

Cryogenic Engineering
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Lecture No. # 14
Gas Liquefaction and Refrigeration Systems

So, welcome to the fourteenth lecture on cryogenic engineering and what we have been dealing is related to gas liquefaction and refrigeration. In the earlier lectures, we have covered several topics related to this particular gas liquefaction topic.

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Earlier Lecture

- In the earlier lectures, we have seen an Ideal Thermodynamic cycle, in which all the gas that is compressed is liquefied.
- In a Linde - Hampson system, a heat exchanger is used to conserve cold. In this system, only a part of the gas that is compressed is liquefied.
- In a Precooled Linde - Hampson system, the liquid yield and FOM are improved by precooling the working fluid using an independent refrigerating system.

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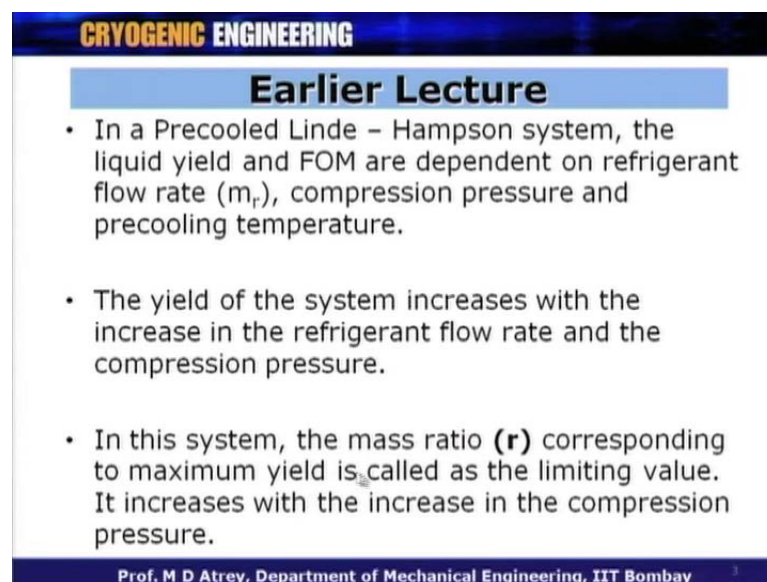
In the earlier lecture, we talked about ideal thermodynamic cycle in which what we know is whatever gas is compressed gets liquefied. So, all the gas which gets compressed, gets liquefied and that is why it is called ideal thermodynamic cycle? Which normally cannot be realized in practice?

The first modification of that happened in a cycle called Linde -Hampson system. It has got heat exchanger in it and this heat exchanger is used to conserve the cold. In this system, only a part of the gas that is compressed is liquefied. So, if you compress x gas only 5 to 10 percent of that gas will get liquefied and the remaining gas in the cold condition will go back to this heat exchanger and it gives the cold to the incoming compressed gas and thus the cycle continues.

So, Linde - Hampson cycle or system was the first modification from the ideal thermodynamic cycle and then can several modifications of the Linde - Hampson system itself and one of which is Precooled Linde - Hampson system. In which, the liquid yield and the Figure of Merit gets improved by precooling the working fluid using an independent refrigerating system.

So, this Precooled Linde - Hampson system what happens they got one more auxiliary system to the major system and this auxiliary system runs a refrigeration cycle with precools the gas, when it enters the heat exchanger. So, basically what is done is precooling and thereby the Liquid yield and the figure of Merit get improved.

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Earlier Lecture

- In a Precooled Linde - Hampson system, the liquid yield and FOM are dependent on refrigerant flow rate (\dot{m}_r), compression pressure and precooling temperature.
- The yield of the system increases with the increase in the refrigerant flow rate and the compression pressure.
- In this system, the mass ratio (**r**) corresponding to maximum yield is called as the limiting value. It increases with the increase in the compression pressure.

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This was studied during the last lecture and in a Precooled Linde - Hampson system, the liquid yield y what we call as and the Figure of Merit are dependent on the refrigerant flow rate \dot{m}_r is what is the refrigerant flow rate? We call as in auxiliary circuit also the compression pressure and the precooling temperatures. So, there are 3 major parameters which is \dot{m}_r compression temperature and the precooling temperature.

The yield of the system increases with the increased in the refrigerant flow rate and the compression pressure. So, this is a natural output which we understood from the earlier lecture that the y or the yield increases with increase in the refrigerant flow rate. Naturally, increase refrigerant flow rate means increased cooling effect and the compression pressure, if we increase the compression pressure also in that case, also the

yield increases. So, one can understand that as the flow rate increases, we get more benefit in terms of having y .

In this system, however, the mass ratio r corresponding to maximum yield is called as limiting value; that means, one cannot go on increasing the refrigerant flow rate $m \dot{r}$. It has got some limiting value and we are talked about in detail what this limiting value is all about? So, this limiting value is a function of compression pressure also. So, if we increase a compression pressure in the major system.

The value of limiting ratio also increases that we can have more and more mass flow rate in the auxiliary circuit or in the precooling circuit. So, the relationship between the value of r and P_2 on the compression pressure was shown in the last lecture. It was done using a problem or a tutorial and we could draw various curves, we could understand what is this limiting value of r ? As a function of this compression pressure.

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Earlier Lecture

- Work/unit mass of gas compressed increases with the increase in the refrigerant flow rate and compression pressure.
- Work/unit mass of the gas liquefied decreases with the increase in the refrigerant flow rate and compression pressure.
- Figure of Merit (FOM) increases with the increase in the refrigerant flow rate and the compression pressure.

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From this we understood that, work per unit mass of gas compressed increases with the increase in refrigerant flow rate and compression pressure. This is very directly one can understand from this as the flow rate increases, we have got 2 compresses. Now, one is a major system, one is a precooling circuit. So, as the refrigerant flow rate increases the work per unit mass of gas will increase also, if the compression pressure increases work per unit mass of gas compressed will increase.

But the work per unit mass of gas liquefied decreases with the increase in the refrigerant flow rate and the compression pressure. This is the most important thing why are doing all this thing? Because we want to decrease the value of W upon $m \dot{f}$ that is work per unit mass of gas liquefied and that is the whole objective of this precooling circuit. Also, the Figure of Merit increases with the increase in the refrigerant flow rate and the compression pressure. So, whole exercise was basically done, in order to decrease the value of work per unit mass of gas liquefied or increase the Figure of Merit with the increase in the refrigerant flow rate and the compression pressure.

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Outline of the Lecture

Topic : Gas Liquefaction and Refrigeration Systems (contd)

- Linde Dual – Pressure System
 - Liquid yield
 - Work requirement
- Parametric study

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With this background, what we have understood? What we have done during the last lectures? The outline of this present lecture is again on the same topic of Gas Liquefaction and Refrigerant System. In this particular lecture, what we are going to talk about is Linde Dual - Pressure System? As a name suggests it has got Linde 2 pressure systems or we got 2 devices or 2 compresses and 2 expanders to be specific in this particular cycle.

It is also again is a modification of the Linde - Hampson cycle with dual units existing **in the cycle** in the system. What we are going to study about in this particular lecture is? What is the liquid yield in this particular case? What is the work requirement in this particular case? There should be some advantages and we are going to study all these advantages and lastly there are various parameters and will do the parametric study. But

this parametric study, I am going to carry out again as we have done previously using a tutorial, because here we understand with the values how to solve the problem at the same time we can draw various curves ourselves. So, this is what is going to be studied in this particular lecture?

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Introduction

- Mathematically, the work requirement for an ideal isothermal compression process is given by,
$$\dot{W} = \dot{m} r T_1 \ln \left(\frac{P_2}{P_1} \right)$$
- The work requirement decreases either with the decrease in the mass flow rate or with the decrease in the compression ratio.
- In a Linde Dual – Pressure system, the work requirement decreases, when the compression of fluid is done in two stages and for different mass flow rates (\dot{m}).

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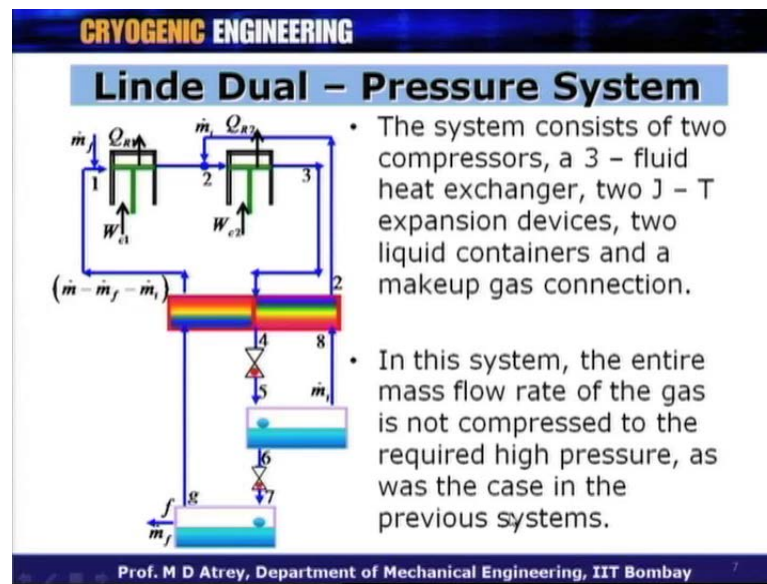
Mathematically, the work requirement for an ideal isothermal compression is process given by \dot{W} is equal to $\dot{m} r T_1 \ln \left(\frac{P_2}{P_1} \right)$. This is the work of compression for an isothermal process. What does this mean? The work requirement decreases either with the decrease in mass flow rate or with the decrease in compression ratio. In this equation, you can see there are 3 parameters \dot{m} , T_1 and $\frac{P_2}{P_1}$.

Suppose, the compression process is being carried out at given temperature or at room temperature only. We will not consider T_1 option open to us, because we want to carry out the process of compression always at room temperature. In this case, if I want to decrease the value of \dot{W} that is work of compression I can touch upon 2 parameters \dot{m} and $\frac{P_2}{P_1}$. So, the work requirement decreases either with the decrease in mass flow rate that is \dot{m} or with the decrease in the compression ratio actually, this is a well known thing of our thermodynamics students.

This is what exactly is being exploited in a Linde Dual - Pressure System. In a Linde Dual - Pressure System, the work requirement decreases, when the compression of fluid is done in two stages and for different mass flow rates. So, if I want to go for a very high

pressure, I will not go to that high pressure in one shot, but I will go in 2 stages and this is again known as a compounding of compressor or having 2 or 3 different stages of compression and also in each stage we will control the value of \dot{m} . So, we want to minimize the work input to the compressor in such a way or by playing with this parameters \dot{m} and P_2 by P_1 and this is a real basics of this Linde Dual - Pressure System. Now, how it is done? We will see in the coming slides.

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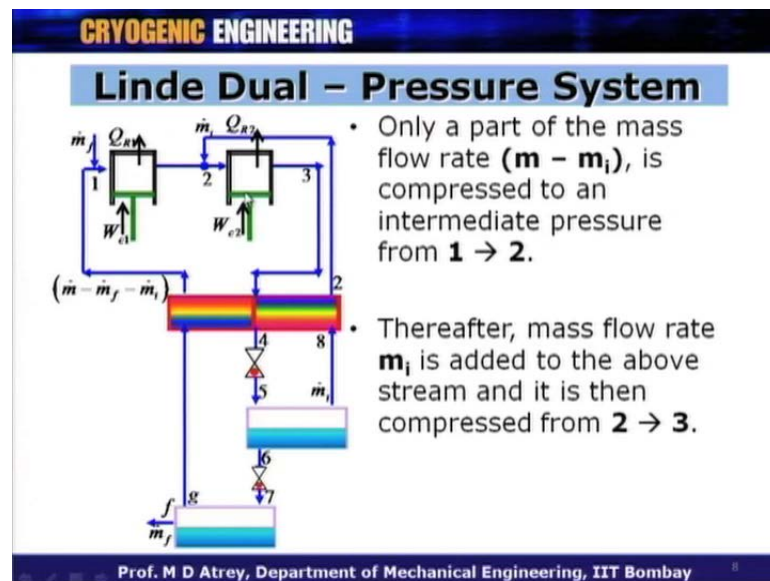
So, here is a slide which shows you how a Linde Dual - Pressure System works. So, as I said dual means 2 and we have got 2 compressors. This is what you can see from this particular schematic? There are 2 compressors and 2 expanders. So, we have got 2 compression processes and we have got 2 expansions over here at the same time, we have got 2 containers also.

So, one can see Linde - Hampson system having 1 heat exchanger and compressor process and an expansion process. However, what we have is a two compressors, two expanders and two containers and this is what a highlight of a Linde Dual - Pressure System is? So, what does Linde Dual - Pressure System has? The system consists of two compressors, a 3-fluid heat exchanger and you can see in this heat exchanger, you got one flow which is going down and one stream is going up and one stream is going up and this is very critical heat exchanger in this case and this is called as 3-fluid heat exchanger.

So, you got two compressors, 3-fluid heat exchanger, 2 joule Thomson - expansion devices. So, I got a one device over here and we got a one device over here, making it dual expansion kind of a system and again two liquid containers and a makeup gas connection. So, you got two containers, one over here, one over here and here you got a makeup gas connection $m \cdot f$ at this point. So, this is what a Linde Dual - Pressure System is all about?

In this system, the entire mass flow rate of the gas is not compressed to the required high pressure as was the case in the previous systems and that is what a highlight of having a two stage compressor, we do not compress the gas to the final pressure in one shot, but we do it with two compressors. In each of this compressor, we are playing with the mass flow rate also as I said in the earlier slide, we can vary the mass flow rate as well as the pressure ratio across this compressor. So, got a compressor number 1 and compressor number 2 over here.

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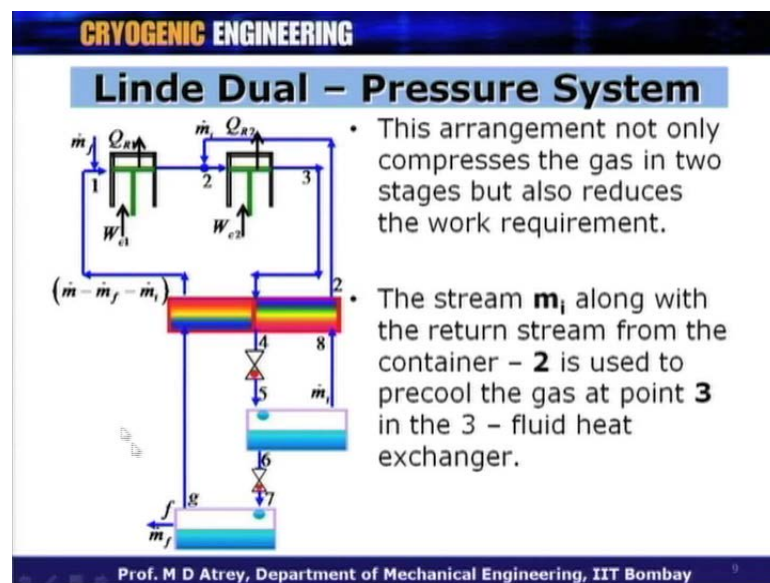


Now, as you see I am talking about now the mass flow rates in each of these compressors. Only a part of the mass flow rate that is m minus $m \cdot i$ which is coming from this side is being compressed in a compression of number 1 is compressed to an intermediate pressure here P_i . So, we have got two compressors and at this point I get the final pressure that is what is in my mind? I compress the gas equivalent to m minus $m \cdot i$ at this point and compressing to some intermediate pressure P_i and the next

compressor compresses this gas at P_1 to the final pressure at this point all right. So, this is what a compression process takes place.

Thereafter, that means at point 2, the mass flow rate \dot{m}_i is added to the above stream and is then compressed from 2 to 3. So, here you can see I got one more stream coming from here, after the first expansion, it joins at this point the gas flow which is compressed at this point is $\dot{m} - \dot{m}_i$ having \dot{m}_i joining this stream what you have is a \dot{m} over here. So, I am compressing full mass flow rate in the second compressor from 0.2 to 0.3 or from intermediate pressure P_i to final pressure P_3 at this point. So, this is what is to be understood that mass flow rate here is $\dot{m} - \dot{m}_i$ and mass accelerated in this second compressor it is \dot{m} , the compression is from P_1 to P_i or P_2 and from P_2 or P_i to P_3 in the second compressor. This is the basics of Linde - Dual Pressure System.

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This arrangement not only compresses the gas in two stages, but also reduces the work requirement and this is why we are doing all this thing? Basically, we want to reduce the work done per unit mass of gas which is compressed. The stream \dot{m}_i along with the return stream from the container number 2 here, is used to precool the gas at point 3 in the 3-fluid heat exchangers.

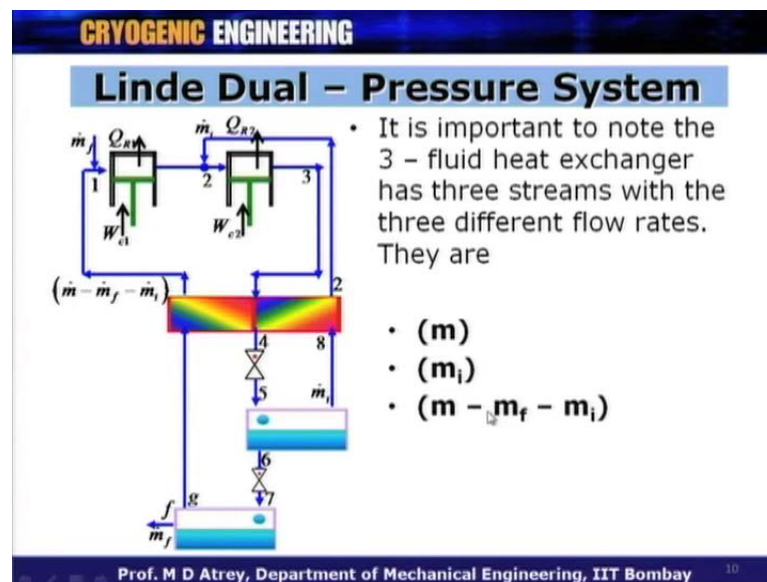
Now, the circuit is the gas when compressed up to the maximum value P_3 gets precooled by the return stream over here and by return stream over here. What is this return stream? This gas which is going to pass or \dot{m} which is compressed to the full

pressure is precooled for this in heat exchanger and it undergoes the first expansion and this expansion happens from value of P_3 up to a P_i and the part of the gas or \dot{m}_i goes back.

So, the return stream having \dot{m}_i goes back and it pre-cools the gas coming over here all right. And from this gas at this intermediate temperature and pressure next expansion happened from P_i to P_f or the lowest pressure which is P_1 and here what you get is a \dot{m}_f , that is the fluid which is getting liquefied the gas which is getting low liquefied. So, this return stream will have $\dot{m} - \dot{m}_f - \dot{m}_i$, because \dot{m}_i has already gone back \dot{m}_f has been already taken off.

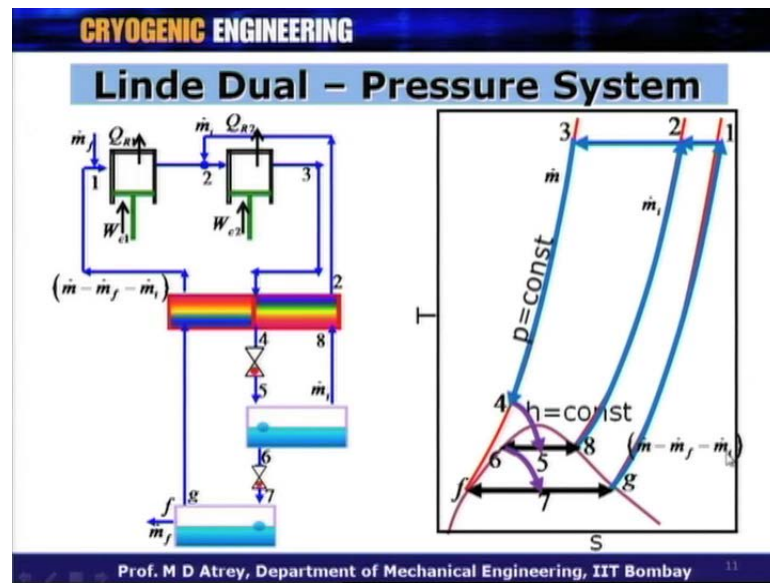
The return stream will have $\dot{m} - \dot{m}_f - \dot{m}_i$ and this is going to get compressed, when \dot{m}_f is added over here as a makeup gas and this is how the whole cycle works. So, the pre-cooling heat exchanger is basically 3-fluid heat exchanger, you get in pre-cooled by this return stream at intermediate pressure and this return stream at the lowest pressure at P_1 .

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It is important to note that 3-fluid heat exchanger has three streams with the three different flow rates. They are as I just talked about \dot{m} which is coming from here \dot{m}_i which is going back and $\dot{m} - \dot{m}_f - \dot{m}_i$ which is going through this point.

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Now, most important thing if I want to plot the whole cycle here. This is my T - S diagram or temperature entropy diagram. The gas is compressed from 1 to 2 intermediate pressures from 2 to 3 to the final destination pressure, then gas is getting precooled up to point 4 which is over here at this point. The gas gets expanded from P 4 to P i or P 2 value here and you can see the expansion process is isenthalpic. The return gas \dot{m}_i goes back and joins the main stream over here, the remaining gas which is $\dot{m} - \dot{m}_i$ gets expanded further from point 6 say 1 over here, we get \dot{m}_f and the return stream has $\dot{m} - \dot{m}_f - \dot{m}_i$ which goes back and the cycle continues.

Now, this is the most important thing that we have got two compression process 1 to 2 and 2 to 3 and we have got two expansion processes from 4 to 5 and 6 to 7. We got a return stream which is going at \dot{m}_i at point 8 and return stream at $\dot{m} - \dot{m}_f - \dot{m}_i$ at point and this is how the whole T - S diagram would look like. So, most important thing is to plot this T - S diagram correctly.

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Linde Dual – Pressure System

- Consider a control volume for this system as shown in the figure.

IN	OUT
$m \text{ @ } 3$	$m_i \text{ @ } 2$
	$m - m_f - m_i \text{ @ } 1$
	$m_f \text{ @ } f$

Applying the 1st Law, we have

$$\dot{m}h_3 = \dot{m}_i h_2 + (\dot{m} - \dot{m}_f - \dot{m}_i) h_1 + \dot{m}_f h_f$$

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Now, we will come to our routine job to sort of get expressions for the work input and a value of y . So, consider a control volume as shown over here and see what is going in? And what is going out of this system? So, what is going in here is $m \dot{m}$ at 0.3, what is going out is $m \dot{m}_i$ at point 2 m minus m_f minus m_i at point 1 and $m \dot{m}_f$ at f over here. Applying the first law, we know all these things getting the respective enthalpy values at those points, we had this expression. I will not go through this all the enthalpy values are associated with all the mass flow rate over here and if we reorganize the whole thing.

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Linde Dual – Pressure System

- Rearranging the terms, we have

$$\frac{\dot{m}_f}{\dot{m}} = \left(\frac{h_1 - h_2}{h_1 - h_f} \right) \left[\frac{\dot{m}_i}{\dot{m}} \left(\frac{h_1 - h_2}{h_1 - h_f} \right) \right]$$

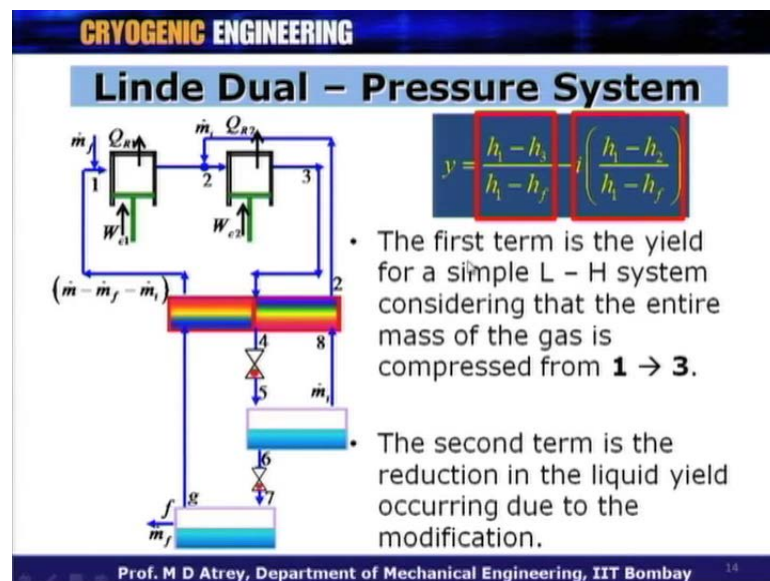
- Denoting the intermediate mass ratio $\frac{\dot{m}_i}{\dot{m}} = i$
- We have the liquid yield as,

$$y = \frac{h_1 - h_2}{h_1 - h_f} - i \left(\frac{h_1 - h_2}{h_1 - h_f} \right)$$

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What we get over here is? \dot{m}_f upon \dot{m} which is nothing, but y is equal to $\frac{h_1 - h_3}{h_1 - h_f}$ minus i times $\frac{h_1 - h_2}{h_1 - h_f}$. So, a very important expression and you can see that **the** there are two terms, again if you could denote \dot{m}_i upon \dot{m} as intermediate mass ratio \dot{m}_i upon \dot{m} as i . So, it could be called as i which is intermediate mass ratio. We have the liquid yield e as \dot{m}_f upon \dot{m} is nothing, but y is equal to $\frac{h_1 - h_3}{h_1 - h_f}$ minus i times $\frac{h_1 - h_2}{h_1 - h_f}$.

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That means, that y is equal to this expression and what you have is the first term is the yield for a simple Linde - Hampson system considering that entire mass of the gas is compressed from 1 to 3. So, if you look at this first term it is nothing, but this that is a simple Linde - Hampson system. Where, the gas is compressed within 1 compressor from point 1 to point 3 and the yield in that case would have been this.

However, this is not a Linde - Hampson system which is compressed in the gas from 1 to 3, but we have got 1 to 2 and 2 to 3 and we have got \dot{m}_i joining at an intermediate point at an intermediate pressure and therefore, we got a term called i . So, this second term takes into consideration the Linde Dual - Pressure System and it differentiates otherwise, from Linde - Hampson system anyway the compression will happen from 1 to 3 processes.

So, the second term is a reduction in the liquid yield occurring due to modification of the dual pressure system all right. So, if one wants to understand the first term indicates what would be the yield, if the gas is compressed from point 1 to point 3 directly and the second term actually bring it to the actual case Linde Dual - Pressure System. It deducts the value of y, because it is a dual pressure system all right. So, you will get less yield as compared to what it would otherwise; have got the Linde - Hampson system compress in the gas using 1 compressor from point 1 to point 3 directly.

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Linde Dual - Pressure System

- For the work requirement, consider a control volume for the compressor - **1** as shown in the figure.

IN	OUT
$m - m_i @ 1$	$m - m_i @ 2$
$-W_{c1}$	$-Q_{R1}$

- Using 1st Law for the above table, we get

$$E_{in} = E_{out}$$

$$(\dot{m} - \dot{m}_i)h_1 - W_{c1} = (\dot{m} - \dot{m}_i)h_2 - Q_{R1}$$

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For the work requirement now, what we have done till now is a finding an expression of y. But what is most important is? What is my W_{c1} and W_{c2} ? Which is the work input to the compressor number 1 and the work input to the compressor number 2 and this is what will basically bring about the advantages, we get in a Linde - Dual pressure system.

So, we should find out what is my W_{c1} and W_{c2} ? What you understand from here is for W_{c1} ? What you have is a mass flow rate of $m - m_i$, corresponding to that is a Q_R value which is minus Q_{R1} . What is going in a system is $m - m_i$? What is coming out from that is $m - m_i$ at point 2? What is going in is minus W_{c1} ? What is coming out is minus Q_{R1} ?

This is according to the direction and science convection; we are going to follow with W_{c1} and Q_{R1} as you all know. So, using first law what we have is an energy in is equal to

energy out. So, what is going in is $\dot{m} h_1 - W_{c1}$ is equal to what is coming out $\dot{m} h_2 - Q_{r1}$, which is going to come out.

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Linde Dual – Pressure System

- Rearranging the terms, we have

$$Q_{R1} - W_{c1} = (\dot{m} - \dot{m}_i)(h_2 - h_1)$$
- By 2nd Law, the Q_{R1} is given by,

$$Q_{R1} = (\dot{m} - \dot{m}_i)T_1(s_2 - s_1)$$
- Combining the above equations, we have

$$-W_{c1} = (\dot{m} - \dot{m}_i)(T_1(s_1 - s_2) - (h_1 - h_2))$$

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Hearing in these terms, what we have is a $Q_{r1} - W_{c1}$ is equal to $\dot{m} h_2 - \dot{m} h_1$. By second law, we got a value of Q_{r1} and we know that Q_{r1} is equal to $\dot{m} T_1 (s_2 - s_1)$ basically multiplied by the corresponding mass flow rate. If you put this expression of Q_{r1} in this, what we get is W_{c1} which is equal to $\dot{m} (T_1 (s_1 - s_2) - (h_1 - h_2))$.

So, we can see that what is the mass flow rate in the first compressor $\dot{m} - \dot{m}_i$ multiplied by temperature into Δs minus $h_1 - h_2$. This expression is similar to what we have had earlier for compression process. If I go to the second compressor now, what is the flow rate through that is \dot{m} ? The second compression have full mass flow rate of \dot{m} and corresponding value of entropies and enthalpy have to be taken into account.

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Linde Dual – Pressure System

- The mass flow rate across the compressor – **2** is **(m)**.
- Following the similar procedure for the work requirement for the compressor – **2**, we have,

$$W_{c2} = \dot{m}(T_1(s_2 - s_3) - (h_2 - h_3))$$

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The mass flow rate across the second compressor is \dot{m} . Following the similar procedure for the work requirement for the compressor 2, what we get is? W_{c2} is equal to $\dot{m} [T_1 (s_2 - s_3) - (h_2 - h_3)]$, because the process W_{c2} compression processes happening across point 2 and point 3 and we are taking the entropies and enthalpies related to those point. So, this will form the work of compression for the second compressor.

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Linde Dual – Pressure System

- The total work requirement is given by

$$W_c = W_{c1} + W_{c2}$$

- Denoting the ratio $\frac{\dot{m}_i}{\dot{m}} = i$
- We have, the work/unit mass of gas compressed as given by

$$\frac{W_c}{\dot{m}} = T_1 (s_1 - s_2) - (h_1 - h_2) - i (T_1 (s_1 - s_2) - (h_1 - h_2))$$

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Now, the total work requirement is given by W_c is equal to W_{c1} plus W_{c2} . So, the total work is now going to be determined by what is the value of W_{c1} ? And what is the value of W_{c2} ? W_{c1} depends on what is the amount of gas which is getting compressed in the compressor number 1 and what is the pressure ratio across point 1 and 2 all right P_2 by P_1 .

Similarly, W_{c2} depends on what is the mass total \dot{m} going into it and what is the pressure ratio across it 2 and 3. It is this W_{c1} and W_{c2} , which makes a difference to the value of W_c of the total work of compression and this is the most important to understand, because by doing this Linde Dual - Pressure System, we are basically trying to minimize the work of compression in this case as compared to a simple Linde - Hampson cycle.

Denoting, the ratio know if you add both the expressions together and if you put the value of I in that expression, we get work per unit mass of gas compressed that is W_c by \dot{m} as this expression that is $T_1 \ln \frac{s_1}{s_3} - h_1 + h_3$ as if the whole thing is getting compressed from 1 to 3 part which is what we saw in a expression for y . In the similar line, again you got expression for W_c by \dot{m} first term denotes as if the whole thing is compressed from 1, 2, 3 and the second term denotes depending on the value of I what is to be deducted from 8 that is $I \ln \frac{T_1}{T_2} - s_1 + s_2 - h_1 + h_2$. This much of work will be done less as compared to the process of compression carried from 1, 2, 3 process.

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Linde Dual – Pressure System

$$-\frac{W_c}{\dot{m}} = T_1(s_1 - s_3) - (h_1 - h_3)$$

$$-i(T_1(s_1 - s_2) - (h_1 - h_2))$$

- The first term is the work requirement for simple system considering that the entire mass of the gas is compressed from **1** → **3**.
- The second term is the reduction in the work requirement occurring due to the modification.

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So, same thing is written in the first term, is the work requirement for simple system considering that the entire mass of the gas is compressed from 1 to 3.

The second term is the reduction in the work requirement occurring due to the modification and this is what we basically, I am going to exploit in order to reduce the work done per unit mass of gas which is compressed. With this back ground on Linde Dual - Pressure System, as a said you have just derived expression for a why and the power input I want to know take you to understand what is the effect of varies parameter.

So, what I am going to do is? I am basically giving you a tutorial and will solve this tutorial and understand from this how these parameters play a very important role. In order to, get expressions for what is the work done? What is the work of compression per unit mass of gas which is compress and thinks like that?

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CRYOGENIC ENGINEERING

Tutorial

- Determine W/m_f & **FOM** for a Linde Dual – Pressure System with Argon as working fluid for the following intermediate pressures. The system operates between 1.013 bar (1 atm) and 121.5 bar (120 atm). The intermediate mass ratio i is **0.6**.

Ar	Int. Pr. 2
I	4.05 bar
II	20.3 bar
III	75.9 bar
IV	101.3 bar

- Repeat the above problem for $i = 0.7$. Plot the data graphically and comment on the nature of y , W/m_f , **FOM** versus i .

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So, this tutorial is the most important tutorial. So, what is the problem to understand is? Determine W by $m \cdot f$ and Figure of Merit for a Linde Dual - Pressure System with Argon as working fluid for the following intermediate pressures. The system operates between 1 atmosphere and 120 atmosphere and the intermediate mass ratio i is 0.6. There are various intermediate pressure points that is 0.2 and in this problem, we want to understand what is the value of W by $m \cdot f$ and Figure of Merit for this intermediate pressures.

So, I am going to compress the gas first from 1 bar upto 4.05 bar and solve all the values to get what is the W by $m \cdot f$ and Figure of Merit for that case. Then I will get the second value of pressure 20.3, 75.9, 101.3 and in such a way that final pressure always remains at 120 atmosphere all right. So, here I am on to understand what is the effect of this intermediate pressure?

So, first of all we should calculate how much work per unit mass of gas, which is liquefied one gets at different intermediate pressure. The intermediate mass flow ratio i is in this case for point 6. Then my tutorial also want to understand, what will happen? If I go from i value of 0.6 to 0.7. So, repeat the above problem for i is equal to 0.7 and ultimately plot the data graphically and comment on the nature of y , W by $m \cdot f$, Figure of Merit versus i all right.

So, is basically I am trying to understand what is the effect of intermediate pressure on W by $m \dot{f}$ and Figure of Merit and what is the effect of value of I or the mass ratio 0.6 and 0.7 on all this studies, is there are intermediate 2 problems. In order to, understand the effect of intermediate pressure and effect of i on this important parameters all right, this is my problem.

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CRYOGENIC ENGINEERING

Tutorial

Given

Cycle : Linde Dual – Pressure System
 Working Pressure : 1 atm $\rightarrow P_1 \rightarrow$ 120 atm
 Working Fluid : Argon
 Temperature : 300 K
 Intermediate mass ratio : $i = 0.6$ & 0.7

For above System, Calculate

1 Work/unit mass of gas liquefied and FOM

Ar	Int. Pr. 2
I	4.05 bar
II	20.3 bar
III	75.9 bar
IV	101.3 bar

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So, what is given in this particular problem is? We want to work on a Linde Dual - Pressure System. The working pressures are from 1 atmosphere to 120 atmosphere. The working fluid is Argon. This is very important at one should have all the properties of Argon. Temperature is 300 Kelvin; that means, the process of compression is carried out at 300 Kelvin. The intermediate mass ratio is 0.6 and 0.7 for above system calculate work per unit mass of gas liquefied it is W by $y m \dot{f}$ and Figure of Merit and the intermediate pressure are 4, 20, 75, 101 atmospheres all right. So, this is my problem.

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CRYOGENIC ENGINEERING

Methodology

- The two mass ratio (**i**) conditions under study are 0.6 and 0.7.
- In this tutorial, the liquid yield and work/unit mass of gas liquefied are calculated only for **i = 0.6** and **4.05 bar** as intermediate pressure condition.
- All other calculations pertaining to **i = 0.6 & 0.7** and for all other intermediate pressure conditions are left as an exercise to students.

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What is the methodology? The methodology is for solving in this particular class is, the two mass ratio i condition under study are 0.6 and 0.7. In this particular tutorial, I am not going to repeat the calculation for each of these values. I am going to demonstrate to you how to calculate this values for one pressure and for one i value for expect to you do is? Then carry out this exercise is further for all the pressures and for both the values of i is equal to 0.6 and 0.7.

So, in this tutorial, the liquid yield w and work plus unit mass of gas liquid in W by $m \cdot f$ are calculated only for i is equal to 0.6 and 4.05 bar or 4 atmosphere as intermediate pressure conditions. So, I am just solving 1 problem for i is equal to 0.6 and P_2 or P_i is equal to 4 atmosphere while what I am living it to you is? To calculate i is equal to 0.6 and all the pressure and then i is equal to 0.7 and all the pressures for your selves.

However, final slides I got values for all this calculations. So, we have done all this calculation ourselves and what we want to do is to check your calculations. So, that you also get those values saying that your calculations are correct; it is basically in a exercise for you. All other calculations pertaining to value of i is equal 0.6 and 0.7 and for all other intermediate pressure conditions are left for you as an exercise. Let us go for i is equal to 0.6 and P_2 or P_i is equal to for atmosphere.

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CRYOGENIC ENGINEERING

Tutorial

• **Ideal Work Requirement**

$$-\frac{\dot{W}_c}{\dot{m}} = T_1 (s_1 - s_f) - (h_1 - h_f)$$

	1	f
p (bar)	1.013	1.013
T (K)	300	87.3
h (J/g)	349	75
s (J/gK)	3.85	1.4

$$-\frac{\dot{W}_c}{\dot{m}} = 300(3.85 - 1.4) - (349 - 75) = 461 \text{ J/g}$$

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So, before as you all now with the earlier problem, first we want the ideal cycle analysis from which will get ideal work requirement and this is very important to calculate Figure of Merit. So, from ideal work requirement, this is the formula $W_c / \dot{m} = T_1 (s_1 - s_f) - (h_1 - h_f)$ and here as you know whatever is compressed is getting liquefied, we know the point 1, we know the point f also.

So, pressure is 1.013 bar, temperature is 300 and 87.3 which is the boiling point of Argon at 1 atmosphere, then enthalpy values are given at point 1 and f the entropy values are given at 1 and f. If you put these values in this equation, what you get is? W_c / \dot{m} work of compressor per unit mass of gas liquefied is equal to 461 joules per gram. So, this is my first basic calculation which we do almost for all the problems.

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CRYOGENIC ENGINEERING

Tutorial

• The enthalpies and entropies are as given below.

	1	2
p (bar)	1.013	4.05
T (K)	300	300
h (J/g)	349	348
s (J/gK)	3.85	3.6
	3	f
p (bar)	121.5	1.013
T (K)	300	87.3
h (J/g)	326	75
s (J/gK)	2.84	1.4

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Now, coming to our problem that is Linde Dual - Pressure System and this is our schematic. In this schematic, the enthalpies and entropies one should get from the T - S charts for Argon all right. So, get those charts for Argon and get the enthalpy values. So, here you can see point 1, 2, 3 and f, these are the values which are required for our formula where we require enthalpy values and entropy values. Now, where are these points, the point 1 is at 1 bar 300 Kelvin at this.

Where $m - \dot{m}_i$ will be getting two compressor number 1 point to is the intermediate pressure or after the first compressor which is this point. The point 3 is over here and the corresponding properties are given now, point 2 is going to be at 4.05 bar, while point 3 is going to be at maximum pressure that is 1 to atmosphere or 121.5 bar and again point f is going to be the last point at this which is going to be at again at 1 bar. So, one should check all these values at 1 bar, 4 bar, 300 Kelvin, 300 Kelvin, 300 Kelvin here, while at point f it is 87.3 Kelvin, corresponding value of enthalpies 349, 348 and 326. One can understand that as the pressure increases the enthalpy at the same temperature at 300 Kelvin temperatures are same over here the enthalpy decreases.

Corresponding to again similar thing happens to the entropy as the pressure increases, the enthalpy and entropy values are decrease over here. So, these are the 4 different points and now, what we want you to see is? Take the enthalpies and entropies of these values and put in the equation.

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CRYOGENIC ENGINEERING

Tutorial

Ar	i	Int. Pr. 2
I	0.6	4.05 bar

- The T - s diagram for a Linde Dual - Pressure system is as shown.
- The compression process is from **1 atm** → **4 atm** → **120 atm**, As shown in the figure.

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So, corresponding temperature entropy diagram is 1, 2, 3, 4. So, one compression from 1 to point 2, 1 atmosphere to 4 atmosphere, which is intermeditation (C) 4 atmosphere to 120 atmosphere, which is maximum pressure for here. The T - S diagram for a Linde Dual - Pressure System is as shown here. The compression process is from 1 atmosphere to 4 atmosphere and then from 4 atmosphere to 120 atmosphere. So, you get a W_c 1 over here from 1 to 2 and W_c to here from 2 to 3 in this case and respect to points are given over here in this T - S diagram.

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CRYOGENIC ENGINEERING

Tutorial

• **Liquid yield**

$$y = \frac{\dot{m}_f}{\dot{m}} = \frac{h_1 - h_2}{h_1 - h_f} - i \left(\frac{h_1 - h_2}{h_1 - h_f} \right)$$

Ar	i	Int. Pr. 2
I	0.6	4.05 bar

	1	2	3	f
p (bar)	1.013	4.05	121.5	1.013
T (K)	300	300	300	87.3
h (J/g)	349	348	326	75
s (J/gK)	3.85	3.6	2.84	1.4

$$y = \frac{(349 - 326)}{(349 - 75)} - 0.6 \frac{(349 - 348)}{(349 - 75)} = \frac{(23)}{(274)} - 0.6 \frac{(1)}{(274)} = 0.0817$$

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Now, let us first calculate the liquid yield which is this expression as you know, because of the first term and second term. What you want to know is? Now, enthalpy at point 1, 2, 3 and f and we have already made those tables will get the points enthalpies at h 1, h 2, and h f and of course, h 3 also.

So, the first is Argon I is equal to 0.6 at this point which is intermediate pressure of 4.05 bar and the corresponding values are given over here 1, 2, 3, 4 all the temperature 300, 300, 300, 87.3 corresponding enthalpy values, corresponding entropy values. If you put those values in this formula what you get is 0.0817. So, y for I is equal to 0.6 and intermediate pressure is equal to 4 atmosphere, we get value of y to be equal to 0.0817 that is my calculation and I am as a said I am going to do this calculation for only one I and 1 pressure which of this 2 values over here. Now, let us complete all the calculation for these particular values.

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CRYOGENIC ENGINEERING

Tutorial

• **Work/unit mass of Ar compressed** $i = 0.6$

$$-\frac{W_c}{\dot{m}} = T_1(s_1 - s_3) - (h_1 - h_3) - i(T_1(s_1 - s_2) - (h_1 - h_2))$$

	1	2	3	f
p (bar)	1.013	4.05	121.5	1.013
T (K)	300	300	300	87.3
h (J/g)	349	348	326	75
s (J/gK)	3.85	3.6	2.84	1.4

$$-\frac{W_c}{\dot{m}} = \frac{300(3.85 - 2.84) - (349 - 326)}{-0.6(300(3.85 - 3.6) - (349 - 348))} = 235.6 \text{ J/g}$$

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So, now let us calculate work per unit mass of Argon compressed for I is equal to 0.6 and this is the expression. Again, we are T 1 in to s 1 s 3 minus h 1 minus h 3 minus I times T 1 into s 1 minus s 2 minus h 1 minus h 2 putting this values from this table, if I put this value I get W c upon m dot is equal to 235.6 joule per gram. This is work done per unit mass of gas Argon compressed.

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CRYOGENIC ENGINEERING

Tutorial

• **Work/unit mass of Ar liquefied**

$-\frac{W_c}{\dot{m}} = 235.6$

$y = 0.0817$

$$-\frac{W_c}{\dot{m}_f} = -\frac{W_c}{y\dot{m}} = \frac{235.6}{0.0817} = 2883.7 \text{ J/g}$$

• **FOM**

$-\frac{W_l}{\dot{m}_f} = 461$

$$FOM = \frac{W_l}{\dot{m}_f} \bigg/ \frac{W_c}{\dot{m}_f} = \frac{461}{2883.7} = 0.1598$$

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Now, I want to calculate work done per unit mass of Argon liquefied for which what I want to know is? W_c upon \dot{m} and I want to know what is the value of y ? Which is 0.0817 which is what we have just calculated? What we know is? W_c upon \dot{m}_f is equal to W_c upon \dot{m} upon y all right. So, what we have is both the values? This upon this will give you W_c upon \dot{m}_f .

So, I will do 235.6 upon 0.0817 which is equal to 2883.7. This is my work done per unit mass of gas Argon liquefied which is rather high as compared to the ideal work input which we had seen earlier in the ideal thermal dynamic cycle. The Figure of Merit is equal to ideal W_l upon \dot{m}_f divided by actual W_c upon \dot{m}_f . W_l upon \dot{m}_f is 461 from ideal thermal dynamic cycle and if I put those values what you get is? Figure of Merit is 0.1598. So, here we have got the all the values calculated. So, for that is y and Figure of Merit y and W_c y and total work done.

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CRYOGENIC ENGINEERING

Tutorial

- Tabulating the results for $i = 0.6$, we have the following comparison for the various values of
- Intermediate pressure.

	Int. Pressure	y	$\frac{W}{m}$	$\frac{W}{m_L}$	FOM
I	4.05 bar	0.0817	235.6	2883.7	0.1598
II	20.3 bar	0.0752	172.6	2295.2	0.2008
III	75.9 bar	0.0512	118.0	2304.6	0.2000
IV	101.3 bar	0.0424	111.4	2627.4	0.1754

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I would like to calculate all these values for all the intermediate pressures and without giving the details of those calculations, I will give you a table which will have all these values. So, tabulating the result for i is equal to 0.6, we have the following comparison for the various values of intermediate pressure. So, intermediate pressure is 4.05 bar. So, case number 1, 2, 3 and 4. This table gives for all the intermediate pressures that is 4 atmosphere, 20, 75, 101 pressure as the value of P_2 . The final value of the P_3 is always 1 twentyeth atmosphere and you got a value of y , W by m dot work done per unit mass of gas compressed, W by m dot f Work done per unit of gas liquefied and Figure of Merit.

What you can see from this table is? I will talk about each of these vertical columns in the next slides. But what we can see is? If the entry pressure and exit pressure remain the same and if the value of intermediate pressure, if it is increased the value of y as started decreasing. So, 0.08, 0.07, 0.05 and 0.04 are the values of y ; that means, the value of m dot f upon m dot **has decreases** the yield has decreased, if we increase the intermediate pressure.

But the value of W by m dot has decreased 235, 172, 118, 111. So, this value has decreased what is interesting is to see is the value of W upon m dot f that is work done per unit mass of gas liquefied what you can see? This has come down from 2883, 2295 and again started increasing. So, it is decreases over here, but then there is increase over

here, it means that it has gone through a minima and similarly, to that what we get FOM Figure of Merit. We have got 0.15, 0.2008 and 0.2 and 0.17; that means, FOM goes through a maximum and this is what we see in the next slide? So, what you understand from here?

As the intermediate pressure increases, y decreases, W by $m \dot{m}$ decreases, but W upon $m \dot{m} f$ goes through a minima and Figure of Merit goes through a maxima. This is what is the output of all this value for i is equal to 0.6 of the same study is done at 0.7 also. When i is equal to made equal to 0.7, we have carried out the same studies and the results for i is equal to 0.7 for the various intermediate values is this.

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CRYOGENIC ENGINEERING

Tutorial

- Similarly, calculating the results for $i = 0.7$, we have the following comparison for the various values of
 - Intermediate pressure.

	Int. Pressure	y	$\frac{W}{m \dot{m}}$	$\frac{W}{m \dot{m} f}$	FOM
I	4.05 bar	0.0814	228.2	2803.4	0.1644
II	20.3 bar	0.0738	154.7	2096.2	0.2199
III	75.9 bar	0.0457	91.0	1991.2	0.2315
IV	101.3 bar	0.0355	83.3	2346.5	0.1964

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So, again you can see now as a intermediate pressure increases from 4 atmosphere to 20 to 75 to 101, y again decrease from 0.0874 and 3. W upon $m \dot{m}$ that is work per unit mass of gas compressed decreased from 228 to 154, 91 to 83, but W upon $m \dot{m} f$ work per unit mass of gas liquefied. Again, went through a minima as the minima is happening possible between gas 2 and gas 3 and it is a maxima what here goes down reaches a minima and starts going up over here.

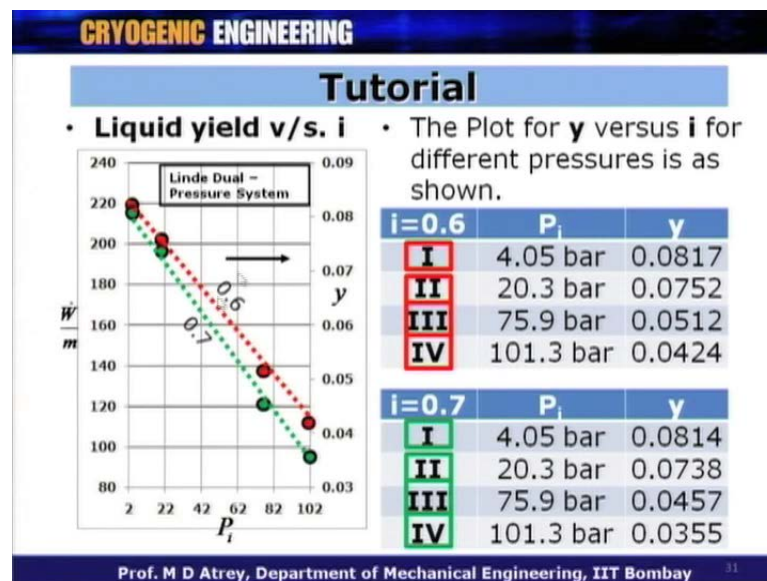
The Figure of Merit again goes through a maximum somewhere I (()) in case 2 and case 3. So, again it follows absolute the same train as it was for i is equal to 0.6. Now, here as I said we have carried out only one exercise for 0.6 and 4.05 bar. One, I expect all the students to do is gate this table calculated by yourself, get the earlier table also calculated

by yourself and then a plot it and then you start understanding what exactly happens? And why it happens? All right.

So, all this thing has been late for you as an exercise while, I have just got all this value calculated I am putting in front of you. So, this may serve as the answers for you while I expect you to solve this as a problem or as and assignments for you to see that you can read the values of enthalpy and entropy properly. Now, what is important is? To understand why it happens? I mean plotting this are calculating this values not a big deal, but what I am want to understand is?

What happens when I go from 0.6 to 0.7 as I values and for a 1 I y if the intermediate pressure goes on increasing. y the value of y decreases, y the value of W by m dot decreases, y the value of W by m dot goes through a minima and y the value of FOM goes through a maxima all right. So, this is the most important thing for the engineers to understand.

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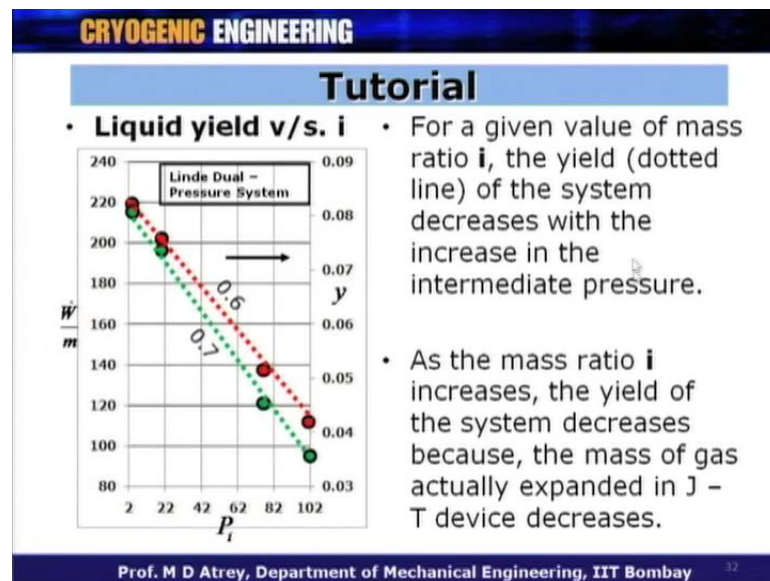
So, let us now therefore, whatever table I have just shown you. Let us try to plot this table graphically and therefore, the understanding will be much much better. So, first table will calculate is liquid yield y versus i . So, what I am showing here you are the two access on the left side you got W by m dot and the right side you got y on the x axis what you have is intermediate pressures? Which are varying from 2 bar to 102 bar as you increases intermediate pressure what happens to the value of y which is on the right axis,

right y axis do not see of the left y axis. So, see whatever points, we have got let us plot those y values versus the intermediate pressures. So, this is my table for i is equal to 0.6.

So, if i plot this points the first point 4.05 point 0.8. So, if you see on the right axis 0.08 value and 4 bar. The second value, third value, the fourth value all right. So, if you join them together you can see kind of linear variations when you plot y with respect to the value of intermediate pressure and this is the value for I is equal to 0.6.

We are going to second table for i is equal to 0.7 again get those values of case number 1 over here, case number 2, case number 3 and case number 4 . So, if you could plot this values and connect them you can see this. So, what we understand from this particular graph that as you go on increase in the intermediate pressure, the value of y decreases. This is exactly what we saw in the earlier table.

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So, for a given value of mass flow ratio i , the yield of a system decreases with the increase in intermediate pressure. Why does it happen? Now, if you recollect the expression for y , we have got the first term and the second term. What you understand second term, has a negative value which has numerator as h_1 minus h_2 as you understand that as the value of P_2 of the P_i increases, this second bracket starts increasing and therefore, the value of y decreases all right.

If the intermediate pressure increases, it means that mathematically the expression for why whatever we had? The second part which was i times a one bracket that bracket has a numerator which was h_1 minus h_2 and this values starts increasing all right and therefore, the negative parts start increasing and therefore, the value of y decreases as you go on increases the intermediate pressure.

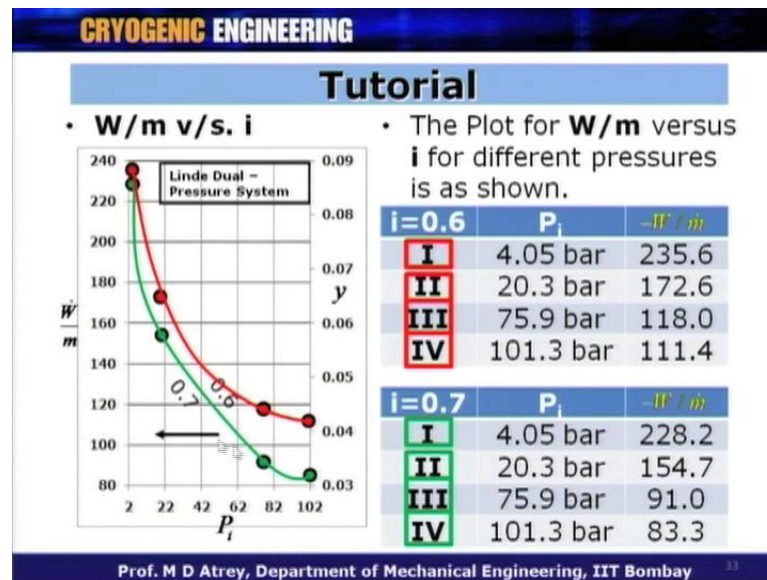
The second aspect to understand for this is as the intermediate pressure increases, the intermediate temperature in the Dom also increases all right. If you remember this has the Dom and this intermediate temperatures starts moving to the critical temperature value or it is starts going up the Dom and therefore, when the second expansion happens?

It will happen from this intermediate temperature and again at this point the point after the expansion will fall towards the gas region and therefore, if you increase the intermediate pressure, the intermediate temperature from which the second expansion happens that also gets lifted up corresponding to which the point after the second expansion falls near the gas side therefore, again you will get the less y value and this explains y , if you increase the intermediate pressures you will get less y in those cases.

Also, as the mass ratio i increases; that means, if I go from 0.6 to 0.7, the yield of the system decreases. Now, understand again if I intermediate gas ratio increases for example, i is equal to 0.7, 70 percent of the mass flow is going back, it means that only 30 percent of the gas is going for the second expansion all right and the liquid yield is going to come from this 30 percent only. It means that more the years goes back, more the value of i is the $m \cdot f$ is always going to be less, because the second expansion has only 30 percent of the gas which is compressed all right. So, as the value of i increases less and less gas is going to go for the second expansion which means less and less yield all right.

So, corresponded to however, work per unit mass of gas which is going to compresses going to became a less and less. But the value of y effectively is going on the reducing in the value of i increases. So, as the mass ratio i increases, the yield of the decreases, because the mass of the gas actually expanded in J - T device decreases and this is the reason that as i increases the y valuable decreases.

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Now, let us plot the second parameter which is W by m dot versus i all right. So, the plot between W by m dot versus i for this value is over here. So, we got a various values and you can see that as you gone increasing the value of intermediate pressure, the W by m dot decreases. What is important node is? The slop of this is much higher initially, while it is less and less as you go for higher value of intermediate pressure.

The maximum value of P_3 is always 120 atmosphere. This was the case for i is equal to 0.6 as you go for i is equal to 0.7 again, similar trends you can see, but here W by m dot is steel less; that means, if you increase the intermediate ratio 0.7 to 0.7, the W by m dot further decreases. But again you can see that the slow here is very high initially and it flattens down as you go for higher intermediate pressures all right.

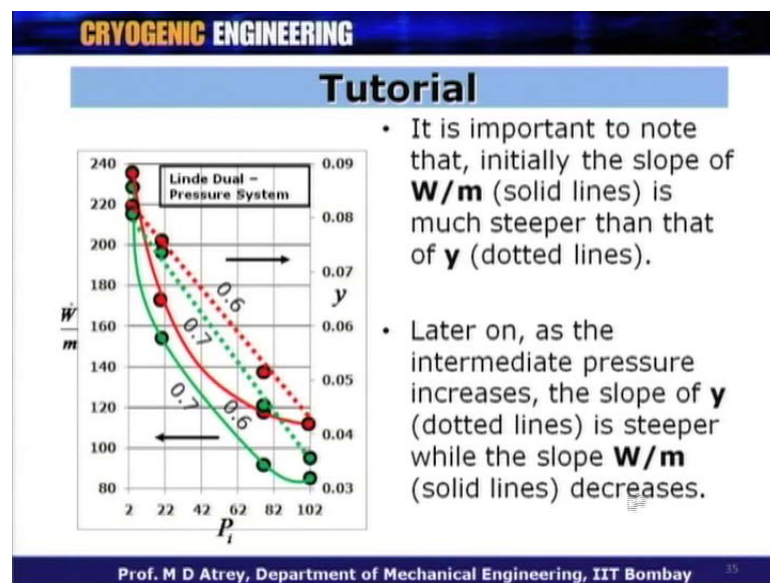
So, this can be definitely understood that as you go on increasing the intermediate pressure your W_{c1} and W_{c2} will change. The W_{c2} will have only m minus m dot i which is compress while, W_{c1} will have the entire m dot which is going to be compressed. So, as you go on increase in the P_1 , the pressure ratio for the second compress is less and less while the pressure ratio for the first compressor increases. Corresponding to that the value of W_{c1} and W_{c2} change, but, if you go on increase in the value of intermediate pressure all right.

The whole mass which is getting compress actually is in the compressor number 2. So, the second compressor is not subjected to less pressure ratio and therefore, the value of

W_{c2} in this case, gets less and less all right. So, as you go on increasing the intermediate pressure ratio, the value of W_{c2} decreases and therefore, if you add both of them together W_{c1} plus W_{c2} . The value of work done per unit mass of gas compressed goes on decreasing, if you go on increase in the intermediate pressure.

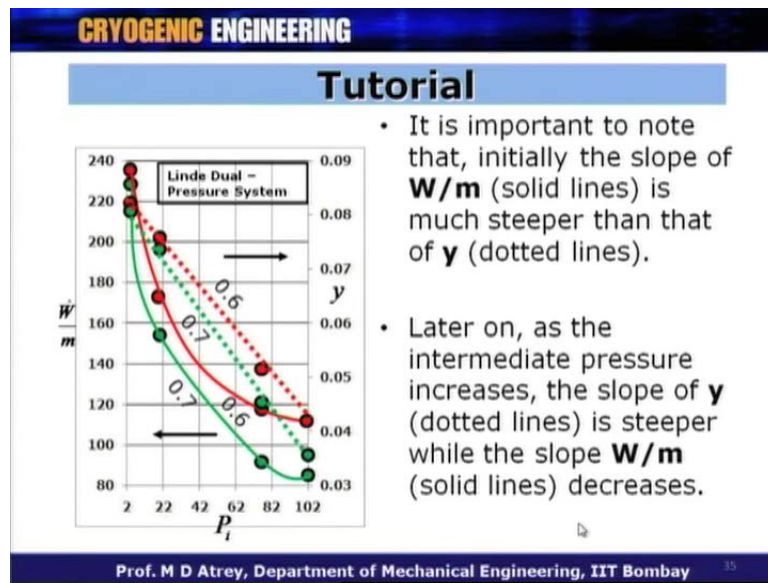
This we will understand with all the logistics which I am just talking about what happens should W_{c1} ? What happens to W_{c2} ? As you go on increasing the intermediate pressure ratio though, the pressure ratio for the first compressor increases, but it is subjected to less mass flow rate. The while, the pressure ratio for the second compressors gets reduced, but it subjected to maximum mass flow rate. So, it is net addition of W_{c1} and W_{c2} we shall determine what is the value of W by m dot in this case.

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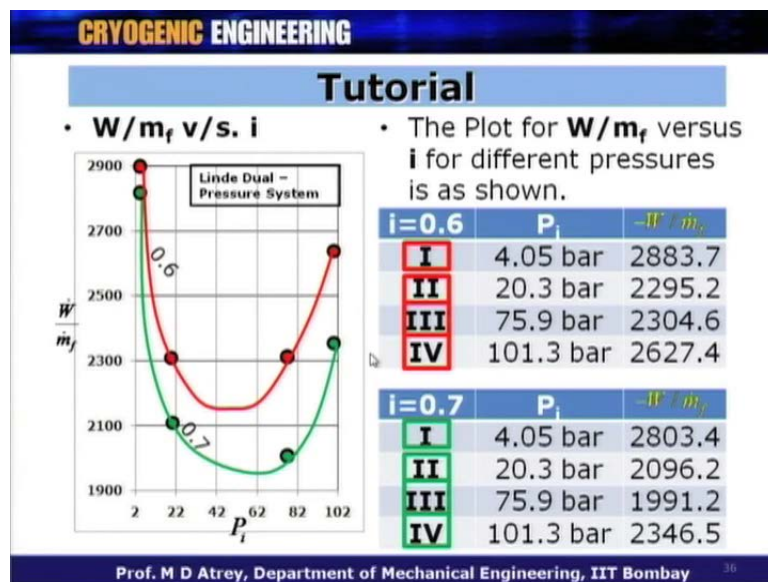
Now, for a given value of mass i W by m dot of the system decreases with the increase in the intermediate pressure. As the mass ratio of i increases, the W by m dot decreases, because the more of the mass flow rate is bypassed from compressor number 1. So, if we increases i know your work W_{c1} . Now, in this case will decrease while W_{cT} will remain the same, but W_{c1} will decrease, because the least mass is going to the compressor number 1. So, you can see the relative curve in both the cases that is y curve and W by m dot.

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It is important to note that, initially the slope of W by m dot is much steeper than that of y . Later on, as the intermediate pressure increases, the slope of y is steeper while the slope W by m dot decreases in this case.

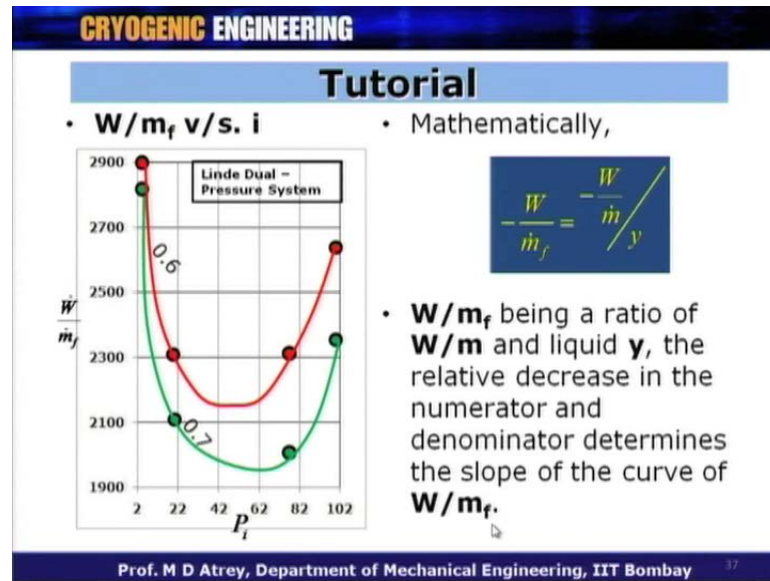
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What is important now is to understand? W by m dot f versus i which depends on both this waver of y and W by m dot and this is the values. So, if I plot these values 1, 2, 3 and 4 and if I join them what you can see is? The clear cut minima between this point over in this 2 case and again for i is equal to point 7, if I put those values all right.

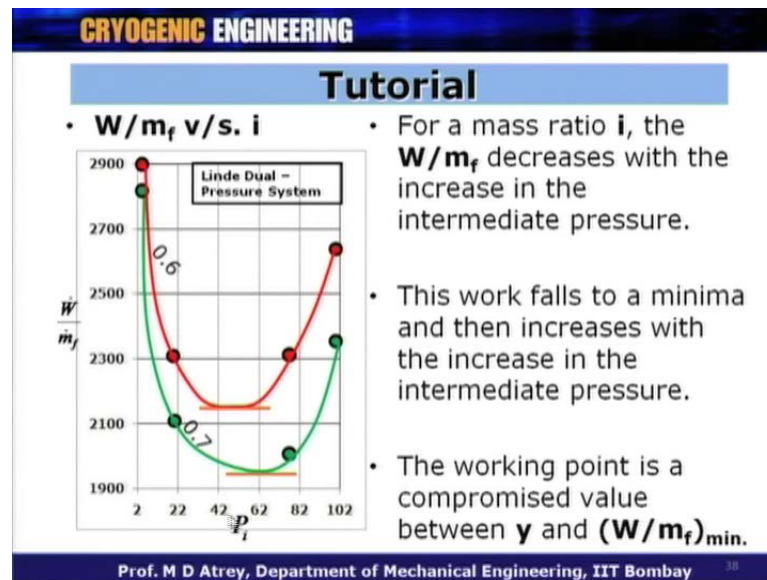
So, this is the whole exercise is carried out in order to choose a point of a intermediate pressure for which W by \dot{m} is going to be minimum all right. So, this is the 0.6 case and this is the 0.7 case all right. This is the most important curve which we get from the values of division of W by \dot{m} divided by y , because the plot of W by \dot{m} value is determined by W by \dot{m} divided by y .

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So, mathematically as you understand W by \dot{m} is nothing, but W by \dot{m} upon y . W by \dot{m} being a ratio of W by \dot{m} and y the relative decrease in the numerator and denominator determines the slope of the value of W by \dot{m} and that is y on the curve nature is like this.

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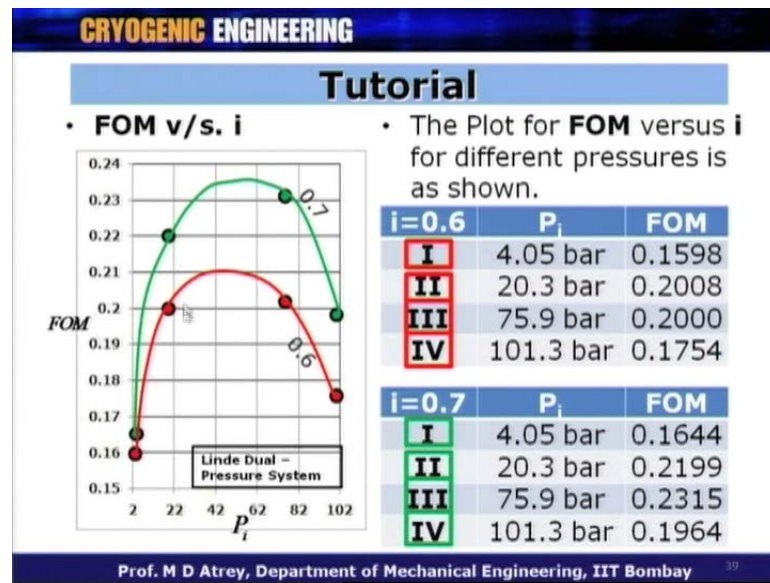


For a mass ratio of i , W by $m \dot{f}$ decrease with the increase in the intermediate pressure. It decreases first goes through minima and again starts increasing. The work falls to minima and then increases with the increase in the intermediate pressure and you can find that for this case 0.6, one should operate at a pressure somewhere around 50 bar, 50 atmosphere while somewhere on 62 is a corresponding optimum pressure for i is equal to 0.7.

The working point is compromised value of y and $m \dot{f}$. So, as you know that as you go on increase in the intermediate pressure, the value of y force down while the W by $m \dot{f}$ is touching minimum at this 2 values. Therefore, what value of P_i one should select is actually going to be compromise between what y you want corresponding to that what W by $m \dot{f}$ you might get. So, what is your liquefaction requirement and what should we you are W by $m \dot{f}$. This is the compromise value minus to select in this particular case all right.

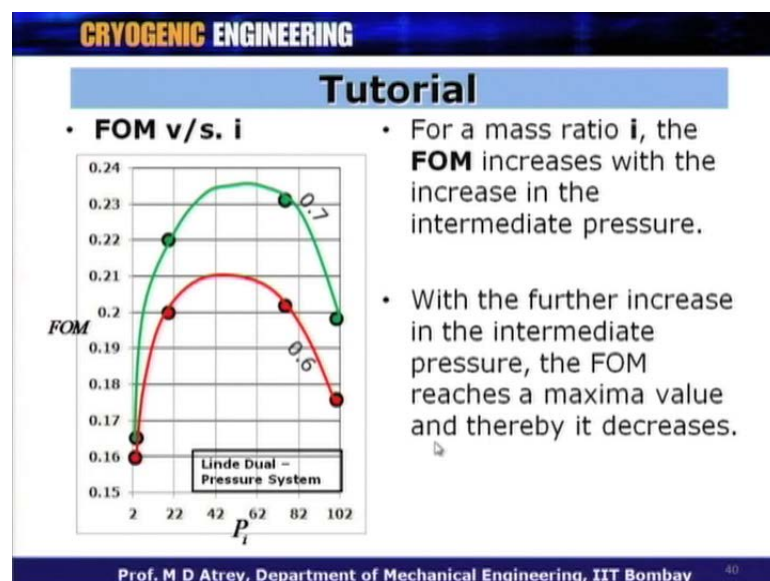
So, depending on your liquefaction requirement corresponding W by $m \dot{f}$ minimum, we should be selected.

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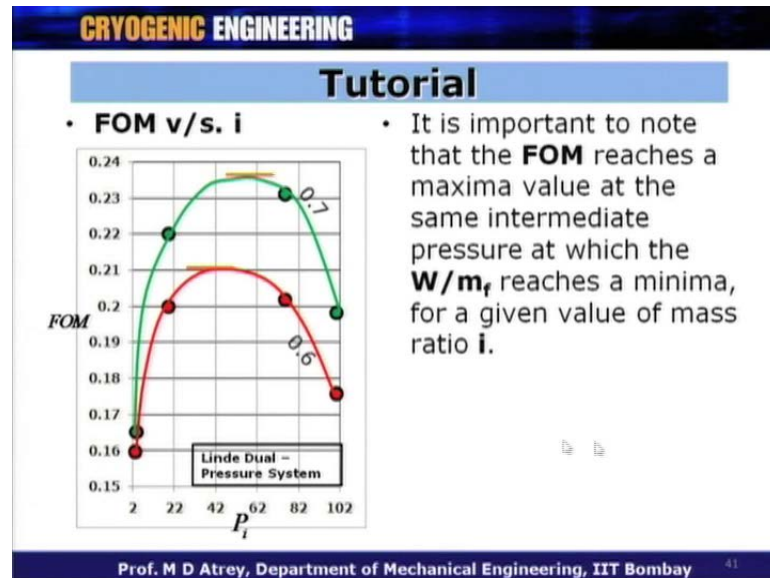
The figure of merit is also plotted from the same thing and exactly reverse will happen. Wherever, there is the W by $m \dot{f}$ minimum definition of Figure of Merit, you will get maximum W by $m \dot{f}$ here and for 0.7 case i is equal to 0.7 case. You will get a value of Figure of Merit as maximum around 62 bar somewhere here and other said, around 50 bar somewhere at 0.6 and this is the value one should select, if one has choice or one has to compromise and on this top position and corresponding to those values the value of p_i should be shows on all right.

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So, for mass ratio of i the Figure of Merit increases with the increase in the intermediate pressure. With the further increases in the intermediate pressure, the Figure of Merit reaches a maxima value and thereby it decreases.

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It is important to note that the Figure of Merit reaches a maxima value at the same intermediate pressure at which the W by m dot, reaches a minima for a given value of mass ratio i . This is just talked about with this background, the way the tutorial has been solved.

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CRYOGENIC ENGINEERING

Assignment

- Determine y , W/m , W/m_r and **FOM** for a Linde Dual – Pressure System with Argon as working fluid. The system operates between 1.013 bar (1 atm) and 202.6 bar (200 atm). The intermediate mass ratio **$i=0.6$** .
- Ans : 0.09115, 155 J/g, 1700.49 J/g, 0.2711.

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I got a assignment for you and assignment solving only this will make you perfect. So, what I want to you understand solve the problem again, for Argon for different pressures of 1 bar and 200 bar. Now, intermediate mass ratio is taken as 0.6 and we have go in the answers also for this case for y W by m dot W by m dot f and Figure of Merit. So, please do this in this size and check your answers, check your calculations to get these answers.

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CRYOGENIC ENGINEERING

Summary

- Linde Dual – pressure system is a modification of a Simple Linde – Hampson system in order to reduce the work requirement.
- In this system, the entire mass flow rate of the gas is not compressed to the required high pressure, as was the case in the previous systems.
- In this system, the work requirement decreases when the compression of fluid is done in two stages and for different mass flow rates.

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In summary on this particular lecture, the Linde Dual - Pressure System is a modification of a simple Linde - Hampson system in order to reduce the work requirement all right.

The Linde Dual - Pressure System is they were to increase yield all right. In fact, you know that the yield decreases with the intermediate pressure increasing. The Linde Dual - Pressure System is basically aimed at reducing the work requirement per unit mass of gas compressed or per unit mass of gas liquefied. In this system, the entire mass flow rate of the gas is not compressed to the required high pressure, as was the case in the previous systems all right. In this system, the work requirement decreases when the compression of fluid is done in two stages and for different mass flow rates. This is the most important thing.

The pressure ratios for the compressor number 1 and compressor number 2 or going to be different at the same time, the mass flow rates in the compressor number 1 and 2 are going to be different.

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CRYOGENIC ENGINEERING

Summary

- The yield of the system is given by the following equation.
$$y = \frac{h_1 - h_s}{h_1 - h_f} - i \left(\frac{h_1 - h_2}{h_1 - h_f} \right)$$
- The work requirement is given by
$$-\frac{W_c}{\dot{m}} = T_1(s_1 - s_s) - (h_1 - h_s) - i(T_1(s_1 - s_2) - (h_1 - h_2))$$
- In the above equations, the first term corresponds to the L - H system and the second term is the reduction occurring due to the modification.

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The yield of the system is given by the following equation, which is this as you know y is equal to $h_1 - h_s$ upon $h_1 - h_f$ minus i times $h_1 - h_2$ upon $h_1 - h_f$. The work requirement also is given by this system. This is expression as if we are compressed from 0.12, 0.3 and then the reducing the work in the ratio of i from 1 to 2. In the above expressions **in the above expressions**, of the equations the first term corresponds to the Linde - Hampson system and the second term is the reduction occurring due to the modification.

So, basically the first term in both the cases, is as if we have Linde - Hampson system from 0.12, 0.3, which is normally not possible. One can at compressed from 0.1 to 0.3 directly and therefore, what we are doing is reduction? Which is happening in value of y and W_c by \dot{m} , because of the modifications of the Linde - Dual Pressure System?

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CRYOGENIC ENGINEERING

Summary

- For a given value of mass ratio i , the y and W/m of the system decreases with the increase in the intermediate pressure.
- For a mass ratio i , the W/m_f passes through a minima as the intermediate pressure increases.
- On the other hand, for a mass ratio i , the FOM passes through a maxima with the increase in the intermediate pressure.

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For a given value of mass ratio i , the y and W by m dot of the system decreases with the increase in the intermediate pressure. This is the just what we saw in the tutorial answer sheet? Where as you go on increase in the intermediate pressure, the value of y and W by m dot decreases. For a value of mass ratio of i , W by m dot f passes through a minima as the intermediate pressure increases. So, this is the minima which we want to attack basically and what intermediate pressure one gets minimum work done per unit mass of gas which is liquefied.

We want to attach that minima, which will ensure that you adding minimum work of compressor to get the work of liquefaction per unit mass of gas which is going to be liquefied. On the other hand, for a mass ratio of i , the Figure of Merit passes through a maxima with the increase in the intermediate pressure **right** all right. So, when you attain minima over here at the same particular pressure we at in maxima for the Figure of Merit.

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CRYOGENIC ENGINEERING

Summary

- The operating point of the system is a compromised value between the y and the $(W/m_f)_{\min}$.
- It is important to note that the **FOM** reaches a maxima value at the same intermediate pressure at which the W/m_f reaches a minima, for a given value of mass ratio i .

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The operating part of the system is a compromised value between y and W upon $m \dot{f}$, as you know as you go on increasing the intermediate pressure the value of y decreases. But the value of W by $m \dot{f}$ will fall through minima. So, one has to taken optimum value between 2 between in this 2. In fact, why should design to gate W by $m \dot{f}$ minimum corresponding to that your desired y value should match.

So, whatever my desired value of y is i should get W by $m \dot{f}$ minimum at. It is important to note that the Figure of Merit reaches a maxima value at the same intermediate pressure at which the W by $m \dot{f}$ reaches a minima, for a given value of i . Thank you very much.