Cryogenic Engineering
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Lecture No. # 15
Gas Liquefaction and Refrigeration Systems

So, welcome to the fifteenth lecture of cryogenic engineering, under the NPTEL program.

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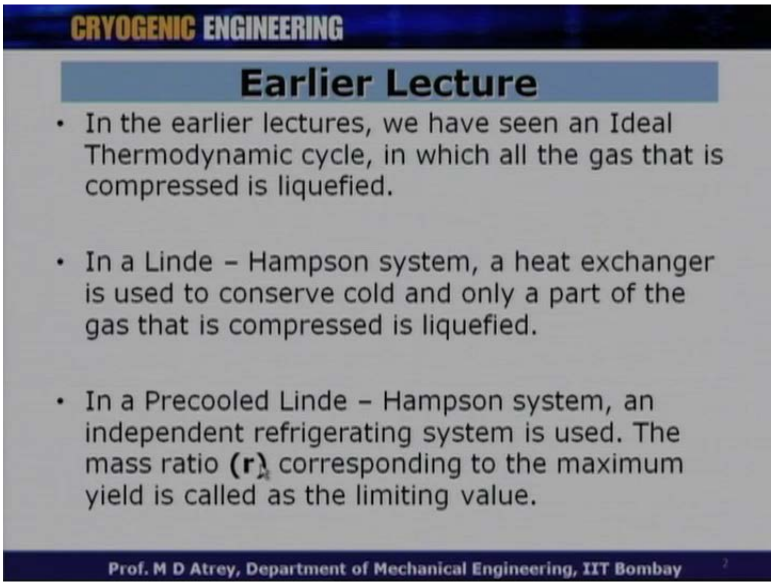

CRYOGENIC ENGINEERING

Earlier Lecture

- In the earlier lectures, we have seen an Ideal Thermodynamic cycle, in which all the gas that is compressed is liquefied.
- In a Linde – Hampson system, a heat exchanger is used to conserve cold and only a part of the gas that is compressed is liquefied.
- In a Precooled Linde – Hampson system, an independent refrigerating system is used. The mass ratio **(r)** corresponding to the maximum yield is called as the limiting value.

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In the earlier lectures on gas liquefaction, we have talked about various cycles and I will just go in brief about those cycles. We have seen an ideal thermodynamic cycle, in which all the gas that is getting compressed is getting liquefied. In the ideal cycle, whatever gas gets compressed, it gets liquefied 100 percent.

Then we came from this ideal cycle to a little practical cycle, which was a Linde - Hampson system. So, in a Linde - Hampson system, a heat exchanger is added to the thermodynamic cycle and so, whatever cooling effect is produced the cold is conserved. So, a heat exchanger is used to conserve the cold and only a part of the gas that is compressed is liquefied.

So, in first case all the 100 percent gas which has compressed got liquefied, here only a part of the gas that was compressed got liquefied. Then we went for the second modification, which is the Percooled Linde - Hampson system, it is an independent refrigerating system is used to precool the gas before it enters the heat exchanger.

The mass ratio of this refrigerating cycle, corresponding to the maximum yield is called limiting value meaning which that one cannot go on increasing, the refrigerant flowing the auxiliary cycle. Because after a particular r value addition of that r value there is no significant effect or there is no effect on the yield and therefore, this limiting value of the r or the amount of refrigerant that is flowing through the auxiliary refrigerating system. We have seen this system in detail, in the earlier lecture.

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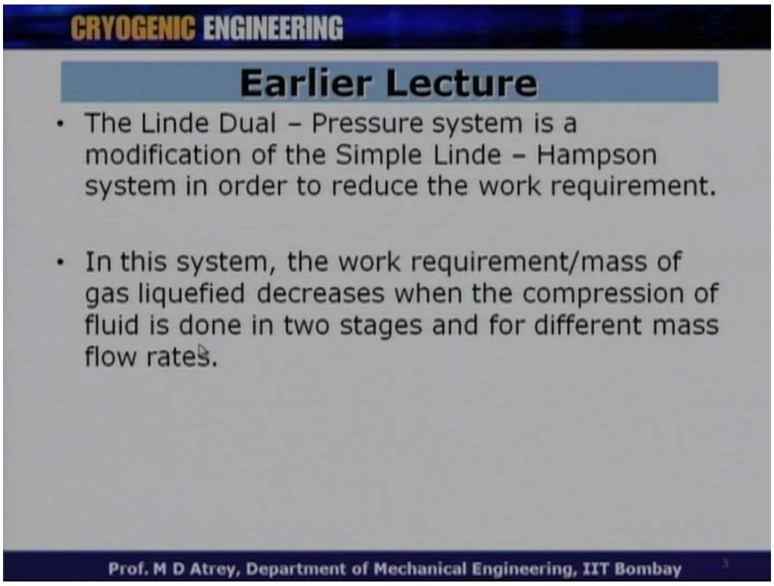
CRYOGENIC ENGINEERING

Earlier Lecture

- The Linde Dual – Pressure system is a modification of the Simple Linde – Hampson system in order to reduce the work requirement.
- In this system, the work requirement/mass of gas liquefied decreases when the compression of fluid is done in two stages and for different mass flow rates.

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Then we went for a Linde Dual - Pressure system wherein, we used two compressors. The Linde Dual - Pressure system is a modification of the simple Linde - Hampson system in order to reduce the work requirement. In this system, the work requirement per mass of gas liquefied decreases when the compression of the fluid is done in two stages and for different mass flow rates. I think this is very straight forward and this is what we did during the last lecture?

So, we had two compressors over there and both the compressors had different flow rates, as part of the gas which was compress in the first, was the second stream joints there and the total amount aim is getting compressed only in the second compressor, while the first compressor had less amount to compress. Now, going ahead from there, we come to a different cycle.

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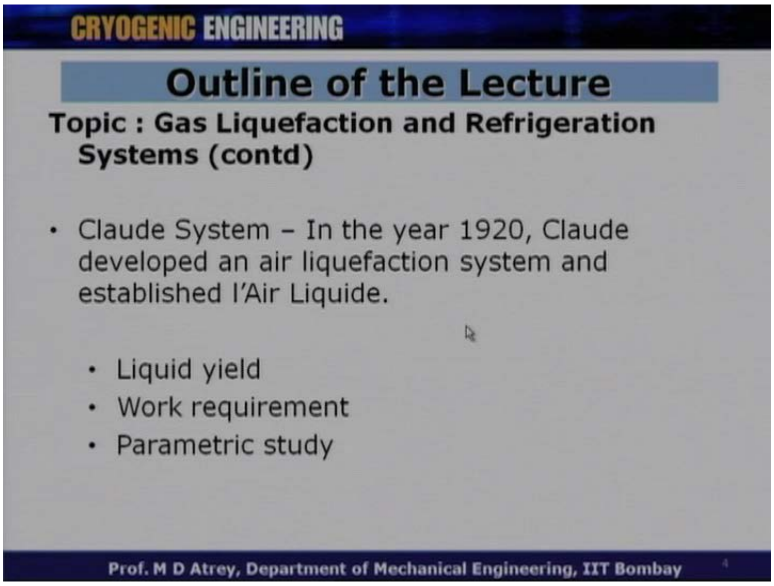

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Outline of the Lecture

Topic : Gas Liquefaction and Refrigeration Systems (contd)

- Claude System – In the year 1920, Claude developed an air liquefaction system and established l'Air Liquide.
  - Liquid yield
  - Work requirement
  - Parametric study

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Now, and the outline of this lecture which is I am going to talk about is again continuing of the same topic Gas Liquefaction and Refrigeration System. What I am going to talk today about is? The Claude cycle or sometimes called as Claude cycle, I will call is Claude cycle. 1920 Claude developed an air liquefaction system, which is famously known as air liquid today a French company. And during my first lecture, when I talk about the chronological event in the field of cryogenic engineering, Claude figured in that list also. This is a very big company as of now, which makes helium liquefier Nitrogen liquefiers.

Under this Claude system, I am going to talk about the expressions for liquid yield, the work requirement and also the parametric study which is a very important constituent of the Claude cycle.

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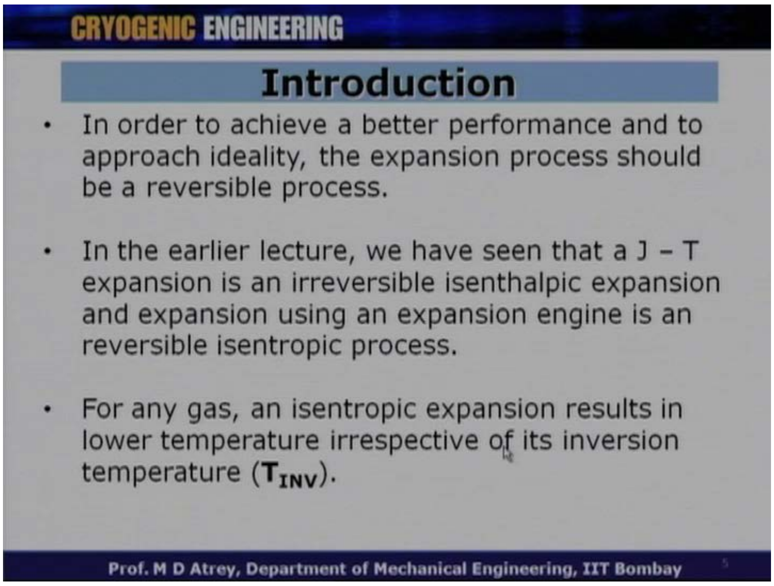

Introduction

- In order to achieve a better performance and to approach ideality, the expansion process should be a reversible process.
- In the earlier lecture, we have seen that a J – T expansion is an irreversible isenthalpic expansion and expansion using an expansion engine is an reversible isentropic process.
- For any gas, an isentropic expansion results in lower temperature irrespective of its inversion temperature ($\mathbf{T_{INV}}$).

So, introduction to the Claude cycle, in order to achieve a better performance and to approach ideality, the expansion process should be a reversible process. Till now, what you have was a Joule Thomson expansion, which is an irreversible process.

In the earlier lecture, we have seen that a J - T expansion or a Joule Thomson expansion is an irreversible isenthalpic expansion and expansion using an expansion engine is a reversible isentropic process. We have talked about this earlier in one of the earlier lectures that expansion could be assured by in isenthalpic expansion or an isentropic expansion.

In a domestic refrigerator for example, what we have is a Joule Thomson expansion achieved using a capillary tube. Till now, whenever we have expanded the gas in the earlier cycle, we have found that a J - T expansion is used which is an isenthalpic expansion and it is an irreversible process and therefore, we have to worry about the inversion temperature of this gas. Below which only when the J-T expansion is carried out would result in cooling.

Now, in this Claude cycle, we will talk about an expansion engine where, you got a piston and cylinder kind of an arrangement. Where, the work also is done by the system in an expansion engine and in this case, when the expansion is carried out using such an engine. The process is a reversible isentropic process ideally and this would always result in cooling and one would not have to worry about the inversion temperature of the gas.

So, for any gas, an isentropic expansion results in lower temperature irrespective of it is inversion temperature that is T INV. So, this is clear now, that isentropic expansion is always preferred and is always basically, a result in lowering of temperature which is not so, in case of a J - T expansion.

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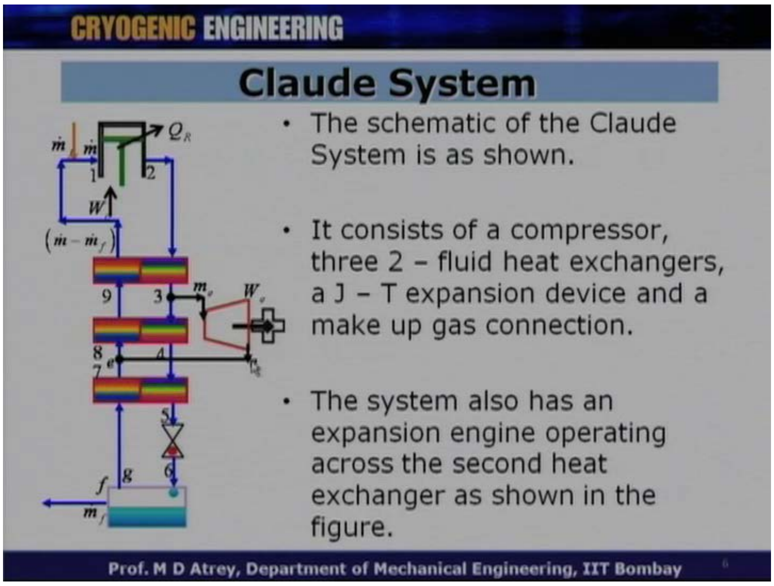

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Claude System

- The schematic of the Claude System is as shown.
- It consists of a compressor, three 2 – fluid heat exchangers, a J – T expansion device and a make up gas connection.
- The system also has an expansion engine operating across the second heat exchanger as shown in the figure.

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So, this is a schematic of a Claude cycle and what you can see from here. It has got a compressor over here and it has the… What an expander over here? And we got a heat exchangers 1, 2 and 3 and there is the J - T wall at the end whether liquid is yielded in a container at this point here.

So, this schematic Claude cycle is as shown here. It consists of a compressor, which is this and a three 2-fluid heat exchanger. So, there are 3 heat exchangers 1, 2 and 3 and there are 2-fluids. As in earlier cases, we have found that there were 3-fluid heat exchangers also and a J - T expansion device and a makeup gas connections. So, after getting the liquid at this point, the gas would go back and whatever amount m dot f as you get at this point makeup gas is added equal to m dot f at this point.

The system has an expansion engine operating across the second heat exchanger as shown over here. So, what you can see from here is? When the gas is getting compressed is goes to the heat exchanger and a part of the gas that is m e is taken off from here. The gas is expanded to the expansion engine over here, which produces cold and this gas the cold gas at the end of the expansion joins the return stream is coming basically from the gas which has not got liquefied at this point.

Whatever, amount of gas is coming from 6 off which only m dot f gets liquefied and the return stream will have the remaining gas this gets joined to m dot e, which is coming from expansion engine and a whole gas goes back and gets compressed with a makeup gas connection at this point?

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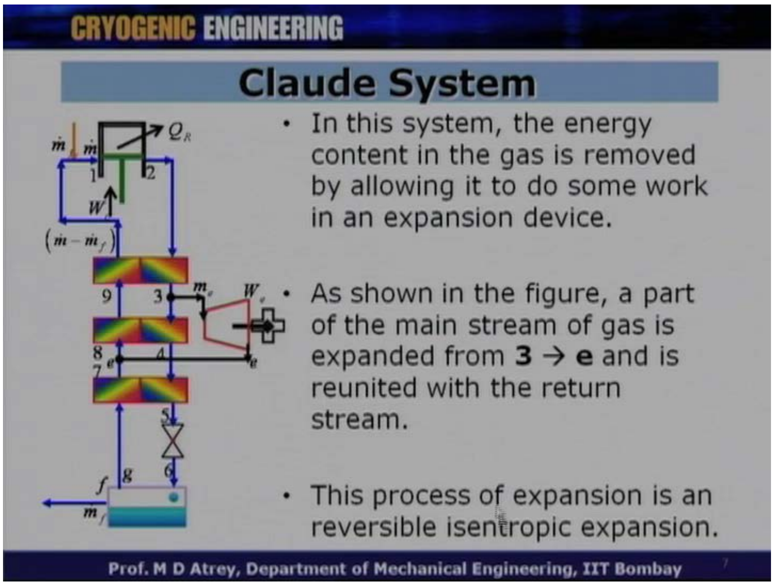

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Claude System

- In this system, the energy content in the gas is removed by allowing it to do some work in an expansion device.
- As shown in the figure, a part of the main stream of gas is expanded from **3 → e** and is reunited with the return stream.
- This process of expansion is an reversible isentropic expansion.

So, in this system, the energy content in the gas is removed by allowing it to do some work in an expansion engine. So, what you get at the end of the expansion over here is W e, which is the expansion work that is the work done by the expansion engine. So, here on the compressor, we are doing the work while at this point here during expansion we get W e as the work done by the system.

As shown in the figure, a part of the main stream of gas is expanded from 3 to e. So, a part of the gas, which is m e is taken from this m gas which is compressed and this expands from point 3 to point e where it joins the return stream and this is the stream, which is going to produce cold. So, that you go near the dome and then we have a J - T expansion resulting in getting liquid at this point.

This process of expansion is a reversible isentropic expansion and that is what we are talking about ideal expansion in this case. However, it will have its own efficiency, and therefore, it will not be a clearly 100 percent isentropic expansion. But as of now, we will take it as 100 percent expansion and we will worry about the irreversibilities later.

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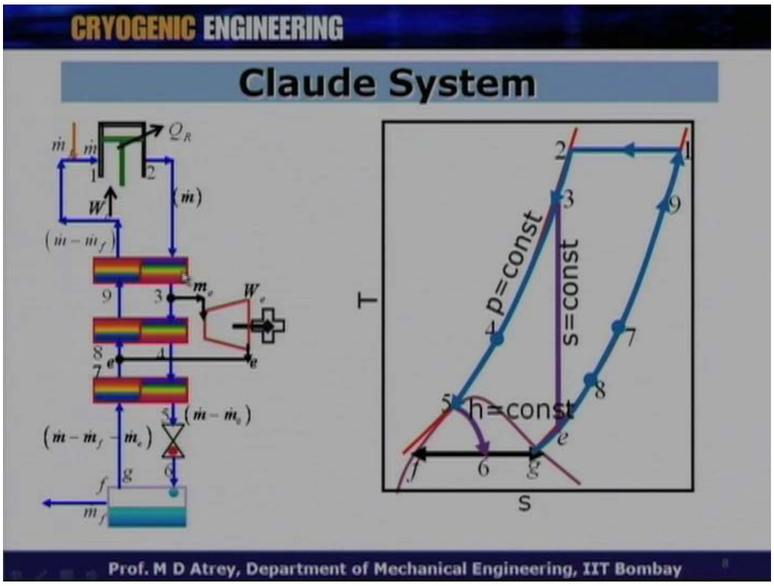

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Claude System

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See, if I plot this cycle, the Claude cycle on a temperature-entropy diagram, let us try to understand this. The 1 to 2 process is a compression, which is what you can see here; 1 to 2 processes is an isothermal compression process. In the heat exchanger, number 1, this m dot is cooled by the return stream from point 2 to point 3, and you can see from point 2 to point 3, and this temperature matches with the return stream at this point, which is 9 over here. Assuming that, this heat exchanger is a 100 percent efficient heat exchanger.

Now, at this point, now from where the gas is expanded, is a very important question in case of Claude cycle. So, why I should expand the gas from only 3? Why not it should be at still lower temperature and things like that? That has to be varied and the effect of this point 3, we can understand in the subsequent slides.

So, the gas is taken off from this main circuit at point 3 and it is expanded isentropically from 3 to e. And this you can see on the T-s diagram, that gas is taken from high pressure line at point 3 and it is expanded from point 3 to point e. So, what you can see, being an isentropic expansion, you can have a vertical line over here on a T-s diagram. So, one has to locate this point on a T - s diagram, when when this isentropic expansion what you have to do is? To draw a vertical line from point 3 and see where, it intersects on the lower pressure return line and that is the point where the point e would lie, because this is a completely 100 percent isentropic expansion.

So, the gas expands from point 3 and joints the return stream at point e, where it gets mixed up with the return stream where it joints the return stream. Coming to the higher pressure line again, at point 3 the gas which is going to be here at this point is m minus m e, because m e has been diverted through the expansion engine. So, what is going ahead after a point 3 is m minus m e. This gas further getting percooled from 3 to 4 and this is what you can see on the T-s diagram, it gets percooled from 3 to 4 and this 4 would match the outlet temperature of this heat exchanger which is 7 all right. Being 100 percent heat exchanger effectiveness, we can see that this point matches over here.

Now, this gas will get in further percooled from 4 to 5. This gas amounting to m minus m e is getting further percooled from 4 to 5 by the return stream, which is going after liquefaction. So, what is the return stream will have the m minus m e is the gas which is getting expanded from 5 to 6, it will for in a dome and corresponding to length of the 6 to g you will get liquefaction. Depending on the point where it falls, here the m dot f is collected from the container and the return stream would therefore, have m minus m e minus m f.

So, out of the gas which is m is compressed m e is going to be expanded through the expansion engine and it joins here at point e, what is going to come ahead beyond this point is m minus m e out of which m dot f is collected. So, what is remaining? And what is going back in the return stream is m minus m f minus m e. This is very important, because we have to understand about this in subsequent slides.

The gas which is going back is m minus m f minus m e, which is at point g. Which is the gas over here, at this point the gas comes out at 7. So, you can see from here, it comes to 7 which is on a higher temperature side, because it matches with the temperature 4. However, the cold gas which is expanded from 3 to e comes at a much lower temperature as you can see; it will depend on what is the point number 3. Where the point number 3 is? In a T - s diagram where the point 3 is depending on that the point e would be decided and this stream which is coming at point 7 will combine with with the m e flow rate and coming out at temperature at point e.

So, as a result of which as you can see, the heat exchanger output is actually at T 7, while the expansion engine output as T e combining both these, the point at the entry of this second heat exchanger is coming at point 8; this is because of the cold which is going to be generated during the expansion 3 to e, I would this is clear.

Depending on the expansion point e and depending on the point 7, which comes out of this heat exchanger? These two streams will be get combined and they will be getting combined by enthalpy rule; that means, they have to multiplied by their respective flow rates that is m minus m f minus m e into s 7 plus m e into enthalpy at this point e, which would result in a some enthalpy, which will corresponding to point number 8, the temperature corresponding to point number 8.

Now, the gas enters at point 8 and it would come out at point 9 and this now gas will be m minus m f over here, because m e has joined return stream. The stream going to the second heat exchanger now will have m minus m f as the mass flow rate. So, this gas entering at 8 will get warm up to point 9, which matches with point 3 and further this gas point 9 enters, after the first heat exchanger comes to point 1 that the makeup gas would join amounting to m dot f and the cycle continues. I hope this T - S diagram which is the very important diagram to understand has been understood by you. What is important to understand is? What is that m e has been drawn from the main circuit at what T 3 temperature it has been drawn off and how much amount of m dot f or what is your return stream gas is left?

What you can see is all these heat exchangers are basically having different flow rates. For example, in this heat exchanger you have got a m dot flow rate over here, while it is the return stream have m minus m f. This has m minus m e while this has m minus m f and the third heat exchanger will have m minus m e over here while it will have m minus m f minus m e over here.

So, these heat exchangers are kind of imbalance heat exchanger means that the flow rates in both the directions the hot stream and the cold stream flow rates are different. If you have understood this diagram, we can go ahead now in order to compute the expressions for the yield.

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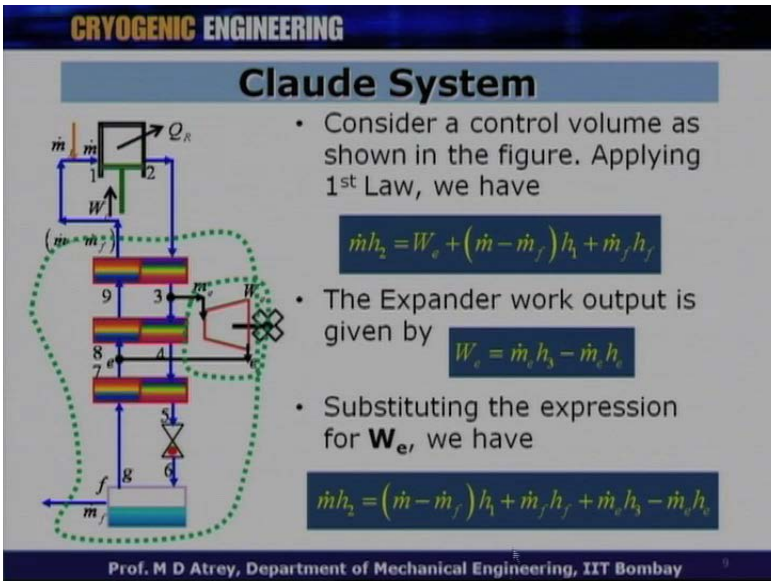

CRYOGENIC ENGINEERING

Claude System

- Consider a control volume as shown in the figure. Applying 1st Law, we have

$$\dot{m}h_2 = W_e + (\dot{m}-\dot{m}_f)h_1 + \dot{m}_f h_f$$

- The Expander work output is given by $W_e = \dot{m}_e h_3 - \dot{m}_e h_e$
- Substituting the expression for **W_e**, we have

$$\dot{m}h_2 = (\dot{m}-\dot{m}_f)h_1 + \dot{m}_f h_f + \dot{m}_e h_3 - \dot{m}_e h_e$$

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So, consider a control volume as shown in the figure and applying first law, what we have is? We have this we have done plenty number of times earlier for various cycles. So, I will not go into the details of these calculations, but I will just show you the final expression which comes out in 1 or 2 steps.

So, what is going in has to come out that is what your first law says. So, what is entering at this point is m dot h 2 and what is coming out of this control volume is W e. So, m dot h 2 is equal to W e plus the return stream which is coming to this point which is m minus m f into h 1 plus m f h f which is going to come at this point all right.

The expander work output that is W e is given by what is entering the expansion engine is m e into h 3 while what is leaving is m e into h e. So, W e is equal to m e h 3 minus m e h e. So, basically the flow rates are same, it depends on the enthalpy difference between h 3 minus h e or T 3 minus T e on a T - s diagram. So, if you put this value of m e over here what you get is? m dot h 2 into m minus m f h 1 plus m f h f plus m e h 3 minus m e h e.

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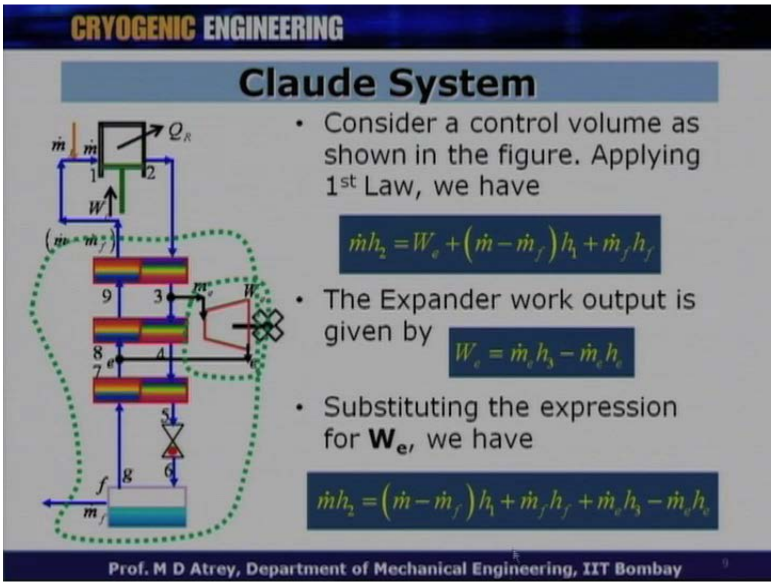

CRYOGENIC ENGINEERING

Claude System

- Consider a control volume as shown in the figure. Applying 1st Law, we have

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- Substituting the expression for $\mathbf{W_e}$, we have

$$\dot{m}h_2 = (\dot{m}-\dot{m}_f)h_1 + \dot{m}_f h_f + \dot{m}_e h_3 - \dot{m}_e h_e$$

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By reorganizing this or rearranging this, what we get ultimately is this? Why which is nothing, but m dot f upon m dot is equal to h 1 minus h 2 upon h 1 minus h f, which is what we are familiar with this expression. Now, plus x time h 3 minus h e upon h 1 minus h f and what is x, x is nothing, but the mass flow fraction of the stream which gets expanded. So, m dot e divided by m dot. So, depending on how much mass fraction of the total mass flow rate has been taken off for expansion. This value of x will be determined.

So, basically if the more gas goes through it more and more cold will will get generated. However, if you take lot of gas from here as you can understand less gas is going to in front. So, m minus m e which is going front is basically some gas out of that stream is going to get liquefied. One has to be very careful about how much gas is taken and at what temperature is taken and this is what we are going to see in subsequent slides.

So, where the expander mass flow rate is denoted by x over here. So, what you can see is h 1 minus h 2 upon h 1 minus h f is the very familiar expression for you and the Claude cycle brings this additional component or the next bracket, because of which y gets increased.

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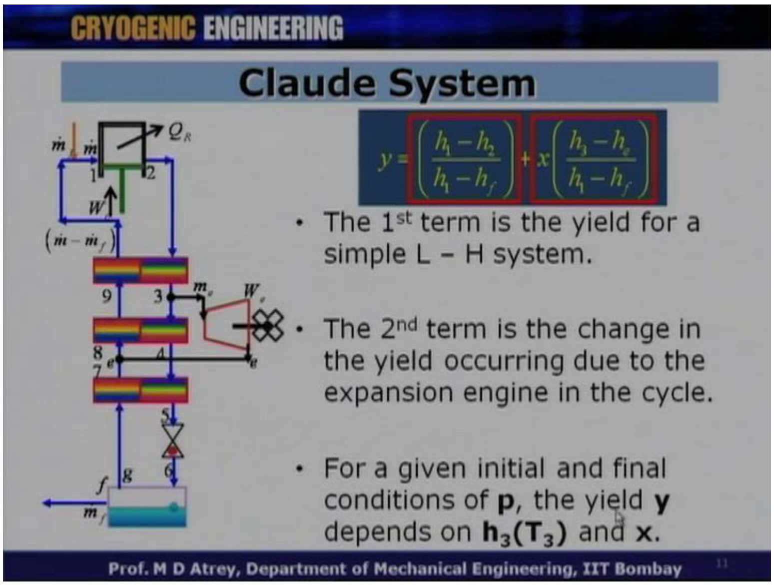

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Claude System

$$y=\left(\frac{h_1-h_2}{h_1-h_f}\right)+x\left(\frac{h_3-h_e}{h_1-h_f}\right)$$

- The 1st term is the yield for a simple L – H system.
- The 2nd term is the change in the yield occurring due to the expansion engine in the cycle.
- For a given initial and final conditions of **p**, the yield **y** depends on **$h_3(T_3)$** and **x**.

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So, the first term is the yield of a simple Linde - Hampson cycle, which we have seen that it depends only on the point 1 and 2, the compression process 1 and 2. The second term which is an addition to the simple Linde - Hampson cycle is basically coming, because of the expansion engine over here. So, second term is the change in the yield occurring due to the expansion engine in cycle.

For a given initial and final conditions of p; that means, if the point 1 and 2 are fixed, the yield y depends on h 3. So, if you see this expression, the y value depends on 1 and 2 and 3, because as soon as your 3 is decided the point e is also decided immediately. Enthalpy 3 will depend on what is the corresponding temperature at which the expansion starts all right. If your 1 and 2 points; that means the pressures are fixed, it will depend on the value of x and the temperature T 3 from where the expansion takes place all right.

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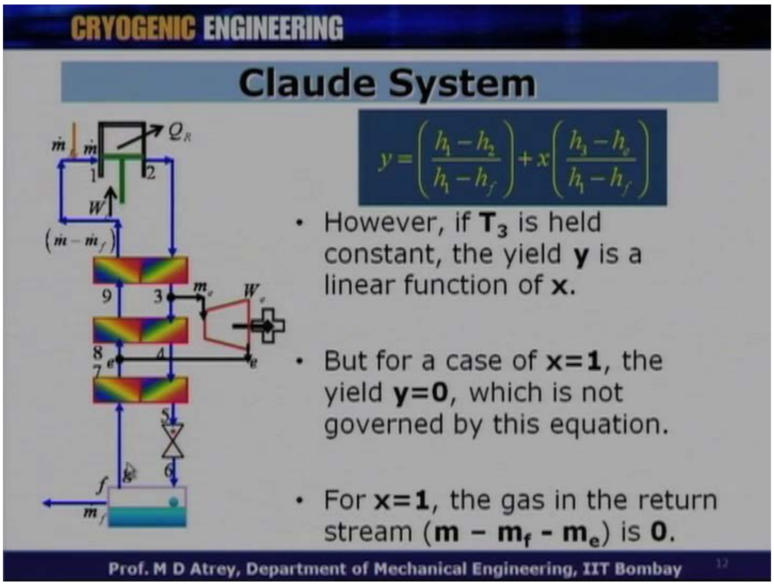

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$$y = \left(\frac{h_1 - h_2}{h_1 - h_f}\right) + x\left(\frac{h_3 - h_e}{h_1 - h_f}\right)$$

- However, if $\mathbf{T_3}$ is held constant, the yield **y** is a linear function of **x**.
- But for a case of **x=1**, the yield **y=0**, which is not governed by this equation.
- For **x=1**, the gas in the return stream ($\mathbf{m - m_f - m_e}$) is **0**.

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And if we see that, if the T 3 also is held constant, then the yield depends on only x. So, this is constant, if your point 1 and 2 are fixed and on your pressure number 2, on your p h value your temperature is also fixed, because the (( )) your h 3 minus h e also gets fixed. This becomes a linear function of x now, because this is a constant, this also becomes constant and the value of x however, could be variable.

So, in this case y will be a linear function of x, when the temperature T 3 is fixed. I can go on increase in the value of x, I am sure I will get y increased. So, I can theoretically I can go on increase in the value of x. But take a limiting case that if I make x is equal to 1; that means, whatever gas is coming from here is expanded; that means, no gas is going to go in front. In that case, y should be equal to 0 when x is equal to 1, I will not get the value of y to be equal to 0 y, because that particular case has not been taken into account in this expression all right.

So, what happens when x is equal to 1, the gas in the return stream which is m minus m f minus m e is equal to 0. Actually, the gas which comes over here also equal to 0 basically. So, if we want that the heat exchanger has to function; that means, they should be return stream you got a onward stream and you should have a return stream also and in order that return stream is finite value your m minus m f minus m e should not be equal to 0 all right.

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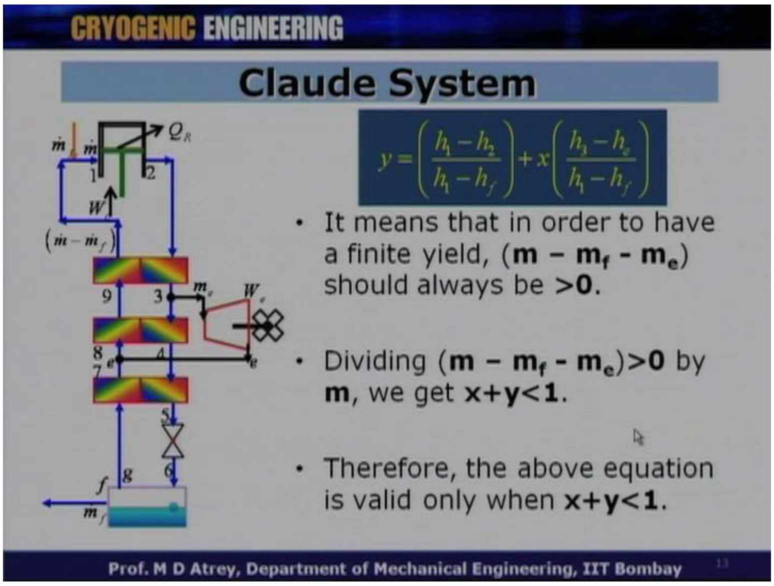

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$$y=\left(\frac{h_1-h_2}{h_1-h_f}\right)+x\left(\frac{h_3-h_e}{h_1-h_f}\right)$$

- It means that in order to have a finite yield, **($\mathbf{m - m_f - m_e}$)** should always be **>0**.
- Dividing **($\mathbf{m - m_f - m_e}$)>0** by **m**, we get **x+y<1**.
- Therefore, the above equation is valid only when **x+y<1**.

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So, it means that in order to have a finite yield; that means, in order to have a finite value of y at this point your m minus m f minus m e which is nothing, but the return stream at point g has to be always more than 0; that means, it should be some finite value so, that the heat exchangers function.

So, if we divide the whole thing by m what we get is 1 minus y minus x, which is more than 0 and if I rearrange that what I should get is x plus y should always be less than 1. It means that in order that heat exchangers function, in order that I should get some finite return stream. I should see to it that x plus y is always less than 1, if your x plus y is less than 1 all these things possible and least expression of y is equal to this bracket plus x time this bracket is valid only, when your x plus y is less than 1. When x plus y is equal to 1; that means, again there is no return stream whatever x has been directed and remaining y is equal to 1, then I will not get any return stream at this point.

So, one should ensure that x plus y is always less than of (( )). However, from this expression I can go on increased the value of x and I can get y also as increased value. So, it has no meaning. So, therefore, as soon as this particular expression start giving x plus y as more than 1, we should not use this expression. This expression is valid for x plus y less than 1 only. Therefore, the above equation is valid only when x plus y is less than 1.

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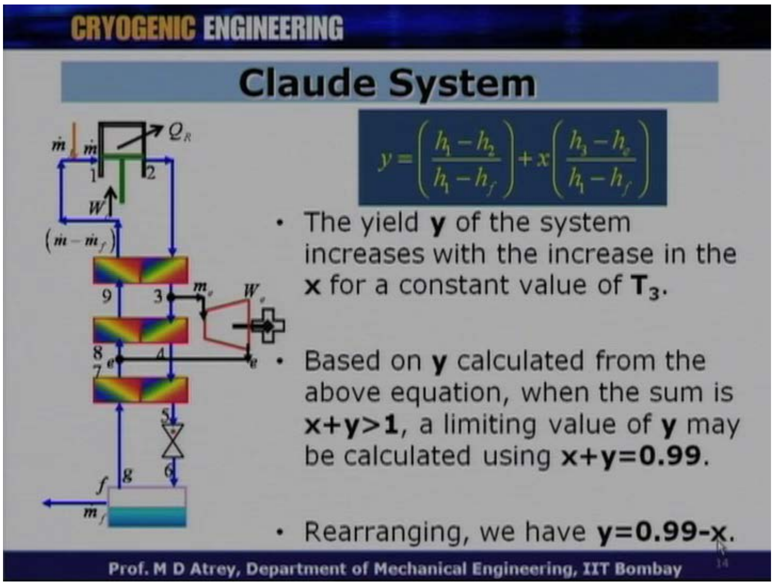

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Claude System

$$y=\left(\frac{h_1-h_2}{h_1-h_f}\right)+x\left(\frac{h_3-h_e}{h_1-h_f}\right)$$

- The yield **y** of the system increases with the increase in the **x** for a constant value of **T_3**.
- Based on **y** calculated from the above equation, when the sum is **x+y>1**, a limiting value of **y** may be calculated using **x+y=0.99**.
- Rearranging, we have **y=0.99-x**.

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The yield y of the system increases with the increase in the value of a x for a constant value of T 3, that is what we have seen that. If we start increase the value of x from let say point 1 at a particular value of T 3, it will start increasing with a value of x in a linear fashion.

Based on y calculated from the above equation, when x plus y becomes more than 1 that is why I just talked about? When utilizing this equation when we reach a condition that x plus y is more than 1, we know that a limiting value of y must be calculated now and what we say, as we say that x plus y has to be less than 1 let us compute a limiting value that is x plus y is equal to 0.99. When my x plus y happens to be more than 1 in this case, I will calculate the y is equal to 0.99 minus x, this could be 0.999 also minus x, but I am taking only 2 digits. So, y is equal to 0.99 minus x, when we reach a condition that x plus y is more than 1.

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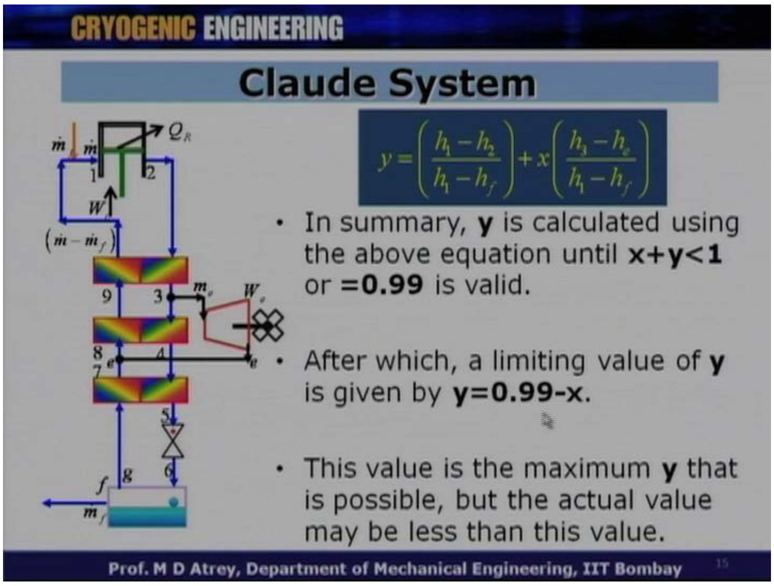

CRYOGENIC ENGINEERING

Claude System

$$y=\left(\frac{h_1-h_2}{h_1-h_f}\right)+x\left(\frac{h_3-h_e}{h_1-h_f}\right)$$

- In summary, **y** is calculated using the above equation until **x+y<1** or **=0.99** is valid.
- After which, a limiting value of **y** is given by **y=0.99-x**.
- This value is the maximum **y** that is possible, but the actual value may be less than this value.

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In summary, how do I calculate y. Y is calculated using the above equation until x plus y is less than 1 or 0.99. So, this expression is valid, when x plus y is equal to 0.99. After which, a limiting value of y can be calculated by y is equal to 0.99 minus x and this is what a general principle, we are going to follow to calculate the yield choosing Claude cycle I would this clear to you.

This value is the maximum y value, because I am taking 0.99. This is the maximum y value that is possible, but actual value could be less than this value. The actual value could be less than this value, but this is the limiting value; that means, the y cannot exceed this value in any case.

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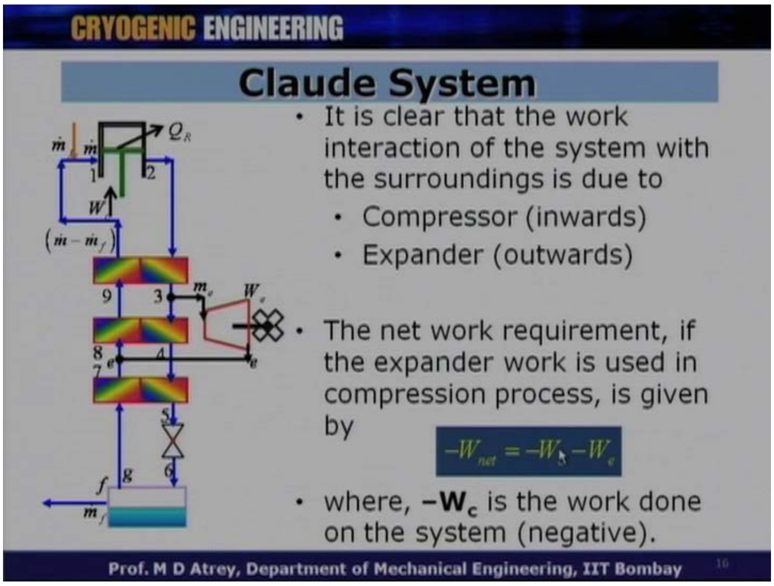

CRYOGENIC ENGINEERING

Claude System

- It is clear that the work interaction of the system with the surroundings is due to
  - Compressor (inwards)
  - Expander (outwards)
- The net work requirement, if the expander work is used in compression process, is given by $-W_{net} = -W_c - W_e$
- where, $-\mathbf{W_c}$ is the work done on the system (negative).

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It is clear that the work interaction of the system with the surrounding is due to let us calculate the work requirement for the system and where, the work comes into picture is compressor and expander. Compressor we do the work on the system and the expander we get work output; that means, work is done by the system. So, the net work requirement, if the expander work is used in compression process ideally, whatever work is done by the system it could be transfer to the compressor basically. So, that decreases the net work which has to be done by the compressor.

So, minus W net is equal to minus W c minus W e. The negative minus W c is the work done on the system. So, work done on the system is negative and work done by the system is positive. So, it is a net difference between what is done on the system minus work done by the system.

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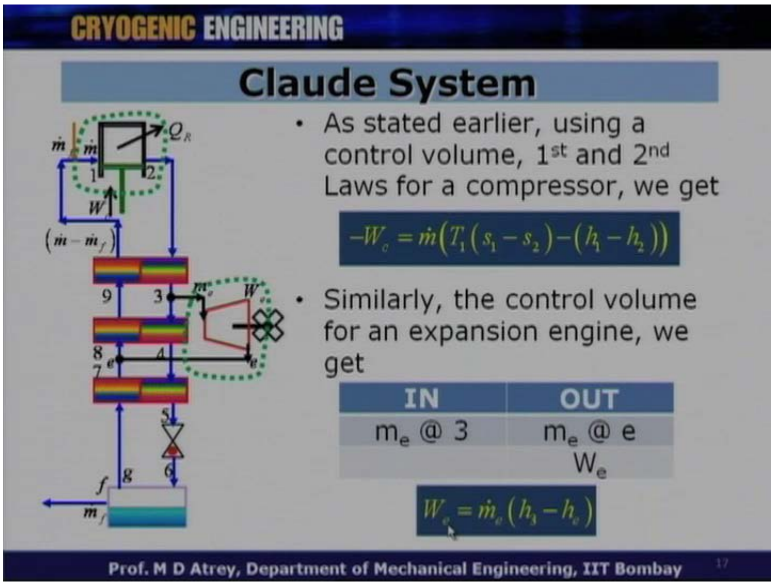

CRYOGENIC ENGINEERING

Claude System

- As stated earlier, using a control volume, 1st and 2nd Laws for a compressor, we get

$$-W_c = \dot{m}\left(T_1\left(s_1 - s_2\right) - \left(h_1 - h_2\right)\right)$$

- Similarly, the control volume for an expansion engine, we get

IN	OUT
m_e @ 3	m_e @ e
	W_e

$$W_e = \dot{m}_e\left(h_3 - h_e\right)$$

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As we have stated earlier, using the control volume first and second law for compressor. This is very standard equation, which we have use almost in the of the cycles that work done on the compression is equal to m dot into T 1 into delta s across 1 and 2 minus delta h across 1 and 2, the very standard, equation which has been used.

Similarly, the control volume for expansion engine also gives us what is entering is m e at point 3, what is leaving is m e at point e at this point and also what we are getting as output is W e meaning which W e is equal to m e into h 3 minus h e, we have talked about T is earlier also.

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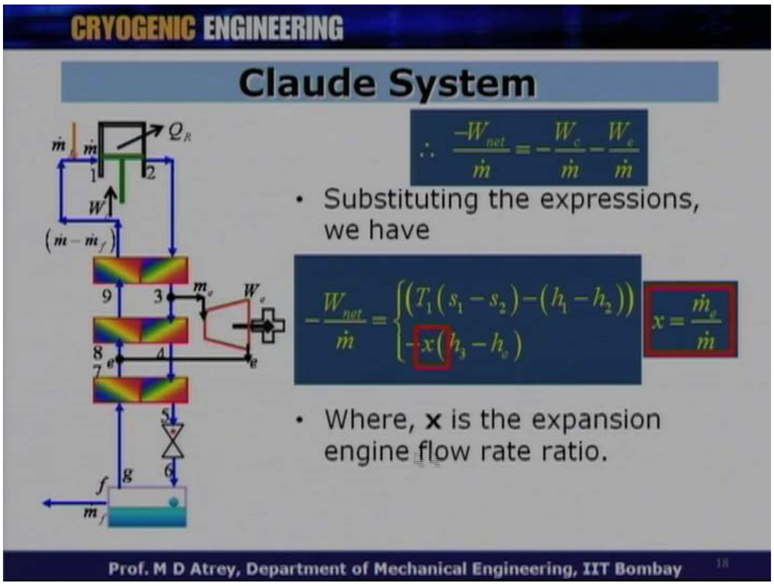

CRYOGENIC ENGINEERING

Claude System

$$\therefore \frac{-W_{net}}{\dot m} = -\frac{W_c}{\dot m} - \frac{W_e}{\dot m}$$

- Substituting the expressions, we have

$$-\frac{W_{net}}{\dot m} = \left\{\begin{array}{l}(T_1(s_1 - s_2) - (h_1 - h_2)) \\ -\boxed{x}(h_3 - h_e)\end{array}\right. \qquad \boxed{x = \frac{\dot m_e}{\dot m}}$$

- Where, **x** is the expansion engine flow rate ratio.

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So, if I put them together, I will get dividing by m dot to the expression. I get W net upon m dot; that means, net work done per unit mass of gas which is compressed is equal to T 1 into s 1 minus h 2 minus h 1 minus h 2 minus x into h 3 minus h. Where, x is equal to m dot e upon m dot again, x is the expansion engine flow rate.

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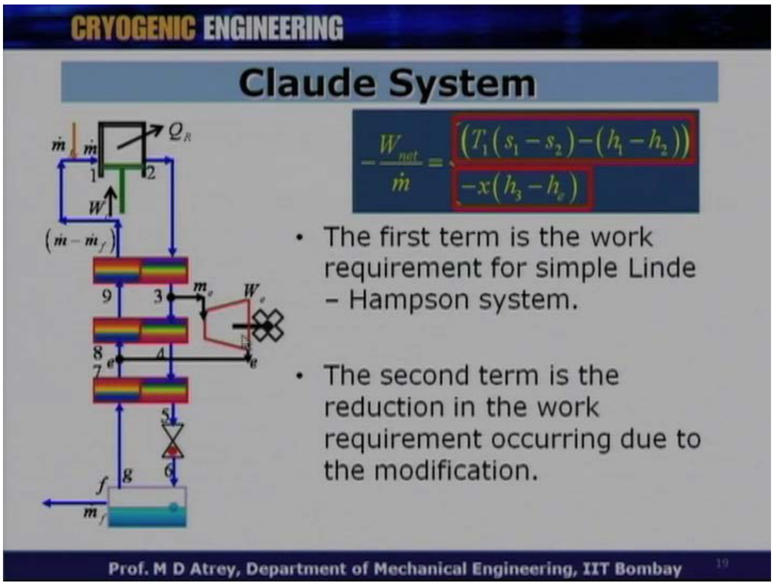

CRYOGENIC ENGINEERING

Claude System

$$-\frac{W_{net}}{\dot m} = \begin{array}{l}\boxed{(T_1(s_1 - s_2) - (h_1 - h_2))} \\ \boxed{-x(h_3 - h_e)}\end{array}$$

- The first term is the work requirement for simple Linde – Hampson system.
- The second term is the reduction in the work requirement occurring due to the modification.

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So, what do we understand from this? The first term is the work requirement for simple Linde - Hampson cycle, where the only compressor comes into picture. While, the second work is the work done by the system and it is coming only, because the expansion engine is in place in the cycle now all right. So, this is the work done by the system amount to W e.

The second term is the reduction in the work requirement occurring due to the modification. But you may understand that the work output what you get at this point, which is expansion engine is much smaller as compared to work done on the system. So, if we are talking about kilo watts over here, what you get here could be 50, 60 watts only over here.

However, sometimes we may consider, sometimes we do not consider also. But theoretically, we can consider that whatever net is available is work done on the compressor minus work you get expansion engine. Although, it could be possible that this work output you get from the expansion engine is very very small as compared to the work done on the compressor.

Based on the... What we have learnt in the Claude cycle, it will be very useful. Now, we take a small tutorial and we we do the parameters studies using this tutorial. Where, we can understand the effect of values of the gas x, the concept x or the gas amounting to W m dot e which is diverted to the expansion engine and also we will try to worry about what happens, if the temperature T 3 from where the expansion happens. So, what is the… What is the role of T 3 in these calculations?

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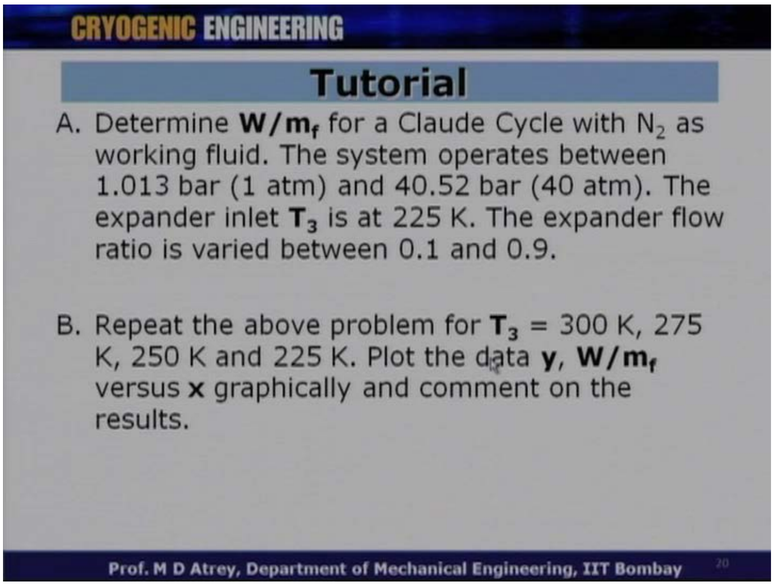

CRYOGENIC ENGINEERING

Tutorial

A. Determine **W/m_f** for a Claude Cycle with N_2 as working fluid. The system operates between 1.013 bar (1 atm) and 40.52 bar (40 atm). The expander inlet **T_3** is at 225 K. The expander flow ratio is varied between 0.1 and 0.9.

B. Repeat the above problem for **T_3** = 300 K, 275 K, 250 K and 225 K. Plot the data **y**, **W/m_f** versus **x** graphically and comment on the results.

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So, from that point of view this tutorial has been framed. The part a is determine W by m f that is work of compression per mass of gas liquefied for a Claude cycle with Nitrogen as a working fluid. So, one can have air Oxygen, Nitrogen etcetera with Nitrogen are the working fluid in this case. The system operates between 1 atmosphere and 40 atmosphere or 1.013 bar and 40.52 bar. The expander inlet T 3 value is 225 Kelvin that means it is below ambient.

The expander flow ratio is varied between 0.1 and 0.9 that means you can see the value of x is varied between 0.1 to 0.9 and what we want to see is? What are the effects of this variation of x on the value of W by m f?

Now, in this problem what we have done value of T 3 has been fixed at 225 Kelvin. In the part b, we want to study repeat the above calculations for T 3 is equal to 300 K, 275 K, 250 K and 225 K. So, here in we can see that what is the role of this T 3 in the entire cycle, if we reduce the value from 300, 275, 250, 225 etcetera how does it change the value of y and W by m f? So, what the problem says is plot the data that is y and W by m f versus x graphically and comment on the results.

So, basically here we want to study the effect of the value of T 3 and the value of x, which is varied from 300 point to 25 K and 0.1 to 0.9 Kelvin respectively and we want to study how do this parameters affect y and W by m f versus x, when we graphically plot we can the nature would be clear.

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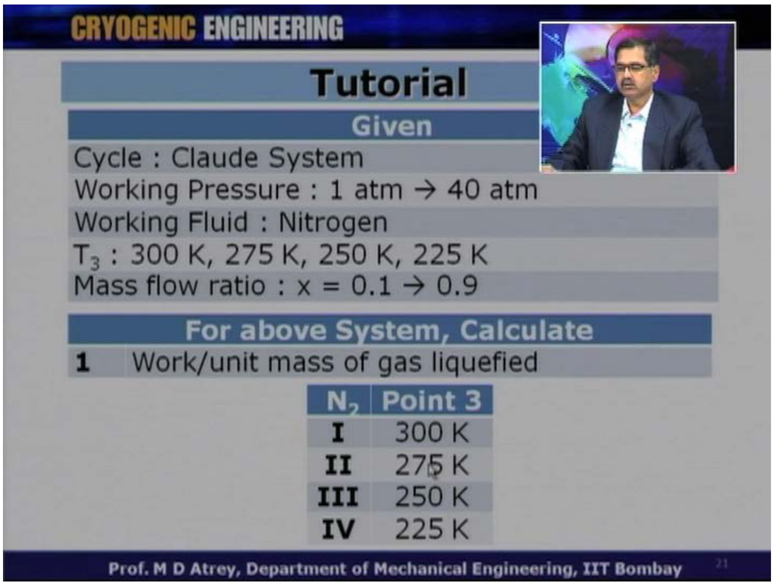

CRYOGENIC ENGINEERING

Tutorial

Given

Cycle : Claude System

Working Pressure : 1 atm → 40 atm

Working Fluid : Nitrogen

T_3 : 300 K, 275 K, 250 K, 225 K

Mass flow ratio : $x = 0.1 \rightarrow 0.9$

For above System, Calculate

1 Work/unit mass of gas liquefied

N_2	Point 3
I	300 K
II	275 K
III	250 K
IV	225 K

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What is the problem? The problem is we got a Claude cycle, got a Claude system. The working pressure is 1 atmosphere to 40 atmosphere here, the working fluid is Nitrogen the value of T 3 variations, what we want in total is from 300 Kelvin to 225 Kelvin. So, 300, 275, 250 and 225 that is in steps of 25 Kelvin. The mass flow fraction that is x is varied from 0.1 to 0.9, as I said earlier x cannot be equal to 1; that means all the gas which comes cannot be diverted through the expansion engine all right. So, it has to be less than 1.

For the above system what is asked from you is? To calculate the work per unit mass of gas is liquefied and for which you have to in the calculations of y. One has to calculate y first and this W by m divided by y will give you W by m f and 1, 2, 3 and 4 for point 3 at 300, 275, 250 and 225 Kelvin and this is what the overall problem is?

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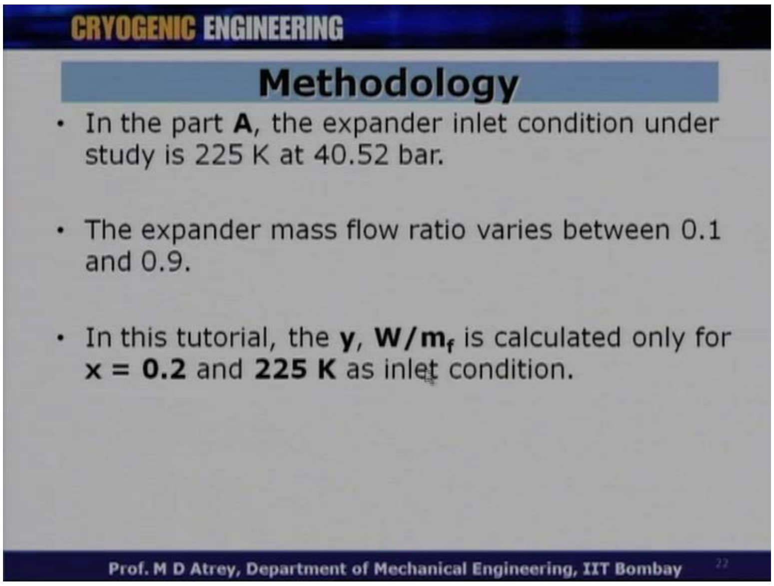

CRYOGENIC ENGINEERING

Methodology

- In the part **A**, the expander inlet condition under study is 225 K at 40.52 bar.
- The expander mass flow ratio varies between 0.1 and 0.9.
- In this tutorial, the **y**, **W/m$_f$** is calculated only for **x = 0.2** and **225 K** as inlet condition.

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So, in the part A, the expander inlet condition under study is 225 Kelvin and at 40.52 bar. The expander mass flow ratio varies between 0.1 and 0.9. What we are going to do now? I am not going to solve all the problems all the for, all the values of x from 0.1 to 0.9. What we are going to do in detail is? To understand, how y and W m f are calculated for x is equal to point 2 for example, and e 3 is equal to 225 Kelvin for example, as in inlet condition. We will give you the complete table and what we expect you that you will solve it out yourself. So, that you can see that you have understood the problem.

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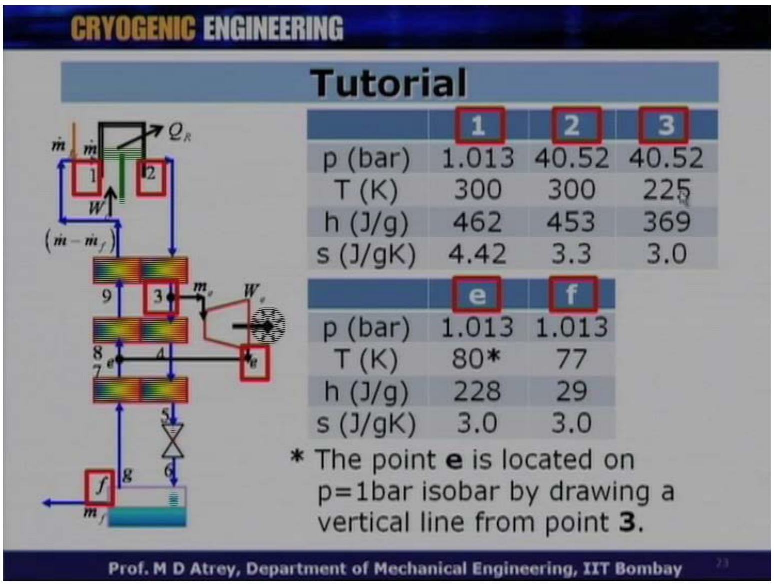
CRYOGENIC ENGINEERING

Tutorial

	1	2	3
p (bar)	1.013	40.52	40.52
T (K)	300	300	225
h (J/g)	462	453	369
s (J/gK)	4.42	3.3	3.0

	e	f
p (bar)	1.013	1.013
T (K)	80*	77
h (J/g)	228	29
s (J/gK)	3.0	3.0

* The point **e** is located on p=1bar isobar by drawing a vertical line from point **3**.

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So, now let see the Claude cycle and here is the Claude cycle having a compressor and expansion engine and what we have is a point 1, 2, 3, 4, e and f. So, one can see that point 1 is over here, let us try to locate the points point 1 is at 1 bar 300 Kelvin corresponding enthalpy entropies are given at point 1.

Point 2 is the after the compression and again at this point 2, we got pressure temperature conditions given over here, with the corresponding enthalpy, entropy values. The point 3 happens to be what is given in the problem? Which is what we have taken as 225 Kelvin and pressure is same as what it is at point 2? Corresponding enthalpy, entropy are again calculated out here.

As soon as the point 3 is fixed on a T - s diagram at 40 and 225 Kelvin, we get the point e which is over here and the temperature at this point e happens to be 80 Kelvin. So, what we do basically? We have got a T - s diagram, we draw a vertical from 225 Kelvin on 40 bar line vertically down where, it intersects the temperature on 1 bar isobaric line and wherever it intersects the corresponding temperature happens to be 80 Kelvin. So, the point e is located on p is equal to 1 bar isobar line by drawing a vertical line from point 3 and this has to be graphically or it from a T - s diagram, it has to be taken.

And the pressure at this point is going to be low pressure line, because this is a return stream. The point f happens to be the boiling point of Nitrogen which is 77 Kelvin so, here f is where form you collect the liquid from where you get the yield value. So, this is the most important part, if you come it any mistake on this, then the whole problem is equal to going to go wrong.

So, one always should have a habit of locating this point 1, 2, 3, e and f and correspondingly one should correctly write the value of enthalpy and entropy. This is the very common mistakes student come it, because they read the T - s diagram very fast, the current points are not located and sometimes what you is? Your yield also happens to be negative and thing like that. So, the enthalpy, entropy value has to be correctly have to be correctly taken from the T - s diagram.

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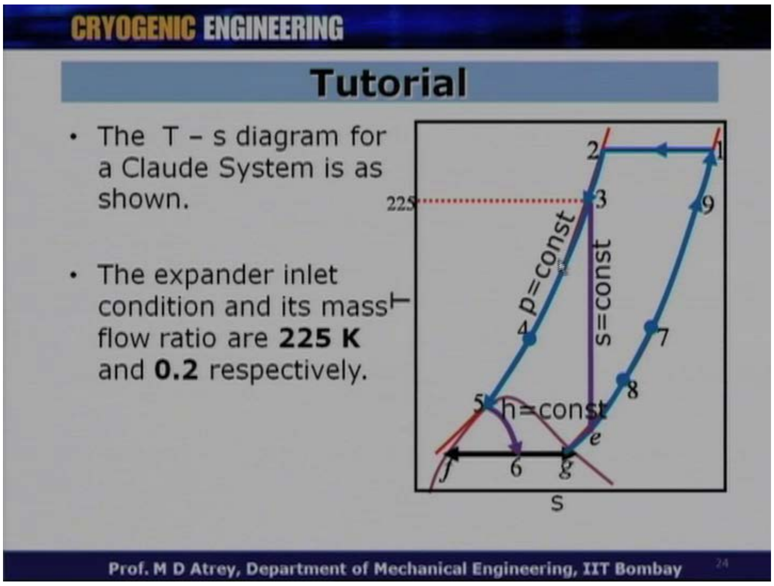

CRYOGENIC ENGINEERING

Tutorial

- The T – s diagram for a Claude System is as shown.
- The expander inlet condition and its mass flow ratio are **225 K** and **0.2** respectively.

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Now, one can see the corresponding T - s diagram here. 1, 2 is a compression, 2 to 3 is a cooling, 3 to e is where you get the point e that is what I just said, that we draw a vertical from high pressure line at point 3 to the low pressure line which intersect this point at e and correspondent to this what you get is 80 Kelvin in this step. In this particular case, temperature at point 3 is 225 Kelvin.

So, the expander inlet condition and it is mass flow rate at are 225 Kelvin and how much gas is going in this particular problem is we are saying x is equal to 0.2. So, point 2 times m dot is the gas which is sent to this expansion engine. So, fraction which is going through the expansion engine is only point 2; that means, what is coming out at this point is 0.8 point at fraction of the total mass flow rate over here.

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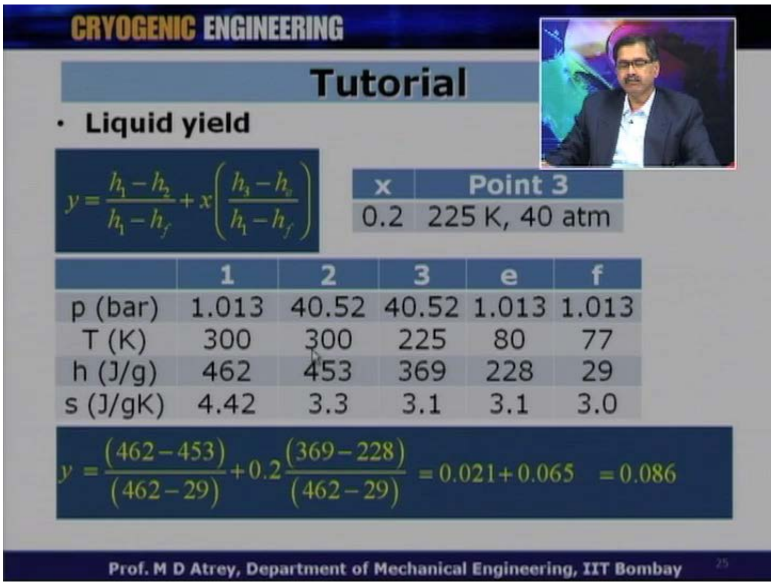

CRYOGENIC ENGINEERING

Tutorial

- **Liquid yield**

$$y = \frac{h_1 - h_2}{h_1 - h_f} + x\left(\frac{h_3 - h_e}{h_1 - h_f}\right)$$

x	Point 3
0.2	225 K, 40 atm

	1	2	3	e	f
p (bar)	1.013	40.52	40.52	1.013	1.013
T (K)	300	300	225	80	77
h (J/g)	462	453	369	228	29
s (J/gK)	4.42	3.3	3.1	3.1	3.0

$$y = \frac{(462-453)}{(462-29)} + 0.2\frac{(369-228)}{(462-29)} = 0.021 + 0.065 \quad = 0.086$$

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So, let us start doing simple calculations and calculate the liquid yield by the formula y is equal to h 1 minus h 2 upon h 1 minus h f plus x time h 3 minus h e upon h 1 minus h f. Where, x is equal to 0.2 in this case, T 3 is equal to 225 Kelvin and 40 atmosphere pressure.

So, taking the respective points from this table from where enthalpy, entropy have to be read let us calculate the value of y is equal to h 1 minus h 2 which is 462 minus 453 upon h 1 minus h f 462 minus 29 plus x time that is 0.2 times h 3 minus h e which is nothing, but a different between this 2 values divided by h 1 minus h f and this will give you 0.086.

So, why that is m dot f upon m dot is only 0.086. So, this much fraction of gas gets liquefied, when point 2 fractions is diverted through the expansion engine and this is what we have calculated? Now, let us calculate even W by m f by the same technique.

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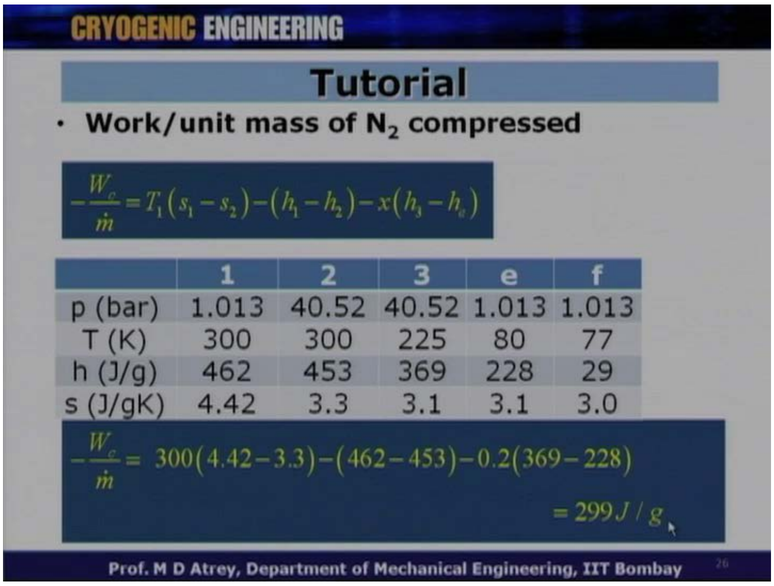

CRYOGENIC ENGINEERING

Tutorial

- **Work/unit mass of N_2 compressed**

$$-\frac{W_c}{\dot{m}}=T_1\left(s_1-s_2\right)-\left(h_1-h_2\right)-x\left(h_3-h_e\right)$$

	1	2	3	e	f
p (bar)	1.013	40.52	40.52	1.013	1.013
T (K)	300	300	225	80	77
h (J/g)	462	453	369	228	29
s (J/gK)	4.42	3.3	3.1	3.1	3.0

$$-\frac{W_c}{\dot{m}}= 300(4.42-3.3)-(462-453)-0.2(369-228)$$

$$=299J/g$$

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So, if I want to calculate work per unit mass of gas compressed. The formula is W c by m dot which is nothing, but T 1 into s 1 minus h 2 minus h 1 minus h 2 minus x times h 3 minus h e. Again taking the different entropy values and enthalpy values as given over here, this is the calculation you do and what you get at the end is 299 joule per grams. So, this is the net work done on the system… This is the work done on the compressor not net work done, but it is the work done on the compressor.

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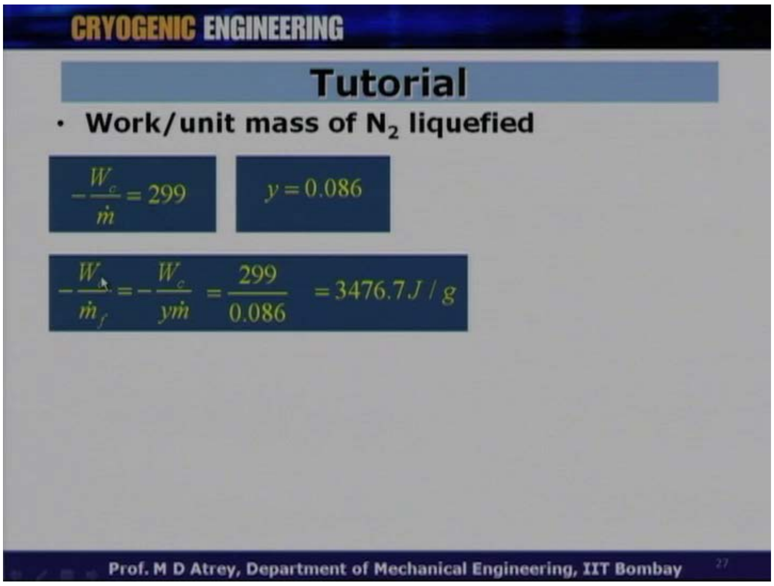

CRYOGENIC ENGINEERING

Tutorial

- **Work/unit mass of N_2 liquefied**

$$-\frac{W_c}{\dot{m}}=299$$

$$y=0.086$$

$$-\frac{W}{\dot{m}_f}=-\frac{W_c}{y\dot{m}}=\frac{299}{0.086}=3476.7J/g$$

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And work done per unit mass of gas liquefied is W c y m dot is equal to 299, y is equal to 0.086 and W c by m dot f is 299 upon 0.086 which is 3476.7 joule per gram all right. So, what you get from here is? The work done on compression per unit mass of gas which is liquefied.

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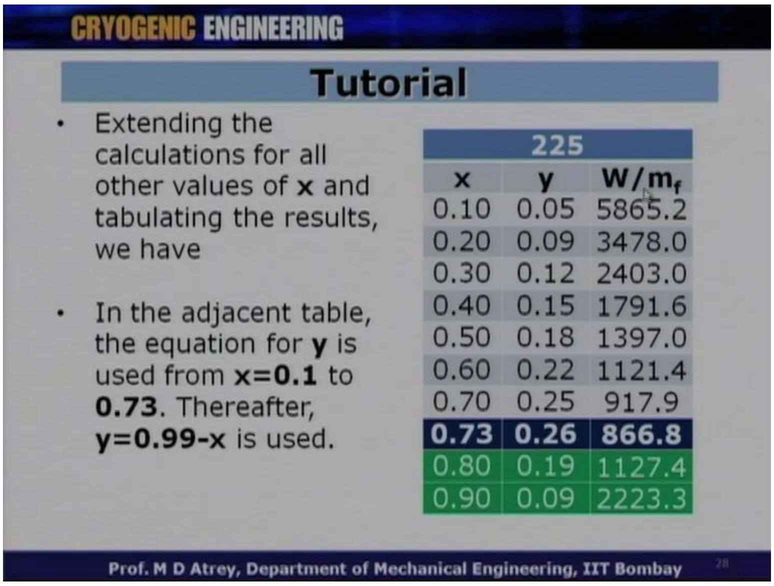

CRYOGENIC ENGINEERING

Tutorial

- Extending the calculations for all other values of **x** and tabulating the results, we have
- In the adjacent table, the equation for **y** is used from **x=0.1** to **0.73**. Thereafter, **y=0.99-x** is used.

225		
x	y	W/m_f
0.10	0.05	5865.2
0.20	0.09	3478.0
0.30	0.12	2403.0
0.40	0.15	1791.6
0.50	0.18	1397.0
0.60	0.22	1121.4
0.70	0.25	917.9
0.73	**0.26**	**866.8**
0.80	0.19	1127.4
0.90	0.09	2223.3

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Extending this calculation for all the values of x and tabulating the results. So, what we have done? I have just calculated the value of y and W by m f for x is equal to 0.2 for the value of T 3 when it is equal to 225 Kelvin. Now, what I will do? I will give a sweep to the value of x from 0.1 to 0.9. Remember all the discussion, what we had earlier regarding where do we use this expression and after limiting value what expression do we used to calculate the value of y?

So, extending this completely and I am not showing all the details what I show is a table, which gives when T 3 is equal to 225 Kelvin, when you vary the value of x from 0.1 to 0.9. The corresponding y values are given over here and corresponding W by m f values are also given over here.

So, what you can see here, as we had seen that as the value of x is increasing. y increases linearly, up to the point when x plus y is limiting value, it has to be less than 1 or in this case is 0.99. So, you can see that when the x is increase from 0.1 to 0.73 and x is equal to 0.73, y goes on increasing and meets limiting value of 0.26. Where after that, if x goes on increasing I cannot use the earlier expression, because x plus y happens to be more than 1. In that case, we have used the limiting value of y which is 0.99, we have did the limiting value of x plus y is equal to 0.99 and therefore, y has been calculated as 0.99 minus x.

So, you can see the sum here happens to be 0.99 all the time. This is the limiting value of y; that means, actual y could be less than this y, but definitely not more than this. Having calculated y in this fashion, we have calculated W by m f also and that also uses this y values and what you can see is over here? The results over here. What does the result show you? The result show you that as the x increases, the y value increases and goes through a maximum of 0.26 and after that it mix a limiting value and it is starts coming down.

Similarly, what you can see on W by m f that as the value of x increases the W by m f value or the work done per unit mass of gas liquefied decreases and that is basically where the principle of use in expansion engine happens, the amount of gas which is transferred or which goes to expansion engine should be such that the W by m f has to be minimum and y has to be maximum for that particular temperature T 3.

So, W by m f the value goes on the reducing and it meets a minimum value at which y happens to be maximum, because W by m f nothing, but W by m upon y. So, wherever y get minimized, the value of W by m f will be maximum over there. So, the value of W by m f reaches the minimum value over and again it is start increasing. So, this y goes to a maximum and W by m f goes to a minimum that is what we understand from this.

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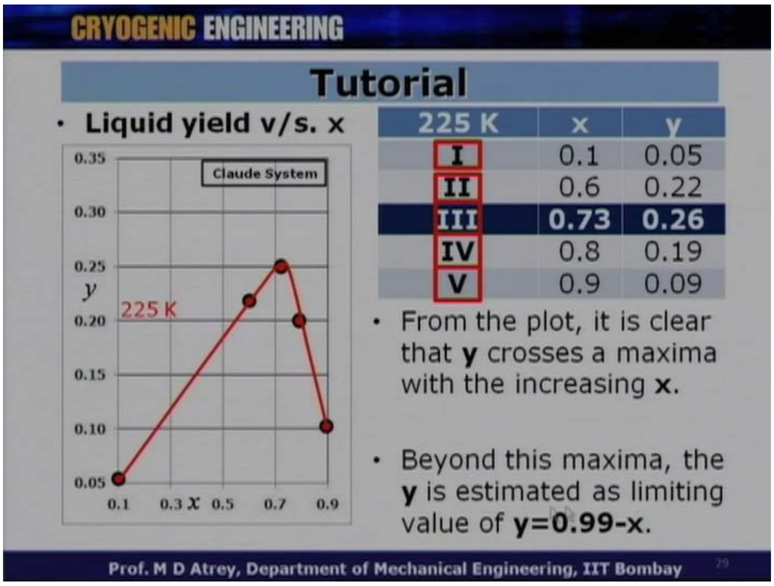

CRYOGENIC ENGINEERING

Tutorial

- **Liquid yield v/s. x**

225 K	x	y
I	0.1	0.05
II	0.6	0.22
III	**0.73**	**0.26**
IV	0.8	0.19
V	0.9	0.09

- From the plot, it is clear that **y** crosses a maxima with the increasing **x.**
- Beyond this maxima, the **y** is estimated as limiting value of **y=0.99-x**.

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So, if I plot this thing that is y versus x what we have just calculated and I just take 4 representative values or 5 representative values over here and if we calculate case number 1, 2, 3, 4 and 5 and what if I join what you say is? This is the x, when the x is varied from 0.1 to 0.9 y goes to a maxima and the maximum value happens to be around 0.25 or 0.26 for T 3 is equal to 225 Kelvin temperature.

If I do this from the plot, it is clear that y crosses maxima when the value of x is increase from 0.1 to 0.9 and the optimum value of x at which the y is maxima happens to be around 0.73. Beyond these maxima, with up to this point we had use the expression which was given as y is equal to h 1 minus h 2 etcetera what we have calculated earlier? Beyond this value, what we have use y is equal to 0.99 minus x is a limiting case of y over here.

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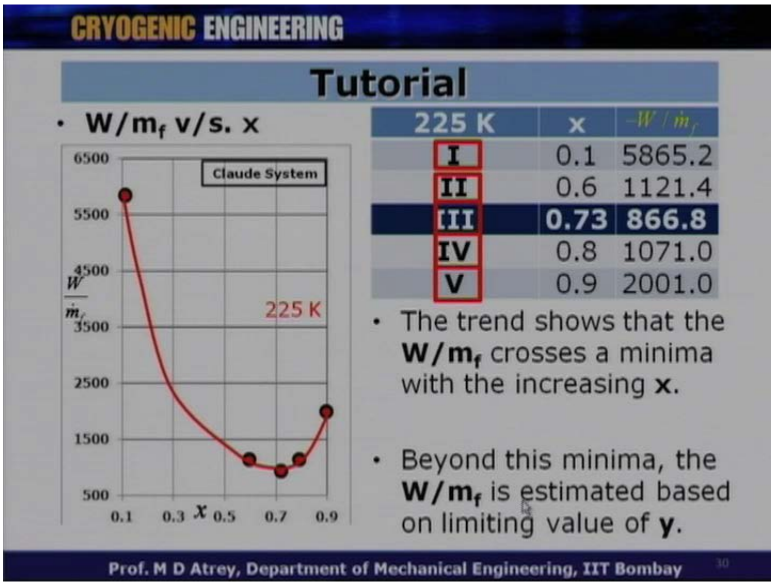

Tutorial

- W/m_f v/s. x

225 K	x	$-W/\dot{m}_f$
I	0.1	5865.2
II	0.6	1121.4
III	0.73	866.8
IV	0.8	1071.0
V	0.9	2001.0

- The trend shows that the W/m_f crosses a minima with the increasing **x**.
- Beyond this minima, the W/m_f is estimated based on limiting value of **y**.

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Similarly, if I compute the value of W by m f versus x I see that the value of W by m f as I said earlier, it is starts coming down it hits the minimum value and then it start going up and if you plot this 1, 2, 3, 4, 5 cases over here. What you see is for T 3 is equal to 225 Kelvin; I get minimum at this point. Again around 0.73 and this condition is for T 3 is equal to 225 Kelvin temperature. The trend shows that the W by m f crosses a minimum with an increasing value of x. Beyond this minima, the W by m f is estimated based on the limiting value of y.

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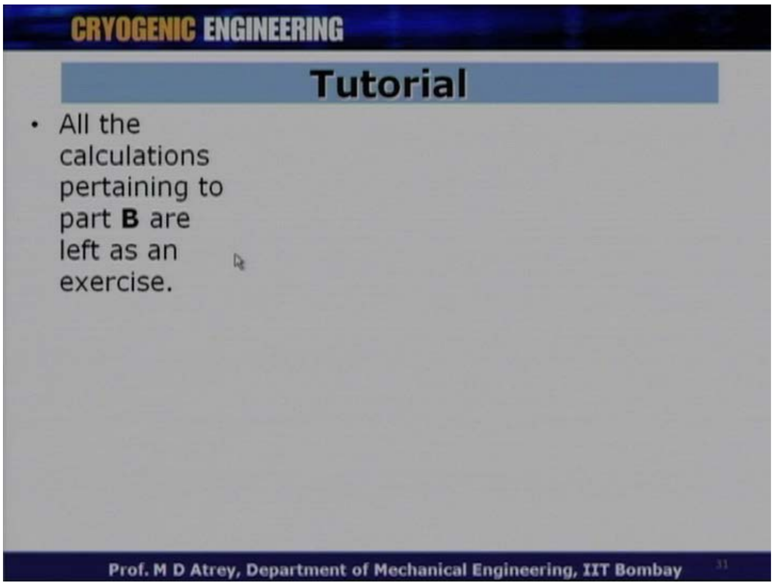

Tutorial

- All the calculations pertaining to part **B** are left as an exercise.

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Now, all the calculations are pertaining to the part b, which is what we have done till now, is part A and it part B now, what we have going to change is the value of the T 3 and we will give a sweep to the value of a T 3 also; that means, we will go for T 3 is equal to 300, 275, 250, 225 etcetera and for every value of T 3 will have a sweep of even x value from 0.1 to 0.9. It means there is a lot of a calculation involved and what we have done is written a small program and from where I will now, show the table the results. The results are calculated shown in here.

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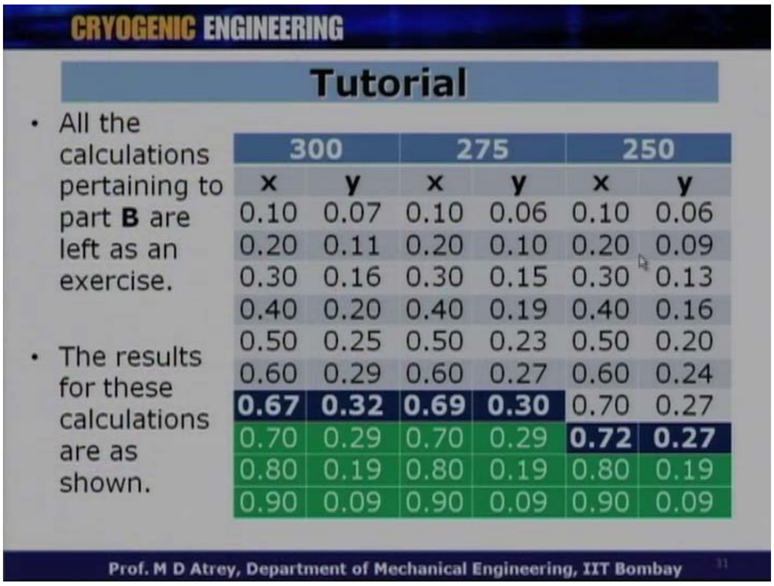

CRYOGENIC ENGINEERING

Tutorial

- All the calculations pertaining to part **B** are left as an exercise.
- The results for these calculations are as shown.

300		275		250	
x	y	x	y	x	y
0.10	0.07	0.10	0.06	0.10	0.06
0.20	0.11	0.20	0.10	0.20	0.09
0.30	0.16	0.30	0.15	0.30	0.13
0.40	0.20	0.40	0.19	0.40	0.16
0.50	0.25	0.50	0.23	0.50	0.20
0.60	0.29	0.60	0.27	0.60	0.24
0.67	**0.32**	**0.69**	**0.30**	0.70	0.27
0.70	0.29	0.70	0.29	**0.72**	**0.27**
0.80	0.19	0.80	0.19	0.80	0.19
0.90	0.09	0.90	0.09	0.90	0.09

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So, what we can see now here, is a the value of T 3 is equal to 300 Kelvin, 275 Kelvin, 250 Kelvin and as I have already done the value calculations for 225 as T 3 value. So, this is the value of for T 3 is equal to 300 Kelvin, this is actually room temperature basically, the expansion happens at room temperature only. So, if a if you see from here, I go in a sweep from 0.1 to 0.9 and correspondingly the value of y are calculated and here you can see that the maximum value of y happens to be for x is equal to 0.67 and the maximum value of y is 0.32.

If I reduce the value of T 3 to 275 Kelvin, the maximum value of y happens to be 0.3 which is little less than this and corresponding x happens to be 0.69; that means, little higher than this. If I come further down from here, T 3 is equal to 250 Kelvin. The maximum value of y is 0.27 which is less than this and the corresponding x value is now 0.72.

And if you remember the 225 Kelvin, it was 0.73 and around 0.26 for T 3 is equal to 225 Kelvin that means, as you go on the reducing the temperature over here. The y value goes on optimum y value or the maximum y value goes on decreasing and correspondingly, the amount of gas which can get transferred to the expansion engine as shown an increase over here 0.67, 0.69, 0.72 and 0.73 for expansion engine when T 3 is equal to 225 Kelvin.

However, please understand that all this calculations have been done for heat exchanger effectiveness 100 percent and expander efficiency is 100 percent. As soon as the inefficiency parameter starts coming in these values can change all right. The optimum can shift according.

So, all the calculations pertaining to part B are again shown for W by m f here. So, again the same thing is done for the T 3 is equal to 300 Kelvin, 275 Kelvin, 250 Kelvin and again, what you see is a minimum is at 0.67, 0.69, 0.72 while W by m f shows an increased value, when the T 3 value is decreased is very difficult to follow a table. If I plot this graphically, it will make more sense and therefore, all this calculations are now shown graphically where some confusion can be directly drawn.

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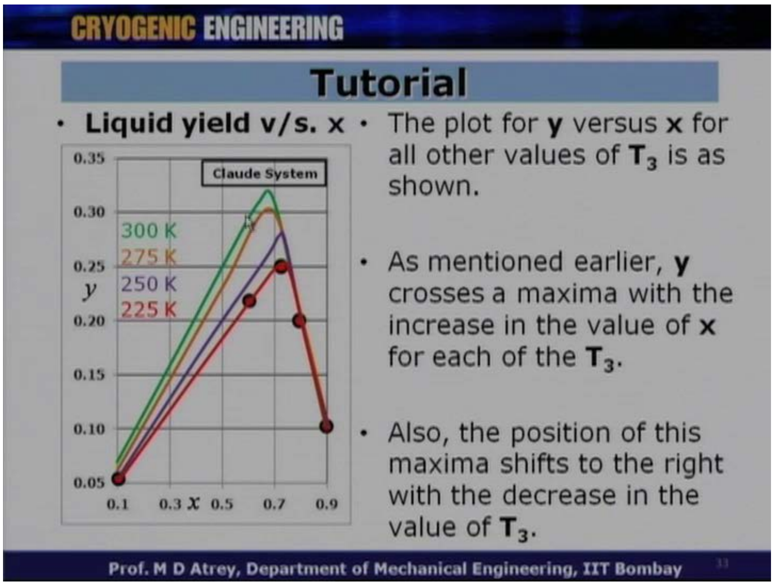

CRYOGENIC ENGINEERING

Tutorial

- **Liquid yield v/s. x**

- The plot for **y** versus **x** for all other values of $\mathbf{T_3}$ is as shown.
- As mentioned earlier, **y** crosses a maxima with the increase in the value of **x** for each of the $\mathbf{T_3}$.
- Also, the position of this maxima shifts to the right with the decrease in the value of $\mathbf{T_3}$.

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So, if I draw this liquid yield y versus x, what you can see is for different T 3 values and this is for 225 Kelvin, the second curve is for T 3 is equal to 250 Kelvin, the third curve is for 275 Kelvin now and forth Kelvin what you can see is for 300 Kelvin.

So, what you can see from here is? As the value of T 3 is increased; that means, 225 to 275 to 300 Kelvin my y or the yield has increased. Theoretically, I can understand that I can carry out the expansion from as high temperature as possible, which is 300 Kelvin in this, is going to be beneficial.

As mentioned earlier, y crosses a maxima with increase in the value of x for each T 3. So, every T 3 what you see? It has gone through a maximum and what you can see from here? The position of this maxima shift to the right, when decreasing temperature value of T 3; that means, if I come down from 300 to 225 Kelvin, the optimum x value has changed. It is a marginal change however, from 0.67 to 0.73 or so; that means, when the temperature decreases down I can send more gas through the expansion engine. I can divert more gas through the expansion engine to get an optimum. Optimum requires more gas to be diverted through the expansion engine.

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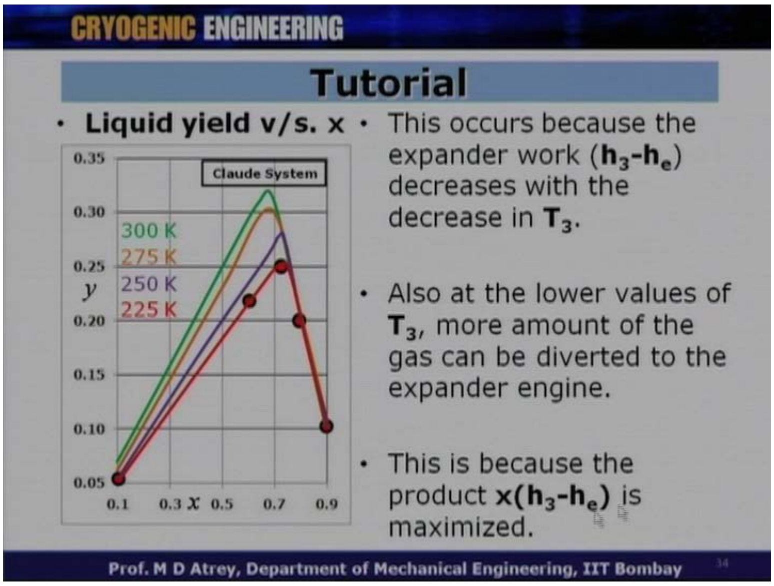

CRYOGENIC ENGINEERING

Tutorial

- **Liquid yield v/s. x**
- This occurs because the expander work (**h_3-h_e**) decreases with the decrease in **T_3**.
- Also at the lower values of **T_3**, more amount of the gas can be diverted to the expander engine.
- This is because the product **$x(h_3$-$h_e)$** is maximized.

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Why this occurs, because the expander work h 3 minus h e decreases with the decrease in temperature T 3. So, as I go down the temperature, what you can see is? The y in the formula is a function of x into h 3 minus h e. If you remember the formula y is equal to h 1 minus h 2 upon h 1 minus h f plus it is time h 3 minus h e upon h 1 minus h f.

So, the product of x into h 3 minus h e I would like to get maximized, because I want to see the optimum associated with those values. So, as soon as I go on lowering the temperature, the value of h 3 minus h e is going to get less and less depending on a T - s diagram for a particular gas. So, because h 3 minus h e gets reduced, I can divert more and more gas. So, that I get an optimum value all right and that is the reason that, I can divert more and more gas when I come down lower in temperature.

Also at the lower values of T 3, more amount of gas can be diverted to the expansion engine that is what I just talked about? This is, because the product x into h 3 minus h e is to be maximized; I hope I am clear on this.

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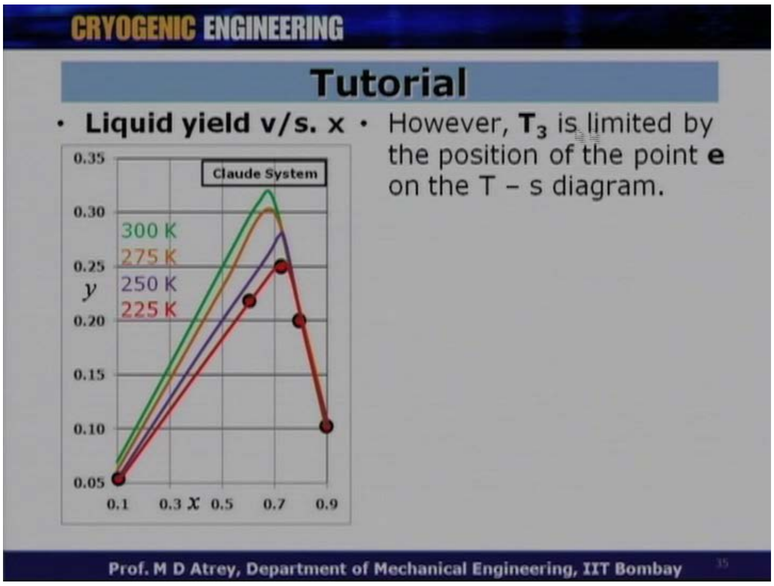

CRYOGENIC ENGINEERING

Tutorial

- **Liquid yield v/s. x**
- However, $\mathbf{T_3}$ is limited by the position of the point **e** on the T – s diagram.

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However, the T 3 is limited by the position of the point e where, one can really come down below particular temperature, because the isentropic expansion from that 0.3 might for in the dome, which is undesirable. Because expansion engine would not like to see the liquid, it wants to be I gaseous stage and therefore, how much it has to be load is limited by the value of isentropic expansion from point T 3.

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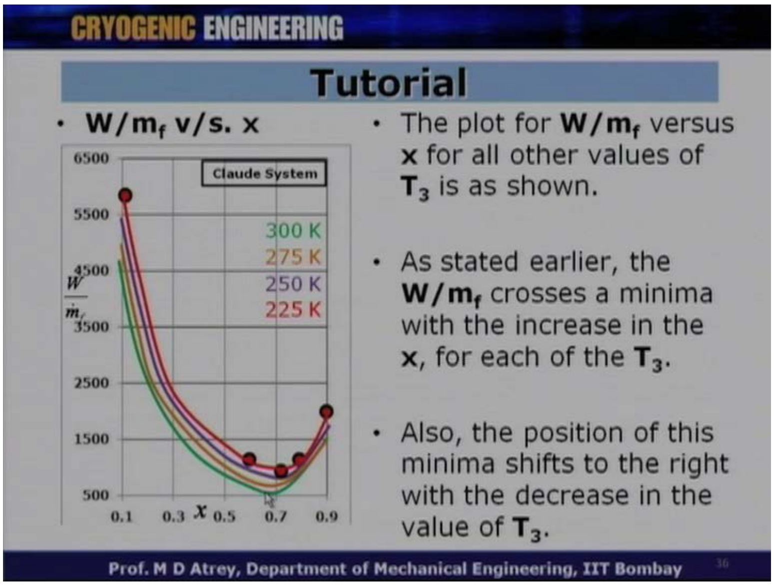

CRYOGENIC ENGINEERING

Tutorial

- W/m_f v/s. x

- The plot for W/m_f versus x for all other values of T_3 is as shown.
- As stated earlier, the W/m_f crosses a minima with the increase in the x, for each of the T_3.
- Also, the position of this minima shifts to the right with the decrease in the value of T_3.

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If I do the similar thing now, to understand W by m f versus x. This is the curve for T 3 is equal to 225 Kelvin and if I draw for 250 Kelvin followed by 275 Kelvin and followed by 300 Kelvin.

Exactly in the similar line what I talked about at 300 Kelvin, because I get maximum y value there I get minimum W by m f over here, while if I lower the temperature my W by m f requirement increases. As stated earlier, the W by m f crosses a minima with the increase in the value of x for each of the T 3 values.

Also, the position of this minima shifts to the right. So, my optimum value shift to the right with decrease in the value of T 3. So, as I decrease the value of T 3, the optimum value shift to the right, because my optimum y happens to be for those respect to x values. This is this can be well understood from what we have studied for the y values? y versus x behavior is absolutely similar to what we are seeing in this particular case.

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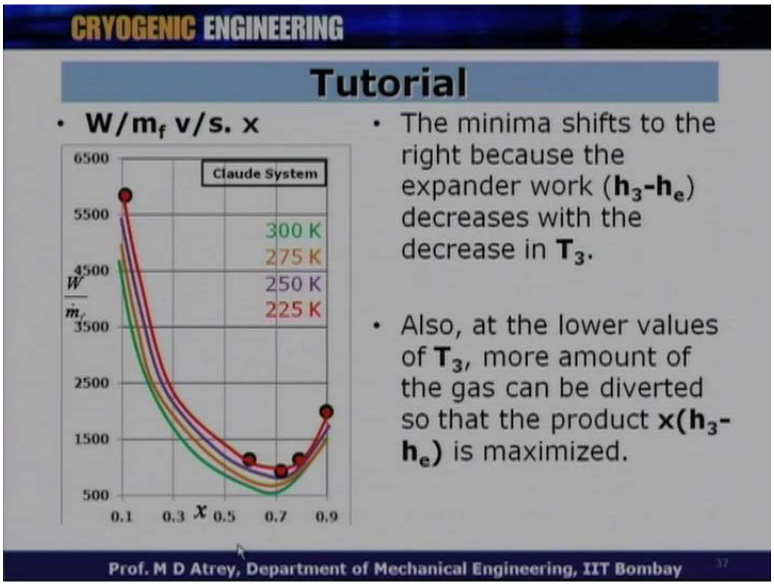

CRYOGENIC ENGINEERING

Tutorial

- **W/m_f v/s. x**

- The minima shifts to the right because the expander work (**h_3-h_e**) decreases with the decrease in **T_3**.
- Also, at the lower values of **T_3**, more amount of the gas can be diverted so that the product **$x(h_3-h_e)$** is maximized.

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The minima shift to the right, because expander work h 3 minus h e decreases with the decrease in the value of T 3. Also, at the lower values of T 3, more amount of the gas can be diverted so, that the product x into h 3 minus h e is maximized. This is what we talked about earlier for the y versus x case? And this is what has to be well understood, that y it goes to a minimum and y it shift to right in this particular case.

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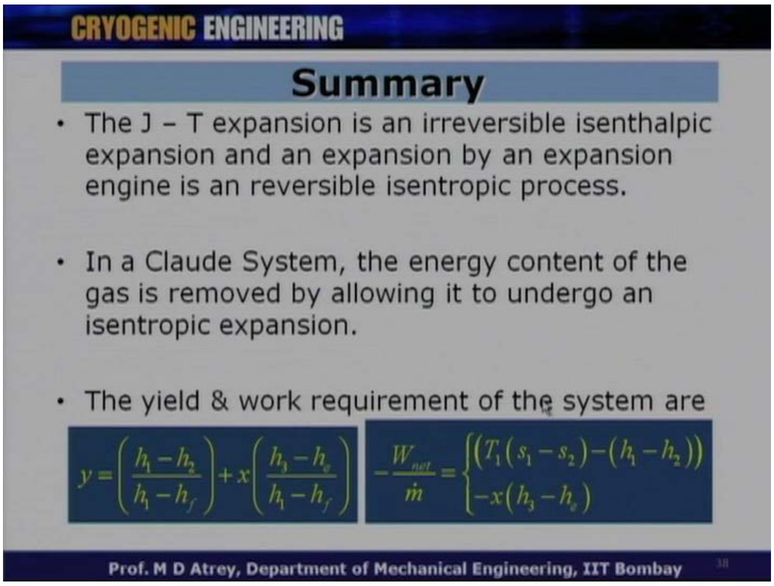

CRYOGENIC ENGINEERING

Summary

- The J – T expansion is an irreversible isenthalpic expansion and an expansion by an expansion engine is an reversible isentropic process.
- In a Claude System, the energy content of the gas is removed by allowing it to undergo an isentropic expansion.
- The yield & work requirement of the system are

$$y=\left(\frac{h_1-h_2}{h_1-h_f}\right)+x\left(\frac{h_3-h_e}{h_1-h_f}\right)$$

$$-\frac{W_{net}}{\dot{m}}=\begin{cases}\left(T_1\left(s_1-s_2\right)-\left(h_1-h_2\right)\right)\\-x\left(h_3-h_e\right)\end{cases}$$

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If I summarize, whatever we have learned today? The J - T expansion is an irreversible isenthalpic expansion and an expansion by an expansion engine is an reversible isentropic process, it is clear. In a Claude system, the energy content of the gas is removed by allowing it to undergo an isentropic expansion. The yield and work requirement of the systems are y is equal to this formula, which I am sure now your conversant with and W net by m dot is equal to this formula of which this is the work done on the compressor, this is work done by the expansion engine.

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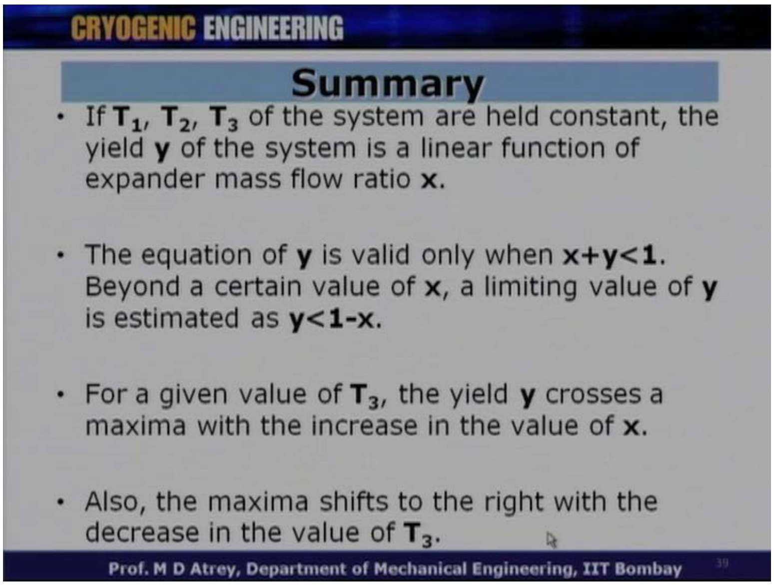

Summary

- If $\mathbf{T_1}$, $\mathbf{T_2}$, $\mathbf{T_3}$ of the system are held constant, the yield **y** of the system is a linear function of expander mass flow ratio **x**.
- The equation of **y** is valid only when $\mathbf{x+y<1}$. Beyond a certain value of **x**, a limiting value of **y** is estimated as $\mathbf{y<1-x}$.
- For a given value of $\mathbf{T_3}$, the yield **y** crosses a maxima with the increase in the value of **x**.
- Also, the maxima shifts to the right with the decrease in the value of $\mathbf{T_3}$.

If T 1, T 2 and T 3 of the system are held constant, the yield y of the system is a linear function of the expander mass flow ratio x. Again, this is clear from the yield expression for y.

The equation for y is valid only when x plus y is less than 1. Beyond a certain value of x, a limiting value of y is estimated as y is less than 1 minus x. I am sure these clear why do we doing this? Because we want the return stream to be having a finite mass flow rate, this m minus m e minus m f as to be more than 0.

For a given temperature of T 3, the yield y crosses maxima with the increase in the value of x. So, if you give a sweep to x, the value of y goes through a maxima and this is where the value of y, we want to attain. Also, the maxima shifts to the right with the decrease in the value of T 3, we have talked about this.

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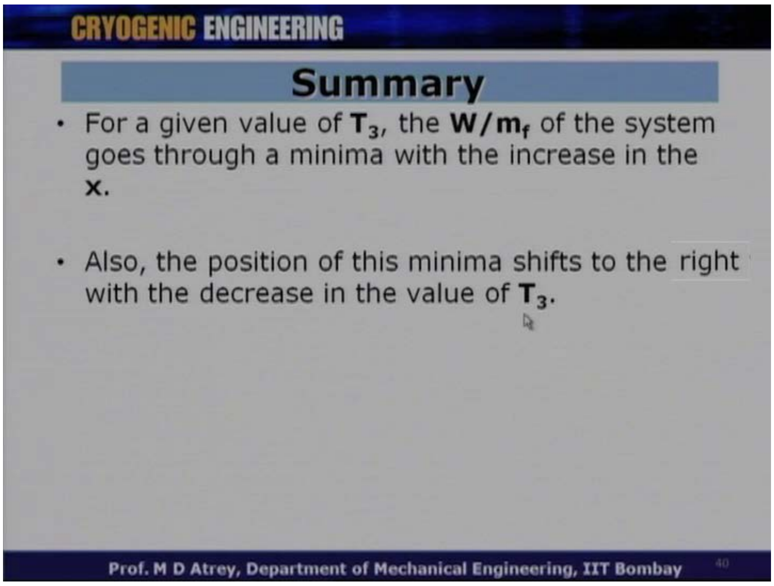

CRYOGENIC ENGINEERING

Summary

- For a given value of T_3, the W/m_f of the system goes through a minima with the increase in the **x**.
- Also, the position of this minima shifts to the right with the decrease in the value of T_3.

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For a given value of T 3, the W by m f of the system goes through a minima with the increase in the value of x and this is again the optimum value of x, we would like to attain or use for design purpose for cycle optimization. Also, the position of this minima shifts to the right with the decrease in the value of T 3.

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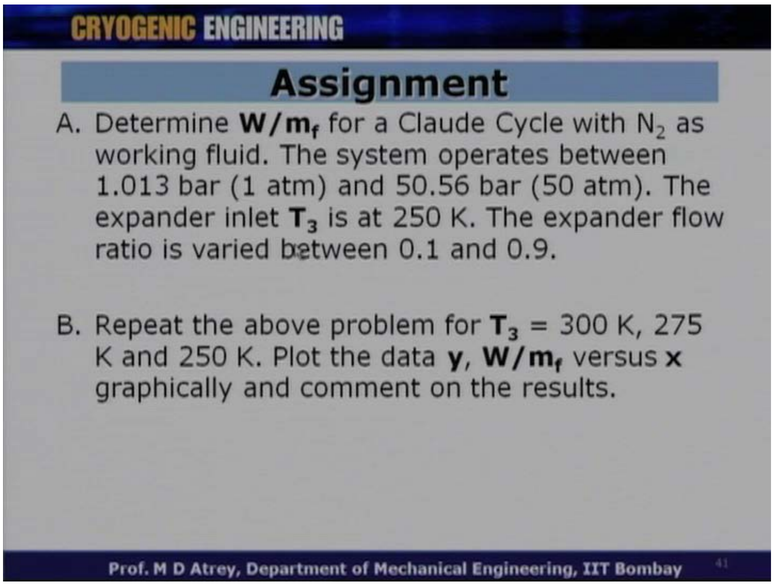

CRYOGENIC ENGINEERING

Assignment

A. Determine W/m_f for a Claude Cycle with N_2 as working fluid. The system operates between 1.013 bar (1 atm) and 50.56 bar (50 atm). The expander inlet T_3 is at 250 K. The expander flow ratio is varied between 0.1 and 0.9.

B. Repeat the above problem for T_3 = 300 K, 275 K and 250 K. Plot the data **y**, W/m_f versus **x** graphically and comment on the results.

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With this background and with this tutorial an assignment is given to you on the similar line (( )) Nitrogen as a working fluid, but we have changed the pressure to 50 atmosphere and again T 3 has to be 250 Kelvin, expander flow rate to be varied from 0.1 to 0.9.

And again I would like you to repeat the cases for temperature, because we have done a case for 40 bar or 40 atmosphere pressure, as soon as you go to 50 bar what happens to optimum y value and what happens to the optimum W by m f value has to be well understood all right.

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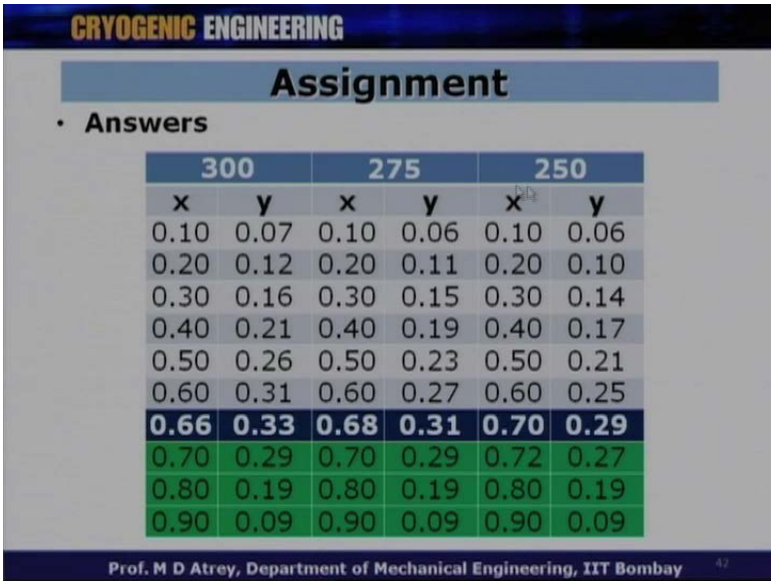

CRYOGENIC ENGINEERING

Assignment

- **Answers**

300		275		250	
x	y	x	y	x	y
0.10	0.07	0.10	0.06	0.10	0.06
0.20	0.12	0.20	0.11	0.20	0.10
0.30	0.16	0.30	0.15	0.30	0.14
0.40	0.21	0.40	0.19	0.40	0.17
0.50	0.26	0.50	0.23	0.50	0.21
0.60	0.31	0.60	0.27	0.60	0.25
0.66	**0.33**	**0.68**	**0.31**	**0.70**	**0.29**
0.70	0.29	0.70	0.29	0.72	0.27
0.80	0.19	0.80	0.19	0.80	0.19
0.90	0.09	0.90	0.09	0.90	0.09

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These are the answers for your questions and I am sure you will take care of y versus x and try to tally your answers.

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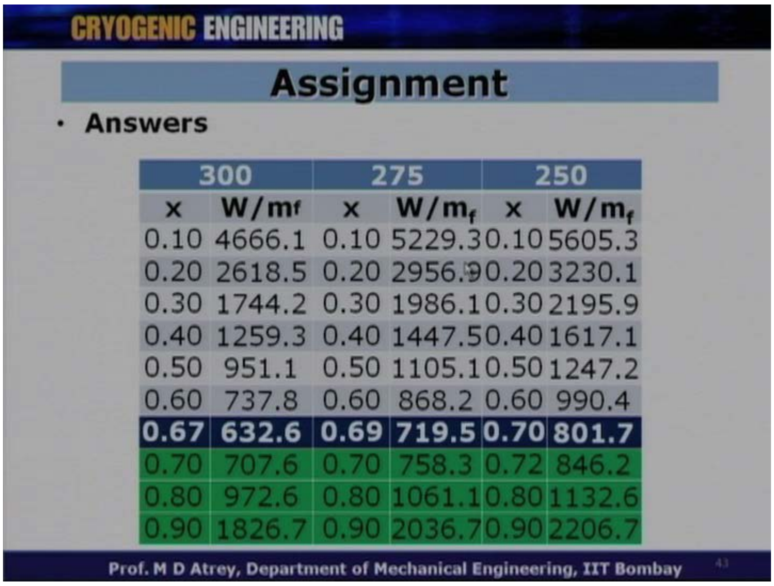

CRYOGENIC ENGINEERING

Assignment

- **Answers**

300		275		250	
x	W/m_f	x	W/m_f	x	W/m_f
0.10	4666.1	0.10	5229.3	0.10	5605.3
0.20	2618.5	0.20	2956.9	0.20	3230.1
0.30	1744.2	0.30	1986.1	0.30	2195.9
0.40	1259.3	0.40	1447.5	0.40	1617.1
0.50	951.1	0.50	1105.1	0.50	1247.2
0.60	737.8	0.60	868.2	0.60	990.4
0.67	**632.6**	**0.69**	**719.5**	**0.70**	**801.7**
0.70	707.6	0.70	758.3	0.72	846.2
0.80	972.6	0.80	1061.1	0.80	1132.6
0.90	1826.7	0.90	2036.7	0.90	2206.7

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Similarly, W by m f upon x variation also has to be has been given for different T 3 values. Please carry out this exercise and try to see that your results are matching with these results.

Thank you very much.