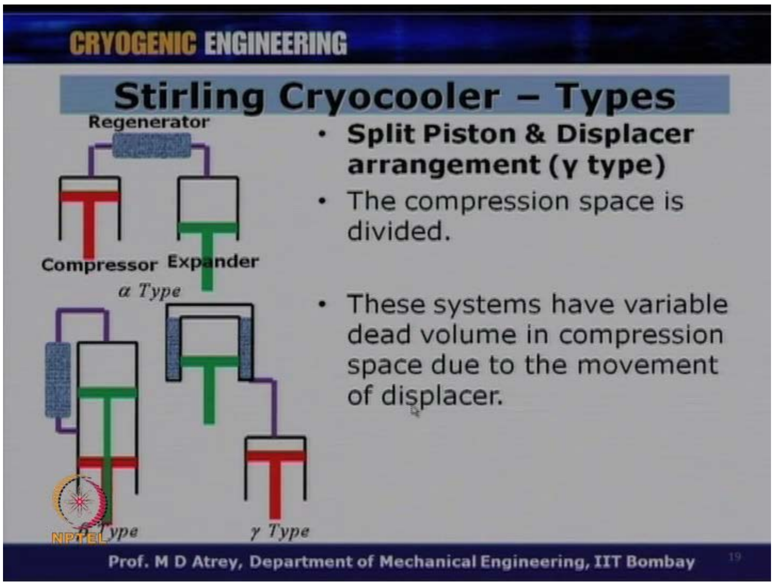

So, in this arrangement, which is the integral arrangement, we can have the same crank shaft or the same crank driving the piston and the displacer. Then we got other unit, which is called as gamma type or it is also called as split type piston displacer arrangement. So, this split unit that means, you got a piston over here; you got a expander displacer over here. So, this is basically displacer; while in this case both are pistons. So, the compressor space in this case, this is the compressor space and the gas may enter through the displacer over here. So, this is the compressor volume, which is connected to the compressor volume over here. So, the compressor space is divided with the compressor volume above the piston and below the displacer in this case, in gamma type arrangement over here.

These systems have variable dead volume in compression space due to the movement of displacer. So, when the displacer starts moving, you will have a different dead volume as compared to what we have in other cases; and therefore, gamma type, split piston type arrangement also may be used many times, and this will have different drive mechanisms, because displacer drive will be different, while the piston drive will be different in this case. So, these are just the ways, how these different mechanisms work; and how they are classified as alpha, beta and gamma arrangements.

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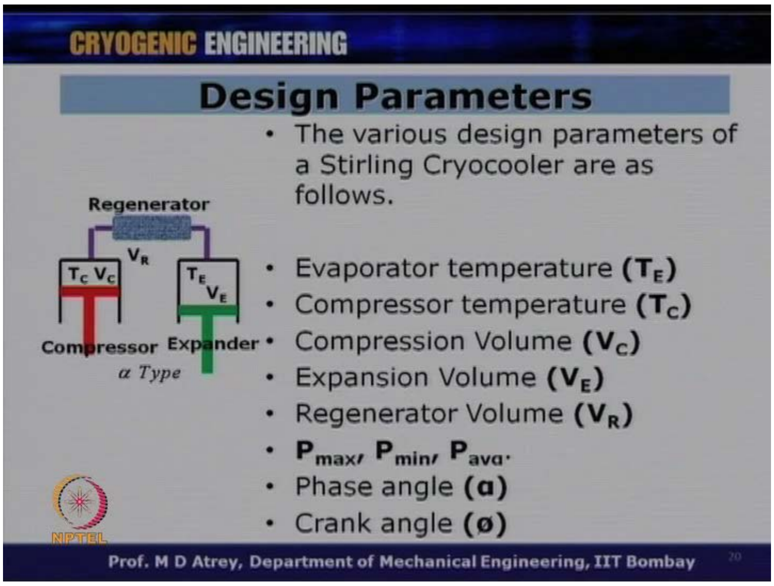

Now, if I were to go for a design of a Stirling Cryocoolers; now this is the very important thing to understand. What do have to do? I have to first understand, what are my design parameters? So, you have got a compressor piston, and you have got a expander displacer, the gas gets compressed over here, it goes to regenerator, where the constant volume process happens, it comes to the expander space volume, and the expansion occurs, and gas gets cooled over here, and you get a cooling effect at this point.

If I were to design, what is my compressor piston diameter should be; what is my expander displacer diameter should be or expander piston diameter should be. I should know how my compression space varies; what is the variation in compression space volume; what is the variation in expansion space volume; corresponding to these volumes, what is the volume of the regenerator. Also, at what temperature do I get cooling, at what temperature, do I get compression; and this is the very important design parameters. And therefore, let us see what this design parameters are; and is a very important, if you as a mechanical engineer, where to go for designing this Stirling Cryocooler.

So, let us see the various design parameters of a Stirling Cryocooler are as follows. Evaporator temperature or expansion space temperature, which is at T E; and this will be at this particular temperature; at this temperature, we got isothermal expansion and therefore, the cooling effect will be generated at this particular temperature. Then we have got a compressor temperature, which is T C. So, here the process of compression happens and we will get the process happening at T C at this point over here, which is isothermal compression process and isothermal expansion process, as we know what happens in ideal Stirling cycle.

Then we have got compression volume, which is V C. So, what is my maximum compression space volume; what is my maximum expansion space volume; also which will come into picture. So, depending on the diameter of this piston, depending on the stroke of this piston, you will have pi by 4 T square into stroke, and that is what your compression space volume will be. Similarly, you will have expansion space volume depending on the diameter and the stroke of the expander piston or a displacer; then we have got to a regenerate volume, which is V R, which is coming over here.

Then we got a, what is my pressure generation, because the gas gets compressed, gas is expanded, so you got a maximum pressure, minimum pressure and average pressure, these are very important values to be known. Then what is my phase angle between the compression space volume and expansion space volume; we just saw that the compression piston and expander piston do not reach top dead center at the same time, but they come after a phase lag of alpha; and therefore, this is a very critical parameter which we will study in the next slides.

So, phase angle is very important alpha and we talk about crank angle; suppose this drives are given by crank, then we got a crank angle also. So, all this together will basically form the design parameters, which are very important; and one has to know that for how much cooling effect is to be obtained at T E, will becomes your starting point.

If I were to design a Stirling Cryocoolers, I should know how much amount of cooling is required to be generated at a particular T E, and corresponding to that depending on all this parameters, I have to design a Stirling Cryocoolers.

So, in order to take care of all these design parameters, Schmidt’s has given his Schmidt’s analysis; and this is as I said is a one of the basic analysis that is used for first guess of different dimensions that could be obtained in order to design a Stirling Cryocooler.

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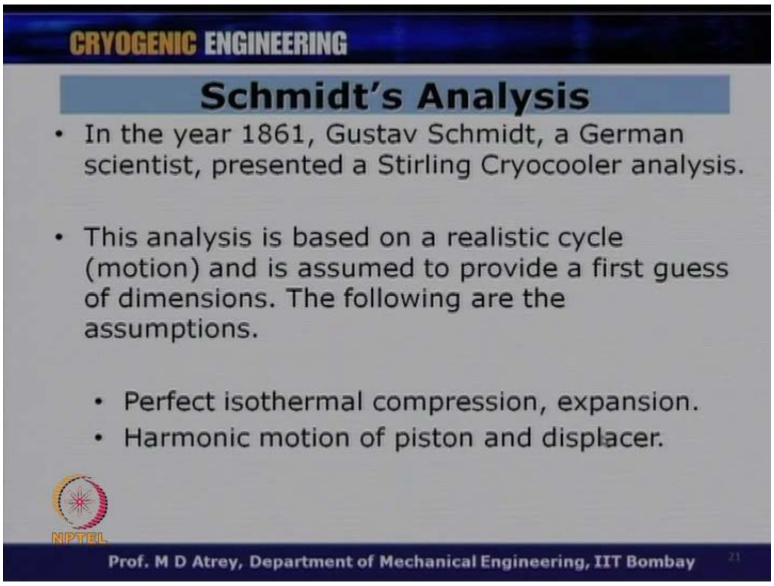

So, in the year 1861, Gustav Schmidt, a German scientist presented a Stirling Cryocooler analysis. This analysis is based on realistic cycle that is from motion point of view as we saw that discontinuous motion is not possible and Schmidt considered a continuous motion or a sinusoidal motion, which is the more realistic kind of a motion. And we assumed that this motion to provide a first guess of dimension; the following are different assumptions.

He assumed the perfect isothermal compression and expansion process as exist in ideal Stirling cycle; he assumed harmonic motion of piston and displacer which is more realistic motion of piston and displacer; he assumed that there is a perfect heat exchange in regeneration; also he assumed there is no pressure drop in systems.

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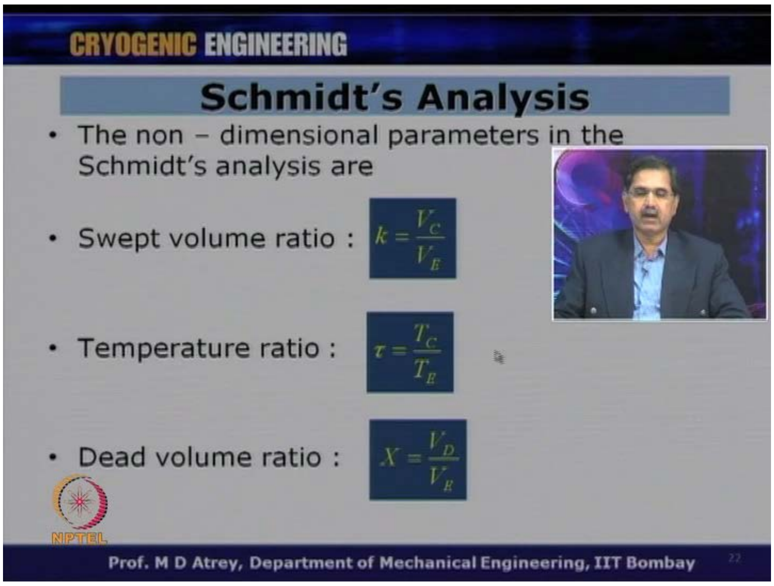

The non - dimensional parameters in the Schmidt's analysis which he considered are swept volume ratio, which is k V C upon V E; what is V c is a swept volume in compression space by the compression piston, and V E is the swept volume in the expansion space by expander displacer; and this ratio V C by V E is called as k. Then we got a temperature ratio, which is T C upon T E compressions temperature divided by expansion temperature and this is called as tau. And we got a dead volume ratio; that means we will have some dead volume in the system, this is called as X, which is equal to V D upon V E, where V D is the dead volume in a system. So, you got a V D by V E, you got a V C by V E and we got T C by T E, so k X and alpha, so k X and tau. In addition to that we have alpha, which is a phase angle between the piston and the displacer, which we will see in the next slide.

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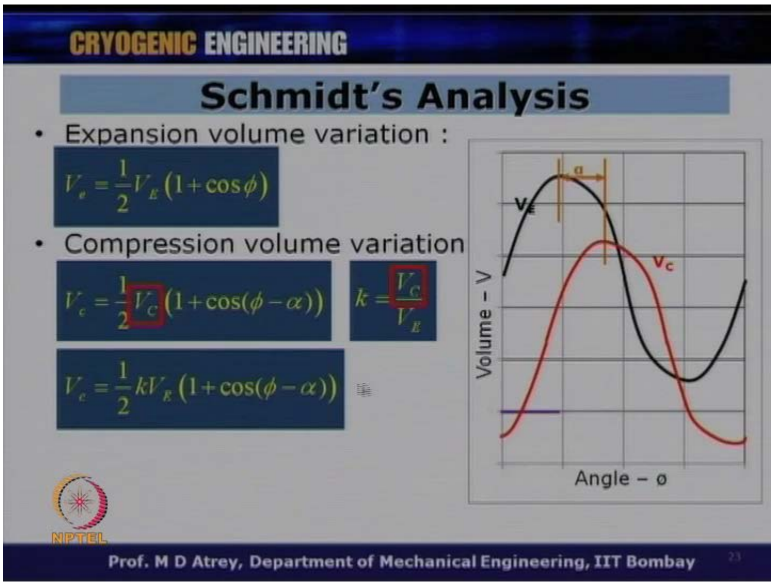

So, expansion space volume variation will be given by V e, the small e shows the variation of expansion space volume with phi, which is the crank volume. When the crank moves, corresponding to that, we will have V e small e as the volume of expansion. When the crank angle is at 180 degree, what you will get is V e is equal to capital V E in that case; when cos phi is equal to 1 we will have 1 plus 1 as 2 and V E is equal to capital V E or maximum expansion space volume; corresponding to that, we have got compression space volume variation, given by this formulation, where we can see that, phi is now phi minus alpha; alpha being the phase difference between V C and V E; this is V C and this V E, and this is maximum swept volume in the compression space.

So, this is highlighting the presence of alpha in the compression and the expansion space variations; and this will be always there. So, you can see, if I were to plot these two variations, we have got a V E variation, which is a sinusoidal; and we have got a V C variation, which is also sinusoidal; and this is the alpha angle between the two. So, V E is leading, the expansion space volume is leading the compression space volume variations, by angle alpha, which is one of the important parameters; you can see later that if alpha is made equal to 0, you will not get any cooling effect. The cooling effect is obtained basically due to this phase difference, we should be optimally designed. So, we got these variations of compression space volume and expansion space volume; and here we can write V C as k into V E in that case; in that case, this formulation will turn out to be this.

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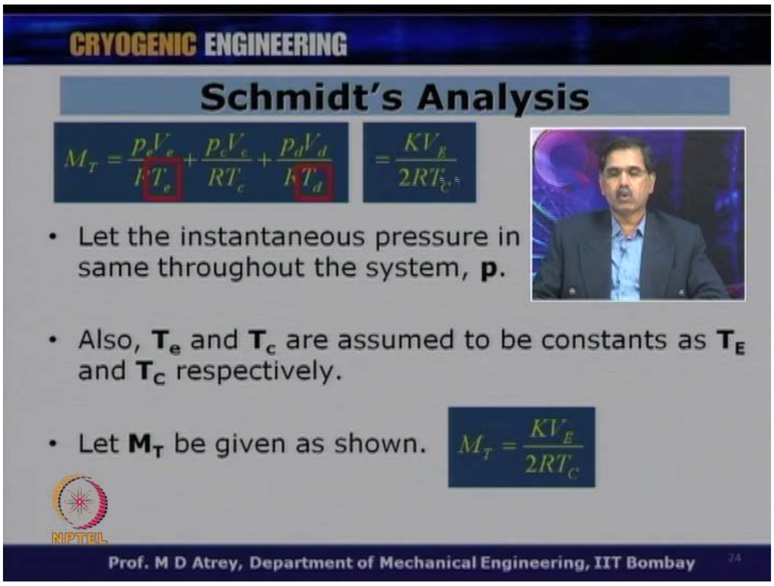

Now, If I were to do the mass balance for entire Cryocooler, we have got a mass M T is equal to p V upon RT, which is the mass fraction in the expansion space, mass fraction in the compression space, and mass fraction in the dead volume, p d V d upon RT d. This is my total mass in a system, at any point of time; let the instantaneous pressure in the system, be same throughout the same; that means, there are no pressure drops in the system; this is assumption in Schmidt's analysis that there is no p e, p c, p d, they are all in the same as p.

And in that case, also assuming that T e, T c are constant temperature, which is what we know, that there isothermal compression process, isothermal expansion process or T e, T c are assumed to be constant as T E and T C respectively; in that case my M T will be given as, I can write this entire m t as some constant into V E upon 2RT c; it is just assumption that there is some constant and I can represent entire this thing as expansion space volume divided by compression space temperature related by KV E upon 2RT c this k takes care of all other things basically. So, now I can write this m t is equal to all this parameters is equal to KV upon 2RT c; now I will manipulate this algebraic T.

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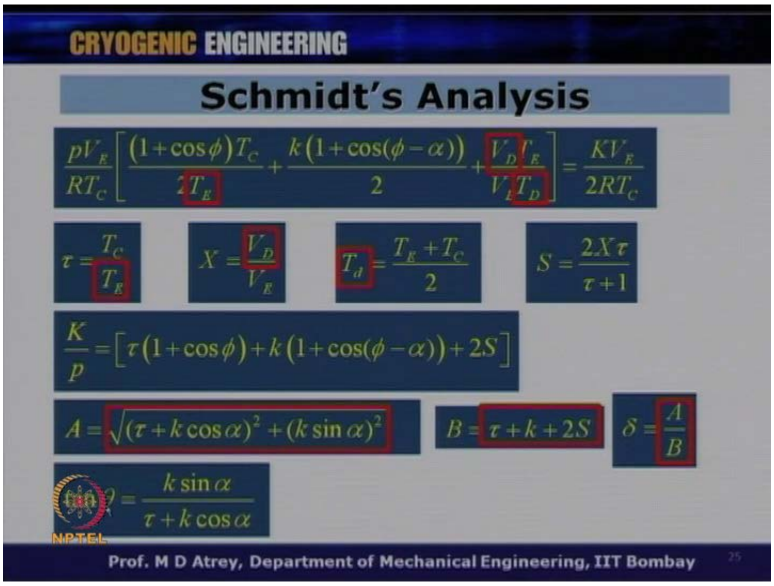

So, if I take p V, we know that there is no p V, p c and all that, p can be taken common, RT c can be taken common and V E can also be taken common; my entire expression now will be will look like this. So, 1 plus cos phi T C upon 2 T E, T C upon T is nothing but tau; then k will come into picture, and phi minus alpha term will be come into picture for compression specification, then we will get dead volume and we know that this is equal to temperature now, this constant into V E upon 2RT c, then putting the value upon T C upon T E as tau, x is equal to V D upon V E as we have earlier decided to have; and assuming that the dead space volume is a mean value between T E and T C. So, T D is equal to T E upon T C by 2, which is what will come over here.

Also we defined one more constant as S, which is 2 x tau upon tau plus 1. So, if we use if we do some earlier algebraic manipulation, this S value also will figure over here. So, I am going to replace all this thing by their non-dimensionless values over here, entire equation now will get reduced to this, K by P is equal to tau into 1 plus cos phi plus K into 1 plus cos phi minus alpha plus 2 S, it is very simplified now, and all the constants are defined over here. If I define further constants as A B and delta as A by B, and putting this values over here in this equation and the mass equation, I will further go as tan theta, I have defined one more angle as tan theta, which is K sin alpha upon tau plus K cos alpha, which is given in this A.

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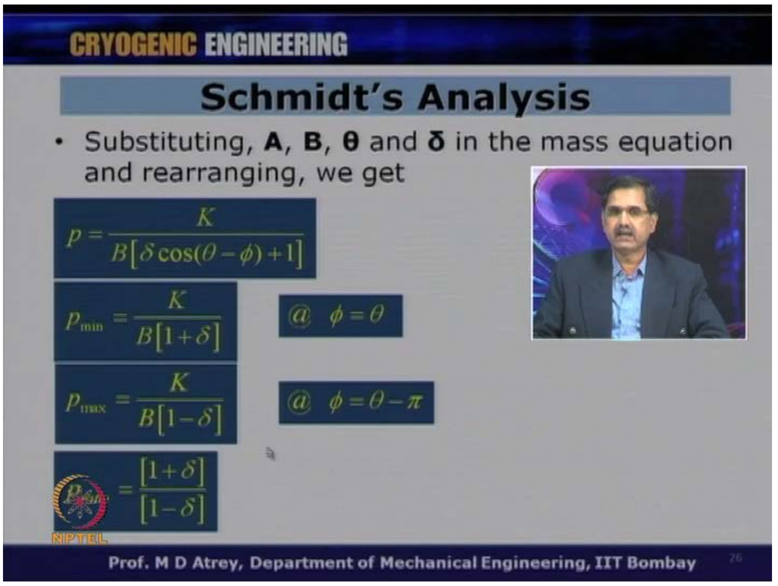

Substituting A, B theta and delta in the mass equation and rearranging them that is algebraic manipulation can be done, what we get is a pressure expression; this is the very important expression for pressure, which is K upon B into this; in this case my P minimum will happen, when my numerator is maximum, and the numerator is maximum when cos is 1; and therefore, I will get 1 plus delta, and this will happen, when phi equal to theta, and I get P max, P is equal to P max, when my denominator is minimum; this will happen when my phi is equal to theta minus phi or when this particular parameter is minus 1 in this case.

So, I get maximum pressure as a function of constant divided by P B into 1 minus delta and I get minimum pressure as K upon B 1 plus delta. So, if I got the pressure ratio that is p max upon p minimum is nothing but 1 plus delta upon 1 minus delta; and what is delta; delta was equal to A by B, where A by B has been defined earlier.

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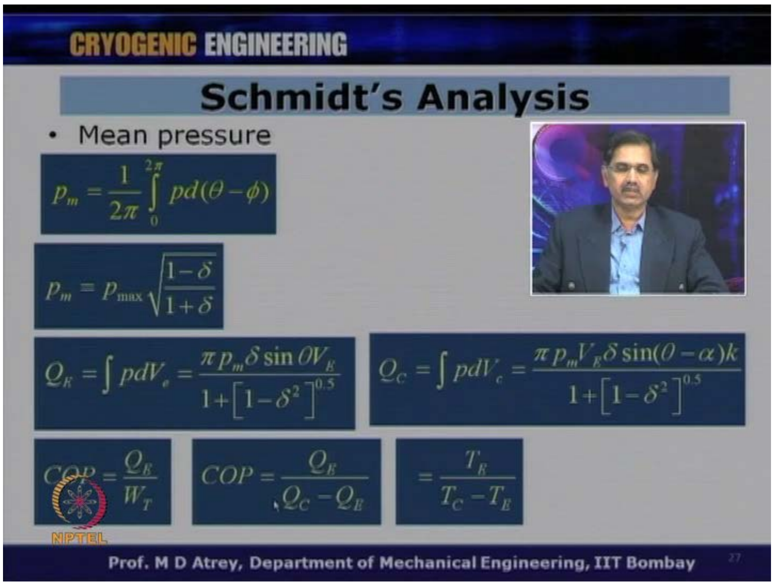

And this is the typical expression for Schmidt's analysis for pressure; and if I integrate that over complete cycle 0 to 2 pi, what I get is a mean pressure, which is defined; and this mean pressure can be expressed as P max into 1 minus delta divided by 1 plus delta under root; and if I were to now calculate cooling effect, which is nothing but integral p d V e, which is what you know, I put the value of pressure in this, and I put the value of d V e in this case, and I will get now what is cooling effect over here; and this term gives me cooling effect.

Similarly, if I were to find out what is the work done during compression, which is integral which is Q C, which is integral p d V c, if I do the similar integration, keeping the value of p here, I get this expression. Now I am to calculate what is the COP of the machine? I know COP of the machine is equal to cooling effect divided by work input, which is equal to cooling effect Q E obtained as these over here divided by Q C minus Q E, and if I were to put all this value at their respective position, I will get COP ultimately equal to T E upon T C minus T E, which proves that in this case also by Schmidt's analysis the COP of the Stirling cycle is same as carnot cycles, T E upon T C minus T E.

By doing all these things, now I am relating pressure generating system, pressure ratio generated in the Cryocooler and it is all related to what is my compression space volume; what is my expansion space volume and thing like that. This is the very important analysis, which relates all this parameters together, and based on all this parameters, we can find out what is the cooling effect, and also what is the power input to the system, which is Q C minus Q E, and what is the COP of the system; again this is based on the assumption, what Schmidt's analysis has assumed.

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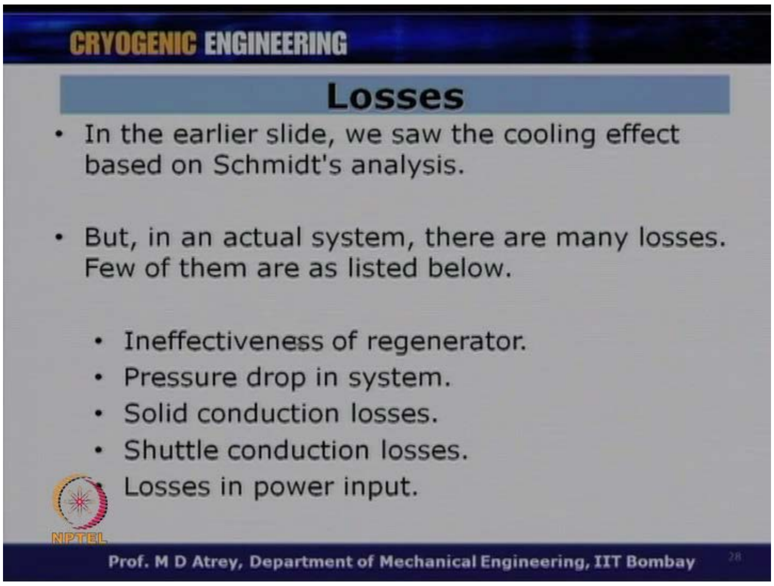

Now, there are different losses in the system, which Schmidt's analysis has not taken into account. So, in the earlier slide, we saw the cooling effect based on Schmidt's analysis, but in actual case, there are many loses as given below. What are these losses in effectiveness of heat exchanger or in effectiveness of regenerator, which has been taken into account; pressure drop in the system, we had assumed all the pressures to be the same; we got solid conduction, because we got a high temperature and low temperature, across the solid members in the system; we got a shuttle conduction, because the motion of the displacer up and down; and we got losses in power input, because of mechanical efficiency. So, all these losses have to be taken into account, in order to get net cooling effect that is available from the system and also net power that is required to be given to the system; and therefore, we will have a net compressed COP of the system also.

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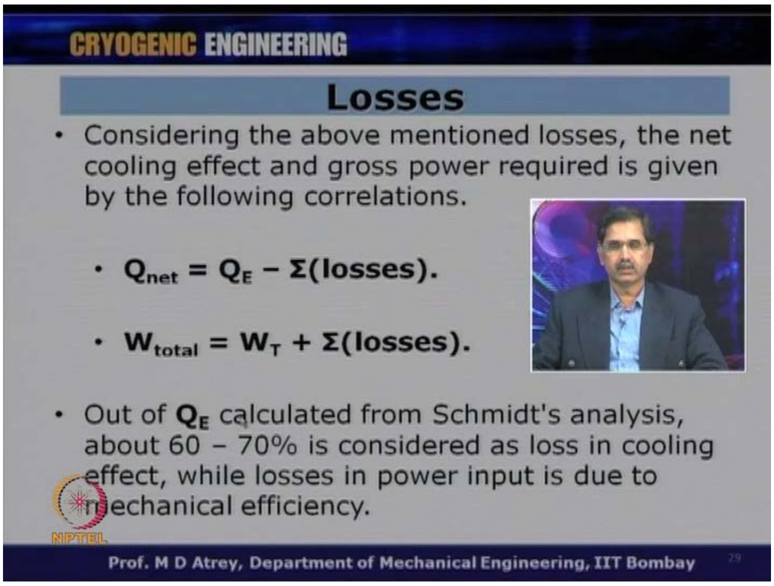

So, normally considering the above mentioned losses, the net cooling effect and gross power, input to the system is given as following correlations; Q net is equal to whatever we have got from the Schmidt's analysis Q E minus sigma losses; this is my net cooling effect. My net power input w total is equal to whatever W T, I have calculated based on Schmidt's analysis plus sigma losses. Now in Schmidt's analysis, we do not calculate all this loses over here, while they are taken as some factor of Q E. So, as I said that this is the first case of analysis, which we use to have first case for the dimensions of the Stirling Cryocoolers; we assumed that out of Q E calculated from the Schmidt's analysis, about 60 to 70 percent is considered as loss in cooling.

So, if I calculate 100 watts as Q E, I will understand that 65 percent is loss as losses, and what is available, Q net is only 35 now. So, first calculate Q E based on various dimensions, assume that 60 to 70 percent is loss and what is net available is Q net after that; similarly I have to take mechanical losses into account, to get to calculate, what is my net power input to the Stirling Cryocooler; and this is what will give me a first case of different dimensions for Stirling Cryocoolers.

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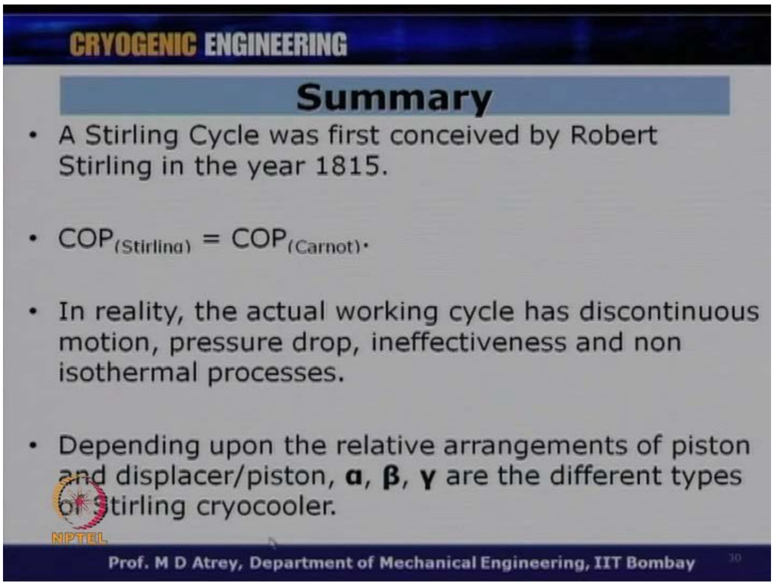

So, in summary, I were to write the summary of this lecture; a Stirling cycle was first conceived by Robert Stirling in the year 1815; we know that COP Stirling is equal to COP carnot; in reality, the actual working cycle has discontinuous motion, pressure drop, ineffectiveness and non isothermal process; depending on the relative arrangement of piston and displacer/piston, alpha, beta, gamma are different types of Stirling Cryocoolers.

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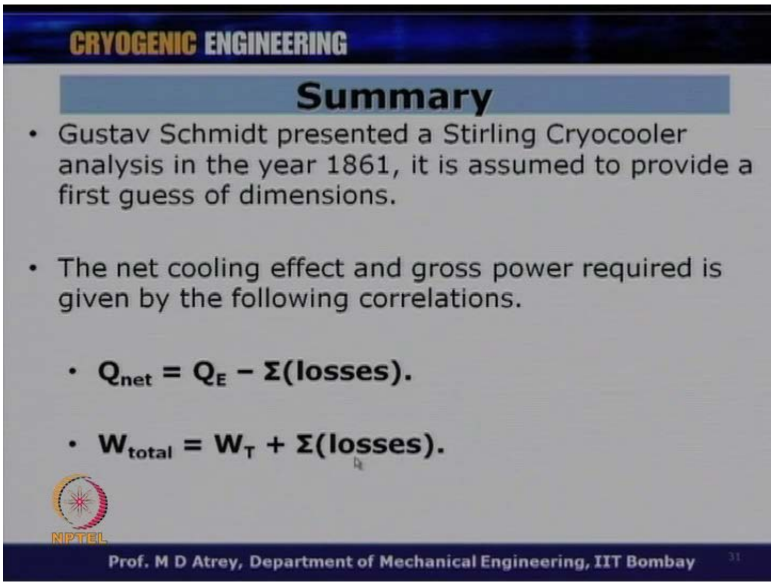

Schmidt's presented a Stirling cycle analysis in the year 1861, it is assumed to provide a first guess of dimensions. The net cooling effect and gross power input is given by following correlations; Q net is equal to Q E minus sigma losses; W total is equal to W T plus sigma losses. A self assessment is given based on this lectures, kindly assess yourself for this lectures.

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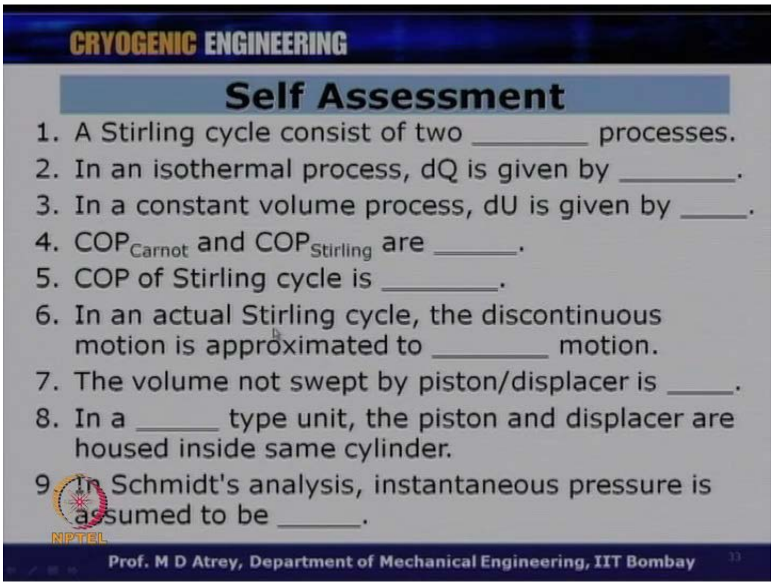

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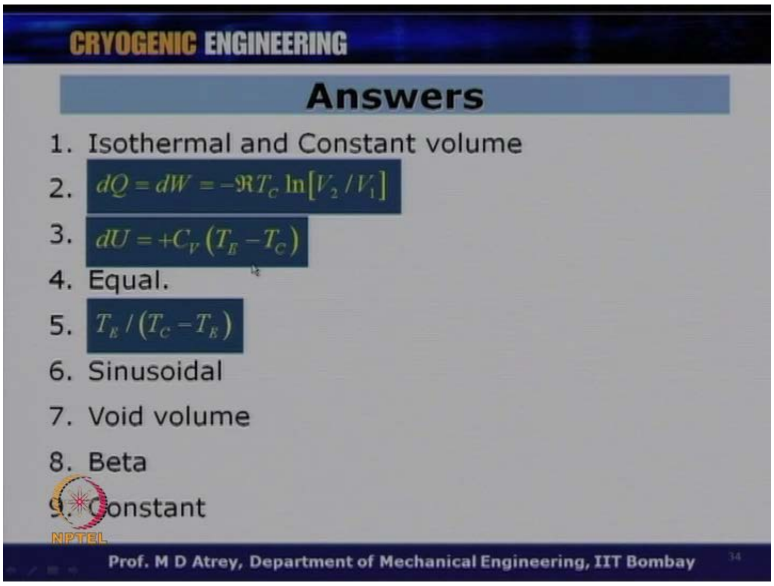

And these are the different questions, please try to answer those. Thank you very much.