Cryogenic Engineering

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Lecture No. # 13

Gas Liquefaction and Refrigeration Systems

So, welcome to the thirteenth lecture of cryogenic engineering under the NPTEL program, to take an over view of the earlier lecture.

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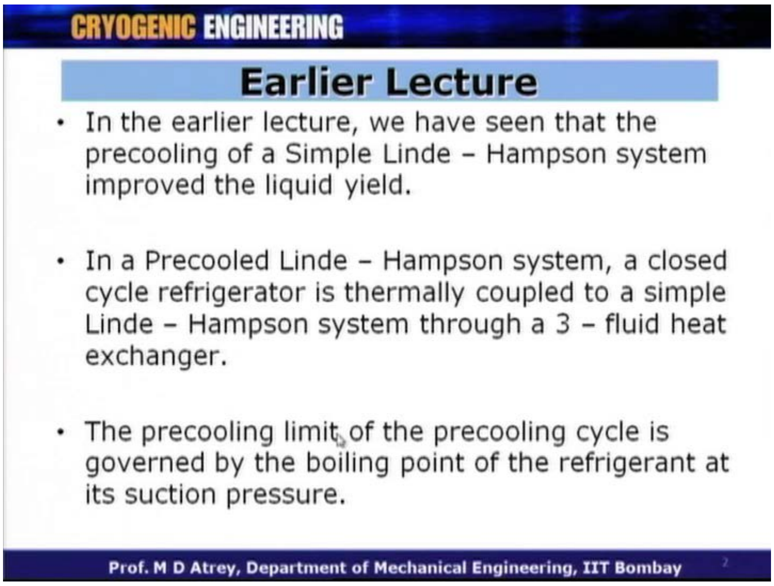

CRYOGENIC ENGINEERING

Earlier Lecture

- In the earlier lecture, we have seen that the precooling of a Simple Linde – Hampson system improved the liquid yield.
- In a Precooled Linde – Hampson system, a closed cycle refrigerator is thermally coupled to a simple Linde – Hampson system through a 3 – fluid heat exchanger.
- The precooling limit of the precooling cycle is governed by the boiling point of the refrigerant at its suction pressure.

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In the earlier lecture, we have seen that the precooling of a Simple Linde - Hampson system improved the liquid yield. Before that we had talked about Simple Linde - Hampson and then we are talking about the precooling of the Simple Linde - Hampson cycle and we found that it improved the liquid yield.

In a Precooled Linde - Hampson system, a closed cycle refrigerator is thermally coupled to a simple Linde - Hampson system through a 3-fluid heat exchanger. So, we have got a precooling arrangement through a 3-fluid heat exchanger. The precooling limit of the precooling cycle is governed by the boiling point of the refrigerant at it is suction pressure. So, we have got a refrigerant moving in a close cycle manner in a precooling circuit and the boiling point of this refrigerant at a suction pressure at low pressure, it will decide the lowest possible precooling temperature that can be obtained for the Linde - Hampson cycle.

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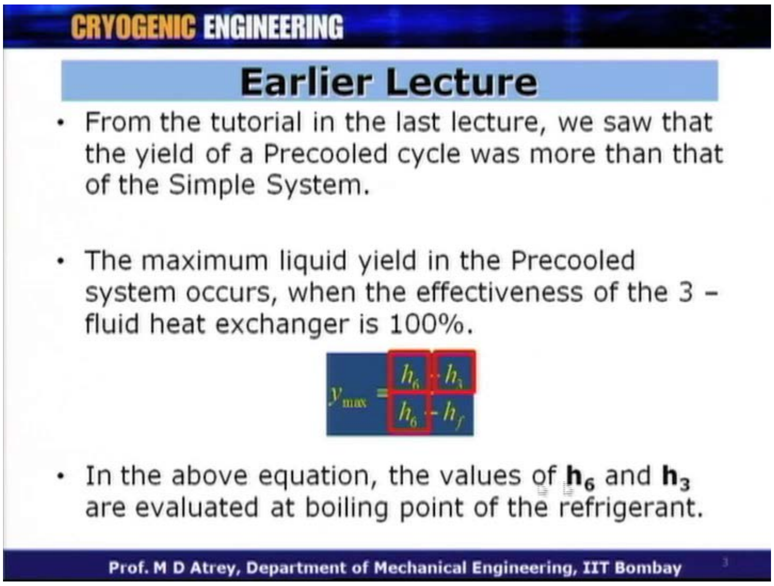

CRYOGENIC ENGINEERING

Earlier Lecture

- From the tutorial in the last lecture, we saw that the yield of a Precooled cycle was more than that of the Simple System.
- The maximum liquid yield in the Precooled system occurs, when the effectiveness of the 3 – fluid heat exchanger is 100%.

$$y_{max} = \frac{\boxed{h_6} - \boxed{h_3}}{\boxed{h_6} - h_f}$$

- In the above equation, the values of **h_6** and **h_3** are evaluated at boiling point of the refrigerant.

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From the Tutorial in the past lecture, we saw that the yield of a Precooled cycle was more than that of the simple system and this is why we do precooling basically, the maximum liquid yield in the Precooled system occurs, when the effectiveness of the 3-fluid heat exchanger is 100 percent, but as you know in practical system, the heat exchanger effectiveness will not be 100 percent. So, the effect of this also has to be considered.

The maximum yield or a y max is obtained by the enthalpy difference what you get after the first heat exchanger? And that is h 6 minus h 3 upon h 6 minus h f when the enthalpy at point 6 and point 3 are evaluated at temperature, which is at the boiling point of the refrigerant. So, in the above equation, the values of h 6 and h 3 are evaluated at boiling point temperature of the refrigerant, then only whatever yield you get is y max.

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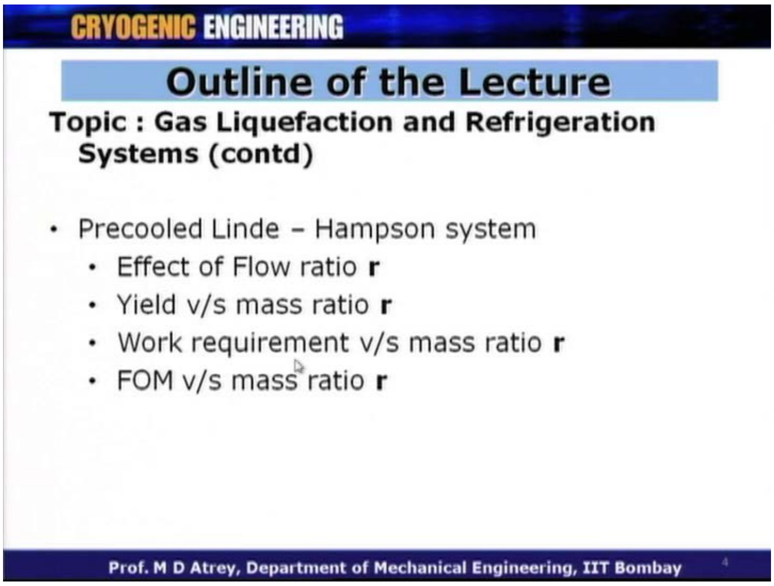

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Outline of the Lecture

Topic : Gas Liquefaction and Refrigeration Systems (contd)

- Precooled Linde – Hampson system
  - Effect of Flow ratio **r**
  - Yield v/s mass ratio **r**
  - Work requirement v/s mass ratio **r**
  - FOM v/s mass ratio **r**

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With this background, the outline of current lecture is continuing on the same topic of Gas Liquefaction and Refrigeration System. We will take this topic ahead, the Precooled Linde - Hampson system where in we will study the effect of different parameters that are effect the performance of this system. What are these different parameters? The effect of flow ratio r, this is the ratio of the mass flow rate, the refrigerant of the refrigerant divided by the mass flow rate of let us say, the gas to be liquefied.

So, m dot r upon m dot is what we call as flow ratio r, then yield versus mass flow ratio, mass ratio r. Yield is y whatever liquefaction yield you get that as a function of this r which is a very important parameter and we will try to understand what happens to this ratio? Then different work requirement, we have got compressor work requirement per gram of mass which is compressed and also the work requirement per gram or k g of gas which is liquefied.

So, you got a work requirement versus mass ratio r and finally, Figure of Merit versus mass ratio r which depends on the above 2 parameters. So, in effect will study all this effect as a function of r which is very very important parameter, if r is equal to 0 it reduce down to a simple Linde - Hampson cycle. So, r is the only parameter which changes the simple Linde - Hampsn cycle to Precooled Linde - Hampson cycle and therefore, to study the effect of this r parameter soon all this different performers parameters is a very important task.

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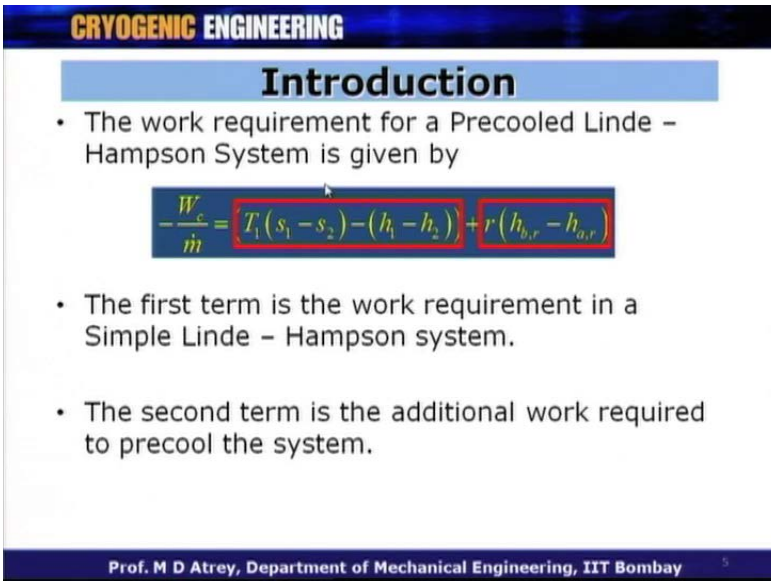

CRYOGENIC ENGINEERING

Introduction

- The work requirement for a Precooled Linde – Hampson System is given by

$$-\frac{\dot{W}_c}{\dot{m}} = \boxed{T_1(s_1 - s_2) - (h_1 - h_2)} + \boxed{r(h_{b,r} - h_{a,r})}$$

- The first term is the work requirement in a Simple Linde – Hampson system.
- The second term is the additional work required to precool the system.

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To introduce what we are going to learn? The Work requirement for a Precooled Linde - Hampson cycle is given by this formula minus W c upon m dot is equal to the first bracket and this is the second bracket. The first term or the first bracket is a work requirement in a simple Linde - Hampson system, we have seen the derivation of this formula also and to this first term or the first bracket what you add is a second term, when we go from a simple Linde - Hampson system to a Precooled Linde - Hampson system.So, when you go from a simple Linde - Hampson to a Precooled Linde – Hampson, this is an additional work that the compressor has to do and this is the Work done by the refrigeration system or the precooling circuit compressor.

Second term is the additional work required to precool the system. So, the first term remain the same as long as the suction pressure and the compressor pressure at the end of compression remain the same, but this is the additional amount of work that has to be done and that comes due to the precooling circuit in the Precooled Linde - Hampson system.

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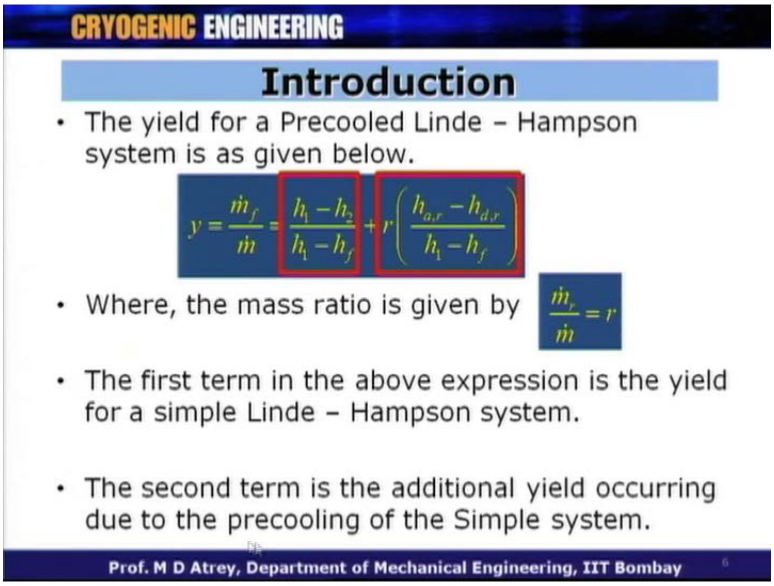

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Introduction

- The yield for a Precooled Linde – Hampson system is as given below.

$$y=\frac{\dot{m}_f}{\dot{m}}=\boxed{\frac{h_1-h_2}{h_1-h_f}}+\boxed{r\left(\frac{h_{a,r}-h_{d,r}}{h_1-h_f}\right)}$$

- Where, the mass ratio is given by $\frac{\dot{m}_r}{\dot{m}}=r$
- The first term in the above expression is the yield for a simple Linde – Hampson system.
- The second term is the additional yield occurring due to the precooling of the Simple system.

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The yield for a Precooled Linde - Hampson system is as given below. y is equal to m dot f upon m dot is equal to h 1 minus h 2 upon h 1 minus h f which is same as simple Linde-Hampson cycle plus the additional yield which come, because the precooling cycle. where the mass ratio r is given by m dot r upon m dot is equal to r. The first term in the above expression is the yield from a simple Linde - Hampson system and the second term is the additional yield occurring due to the precooling of the simple system.

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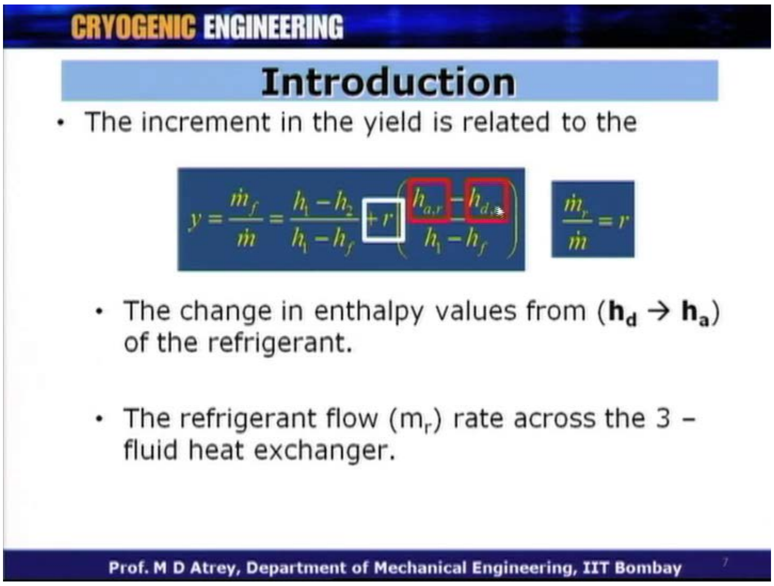

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Introduction

- The increment in the yield is related to the

$$y=\frac{\dot{m}_f}{\dot{m}}=\frac{h_1-h_2}{h_1-h_f}\boxed{+r}\left(\frac{\boxed{h_{a,r}}-\boxed{h_{d,r}}}{h_1-h_f}\right) \qquad \frac{\dot{m}_r}{\dot{m}}=r$$

  - The change in enthalpy values from (**h_d → h_a**) of the refrigerant.
  - The refrigerant flow (m_r) rate across the 3 – fluid heat exchanger.

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The increment in the yield, is related to this bracket we are talking about what is it related to the additional increment the additional yield which come, because of precooling what you can see from this formula, is basically dependent on h a and h d or the difference between h a and h d to change in enthalpy from the value h d to h a of the refrigerant.

So, this is the very important to understand that h a and h d value will determine or the difference in this enthalpy value will determine how much y increment will be occurring when we go from a simple Linde - Hampson cycle to a Precooled Linde - Hampson cycle. At the same time, the other parameter is r which the refrigerant flow rate m dot r rate across the 3-fluid heat exchanger.

So, r is a parameter which is the ratio of m dot r to m dot this basically, it decided by the m dot r value. So, m dot r upon m dot, because m dot remain the same and if we are going to change the value of m dot r, because of which the r changes. So, r and the enthalpy different across the heat exchanger will decide how much increment will occur when we go from a simple system to a Precooled system.

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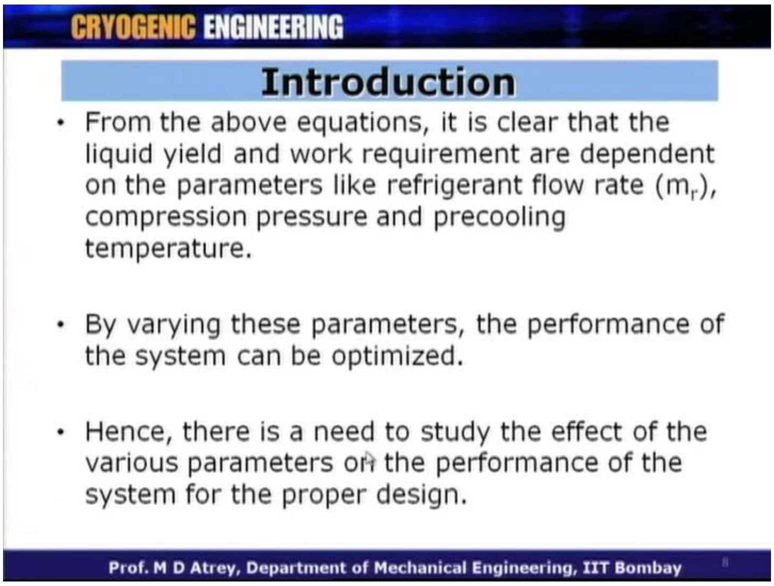

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Introduction

- From the above equations, it is clear that the liquid yield and work requirement are dependent on the parameters like refrigerant flow rate (m_r), compression pressure and precooling temperature.
- By varying these parameters, the performance of the system can be optimized.
- Hence, there is a need to study the effect of the various parameters on the performance of the system for the proper design.

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From the above equations, it is clear that the liquid yield and the work requirement are dependent on the parameters like refrigerant flow rate m dot r, compression pressure and precooling temperature. By varying these parameters, the performance of the system can be optimized. So, we got a various parameters and we got to vary about which parameter we should change? So, that we get more yield should we change m dot r, should we change the suction pressure, should we change the m dot all this will decide how much increase or decrease would there be in terms of in parameters like liquid yield (( )) requirement etcetera.

Hence, there is a need to study the effect of various parameters on the performance of the system for the proper design. So, one need not vary, one need not straight away go and increase the refrigerant m dot r, one should not really play with the suction pressure or a compression pressure unless one understand what is the effect of this parameters on the cycle in (( ))? I mean Precooled cycle plus the real simple Linde - Hampson cycle, the effect of this 2 cycles are actually related.

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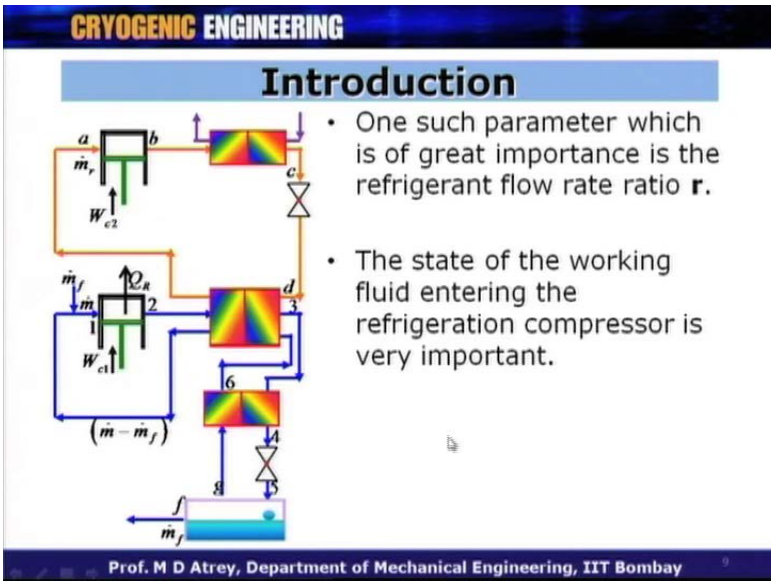

Introduction

- One such parameter which is of great importance is the refrigerant flow rate ratio **r**.
- The state of the working fluid entering the refrigeration compressor is very important.

Now, this is a cycle, this is the precooling circuit and this is the simple Linde - Hampson cycle and this is the 3-fluid heat exchanger. One such parameter which is of great importance is the refrigerant flow rate r, this is what we talked about and r it depends on this m dot r to m dot, the ratio of m dot r to m dot will decide the value of r. The state of the working fluid entering the refrigerant compressor is very important.

So, the value of m dot r also determines at what state m dot r enters the refrigeration compressor all right. So, this is very important as you know in a reciprocating compressor, the entry to the compressor should be in gaseous phase. The value of m dot r will decide or will ensure that the gas which is entering the compress is in proper state.

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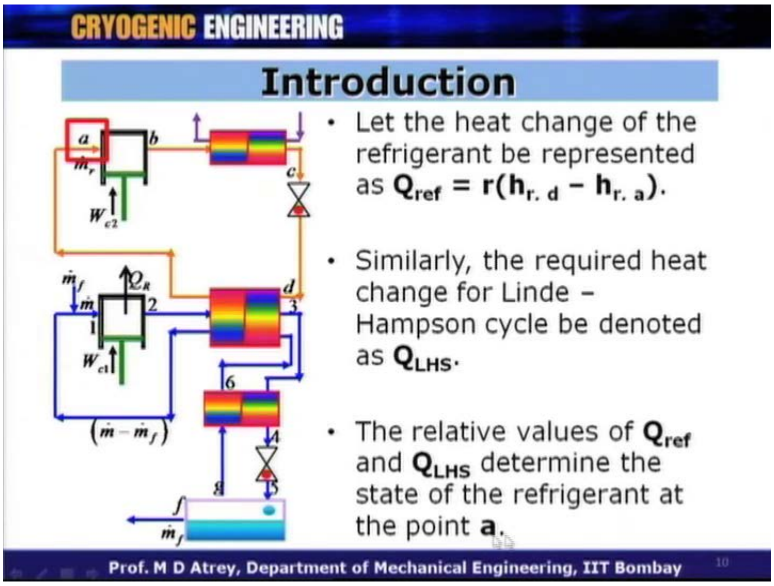

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Introduction

- Let the heat change of the refrigerant be represented as $\mathbf{Q}_{ref} = \mathbf{r}(\mathbf{h}_{r,\,d} - \mathbf{h}_{r,\,a})$.
- Similarly, the required heat change for Linde – Hampson cycle be denoted as $\mathbf{Q}_{LHS}$.
- The relative values of $\mathbf{Q}_{ref}$ and $\mathbf{Q}_{LHS}$ determine the state of the refrigerant at the point **a**.

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Let the heat change of the refrigeration refrigerant we represented by Q ref. So, Q ref is basically the the load on the system or the precooling requirement of this heat exchanger and what you get across is r into the enthalpy difference across this heat exchanger. So, r into h r d minus h r a, this will determine how much cooling load is offered by the cooling circuit all right. Similarly, the required heat change for the Linde - Hampson cycle be denoted by Q LHS all right. So, Q Linde - Hampson cycle system load will be determined by the whole of 3-fluid heat exchanger.

Now, what we want exactly is Q LHS? And what this system offers or the what this Precooled circuit offers is Q ref? This 2 values actually should match, if we have got a optimized value of Q LHS. The relative values of Q ref and Q LHS determine the state of the refrigerant at the point a.

Now, we want for a optimize design of the Precooled Linde - Hampson system what we want is a Q LHS? While the precooling circuit offers you Q ref. Now, depending on the magnitude of Q ref and the requirement of Q LHS, this will decide what is the state of the fluid when it leaves this 3-fluid heat exchanger? What is the state of fluid at point a, which is the point at which it enters the refrigeration compressor or the precooling circuit compressor all right. So, the relative magnitude of these 2 parameters will decide what is the state of the refrigerant, when it enters the refrigerant compressor.

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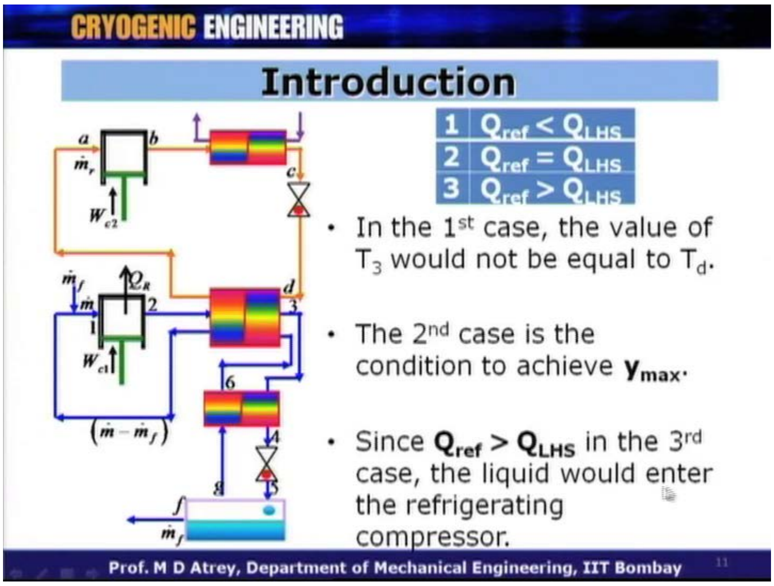

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Introduction

1	$Q_{ref} < Q_{LHS}$
2	$Q_{ref} = Q_{LHS}$
3	$Q_{ref} > Q_{LHS}$

- In the 1st case, the value of T_3 would not be equal to T_d.
- The 2nd case is the condition to achieve $\mathbf{y_{max}}$.
- Since $\mathbf{Q_{ref}} > \mathbf{Q_{LHS}}$ in the 3rd case, the liquid would enter the refrigerating compressor.

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So, there are 3 possibilities, Q ref less than Q LHS, Q ref is equal to Q LHS and the third one is Q ref is greater than Q LHS. In the first case, the value of the T 3 would not be equal to T d what is the first case, when Q ref or the refrigerant effect that is given by this precooling circuit is less than Q LHS. In this case, the gas state or the gas state at point 3 will not be at the same temperature at T d, because the refrigeration or cooling effect that is available in the precooling circuit is less than the desired value of Q LHS. In that case, the temperature at point 3 will not be equal to the point T d; that means, it will not be equal to the boiling point of the refrigerant at this suction pressure.

In the second case, when Q ref is equal to Q LHS what we achieve is a temperature at point T 3 is equal to T d; that means, based on the second law of thermodynamics the lowest temperature that is possible at point 3 is equal to the temperature at point d which is in that case is equal to the boiling point of the refrigerant at the suction pressure. At this pressure pressure at this point pressure after the expansion of the refrigerant here and therefore, in this case what we achieve is the lowest possible temperature at point 3 and therefore, the yield what you get is y max? We have talked about this in the earlier lecture.

However, what is important is a third case? When the refrigeration effect that is available in the precooling circuit is more than required that is Q ref is more than Q LHS. So, in the third case now, the liquid would enter the refrigeration compressor, because the cooling effect that is available that is Q ref is more than required and therefore, the refrigerant which is going to come out at this point.

a is going to be in 2 phase zone or in liquid zone, because it has not got vaporize completely all right and therefore, the state of the refrigerant at this point could be liquid plus gas over here, which is not good from the compressor point of view and this is the very important thing to understand that, if Q ref is more than Q LHS. Then the state of the refrigerant at this point when it enters the refrigeration compressor could be liquid or liquid plus gas and therefore, this is not a desired condition.

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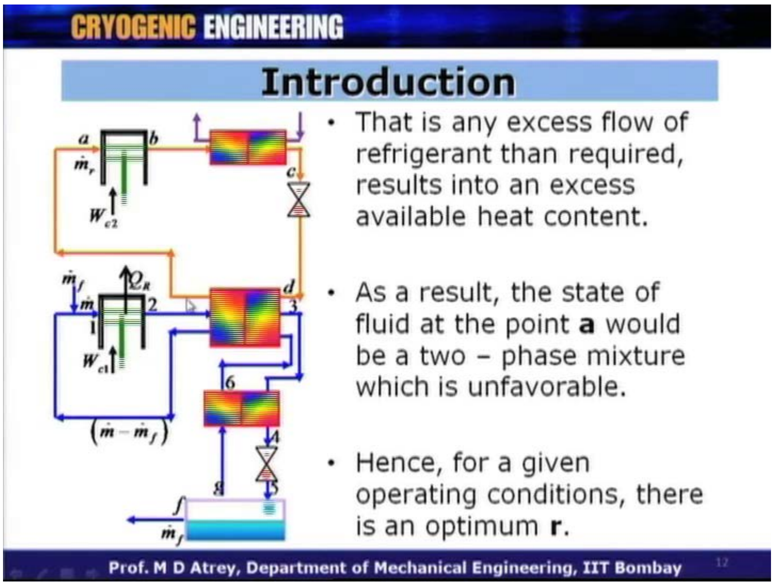

CRYOGENIC ENGINEERING

Introduction

- That is any excess flow of refrigerant than required, results into an excess available heat content.
- As a result, the state of fluid at the point **a** would be a two – phase mixture which is unfavorable.
- Hence, for a given operating conditions, there is an optimum **r**.

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That is any excess flow of the refrigerant than required, results into an excess available heat content. As a result, the state of the fluid at point a would be a two - phase mixture which is unfavorable, which is not desired in this case and hence, for a given operating conditions, there is an optimum value of r. We want to avoid such situation in fact, we should never run a compressor in which the state of refrigerant at this point is liquid or 2 phase and therefore, we should never operating in such a condition and therefore, we should always see that there is optimum value of r which ensures that the refrigerant state at this point a is always in gaseous phase all right.

This is a very important limitation of the system that the corresponding phase at this point or the r value should be the optimize value which will ensure that the refrigerant, when it enter the compressor is in gaseous phase.

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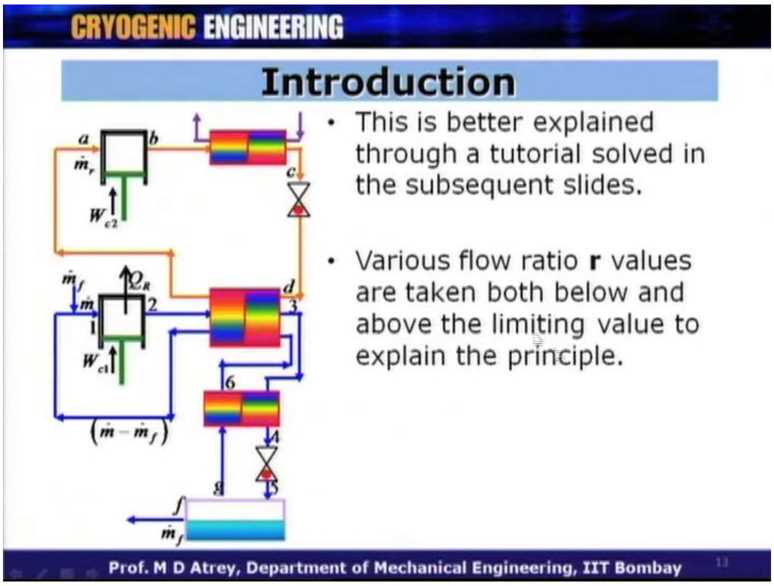

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Introduction

- This is better explained through a tutorial solved in the subsequent slides.
- Various flow ratio **r** values are taken both below and above the limiting value to explain the principle.

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This is better explained through a tutorial solved in the subsequent slides. So, to understand this will solve a problem and from there we can draw conclusions. In which, in this problem I have got a various flow ratio r values are taken both below and above the limiting value to explain the principle of the value of r or the dependence of the entire performance of this circuit on the value of r.

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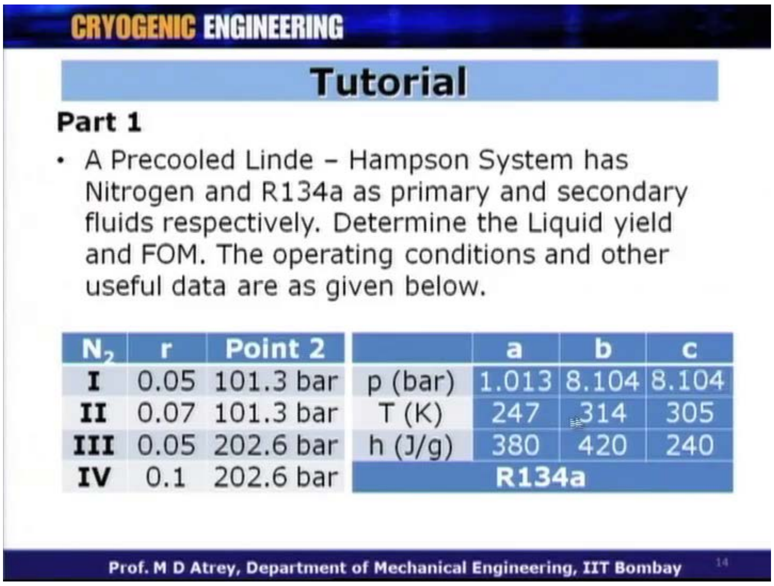

CRYOGENIC ENGINEERING

Tutorial

Part 1

- A Precooled Linde – Hampson System has Nitrogen and R134a as primary and secondary fluids respectively. Determine the Liquid yield and FOM. The operating conditions and other useful data are as given below.

N_2	r	Point 2
I	0.05	101.3 bar
II	0.07	101.3 bar
III	0.05	202.6 bar
IV	0.1	202.6 bar

	a	b	c
p (bar)	1.013	8.104	8.104
T (K)	247	314	305
h (J/g)	380	420	240
R134a			

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So, a tutorial is taken here and let us understands a tutorial and this tutorial has got 2 parts. The part 1 is this, the statement is like this a Precooled Linde - Hampson system has Nitrogen and R134a as primary and secondary fluids respectively, this you understand nitrogen are working fluid R134a is a precoolant. Determine the Liquid yield and Liquid yield is y and Figure of Merit.

The operating conditions and other useful data are given below. The operating conditions are this, we have got 4 cases over here and this is very important to understand where we vary the value of r which is 0.05, 0.07. When we have got a compressor of 101.3 bar for this 2 cases and we also want to study the effect of value of r for different pressures and which cases we have taken the pressure as 202 .6 bar or 200 atmosphere and again here, we value the value of r from 0.05 to 0.1.

So, here in short we are studying the effect of precooling for 2 different pressures 100 atmosphere and 100 atmosphere and also we are studying the effect for 2 different r values 0.05 and 0.07, and 0.05 and 0.1 for 200 atmosphere. The corresponding values for the precooling circuit are given over here, where we are compressing the precoolant of the refrigerant from 1 atmosphere to 8 atmosphere and expanding from 8 atmosphere to 1 atmosphere again and we have got different temperature values and enthalpy values for R134a, which is the refrigerant in this case.

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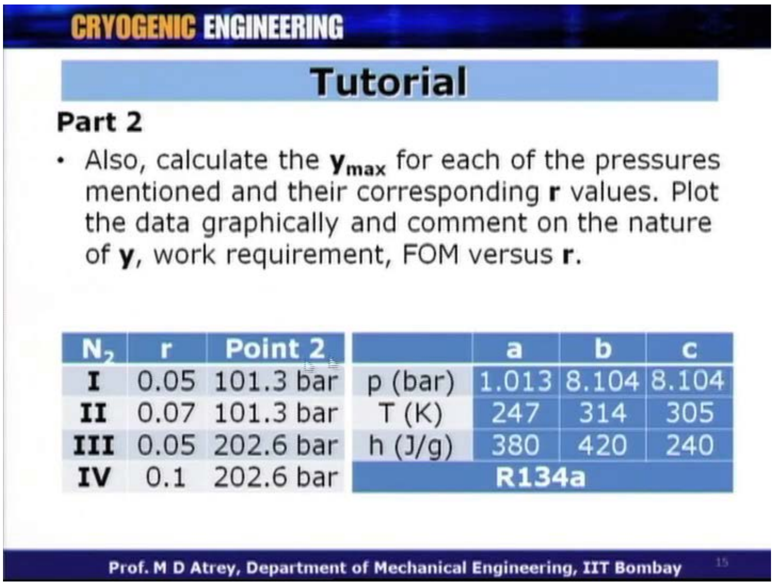

CRYOGENIC ENGINEERING

Tutorial

Part 2

- Also, calculate the y_{max} for each of the pressures mentioned and their corresponding **r** values. Plot the data graphically and comment on the nature of **y**, work requirement, FOM versus **r**.

N_2	r	Point 2
I	0.05	101.3 bar
II	0.07	101.3 bar
III	0.05	202.6 bar
IV	0.1	202.6 bar

	a	b	c
p (bar)	1.013	8.104	8.104
T (K)	247	314	305
h (J/g)	380	420	240
R134a			

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We have got a Part 2 of this problem where we extend this problem definition and here we would like to calculate the y max for each of the pressures mentioned and their corresponding r values.

So, identify the y max for given pressures and corresponding to those values find out the r values or the limiting r values in this case. Plot the data graphically and comment on the nature of y, work requirement, FOM versus r. So, this is what a Part 2 will be. So, let us understand the Part 1 which is a very simple case, we have solved the tutorial in the earlier class for this. Again, the problems in this repeat in the table in order to understand what is basically ask for in these 2 problems or these 2 tutorials.

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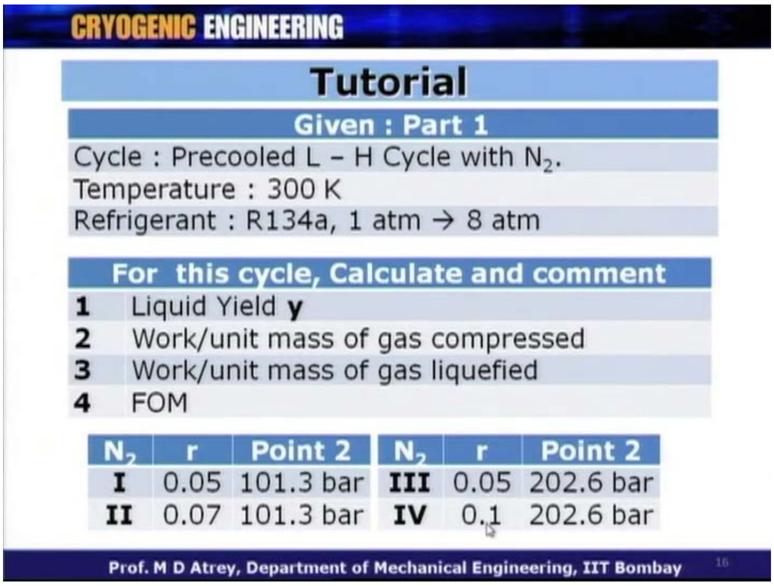

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Tutorial

Given : Part 1

Cycle : Precooled L – H Cycle with N_2.
Temperature : 300 K
Refrigerant : R134a, 1 atm → 8 atm

For this cycle, Calculate and comment

1 Liquid Yield **y**
2 Work/unit mass of gas compressed
3 Work/unit mass of gas liquefied
4 FOM

N_2	r	Point 2	N_2	r	Point 2
I	0.05	101.3 bar	**III**	0.05	202.6 bar
II	0.07	101.3 bar	**IV**	0.1	202.6 bar

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So, what is given us for Part 1 is a… We want to solve the Precooled Linde - Hampson cycle with Nitrogen is a working fluid, the temperature of compression is 300 Kelvin, the refrigerant is R134a where it is compress from 1 atmosphere to 8 atmosphere what we have to do is to find out? Liquid yield y, work per unit mass of gas compressed, work per unit mass of gas liquefied and FOM. The Nitrogen r is at having 2 values point 0.05 to 0.07 for 100 atmosphere and then 0.05 to point 1 r for 200 atmosphere.

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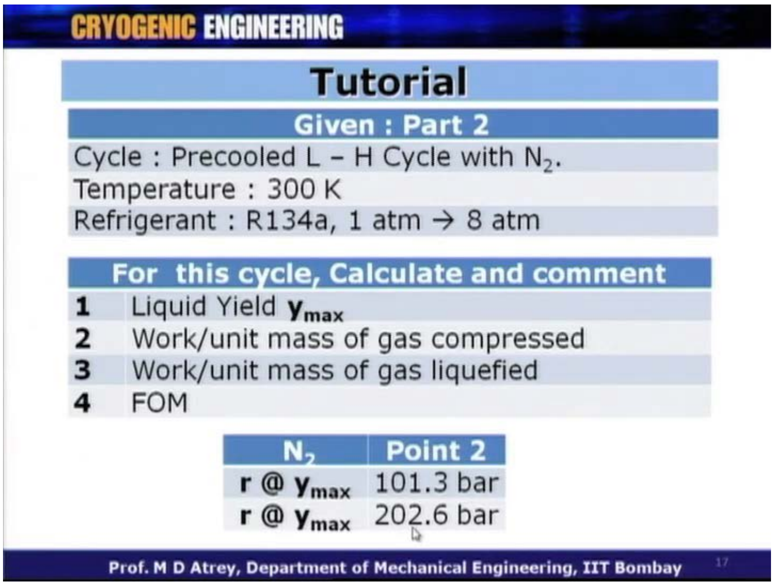

CRYOGENIC ENGINEERING

Tutorial

Given : Part 2

Cycle : Precooled L – H Cycle with N_2.
Temperature : 300 K
Refrigerant : R134a, 1 atm → 8 atm

For this cycle, Calculate and comment

1	Liquid Yield $\mathbf{y_{max}}$
2	Work/unit mass of gas compressed
3	Work/unit mass of gas liquefied
4	FOM

N_2	Point 2
r @ $\mathbf{y_{max}}$	101.3 bar
r @ $\mathbf{y_{max}}$	202.6 bar

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For Part 2 similarly, we have got the same values only in that case, we want to understand the dependence or r at y max value for 100 bar 100 atmosphere again we want to find r at y max value for 200 atmosphere.

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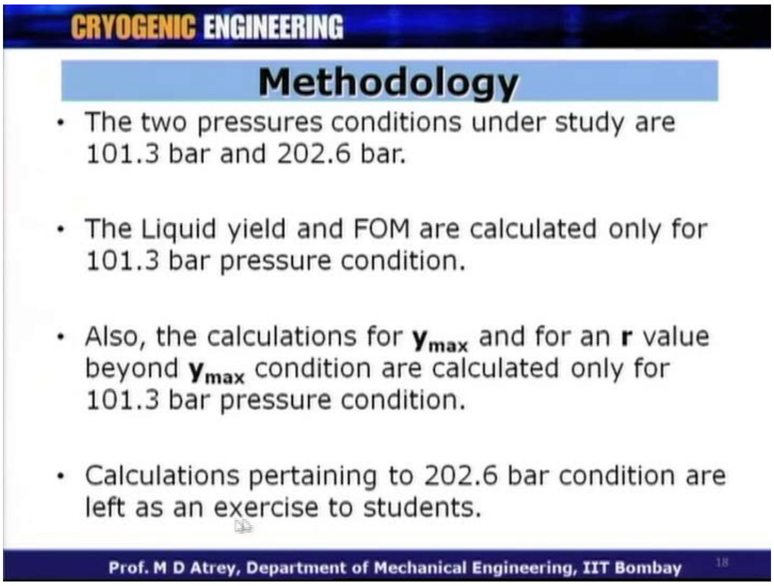

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Methodology

- The two pressures conditions under study are 101.3 bar and 202.6 bar.
- The Liquid yield and FOM are calculated only for 101.3 bar pressure condition.
- Also, the calculations for $\mathbf{y_{max}}$ and for an **r** value beyond $\mathbf{y_{max}}$ condition are calculated only for 101.3 bar pressure condition.
- Calculations pertaining to 202.6 bar condition are left as an exercise to students.

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The methodology, we are going to fallow to solve this tutorial is the 2 pressures what we are taken are 100 bar and 100 bar or 100 atmosphere and 100 atmosphere. The Liquid yield and FOM are calculated only for 100 atmosphere here. So, what I am going to do is solve the problem for 1 pressure and you would expect that you will solve the problem for 200 atmosphere.

Will also, calculate the y max value and corresponded to y max value we find the r value and will also go beyond y max, will go for an r value which is beyond y max condition and again we will do the same calculations for 100 atmosphere or 101.3 bar pressure condition only. Calculations pertaining 200 atmosphere or 202.6 bar conditions are left as an exercise for the students right.

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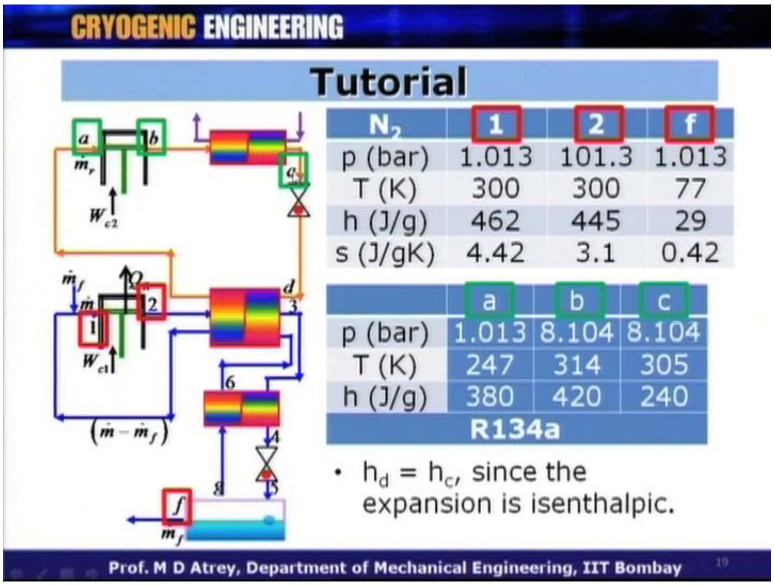

CRYOGENIC ENGINEERING

Tutorial

N_2	1	2	f
p (bar)	1.013	101.3	1.013
T (K)	300	300	77
h (J/g)	462	445	29
s (J/gK)	4.42	3.1	0.42

	a	b	c
p (bar)	1.013	8.104	8.104
T (K)	247	314	305
h (J/g)	380	420	240
R134a			

- $h_d = h_c$, since the expansion is isenthalpic.

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So, we got a circuit here, we got a point 1, 2 and point f as given over here similarly, we have got point a, b and c as given over here. So, we have got a a, b and c conditions given over here for R134a and 1, 2, f as given over here. It is just to basically, make you habituated to locate this points are correspondent those values find out a temperature enthalpy and entropy respectively. Again as you know at c what you have is a h d is equal to h c the enthalpies at point c is equal to enthalpy at point d.

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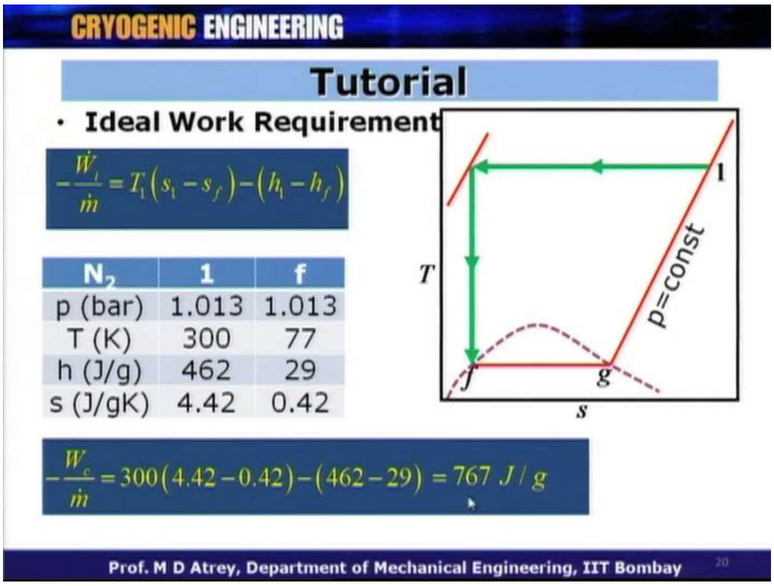

CRYOGENIC ENGINEERING

Tutorial

- **Ideal Work Requirement**

$$-\frac{\dot{W}_i}{\dot{m}} = T_1(s_1 - s_f) - (h_1 - h_f)$$

N_2	1	f
p (bar)	1.013	1.013
T (K)	300	77
h (J/g)	462	29
s (J/gK)	4.42	0.42

$$-\frac{W_c}{\dot{m}} = 300(4.42 - 0.42) - (462 - 29) = 767\ J/g$$

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First, we do the ideal work requirement I will not going the details of this calculations again, because we have done this calculation earlier I will show this calculations. So, as you know the ideal calculation come from this fact that whatever is compressed is getting liquefied using this formula, we get the values for point 1 and point f put this values in this formula what you get is a 761 joule per gram, which is a very standard value for Nitrogen as a working filed.

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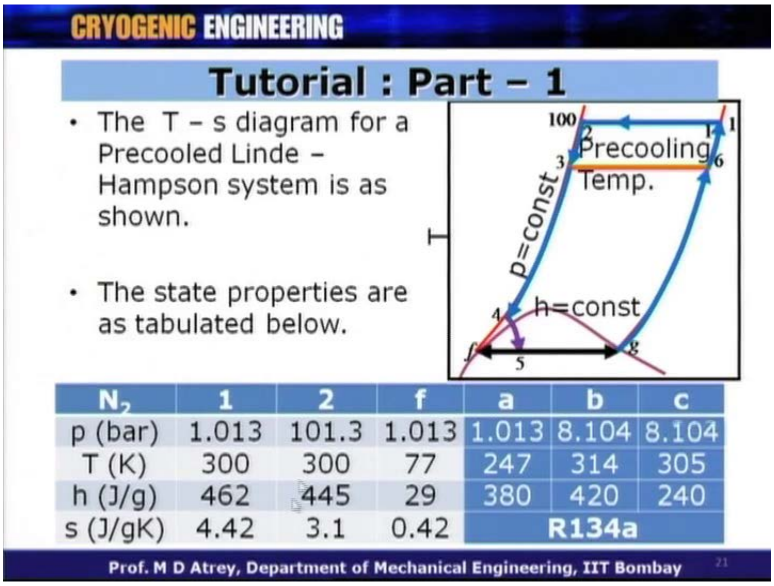

CRYOGENIC ENGINEERING

Tutorial : Part – 1

- The T – s diagram for a Precooled Linde – Hampson system is as shown.
- The state properties are as tabulated below.

N_2	1	2	f	a	b	c
p (bar)	1.013	101.3	1.013	1.013	8.104	8.104
T (K)	300	300	77	247	314	305
h (J/g)	462	445	29	380	420	240
s (J/gK)	4.42	3.1	0.42	R134a		

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Now, this is a T - S diagram for a precooling circuit and this is precooling temperature. The properties are given over here as given in the data.

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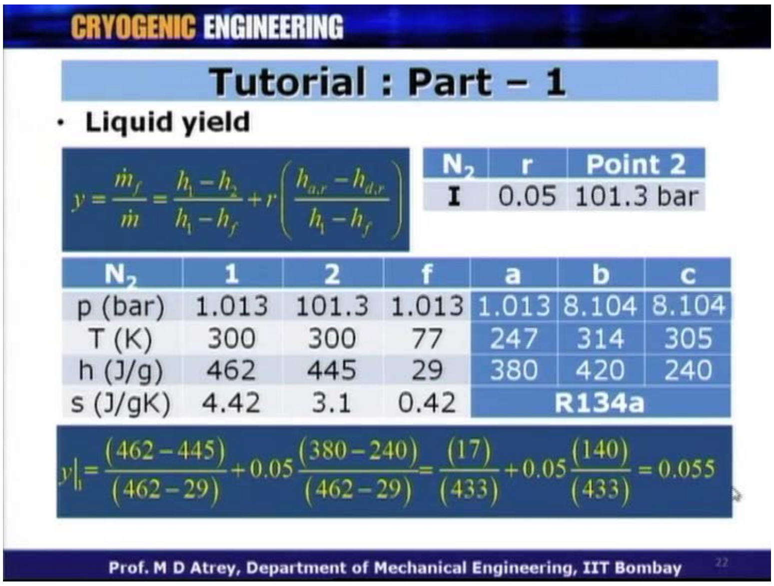

Tutorial : Part – 1

- **Liquid yield**

$$y = \frac{\dot{m}_f}{\dot{m}} = \frac{h_1 - h_2}{h_1 - h_f} + r\left(\frac{h_{a,r} - h_{d,r}}{h_1 - h_f}\right)$$

N_2	r	Point 2
I	0.05	101.3 bar

N_2	1	2	f	a	b	c
p (bar)	1.013	101.3	1.013	1.013	8.104	8.104
T (K)	300	300	77	247	314	305
h (J/g)	462	445	29	380	420	240
s (J/gK)	4.42	3.1	0.42	R134a		

$$y|_I = \frac{(462-445)}{(462-29)} + 0.05\frac{(380-240)}{(462-29)} = \frac{(17)}{(433)} + 0.05\frac{(140)}{(433)} = 0.055$$

Find out the Liquid yield by this formula, we have got a y is equal to m dot f upon m dot and this is a formula which is a r dependent and enthalpy difference across the 3-fluid heat exchanger from refrigerant side putting those values over here. In the first case, we are taking r is equal to 0.05. Put the enthalpy values at different temperatures and calculate the y 1 1 is depicting r is equal to 0.05 for 100 bar. Putting this values of enthalpy and r is equal to 0.05 what you get is a 0.055 as yield value at first case y 1.

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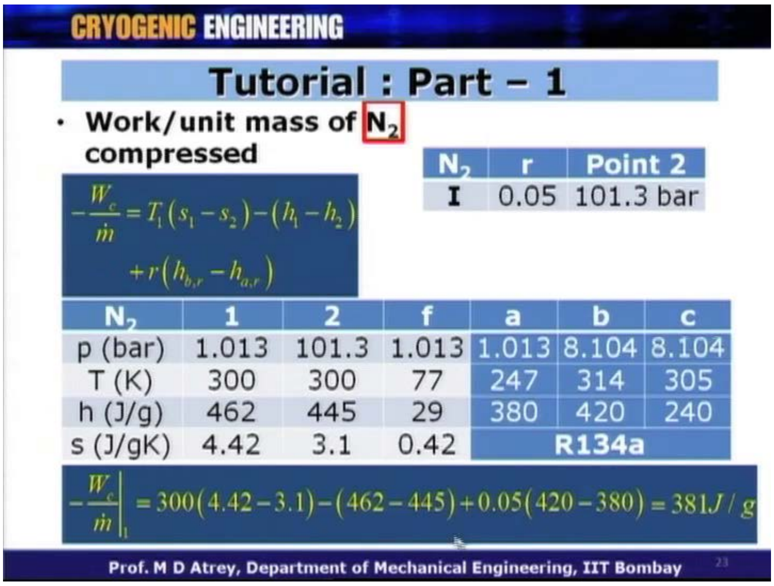

CRYOGENIC ENGINEERING

Tutorial : Part – 1

- **Work/unit mass of N_2 compressed**

$$-\frac{W_c}{\dot m} = T_1(s_1 - s_2) - (h_1 - h_2) + r(h_{b,r} - h_{a,r})$$

N_2	r	Point 2
I	0.05	101.3 bar

N_2	1	2	f	a	b	c
p (bar)	1.013	101.3	1.013	1.013	8.104	8.104
T (K)	300	300	77	247	314	305
h (J/g)	462	445	29	380	420	240
s (J/gK)	4.42	3.1	0.42	R134a		

$$-\frac{W_c}{\dot m}\Big|_1 = 300(4.42 - 3.1) - (462 - 445) + 0.05(420 - 380) = 381 J/g$$

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And then calculating work per unit mass of Nitrogen compressed. This is the formula put the value of r and h b and h a, this is the additional compressor work 1 has to do for the precooling circuit, putting this values over here what you get is a 381 joule per gram as the yield for the first case.

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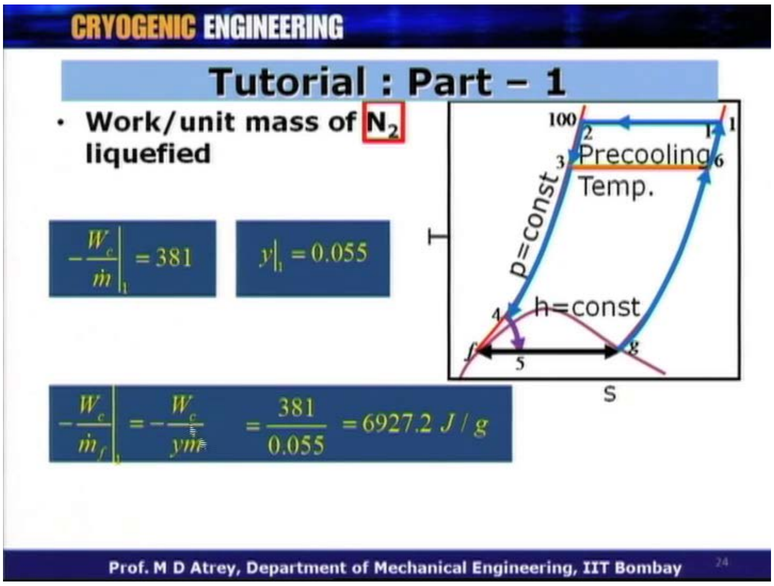

CRYOGENIC ENGINEERING

Tutorial : Part – 1

- **Work/unit mass of N_2 liquefied**

$$-\frac{W_c}{\dot m}\Big|_1 = 381$$

$$y\big|_1 = 0.055$$

$$-\frac{W_c}{\dot m_f}\Big|_1 = -\frac{W_c}{y\dot m} = \frac{381}{0.055} = 6927.2\ J/g$$

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And the work per unit mass of Nitrogen which is liquefied is coming from W c by m dot and y 1 dividing this, what you get is a 6927.2 joule per gram as work per unit mass of nitrogen liquefied.

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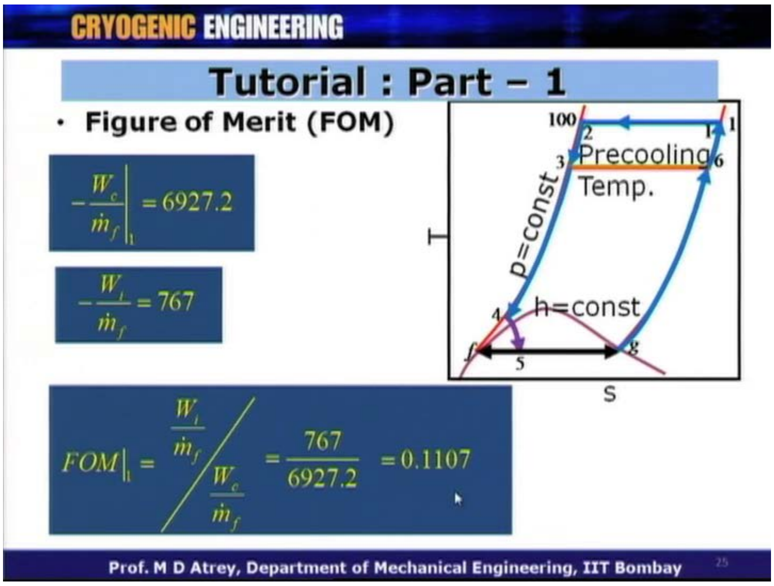

CRYOGENIC ENGINEERING

Tutorial : Part – 1

- **Figure of Merit (FOM)**

$$-\frac{W_c}{\dot{m}_f}\bigg|_1 = 6927.2$$

$$-\frac{W_i}{\dot{m}_f} = 767$$

$$FOM\big|_1 = \frac{W_i}{\dot{m}_f} \Big/ \frac{W_c}{\dot{m}_f} = \frac{767}{6927.2} = 0.1107$$

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The Figure of Merit comes by ideal work per unit mass of gas liquefied divided by the actual work per unit mass of gas liquefied, W i by m dot f is equal to W c by m dot f putting this 2 values the FOM Figure of Merit for the first cases is 0.1107.

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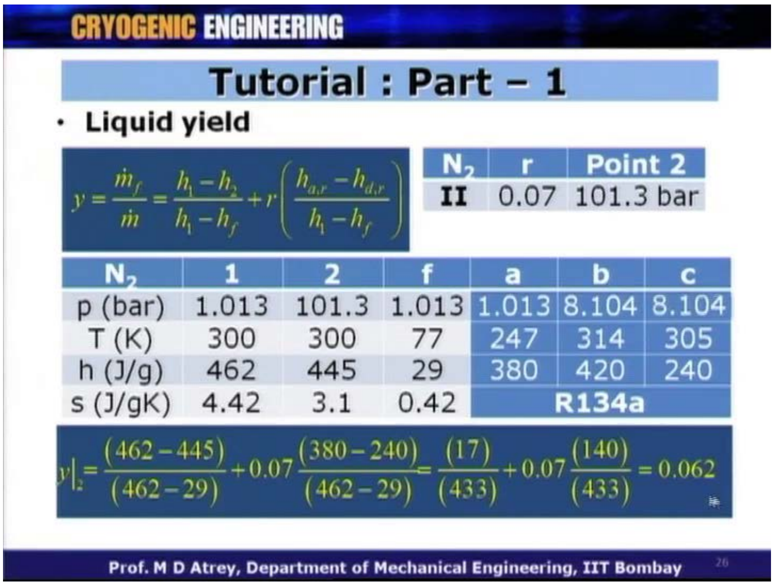

CRYOGENIC ENGINEERING

Tutorial : Part – 1

- **Liquid yield**

$$y = \frac{\dot{m}_f}{\dot{m}} = \frac{h_1 - h_2}{h_1 - h_f} + r\left(\frac{h_{a,r} - h_{d,r}}{h_1 - h_f}\right)$$

N_2	r	Point 2
II	0.07	101.3 bar

N_2	1	2	f	a	b	c
p (bar)	1.013	101.3	1.013	1.013	8.104	8.104
T (K)	300	300	77	247	314	305
h (J/g)	462	445	29	380	420	240
s (J/gK)	4.42	3.1	0.42	R134a		

$$y\big|_2 = \frac{(462-445)}{(462-29)} + 0.07\frac{(380-240)}{(462-29)} = \frac{(17)}{(433)} + 0.07\frac{(140)}{(433)} = 0.062$$

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Now, I want to do the same thing for next r value, we have done all the calculations for r is equal to 0.05 will repeat the same calculation for r is equal to 0.07 and will then compare the results. So, putting the value of r 0.07 at this value at this in this formula what you get is? The y 2 or the yield for the second case, which is y 2 when r is equal to

0.07 for pressure of 101.3 bar the yield you get is 0.062 and if you camper this with y 1 this is more.

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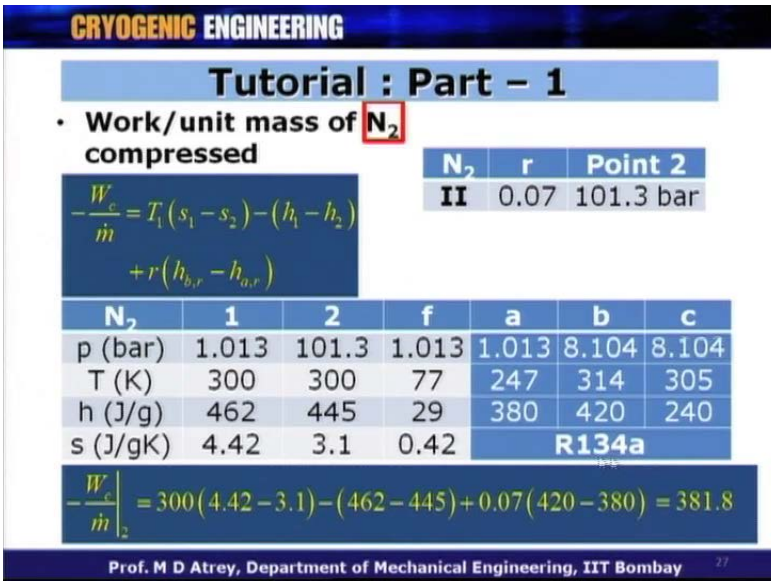

CRYOGENIC ENGINEERING

Tutorial : Part – 1

- **Work/unit mass of N$_2$ compressed**

$$-\frac{W_c}{\dot m} = T_1(s_1 - s_2) - (h_1 - h_2) + r(h_{b,r} - h_{a,r})$$

N$_2$	r	Point 2
II	0.07	101.3 bar

N$_2$	1	2	f	a	b	c
p (bar)	1.013	101.3	1.013	1.013	8.104	8.104
T (K)	300	300	77	247	314	305
h (J/g)	462	445	29	380	420	240
s (J/gK)	4.42	3.1	0.42	R134a		

$$-\frac{W_c}{\dot m}\bigg|_2 = 300(4.42-3.1)-(462-445)+0.07(420-380) = 381.8$$

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And again repeat in the calculations, work per unit mass of nitrogen compressed using this formula for r is equal to 0.07 put the formula over here and the value I get is 381.8, which is... I think little bit more than what you got earlier? And this is, because of fact that this is additional work is coming, because of the increased value of r from 0.05 to 0.07.

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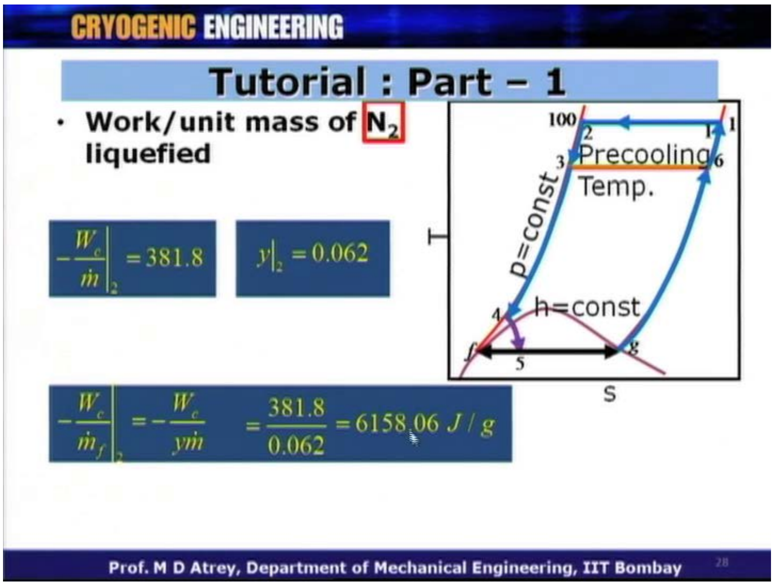

CRYOGENIC ENGINEERING

Tutorial : Part – 1

- **Work/unit mass of N$_2$ liquefied**

$$-\frac{W_c}{\dot m}\bigg|_2 = 381.8 \qquad y\big|_2 = 0.062$$

$$-\frac{W_c}{\dot m_f}\bigg|_2 = -\frac{W_c}{y\dot m} = \frac{381.8}{0.062} = 6158.06\ J/g$$

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And then the Work per unit mass of gas liquefied, this is what you get? So, 6158.06 is the work per unit mass of gas liquefied and this is the value you get.

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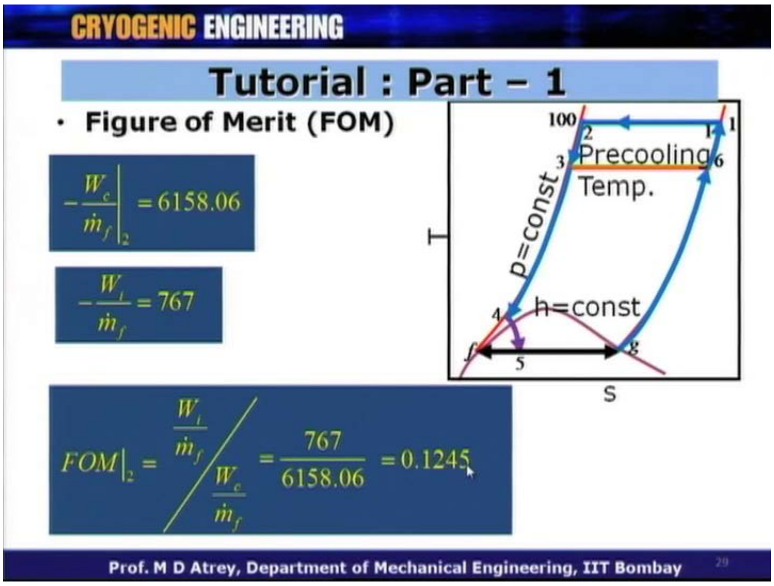

CRYOGENIC ENGINEERING

Tutorial : Part – 1

- Figure of Merit (FOM)

$$-\left.\frac{W_c}{\dot{m}_f}\right|_2 = 6158.06$$

$$-\frac{W_i}{\dot{m}_f} = 767$$

$$FOM|_2 = \frac{W_i}{\dot{m}_f} \Big/ \frac{W_c}{\dot{m}_f} = \frac{767}{6158.06} = 0.1245$$

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And at the end get in the FOM or 0.07 r value which is FOM in second case and what you get is? FOM as 0.1245. Now, let us look at the Part 2 of the tutorial and which is very important what we did in Part 1? It was just the study of the 2 r values for a given compression pressure, which studied the y max value, we study the y value, we calculated FOM exactly for 0.05 and 0.07 r values.

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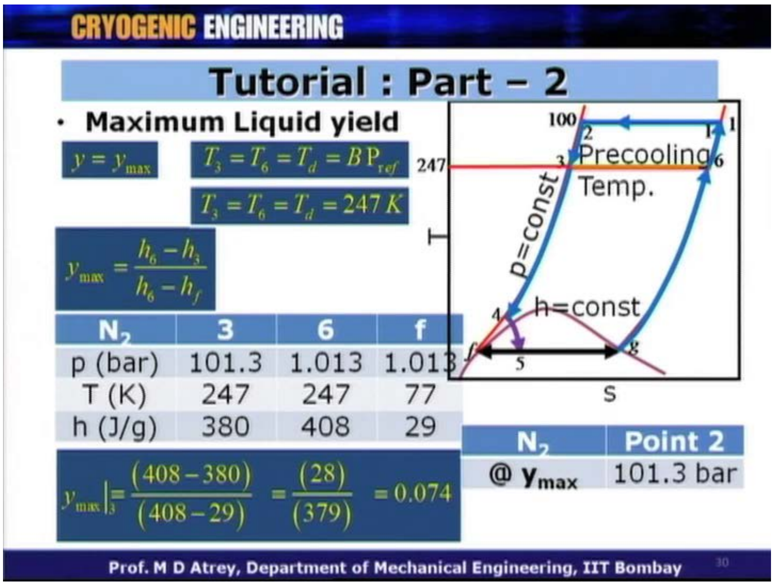

CRYOGENIC ENGINEERING

Tutorial : Part – 2

- Maximum Liquid yield

$y = y_{max}$

$T_3 = T_6 = T_d = BP_{ref}$

$T_3 = T_6 = T_d = 247\,K$

$$y_{max} = \frac{h_6 - h_3}{h_6 - h_f}$$

N_2	3	6	f
p (bar)	101.3	1.013	1.013
T (K)	247	247	77
h (J/g)	380	408	29

$$y_{max}|_3 = \frac{(408-380)}{(408-29)} = \frac{(28)}{(379)} = 0.074$$

N_2	Point 2
@ y_{max}	101.3 bar

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Now, we will talk about the maximum Liquid yield or y max. The y max yield will occur when we got the lowest possible precooling temperature at which the working fluid enters the heat exchanger all right. So, why is equal to y max, when T 3 are the precooling temperature at which the Nitrogen or the working fluid enters the Linde - Hampson cycle heat exchanger.

So, T 3 is equal to T 6 is equal to T d and T d is the temperature of the refrigerant in the precooling circuit and when this T d is equal to the lowest possible temperature which is the boiling point of the refrigerant all right. So, the boiling point of the temperature is T d and when T 6 is equal to T 3 is equal to T d, when this condition occurs whatever y value you get is equal to y max. So, the boiling point of the refrigerant at the corresponding suction pressure of around 1 bar is 247 Kelvin for R134a and therefore, the lowest possible temperature for T 3 is equal to T 6 is equal to T d is equal to 247 Kelvin and this will decide the yield and this yield will be maximum for pressure at point 2 is equal to 101.3 bar. This is the Nitrogen compressor pressure at for which first condition, we have doing all this calculation.

So, y max now, will be equal to h 6 minus h 3 upon h 6 minus h f, we are taking control volume after the first heat exchanger only as we have done in the last lecture and when we solve the problem. This is the formula and here h 6 and h 3 are evaluated at the corresponding temperature of the boiling point of the refrigerant all right. So, these are the enthalpy values for condition 3, 6 and f and you can see here for condition 3 and 6 the temperature T is 247 Kelvin which is the boiling point of the refrigerant R134a and corresponding those the enthalpy values are taken which is 380 and 408.

Now, this is very important to understand. So, please pay attention on getting this values when you want to calculate or when the problem demands the calculation of maximum yield possible from a precooling circuit. So, in this case let us call this as third case now, y max we have done y at 0.05 we have done y at 0.07 and now, we are getting a condition where we get a maximum y value and this y value is h 6 minus h 3 upon h 6 minus h f is coming from this and this y max value happens to be 0.074.

Now, understand that this is a maximum possible of y, maximum possible y value, but corresponding to this we do not know what is the r value. Earlier case, we have calculated y as 0.05 r value and y at 0.07 r value. Now, obviously those y values where less than 0.074 and this is the y maximum which is possible, but corresponding to this we do not know what is the corresponding r value in the precooling circuit. So, we will have to find out the value of r in order to get this y maximum or in order to get y is equal to 0.074.

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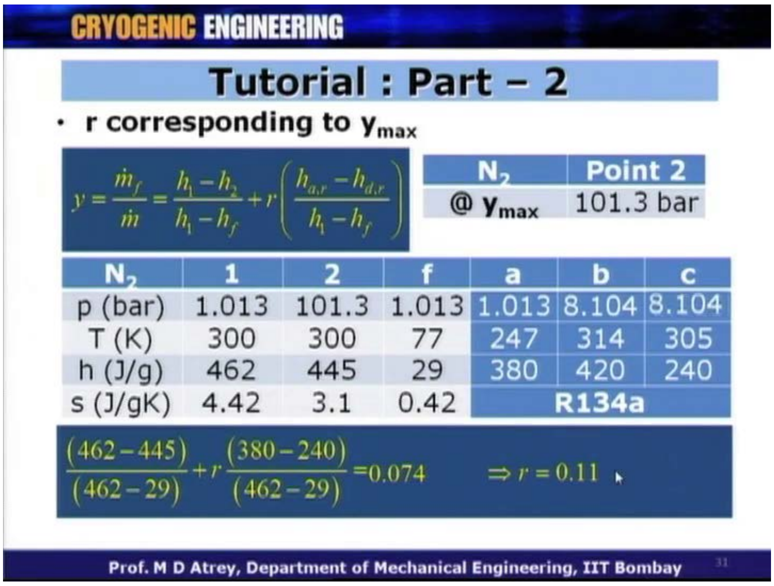

CRYOGENIC ENGINEERING

Tutorial : Part – 2

- **r corresponding to y_{max}**

$$y = \frac{\dot{m}_f}{\dot{m}} = \frac{h_1 - h_2}{h_1 - h_f} + r\left(\frac{h_{a,r} - h_{d,r}}{h_1 - h_f}\right)$$

N_2	Point 2
@ y_{max}	101.3 bar

N_2	1	2	f	a	b	c
p (bar)	1.013	101.3	1.013	1.013	8.104	8.104
T (K)	300	300	77	247	314	305
h (J/g)	462	445	29	380	420	240
s (J/gK)	4.42	3.1	0.42	R134a		

$$\frac{(462-445)}{(462-29)} + r\frac{(380-240)}{(462-29)} = 0.074 \qquad \Rightarrow r = 0.11$$

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So, our next exercise now, is to calculate r value corresponding to this y max. How do you get these? You get this from the simple formula which we are applying throughout. So, here in this case however, now what we know is y max and what we do not known is the value of r everything else is known to us.

So, y max at 101.3 bar is known to us. Correspondent to this, these are again the enthalpy value curve which is what is put over here. The value of a b and c are taken at 247 Kelvin here the value of b is 314 and value of c is 305 temperatures what you get from here, corresponding value of enthalpies are taken over in this point. So, if I put those values and how do we get this values basically, they are coming from the enthalpy values all right.

So, enthalpy and the corresponding pressures will get a temperature for R134a a case putting those enthalpy values over here, for unknown r value what is known here is? y max what you get is r is equal to 0.11 . So, this value is of course, different than what we got earlier? We had got the value of y at 0.05 r, we had got the value of y at 0.07 r. Now, this is the y max condition and corresponding to this y max r value is 0.11 what is the r value? It is the ratio of the refrigerant mass flow rate divided by the mass flow rate in the working field or m dot r upon m dot.

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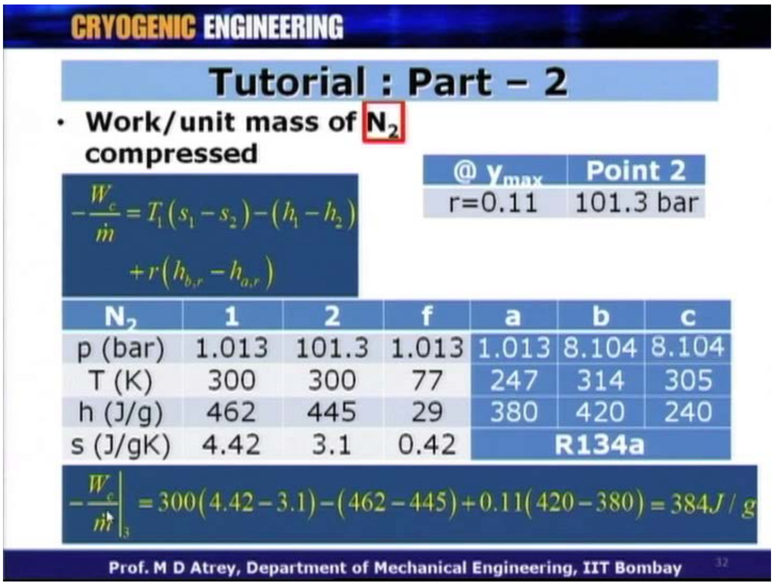

CRYOGENIC ENGINEERING

Tutorial : Part – 2

- **Work/unit mass of N_2 compressed**

$$-\frac{W_c}{\dot{m}} = T_1(s_1 - s_2) - (h_1 - h_2) + r(h_{b,r} - h_{a,r})$$

@ y_{max}	Point 2
r=0.11	101.3 bar

N_2	1	2	f	a	b	c
p (bar)	1.013	101.3	1.013	1.013	8.104	8.104
T (K)	300	300	77	247	314	305
h (J/g)	462	445	29	380	420	240
s (J/gK)	4.42	3.1	0.42	R134a		

$$-\frac{W_c}{\dot{m}}\bigg|_3 = 300(4.42-3.1)-(462-445)+0.11(420-380) = 384J/g$$

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Now, corresponding to this r value will find out what is the work per unit mass of Nitrogen compress and work per unit mass of Nitrogen which is going to be liquefied. So, putting that r value in this equation and getting corresponding to this is the value of h b minus h a. So, r is equal to 0.11 enthalpy values are given over here, putting this values in this equation, you get the value of W c upon m dot in this case again as you realize this additional term is going to decide what is the net value of W c upon m dot. For the third case, which is a y max case or r is equal to 0.11 in this case.

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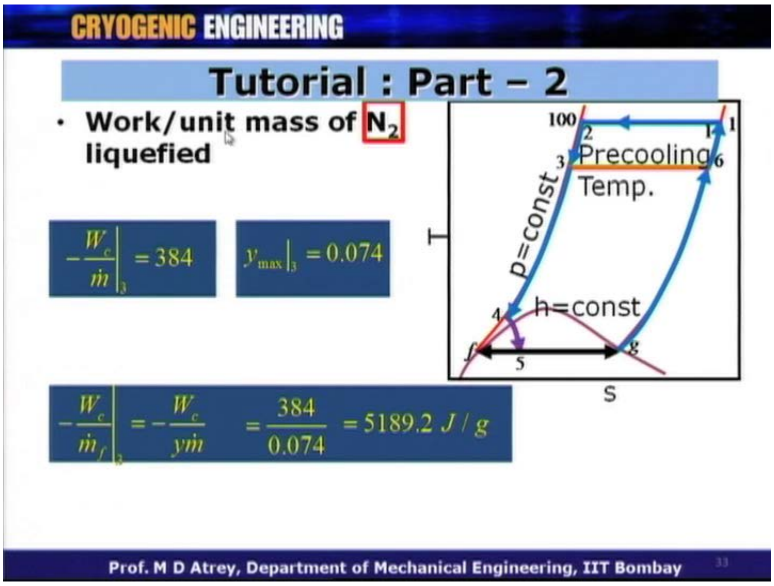
CRYOGENIC ENGINEERING

Tutorial : Part – 2

- **Work/unit mass of N$_2$ liquefied**

$-\frac{W_c}{\dot{m}}\Big|_3 = 384$

$y_{max}\big|_3 = 0.074$

$-\frac{W_c}{\dot{m}_f}\Big|_3 = -\frac{W_c}{y\dot{m}} = \frac{384}{0.074} = 5189.2\ J/g$

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Extending, it further work per unit mass of Nitrogen which is liquefied putting this value and corresponding y max value and dividing this y y max is what you get as 5189.2 joule per gram and this is amount of work done per unit mass of Nitrogen which is going to be liquefied.

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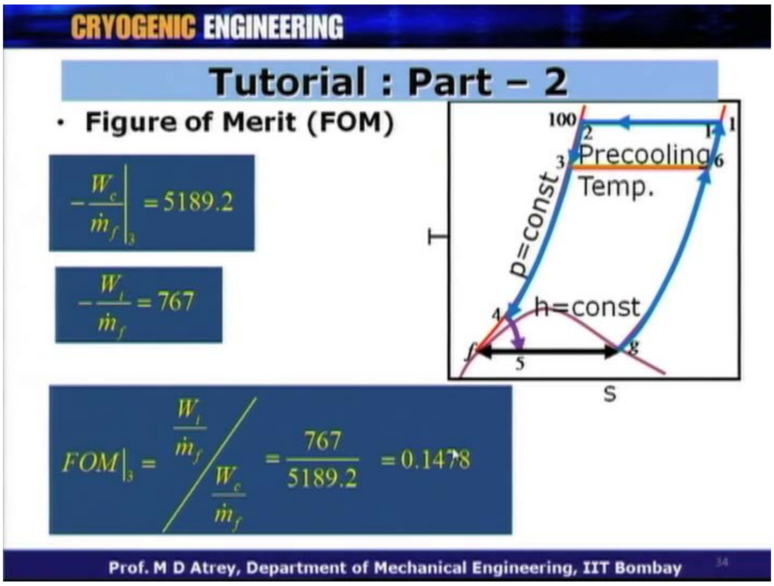
CRYOGENIC ENGINEERING

Tutorial : Part – 2

- **Figure of Merit (FOM)**

$-\frac{W_c}{\dot{m}_f}\Big|_3 = 5189.2$

$-\frac{W_i}{\dot{m}_f} = 767$

$FOM\big|_3 = \frac{W_i}{\dot{m}_f} \Big/ \frac{W_c}{\dot{m}_f} = \frac{767}{5189.2} = 0.1478$

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Also extending it further, to calculate the Figure of Merit what he require is a W c upon m dot f and what you require is a W I upon m dot f, 2 values this divided by this will give you FOM for the third case which is 0.1478 all right.

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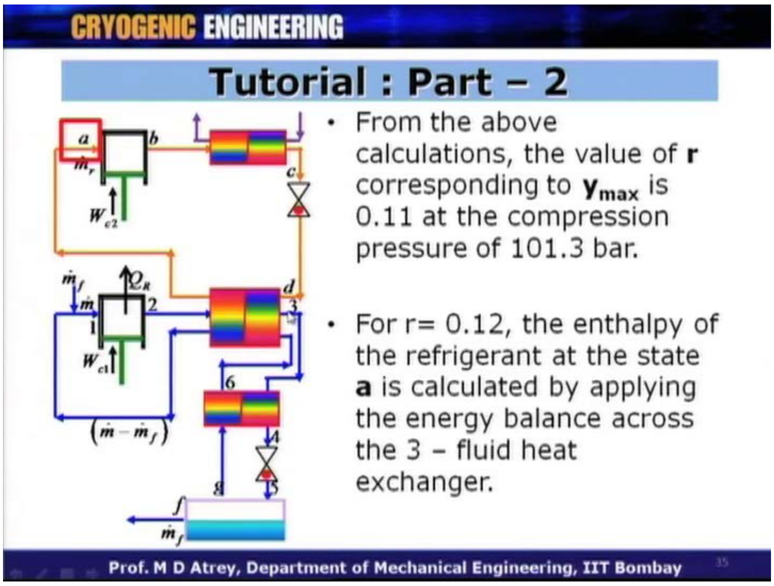

CRYOGENIC ENGINEERING

Tutorial : Part – 2

- From the above calculations, the value of **r** corresponding to $\mathbf{y}_{\max}$ is 0.11 at the compression pressure of 101.3 bar.
- For r= 0.12, the enthalpy of the refrigerant at the state **a** is calculated by applying the energy balance across the 3 – fluid heat exchanger.

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Now, this is the circuit what we have going to talking about from the above calculations, the values of r corresponding to y max is 0.11 at the compression pressure of 101.3 bar. This is what we have calculated? Now, what we want to do is? We want to go ahead and calculate the whole thing for a value which is higher than this r values; that means, will go above the value of 0.11 and let us take for r is equal to 0.12, the enthalpy of the refrigerant at the state a here is calculated by applying the energy balance across the 3-fluid heat exchanger.

Now, understand that we say that the lowest possible temperature that can be obtained at point 3 is equal to the boiling point of the refrigerant at d and which is what we exploited in the earlier case and we got y max condition.

Now, it also depends upon the cooling effect what you get here, is also depends upon r into enthalpy difference over here. Now, I am going to extend go beyond the r value which was which is obtained at 0.11 for a y max condition and now, I want to study what happens? If I extant this r value corresponding to this y max condition beyond 0.11 and let us take 0.12 and I would like to understand now, what happens to the refrigerant which is coming out of this 3-fluid heat exchanger all right.

Because as you understand, the temperature at this point T 3 is not going to change even if now, I change the value of r. Do you understand? The lowest temperature that is possible at point 3 is going to be the boiling point of the refrigerant which we already taken into account for r is equal to 0.11 which is what will determine y max value.

Now, if I further increase the value of r beyond 0.11, the temperature at point 3 is not going to change. However, something is going too happened to the state of the refrigerant when it comes out of this 3-fluid heat exchanger and this is what exactly we want to understand in this problem and therefore, for which we will do an energy balance across this 3-fluid heat exchanger.

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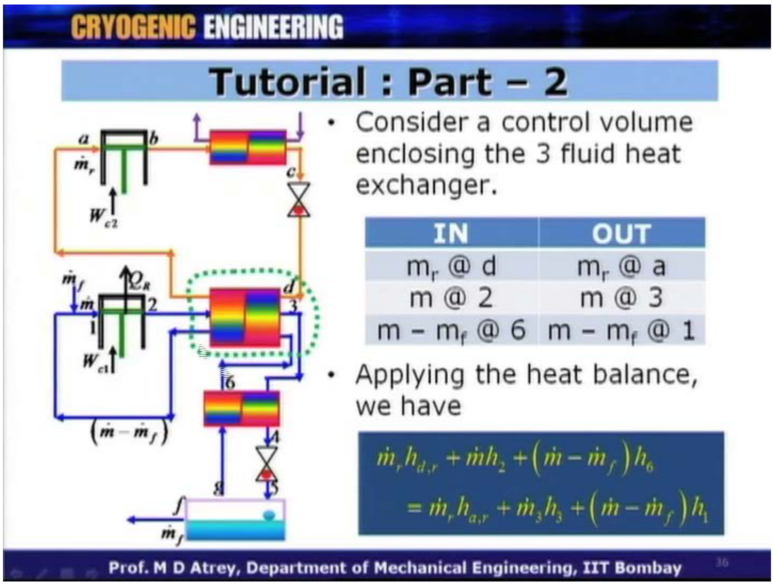

CRYOGENIC ENGINEERING

Tutorial : Part – 2

- Consider a control volume enclosing the 3 fluid heat exchanger.

IN	OUT
m_r @ d	m_r @ a
m @ 2	m @ 3
$m - m_f$ @ 6	$m - m_f$ @ 1

- Applying the heat balance, we have

$$\dot{m}_r h_{d,r} + \dot{m} h_2 + \left(\dot{m} - \dot{m}_f\right) h_6 = \dot{m}_r h_{a,r} + \dot{m}_3 h_3 + \left(\dot{m} - \dot{m}_f\right) h_1$$

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So, consider a control volume enclosing the 3-fluid heat exchange. What is entering the 3- fluid heat exchanger is m dot r at point d, m dot at point 2 and m dot minus m dot f at point 6 all right and what is leaving this heat exchanger is m dot r at point a here, then m dot at point 3 over here, an m dot minus m dot f at point 1 which is at this point. So, the energy balance is whatever is come inside is equal to whatever energy which is living this control volume and therefore, applying the heat balance over here we get m dot r into h d r which is at this point plus m dot h 2 which is at this point plus m dot minus m dot f into h 6 point at point is equal to m dot r into h dot a which is at this point which is entry to the refrigeration compressor and m dot r is the mass flow rate of the refrigerant plus m dot 3 into h 3 which is at this point all right.

Which is the enthalpy of the m dot 3 which is leaving at point 3 and m dot minus m dot f into h 1 which is at this point which is also a condition, which it enters the primary compressor.

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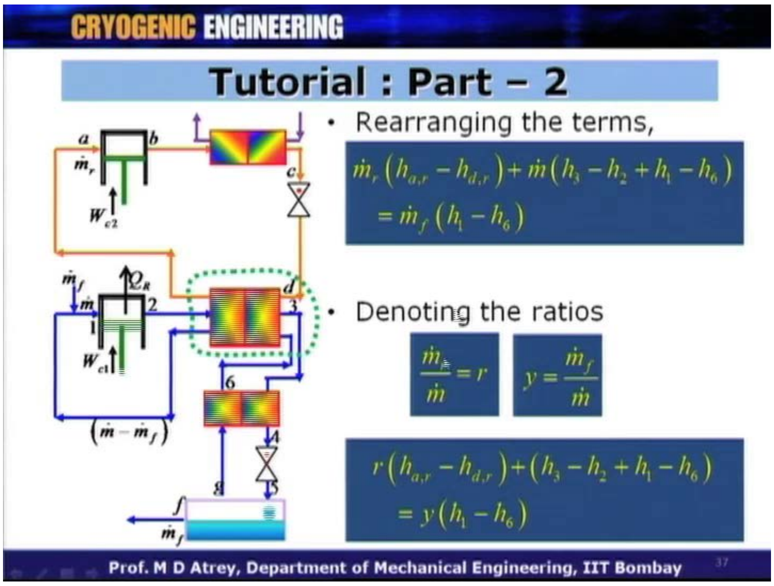

CRYOGENIC ENGINEERING

Tutorial : Part – 2

- Rearranging the terms,

$$\dot{m}_r\left(h_{a,r}-h_{d,r}\right)+\dot{m}\left(h_3-h_2+h_1-h_6\right) = \dot{m}_f\left(h_1-h_6\right)$$

- Denoting the ratios

$$\frac{\dot{m}_r}{\dot{m}}=r \qquad y=\frac{\dot{m}_f}{\dot{m}}$$

$$r\left(h_{a,r}-h_{d,r}\right)+\left(h_3-h_2+h_1-h_6\right) = y\left(h_1-h_6\right)$$

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Reorganize this in this term or rearrange in this terms, what you get is this and then divide the whole thing by m dot what you get m dot r upon m dot is equal to r and m dot f 1 m dot is equal to y and putting this values over here, you get this equation.

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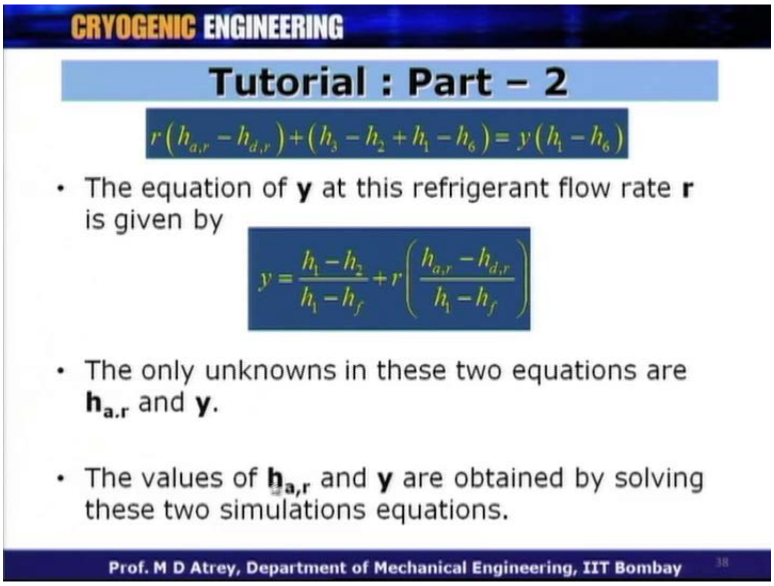

CRYOGENIC ENGINEERING

Tutorial : Part – 2

$$r\left(h_{a,r}-h_{d,r}\right)+\left(h_3-h_2+h_1-h_6\right)=y\left(h_1-h_6\right)$$

- The equation of **y** at this refrigerant flow rate **r** is given by

$$y=\frac{h_1-h_2}{h_1-h_f}+r\left(\frac{h_{a,r}-h_{d,r}}{h_1-h_f}\right)$$

- The only unknowns in these two equations are $\mathbf{h_{a.r}}$ and **y**.
- The values of $\mathbf{h_{a,r}}$ and **y** are obtained by solving these two simulations equations.

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Again, the rearranging this will give you this equation all right. So, 1 equation we are getting, because the energy balance across the 3-fluid heat exchanger. This equation, the equation of y at this refrigerant flow rate r is given by the second equation is going to come from this which is what a well known equation. Now, y is equal to h 1 minus h 2 upon h 1 minus h f, h 1 minus h f plus r times the this value and now, I get a relation between y and r from this equation and also I get relation from y and r from this equation, the only unknown in this 2 equations are enthalpy at point a and y.

So, here y is unknown h a or enthalpy at point a is unknown in this 2 cases and this is what exactly we are going to find out, because the value of enthalpy at point a will decide what is the phase of the… what is the state of the refrigerant at point a is? The value of h a r and y are obtained by solving this 2 simultaneous equations.

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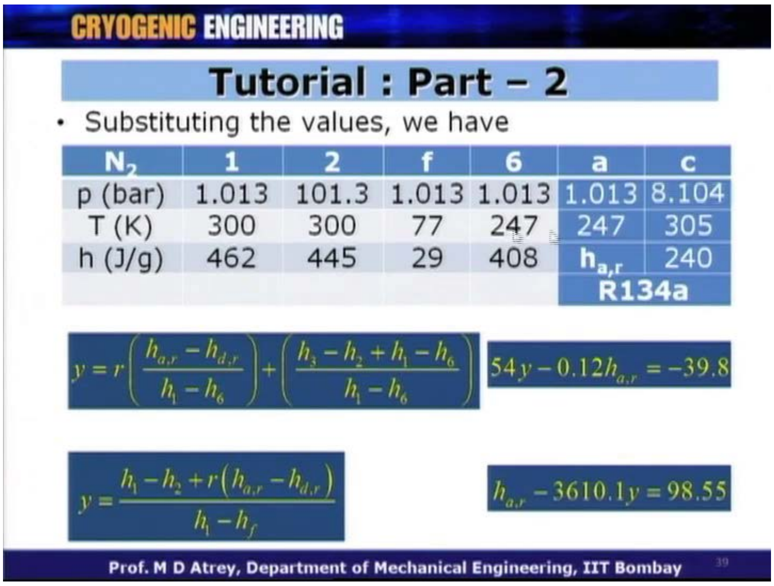

CRYOGENIC ENGINEERING

Tutorial : Part – 2

- Substituting the values, we have

N_2	1	2	f	6	a	c
p (bar)	1.013	101.3	1.013	1.013	1.013	8.104
T (K)	300	300	77	247	247	305
h (J/g)	462	445	29	408	$h_{a,r}$	240
					R134a	

$$y = r\left(\frac{h_{a,r} - h_{d,r}}{h_1 - h_6}\right) + \left(\frac{h_3 - h_2 + h_1 - h_6}{h_1 - h_6}\right)$$

$$54y - 0.12h_{a,r} = -39.8$$

$$y = \frac{h_1 - h_2 + r\left(h_{a,r} - h_{d,r}\right)}{h_1 - h_f}$$

$$h_{a,r} - 3610.1y = 98.55$$

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Substituting the values, corresponding to that put in the enthalpies at h a r and h 1 2 f and 6 points over here. This is the first equation y in terms of r and all the enthalpies over here, the equation what you get in terms of y and h a r are this and the second equation was this putting the values respective enthalpy values in this equation, the second equation what you get is this? Which is also having variables h a and y.

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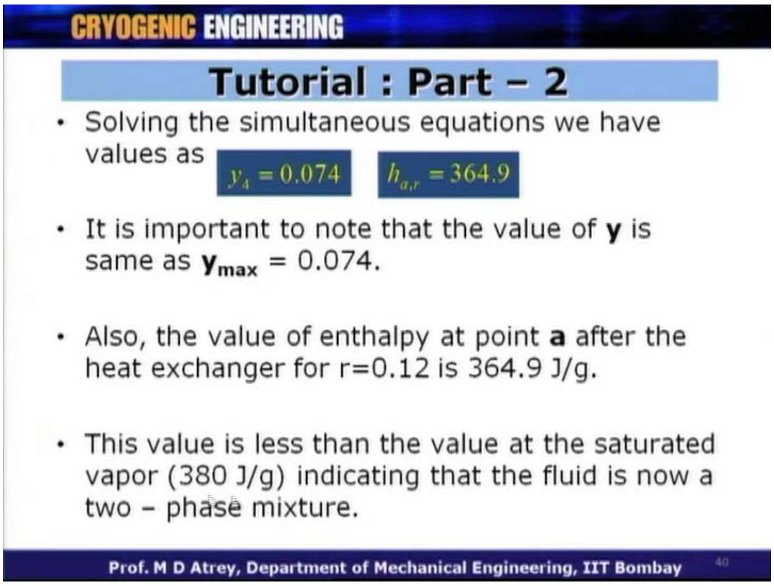

CRYOGENIC ENGINEERING

Tutorial : Part – 2

- Solving the simultaneous equations we have values as $y_4 = 0.074$ $h_{a,r} = 364.9$
- It is important to note that the value of **y** is same as $\mathbf{y}_{\mathbf{max}}$ = 0.074.
- Also, the value of enthalpy at point **a** after the heat exchanger for r=0.12 is 364.9 J/g.
- This value is less than the value at the saturated vapor (380 J/g) indicating that the fluid is now a two – phase mixture.

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Solving this simultaneous equations and is a well known technique to solve this simultaneous equation what you get is? y 4 is equal to 0.074 , this is the case when m dot r. Now, in this case is 0.12 which is a beyond 0.1 corresponding y max value and correspondent to the enthalpy or h a at point a, the enthalpy is 364.9. What you understand from this are fallowing?

It is important to note that the value of y you are getting at this point is same as y max value. The y max value what we had got earlier also was 0.074, that was obtained when m dot r was 0.11. In this particular case, where even if m dot r had been increased from 0.11 to 0.12, the value of y 4 correspondingly has remain the same; that means here, there is some confusion to withdrawn which is what we will do? Now, also the value of enthalpy at point a after the heat exchanger for r is equal to 0.12 is 364.9 which is over here.

This value is less than the value at the saturated vapor condition at (380 joule per gram) for R134a and corresponding pressure what does this indicate? These indicate that the fluid is now a two-phase mixture. The state of the fluid at point a has less enthalpy than the saturated vapor condition, it means that this particular point h a is not a saturated vapor or not in a super saturated condition also. It is definitely therefore, inside the (( )) which means that it is a two-phase mixture, which means that this is a liquid plus gas and which is a most undesirable condition for the compressor to operate in and this is what is coming out of this tutorial now.

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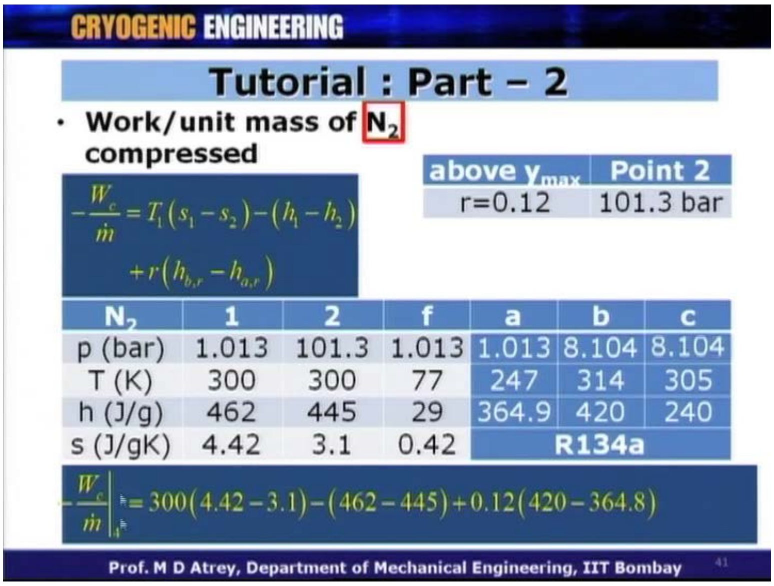

CRYOGENIC ENGINEERING

Tutorial : Part – 2

- Work/unit mass of N_2 compressed

$$-\frac{W_c}{\dot{m}} = T_1(s_1 - s_2) - (h_1 - h_2) + r(h_{b,r} - h_{a,r})$$

above y_{max}	Point 2
r=0.12	101.3 bar

N_2	1	2	f	a	b	c
p (bar)	1.013	101.3	1.013	1.013	8.104	8.104
T (K)	300	300	77	247	314	305
h (J/g)	462	445	29	364.9	420	240
s (J/gK)	4.42	3.1	0.42	R134a		

$$-\frac{W_c}{\dot{m}}\bigg|_4 = 300(4.42 - 3.1) - (462 - 445) + 0.12(420 - 364.8)$$

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Now, correspondent to this problem now, let us calculate for this r value what is work per unit mass of Nitrogen which is compress and again work per unit mass of Nitrogen which is going to be liquefied. So, going ahead applying the same formula taking the r value is equal to 0.12 for this case at the same pressure of compression putting this values over here, what you get is? Now, W c upon m dot for the fourth case of r is equal to 0.12 as 385.6 over here.

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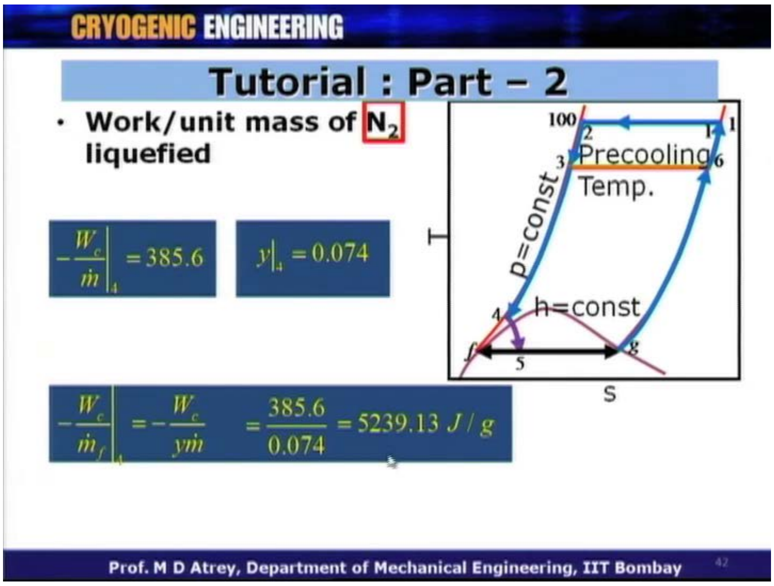
CRYOGENIC ENGINEERING

Tutorial : Part – 2

- Work/unit mass of N_2 liquefied

$-\frac{W_c}{\dot{m}}\Big|_4 = 385.6$

$y|_4 = 0.074$

$-\frac{W_c}{\dot{m}_f}\Big|_4 = -\frac{W_c}{y\dot{m}} = \frac{385.6}{0.074} = 5239.13\ J/g$

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And for the calculating, the work per unit mass of Nitrogen which is liquefied again using the formula over here, which is the repetition of the whole thing and if you divide this by y value what you get is a work per unit of mass nitrogen which is liquefied.

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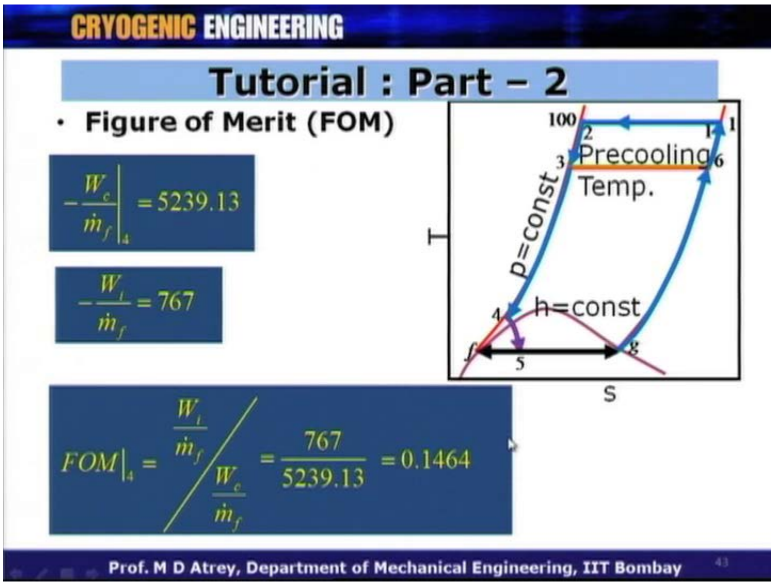
CRYOGENIC ENGINEERING

Tutorial : Part – 2

- Figure of Merit (FOM)

$-\frac{W_c}{\dot{m}_f}\Big|_4 = 5239.13$

$-\frac{W_i}{\dot{m}_f} = 767$

$FOM|_4 = \frac{W_i}{\dot{m}_f} \Big/ \frac{W_c}{\dot{m}_f} = \frac{767}{5239.13} = 0.1464$

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Extending, it ahead more repetitive calculations find a Figure of Merit what you get is this? The Figure of Merit for this forth case is 0.1464 all right. So, now, we have solved 4 cases for different r conditions of 0.05, 0.07, 0.11 and 0.12 and for each value we have calculated different performance parameters and now, we should understand what exactly happened to the performance parameter.

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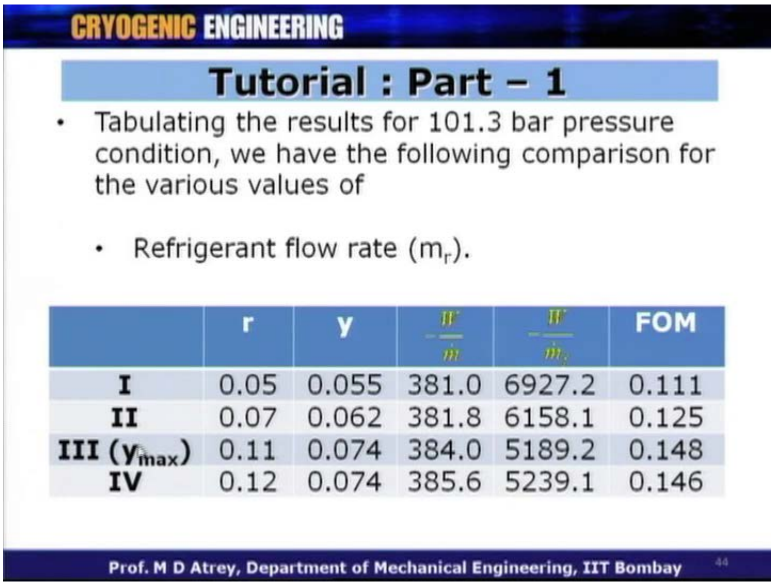

CRYOGENIC ENGINEERING

Tutorial : Part – 1

- Tabulating the results for 101.3 bar pressure condition, we have the following comparison for the various values of
  - Refrigerant flow rate (m_r).

	r	y	$-\frac{\dot{W}}{\dot{m}}$	$-\frac{\dot{W}}{\dot{m}_f}$	FOM
I	0.05	0.055	381.0	6927.2	0.111
II	0.07	0.062	381.8	6158.1	0.125
III (y_{max})	0.11	0.074	384.0	5189.2	0.148
IV	0.12	0.074	385.6	5239.1	0.146

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Tabulating the results for 101.3 bar pressure condition, we have the following comparison for the different values of r. Refrigerant flow rate m dot r now, this is the most significant table and lot of conclusion can be drawn from this. So, what do you understand from this?

We have got first case, second case, third case and the forth case correspondent that we have got different r values. Which is for the first case we had a 0.05, then 0.07. The third case happens to be for a y max condition correspondent to this y max condition, we had calculated the r values which was 0.11 and extending this problem further, we had gone to understand what happens to the y values or different parameters when we cope beyond the r value correspondent to y max which is 0.11 and therefore, we had taken the next r value which is 0.12.

These are 4 different cases and from here what we understand is? The y value increased when we went from 0.05 to 0.07 and to 0.11. However, the y value remain the same when we went beyond the y max value; that means, the maximum value remain the same even if we increase the value of r beyond 0.11 correspondent to the y max value.

What we understand from here is? There is no point in going beyond this r values or whatever be the r value here afterwards, the y max value remain the same or the y value remains equal to the y max value, we will see this graphically. Similarly, what you see here? What happens to work of compressor per unit mass of gas which is compress that is W upon m dot, we see that is a slight increase over here and this increase continuous. Similarly, W upon m dot f or the work of compression per unit mass of gas which is the liquefied and we find that this is decreasing. This is decreasing up to the y max value and then this shows a little bit increase over here. Similarly, the Figure of Merit also increases up to the y max value and then it shows little decrease over here all right.

So, we understand from this table, we have computed all the values which we have calculated till now, although it is sounded repetitive calculations, but in order to understand, the parametric variation and it is effect on this different values, it is very important to calculate this values. And now, we will this let us see this variation or this table in a graphical format which will help us to understand, how the curve looks like?

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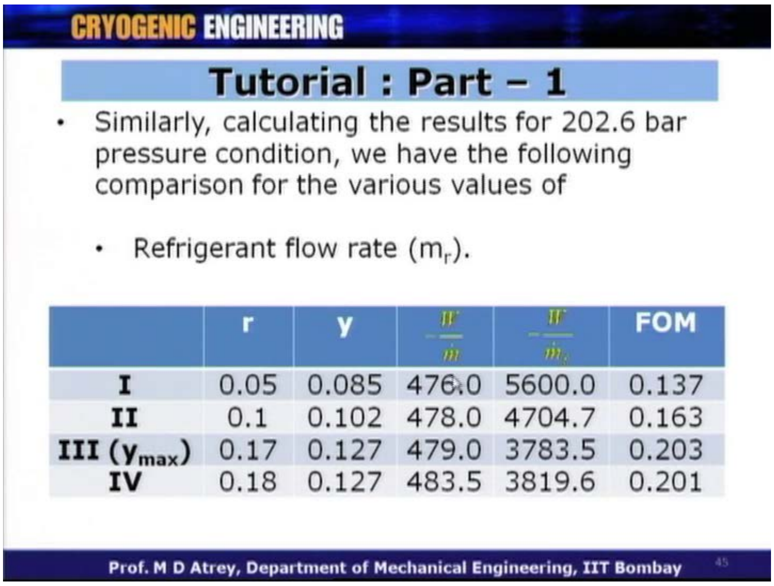

CRYOGENIC ENGINEERING

Tutorial : Part – 1

- Similarly, calculating the results for 202.6 bar pressure condition, we have the following comparison for the various values of
  - Refrigerant flow rate (m_r).

	r	y	$-\frac{W}{\dot{m}}$	$-\frac{W}{\dot{m}_f}$	FOM
I	0.05	0.085	476.0	5600.0	0.137
II	0.1	0.102	478.0	4704.7	0.163
III (y_{max})	0.17	0.127	479.0	3783.5	0.203
IV	0.18	0.127	483.5	3819.6	0.201

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Similarly, calculating the results for 202.6 bar pressure conditions, we have the fallowing comparison. So, we repeated the same calculations which we did earlier for 1.63 bar. Now, we have got a 202.6 or 200 atmosphere as a pressure condition for the for Nitrogen working fluid and we see the variations for different m dot r values, we have not gone into the details of this calculations, but this table shows the variation. We have done these calculations and we are just showing a table, because we do not want to show you the same repetitive calculations again.

Here, also we understand that we varied the r value from 0.05 to 0.1, we have calculated the corresponding r value for a y max condition which we got as 0.17 and then we got a one more condition beyond the r value for corresponding y max value and we took a value of r as 0.18, corresponding to that we have got different y values over here again and we found that the y value also increased over here and remain constant beyond y max.

Similarly, we have understood we have try to compute the W by m dot and W by m dot f over here and also the Figure of Merit again it shows a similar kind of (( )) what we saw earlier all right. So, let us understand this now in the subsequent slides.

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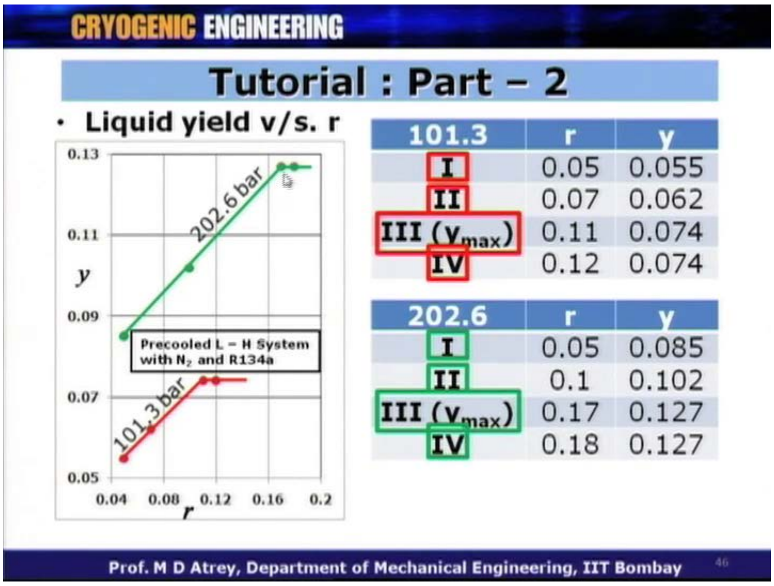

CRYOGENIC ENGINEERING

Tutorial : Part – 2

- Liquid yield v/s. r

101.3	r	y
I	0.05	0.055
II	0.07	0.062
III (y_{max})	0.11	0.074
IV	0.12	0.074

202.6	r	y
I	0.05	0.085
II	0.1	0.102
III (y_{max})	0.17	0.127
IV	0.18	0.127

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Now, if I want to see graphically the effects of variation in r on the y values and let us see for the first case when the pressure is 101.3 and these are the corresponding values. You got a first case over, you got a second case over here, the first case is r is equal to 0.05, the second case is r is equal to 0.07 correspondent those values we have got a y values and then we got a y max value which is 0.11 which is for r is equal to 0.11 and corresponding y values of 0.74, which is coming at this point and the next value is for r is equal to 0.12 and if we join this points what you say is a curve like this?

So, what you see from this is for 100 and 1.3 bar case when you increase the r values, it goes through a y max condition and whatever is a further r value you take the y value remains constant at y is equal to y max. Let us see now, variations for the second pressure which is 202.6, first case, the second case and the third case over here and we compute the 4 case and let us join the point and what you find again is that it goes to a y max value here and then it remain constant, when we took a value of r beyond the y max condition all right.

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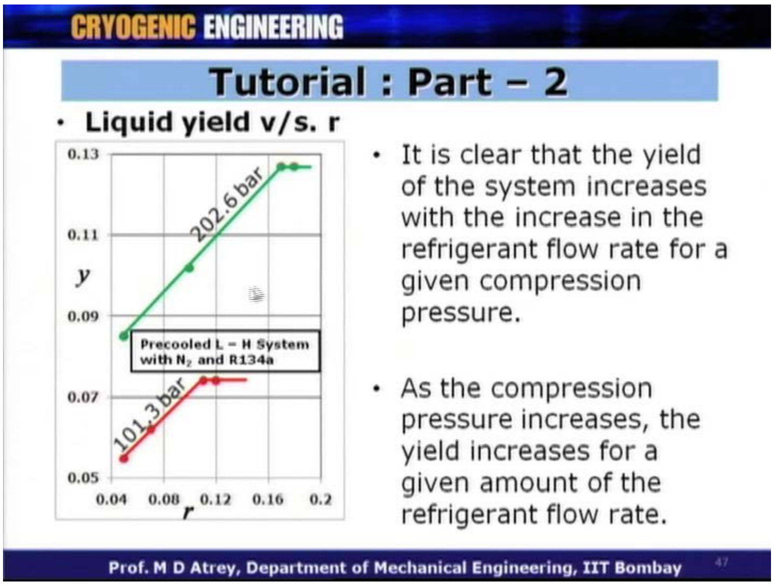

CRYOGENIC ENGINEERING

Tutorial : Part – 2

- **Liquid yield v/s. r**

- It is clear that the yield of the system increases with the increase in the refrigerant flow rate for a given compression pressure.
- As the compression pressure increases, the yield increases for a given amount of the refrigerant flow rate.

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So, different conclusion that could be understood from this r, it is clear the yield of the system increases with the increase in the refrigerant flow rate for a given compression pressure. As you can see from both this condition, as you increase the value of r there is the increase in the yield value. For 1 pressure what you find is that the yield increases with the increase in the value of r also, what you can understand from here is? As the compression pressure increases, in one case we have got 101.3 bar, the second case we have got 202.6 bar. So, as this pressure increases, the yield increases for a given amount the refrigerant.

So, whatever at let us say r is equal to 0.08, you got a particular yield and as soon as you go for a second value of the pressure or increase value of the pressure for the same r value, we got a higher yield; that means, if you increase the compressor pressure for Nitrogen, we got a higher yield even though the value of r remain the same.

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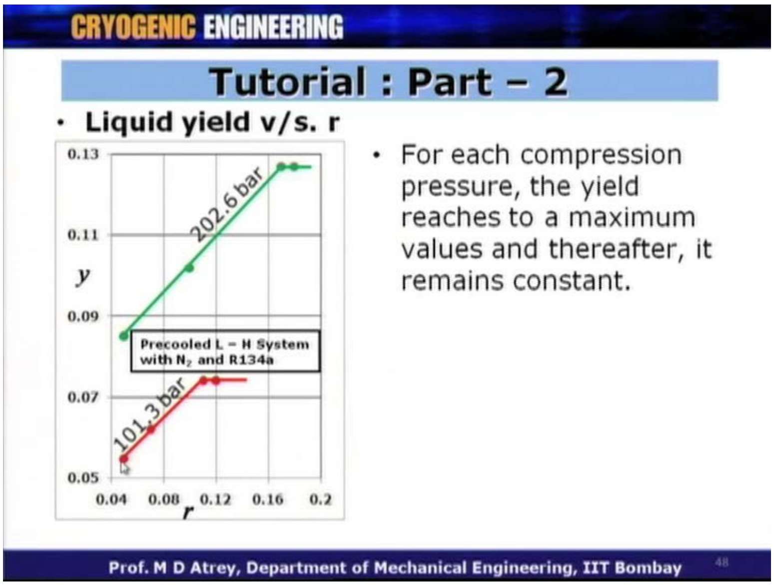

Tutorial : Part – 2

- **Liquid yield v/s. r**

- For each compression pressure, the yield reaches to a maximum values and thereafter, it remains constant.

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For each compression pressure, the yield reaches to a maximum values and thereafter, it remains constant. So, the yield went to a particular value over here, which is what y max is for a given pressure and then it remained the same over here. Similarly, for this pressure also it went up to the maximum and it remain constant throughout and these are the y max conditions and corresponding to this y max, we have got r conditions.

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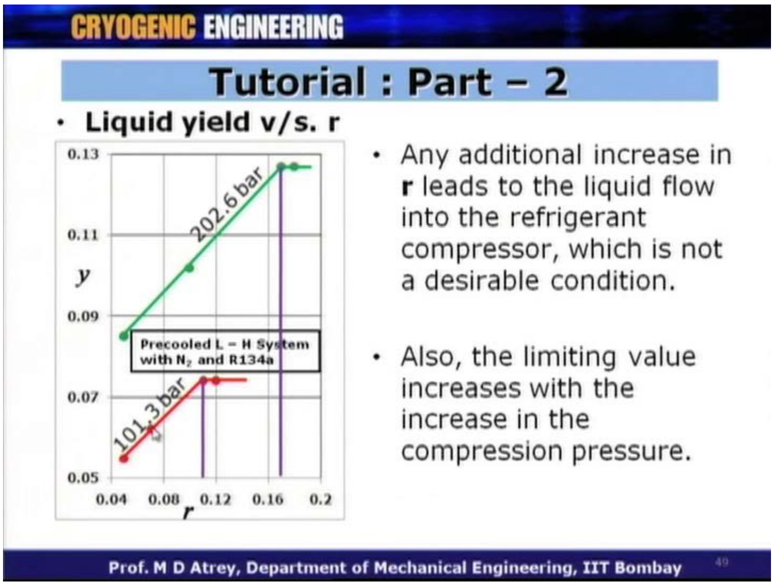

Tutorial : Part – 2

- **Liquid yield v/s. r**

- Any additional increase in **r** leads to the liquid flow into the refrigerant compressor, which is not a desirable condition.
- Also, the limiting value increases with the increase in the compression pressure.

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Any additional increase in the r thereafter, leads to the Liquid flow into the refrigerant compressor which is not a desirable condition, we have seen earlier that if we go beyond this value of r. The y max remain the same, but the state of the refrigerant when it enters the compressor in the precooling circuit is not a gaseous phase, it is a Liquid plus gas and it is not a desirable condition. So, we do not want to Liquid to enter the refrigerant compressor and therefore, this is an undesirable condition. In fact, we should never go beyond this limiting value of r. Also, the limiting value increases with the increase in the compression pressure.

So, we understood that when we went for 202.6 bar, the limiting value of r was 0.17 in this case, it was 0.11. If we increase the pressure of the working fluid, the limiting r value has increased from 0.11 to 0.17 when we went from 101.3 bar to 202.6 bar.

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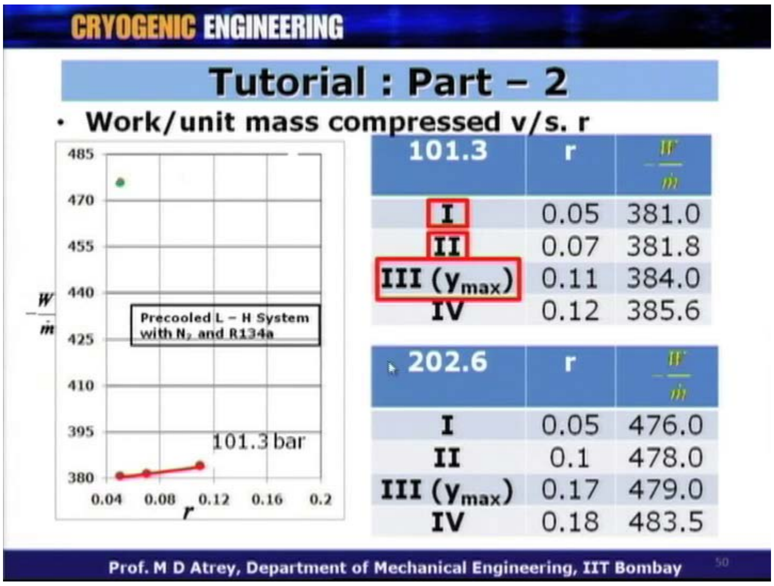

CRYOGENIC ENGINEERING

Tutorial : Part – 2

- Work/unit mass compressed v/s. r

101.3	r	$\frac{W}{\dot{m}}$
I	0.05	381.0
II	0.07	381.8
III (y_{max})	0.11	384.0
IV	0.12	385.6

202.6	r	$\frac{W}{\dot{m}}$
I	0.05	476.0
II	0.1	478.0
III (y_{max})	0.17	479.0
IV	0.18	483.5

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Now, I would like to compute work per unit mass of gas compress verses r these are the conditions. So, you can see that 3 cases are over here and we join this and this seem to varying linearly over here.

So, as you increase the r the this is a slight increase in the work which is understandable, because the second bracket as you know it depends on the value of r, which is what is getting figured over here; that means, the hardly any increase not much increase and now, for the second condition also we have got a similar case, if you increase the value you got an increase there.

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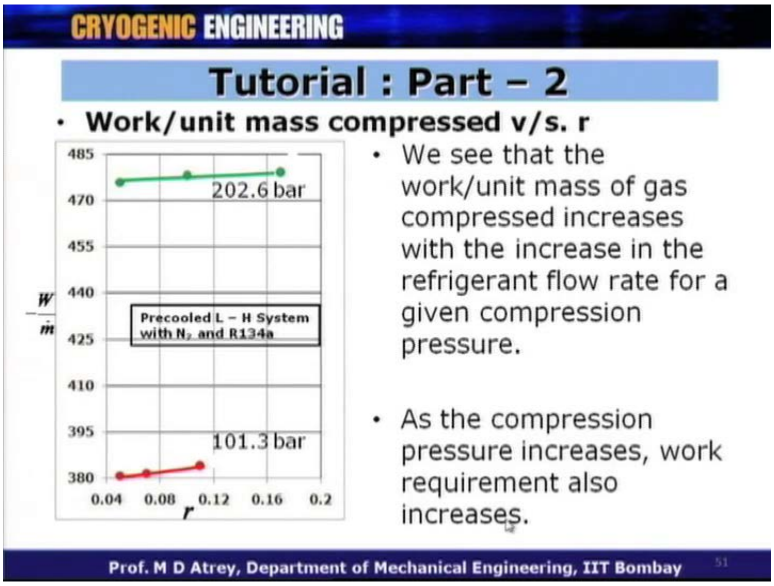

Tutorial : Part – 2

- Work/unit mass compressed v/s. r

- We see that the work/unit mass of gas compressed increases with the increase in the refrigerant flow rate for a given compression pressure.

- As the compression pressure increases, work requirement also increases.

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We see that the work per unit of gas compressed increases with the increase in the refrigerant flow rate for a given compression pressure. As the compression pressure increases, work requirement also increases; it is clear from formula itself.

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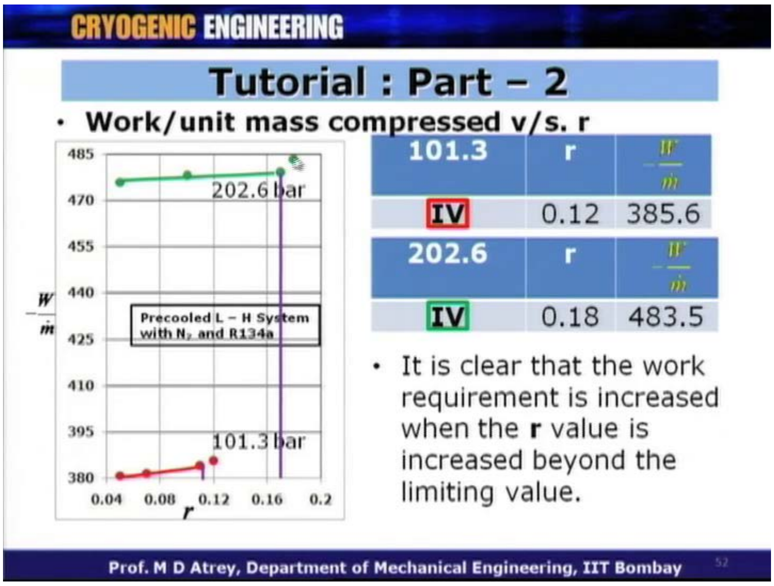

Tutorial : Part – 2

- Work/unit mass compressed v/s. r

101.3	r	$\frac{W}{\dot{m}}$
IV	0.12	385.6

202.6	r	$\frac{W}{\dot{m}}$
IV	0.18	483.5

- It is clear that the work requirement is increased when the **r** value is increased beyond the limiting value.

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If we get the fourth point, we find that the work increases there for this case as well as this case. However, as you know that this point should be avoided. It is clear that the work requirement is increased when the r value is increased beyond the limiting value, but this is absolutely undesirable points and it should not be this is just to show the quantitative value over here, this should be never brought into practice.

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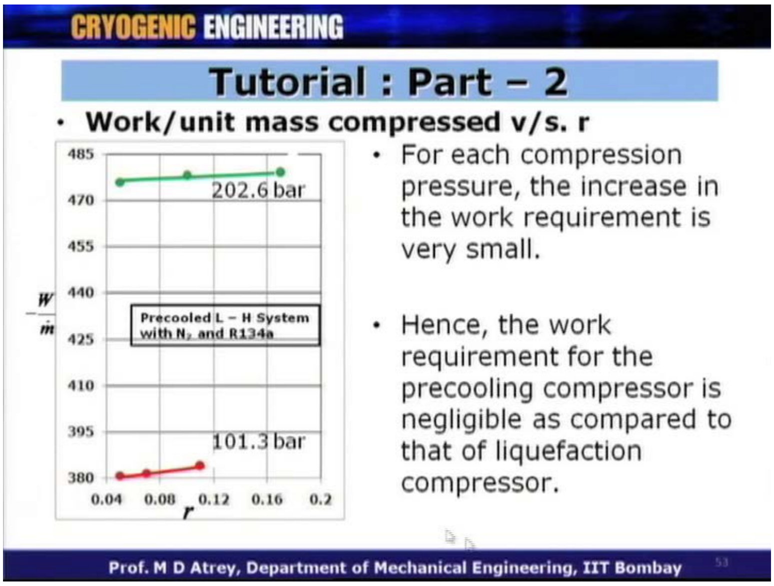

CRYOGENIC ENGINEERING

Tutorial : Part – 2

- Work/unit mass compressed v/s. r

- For each compression pressure, the increase in the work requirement is very small.
- Hence, the work requirement for the precooling compressor is negligible as compared to that of liquefaction compressor.

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For each compression pressure, the increase in the work requirement is very small. Hence, the work requirement for the precooling compressor is negligible as compared to that of liquefaction compressor. The work requirement from the primary compressor is remain the same, because the pressures have been kept the same while, they whatever increases happened here is, because of the precooling circuit compressor or a refrigerating compressor and we find that is the hardly increase of 1 or 2 units over here, which we can neglect also.

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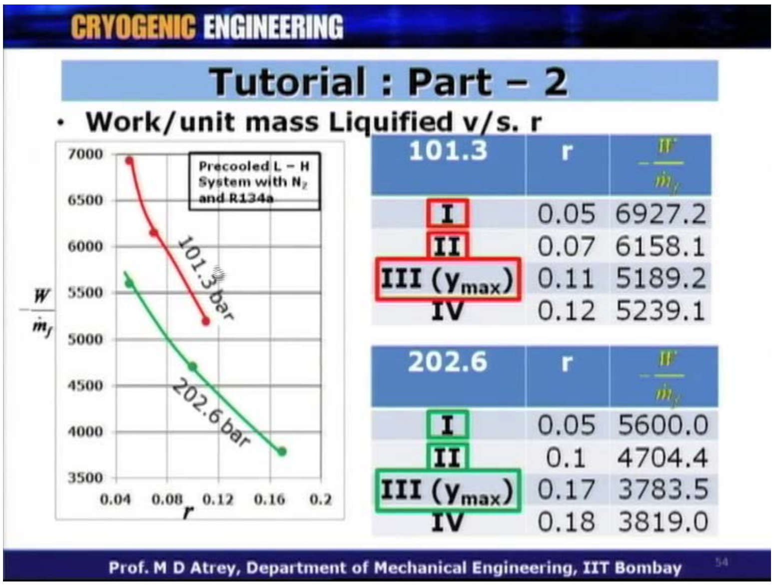

CRYOGENIC ENGINEERING

Tutorial : Part – 2

- Work/unit mass Liquified v/s. r

101.3	r	$\frac{W}{\dot{m}_f}$
I	0.05	6927.2
II	0.07	6158.1
III (y_{max})	0.11	5189.2
IV	0.12	5239.1

202.6	r	$\frac{W}{\dot{m}_f}$
I	0.05	5600.0
II	0.1	4704.4
III (y_{max})	0.17	3783.5
IV	0.18	3819.0

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Now, if you want to complete W upon m dot f, we can find that, as for the 1, 2, 3 case as you go on increase in the value of r. The W upon m dot f comes down and this is basically why we are doing the whole exercise? We want to basically decrease the work of compression per unit mass of gas which is liquefied and the whole exercise is carried out, because of this all right.

So, for 101 bar, this is what (( )) and for the second case, these are 3 points and this is the W upon m dot at has further decrease when we went from 101.3 bar to 202 bar and this is the (( )) of the whole precooling circuit. Precooling is done, because W by m dot f as decreased why it has decreased, because y has increase in this cases.

So, as r increase, the y increase and therefore, W by m dot f has decrease and therefore, work requirement per unit mass of gas which is liquefied decreases.

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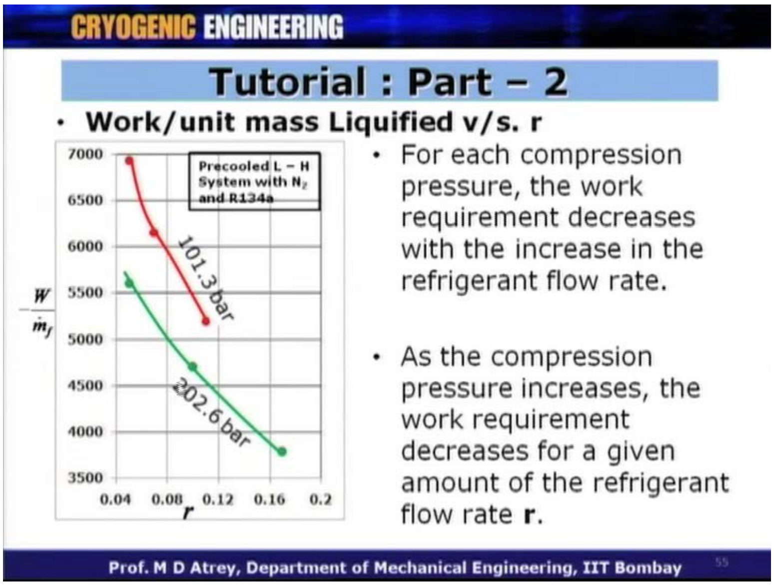

CRYOGENIC ENGINEERING

Tutorial : Part – 2

- **Work/unit mass Liquified v/s. r**

- For each compression pressure, the work requirement decreases with the increase in the refrigerant flow rate.
- As the compression pressure increases, the work requirement decreases for a given amount of the refrigerant flow rate **r**.

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So, for each compression pressure, the work requirement decreases with the increase in the refrigerant flow rate, which is what we understood? As the compression pressure increases, the work requirement decreases for a given amount of the refrigerant. So, if you go from 101 to 202 the work requirement further decreases and therefore, it is better to go for higher and higher pressure and more and more r respectively.

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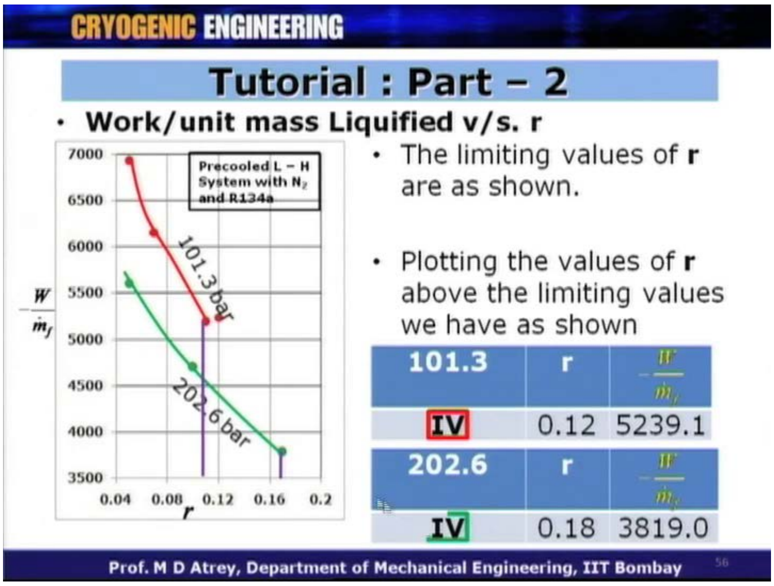

CRYOGENIC ENGINEERING

Tutorial : Part – 2

- **Work/unit mass Liquified v/s. r**

- The limiting values of **r** are as shown.
- Plotting the values of **r** above the limiting values we have as shown

101.3	r	$\frac{W}{\dot{m}_f}$
IV	0.12	5239.1

202.6	r	$\frac{W}{\dot{m}_f}$
IV	0.18	3819.0

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The limiting values of r are shown over here. This is the limiting value; we should not operate beyond these values, plotting the values of r above the limiting values of r as shown all right. The forth case, if you want to see which is undesirable case? The points are shown over here, which should not be actually brought practice.

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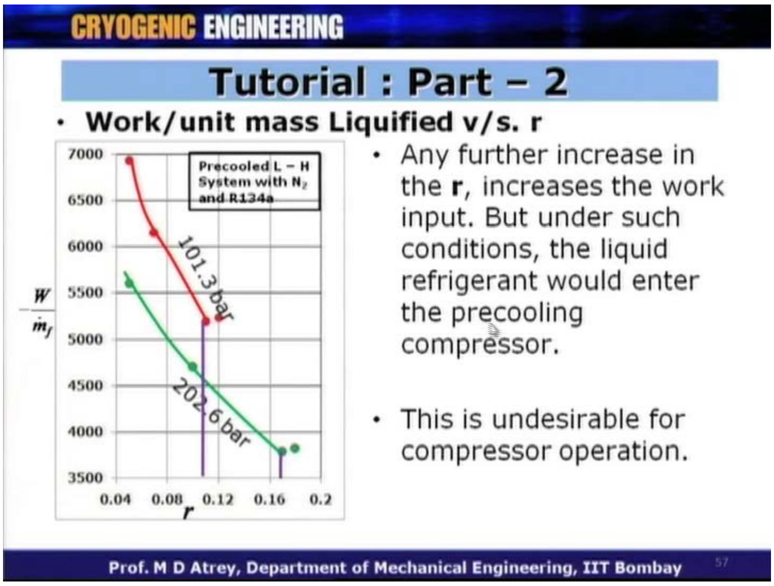

CRYOGENIC ENGINEERING

Tutorial : Part – 2

- **Work/unit mass Liquified v/s. r**

- Any further increase in the **r**, increases the work input. But under such conditions, the liquid refrigerant would enter the precooling compressor.
- This is undesirable for compressor operation.

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Any further increase in the value of r, increases the work input. But under such conditions, the liquid refrigerant would get enter the precooling compressor and therefore, it is not a desirable condition.

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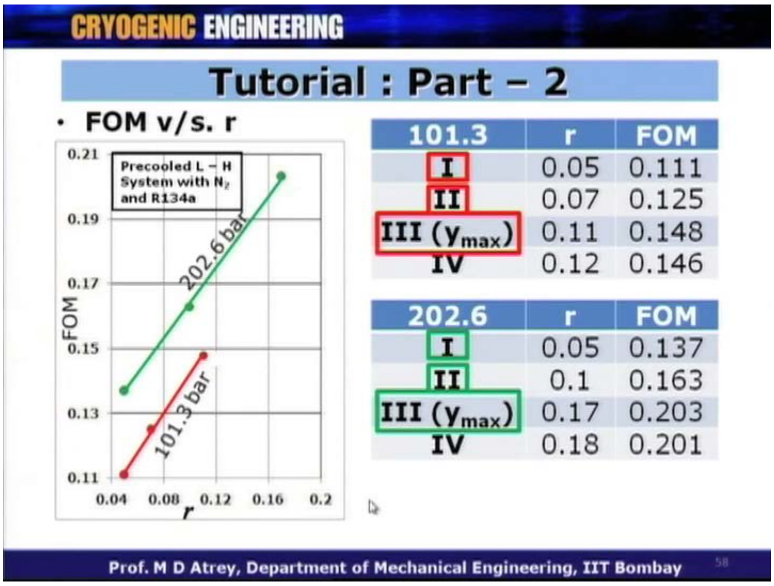

CRYOGENIC ENGINEERING

Tutorial : Part – 2

- **FOM v/s. r**

101.3	r	FOM
I	0.05	0.111
II	0.07	0.125
III (y_{max})	0.11	0.148
IV	0.12	0.146

202.6	r	FOM
I	0.05	0.137
II	0.1	0.163
III (y_{max})	0.17	0.203
IV	0.18	0.201

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And if you plot the Figure of Merit, we find that the figure merit increases for 101.3 bar pressure and again, it show the similar trained for the second case also like this all right.

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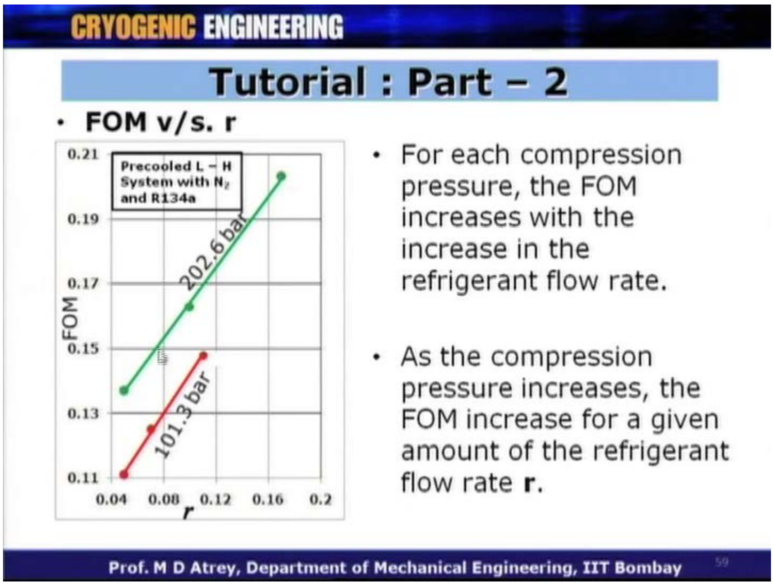

CRYOGENIC ENGINEERING

Tutorial : Part – 2

- **FOM v/s. r**

- For each compression pressure, the FOM increases with the increase in the refrigerant flow rate.
- As the compression pressure increases, the FOM increase for a given amount of the refrigerant flow rate **r**.

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So, for each compression pressure, the Figure of Merit increases with the increase in the refrigerant flow rate. As the compression pressure increases from this to this case, the Figure of Merit increase for a given amount of refrigerant flow rate for any r therefore, as you increase the pressure FOM increases which is what we want basically?

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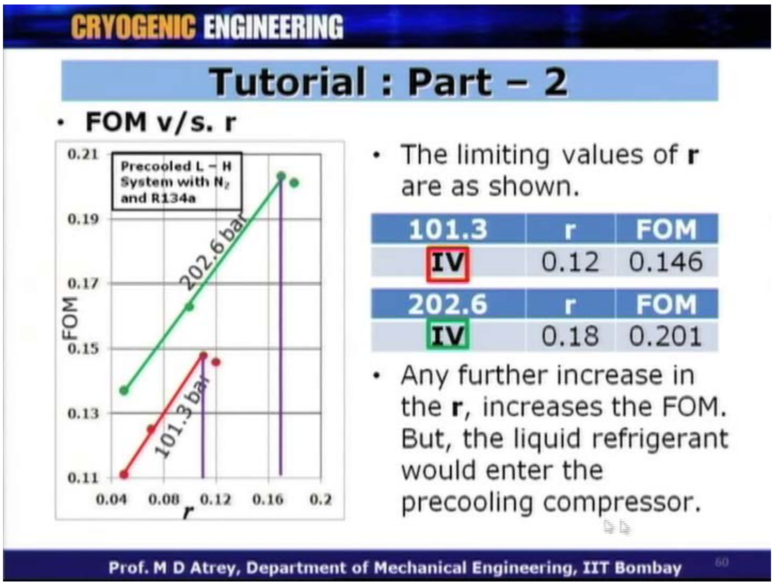

CRYOGENIC ENGINEERING

Tutorial : Part – 2

- **FOM v/s. r**

- The limiting values of **r** are as shown.

101.3	r	FOM
IV	0.12	0.146

202.6	r	FOM
IV	0.18	0.201

- Any further increase in the **r**, increases the FOM. But, the liquid refrigerant would enter the precooling compressor.

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Again there is a limiting value of r as shown in the figure. If we plot those values define FOM has the actually show a decreased over here. Any further increase in the r value increases the FOM. But the Liquid refrigerant would enter the precooling compressor. This is not a desirable condition and therefore, this should not be brought in practice.

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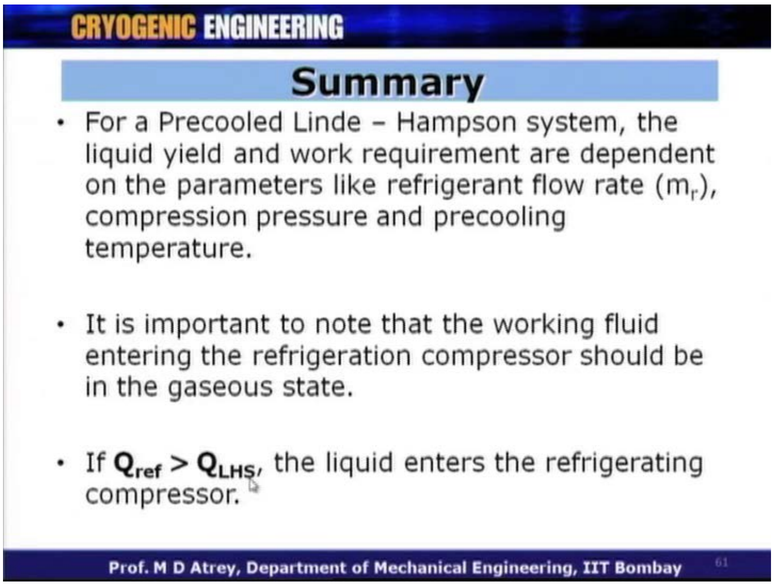

CRYOGENIC ENGINEERING

Summary

- For a Precooled Linde – Hampson system, the liquid yield and work requirement are dependent on the parameters like refrigerant flow rate (m_r), compression pressure and precooling temperature.
- It is important to note that the working fluid entering the refrigeration compressor should be in the gaseous state.
- If $\mathbf{Q_{ref} > Q_{LHS}}$, the liquid enters the refrigerating compressor.

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With this work let us summarize whatever we are learned till now, for a Precooled Linde - Hampson system, the liquid yield and work requirement are dependent on the parameters like m dot r compression pressure and precooling temperature. It is important to note that the working fluid entering the refrigeration compressor should be in the gaseous state. If Q ref is more than Q LHS, the liquid enters the refrigerant compressor. This is what we have talked earlier about those 3 cases.

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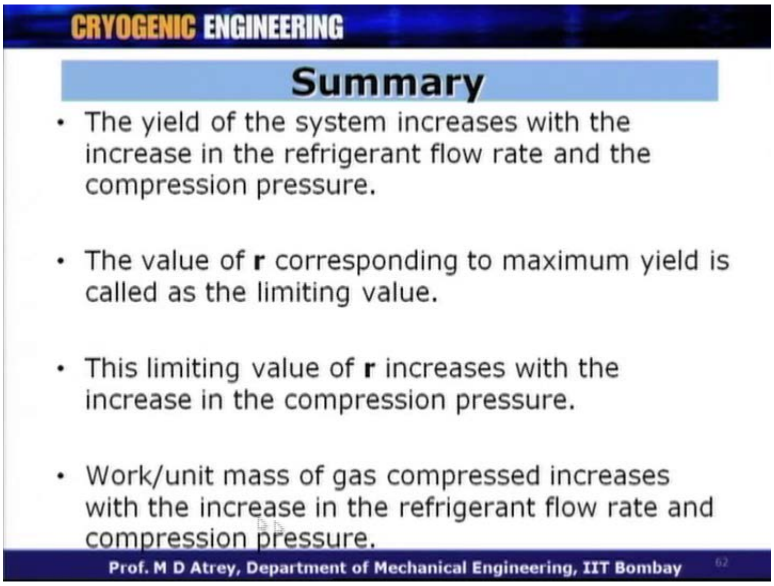

CRYOGENIC ENGINEERING

Summary

- The yield of the system increases with the increase in the refrigerant flow rate and the compression pressure.
- The value of **r** corresponding to maximum yield is called as the limiting value.
- This limiting value of **r** increases with the increase in the compression pressure.
- Work/unit mass of gas compressed increases with the increase in the refrigerant flow rate and compression pressure.

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The yield of the system increases with the increase in the refrigerant flow rate and the compression pressure. The value of r corresponding to the maximum yield is called the limiting value and we should never exceed this limiting value of r. This limiting value of r increases with the increase in the compression. So, if you go from 100 atmosphere to 200 atmosphere, the limiting value of r increases. Work per unit mass of gas compressed increases with the increase in the refrigerant flow rate and the compression pressure.

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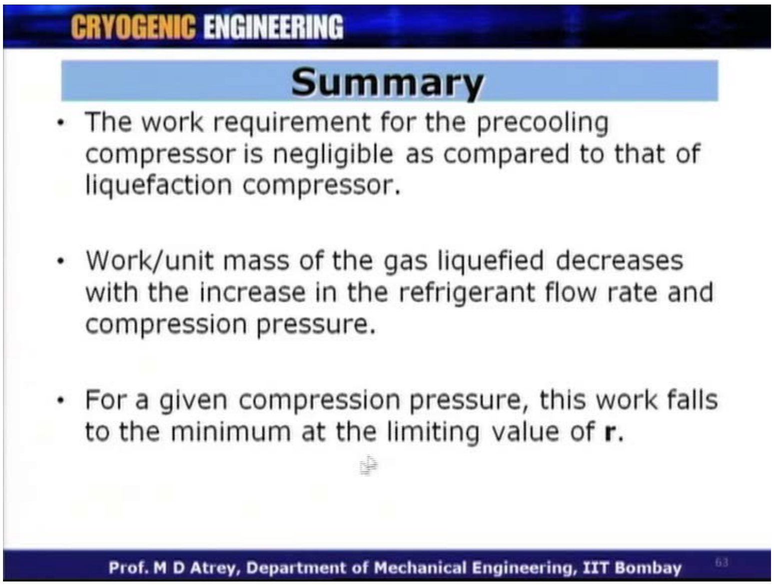

CRYOGENIC ENGINEERING

Summary

- The work requirement for the precooling compressor is negligible as compared to that of liquefaction compressor.
- Work/unit mass of the gas liquefied decreases with the increase in the refrigerant flow rate and compression pressure.
- For a given compression pressure, this work falls to the minimum at the limiting value of **r**.

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The work requirement for the precooling compressor is negligible as compare to that of the liquefaction compressor. The work per unit mass of the gas liquefied decreases, this is why we do the whole exercise with the increase in the refrigerant flow rate and compression pressure. For a given compression pressure of 100 bar or 200 bar, this work falls to a minimum value at the limiting value of r. So, work per unit of mass of gas liquefied falls to a minimum value for the limiting value of r all right.

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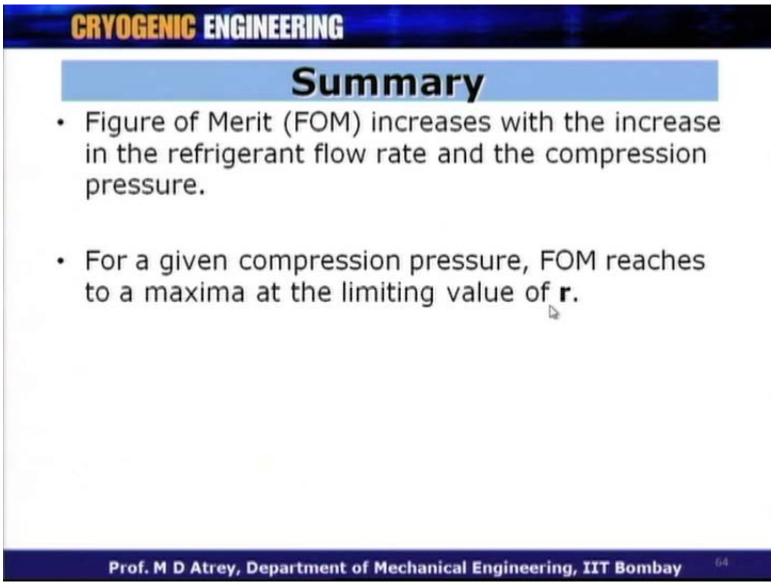

CRYOGENIC ENGINEERING

Summary

- Figure of Merit (FOM) increases with the increase in the refrigerant flow rate and the compression pressure.
- For a given compression pressure, FOM reaches to a maxima at the limiting value of **r**.

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The Figure of Merit increases with the increase in the refrigerant flow rate and the compression pressure for a given a compression pressure. For a given compression pressure, FOM reaches to a maxima at the limiting value of r. These are the conclusions drawn, I am sure you must you must have appreciated the effect of different parameters on the y value on the FOM value etcetera and also understand the limiting r value concept.

Thank you.