

Cryogenic Engineering
Prof. M. D. Atrey
Department of Mechanical Engineering
Indian Institute of Technology, Bombay

Lecture No. #24
Gas Separation

So, welcome to the NPTEL lecture of Cryogenic Engineering, and we are talking about Gas Separation and related to distillation column, where the gas mixture gets separated.

(Refer Slide Time: 00:36)

CRYOGENIC ENGINEERING

Earlier Lecture

- In the earlier lecture, we have seen the working of rectification column with the help of animation.
- **Ponchon & Savarit, McCabe & Thiele** and **Numerical** techniques are used to calculate the theoretical number of plates.
- **McCabe & Thiele** method is less general and is widely used for binary mixtures.
- The major assumption is that the saturated vapor and saturated liquid enthalpies are independent of the mole fraction.

Prof. M D Atrey, Department of Mechanical Engineering, IIT Bombay

Just to take a overview of the earlier lecture; in the earlier lecture we have seen the working of a rectification column with the help of animation. If you remember that we had shown how the feed comes, how the gas components gets separated, the low boiling component, the high boiling component gets separated from a mixture.

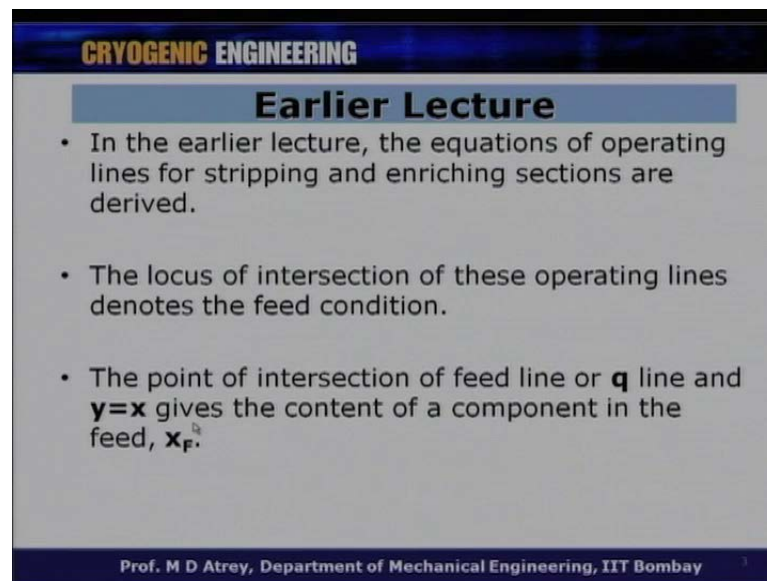
Then we got a Ponchon and Savarit, McCabe and Thiele, and Numerical techniques that are used to calculate the theoretical number of plates in the rectification column. So, every column consists of plates, and a design of rectification column would consist of calculating the theoretical number of plates, and there are several techniques depending on the complexities, and assumptions. And these are three techniques, Ponchon and Savarit, McCabe and Thiele, and Numerical techniques.

Of which, we are going to talk about McCabe and Thiele, because this is a less general and is widely used for binary mixture and is valid at cryogenic temperature. So, it is a simpler techniques of all three has been proved for cryogenic gas separation column.

The major assumption is that, the saturated vapor and saturated liquid enthalpies are independent of mole fractions. We assume here that the capital h and the small h which is the enthalpy of the vapor and the enthalpy of the liquid, saturated vapor and saturated liquid are independent of molar composition. This is what we are seen in the earlier lecture in detail.

In the earlier lecture, the equation for operating like in addition to what we just talked, we also had derived the equations for operating lines for stripping section and enriching section **alright**.

(Refer Slide Time: 01:56)



CRYOGENIC ENGINEERING

Earlier Lecture

- In the earlier lecture, the equations of operating lines for stripping and enriching sections are derived.
- The locus of intersection of these operating lines denotes the feed condition.
- The point of intersection of feed line or q line and $y=x$ gives the content of a component in the feed, x_F .

Prof. M D Atrey, Department of Mechanical Engineering, IIT Bombay

The section about the feed is called enriching section of the column and the section below the feed is called as stripping section we derive a relationship between the vapor fraction which goes up and the liquid molar fraction which comes down and this is what we call as operating line for stripping and enriching section.

The locus of intersection of these two operating line denotes the feed condition. So feed comes under a particular molar composition with a particular quality and the intersection of these two operating lines denotes the feed condition.

The point of intersection of feed line or a q line, this is what we call as quality line or a which denotes the wetness of the feed basically, amount of liquid it brings to the total number of moles that is what we call as q and the intersection of this q line and the y is equal to x line, gives the content of a component, basically it gives the composition of the low boiling component in the mixture and it is called as x_F **alright**.

This particular lecture extending the same knowledge ahead in order to design a distillation column or a rectification column in terms of theoretical plate calculations so, what we want do is talk about graphical solution for column design using McCabe Thiele method.

(Refer Slide Time: 03:14)

CRYOGENIC ENGINEERING

Outline of the Lecture

Topic : Gas Separation (contd)

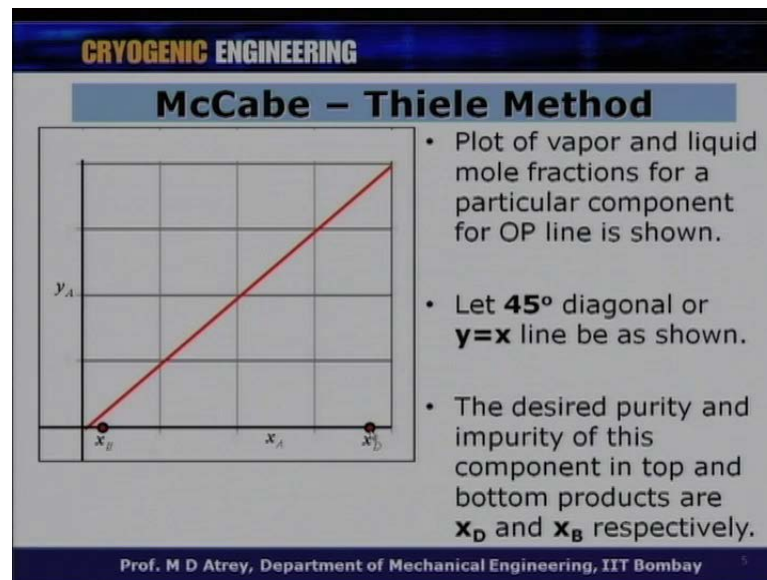
- Graphical solution for column design using McCabe - Thiele method
- Tutorial
- Parametric study

Prof. M D Atrey, Department of Mechanical Engineering, IIT Bombay

So, McCabe Thiele method as we talked about is a simpler of all three and it is basically a graphical solution. You do not have to do lot of calculation over here, the entire column can be represented on a graph and which helps us to calculate how many theoretical plates are required in enriching section, how many theoretical plates are required in the stripping section.

And we can better understand this graphical technique using a tutorial which will solve using a excel sheet and then we can see different parameters which affects this design of the column. So, what are these parameters, how do they affect the design of the column, this we can see, all these three things we can see in this lecture.

(Refer Slide Time: 03:59)

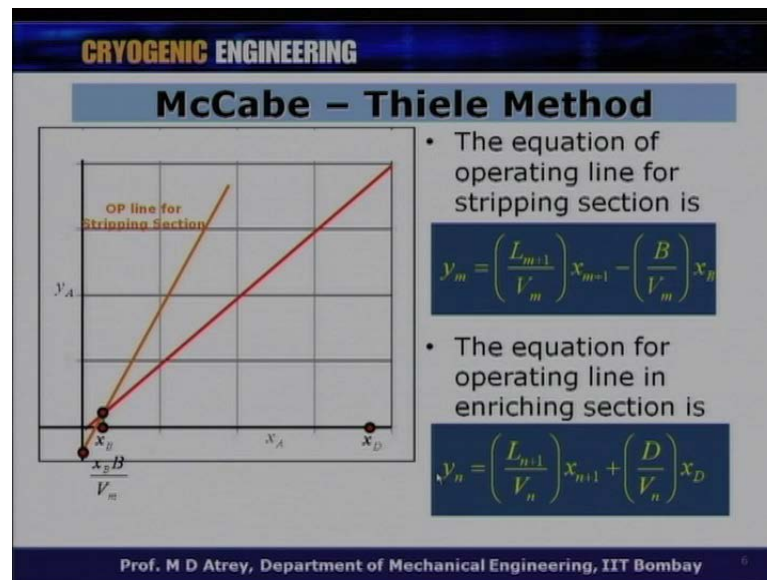


So, taking overview again in order to understand this method in details, just see what we have earlier learnt. So, if we plot the vapor and liquid mole fractions for a particular component for the operating line is shown over here.

Let 45 degree diagonal, this line represents y is equal to x line which is what is known to us. The desired purity and impurity of these components in top and bottom products are x D and x B respectively. We have talked about this. x D denotes the purity requirement of a component in the condensate or D that is obtained from the top of the column, while x B shows amount of that component in the product which is taken out from the boiler side; that means, basic this length becomes the impurity, for example, if you want take oxygen from these this will denoting the amount of nitrogen existing in the oxygen while this will shows amount of nitrogen in the condensate that is rich in nitrogen and it is taken from top of the column.

So, x D denotes the purity while x B denotes the impurity of the same component **from the** from B or the product which is taken from the boiler side.

(Refer Slide Time: 05:13)



The equations of operating line first stripping section is this is what we had derive last time, y_m is equal to L_{m+1} by V_m into x_{m+1} minus B by V_m into x_B . This is the equation for the stripping line and this is what we had plot and we remember that this operating line intersect y is equal to x line at x is equal to x_B because at this point. This is the bottom most plate and therefore, when x_{m+1} becomes x_B , this equation reduced to y is equal to x or y_m is equal to x_B and therefore, we had found earlier, we had derived earlier that if x_{m+1} is replace as x_B we will reduced y is equal to x and this therefore, happens to be the point of intersection of the operating line for stripping section and y is equal to x line.

The equation for operating line in enriching section is y_n . So, m and n represent the stripping section and the enriching section respectively. So, y_n is equal to L_{n+1} by V_n into x_{n+1} plus D by V_n into x_D . This equation represents the operating line for the enriching section which is that top part of the distillation column.

So, this is operating line for the enriching section, this is operating like the stripping section. Also to be seen from these that L_{m+1} by V_m is the slope of this line while L_{n+1} by V_n is the slope of operating line for the enriching section.

So, these equations give the slopes of the two lines L_{m+1} by V_m and it also gives you the y intercept; that means, where this lines intersect the y axis for example, for operating lines for the enriching section we got D by V_n into x_D as the y intercept

and therefore, we know this point and we know this point also. Joining these two points 1 can draw the operating lines for the enriching section once we know the equation of the operating lines for the enriching section.

Similarly, for the stripping section we have got the slope of the line which is L_m by V given by this equation and also we have got a y intercept which is minus B by V_m into x B .

So, what is this y intercept again its points of intersection of the operating line of enriching section with the y axis and which is what you can see in this graph, That this point is going to be negative point, negative side of the y axis and therefore, this point the y intercept can be located on the y axis at this point similarly, we know that this prime intersect y is equal to x at x is equal to x B , knowing these two points we could join this point and extended it further and in this way we will be able to draw the strip operating line for the stripping section.

Similarly, what I told you for the stripping section, we can see the same thing same arguments for the enriching section that the operating lines for the enriching section intersects y is equal to x line at x is equal to x D .

So, in these equation again if I put x_{n+1} equal to x D for the top most plate from where D is taken out; that means, the plate below the condenser, this equation will get reduce to y is equal to x and therefore, we say that the operating line for enriching section intersects y is equal to x line at x is equal to x D .

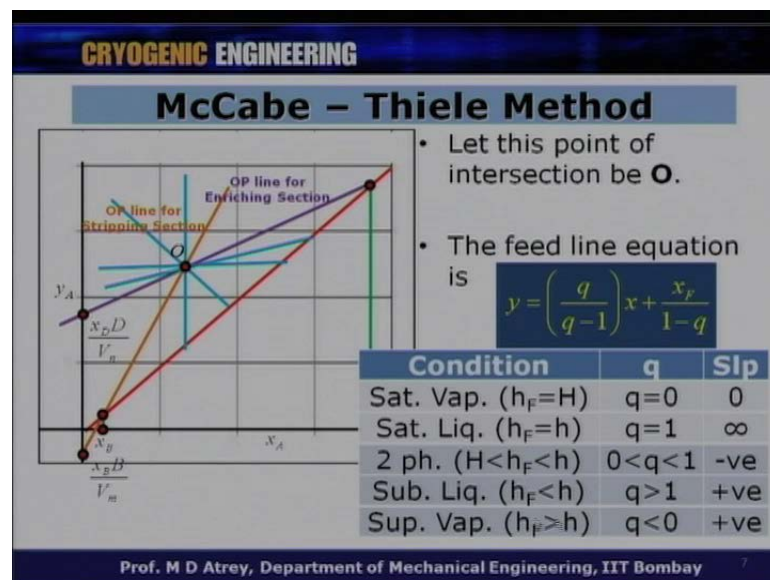
Now, let us assume that these two lines intersect at the point O and this denotes the feed condition. The feed line equation is y is equal to q upon $q - 1$ into x plus x_F upon $1 - q$ and this is what we had seen that slope of the q line or the slope of the feed line is q upon $q - 1$ and it is the value of q which will decide the slope of this feed line while its y intercept will be x_F upon $1 - q$.

Now, everything depends on the feed line condition and the feed line condition is determined by the value of q namely, we got various conditions that if the feed is going to be saturated vapor where h_F is equal to capital H. In this case the q will be equal to 0 and if q is equal to 0 if you put this value q upon q minus 1 the slope also will be equal to 0 and therefore, this line is going to be horizontal or parallel to x axis **alright**.

So, depending on the feed condition, the slope of q line will change. The next condition is saturated vapor when h_F is equal to small h . q is equal to 1, if I put q is equal to 1 in this equation, the slope will be infinity therefore, this line is going to be parallel to y axis.

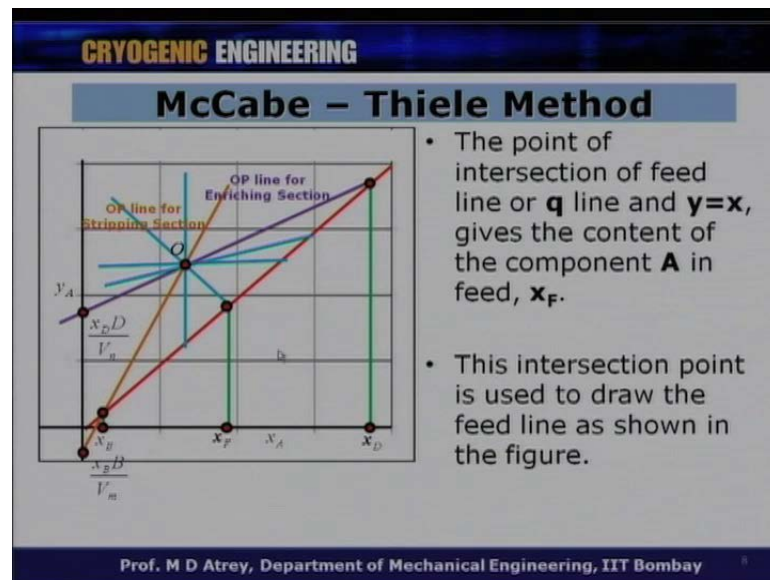
Similarly, we have got a two phase where h_F is going to be between capital h and small h . We have seen all these in detailed again in the earlier lecture. The q therefore, would lie between 0 and 1 and a slope would be negative and if you got a slope negative, the line could be in this direction between this two lines basically.

(Refer Slide Time: 08:30)



Then we got a sub cool liquid possibility and there we got a super cooled liquid possibility and therefore, in this case the slope will be positive and therefore, the slope could be in this direction the line could be in this direction. So, all this conditions which can exist for a feed line have been represented on this graph and this will decide because, all this lines will intersect y is equal to x line at x is equal to x_F and this will decide the number of plates in enriching section and number of plates in the stripping section.

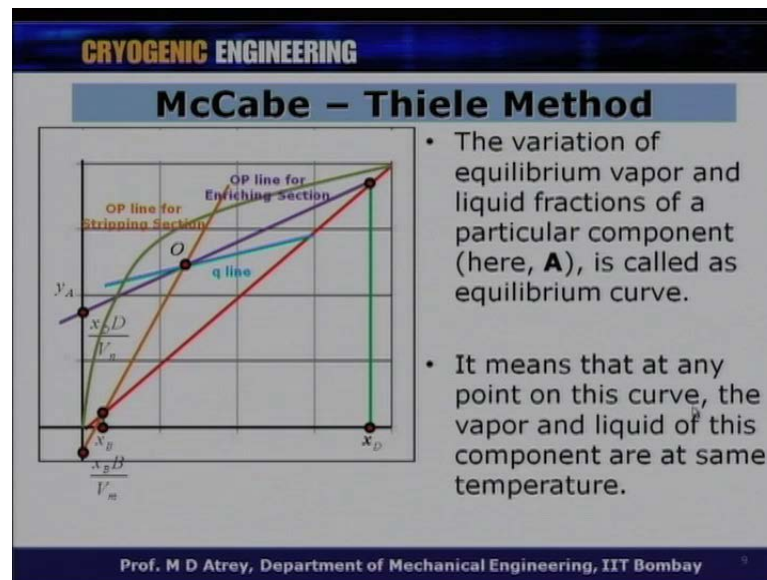
(Refer Slide Time: 10:32)



The point of intersection of feed line or q line and y is equal to x gives the content of the component A in the feed x_F this what we have seen. So, if we take these are feed line the intersection of these line with y is equal to x will give you x_F . This intersection point is used to draw the feed line as shown.

So, what I know? I know the point of intersection, I know the composition of the feed if I locate this two points I can join them and I can draw therefore, the q line on the feed line and this will decide the slope of this line or again this basically represent the condition of the feed.

(Refer Slide time: 11:10)



The variation of equilibrium vapor and liquid fraction for a particular component is called as equilibrium curve. If you remember my earlier lectures, we have talked about this, that there the equilibrium curve exist for every compositions for every mixture for a given composition and pressure **alright**. So, these are the points on which y and x vapor and liquid are in thermal equilibrium; that means, the temperatures are same and they remain in thermal equilibrium.

So, equilibrium curve for a given composition for a given pressure is going to be a unique numbers and if you join all these points, y is equal to x for various compositions. So, you got a unique points and if you could join this points together for different temperatures on at each point on this curve y and x are in thermal equilibrium.

So, if I know that, if suppose this is oxygen or nitrogen at 1 bar then this point is 77 Kelvin, this point possibly going to at 95 Kelvin and therefore, you can join these points across this temperature and at every point here the temperature will be in between 95 to let say 77 and at each point for a given temperature y and x will be an thermal equilibrium. It means that at any point on this curve, this is what I just said. The vapor and liquid at this component are at same temperature they are in thermal equilibrium.

Now, this condition exists on each plate. If you remember again or earlier arguments in the third lecture of gas separation or something like that on each plate vapor and liquid are in thermal equilibrium because, there is a heat transfer between the vapor which is

going through the liquid and liquid which is coming from the top of the plate, on every plate therefore, some vapor leaves and some liquid leaves. This vapor and liquid above this plate are going to be in thermal equilibrium. So, on each plate, vapor and liquid are in thermal equilibrium. Therefore, the plate condition lies on the equilibrium line. That is equilibrium curve gives the relationship between the liquid composition x_n and vapor composition y_n on the same plate. So, it relates y_n and x_n for every plate. This is what it denoted by the equilibrium curve.

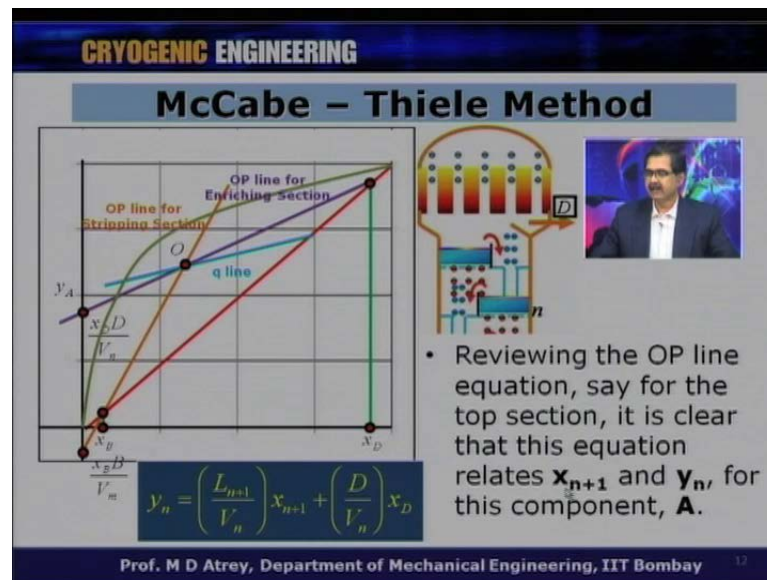
Since the top and the bottom products have different boiling points for example, 77 Kelvin may be the product **product** had 95 Kelvin which is a oxygen. Since the top and bottom products have different boiling points there is gradual variation of temperature across the length of the column **alright**. So, I said 95 should be 90 Kelvin which is oxygen 77 Kelvin which is nitrogen.

So, you have got a 90 Kelvin at the bottom, you got a 77 Kelvin on the top and at any point across the column, at any point in the column at any plate in the column, you have got a temperature in between these two values 90 to 77 Kelvin. So, when you come down from top to bottom you will have a temperature gradient from 77 Kelvin at the top to 90 Kelvin at the bottom.

The operating line relates the variation of liquid and vapor mole fractions of a particular component across the length of the column. So, what is operating line? Operating line is basically giving you relationship between y and x_{n+1} . Basically so, y_n as a function of x_{n+1} and therefore, it relates the liquid and the vapor mole fractions of a particular component across the length; that means, at various temperatures **alright**.

So, operating line gives a relationship between y_n and x_{n+1} at various temperatures across the length of the column while the equilibrium curve is basically give you the relationship between y_n and x_n on every plate who are in thermal equilibrium with each other **alright**. These are the concept for all these lines.

(Refer Slide Time: 14:51)



So, let us now see all these things all the graphical representation with respect to the column. We are just showing you at top of the column where D is taken out, where condensate is taken out and we got some plates over there. And let's concentrate on the nth plate for example. For this nth plate we've got a relationship between y and x n plus 1. So, y n is equal to L n plus 1 by V n x n plus 1 plus D by V n into x D. This is what a relationship which **which** is valid and it relates y n and x n plus 1.

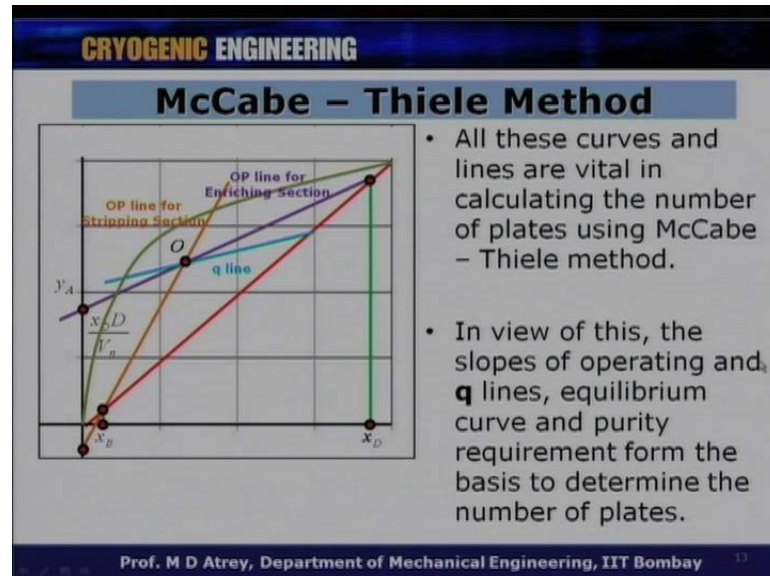
Reviewing the operating line equation, say for the top section it is clear that this equation relates x n plus 1 and y n for this component **A** **alright**. This is what I've just say **alright**.

So, let us see this is the nth plate **alright** and y n into V n is actually going to give you the or y n is the mole fraction of a component which is leaving this plate, nth plate in a vapor condition while x n plus 1 is a molar fraction of the liquid which is coming from top plate above this nth plate **alright**. This is leaving L n plus 1 or x n plus 1 is the mole fraction of the liquid which is leaving the n plus 1 th plate. So, nth plate is receiving liquid from the plate above it and it is denoted by x n plus 1 the mole fraction of that component is by x n plus 1 while y n is the mole fraction of the vapor of a component which is leaving this nth plate.

So, all these curves, the operating line for the enriching section stripping section q line and the equilibrium curve. So, all these curve and the lines are vital in calculation of the

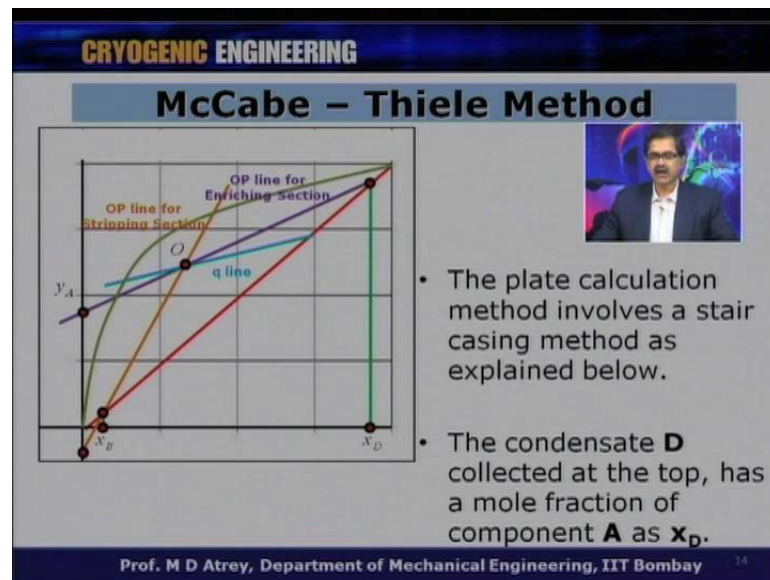
number of plates using McCabe Thiele method and therefore, I wanted to give you the entire glimpse of graphical representation of the McCabe Thiele.

(Refer Slide Time: 16:19)



In view of this, the slopes of the operating line and q line so, what is this line basically? It has got slope it has got y intercepts associated with a physical condition. So, the slope of the operating line and q line the equilibrium curves and the purity requirements form the basis to determine the number of plates. So, all these parameters x_B , x_D , the slopes of operating lines, slope of q line, equilibrium curves are all required in order that we calculate the theoretical number of plates for a given composition for a given separation, gas separation problem **ok**.

(Refer Slide time: 17:12)

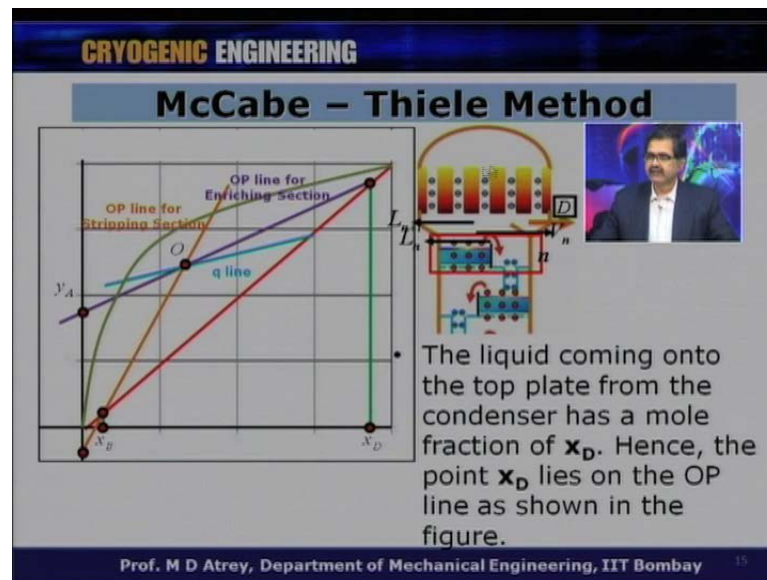


Now, let us start the with this background, with this preamble, now let start calculating or let start understanding the procedure in which we calculate now the number of plates.

The plate calculation methods involve a stair casing method as explained below. So, we are going to talk a method which is normally also called as the stair casing method. Because, we actually climbed down the stair case in order to calculate the number of plates.

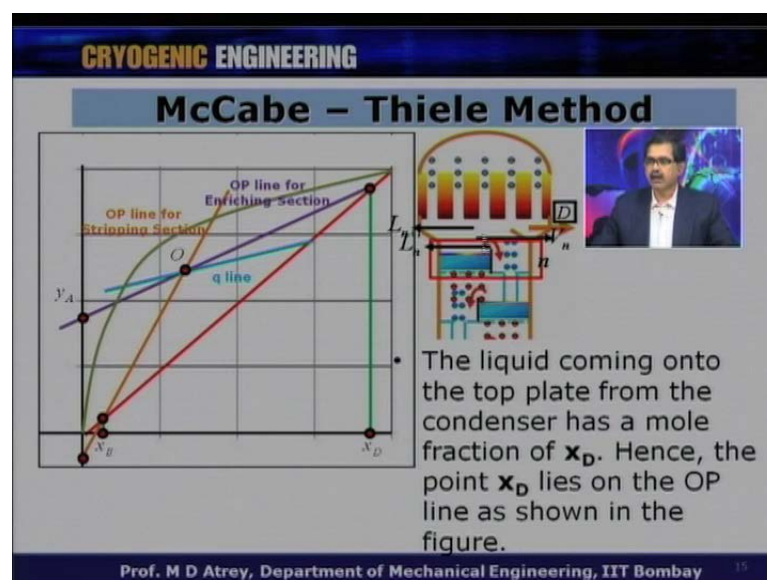
The condensate D which is taken from the top and what is taken from the bottom is called B, what is taken from the top is low boiling component is nitrogen in a mixture of nitrogen and oxygen. The condensate D collected at the top has a mole fraction of a component A that is nitrogen as x_D . So, if I want D to have a 98 percent of nitrogen, it talks about the purity of nitrogen, condensate which are receive from the top and therefore, the value of x_D is going to be 0.98 in that case.

(Refer Slide Time: 18:07)



So, you got a liquid coming from the top, whatever vapor goes up it gets condensed. Out of which I am going to take a D out, I am going to take D moles out. The remaining moles will come down, whatever condense at happen, a part of it is going to taken out. And in order that always the L comes down, in order that always we have got some L which is coming down, remaining L is going to come down of the condensate.

(Refer Slide Time: 18:07)

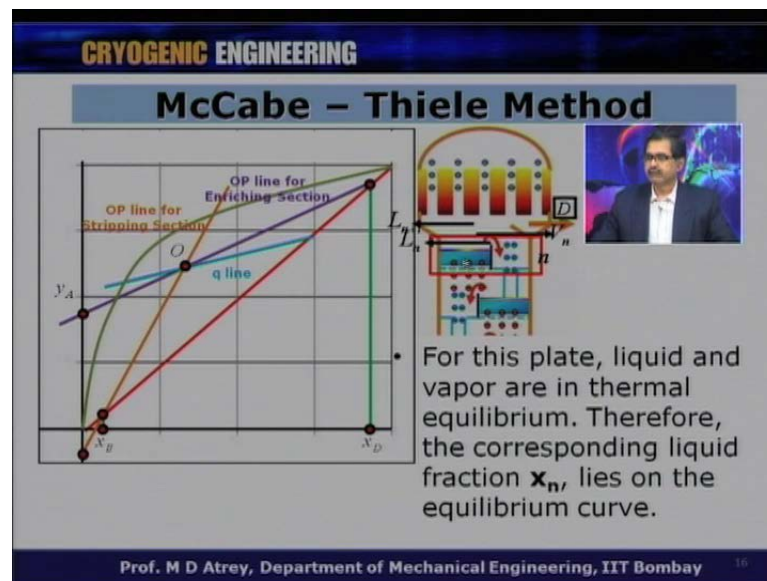


The liquid coming on to the top plate from the condenser has a mole fraction of x_D . Hence point x_D lies on the operating line as shown in the figure. So, operating line will

have also have a point x_D which is what we have proven for the top most plate. So, this plate receives the liquid as L_{n+1} for which lies the point x_D .

Now, let us come to this plate. Let us call this as n th plate which is the top most plate. As a result of whatever come from the top, it has got its own vapor and liquid which is going to leave this plate.

(Refer Slide Time: 18:54)



For this plate, liquid and vapor are in thermal equilibrium therefore, the corresponding liquid fraction x_n for this plate. This is my n th plate now. Whatever liquid leaves, whatever mole fraction leaves this plate now will be called as x_n . whatever vapor leave this place is going to be called as y_n . And as we just talk, this y_n and x_n are going to be in thermal equilibrium because now we are going to talk about this plate only.

So, the corresponding liquid fraction, what we received from the top was x_{n+1} which has got a x_D component, but for this x_D now, I would like to find as a result of the heat transfer between vapor and the liquid. The vapor which is leaving at this place is going to y_n , what is corresponding x_n on this plate which is in thermal equilibrium **alright**.

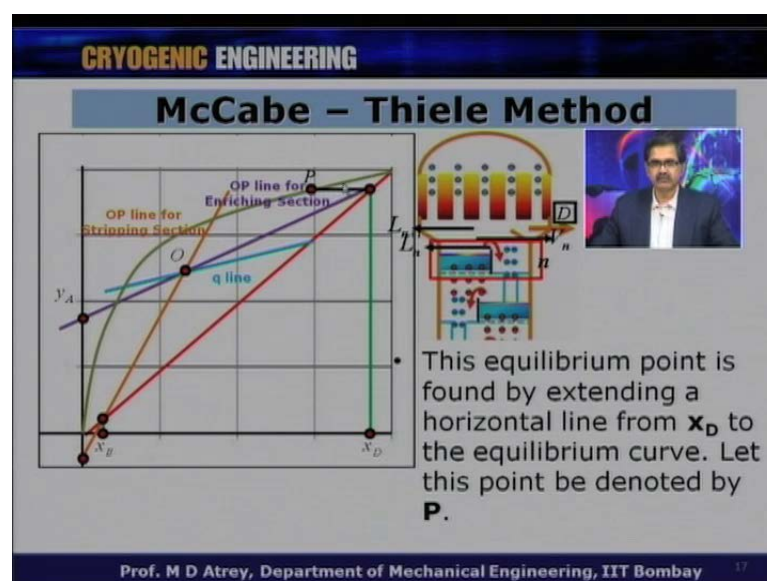
So, here I got a y_n which is known to me. Because y_n and x_{n+1} , I know a relationship between which is operating on operating line. So, I would like to find out for this y_n what is corresponding x_n which is in thermal equilibrium. So, what do I do?

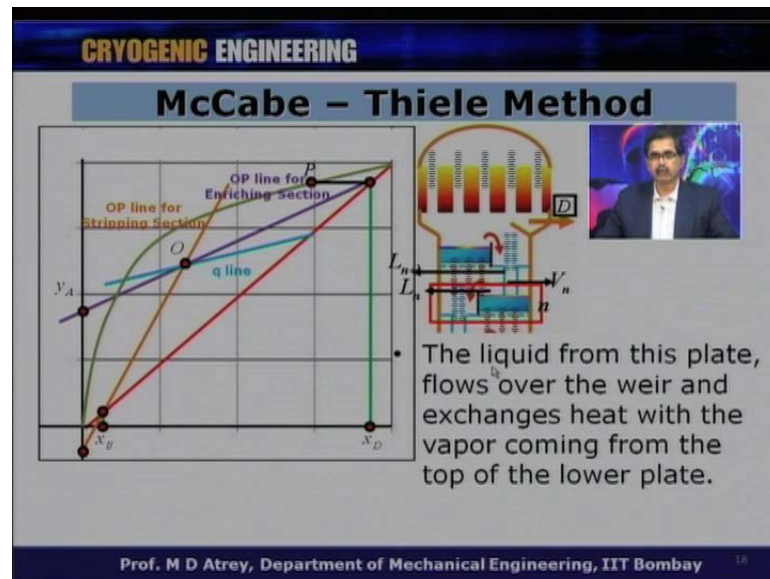
This equilibrium point; that means, for this y_n , the equilibrium point y_n x_n is going to be line on the equilibrium curve which is what we just talk. The equilibrium curve will show the equilibrium of y_n and V_n which exist on every plate. So, for this y_n the corresponding x_n can be found out by keeping y_n constant; that means, I have to move horizontally, keeping y in the same and the point of intersection of this horizontal line with the equilibrium curve is going to give me the condition which exist on this top most n th plate at the moment **alright**.

So, this point is going to give me. This equilibrium point is found by extending a horizontal line from x_D to the equilibrium curve; let this point be denoted by p . So, p denotes thermal equilibrium existing on this n th plate or which is the top most plate right now, where y_n and x_n for this plate are in thermal equilibrium. I hope you understood about this, very important argument.

Now, we have located y_n , V_n for this top most plate. And now let us go the plate below this. Now what was x_n earlier which is existing on this plate, if I am now my n th plate is now this. My attention has come down from top most plate where I located this y_n , x_n . I will now come to the plate below and now this plate is going to receive x_n plus 1 from the top; that means, this is the point. This happens to be the x_n plus 1 as far as my plate below is considered **alright**.

(Refer Slide Time: 20:10)





So, the liquid from this plate flows, this plate means top most plate, flows over the weir. whenever the liquid column goes above this weir. This liquid will fall on the plate which is below it, it goes over the weir and exchanges heat with the vapor coming from the top of the lower plate.

So, my attention has come to this lower plate for which this becomes x_{n+1} . So, I've got a relationship between y_n which is the vapor leaving at this point, the mole fraction of the vapor which is leaving this point. And I've got a relationship between this y_n leaving and x_{n+1} now which is coming from top of this plate **alright**.

So, we know that an operating line relates x_{n+1} with y_n . So, now, y_n is the vapor leaving this plate and x_{n+1} is the point given by p which was existent earlier for the top most plate. So, if I want to find for this x_{n+1} corresponding y_n which exist on this plate, what I will do, I will keep x_{n+1} constant and come down and the point where it intersect the operating line because, operating line gives the relationship between y_n and x_{n+1} .

Now, I am going to come down, keeping x_{n+1} constant point where it intersect the operating line for the enriching section is going to give me corresponding y_n because it relates a y_n with x_{n+1} for this plates for this n th plate now. Hence for this plate the corresponding vapor fraction lies on the operating line because it relates y_n with x_{n+1} for this plate.

The vapor composition y_n for this plate is found by extending a vertical line from p onto the operating line, let this point be denoted by q and therefore, I draw a vertical and I am come down. So, can you understand from here that as soon as I have come down from this plate to this plate, I derived a vertical line over here.

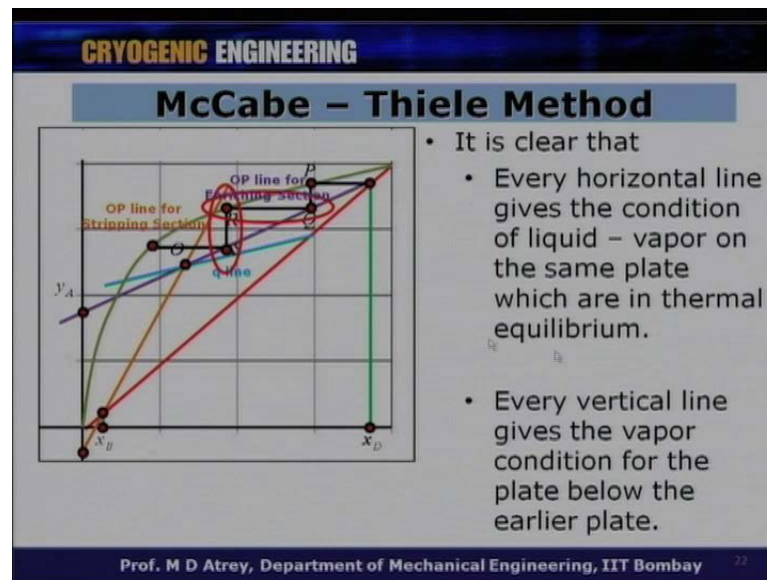
Now, I do the similar argument now have come here and I have located my y_n and now I would like to find out what is x_n for this plate; that means, y_n and x_n are going to be in thermal equilibrium and if I have want to locate that point what I will do? I will again for this y_n I would like to locate the point on equilibrium curve and the point where this line, horizontal line will intersect the equilibrium curve, this will locate y and x_n for this plate because, y_n and x_n will now be in thermal equilibrium and by drawing a horizontal, I will get the location like the wave we have done earlier for p , I will get 1 more point with basically depict the y_n and V_n which are in thermal equilibrium on this plate.

Having done that, I will again go vertically down now to go to the next plate. So, this stair case it will continue and this is what I will show you now.

The stair casing should be continued till the stairs cross the point O . So, how long I will do like this? I will do depending on the curves of this two operating line, till I cross my intersection point O and this as I said number of vertical line would then denote the plate requirement. So, here for example, 1.2 and may be 2 and half. So, I can round it to 3. So, maybe I can say that 3 plates. In fact, I could say 2.5 plates, but 2.5 plates cannot exist therefore, I will say 3 plates are required in the enriching section.

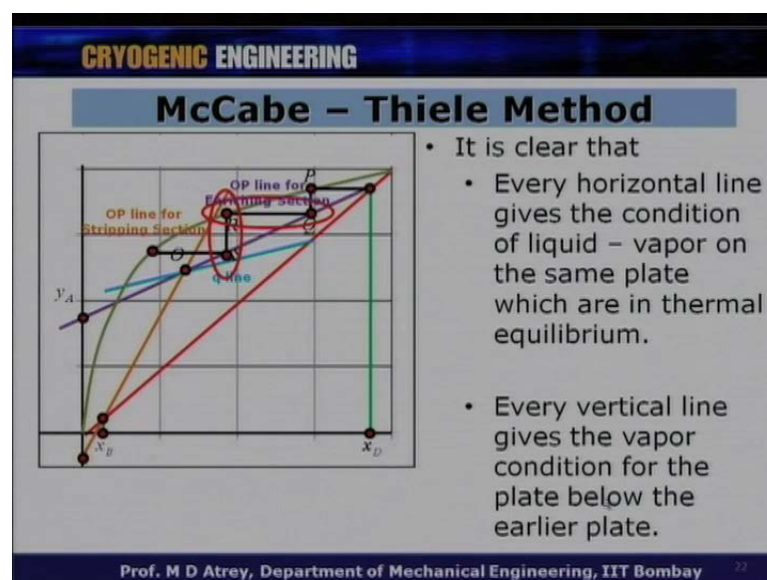
What is important is, have you understood this, why do I have horizontally, why do I've come vertically, please understand that and please that is the most important physics behind of this problem.

(Refer Slide Time: 25:12)



So, the stair casing should continue till the stair cross the point O that is the intersection of operating line as shown in the figure. It is clear that every horizontal line gives the condition of liquid vapor on the same plate which are in thermal equilibrium while every vertical line gives the vapor conditions for the plate below the earlier plate. This is what we just talked about.

(Refer Slide Time: 25:12)



It means that every vertical line indicates the need for the plate in the enriching section till the stair casing crosses the point O; that means, we will go on doing this stair casing

till the time be hit point O and this are the number of plates required in order to get particular purity depending on the slopes of operating line and the q line.

The same exercise could be done for lower section with x_D as the desired impurity of the component A in the bottoms. I will do the same thing I will go vertical over here, locate the point of equilibrium, go down here locate y_n , correspondent y at locate x_n plus 1 on thermal equilibrium and go down here.

So, these are the number of plates vertical lines in the stripping section, these are the number of plates I would get in the enriching section. This will be more clear when I take a tutorial, but, what is important is you have to understand why do above this horizontal and vertical what is this business all about basically. That is the most important thing. Having understood that the problem solving remain is only a kind of algebra.

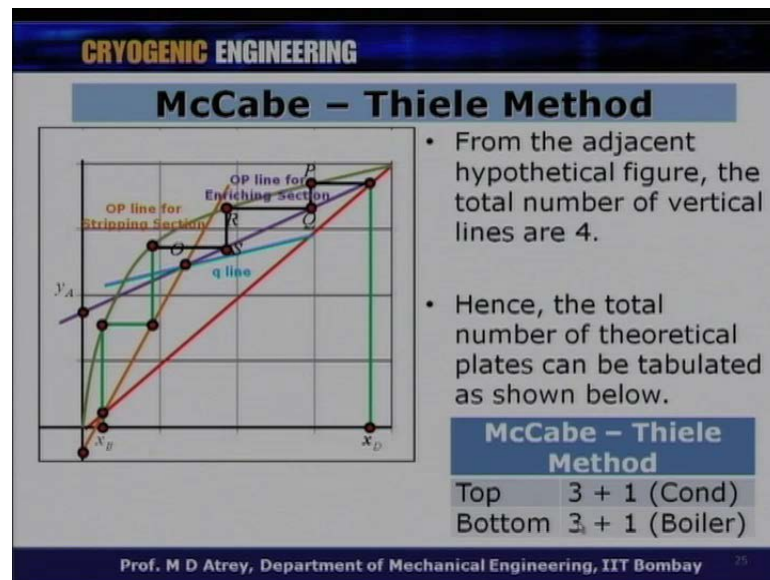
As mentioned earlier, each vertical line indicating a plate, the total number of vertical lines in top and bottom section together with boiler and condenser surfaces gives the total number of theoretical plates required.

So, these are the plates in the column. In addition to that we have got a condenser surface and you have got a boiler surface. Complete all these things will give you theoretical plate requirements. So, I can say 3 plates plus condenser as a requirement in enriching section similarly, 3 plates in the bottom the stripping section plus boiler is a requirement for the stripping section.

From the adjacent hypothetical figure, the total number of vertical lines are 4. These are the just the number of point, if I see the vertical lines; however, you have to see that this is the half line also. Therefore, we have to take the integral of that basically you have to take the round it off. Hence the total number of theoretical plates can be tabulated as from these techniques the top is 3 plus 1, that is 3 number of plates plus condenser plate, bottom is 3 number of plates plus the boiler plate.

So, these are theoretically calculated number of plates by this technique, by this graphical technique given by McCabe Thiele in the top and the bottom or enriching section and the stripping section **alright**. I think this is the theory behind the calculation of number of plates.

(Refer Slide Time: 27:00)



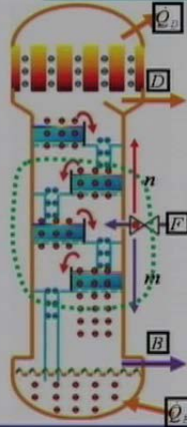
From the adjacent figure, it is important to note that, during the heat exchange process that is along the vertical line the liquid composition x is constant. So, when you come down, x remains constant when you go horizontal, y remains constant.

Similarly, along the horizontal line the vapor composition y remains constant. Also note that in moving from top to bottom the McCabe Thiele diagram starts with a horizontal line here on the top while it ends in the vertical line. Why does this happen? This happens because the liquid will come from top to bottom. This is because liquid flows downwards and it is represented by a vertical line and therefore, we do not have a liquid going down from here because it is no plate below; however, this plate is the top. So, this is the argument behind we go horizontal here we go vertical at the end.

(Refer Slide Time: 28:27)

CRYOGENIC ENGINEERING

McCabe – Thiele Method



- In the earlier lecture, we have balanced moles and enthalpy for top, bottom and mid-section separately.
- But considering the column as a whole, the following equation hold true for mole balance. $F = B + D$
- Multiplying the above equation with the mole fraction of a particular component, we get mole balance for that component.

Prof. M D Atrey, Department of Mechanical Engineering, IIT Bombay

With this background now, let us do the entire column mass balance and energy balance which is required in every problem. In the earlier lecture, we have balanced mole and enthalpy for top bottom and mid section respectively but, consider the column as a whole the following equation hold true what is coming is F, what is leaving is D, what is leaving is B, what are the energies the condensation that the cooling effect (()) effect required is $q D$ and whatever energy is given for boiler is $q B$. These are all energies, these are all mass which is entering and leaving the column.

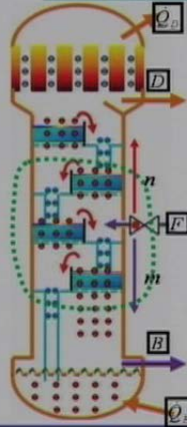
So, simple mass balance is F is equal to B plus D. Whatever feed number of moles B and D are leaving multiplying the above equation with the mole fractions of a particular components, suppose I want to do balance for only nitrogen or only oxygen correspondent to that I have to multiply by the mole fraction of that component x F into F let us say for nitrogen then $x B$ B into x plus $x D$ D is giving you mole balance for this component.

Similarly, for the entire column the enthalpy or the energy balance can be written as what is entering and what is leaving. So, $q B$ is entering $h F$ F is entering $q D$ is the energy which is taken out. So, where the condensation happens and D and B are leaving. So, correspondent enthalpies $h D$ D and $h B$ B.

(Refer Slide Time: 29:27)

CRYOGENIC ENGINEERING

McCabe – Thiele Method



- Similarly, for the entire column, the enthalpy or the energy balance can be written as

IN	OUT
Q_B	Q_D
$h_F F$	$h_D D$
	$h_B B$

$$\dot{Q}_B + h_F F = \dot{Q}_D + h_D D + h_B B$$

- All the arguments regarding the plate calculations would be clear in tutorial as explained in next slide.

Prof. M D Atrey, Department of Mechanical Engineering, IIT Bombay

So, if I do the mass energy balance, these are in is equal to out. This is what equation and therefore, all the arguments regarding the plate calculations would be clear in the tutorial. We have to consider this energy balance; we have to consider this mass balance. Having calculated these things we have to now understand the operating line equations q line equations equilibrium curves and then go for the tutorial which is explained from the next slide from where we will understand how to calculate the theoretical number of plates it. So, now, let us come to the tutorial.

(Refer Slide Time: 30:19)

CRYOGENIC ENGINEERING

Tutorial

- Consider a rectification column for N_2 and O_2 separation operating at 1 atm. Determine the number of theoretical plates required to yield **97% N_2** at top and **95% O_2** at bottom. Feed stream is **50% N_2** and **50% O_2** . Molar fraction of liquid in feed stream is **0.7** mole liquid/mole mixture. The desired flow rate at the bottom product is **20** mole/sec and the heat removed in the condenser at top of the column is **500** kW.

Prof. M D Atrey, Department of Mechanical Engineering, IIT Bombay

Consider a rectification column for nitrogen and oxygen separation plant operating at 1 atmosphere. Determine the number of theoretical plates required to yield 97 percent nitrogen at top and 95 percent oxygen at the bottom which is talking about the purity curve requirement. Feed stream is 50 percent this is x_F . Feed stream is 50 percent nitrogen and 50 percent oxygen. Mole fraction of liquid in feed stream is 0.7 this is talking about quality q , 0.7 mole liquid per mole mixture. The desired flow rate at the bottom which is B is 20 mole per second and the heat removed in the condenser at top is 500 kilo watt.

So, various parameters are given. We've got a purity requirement given, we've got x_F requirement given, we have got a quality given, we have got a desired flow rate from the bottom and the top have been given, we have to calculate and we've got a condenser cooling effect also given as 500 kilo watt. What you had calculate is theoretical number of plates for this.

So, let us convert that into what is given, working pressure 1 atmosphere, mixture is of nitrogen oxygen feed stream 50 percent nitrogen 50 percent oxygen, bottom flow rate 20 moles per second which is B , feed line is 0.7 as q . So, we know in fact, the slope of the line.

For the above mixture the requirement of nitrogen is x_D that is 97 percent which is x_D requirement of oxygen is 95 percent and therefore, x_B happens to be amount of nitrogen in this oxygen which is 0.05. So, this happens to be kind of impurity for oxygen. So, calculate total number of theoretical plates for this problem. So, how do we do?

Start with a molar balance F is equal to B plus D ; we've got a mole balance for particular component also let us say for nitrogen. So, for nitrogen we've got a 50 percent nitrogen in the feed which is x_F 0.5, x_B is the oxygen requirement the bottom which is 0.05 that this is amount of nitrogen in oxygen which is B 0.05 and x_D is the amount of nitrogen in condensate which is 0.97 actually talks about purity of nitrogen condensed to be taken from the top and B is equal to 20.

So, F is equal to 20 plus D from this equation, similarly, I've got a expression for the mole balance of nitrogen putting the values over here. Solving these two equations what we get is F is equal to 39.14 moles per second; D is equal to 19.14 moles second.

(Refer Slide Time: 31:52)

CRYOGENIC ENGINEERING

Tutorial

$F = B + D$ • Mole balance

$x_F F = x_B B + x_D D$ • Mole balance for N_2

Data

x_F	0.5
x_B	0.05
x_D	0.97
B	20

$F = 20 + D$ $0.5F = (0.05)(20) + (0.97)D$

- Solving, we have
- $F = 39.14$ mole/sec, $D = 19.14$ mole/sec.

Prof. M D Atrey, Department of Mechanical Engineering, IIT Bombay

So, F is coming as 39 moles what is coming out is 19 as condensate, what is coming on the bottom is twenty moles per second. So, we have got F B and D values.

Now, what we do energy balance. So, enthalpy balance. Calculate the value of q B because q D is given to us as 500 kilo watts we know, we have to calculate enthalpies for these values, calculate enthalpies for the feed depending on the composition and get the value of q b.

(Refer Slide Time: 32:52)

CRYOGENIC ENGINEERING

Tutorial

- Enthalpy balance

$\dot{Q}_B = \dot{Q}_D + h_D D + h_B B - h_F F$

- Fraction of stream in feed

$q = \frac{H - h_F}{H - h}$

- Rearranging, we have

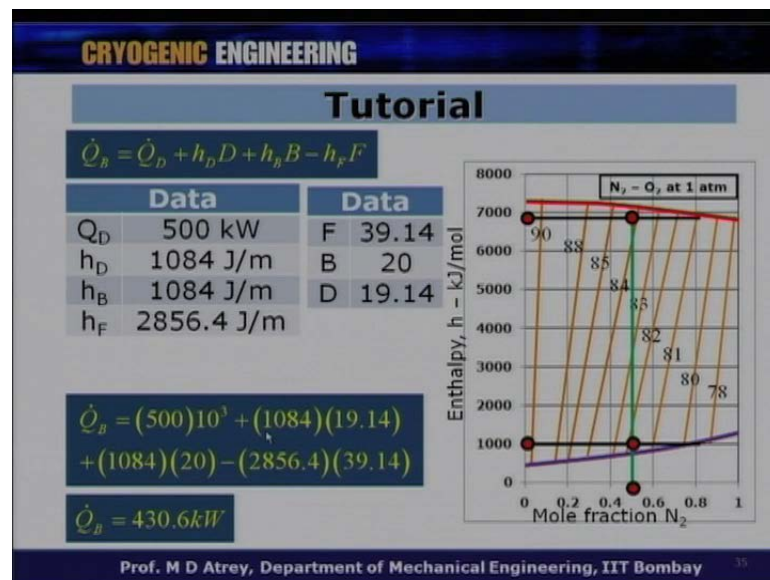
$h_F = qh + (1 - q)H$

Prof. M D Atrey, Department of Mechanical Engineering, IIT Bombay

So, fraction of stream in feed is q which is known to us, enthalpy h_F is equal to $q h$ plus 1 minus $q h$ which is coming from this quality expression. Now let us find out enthalpy, let us locate the point as 0.5 on the bottom 50 percent nitrogen 50 percent oxygen corresponding to this point, find out enthalpy capital h and small h . So, small h is 1084 and capital h is 6992 .

And this will not change and therefore, we will assume that the capital h and small h are now constant. So, related to this if we calculate h_F , putting these values over here, h_F is equal 2856 . So, point lies somewhere in between it is a two phase flow isn't it, because it is a liquid content anyway that is been given correspond that corresponded to 0.7 liquid which is a q we got a h_F as a enthalpy of the feed.

(Refer Slide Time: 33:35)



Putting these values we've got a q_D as 500 kilo watt h_D 1084 h_B is same irrespective of the composition h_F is these, $F B D$ we know, putting these values over here what you get q_B is 430.6 kilo watt. So, we know q_B we know q_D now.

(Refer Slide Time: 34:17)

CRYOGENIC ENGINEERING

Tutorial

• Operating line for **Enriching Section**

$$\frac{D}{V_n} = \frac{H_n - h_{n+1}}{\frac{Q_D}{D} + h_D - h_{n+1}}$$

Data	
Q_D	500kW
H_n	6992 J/m
h_{n+1}	1084 J/m
h_D	1084 J/m
D	19.14 mol/s

$$\frac{D}{V_n} = \frac{6992 - 1084}{\frac{500000}{19.14} + 1084 - 1084}$$

$$\frac{D}{V_n} = 0.226$$

$$\frac{L_{n+1}}{V_n} = 1 - \frac{D}{V_n}$$

$$\frac{L_{n+1}}{V_n} = 1 - 0.226$$

$$\frac{L_{n+1}}{V_n} = 0.773$$

Prof. M D Atrey, Department of Mechanical Engineering, IIT Bombay

So, we want to ultimately find out the operating line for enriching section, operating line for stripping section and q line. So, what is operating line for enriching section? For which I should know the slope of the line I should know the y component of this line.

For calculating the operating line, equation, I should know D by V D by V given by this equation because, D by would give me L by V, if you know they are related D by V is related to L by V. So, these are the different values we have just calculated. Putting those values over here I get D by V equal to 0.226. Once I get D by V, I know L by V or L n plus 1 by V n is equal to 1 minus D by V n and therefore, L n plus by V n is 0.773.

Now, this is nothing but the slope of the operating line for the enriching section. So, L by V is known for the enriching section as 0.773.

So, if I put these values and if I get D by V value putting over here I've got a equation for x D is 0.97 put all those over here L by V is 0.773 x n plus 1 D by V x D. This is my enriching section operating line equation y is equal to 0.773 x n plus 1 plus 0.22, slope of this line is 0.773, y intercept is 0.22. This line intersect y is equal to x at x is equal to x D, I know both these points y intercept and x is equal to x D, I can draw graphically the operating line for the enriching section.

(Refer Slide Time: 35:09)

CRYOGENIC ENGINEERING

Tutorial

- Operating line for **Stripping Section**

$$\frac{B}{V_m} = \frac{H_m - h_{m+1}}{\frac{Q_B}{B} - h_B + h_{m+1}}$$

Data	
Q_B	430.6 kW
H_m	6992 J/m
h_{m+1}	1084 J/m
h_B	1084 J/m
B	20 mol/s

$$\frac{B}{V_m} = \frac{6992 - 1084}{\frac{(430.6)10^3}{20} - 1084 + 1084}$$

$$\frac{B}{V_m} = 0.274$$

$$\frac{L_{m+1}}{V_m} = 1 + \frac{B}{V_m}$$

$$\frac{L_{m+1}}{V_m} = 1 + 0.274$$

$$\frac{L_{m+1}}{V_m} = 1.274$$

Prof. M D Atrey, Department of Mechanical Engineering, IIT Bombay

Having done this now, I would like to do the same thing with the stripping section. So, operating lines for stripping section for which I will have to calculate B by V m now. Put all those values which we have just calculated, q B, enthalpies, B etcetera. Put these values over here. Calculate B by V m, B by V has come out here. Correspondent to that find out L m by V m which is nothing but, the slope of the stripping section. These are all calculations now. What is important to calculate B by V, once you get B by V calculate L by V.

So, now I know the slope of the stripping section line, operating line of the stripping section, I will put these values as I have done for the earlier values, putting all these values what I get equation for the operating of the stripping section. So, this has got a slope of 1.274 and y intercept of minus 0.013. So, again I could draw stripping section line, operating line for the stripping section.

(Refer Slide Time: 36:50)

CRYOGENIC ENGINEERING

Tutorial

- Equation of **Feed Line**

$$q = \frac{H - h_f}{H - h}$$
$$y = \frac{q}{q-1}x + \frac{x_F}{1-q}$$
$$y = \frac{0.7}{0.7-1}x + \frac{0.5}{1-0.7}$$
$$y = -2.34x + 1.67$$

Data	
H	6992 J/m
h_f	2856.4 J/m
h	1084 J/m
x_F	0.5

Prof. M D Atrey, Department of Mechanical Engineering, IIT Bombay

The third is a equation for the feed line. The feed is given by this formula; the q is given by this formula. I've got all other variables available with me. The equation for the feed line is q upon q minus 1 x plus these, putting these values over here, I get y is equal to minus 2.34 x and plus 1.67. That means the slope of the feed line or the q line is minus 2.34. This gives you the direction of the q line is the y intercept of the q line. And again we know that this line intercept y is equal to x at x is equal to x F, x F is equal to, 0.5 if you remember.

So, I have got all the three lines. So, in a summary now before I go to a graphical solution now, I need to know what are my equations. So, operating line for enriching section is given by this equation, slope 0.773, y intercept 0.22.

(Refer Slide Time: 37:29)

CRYOGENIC ENGINEERING

Tutorial

- Summarizing, we have the following.
- OP line for enriching section : $y_n = 0.773x_{n+1} + 0.22$
- OP line for stripping section : $y_m = 1.274x_{m+1} - 0.013$
- q line : $y = -2.34x + 1.67$
- The stair casing procedure is shown on an excel sheet to have a better understanding of the method.

Prof. M D Atrey, Department of Mechanical Engineering, IIT Bombay 41

Operating line for stripping section is given by these slope 1.274, y intercept is minus 0.013 and q line y is equal to minus 2.34 plus 1.67. So, I now know the equation of q line also.

With this now, I am ready to go for my stair casing business, provided I know the equilibrium curve for this particular composition and pressure also. So, the stair casing procedure is shown on excel sheet to have a better understanding of the method now. I do not want to draw it on here, but, I will now go to the excel sheet. You will have to use your graphs paper to calculate these because; it cannot be done on paper. So, please have a graph paper, use the graph paper, plot these lines and then you will be able to do the stair casing which will ultimately give you the number of plates in stripping section and number of plate in the enriching section.

(Refer Slide Time: 38:36)

CRYOGENIC ENGINEERING

Tutorial

- From the excel sheet, it is clear that the total number of vertical lines are **10**.
- Therefore, the total number of theoretical plates for this column can be tabulated as shown below.

McCabe - Thiele Method	
Enriching Section	4 + 1 (Condenser)
Stripping Section	6 + 1 (Boiler)

Prof. M D Atrey, Department of Mechanical Engineering, IIT Bombay

So, let us see the excel sheet now. So, from the excel sheet, it is clear that the total number of vertical lines are 10. You've just seen that what are the vertical lines over in the enriching section and the stripping section and we found that number of vertical lines are 10. Therefore, the total number of theoretical plates for this column can be tabulated as shown over here. Number of plates will be going to be more than this 10 basically, but, vertical lines are 10.

So, what I get is McCabe Thiele method. The enriching section has 4 plus 1 that is 4 number of plates in the enriching section plus 1 condenser and the stripping section has 6 number of plates and 1 for the boiler **alright**.

Having understood this now, having understood how to draw this excel sheet, how to draw this on the graph paper, how to calculate number of plates in enriching section and stripping section, now let us see different parameters which affects the number of plate and that is the most important thing.

So, let us see the number of plates, how do they vary with different parameters. In general the equation for an operating line is given as y is equal to L by V x and K . This is what we know. So, definitely L by V plays a very important role. The slope of the line plays a very important role.

(Refer Slide Time: 39:40)

CRYOGENIC ENGINEERING

Parametric Study

- In general, the equation for an OP line is given as
$$y = \left(\frac{L}{V}\right)x + K$$
- It is clear that the slope of an OP line is a function of **L** and **V**. For an enriching section, the slope can be reduced by decreasing **L**. This is possible by
 - a) Drawing out more liquid from condenser, that is by increasing **D**.
 - b) Reducing the condenser heat load **Q_D**.

Prof. M D Atrey, Department of Mechanical Engineering, IIT Bombay 43

It is clear that the slope of an operating line is a function of L and V, for an enriching section the slope can be reduced. So, if I want to change the slope, I can change the slope by either changing L or by V. For an enriching section what we have is a condenser. This condenser we will decide depending on the available cooling effect or depending on the value of D because, D is the number of moles in the condenser taken out

So, how do I change the value of L? The slope can be reduced by decreasing L. This is possible by drawing out more liquid from condenser that is increasing D. So, I have **I have** kept the cooling effect the same; that means, I get condensate, some condensate of which now I am taking more D out, instead of taking some quantity, I am taking more D out; that means, less liquid will go down. So, L is going to decrease and when L decreases L by V also would decrease.

The other thing is instead of taking not more than D, I will do what I do reducing the condenser heat load; that means, I've got less cooling effect available.

So, whatever vapor comes on the top not I do not have available cooling effect. I have got a less cooling effect and therefore, depending on q D, depending on the available cooling effect, 300 kilo watts, 400 kilo watt. I will decrease the condensate overall condensate only; that means, the value of L will decrease.

So, now let us see the effect of how do this things happen on the excel sheet. Let us come back to the excel sheet to understand these. So, we will see this effect and similar this thing can be understood for the stripping section also. So, in stripping section the slope is reduced by increasing V. So, I can control the slope now depending on what do I do on the boiler.

(Refer Slide Time: 41:23)

CRYOGENIC ENGINEERING

Parametric Study

- Similarly for the stripping section, the slope is reduced by increasing **V**. This is possible by
 - Drawing out less liquid from the boiler, that is by decreasing **B**.
 - By increasing the boiler heat input **Q_B**.

Enriching Section			Stripping Section		
D	L/V	Plates	B	L/V	Plates
19.15	0.77	4 + 6	20	1.27	4 + 6
28.76	0.66	4 + 8	10	1.13	3 + 5
Q _D (kW)	L/V	Plates	Q _B (kW)	L/V	Plates
500	0.77	4 + 6	430	1.27	4 + 6
300	0.62	5 + 8	630	1.19	3 + 5

Prof. M D Atrey, Department of Mechanical Engineering, IIT Bombay

So, I got the q_B is the heat which is supplied to the boiler. The slope can be reduced by increasing V so L by V can be decreased by increasing the denominator, by increasing the V and how do I do this thing? This is possible by drawing out less liquid from the boiler that is by decreasing B. So, I can take less B and therefore, a more V is going to back **ok**.

I will not use all the liquid which is coming down, but I am going to take less liquid and therefore, remaining will get vaporized and therefore, V will increased. So, this is one of the ways. The other way is by increasing the boiler heat q_B or either supply a more heat and therefore, V will increased more and more liquid will get vaporized therefore, as a result of this L by V will decrease.

So, depending on what we just saw, the excel sheet, I am just summarizing here what did we see. We saw that if I my D earlier above 19.15 correspondent to these my L by V was 0.77 and number of plates therefore, were 4 and 6, 4 in enriching section and 6 in stripping section. If I increase the value of D which is what we just saw, if I increase the

D, amount of condensate taken from the condenser, my L by V is going to decrease **alright**. So, **77** 0.77 got decrease to 0.66 as a result of L by V decrease the line shifted towards the equilibrium section. Because of the slope change the operating line shifted towards the equilibrium line, equilibrium curve which resulted in increased number of plates.

So, now I need 4 plus 8; that means, I need 12 number of plates instead of 10 plates over here. So, this was a effect which we can see what happens with the decrease in the slope, **what happens in the decrease in the slope** of the enriching section. So, as L by V decrease the operating line shifted towards the equilibrium curves which resulted in the increase of number of plates.

So, simple fact. If your lines slope shift towards the equilibrium curve, the number of plates would increase. The second parameter which we just saw was q D. If I decrease q D from 500 to 300; that means, if I decrease the condensation, cooling effect which is available here, if I decrease **500** from 500 to 300 my L by V decrease from 0.77 to 0.62. Substantial decrease, this also shifted the lines towards the equilibrium curve and which resulted in increasing number of plates from 10 to 30.

So, here I have to put 3 additional plates look. One each additional plate is a very important design change is a very important investment that has to be done on a addition of column basically. So, this has to be understood very much.

So, decrease D resulted in increased number of plate. Stripping section I can decrease B this is what we say. I can decrease L by V by decreasing B. So, what I did, I decrease B from 20 to 10, L by V decreased from 1.27 to 1.13. The number of plates now the slope has decreased for the stripping section, number of plates change from 10 to 8, number of plates were 10, now it has come to 8 only; that means, the stripping sections slope, if you decrease the stripping section slope, the stripping line moves towards the y is equal to x line. It goes away from the equilibrium section which results in the decrease of the plate in case of stripping section **alright**.

Similarly, what I could do is increase the value of q B; that means, increase the amount of to be given which will increase V and therefore, L by V decrease. So, if I increase the q B from 430 to 630, my L by V decreases from 1.27 to 1.19. Again this moves the curves towards the y is equal to x line away from equilibrium curve and therefore,

number of plates get reduced from 10 to 8. So, please understand the what happens if I do L by V change enriching section and if I do L by we change in a stripping section, this is what it shows.

(Refer Slide Time: 45:49)

CRYOGENIC ENGINEERING

Parametric Study

- From the excel sheet, it is clear that
 - The minimum possible slope of OP line for stripping section is 1.
 - The maximum possible slope of OP line for enriching section is 1.
- For the slope of OP lines as 1, it is obvious that the lines coincide with the **y=x** diagonal.
- Also, as these lines approach **y=x** diagonal, the column has minimum number of plates.

Prof. M D Atrey, Department of Mechanical Engineering, IIT Bombay 65

Extending parameters study from the excel sheet it is clear that the minimum possible slope for operating line for stripping section is 1 which is when it approaches the y is equal to x line for the stripping section. While the maximum possible slope for operating line is go to be enriching section is going to 1. So, for operating line for enrich section, it will have a maximum slope to be equal to 1 while for the operating line because if I reduce the slope, it will come the minimum value that can come is 1 **ok**.

So, here it is minimum value for the stripping section, here it is going to be maximum value for the enriching section. for the slope of operating line as equal to 1, suppose the slope is 1 for both, minimum for these and maximum for these. Both the lines the operating line will get merge to y is equal to x which is diagonal and in this case, as these lines approach to y is equal to x diagonal the column will have minimum number of plates.

This is just hypothetical situation basically L by V is equal to 1; that means, you are not getting any product. Whatever V is going up everything gets condensed, but, just to understand that if I do not to want any product and I just want to operate the column in a steady state manner for a both the operating line will get merge to y is equal to x as a

diagonal the column has a minimum number of plate and that is what I am saying that if
if this operate.