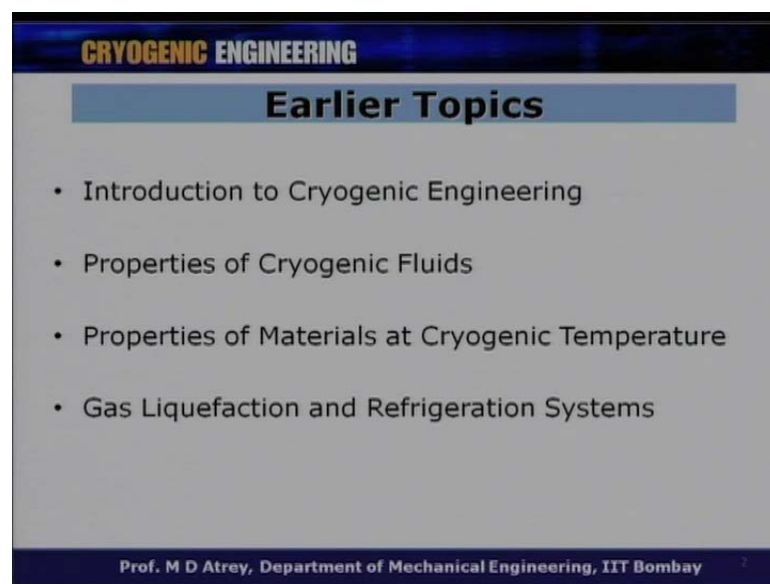


**Cryogenic Engineering**  
**Prof. M. D. Atrey**  
**Department of Mechanical Engineering**  
**Indian Institute of Technology, Bombay**

**Lecture No. #18**  
**Gas Separation**

So, welcome to the eighteenth lecture of the NPTEL course on cryogenic engineering.

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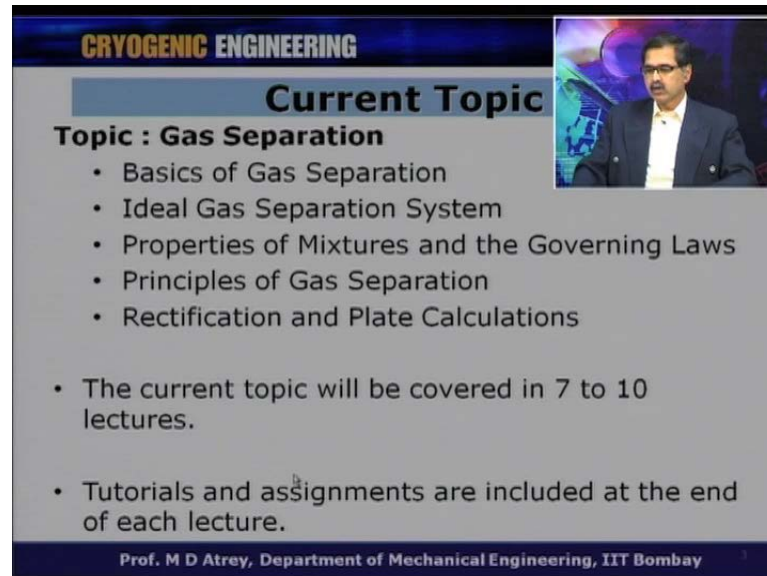


We have done various topics still now, and the broad topics, we can classify them as first the introduction to cryogenic engineering. We studied the properties of different cryogens. So, properties of cryogenic fluids then we studied the properties of materials at cryogenic temperature, and the last topic which we covered was gas liquefaction and refrigeration System.

Having done all these four topics, now I will be changing some gas and we will be going to now gas separation, which is a very important topic of cryogenic engineering. Having liquefied **gas having liquefied** gases together and air being a mixture of various gases, whenever we liquefy air; we got lot of gases in liquefied form in air. And now we have to separate oxygen, nitrogen and other gases may be argon from liquid air, and therefore, we have to use a gas separation technique. And this is the most important thing, because

gases are needed by every industry now days, and therefore, this topic has got a very special relevance from uses point of view from gas user point of view.

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**CRYOGENIC ENGINEERING**

**Current Topic**

**Topic : Gas Separation**

- Basics of Gas Separation
- Ideal Gas Separation System
- Properties of Mixtures and the Governing Laws
- Principles of Gas Separation
- Rectification and Plate Calculations

• The current topic will be covered in 7 to 10 lectures.

• Tutorials and assignments are included at the end of each lecture.

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So, topic is now gas separation, what we want to study under these are different sub topics, which are basics of gas separation then ideal gas separation system, just as we got an ideal system for every system to compare with we got an ideal gas separation system also.

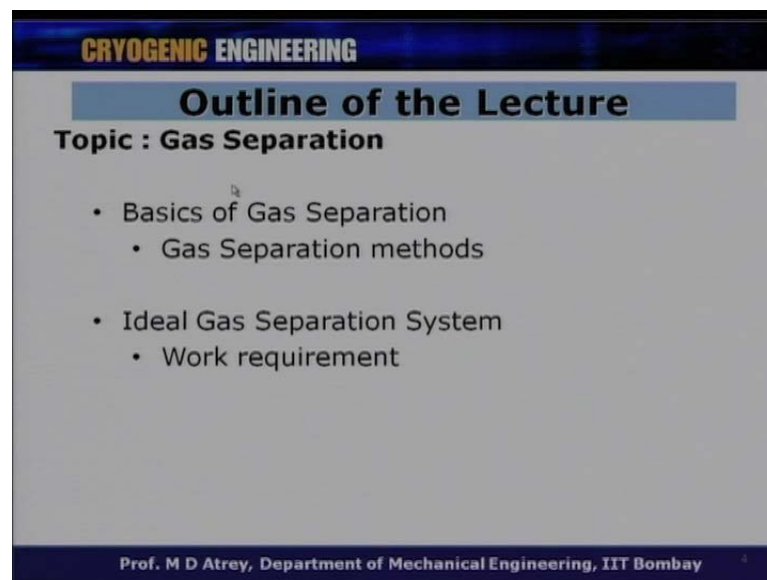
Then we will study properties of mixture and the governing laws. Now, as you possibly know that we got several laws, which govern the mixture and they help us basically in understanding the partial pressure business and the total pressure and the relationship. And they are very important to analyze these mixtures, so, as to get properties of mixture at different temperatures and pressures.

Then will study principles of gas separation in broad. And then separation process itself the rectification and plate calculations. So, plate calculations actually essentially mean that **that** is the column, which has got lot of plates and we have to calculate number of plates involved in that. So, this is the very important topic and which involves, actually it covers bit of chemical engineering also, because it has got some mass transfer aspects associated with it.

We will keep it of course, for mechanical engineers; however, and therefore, from this point of view, I have **I have** basically going to cover this topic from mechanical engineering point of view.

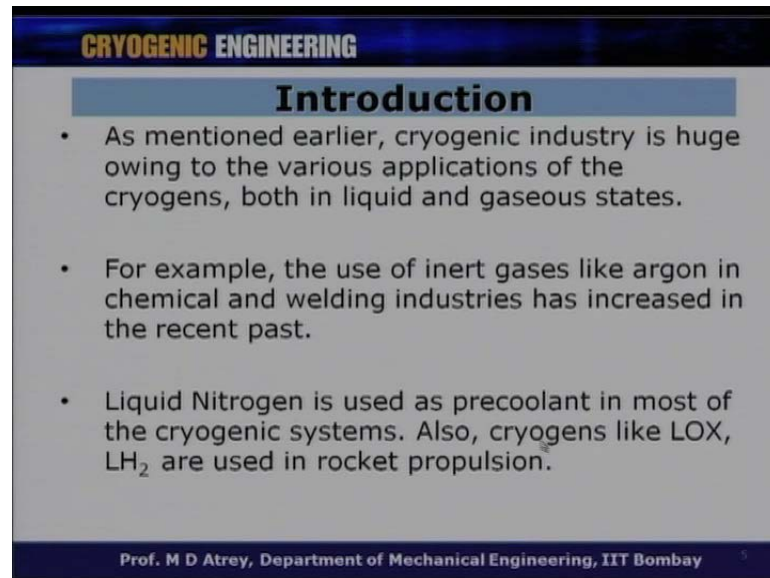
The current topic will be covered in around 7 to 10 lectures, just as we had liquefaction in 7 to 10 lectures. Similarly, as we precede ahead the number of lectures possibly will be around 7 to 10. And the tutorials and assignments are also included at the end of each lecture.

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So, in this topic, what we want to learn is? The outline of today's lecture will be the basics of gas separation. In which, we will study some Gas Separation Methods and then we will find what is the Ideal Gas Separation System and we will find what is the ideal Work Requirement for a ideal gas separation system? This two sub topics will be considered will be **(( ))** in these lecture of first lecture of gas separation **right**.

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**CRYOGENIC ENGINEERING**

### Introduction

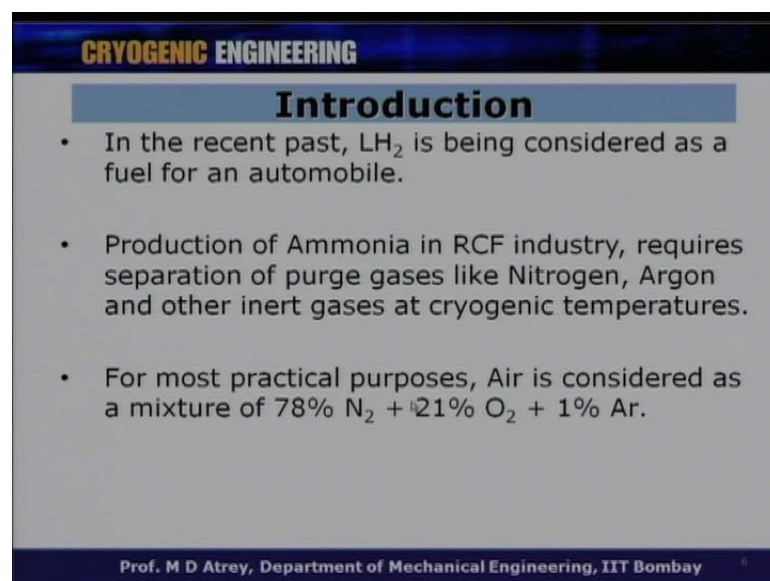
- As mentioned earlier, cryogenic industry is huge owing to the various applications of the cryogens, both in liquid and gaseous states.
- For example, the use of inert gases like argon in chemical and welding industries has increased in the recent past.
- Liquid Nitrogen is used as precoolant in most of the cryogenic systems. Also, cryogens like LOX, LH<sub>2</sub> are used in rocket propulsion.

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As mentioned earlier, the cryogenic industry is huge owing to the various applications of the cryogens, both in liquid and gaseous states. For example, the use of inert gases like argon in chemical and welding industries has increased in the recent past. Basically, I am highlighting the utility of these gases in various industries.

Liquid Nitrogen is used as precoolant in most of the cryogenic systems. Also, cryogens like Liquid Oxygen, Liquid Hydrogen are used in rocket propulsion.

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**CRYOGENIC ENGINEERING**

### Introduction

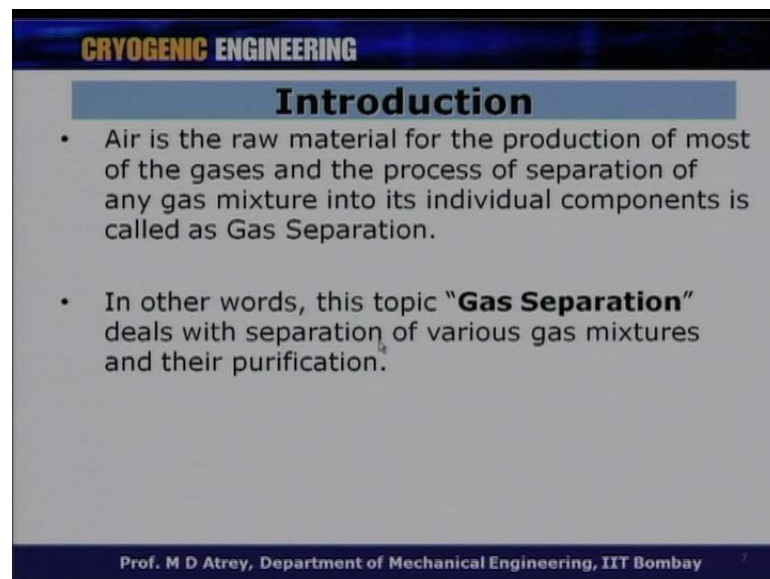
- In the recent past, LH<sub>2</sub> is being considered as a fuel for an automobile.
- Production of Ammonia in RCF industry, requires separation of purge gases like Nitrogen, Argon and other inert gases at cryogenic temperatures.
- For most practical purposes, Air is considered as a mixture of 78% N<sub>2</sub> + 21% O<sub>2</sub> + 1% Ar.

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In the recent past, Liquid Hydrogen is being considered as a fuel in an automobile. The production of ammonia in RCF industry for example, the fertilizer industry requires separation of purge gases like Nitrogen, Argon and other inert gases, this production has to be done at cryogenic temperature and therefore, separation of gases play a very important role in fertilizer industry also. For most practical purposes, Air is considered as a mixture of 78 percent Nitrogen, 21 percent Oxygen and 1 percent Argon. So, if you want use Nitrogen, Oxygen and Argon, we have to basically separate them out from air.

The other ingredients are Helium, Neon, Krypton which occurs in negligible quantities. So, normally as for as Helium, Neon, Krypton are considered **the** you know take them out from air becomes rather difficult. So, normally air will be used to obtain Nitrogen, Oxygen and Argon, which are abundantly used in abundantly required in the industries.

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**CRYOGENIC ENGINEERING**

### Introduction

- Air is the raw material for the production of most of the gases and the process of separation of any gas mixture into its individual components is called as Gas Separation.
- In other words, this topic "**Gas Separation**" deals with separation of various gas mixtures and their purification.

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So, air serves as a raw material. Air is a raw material for the production of most of the gases and the process of separation of the gas mixture into its individual component is called Gas Separation, that is what normally will be defined as? In other words, this topic of Gas Separation deals with separation of various gas mixtures and their purification.

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The slide is titled "CRYOGENIC ENGINEERING" in a blue header. Below it, the main title "Gas Separation" is centered in a white box. A bulleted list follows, detailing four common gas separation techniques. To the right of the list is a small inset photograph of a man in a suit, identified as Prof. M D Atrey. The footer of the slide identifies him as "Prof. M D Atrey, Department of Mechanical Engineering, IIT Bombay".

- Different techniques of gas separation commonly used are
  - Synthetic membranes
  - Adsorption
  - Absorption
  - Cryogenic distillation

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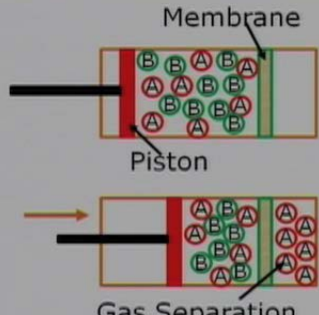
Now, how do we separate GAS mixture? There are various techniques available, we will just touch upon those techniques and (( )) come down to the technique of cryogenic gases separation. So, what are the different techniques? To separate various constituents of a gas mixture, one is using synthetic membranes; we will see each of these techniques in brief to support, to understand how the separation occurs using synthetic membranes? The second technique is adsorption, which is the physical phenomenon as possibly most of you know. The absorption other technique of separation of a gas mixture and then the cryogenic distillation.

So, broadly there are four methodologies of which the first three are normally at room temperature. While the third the last one is cryogenic distillation or separation at cryogenic temperature or at low temperature. So, we will first study what these three techniques are? And then we would go to cryogenic distillation in detail.

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**CRYOGENIC ENGINEERING**

### Gas Separation



- Synthetic membranes are the porous media which allow only a certain gas molecules to pass through.
- The membrane in the figure allows only **Gas A** to pass and hence the separation occurs.
- For example, a thin sheet of palladium allows **H<sub>2</sub>** to pass through.

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So, let us see what synthetic membranes mean? And how does

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So, we can see from this figure, this figure shows there the membrane in some cylinder and this membrane allows to separate a mixture of A and B molecules or a mixture of A and B components of gases and there is the piston which you see from left side.

Synthetic membrane of the porous media, which allows only a certain gas molecules to pass through. The membrane in the figure allows only GAS A to pass and hence the separation occurs. So, in this particular figure, what you say this membranes allows only A to pass through it while B cannot pass through it. So, what will happen? If I go on pushing this piston on the right side, the A will get A will pass through the membrane and on the right side of A, as you can see right side of membrane, what you can see is? Only the component A; that means, only the gas molecule with gas B gas will come through this membrane, while the B gas would be always on the left hand side, because B cannot pass through the synthetic membrane.

For example, a thin sheet of palladium allows only Hydrogen gas to pass through. So, if I use palladium material as membrane material and then have a mixture of various gases, in which one of the components is Hydrogen and then if, I use such kind of a arrangement. Only hydrogen will be in this particular location, where A is right now,



because this membrane will allow only Hydrogen to pass through it while other gas members will not be able to pass through it. So, this is the way the membranes can be used. There being used various membranes, they got a preferential treatment preferential passage for particular gas and according to which they could be used.

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**CRYOGENIC ENGINEERING**

### Gas Separation

- Adsorption is the physical processes in which only a certain kind of gas molecules are adhered to the adsorbing surface.
- The adsorbate in the figure adheres only **Gas A** to the surface and hence the separation occurs.
- For example, finely divided Nickel adsorbs hydrogen on to its surface.

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The second technique is adsorption. So, adsorption is a physical process in which only a certain kind of gas molecules are adhere to the adsorbing surface. Most of you know this adsorbent that you got is surface phenomena and if you got A and B gases, then only one of the gases can adhere to the mixture or it can get adsorbed on this adsorbent while other gas cannot.

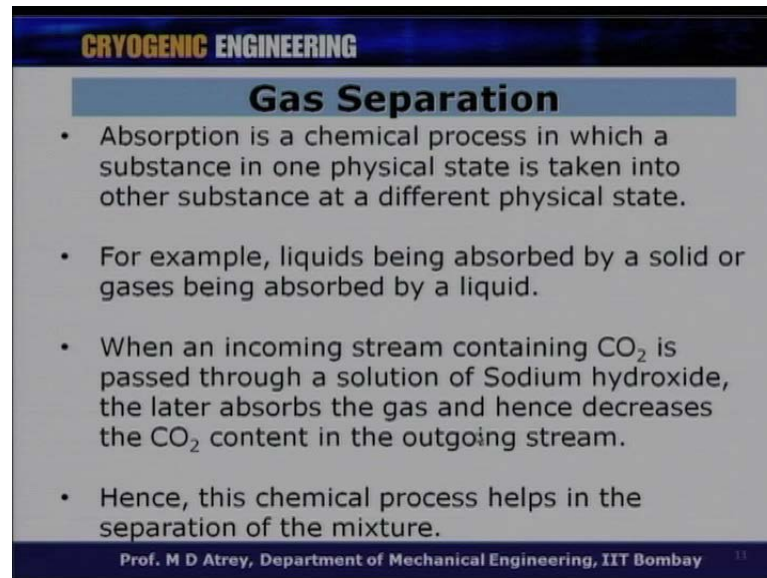
So, adsorbate in the figure adheres only gas A to this adsorbate and it the gas only can get adsorb on the surface on this adsorbate. So, it allows only A to adhere to the surface while B cannot adhere to the surface. In a way therefore, a mixture of A and B will get separated, A will adhere to the gas the adsorbate surface while B will be freely lying over here and then in a way this as separation of this gases A and B.

So, the adsorbate ate in the figure adheres only GAS A to the surface hence the separation occurs. So, what you can see here? The layer here will have only a components adhere to it, while all the B components eventually will be all the land is would be getting adsorbed, while B would remain there meaning which A and B have got separated.



For example finally, divided Nickel Adsorbs Hydrogen on its surface, if I want to separate Hydrogen I can use Nickel as an adsorbent and Hydrogen will get adsorbed on the surface of this Nickel.

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**CRYOGENIC ENGINEERING**

### Gas Separation

- Absorption is a chemical process in which a substance in one physical state is taken into another substance at a different physical state.
- For example, liquids being absorbed by a solid or gases being absorbed by a liquid.
- When an incoming stream containing CO<sub>2</sub> is passed through a solution of Sodium hydroxide, the latter absorbs the gas and hence decreases the CO<sub>2</sub> content in the outgoing stream.
- Hence, this chemical process helps in the separation of the mixture.

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Second is absorption which we know that we got a sponge basically. So, we know that absorption is a chemical process in which substance in one physical state is taken into another substance at a different physical state. So, a solid can take liquid or a liquid can take gas and one physical state is taken into another substance at a different physical state. For example, liquids being absorbed by a solid or gases being absorbed by a liquid.

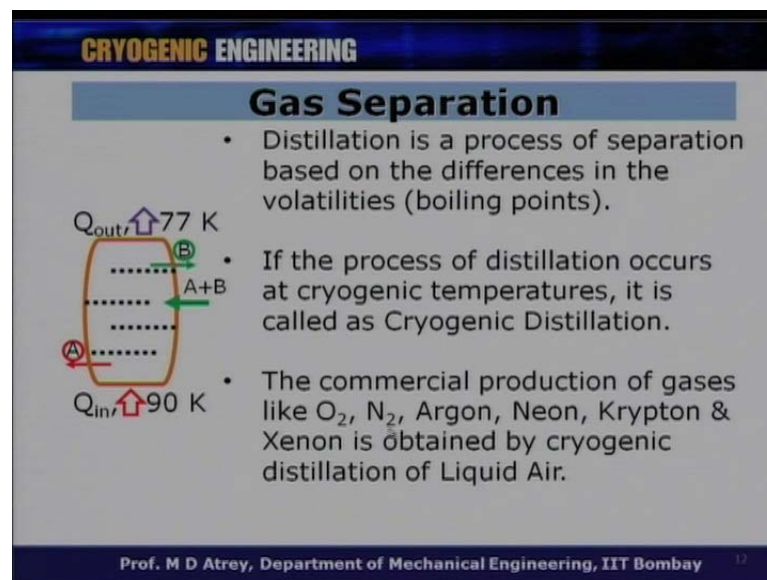
So, what is the example? When an incoming stream containing Carbon Dioxide is passed through a solution of Sodium hydroxide, the latter absorbs the gas and hence decreases the CO<sub>2</sub> content in the outgoing stream.

So, if I got a Sodium hydroxide, Sodium hydroxide will absorb CO<sub>2</sub>. They will actually a chemical reaction will happen between sodium hydroxide and carbon dioxide meaning which if an incoming stream has CO<sub>2</sub>, the quantity of CO<sub>2</sub> can be reduced from this mixture and in a way separation occurs.

So, if I got a sufficient quantity of sodium hydroxide. All the CO<sub>2</sub> from the incoming gases can be removed, because Sodium hydroxide will absorb CO<sub>2</sub> there will be a chemical reaction and maybe some other compounds get formed during this chemical

reaction. In a way, this is the method of separation and therefore, these absorption is basic is used abundantly in the chemical reaction. Hence, this chemical process helps in the separation of the mixture. So, in this case I am removing CO<sub>2</sub> from the mixture by absorbing CO<sub>2</sub> by sodium hydroxide.

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**CRYOGENIC ENGINEERING**

### Gas Separation

- Distillation is a process of separation based on the differences in the volatilities (boiling points).
- If the process of distillation occurs at cryogenic temperatures, it is called as Cryogenic Distillation.
- The commercial production of gases like O<sub>2</sub>, N<sub>2</sub>, Argon, Neon, Krypton & Xenon is obtained by cryogenic distillation of Liquid Air.

Diagram: A schematic of a distillation column. At the bottom, an inlet labeled 'A+B' is shown with a red arrow pointing up. The bottom temperature is labeled 'Q<sub>in</sub> ↑ 90 K'. At the top, an outlet labeled 'B' is shown with a green arrow pointing up. The top temperature is labeled 'Q<sub>out</sub> ↑ 77 K'. The column is represented by a vertical rectangle with horizontal dashed lines inside, indicating trays or stages.

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So, having done membrane adsorption and absorption having understood the basics of this now, let us come to the distillation. Now, distillation is a process of separation based on the differences in volatilities or the boiling points.

So, I have got different gases and all these different gases have got their boiling points and they are different. Depending on the characteristics of these gases, the boiling points should be decided and then the unique boiling point for each gas. Choosing these volatilities or the differences in the boiling point, we have got a process that is called as distillation.

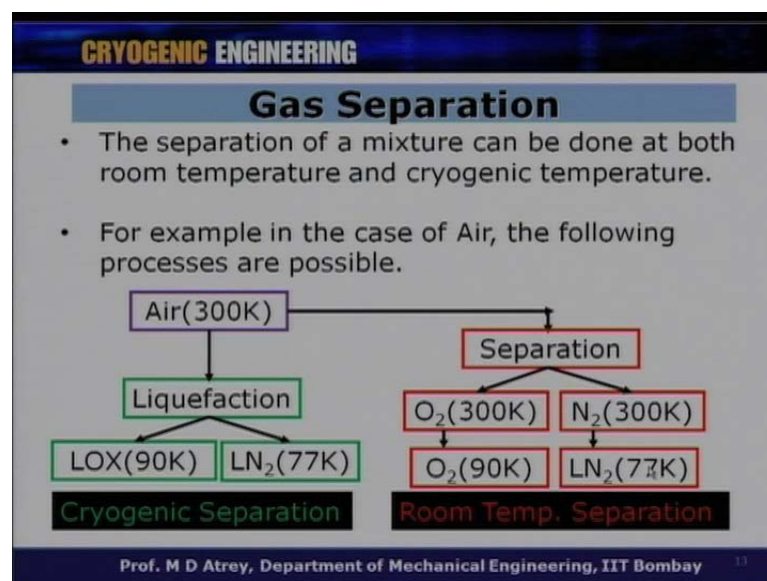
So, here you can see that the something called a schematically I will shown here, something called a distillation column. To which a mixture of A and B, gas A has got a different boiling point, GAS B has got a different volume point and if this mixture enters this distillation column, what you can see is? The mixture A is at the bottom at the end and mixture B is at the top. So, one which has got a higher boiling point of around 90 Kelvin, which is Oxygen boiling point you will have it at the bottom and the liquid you

will get for gas A at the bottom and one can convert this liquid to gas, if one (()) gas or one can get this liquid one (()) liquid a also.

Similarly, on the top what you get is? The lower temperature here the Q is removed is the Q is added. So, you can see that Q in and Q out over here and the lower temperature gas, the lower temperature of boiling point gas. For example, the Nitrogen which has got a boiling point of 77 Kelvin, we will be obtained at the top of this distillation (()), this is just a schematic. So, a mixture of A and B enters the column and at the end, this is now a cryogenic separation. Because the boiling points of this two gases A and B are different, they can get separated A is coming at the bottom, B is going at the top of this column. So, if the process of distillation occurs at cryogenic temperature, this process is called as Cryogenic Distillation.

The commercial production of gases like Oxygen, Nitrogen, Argon, Neon, Krypton and Xenon are obtained by cryogenic distillation of Liquid Air. So, Liquid Air actually a source of all these gases and therefore, if you liquefy air; that means, all these gases are in the liquefied form in the mixture and this mixture can be fade to the column and according to the difference in the boiling points of this gases they can be separated and this is what we going to see in the coming slides and coming lectures?

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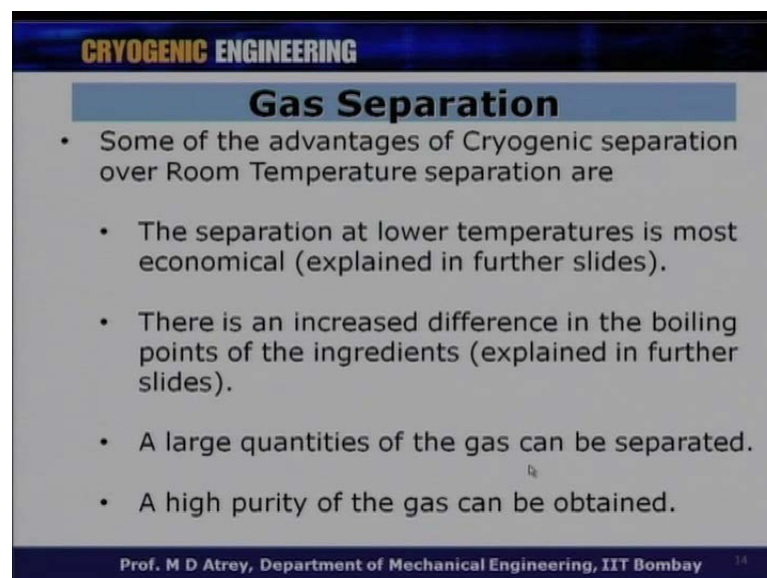
The separation of a gas, separation of a mixture can be done at both room temperature and at cryogenic temperature as we just saw. For example in the case of Air, the

following processes are possible. So, I will just show an example where the cryogenic distillation is done, but there are possibilities that one can do this separation at room temperature also.

So, if I have got air at 300 Kelvin I can liquefy this air at 70 at Kelvin, which is the boiling point of air and from where I can get Liquid Oxygen and Liquid Nitrogen, which I have got boiling point of 90 Kelvin and 77 Kelvin and therefore, because this separation is occurring at low temperature. First I am liquefying air and then I am separating the two gases from liquid air and I call these processes at cryogenic separation.

The other process is now, I have got air first at 300 degree Kelvin and first at room temperature itself at 300 Kelvin itself, I am dividing or I am separating this mixture. Assuming that A is the mixture of Nitrogen and Oxygen only, I have got oxygen at 300 Kelvin and I have got Nitrogen at 300 Kelvin; that means, I am doing a separation of air at room temperature itself and then I get Oxygen at 90 Kelvin and Nitrogen at 77 Kelvin if I want Liquid Nitrogen and Liquid Oxygen. But you can see from here that the separation has occurred at room temperature for which I use special technique called pressure (( )) adsorption, temperature (( )) adsorption and there basically adsorption techniques of separating Oxygen, Nitrogen. These are more widely use these days, it has got it is plus points and negative points.

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**CRYOGENIC ENGINEERING**

### Gas Separation

- Some of the advantages of Cryogenic separation over Room Temperature separation are
  - The separation at lower temperatures is most economical (explained in further slides).
  - There is an increased difference in the boiling points of the ingredients (explained in further slides).
  - A large quantities of the gas can be separated.
  - A high purity of the gas can be obtained.

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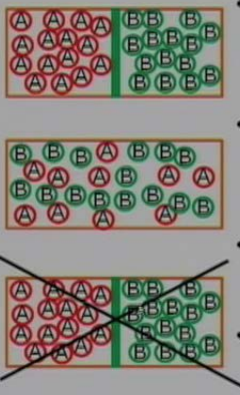
Similarly, cryogenic separation has got its plus points and negative points. Some of the advantages of Cryogenic separation over Room Temperature separation are, The separation at lower temperature is most economical, we will find later that as a separation temperature decreases the power input requirement or a work of separation requirement lessens. So, here we can understand that, there is an increased difference in the boiling point of the ingredients. So, we can do that separation is effective, because of the difference in the volatilities.

A large quantity of the gases can be separated in the Cryogenic distillation and a high purity of the gas can be obtained. If you do the separation at lower temperature, we can get high purity of the gas. We can obtain high purity of the gas at cryogenic temperature. These are the certain advantages of cryogenic separation.

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**CRYOGENIC ENGINEERING**

### Is Gas Mixing Reversible?



- Consider a closed chamber filled with **Gas A** and **Gas B** as shown in the figure.
- Initially, the gases are separated by an impervious wall.
- If the wall is removed, the gases would mix.
- However, the replacement of wall would not result in the separation of gases.

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Let us understand is gas mixing is the reversible process, what do I mean by that? That means, if I mix the gas can I unmix it and can I mix them together. So, meaning which is this process a reversible process.

So, consider a closed chamber filled with Gas A and Gas B as shown here and initially, the gases are separated by impervious wall; that means, A is here and B is here and there is the impervious wall. We should not allow A and B to pass through, they are separate entities in this chamber. If the wall is removed now, the gases would mix. So, suppose I remove this wall we find that A and B are together. Now, if I go back and put this wall

again can I get A and B separated is it possible? This is not possible and therefore, I will say that the replacement of the wall would not result in separation of gas and therefore, I can (( )) that this process is not reversible.

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**CRYOGENIC ENGINEERING**

### Gas Separation

- It is clear that the mixing of two different gases is an irreversible process because unmixing or separation of the mixture requires work input.
- The system in which all the processes are reversible is called as an Ideal System.
- Although in reality such a system does not exist, a system can be conceived to serve the required purpose as explained in the next slide.

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It is clear that the mixing of two different gases is an irreversible process, because unfixing or separation of the mixture requires work input. I have to do some work on this, then only I can unmix the gas. The system in which all the processes are reversible is called Ideal System as you know this. So, mixing and unmixing, if it is possible if it is reversible process, then I will call this Ideal Separation System.

Although, in reality such a system does not exist, a system can be conceived. As you know that every time normally, we deal with an ideal system of separation and therefore, we can conceive such a system where in if I calculate the work of separation or work of mixing and unmixing I have to first conceive an ideal system.


I am going to conceive a system, conceived to serve the required purpose as explained in the next slide. So, I am basically having an Ideal System of gas separation now and how does this work ideal gas separation system?



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**CRYOGENIC ENGINEERING**

### Ideal Separation System



- Consider a closed chamber filled with a mixture of **Gas A** and **Gas B** as shown.
- The temperature and mixture pressure are  **$T_m$**  and  **$p_m$**  respectively.
- The partial pressures of **Gas A** and **Gas B** are given by  **$p_{1a}$**  and  **$p_{1b}$**  respectively.

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So, consider a closed chamber filled with a mixture of Gas A and Gas B as shown over here and there are two pistons one is red and one is black and again I have got a mixture of Gas A and Gas B. These two pistons as showed by arrows can be moved inside and can be taken outside also.

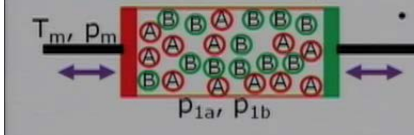
The temperature of this gas mixture is  $T_m$ , the pressure of this gas mixture  $p_m$ . The temperature and the mixture pressure are  $T_m$  and  $p_m$  respectively. The partial pressure of Gas A and Gas B are  $p_{1a}$  and  $p_{1b}$ . Depending on the amount of Gas A and Gas B, we got a partial pressure of Gas A, Gas B termed as  $p_{1a}$  and  $p_{1b}$ .



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**CRYOGENIC ENGINEERING**

### Ideal Separation System



- The chamber has two frictionless opposing pistons made of semi-permeable membranes as shown in the figure.
- As seen earlier, a semi-permeable membrane is a film which allows only one kind of gas to pass through but not the other.

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The chamber has two frictionless opposing pistons made from semi permeable membranes as shown the figure. So, these are not only pistons, there also membranes and they are semi permeable membrane meaning which they allow only one type of Gas A or B to pass through it as we had seen earlier. So, we got a piston which will compress this GAS at the same time, it will separate one of the components of this mixture.

As seen earlier, a semi - permeable membrane is a film which allows only one kind of gas to pass through, but not the other. So, we will this membrane will allow only one type of gas to pass through it.

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**CRYOGENIC ENGINEERING**

### Ideal Separation System

- The left piston (**red**) allows only the **Gas A** to pass through, but not the **Gas B**.
- Similarly, the right piston (**green**) allows only the **Gas B** to pass through, but not the **Gas A**.
- When both pistons are moved inward, the mixture is separated.

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The left piston which is a red, it will allow only a Gas A to pass through it and it will not allow the Gas B to pass through it. So, basically when I start pressing this left piston to right it will let A go through it, but it will not allow B to go through it. Similarly, when I press B on the left side, it will allow only B to pass through it and it will not allow A to pass through it.

Similarly, the right piston allows only Gas B to pass through, but not Gas A. This is what we call as the semi permeable membrane working like a piston in this close chamber. So, as if I go on pressing this two membranes or pistons you can find the separation as started occurring. So, I am getting on the left side of the membrane only A and I am getting right side of this green piston or green membrane only the component B.

So, when I basically reach both the piston reach together A and B would get separated. When the both the pistons are moved inward, the mixture is separated and it would look like this understood. So, I have got a mixture A, I got a Gas A and Gas B separated from mixture of gases A and B as seen here.

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**CRYOGENIC ENGINEERING**

### Ideal Separation System

• Since the processes are reversible, the system interacts with the surroundings to maintain a constant temperature.

The work of separation is the work required to compress each gas from  $p_{1a}$  or  $p_{1b} \rightarrow p_m$  at a constant temperature  $T_m$ .

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Since the processes are reversible, we have (( )) this particular process to be reversible. What should happen? The system interacts with the surroundings to maintain a constant temperature. So, all these processes are happening at temperature being constant  $T_m$ . So, what I am doing? Basically, I am pushing this piston B and piston A inwards. So, I have to do some work  $W_a$  and  $W_b$  respectively and at a same time, because the compression process occurring some  $Q_R$  is being release, some heat generated being released. So, that the temperature  $T_m$  is maintained, I am saying that the temperature  $T_m$  is all through constant and because of this  $Q_R$  Temperature  $T_m$  is being maintained, because of this interaction with the surrounding the constant temperature is maintained.

The work of separation is the work required to compress the compress each gas from it is partial pressure  $P_{1a}$  to  $P_m$  and  $P_{1b}$  to  $P_m$ . The piston A is compressing the Gas B while the piston B is compressing the Gas A. So,  $W_b$  is the work done on the piston B while  $W_A$  is the work done on piston A and what is this compressing? Compressing Gas B from partial pressure  $P_{1b}$  to the mixture pressure  $P_m$  and whole process is carried out at temperature  $T_m$ .

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**CRYOGENIC ENGINEERING**

### Ideal Separation System

• Since the left piston is permeable to **Gas A**, the **Gas A** exerts no pressure on the left piston.

Similarly, the **gas B** exerts no pressure on the right piston.

• When both the pistons are moved inward, the mixture is separated at constant  $T_m$ .

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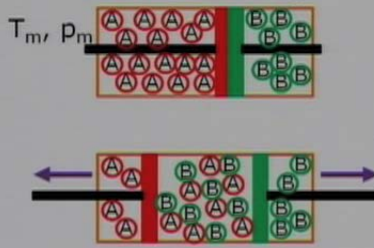
So, here you can see that the gases A and B are being separated here. Since the left piston is permeable to Gas A, the Gas A exerts no pressure on the left piston. When this piston moves the pressure is exerted on this piston by Gas B and not by Gas A. Similarly, the Gas B exerts no pressure on the right piston, it will be Gas A which will exert pressure. Gas B will just pass through this Gas A just passes through this.

When both the pistons are moved inward, the mixture is separated at constant temperature  $T_m$ , because we are calling these the temperature remaining constant in this gas.

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**CRYOGENIC ENGINEERING**

### Ideal Separation System



- The entire processes are assumed to be reversible.
- The process is reversed due to the difference in the concentrations of **Gas A** and **Gas B**.
- Hence, the mixing of the gases would move the pistons away and produce work.

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So, this is the separated gases A and B the pressure here is  $P_m$  and temperature being  $T_m$ , the pressure being  $T_m$  and temperature being  $P_m$ . The entire processes are assumed to be reversible. So, now, I find that the mixture has been separated, but because this process is reversible now, this process should also reach back to the earlier point where it is started from.

The process is reversed due to the difference in the concentration of Gas A and Gas B. So, slowly Gas A will start diffusing through piston A and Gas B, we will start diffusing through the piston B, because they are semi permeable membrane.

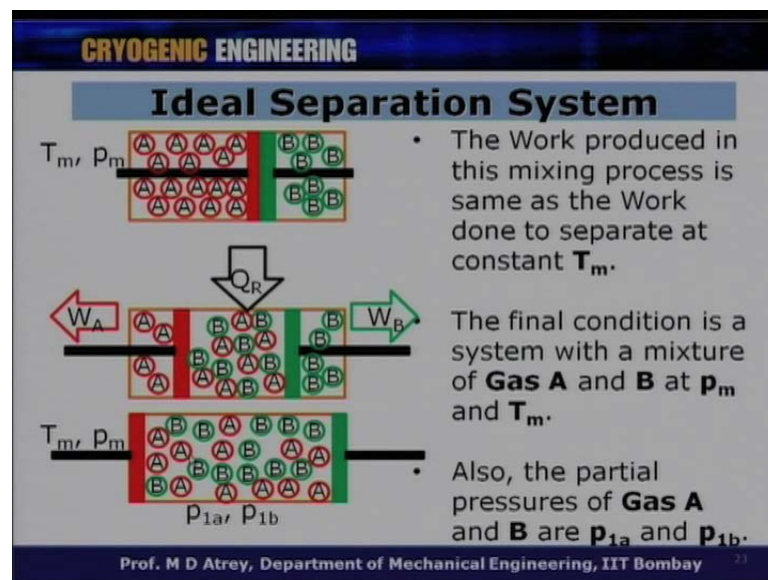
And slowly, the concentration balance will get created slowly. There will way difference in concentration on the left side of the piston and right side of the piston for Gas A as well piston A as well as for piston B and slowly the process of diffusion will start and slowly this piston would start moving back. It will take lot of time, but because the process being reversible. The process would start and therefore, meaning which the Gas A will start getting combined with Gas B; that means, the mixing processes will start a reversible process of mixing will get start.

First we have unmixed or separated A and B , but now slowly Gas A will start coming in Gas B start coming in and process of mixing will get started. Hence, the mixing of the gases would move the pistons away and produce works. So, here bit time the Gas A will start coming in the gap between piston A and B and it is start pushing this piston A back

and piston B back; that means, the work will now get produced by the system. Earlier, we had putting the work, we had work done on the system  $W_a$  and  $W_b$ . Now, the same work will get created by on piston A and piston B by the gas, because now the gas is going to work on the piston.

So, here what you can see is piston A is moved back, piston B is moved back and slowly a mixture of A and B getting form in this space. So, this is basically the reversible process which is what we have seen here occurring now?

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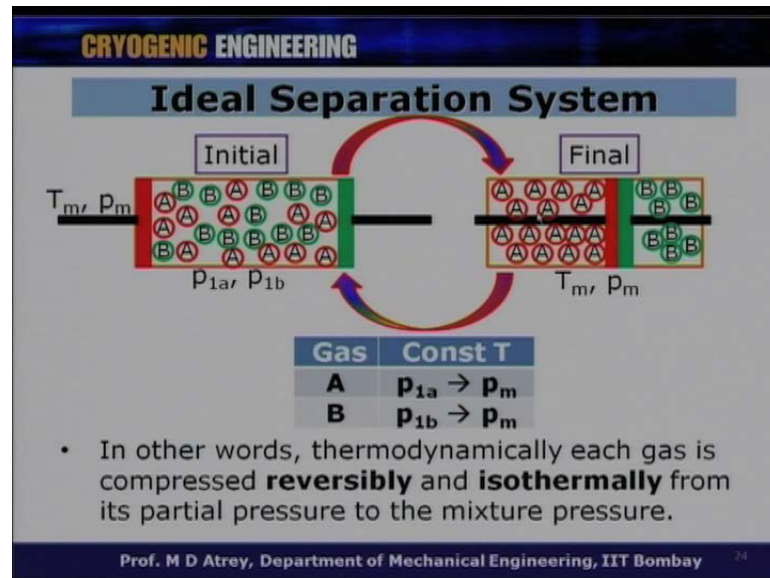
The work produced in this mixing process is same as the work done to separate at constant temperature  $T_m$ . The final condition is system with a mixture of Gas A and B at  $P_m$  and  $T_m$ . So, ultimately I will get mixture of Gas A and B at  $P_m$  and  $T_m$ .

So, what you see here now, in which again now, because the work is kind of expansion of the gas is I am supplying  $Q_R$ . The temperature being kept constant, the surrounding is now putting some heat inside. So, the  $T_m$  is kept constant temperature is maintained and  $W_a$  and  $W_b$  is the work done by the system, work which is produced by the system during mixing. Also, the partial pressures of Gas A and Gas B will be  $P_{1a}$  to  $P_{1b}$ . So, ultimately in this mixture, it will have partial pressure of  $P_{1a}$  and  $P_{1b}$  as it was previously.



So, ultimately I will get now a mixture of gases first I have unmixed the mixture and again mixture occurred automatically and if I put in the work  $W_a$  and  $W_b$  again the process would repeat. So, this is what we call as a reversible process, we are conceiving a system and wherein we can call this particular process as a reversible process of gases separation.

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So, this is my initial condition when I got a mixture A and mixture B Gas A and Gas B. In mixture form, **this** my final condition where I got mixture separated in Gas A and Gas B. The moment I say they are reversible process I should have the arrows like this also. So, I can go from initial to final and final to initial in both the cases temperature  $T_m$  is maintained constant.

So, basically the Gas A is getting compressed from  $p_{1a}$  to  $p_m$ , when I am pushing this piston and Gas B is getting compressed from  $p_{1b}$  or the partial pressure of b to  $p_m$ .

In other words, thermodynamically each gas is compressed reversibly and isothermally from its partial pressure to the mixture pressure. So, **what is** what I am doing? Basically, well separating this mixture, I am compressing reversibly and isothermally Gas A and Gas B from its respective partial pressure to the mixture pressure. This is what basically I am doing in separating this gas mixture.



In order to understand, the process of compression say Gas A from P 1 as to P m, the following analysis is done. Now, basically I would like to calculate the work of separation of Gas A and Gas B. When I say work of compression is nothing, but work of separation and this will give me ideal work of separation. This is what I am going to calculate now?

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**CRYOGENIC ENGINEERING**

**Ideal Separation System**

- Let the mol. wt. of **Gas A** and **Gas B** be  $mol_a$  and  $mol_b$  respectively.
- Number of moles of **Gas A** is given by  $n_a = m_a / mol_a$
- Similarly, number of moles of **Gas B** is  $n_b = m_b / mol_b$
- Then total number of moles in the mixture  $n_m$  is  $n_m = n_a + n_b$
- Then the ratios  $y_a = n_a / n_m$  and  $y_b = n_b / n_m$  are the mole fractions of **Gas A** and **Gas B** respectively.

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So, now let us see the ideal separation of the system. Let us do the calculations to understand, what is the work of separation? If one goes for an ideal separation of a system. Let the molecular weight of a Gas A and Gas B be denoted as mol a and mol b respectively. The number of moles of Gas A will be given by this is very simple mixture formally, which I am going to just kind of a device and go ahead.

So, if I want to calculate number of moles, it will depend on how much grams of gas is there divided by molecular weight that will give me moles. So, n a is a moles of Gas A in a mixture of A and B. So, n a will be given as m a upon molecular weight of a. Similarly, n b will give the moles of b that is m b upon molecular weight of b.

The total number of moles in the mixture n m. Suppose, the mixture number of moles in the mixture are n m, you know n m is equal to n a plus n b and the ratios now, I call as the y a which is basically mole fraction of Gas A, y b is a mole fraction of Gas B. The mole fraction of Gas A is called y a which n a upon n m and y b is n b upon n m. So, y b and y a are basically the mole fraction of Gas B and Gas A respectively.

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**CRYOGENIC ENGINEERING**

### Ideal Separation System

Initial

$T_m, P_m$

$P_{1a}, P_{1b}$

$V_{tot}$

Final

$T_m, P_m$

$V_a, V_b$

$$P_{1a} V_{tot} = n_a \mathcal{R} T_m$$

$$P_{1b} V_{tot} = n_b \mathcal{R} T_m$$

$$(P_{1a} + P_{1b}) V_{tot} = (n_a + n_b) \mathcal{R} T_m$$

$$P_{1a} + P_{1b} = P_m$$

$$P_m V_a = n_a \mathcal{R} T_m$$

$$P_m V_b = n_b \mathcal{R} T_m$$

$$\frac{V_a}{V_b} = \frac{n_a}{n_b}$$

- The volume occupied by each of the gas is directly proportional to its number of moles.

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Now, this is whatever is a initial condition was of A and B when they were a mixture at  $T_m$  and temperature of  $T_m$  and  $P_m$  pressure and the partial pressure were  $P_{1a}$  and  $P_{1b}$ . The corresponding volume occupied by the mixture was  $V_{tot}$  and I can apply now simple GAS laws, which is  $P_{1a}$  into  $V_{tot}$ . This is for Gas law ideal for Gas A and similarly, I will have ideal Gas law for Gas B.

So, what is the pressure of Gas A  $P_{1a}$  into  $V_{tot}$  is equal to  $n_a \mathcal{R} T_m$  where,  $\mathcal{R}$  in this format is what is the call as the universal GAS constant? Similarly, I will write the same equation for Gas B also. If I combine them together  $P_{1a}$  plus  $P_{1b}$  into  $V_{tot}$  is equal to  $n_a$  plus  $n_b$   $\mathcal{R} T_m$ .

So,  $P_{1b}$  is equal to  $n_b \mathcal{R} T_m$  is basically, I am applying for Gas A and Gas B and also a mixture of Gas A plus Gas B note that the volumes are always the same, because A and B together are occupying the same volume and individually also are occupying the same volume.

Meaning, which  $P_{1a}$  plus  $P_{1b}$  is equal to  $P_m$  that is what we have been talking about? And now, is a final when I want a separation has a curved and I have got now particular volume of  $V_a$  associated with Gas A and particular volume of  $V_b$  associated with Gas B.

While the temperature is same as  $T_m$  and now the pressure of the Gas A here is  $P_m$  is the pressure of Gas B here in  $V_b$  volume is  $P_m$ . So, again I apply the gas law the ideal gas law for Gas A and Gas B. Now, the pressure of Gas A in this volume of  $V_a$  is equal to  $P_m$ . So,  $P_m V_a$  is equal to  $n_a R T$  for Gas A, which is  $P_m V_a = n_a R T$ . Same thing, I will do for Gas B and if I now see that if I divide these by these what I get is  $V_a$  upon  $V_b$ . This will basically give the ratio of how much volume is occupied by  $V_a$  and  $V_b$ , it will basically a function depending on what is the ratio of **molecules of n a** moles of n a divide by moles of n b.

So, how many moles of n a **(())** are there how many moles of b will decide what is my  $V_a$  and  $V_b$  are? The volume occupied by each of the gas is the directly proportional to the number of moles that is what we have seen?

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**CRYOGENIC ENGINEERING**

### Ideal Separation System

- From the earlier lectures, the work requirement for a unit mass of gas compressed isothermally is given by 
$$\frac{-W_i}{m} = T_m (s_1 - s_2) - (h_1 - h_2)$$
- The net ideal work requirement of the separation process is the sum of the ideal work requirement by **Gas A** and **Gas B**.
- Mathematically, 
$$-W_i = (-W_{i,a}) + (-W_{i,b})$$
- Dividing the above equation by the mass of the mixture  $m_m$ , we get 
$$\frac{-W_i}{m_m} = \frac{-W_{i,a}}{m_m} + \frac{-W_{i,b}}{m_m}$$

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From the earlier lectures, the work requirements for a unit mass of gas compressed isothermally is given by,  $\frac{-W_i}{m}$  which is  $T_m (s_1 - s_2) - (h_1 - h_2)$ , this what we have done? We have derive this equation in the gas liquefaction cycle. So, basically  $\frac{-W_i}{m}$  is a ideal work done of compression basically, because as we earlier said as earlier pointed out that separation is nothing, but compress in the gas from partial pressure to the mixture pressure  $P_a$  to  $P_m$  or  $P_b$  to  $P_m$ .

So,  $\frac{-W_i}{m}$  is equal to  $T_m (s_1 - s_2) - (h_1 - h_2)$ , which is entropy difference minus  $h_1 - h_2$  enthalpy difference. The net ideal work requirement of the separation process

is the sum of the ideal work requirement for Gas A and Gas B. So, I can write mathematically, the  $W_I$  is equal to  $W_{I,a}$  plus  $W_{I,b}$  the total work of separation is equal to work of separation of Gas A plus work of separation of Gas B and dividing by  $m_m$  I get this  $W_I$  by  $m_m$  divided by  $W_{I,a}$  by  $m_a$  plus  $W_{I,b}$  divided by  $m_b$  is nothing, but the mass of the mixture.

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**CRYOGENIC ENGINEERING**

### Ideal Separation System

$$\frac{-W_I}{m_m} = \frac{-W_{I,a}}{m_a} + \frac{-W_{I,b}}{m_b}$$

- The total mass of mixture  $m_m$  is the sum of mass of **Gas A** and **Gas B**.
- Mathematically, we have  $m_m = m_a + m_b$
- Rearranging the terms, we can write the above equation as

$$\frac{-W_I}{m_m} = \left(\frac{-W_{I,a}}{m_a}\right)\left(\frac{m_a}{m_m}\right) + \left(\frac{-W_{I,b}}{m_b}\right)\left(\frac{m_b}{m_m}\right)$$

- Here,  $m_a$  and  $m_b$  are the mass of the **Gas A** and **Gas B** respectively.

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So, this is what we have just seen? The total mass of the mixture is  $m_m$  and is the sum of Gas A and Gas B. We can write  $m_m$  is equal to nothing, but  $m_a$  plus  $m_b$  absolutely clear till now.

Rearranging the terms, now I am just writing these terms in a different format as given here. So,  $W_I$  by  $m_m$  is equal to  $W_{I,a}$  upon  $m_a$  this is nothing, but work of separation for Gas A multiplied by  $m_a$  by  $m_m$  plus work separation of Gas B divided  $m_b$   $W_{I,b}$  upon  $m_b$  into  $m_b$  by  $m_m$ .

So, I am just writing  $m_m$  as  $W_I$   $m_a$   $m_a$  by  $m_a$   $m_a$   $m_a$  gets cancel what I will gets?  $W_I$  by  $m_a$  which is what you see here?

So, what I am basically? You know writing this expression As  $W_I$  by  $m_m$  is equal to work of separation of  $m_a$  into this component plus work of separation of  $m_b$  into this component. Here  $m_a$  and  $m_b$  are the masses of Gas A and Gas B respectively.

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**CRYOGENIC ENGINEERING**

### Ideal Separation System

$$\frac{-W_i}{m_m} = \left( \frac{-W_{i,a}}{m_a} \right) \left( \frac{m_a}{m_m} \right) + \left( \frac{-W_{i,b}}{m_b} \right) \left( \frac{m_b}{m_m} \right)$$

- The work requirement for each of the individual gas is given by the following equations.

$$\frac{-W_{i,a}}{m_a} = T_m (s_{1a} - s_{2a}) - (h_{1a} - h_{2a}) \quad \frac{-W_{i,b}}{m_b} = T_m (s_{1b} - s_{2b}) - (h_{1b} - h_{2b})$$

- Substituting and rearranging, we get

$$\frac{-W_i}{m_m} = T_m \left( \left( \frac{m_a}{m_m} \right) ((s_{1a} - s_{2a}) - (h_{1a} - h_{2a})) + \left( \frac{m_b}{m_m} \right) ((s_{1b} - s_{2b}) - (h_{1b} - h_{2b})) \right)$$

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So, this is what we have just seen? Now, I can write an expression for  $W_i$  by  $m_a$  also. The work requirement for each of the individual gas is given by the following equation. So,  $W_i$  upon  $m_a$  is equal to  $T_m$  into  $s_{1a}$  minus  $s_{2a}$  minus  $h_{1a}$  minus  $h_{2a}$ . This is the expression, we have just seen of work of separation. When I compress the gas from one  $a$  to two  $a$  position for Gas A and similarly, I can write the same expression for Gas B also.

Substituting and rearranging the term, if I put this value over in this equation, I will get a very long equation. So, substituting this by this and substituting  $W_i$  upon  $m_b$  by this what I get is? This a very big expression over here. So, the top term is nothing, but for Gas A and the bottom term is nothing, but Gas B  $T_m$  is the temperature of mixture which is remaining constant.

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**CRYOGENIC ENGINEERING**

### Ideal Separation System

$$\frac{-W_t}{m_m} = T_m \left[ \left( \frac{m_a}{m_m} \right) ((s_{1a} - s_{2a}) - (h_{1a} - h_{2a})) + \left( \frac{m_b}{m_m} \right) ((s_{1b} - s_{2b}) - (h_{1b} - h_{2b})) \right]$$

- It is clear that the work requirement decreases with the decrease in the temperature.
- Hence, the separation of mixtures at the cryogenic temperatures is most economical.
- The subscripts **1** and **2** denote the initial and the final conditions respectively.

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Now, let us find some expression for this entropy difference and enthalpy differences. So, this is my expression what you can see from here that? The  $T_m$  which is the mixture temperature figuring out here and this is the work of separation of the gas and as you can see that if the  $T_m$  is low or the mixture temperature is low  $W_t$  by  $m$  will be the work of separation will be very less that shows directly that if the temperature is very low or in cryogenic temperature region the work of separation will be very very less.

It is clear that the work requirement decreases with the decrease in temperature. Hence, the separation of mixture at the cryogenic temperature is most economical and one point is proved mathematically from here, that if  $T_m$  is less, if a mixture temperature is less, the work of separation will be very very less, the subscripts one and two denotes the initial and the final condition respectively, what does it mean? What is the about?



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**CRYOGENIC ENGINEERING**

### Ideal Separation System

$$\frac{-W_t}{m_m} = T_m \left( \frac{m_a}{m_m} \left( (s_{1a} - s_{2a}) - (h_{1a} - h_{2a}) \right) + \frac{m_b}{m_m} \left( (s_{1b} - s_{2b}) - (h_{1b} - h_{2b}) \right) \right)$$

- It means that for each gas,  $s_1$  and  $h_1$  are at the partial pressure before the separation. And  $s_2$  and  $h_2$  are at mixture pressure after the separation of the mixture.
- For the sake of understanding, let us first evaluate only **entropy** and **enthalpy** terms for each of the gases.

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It means that for each gas,  $s_1$  and  $h_1$  are at the partial pressure before separation, because each gas is being compressed from its partial pressure to mixture pressure. So, one condition the  $s_1$  and  $h_1$  are basically conditions prevailing at partial pressure before separation while  $s_2$  and  $h_2$  are at the mixture pressure, because the gases are being compressed from their partial pressure to the mixture pressure after the separation of the mixture.

For the sake of understanding, let us first evaluate the entropy and enthalpy terms for this. So, let us see if you could substitute by some simple expression for this enthalpy and entropy difference is for Gas A and Gas B respectively.



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**CRYOGENIC ENGINEERING**

### Ideal Separation System

- For an ideal gas, the specific entropy **s** and specific enthalpy **h** can be expressed as
$$s = c_p \ln T - R \ln p + s_r$$
$$h = c_p T + h_r$$
- where, **s<sub>r</sub>** and **h<sub>r</sub>** are some reference values.
- Hence, **s** and **h** for **Gas A** are given by
$$s_{1a} = c_{pa} \ln T_m - R_a \ln p_{1a} + s_{ra}$$
$$h_{1a} = c_{pa} T_m + h_{ra}$$
$$s_{2a} = c_{pa} \ln T_m - R_a \ln p_{2a} + s_{ra}$$
$$h_{2a} = c_{pa} T_m + h_{ra}$$

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For an ideal gas, the specific entropy  $s$  and specific enthalpy  $h$  are expressed as below. So,  $S$  is equal to  $c_p \ln T - R \ln p + s_r$  plus some reference entropy term. Similarly, enthalpy is equal to  $c_p T + h_r$  plus this reference enthalpy term. Basically, any temperature and any pressure one can select and what we meant to say that it should remain constant throughout calculations.

Where,  $s_r$  and  $h_r$  are some reference values. They could be added 77 Kelvin or boiling point or 1 bar and 0 Kelvin or whatever, because there are various temperature entropy diagrams, which can have different reference values and what we meant to say that? Whenever we refer to such different diagrams, we should keep the same reference values.

Hence,  $s$  and  $h$  for each Gas A are given as, if I put now these general expressions for Gas A I will get this expression while similarly, I will get for  $s_{2a}$  that is after at a temperature  $T_m$  and mixture  $P_m$  I will get these expressions. Similarly, enthalpy values at 1a and 2a conditions are given by these expressions.

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**CRYOGENIC ENGINEERING**

### Ideal Separation System

- The entropy and enthalpy term for **Gas A** is as given below.  $((s_{1a} - s_{2a}) - (h_{1a} - h_{2a}))$
- Substituting, we get
 
$$\left( \cancel{c_{pa} \ln T_m} - R_a \ln p_{1a} + \cancel{s_{fa}} - \cancel{c_{pa} \ln T_m} + R_a \ln p_m + \cancel{s_{fa}} \right) - \left( \cancel{c_{pa} T_m} + \cancel{h_{fa}} - \cancel{c_{pa} T_m} - \cancel{h_{fa}} \right)$$

$$((s_{1a} - s_{2a}) - (h_{1a} - h_{2a})) = R_a \ln \left( \frac{p_m}{p_{1a}} \right)$$
- Also, for **Gas B**

$$((s_{1b} - s_{2b}) - (h_{1b} - h_{2b})) = R_b \ln \left( \frac{p_m}{p_{1b}} \right)$$

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The entropy and enthalpy terms are given, what I want to basically find out is a difference in  $((s_{1a} - s_{2a}) - (h_{1a} - h_{2a}))$  and if I put those values over here. I get this expression this is for entropy difference and this is for enthalpy difference.

And if you see now, because the temperatures remaining same from 1 a to 2 a this terms can get canceled. So, this term will get canceled similarly, the reference values are same they will get cancelled over here and in this case again, the temperature remaining constant this terms will get cancels.

So, what remains here basically  $R_a \ln p_{1a}$  minus sign plus  $R_a \ln p_m$  and it is basically nothing, but logarithmic difference which is nothing, but  $R_a \ln \frac{p_m}{p_{1a}}$ . So, entire  $(s_{1a} - s_{2a}) - (h_{1a} - h_{2a})$  get reduced to  $R_a \ln \frac{p_m}{p_{1a}}$ . Similarly, I can do the same thing for Gas B and I will get expression  $(s_{1b} - s_{2b}) - (h_{1b} - h_{2b})$  is equal to  $R_b \ln \frac{p_m}{p_{1b}}$ .

So, I have done a major calculation and whatever expressions, we have first got to calculate the work of separation, varying this entropy difference and enthalpy difference figure. This complete term now, we will get replaced by this and for Gas A and Gas B complete term will get to replaced by this.

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**CRYOGENIC ENGINEERING**

### Ideal Separation System

$$\left( (s_{1a} - s_{2a}) - (h_{1a} - h_{2a}) \right) = R_a \ln \left( \frac{P_m}{P_{1a}} \right)$$

$$\left( (s_{1b} - s_{2b}) - (h_{1b} - h_{2b}) \right) = R_b \ln \left( \frac{P_m}{P_{1b}} \right)$$

• Substituting, we get the ideal work requirement as

$$\frac{-W_l}{m_m} = T_m \left( \left( \frac{m_a}{m_m} \right) \left( (s_{1a} - s_{2a}) - (h_{1a} - h_{2a}) \right) + \left( \frac{m_b}{m_m} \right) \left( (s_{1b} - s_{2b}) - (h_{1b} - h_{2b}) \right) \right)$$

$$\frac{-W_l}{m_m} = T_m \left( \left( \frac{m_a}{m_m} \right) R_a \ln \left( \frac{P_m}{P_{1a}} \right) + \left( \frac{m_b}{m_m} \right) R_b \ln \left( \frac{P_m}{P_{1b}} \right) \right)$$

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Now, let us find what is this P m upon P 1 a mean or what is P m upon P 1 b mean? So, this is what we got an expression now? This is what we got for a Gas B? So, I can put this expression back in my W I by m m is equal to T m into this and expression will be now replace this term by this in this expression and replace this enthalpy, entropy difference by this and my final expression therefore, will be like this.

So, I get W I upon m m is equal to T m into m a upon m m like this. This entire term get replaced by R a into log P m by a plus m b upon m m comes over here and this entire term of enthalpy and entropy difference get replaced by R b log P m (()).

So, my expression becomes very simple and now, I would like to understand what is my p m upon P 1 is? What is my m a are upon m m is I would like to understand those quantities.

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**CRYOGENIC ENGINEERING**

### Ideal Separation System

- Since the process occurs at constant volume  $V_m$ , using an ideal gas equation we can write
 
$$p_m V_m = n_m R T_m \quad p_{1a} V_m = n_a R T_m \quad p_{1b} V_m = n_b R T_m$$
- Dividing one over the other, we have
 
$$\frac{p_m V_m}{p_{1a} V_m} = \frac{n_m R T_m}{n_a R T_m} \quad \frac{p_m V_m}{p_{1b} V_m} = \frac{n_m R T_m}{n_b R T_m}$$

$$\frac{p_m}{p_{1a}} = \frac{n_m}{n_a} = \frac{1}{y_a} \quad \frac{p_m}{p_{1b}} = \frac{n_m}{n_b} = \frac{1}{y_b}$$
- Where  $y_a$  and  $y_b$  are the mole fractions of **Gas A** and **Gas B** respectively.

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Since the process occurs at constant volume  $V_m$ , using ideal gas equation, which is  $P V$  is equal to  $n R T$ , I can write this expression which we have done earlier,  $p_m V_m$  is equal to  $n_m R T_m$ . So, I can write the same expression for GAS A and GAS B. So, this is the mixture equation for the mixture equation for Gas A and equation for Gas B.

Dividing one over the other, what we get is?  $\frac{p_m V_m}{p_{1a} V_m}$  is equal to  $\frac{n_m R T_m}{n_a R T_m}$ ,  $R T_m$  and  $R T_m$  get cancel  $V_m$  and  $V_m$  get cancel. So,  $\frac{p_m}{p_{1a}}$  is equal to  $\frac{n_m}{n_a}$  is equal to  $\frac{1}{y_a}$ . We know that  $\frac{n_a}{n_m}$  basically moles of a divide by moles of mixture is equal to mole fractions of Gas A. So,  $\frac{p_m}{p_{1a}}$  ultimately gets reduced to  $\frac{1}{y_a}$ .

Similarly, I can do for ratio of mixture and Gas B also and I get the same thing that  $\frac{p_m}{p_{1b}}$  is nothing, but  $\frac{n_m}{n_b}$  is equal to  $\frac{1}{y_b}$ . This is what we get from the here now?

So, I have got a ratio of  $\frac{p_m}{p_{1a}}$  figuring in the earlier station, I can replaced that by  $\frac{1}{y_a}$ . Where  $y_a$  and  $y_b$  are the mole fractions of Gas A and Gas B respectively.

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**CRYOGENIC ENGINEERING**

### Ideal Separation System

- The ideal gas equation can also be expressed in terms of the mass of the gas as shown below.

$p_m V_m = n_m \mathfrak{R} T_m$

$p_{va} V_m = n_a \mathfrak{R} T_m$

$p_{vb} V_m = n_b \mathfrak{R} T_m$

$p_m V_m = \frac{m_m}{mol_m} \mathfrak{R} T_m$

$p_{va} V_m = \frac{m_a}{mol_a} \mathfrak{R} T_m$

$p_{vb} V_m = \frac{m_b}{mol_b} \mathfrak{R} T_m$

$p_m V_m = m_m R_m$

$p_{va} V_m = m_a R_a$

$p_{vb} V_m = m_b R_b$

- In general,  $R_a = \frac{\mathfrak{R}}{mol_a}$  and  $\mathfrak{R} = 8.314 J / mol - K$
- Here  $\mathfrak{R}$  and  $R$  are the **Universal Gas Constant** and **Specific Gas Constant** respectively.

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The ideal GAS equation can also be expressed in terms of mass of the GAS as shown below. So, I can write this expression P V is equal to n R T for GAS mixture, GAS A and GAS B. I can write this n m or the molar fraction as the mass of gas mass of mixture divided by molecular weight of mixture; this is basically definition of n m. I am just replacing this n m by mass of m m upon molecular weight of (( )). So, P m V m is equal to m upon molecular weight of m R T m and same thing I am doing it now for GAS A and same thing I am doing for GAS B. Here m a upon molecular weight of a, here m b upon molecular weight of b will figure.

So, why I am doing all these things basically, I know now that P m V m is equal to m R T now, here R by molecular weight m is nothing, but now specific GAS constant. Here universal GAS constant are divided by molecular weight of a is specific GAS constant of a and here also similarly, the universal GAS constant divided by molecular weight of b becomes specific GAS constant R b as shown over here.

So, basically R upon molecular weight of m are universal GAS constant R upon molecular weight of m will come as this are which is specific GAS constant similarly, I will do for GAS A similarly, I will do for GAS B. Why in general R a is equal to universal GAS constant R upon molecular weight of a where universal GAS constant is 8.314 joules per mol Kelvin.

So, basically now I would like to express **the expire** have the earlier expression in terms of mass of GAS A and mass of GAS B also. So, universal GAS constant R and specific GAS constant R are the universal GAS constant and specific GAS constant respectively.

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**CRYOGENIC ENGINEERING**

### Ideal Separation System

- From the earlier slide, using the ideal gas equation in terms of the gas mass, we have
 
$$p_m V_m = m_m R_m T_m \quad p_{1a} V_m = m_a R_a T_m \quad p_{1b} V_m = m_b R_b T_m$$
- Dividing one over the other, we have
 
$$\frac{p_m V_m}{p_{1a} V_m} = \frac{m_m R_m T_m}{m_a R_a T_m} \quad \frac{p_m V_m}{p_{1b} V_m} = \frac{m_m R_m T_m}{m_b R_b T_m}$$

$$\frac{p_m}{p_{1a}} = \frac{m_m R_m}{m_a R_a} = \frac{1}{y_a} \quad \frac{p_m}{p_{1b}} = \frac{m_m R_m}{m_b R_b} = \frac{1}{y_b}$$

$$\frac{m_a R_a}{m_m} = R_m y_a \quad \frac{m_b R_b}{m_m} = R_m y_b$$

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From the earlier slide, using the ideal GAS equation in terms of GAS mass, what we get is these. So, I am now writing  $p_m V_m = m_m R_m T_m$ ; this is what we just  $p_{1a} V_m = m_a R_a T_m$ ,  $p_{1b} V_m = m_b R_b T_m$ .

Dividing one over the other, I will get  $\frac{p_m V_m}{p_{1a} V_m} = \frac{m_m R_m T_m}{m_a R_a T_m}$ . Now, I can cancel both R and T, because  $T_m$  and  $T_m$  will get canceled over here  $V_m$  and  $V_m$  will get canceled over here, what I get is  $\frac{p_m}{p_{1a}} = \frac{m_m R_m}{m_a R_a} = \frac{1}{y_a}$ . Which is nothing, but  $1/y_a$ , because  $p_m$  upon  $p_{1a}$  is  $1/y_a$ , we are just now earlier seen that what does it mean? Same thing I am doing it for GAS B  $\frac{p_m}{p_{1b}} = \frac{m_m R_m}{m_b R_b} = \frac{1}{y_b}$  nothing, but  $1/y_b$ . So, what I get from here is equal to  $1/y_b$ .

Ultimately, I will get  $\frac{m_a R_a}{m_m} = R_m y_a$ ,  $\frac{m_b R_b}{m_m} = R_m y_b$  if I just a rearrange this. I want an expression basically for this which is what is figuring in my expression for work similarly, for GAS B.



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**CRYOGENIC ENGINEERING**

### Ideal Separation System

$$\frac{p_m}{p_{1a}} = \frac{1}{y_a} \quad \frac{p_m}{p_{1b}} = \frac{1}{y_b} \quad \frac{m_a R_a}{m_m} = R_m y_a \quad \frac{m_b R_b}{m_m} = R_m y_b$$

$$\frac{-W_i}{m_m} = T_m \left( \left( \frac{m_a}{m_m} \right) R_a \ln \left( \frac{p_m}{p_{1a}} \right) + \left( \frac{m_b}{m_m} \right) R_b \ln \left( \frac{p_m}{p_{1b}} \right) \right)$$

- Substituting, we have

$$\frac{-W_i}{m_m} = R_m T_m \left( y_a \ln \left( \frac{1}{y_a} \right) + y_b \ln \left( \frac{1}{y_b} \right) \right) \quad p_m V_m = m_m R_m T_m = n_m \mathcal{R} T_m$$

$$\frac{-W_i}{n_m} = \mathcal{R} T_m \left( y_a \ln \left( \frac{1}{y_a} \right) + y_b \ln \left( \frac{1}{y_b} \right) \right)$$

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So, putting all these things  $P_m$  upon  $P_{1a}$  is equal to  $1$  upon  $y_a$ ,  $p_m$  upon  $P_{1b}$  is equal to  $1$  upon  $y_b$  and  $m_a R_a$  upon  $m_m$  is equal to  $R_m y_a$  and  $m_b R_b$  upon  $m_m$  is equal to  $R_m y_b$ , I will put this expression in my this expression which I have derived earlier substituting all this. So, I will these replace  $P_m$  by  $P_{1a}$  by one upon  $y_a$  and here I will replace these similarly, I will replace these term by  $R_m y_a$  similarly, I will replace these term by  $R_m y_b$ . So, if I have put those expressions I will get this equation.

So,  $W_i$  upon  $m_m$  is equal to  $R_m T_m y_a \log \frac{1}{y_a} + y_b \log \frac{1}{y_b}$ . The expression looks very simple now I have got a  $R T$  mixture temperature and  $R_m y_a$  upon  $1$  upon  $\log \frac{1}{y_a} + y_b \log \frac{1}{y_b}$ . Where  $P_v$  is equal to, if I will just replaced by  $R_m T_m$  and  $m_m$  here and want to convert it to the molar form. I can write work of separation per mole of GAS mixture  $W_i$  by  $n_m$  and here immediately universal GAS constant figure please understand this. Please understand the relationship between the universal GAS constant and specific GAS constant, because this is the work of separation per gram of mixture, per mass basically and this is a molar expression. So, we got a mass expression and a molar expression also.

As soon as the molar expression comes here universal GAS constant comes, as soon as the mass comes over here specific GAS constant comes over here.

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**CRYOGENIC ENGINEERING**

### Ideal Separation System

- The ideal work of separation per mole of mixture (**Gas A** and **Gas B**) is given by
$$\frac{-W_i}{n_m} = \mathcal{R}T_m \left( y_a \ln \left( \frac{1}{y_a} \right) + y_b \ln \left( \frac{1}{y_b} \right) \right)$$
- On the similar lines, if the mixture is composed of three different gases, say **Gas A**, **Gas B** and **Gas C**, the ideal work of separation per mole of mixture is given by
$$\frac{-W_i}{n_m} = \mathcal{R}T_m \left( y_a \ln \left( \frac{1}{y_a} \right) + y_b \ln \left( \frac{1}{y_b} \right) + y_c \ln \left( \frac{1}{y_c} \right) \right)$$

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So, the ideal work of separation per mole of mixture GAS A GAS B will be given as is a molar expression, ideal work of separation, per mole of mixture  $\left( \frac{-W_i}{n_m} \right)$  is equal to  $\mathcal{R} T_m y_a \log \frac{1}{y_a} + y_b \log \frac{1}{y_b}$ . This is for a mixture GAS A plus GAS B. I can extend this a, b, c or a, b, c, d, we can have different components in a mixture and similarly, I can show an expression for a, b and c mixture.

On the similar line, if the mixture is composed of three different gases GAS A, GAS B and GAS c, the ideal work of separation per mole of mixture is given by  $\frac{-W_i}{n_m} = \mathcal{R} T_m y_a \log \frac{1}{y_a} + y_b \log \frac{1}{y_b} + y_c \log \frac{1}{y_c}$ . If you got a d will get  $y_d \log \frac{1}{y_d}$ , what are  $y_a$ ,  $y_b$  and  $y_c$  is a molar fraction of GAS A, GAS B and GAS c in the mixture.

What is  $T_m$  temperature of mixture universal GAS constant? So, this is basically work of separation of per mole of the mixture **work of separation per mole of mixture**, what I am doing is? Basically find out work of separation is nothing, but work done to compressed each of this component GAS A, GAS B and GAS C from their respective partial pressure to the mixture pressure as we have seen in earlier reversible GAS separation process and this is what ideal work of GAS separation will be? The actual work of separation however, as you know will be different and therefore, we introduced a term in the next slide called Figure Of Merit, which we have already introduce in a GAS liquefaction also.

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**CRYOGENIC ENGINEERING**

### Ideal Separation System

- Generalizing the above equation for a mixture of **N** constituents, we have

$$\frac{-W_i}{n_m} = \mathcal{R}T_m \sum_{j=1}^N y_j \ln \left( \frac{1}{y_j} \right)$$

- where  $y_j$  is the mole fraction of **j**<sup>th</sup> component.
- Similar to the Liquefaction systems, the **Figure of Merit (FOM)** is defined as given below.

$$FOM = \frac{-W_i / n_m}{-W / n_m} = \frac{-W_i / m_m}{-W / m_m}$$

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So, generalizing the above equation for a mixture of N constituents, we just showed for A and B, we also showed for a, b and c. But there the mixture which has possibly has got n constituents, what will the formula be? The formula will be  $W_i$  upon  $n_m$  that is work of separation per mole of GAS mixture is equal to  $\mathcal{R} T_m$  into  $\sum_{j=1}^n y_j \ln \left( \frac{1}{y_j} \right)$ . So, if I got  $j$  is equal to 1 to 2  $n$  is equal to 2, I get  $y_1 \ln \left( \frac{1}{y_1} \right) + y_2 \ln \left( \frac{1}{y_2} \right)$ , if 1 and 1 are respective gases.

I can have n gases and therefore, I can have  $\ln \left( \frac{1}{y_1} \right) + \ln \left( \frac{1}{y_2} \right) + \ln \left( \frac{1}{y_3} \right) + \dots$  same terms will repeat and the summation of all these thing will give, the work of separation per mole of GAS mixture. Where  $y_j$  is the mole fraction of the  $j$  th component. So, basically the sigma expression comes normally, we will have two component or three component or maximum four components not more than that.

So, from the same thing this is the ideal work of separation, I define one more term as I said earlier. Similar to the liquefaction system, the Figure of Merit or FOM is also defined as figure of merit is equal to ideal work divided by actual work. The ideal work could be in molar basis or on a mass basis also say,  $W_i$  by  $n_m$  is divided by  $W$  by  $n_m$  you know the negative sign is basically the work done on the system that is why.

Similarly,  $W_i$  by  $m_m$  which is on a mass basis now divided by actual work of separation, actual work of separation must be much higher than the ideal work of separation and therefore, the figure of merit will always be less than one.

So, this is what we have learnt, we have basically got an expression for work of separation we understood, what a figure of merit would be of a given system. So, just to summarize what we did in the first lecture of this topic of GAS separation I will just take it through a some summary points.

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**CRYOGENIC ENGINEERING**

### Summary

- Different techniques employed are Synthetic membranes, Adsorption, Absorption and distillation.
- The separation can be done at both room temperature and cryogenic temperature.
- In an Ideal system all the processes are reversible and the work requirement in an ideal gas separation is called as an Ideal Work.

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We just discuss different techniques employed for GAS separation, which are Synthetic membranes, Adsorption, Absorption and Distillation. The separation can be done at both room temperature and cryogenic temperature. When I use a room temperature technique, I use normally the method of adsorption or I can use membranes or I can use absorption while the distillation normally done at cryogenic temperature.

In an Ideal system all the processes are reversible and the work requirement in an ideal GAS separation is called Ideal works. We just got an expression for Ideal GAS separation of work done in order to are work requirement for Ideal GAS or Ideal work requirement for GAS separation and what is that expression?

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**CRYOGENIC ENGINEERING**

### Summary

- Ideal work requirement per mole of mixture to separate a mixture with **N** constituents is given by

$$\frac{-W_i}{n_m} = R T_m \sum_{j=1}^N y_j \ln \left( \frac{1}{y_j} \right)$$

- where  $y_j$  is the mole fraction of **j<sup>th</sup>** component.

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The Ideal work requirement per mole of GAS mixture to separate a mixture with N constituents is given by this general formula. So, general formula tells you  $W_i$  by  $n_m$  is equal to  $R T_m \sum_{j=1}^N y_j \ln \left( \frac{1}{y_j} \right)$ , when  $j$  is going from 1 to  $n$  constituents  $n$  equal to 2, then and therefore, A and B are the two gases, then  $y_a \ln \left( \frac{1}{y_a + y_b} \right) + y_b \ln \left( \frac{1}{y_a + y_b} \right)$ .

Where  $y_j$  is the mole fraction of the  $j$  the component. (( )) got a small self assessment exercise to given please go through that thing and kindly asses yourself for this. Best done whatever we have learnt in this lecture, I am devoting more time in the lecture two for solving tutorials. So, that you understand the relevance of the formula, which we have just derived in order to calculate the Ideal work of separation. And this is a small self assessment test please go through it and asses yourself the answers given at the end. Thank you very much.