Cryogenic Engineering

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Lecture No. # 11
Gas Liquefaction and Refrigeration Systems

So, welcome to the eleventh lecture of cryogenic engineering under the NPTEL program. Just to take over view of the earlier lecture.

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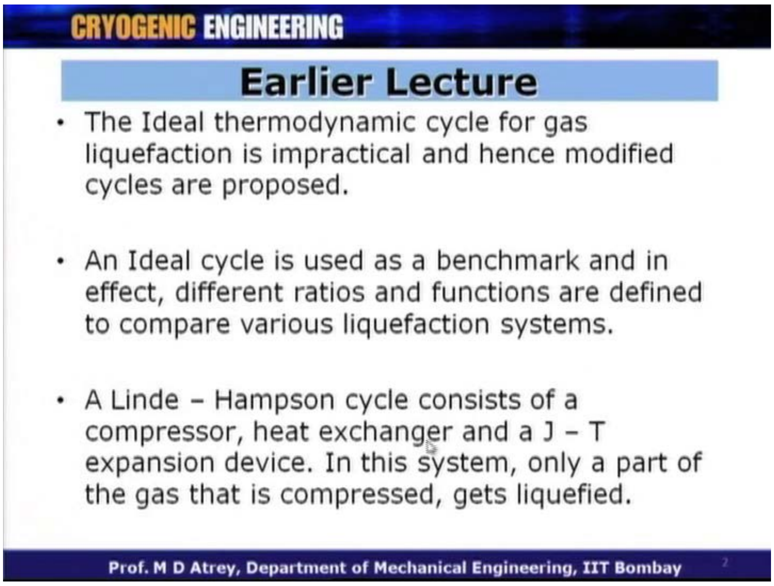

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Earlier Lecture

- The Ideal thermodynamic cycle for gas liquefaction is impractical and hence modified cycles are proposed.
- An Ideal cycle is used as a benchmark and in effect, different ratios and functions are defined to compare various liquefaction systems.
- A Linde – Hampson cycle consists of a compressor, heat exchanger and a J – T expansion device. In this system, only a part of the gas that is compressed, gets liquefied.

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We talked about ideal thermodynamic cycle for gas liquefaction in the earlier lecture and we found that the implementation of ideal thermodynamic cycle for gas liquefaction is impractical and hence modified cycles are proposed. The impracticality came due to the fact that the pressures required to liquefy all the gas that is compressed are very very high and therefore, this pressures cannot be obtained using conventional compresses, which are available. And therefore, this cycles cannot be realized in practice.

However, this cycles form a very good basis of comparison for other cycles. Which is what the next conclusion was. An ideal cycle is used as a benchmark and in effect, different ratios and functions are defined to compare various liquefaction cycles. So, if I am having any other cycle for liquefaction, the results obtained from this cycle or the liquefaction obtained from this cycle can be compared with liquefaction that could be obtained from an ideal cycle. It becomes a basis for comparison for various cycles and therefore, their performance of various cycles can be obtained. The performance parameters can be obtained, when we compare those parameters with the ideal thermodynamic cycle.

Then we talked about Linde - Hampson cycle and it consists of a compressor, heat exchanger and a Joule Thomson expansion device. An ideal cycle had only compressor and J - T expansion device while a heat exchanger is added to the ideal cycle and in this system, only a part of the gas that is compressed, gets liquefied, as against the fact in the ideal cycle whatever amount of gas is compressed it gets liquefied. In fact, all the gas gets liquefied well in this case only a part of the gas, this part of the gas could be only 5 percent or 10 percent or 15 percent. So, only a part of the gas that is compressed gets liquefied. A very important difference between the two cycles and that is why this cycle becomes a kind of practical cycle.

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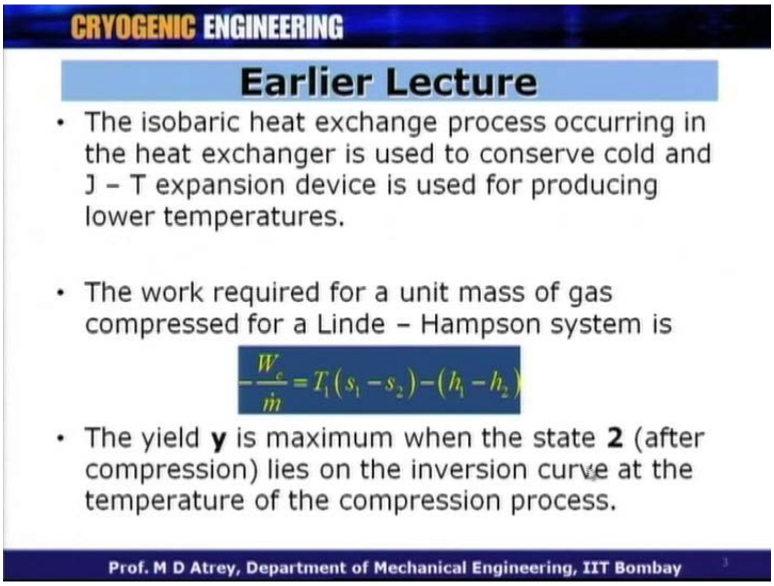

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Earlier Lecture

- The isobaric heat exchange process occurring in the heat exchanger is used to conserve cold and J – T expansion device is used for producing lower temperatures.
- The work required for a unit mass of gas compressed for a Linde – Hampson system is

$$-\frac{W_c}{\dot{m}} = T_1(s_1 - s_2) - (h_1 - h_2)$$

- The yield **y** is maximum when the state **2** (after compression) lies on the inversion curve at the temperature of the compression process.

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The heat exchange process occurring in heat exchanger and this process occurs at constant pressure and that is why we call it isobaric heat exchange process. So, the isobaric heat exchange process occurring in the heat exchanger is used to conserve the cold and J - T expansion device is used for producing lower temperatures. So, the heat exchanger which was a change from the earlier ideal cycle, it basically conserves the cold that is produced at the end. The cold gas which goes back to the compressor precooks the incoming hot gas and that is why we say that whatever cold is produced is conserved in this cycle and J - T expansion valve of course, is used for reducing the temperature with the J - T expansion process.

The work required for unit mass of gas compressed for a Linde - Hampson cycle is given by the formula minus W c upon m dot, m dot is mass unit of the gas which is being compressed is equal to T 1 into (s 1 minus s 2 minus) in bracket (h 1 minus h 2). The 1 and 2 basically denote the state of the points before compression and after compression and this process of compression is carried out isothermally at a temperature T 1. So, this is the work required for unit mass of gas, which is compressed which is m dot.

Now, we found that the yield y, the yield is nothing, but ratio of the gas which is liquefied divided by gas which is compressed that is m dot f upon m dot. The yield y is maximum when the state 2 (after the compression) lies on the inversion curve at a temperature of the compression process. This particular part has been discussed in detail in the last lecture and we found that the yield y is maximum when the state 2 lies on the inversion curve at the temperature of the compression process.

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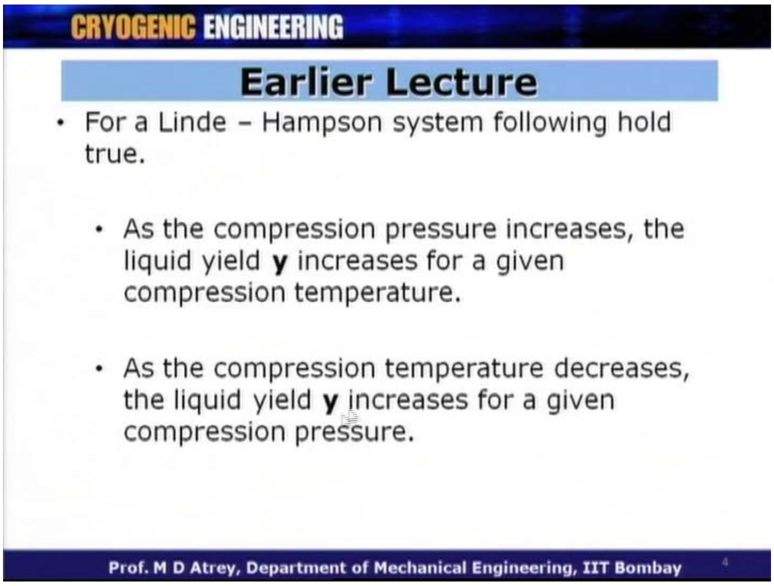

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Earlier Lecture

- For a Linde – Hampson system following hold true.
  - As the compression pressure increases, the liquid yield **y** increases for a given compression temperature.
  - As the compression temperature decreases, the liquid yield **y** increases for a given compression pressure.

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For a Linde - Hampson cycle following hold true and as the compressor pressure increases, the liquid yield y increases for a given compression temperature. So, we had seen that as the pressure increases as the pressure p 2, the gas which is compressed from state 1 to state 2 if you compress instead of 100 bar to 200 bar; that means, the pressure ratio is going to be very very high, the compression work also is going to be very very high, but when you compress at a very higher pressure, the liquid yield y increases for a compression temperature.

So, compression temperature could be kept constant at 300 Kelvin, which is room temperature and if we increase the pressure at the end of the compression y increases similarly, as the compression temperature decreases. So, instead of carrying the compression process at 300 Kelvin, if I carry out the process at 200 Kelvin, then the liquid yield y increases for a given compression pressure and both this conclusions we had seen last time using some problems or tutorials in the last lecture.

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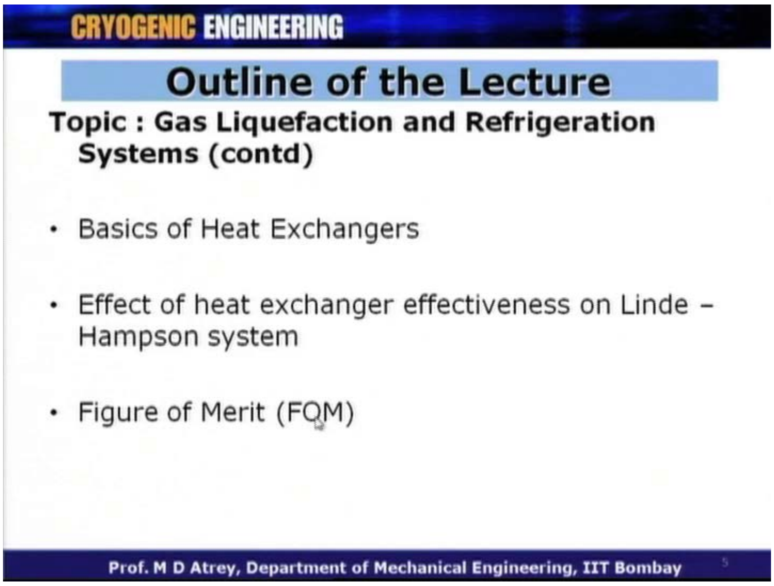

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Outline of the Lecture

Topic : Gas Liquefaction and Refrigeration Systems (contd)

- Basics of Heat Exchangers
- Effect of heat exchanger effectiveness on Linde – Hampson system
- Figure of Merit (FOM)

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So, if you understand this now, we will go the outline at today’s lecture. So, today’s lecture is the continuation of the earlier lecture, which is gas liquefaction and refrigeration system and what we are going to learn today is the basics of heat exchangers as you know in Linde - Hampson cycle. The heat exchanger is the only addition to the ideal thermodynamic cycle and therefore, heat exchanger place a very very crucial role in determine the output from this cycle.

So, we will study what is the heat exchanger? What are the basics related to that, what are the performance parameters? And then we will proceed to understand the effect of heat exchanger effectiveness on Linde - Hampson system. So, today we will continue the study of the earlier cycle, the Linde - Hampson cycle; however, what we will see? And study here is the role of heat exchanger. The heat exchanger performance is very very important and we will give some quantification as to what exactly happens with deterioration in the heat exchanger effectiveness and finally, we will understand what is the figure of merit? And this is a very important term, which is obtained to compare a various cycle. What is FOM figure of merit and all this things will try to understand in today’s lecture.

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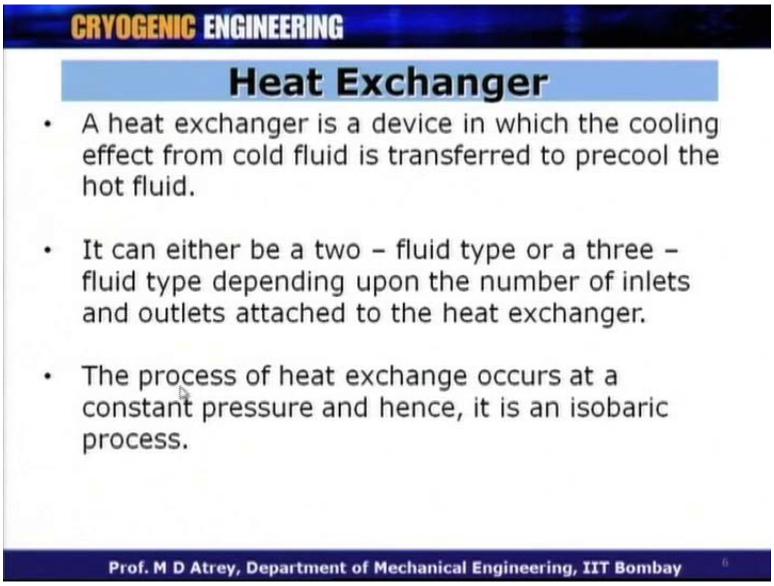

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Heat Exchanger

- A heat exchanger is a device in which the cooling effect from cold fluid is transferred to precool the hot fluid.
- It can either be a two – fluid type or a three – fluid type depending upon the number of inlets and outlets attached to the heat exchanger.
- The process of heat exchange occurs at a constant pressure and hence, it is an isobaric process.

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So, let us come to the basics of heat exchanger. So, what is a heat exchanger? A heat exchanger is a device in which the cooling effect from the cold fluid is transferred to precool the hot fluid. So, basically the hot fluid while travelling to the heat exchanger will become cold and the cold fluid while traveling to the heat exchanger will become hot, because this cold is taken by the hot fluid and the hot fluid gets precooled.

Now, this heat exchanger could be a two - fluid; that means, we can have one hot and one cold fluid or we can have three - fluid type depending upon the number of inlets and outlets attached to the heat exchanger. Normally, the heat exchangers are having two - fluids, but there are various cases which we will see in the next lectures also that a heat exchanger can have two inlets and one outgoing stream or thing like that. So, we can have or one inlet and two outgoing fluids, any combination that is possible depending on the different working fluids that are passing through the heat exchanger and what is the end usage of this particular heat exchanger is?

The process of heat exchanger occurs at a constant pressure and hence, it is an isobaric process. This is an assumption. In fact, there will be some pressure drop across the length of the heat exchanger, but in an ideal heat exchange process we can assume that the pressure remains constant and therefore, we can assume that the process is isobaric.

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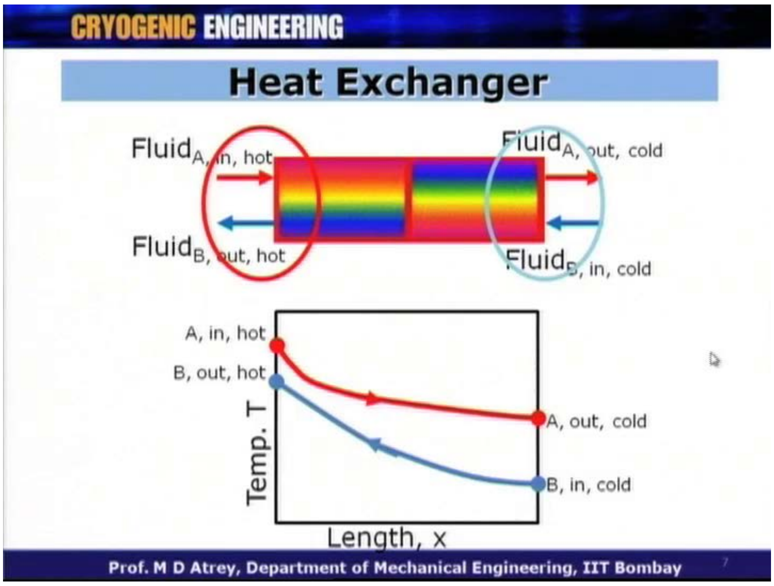

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Heat Exchanger

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So, this is a heat exchanger or this is schematic shown, which we are going to use in this particular lecture and about this all this schematic that are going to be used in this gas liquefaction, we already shown that. This is the heat exchanger and here I have got 2 schematics here, this is the heat exchanger schematic and here, I am talking about the temperature variations across the Length of the heat exchanger.

What you can see that? The fluid A which is a hot fluid in fluid A, in, hot and fluid A, out, cold, so, while travelling through the heat exchanger. This hot fluid becomes cold and how does it become cold? Because there is other fluid coming from the opposite direction meaning which this could be a counter flow heat exchanger. In the counter flow heat exchanger you will have fluids coming in opposite direction.

So, you got a fluid A and fluid B. The fluid B is cold fluid entering from this side while the hot fluid fluid A enters from the other side and during the travel across the length of the heat exchanger; the temperature of the hot fluid A comes down. So, we can see that temperature has come from A, in, hot to A, out, cold while the temperature of the fluid B increases, when it travels across the length to the heat exchanger. So, this because it takes heat from the hot fluid and temperature of the cold fluid now, increases in this way, the B, in, cold and it goes to B, out, hot. So, you can see that the hot fluid becomes cold while the cold fluid becomes hot during the travel all right.

Lot depends on now, how much heat it get in transferred? And what is the A out temperature? What is the B out temperature? Let us see those things now. So, this is basically the hot side of the heat exchanger and this we can call as a cold side of the heat exchanger.

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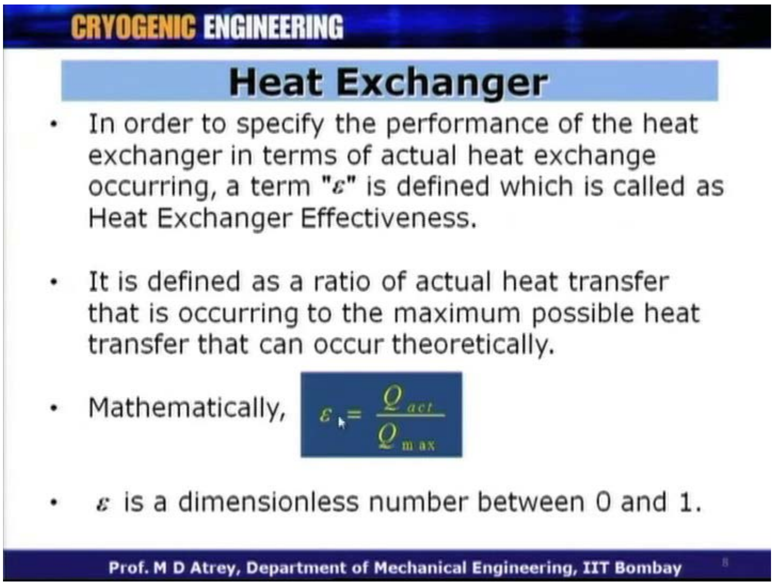

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Heat Exchanger

- In order to specify the performance of the heat exchanger in terms of actual heat exchange occurring, a term "ε" is defined which is called as Heat Exchanger Effectiveness.
- It is defined as a ratio of actual heat transfer that is occurring to the maximum possible heat transfer that can occur theoretically.
- Mathematically, $\varepsilon = \frac{Q_{act}}{Q_{max}}$
- ε is a dimensionless number between 0 and 1.

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Now, in order to specify the performance of the heat exchanger in terms of actual heat exchange occurring, a term epsilon is defined which is called as Heat Exchanger Effectiveness. Now, in ideal case whatever heat is given by the hot fluid should be taken by the cold fluid, but no it does not happen like that. So, whatever heat is given by the hot fluid only a part of that will be taken by a cold fluid and therefore, the actual effectiveness of the heat exchanger is going to be different than what it is ideally.

In order to understand what are the losses in the system? Why the heat exchanger does not performance to 100 percent capacity? A term comes into picture, which is called as effectiveness of heat exchanger a very commonly known term and it defined as the ratio of actual heat transfer that is occurring in the heat exchanger to the maximum possible heat transfer that can occur theoretically. So, actual heat exchange upon maximum possible heat exchanges.

Mathematically, one can the write epsilon or heat exchanger effectiveness is equal to Q actual the actual heat transfer divided by Q max, the maximum possible in heat exchanger. Naturally, the maximum value of Q actual could be Q max value and therefore, the maximum value of epsilon also can be 1, the epsilon parameter is definitely dimensionless and therefore, it lies between 0 and 1.

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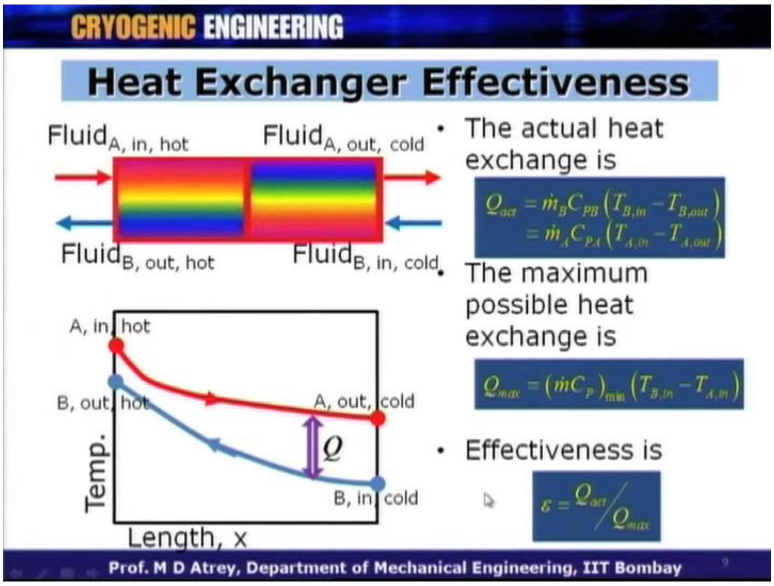

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Heat Exchanger Effectiveness

- The actual heat exchange is

$$Q_{act} = \dot{m}_B C_{PB}\left(T_{B,in} - T_{B,out}\right) = \dot{m}_A C_{PA}\left(T_{A,in} - T_{A,out}\right)$$

- The maximum possible heat exchange is

$$Q_{max} = \left(\dot{m} C_P\right)_{min}\left(T_{B,in} - T_{A,in}\right)$$

- Effectiveness is

$$\varepsilon = Q_{act}/Q_{max}$$

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Here is the parameter, which is Q where the heat is transferred from the hot fluid to the cold fluid. The actual heat exchange now, in mathematical term calculation will be done by how much heat is given by hot fluid? And how much heat is taken by the cold fluid? So, amount of heat that is given by the hot fluid is m into C P into delta T of that particular fluid. So, if I talk about the B fluid we talk about m dot B, which is mass flow rate of the B fluid into the specific capacity at constant pressure of the B fluid into the temperature difference of the b fluid, which is T B, in minus T B, out.

Similarly, we assume that the heat which is given by the hot fluid is m dot A into C P A into delta T of the fluid. So, m dot A into C P A into T A, in minus T A, out. So, this is the Q actual. So, this is the amount of heat which is taken by the B fluid and this is the amount of heat which is given by the A fluid. Now, the maximum possible heat exchange is the B fluid can the reach maximum temperature equal to A, in, hot all right. If the heat exchange is 100 percent if there are no problems, then this B fluid in principle could have reached up to A, hot or the A stream can the reach the lowest temperature of B in cold alright.

So, the maximum heat exchange is going to occur between this two temperatures A, in, hot minus B, in, cold and therefore, Q max is equal to m dot C P minimum and this maximum change of temperature will happen for the fluid, which has got minimum heat capacity. The heat capacity is equal to m dot into C P minimum of the two fluids multiplied by maximum temperature difference that is T B in minus T A in all right.

So, this is a very important thing that is the maximum temperature drop for anything that is possible multiplied by m dot C P or the heat capacity of the fluid, which has got minimum heat capacity m dot c minimum, because the one which has got minimum m dot C P will experience maximum delta T. So, effectiveness is epsilon is equal to Q actual upon Q max. So, Q actual is given by this formula and the Q max can be calculated by this formula and this will give what is the epsilon? Or what is the effectiveness of the heat exchanger is?

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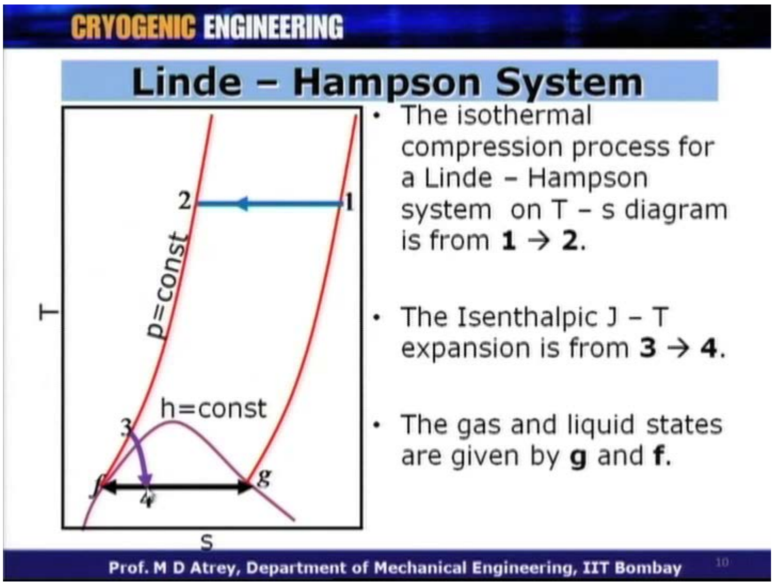

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Linde – Hampson System

- The isothermal compression process for a Linde – Hampson system on T – s diagram is from **1** → **2**.
- The Isenthalpic J – T expansion is from **3** → **4**.
- The gas and liquid states are given by **g** and **f**.

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Now, if I have want to understand, because of this ineffectiveness or effectiveness being a finite value or not hundred percent how exactly it looks on TS diagram for a Linde-Hampson system. So, we know that the isothermal compression process of a Linde - Hampson system is 1 to 2 is a compression process at any temperature, which is constant. Similarly, the isenthalpic process occurs from 3 to 4 like this and this is the J - T expansion process.

So, the gas gets expanded by Jule Thomson effect from 3 to 4, where this length 4 to g represents the amount of liquid that is generated, because of this expansion while the length f to 4 or 4 to f denotes the amount of gas which is going back. So, if this length is more you will get more liquid, if this length is small less gas goes back. So, the gas and the liquid states are given by G and F the gas goes back from this and the liquid is collected at point f.

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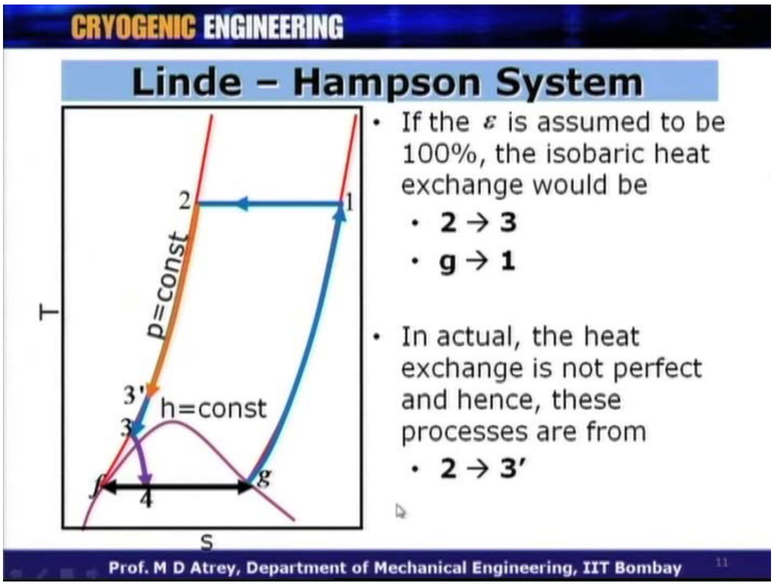

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Linde – Hampson System

- If the ε is assumed to be 100%, the isobaric heat exchange would be
  - **2 → 3**
  - **g → 1**
- In actual, the heat exchange is not perfect and hence, these processes are from
  - **2 → 3'**

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Now, if the heat exchanger effectiveness or epsilon is assumed to be hundred percent, the isobaric heat exchange would be from 2 to 3 process, the hot fluid is getting precool from 2 to 3 and after that it is getting subjected to J - T expansion, similarly, the low pressure sides or the cold fluid experiences g to 1 process. The temperature will increase from point g to point 1, because of which the temperatures the hot fluid will decrease from 2 to 3 and this cycle exists, when the epsilon is 100 percent.

Now, in actual case the heat exchange is not a perfect process and hence the processes are from 2 to 3 dashes. Instead of having heat exchanger process from 2 to 3 we will have it from 2 to 3 dash, it will not come up to 3 point for a hot fluid similarly, for the cold fluid or the low pressure fluid instead of reaching from temperature at point g up to

1, it will go up to only 1 dash. So, it will not go up to 1, but it will go up to 1 dash. This is what is happening, because of the heat exchanger effectiveness term.

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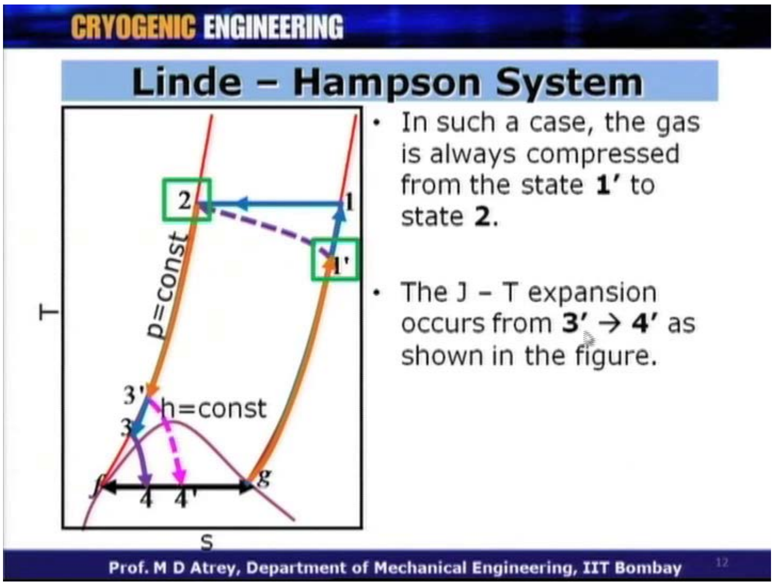

Linde – Hampson System

- In such a case, the gas is always compressed from the state **1'** to state **2.**
- The J – T expansion occurs from **3' → 4'** as shown in the figure.

Now, the gas is leaving the heat exchanger on the low pressure side at one dash and it enters the compressor at temperature one dash. So, naturally the compressors have to do the process of compression from 1 dash to 2. So, the compression occurs from 1 dash to 2, a compressor has to do additional work for bringing it is temperature to 1 dash to 1 first and then isothermally compress the gas from 1 to 2.

So, in essence what you can understand is that? The compressor has to do more work equal to the length from 1 to 1 dash all right. So, in such a case the gas is always compressed from the state 1 dash to state 2 and similarly, the J - T expansion instead of happening from 3 to 4 process, it happens from 3 dash to 4 dash and as I said that this links form 4 dash to g denotes the amount of liquid or amount of liquefaction that is occurring in this cycle. We can see from here, that this link got reduced from 4 to g to 4 dash to g; it means that the liquefaction has reduced, because of the ineffectiveness of the heat exchanger.

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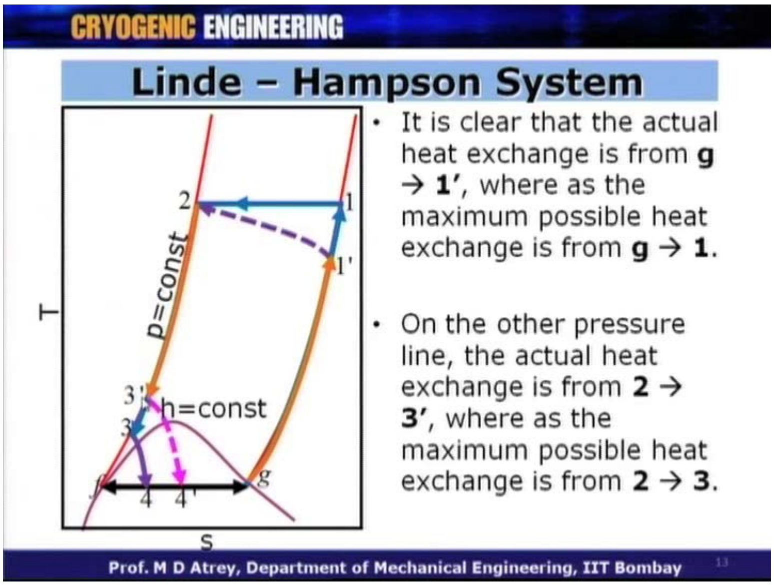

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Linde – Hampson System

- It is clear that the actual heat exchange is from **g** → **1'**, where as the maximum possible heat exchange is from **g** → **1**.
- On the other pressure line, the actual heat exchange is from **2** → **3'**, where as the maximum possible heat exchange is from **2** → **3**.

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So, the J - T expansion is occurring from 3 dash to 4 dash as shown in the figure, it is clear that the actual heat exchange is form g to 1 dash all right. Whereas the maximum possible heat exchange is from g to 1 in principle one should have come from g to 1, because the temperature of the point 2 could have attained on this pressure line, but not the fluid could not reach up to 1 it has reach up to 1 dash.

On the other pressure line, on the high pressure line the actual heat exchange process is from 2 to 3 dash, whereas the maximum possible heat exchange should have been from 2 to 3. So, maximum possible heat exchange should have been from 2 to 3 while it has reached up to 3 dash.

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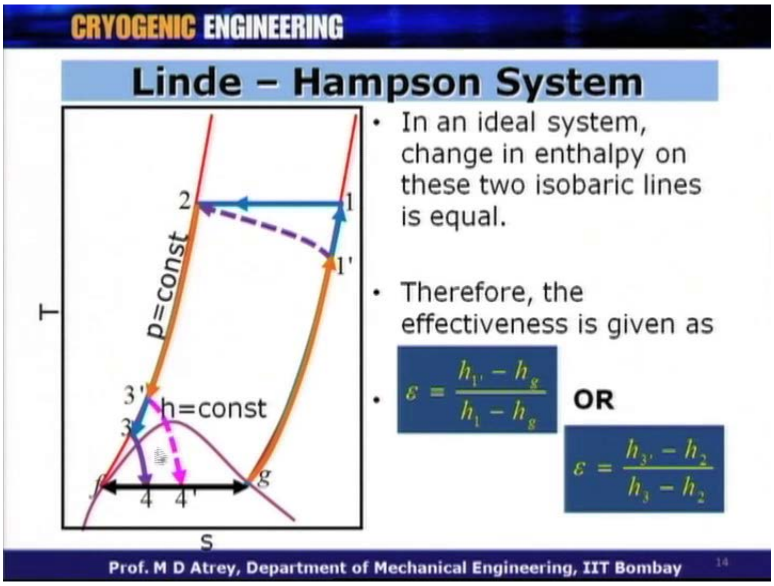

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Linde – Hampson System

- In an ideal system, change in enthalpy on these two isobaric lines is equal.
- Therefore, the effectiveness is given as
- $\varepsilon = \frac{h_{1'} - h_g}{h_1 - h_g}$ **OR** $\varepsilon = \frac{h_{3'} - h_2}{h_3 - h_2}$

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In an ideal system changing enthalpy on these two isobaric lines is equal. Therefore, the effectiveness is given by epsilon is equal to actual heat transform that is h 1 dash minus h g, the enthalpy at these two point, which is actual heat transfer divided by maximum possible heat transfer which is h 1 minus h.

So, maximum enthalpy different that could have obtained, if the heat exchanger effectiveness 100 percent is h 1 minus h g; however, the actual enthalpy difference that occurred is basically, h 1 dash minus h g. So, effectiveness could be return as h 1 dash minus h g upon h 1 minus h. This is on the low pressure side, if I want to give the same definition on this side, then the epsilon in this case is equal to h 3 dash minus h 2 actual enthalpy difference divided by h 3 minus h 2, which is the maximum possible heat transfer that is possible for the high pressure line.So, both these terms will indicate the heat exchanger effectiveness for a given heat exchanger.

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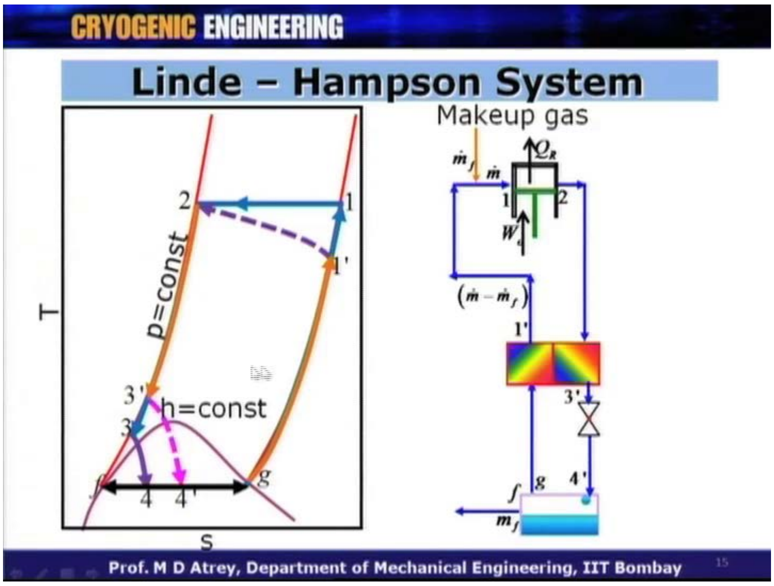

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Linde – Hampson System

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Now, if I want to understand all this points here on this cycle, the gas enters the heat exchanger at point 2, which is this. And instead of coming at 3, it comes at 3 dash and instead of expanding from 3 to 4, it expands from 3 dash to 4 dash across the J - T expansion work. This is a very important to understand, the gas does enter the heat exchanger at point g, but instead of coming at point 1, it is coming out at 1 dash all right. So, this is what is shown over here.

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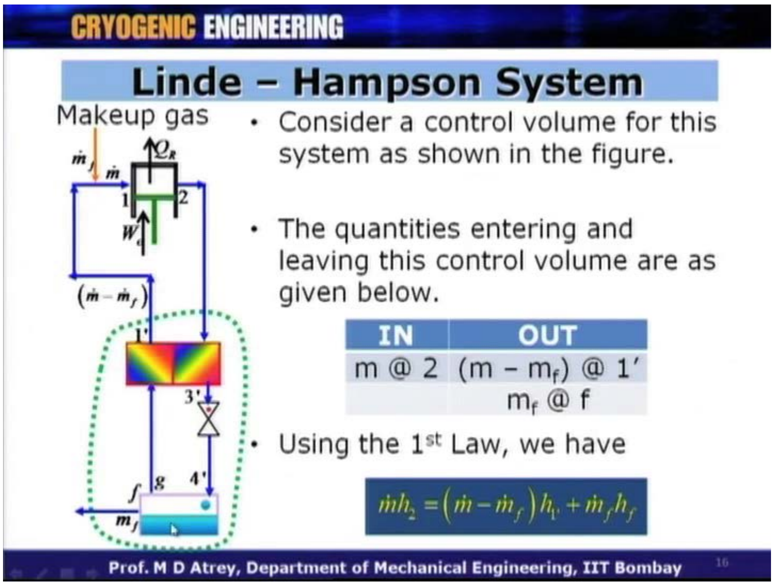

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Linde – Hampson System

- Consider a control volume for this system as shown in the figure.
- The quantities entering and leaving this control volume are as given below.

IN	OUT
m @ 2	(m – m_f) @ 1′
	m_f @ f

- Using the 1st Law, we have

$$\dot{m}h_2 = \left(\dot{m}-\dot{m}_f\right)h_{1'} + \dot{m}_f h_f$$

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Now, if I were to do the heat balance in this cycle as we have done earlier I will consider this control volume. So, consider a control volume for this system as shown in the figure, the quantities entering and leaving this control volume are as given below. So, what is entering this control volume is the mass flow rate m dot at point 2 and what is leading the system are m dot minus m dot f at 1 dash and at same time what is leaving is m dot f that is liquid here, which is m dot f at point f. And if I have the first law of energy balance across this control volume I can write that whatever enters is equal to whatever leaving the system or whatever leaving the control volume. So, whatever is entering is m dot h 2 and whatever leaving is m dot minus m dot f h 1 dash plus m dot FHF. This is what you see from this in our table.

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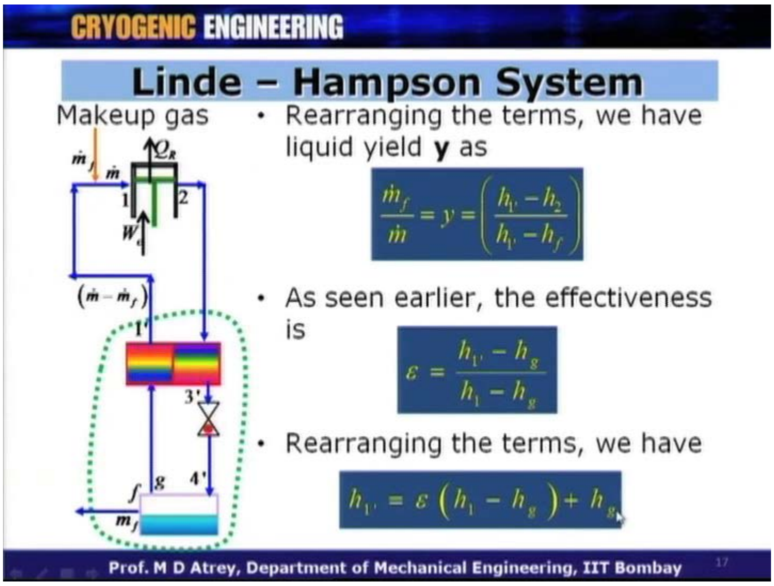

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Linde – Hampson System

- Rearranging the terms, we have liquid yield **y** as

$$\frac{\dot{m}_f}{\dot{m}} = y = \left(\frac{h_{1'} - h_2}{h_{1'} - h_f}\right)$$

- As seen earlier, the effectiveness is

$$\varepsilon = \frac{h_{1'} - h_g}{h_1 - h_g}$$

- Rearranging the terms, we have

$$h_{1'} = \varepsilon\left(h_1 - h_g\right) + h_g$$

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If I rearrange this I can now, develop a correlation for y which is nothing, but m dot f upon m dot, which is equal to h 1 dash minus h 2 upon h 1 dash minus h f. So, you can see now, in earlier case it was h 1 minus h 2 divided by h 1 minus h f when the effectiveness is 100 percent, but because the effectiveness is not 100 percent, we have got h 1 replaced by h 1 dash and therefore, y is defined like this now.

As seen earlier the effectiveness is epsilon is equal to h 1 dash minus h g upon h 1 minus h g. This is what we have calculated earlier. We have developed a correlation for this, if I rearrange this I will get h 1 dash if I want to have an expression in terms of h 1 dash. So, I can write h 1 dash is equal to epsilon into h 1 minus h g plus h g on this side and what I am going to do now, is replace this h 1 dash here by this particular value and rearrange the equation for y.

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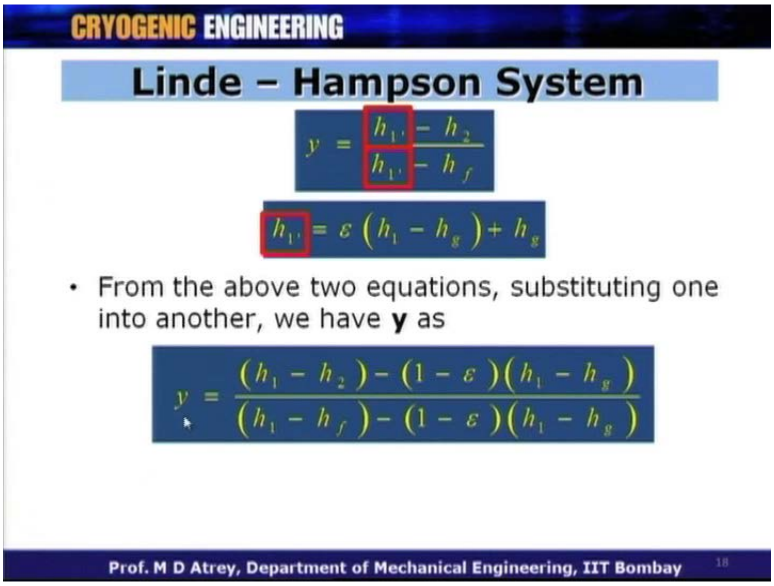

CRYOGENIC ENGINEERING

Linde – Hampson System

$$y = \frac{\boxed{h_{1'}} - h_2}{\boxed{h_{1'}} - h_f}$$

$$\boxed{h_{1'}} = \varepsilon\left(h_1 - h_g\right) + h_g$$

- From the above two equations, substituting one into another, we have **y** as

$$y = \frac{\left(h_1 - h_2\right) - \left(1 - \varepsilon\right)\left(h_1 - h_g\right)}{\left(h_1 - h_f\right) - \left(1 - \varepsilon\right)\left(h_1 - h_g\right)}$$

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This is what my expression for y is. And this what my expression for h 1 dash is. So, if I replace h 1 dash over here, you can get an expression like this. So, y is equal to h 1 minus h 2 minus 1 minus epsilon h 1 minus h g divided by h 1 minus h f minus 1 minus epsilon h 1 minus h g. So, what we can see from here is, the real expression for y, which is h 1 minus h 2 upon h 1 minus h f as numerators and denominators. Both the numerators and denominators are getting lessened by the value equivalent to this. So, naturally as the result of which y as compared to what it is going to be less in this case algebraically.

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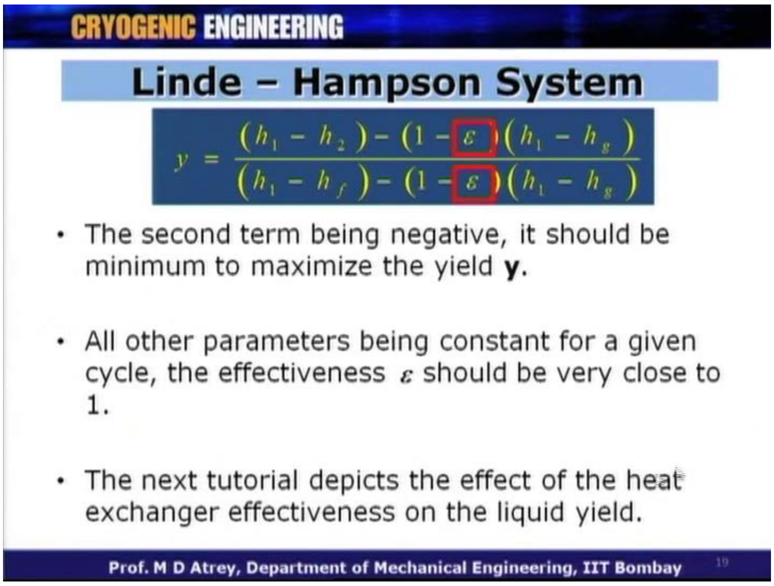

CRYOGENIC ENGINEERING

Linde – Hampson System

$$y = \frac{(h_1 - h_2) - (1 - \boxed{\varepsilon})(h_1 - h_g)}{(h_1 - h_f) - (1 - \boxed{\varepsilon})(h_1 - h_g)}$$

- The second term being negative, it should be minimum to maximize the yield **y**.
- All other parameters being constant for a given cycle, the effectiveness ε should be very close to 1.
- The next tutorial depicts the effect of the heat exchanger effectiveness on the liquid yield.

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So, expression for a while is this and what you can see is, now, the second term being negative, it should be minimum to maximize the y. If I want to have maximum y, this negative term should be as minimum as possible and what you can see now, from this all the parameters are constant. Once my 1 and 2 parameters are fixed state 1 and state 2 pressures are fixed the only variables in the whole expression is only epsilon, which is the effectiveness of heat exchanger.

So, all other parameters being constant for a given cycle, the effectiveness epsilon should be very close to 1. So, everything depends on what is the value of epsilon. If the value of epsilon is equal to 1, you get 1 minus 1 is equal to 0 and the whole expression reduces down to what it was earlier and therefore, one has to ensure that this epsilon value should be as close to 1 as possible. We will several tutorials to understand, the effect of heat exchanger effectiveness on the liquid yield.

So, based on whatever we have learned till now, these tutorials will help you understand all those theoretical aspects, which are just talked about and therefore, I have planned for almost 4 tutorials. So, kindly go through those tutorials in detail to understand every part of it and what is most important is to interpret the results. It is just not mathematical, but the understanding should not be mathematical terms while what is important is to understand the output of these tutorials, the results of these tutorials or the conclusions that are drawn from these tutorials, that is the most important thing of this tutorials.

Then only you can sort of give some quantitative understanding, you can have some number fill for all this cycles and their results.

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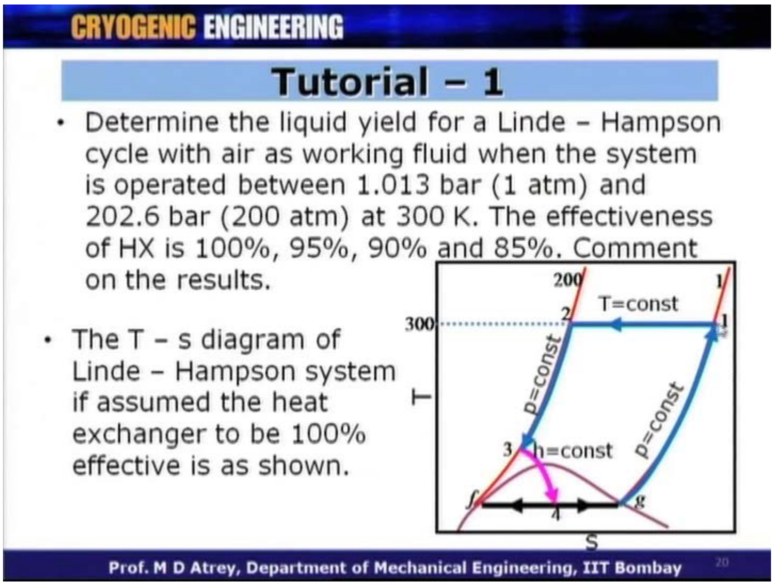

CRYOGENIC ENGINEERING

Tutorial – 1

- Determine the liquid yield for a Linde – Hampson cycle with air as working fluid when the system is operated between 1.013 bar (1 atm) and 202.6 bar (200 atm) at 300 K. The effectiveness of HX is 100%, 95%, 90% and 85%. Comment on the results.
- The T – s diagram of Linde – Hampson system if assumed the heat exchanger to be 100% effective is as shown.

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So, this is the first tutorial and again read the tutorial problem definition properly. Determine the liquid yield for a Linde - Hampson cycle with air as working fluid. The moment I say air as working fluid is to have temperature entropy diagram or TS diagram of air available with you. When the system is operated between 1.013 bar or 1 atmosphere and 202.6 bar or 200 atmosphere at 300 Kelvin.

This is the compression process state 1 and state 2 approximately, 1 bar and 200 bar and the compression temperature is 300 Kelvin. The effectiveness of heat exchanger is 100 percent, 95 percent, 90 percent and 85 percent. Comment on the results.

So, what basically I want you to do is comment on the liquefaction yield unit, when the heat exchanger effectiveness is 100 percent, then 95 percent then, 90 percent and 85 percent and then comment on the results. So, there are actually 4 problems, in this solve this problems to get liquid yield when this is a ideal Linde - Hampson cycle, then we have got effectiveness of 95 percent, 90 percent and 85 percent of the heat exchanger.

So, the first thing is to get a T - S diagram. So, the T - S diagram of Linde - Hampson system if assumed the heat exchanger to be of 100 percent effective, then it is as shown here. So, this as you now, know 1 to 2 is a compression process, 2 to 3 is a heat exchange process, 3 to 4 is a J - T expansion process, J 2 1 is a heat exchanger process, J 2 1 precools to 2 to 3 all right.So, this is what is happening? When the heat exchanger effectiveness is 100 percent, this is the state 1 at 1.013 bar or one atmosphere. This is the state 2; the process 1, 2 is a compression process occurring at 300 Kelvin.

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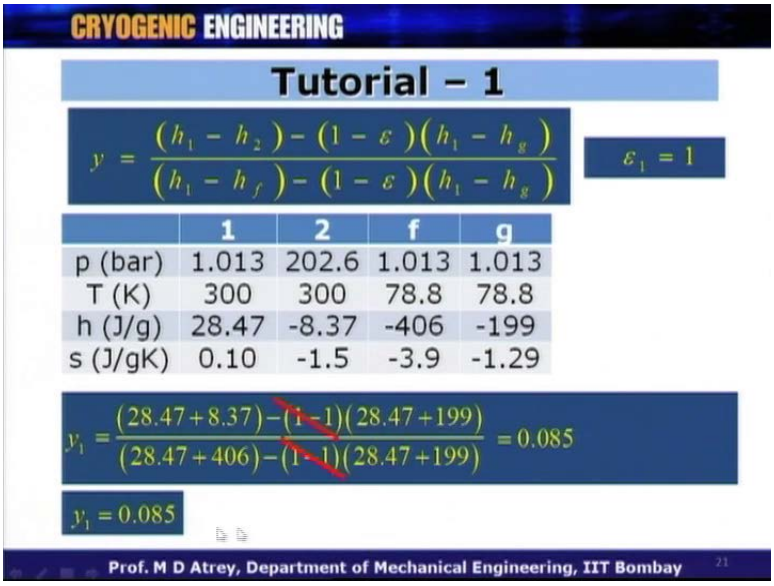

CRYOGENIC ENGINEERING

Tutorial – 1

$$y = \frac{(h_1 - h_2) - (1 - \varepsilon)(h_1 - h_g)}{(h_1 - h_f) - (1 - \varepsilon)(h_1 - h_g)}$$

$\varepsilon_1 = 1$

	1	2	f	g
p (bar)	1.013	202.6	1.013	1.013
T (K)	300	300	78.8	78.8
h (J/g)	28.47	-8.37	-406	-199
s (J/gK)	0.10	-1.5	-3.9	-1.29

$$y_1 = \frac{(28.47 + 8.37) - \cancel{(1-1)}(28.47 + 199)}{(28.47 + 406) - \cancel{(1-1)}(28.47 + 199)} = 0.085$$

$y_1 = 0.085$

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So, first is the formula which we want to apply, which is y equal to h 1 minus h 2 minus 1 minus epsilon h 1 minus h g divided by h 1 minus h f minus 1 minus epsilon h 1 minus h g. This is a general formula and if you put the first case now, this is the formula which we get from a Linde - Hampson cycle having a heat exchanger, which has effectiveness value epsilon.

The first case is when you want to have epsilon 1 is equal to 1 or 100 percent. If you put this value if you get the value of I for the first case, when epsilon is 100 percent. The most important thing is now, locate the point 1, 2, f and g on the T - S diagram, the 1 is at one bar approximately and 300 Kelvin, the 0 .2 is at 200 bar and 300 Kelvin, the point f is 1 bar and 78.8 Kelvin, which is the boiling point of air, because air is the working fluid and the point g is on the gas side and point f is on the liquid side. So, point g and f lie on the same temperature, which is 78.8 Kelvin which is nothing, but the boiling point of air. Get the corresponding enthalpy and entropy and the first step to do any problem is to make this stable. So, that all these values are available to you.

Now, with my experience I can tell you, if you make mistake in getting even 1, value the whole result will change and you will not be able to get a correct confusion drawn from this. So, the most important thing is keeping calm go to the T - S chart locate this point and get correct enthalpy and entropy value, in accuracy of plus minus 5 percent is acceptable if you write instead of 28.47 as 30 no problems or 26 no problem, but do not go from 28 to 40 or 35 that far. So, having got these values from this T - S diagram, the important thing is now, put these values of h 1, h 2, h g and h f in this formula for a while.

So, the first case is, when epsilon is 100 percent or epsilon 1 in this case is 1. So, if I put those values h 1, h 2, h 1, h f and here I got epsilon is equal to 1. So, in principle the whole this side gets equal to 0. So, 1 minus is, 1 is equal to 0 on the numerator as well as denominator and expression gets reduced to h 1 minus h 2 upon h 1 minus h f, which is what we know when the heat exchanger effectors is 100 percent. If you put this values y 1 you get is equal to 0.0085 all right. So, first case for epsilon 1 is equal to 1, you get y 1 is equal to 0.0085.

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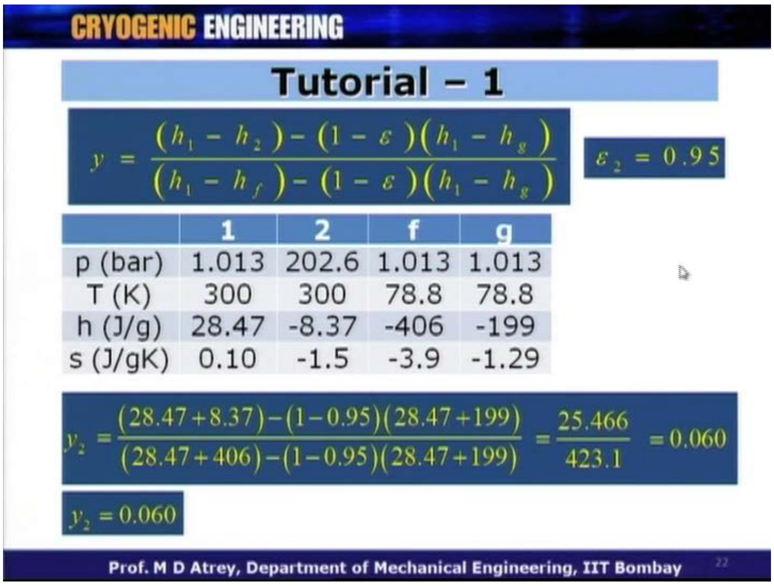

CRYOGENIC ENGINEERING

Tutorial – 1

$$y = \frac{(h_1 - h_2) - (1 - \varepsilon)(h_1 - h_g)}{(h_1 - h_f) - (1 - \varepsilon)(h_1 - h_g)}$$

$\varepsilon_2 = 0.95$

	1	2	f	g
p (bar)	1.013	202.6	1.013	1.013
T (K)	300	300	78.8	78.8
h (J/g)	28.47	-8.37	-406	-199
s (J/gK)	0.10	-1.5	-3.9	-1.29

$$y_2 = \frac{(28.47+8.37)-(1-0.95)(28.47+199)}{(28.47+406)-(1-0.95)(28.47+199)} = \frac{25.466}{423.1} = 0.060$$

$y_2 = 0.060$

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Now, let us go to the second case when epsilon 2 is equal to 0.95 or 95 percent is the effectiveness of the heat exchanger, again go to the same table the values will remain the same here, put the values correctly calculate these values and now, here you get y 2 is equal to 0 .060. So, if you compare with earlier values you can suddenly find that the value of y 2 is less than what you got in earlier case. As soon as you came down on the effectiveness from 100 percent to 95 percent.

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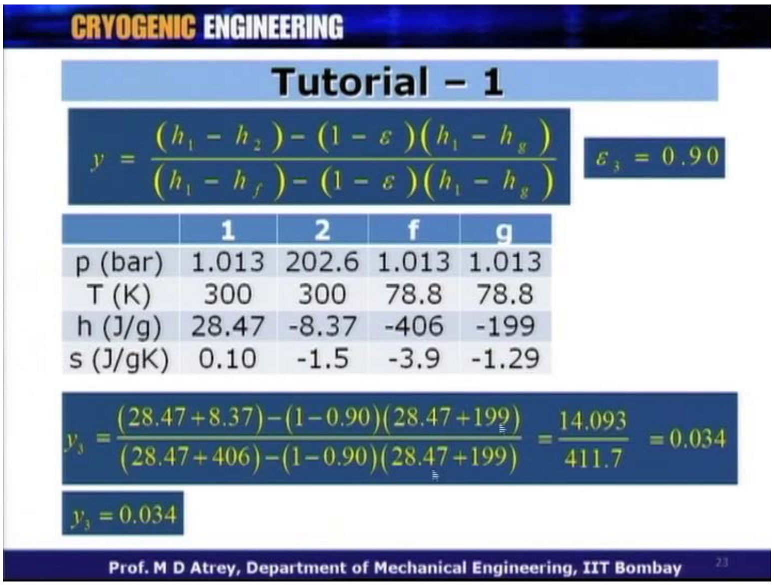

CRYOGENIC ENGINEERING

Tutorial – 1

$$y = \frac{(h_1 - h_2) - (1-\varepsilon)(h_1 - h_g)}{(h_1 - h_f) - (1-\varepsilon)(h_1 - h_g)}$$

$\varepsilon_3 = 0.90$

	1	2	f	g
p (bar)	1.013	202.6	1.013	1.013
T (K)	300	300	78.8	78.8
h (J/g)	28.47	-8.37	-406	-199
s (J/gK)	0.10	-1.5	-3.9	-1.29

$$y_3 = \frac{(28.47+8.37)-(1-0.90)(28.47+199)}{(28.47+406)-(1-0.90)(28.47+199)} = \frac{14.093}{411.7} = 0.034$$

$y_3 = 0.034$

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Now, let us go to the third case where the epsilon 3 value is 90 percent; that means, the heat exchanger effectiveness in this case is now, 90 percent. Apply the same formula again get the same values again for 1, 2, f and g put the values for y 3. See if I put those value and put the value of effectiveness of heat exchanger as 90 percent or 0.9 in this case calculate the whole thing and y 3 has come out to 0.034, which is again less than what you got for 95 and 100 percent effectiveness of the heat exchanger. So, in this case y3 is equal to 0.034.

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The forth case now, we are going is for epsilon 4 is equal to .85 0.85 or 85 percent effectiveness of the heat exchanger, again I put the values from this table and I get the values of y 4 and if I put all those values with effectiveness of the heat exchanger equal to 0.85 in this expression, I get the value of y (( )) B 0.006 very close to 0.So, here we have got 4 cases and we went on reducing the heat exchanger effectiveness from 100 percent to 85 percent.

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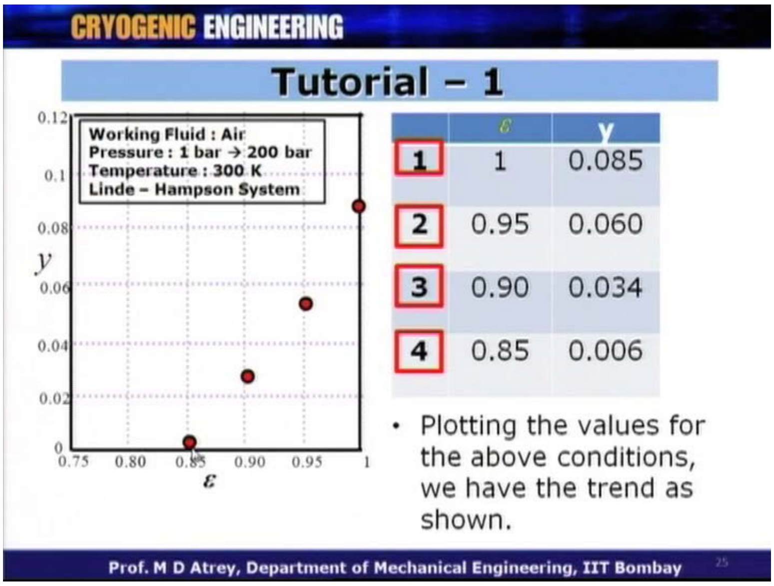

CRYOGENIC ENGINEERING

Tutorial – 1

	ε	y
1	1	0.085
2	0.95	0.060
3	0.90	0.034
4	0.85	0.006

- Plotting the values for the above conditions, we have the trend as shown.

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So, in summary we want to show the schematic in graphical form, the y values for different effectiveness of heat exchanger. So, if I do that for working fluid of air keeping the pressure from one bar to 200 bar of compression process, the temperature of compression is 300 Kelvin for a Linde - Hampson cycle.

The 0.1 is effectiveness of 1, the y value is 0.85 which I have plot over here. So, 0.1 for 1 what you get is 0.085, 0.2 for 95 percent effectiveness of heat exchanger what is you get is .06, then 0.3 is for 0.9 and what you get is 0.034 as y and 0.4 is effectiveness of 0.85 what you get is 0.006, which is very close to 0 plot and connect by line what you get is this.

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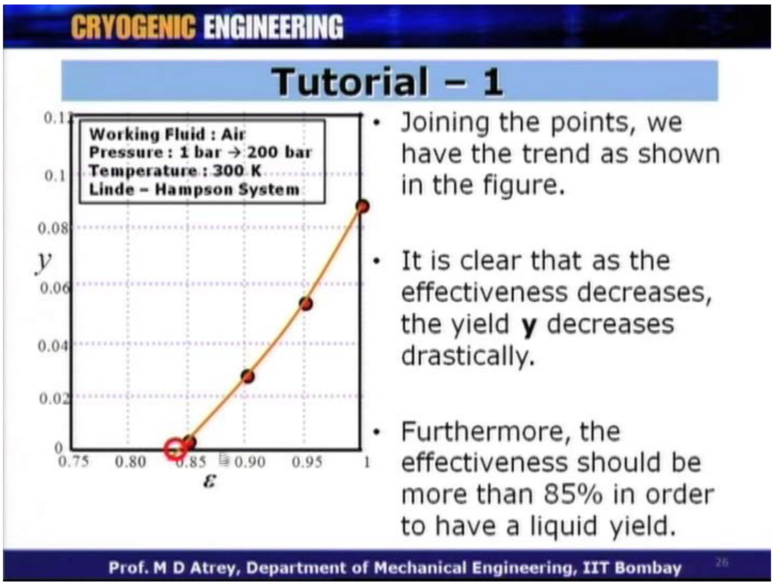

CRYOGENIC ENGINEERING

Tutorial – 1

- Joining the points, we have the trend as shown in the figure.
- It is clear that as the effectiveness decreases, the yield **y** decreases drastically.
- Furthermore, the effectiveness should be more than 85% in order to have a liquid yield.

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So, joining these points, we have the trend as shown in the figure, which you see that as the effectiveness of the heat exchanger decreases the value of y decreases. It is clear that as the effectiveness decreases, the yield y decreases drastically and what you also can see from here is. If the heat exchanger is now, less than 85 percent, the value of the y is almost 0 there is no yield what does it mean? It means that if the heat exchanger effectiveness is less than 85 percent.

The J - T expansion process will never fall in the dome, it will fall outside the dome meaning which there is no liquefaction. When I am having J - T expansion process happening outside the dome, it means that before the J - T expansion there is a gas and after the J - T expansion when we want to have 2 phase fluid what you get at the end of the expansion is again gas and there is no liquid available.

So, the point is that the heat exchanger effectiveness, if it is less than 85 percent. The J - T expansion process will never fall in the dome. the point before the J - T expansion will be far away from the dome and therefore, there will not be any liquefaction in this case. So, one has to be sure that the effectiveness of the heat exchanger should be much higher than 85 percent. In fact, in order to get proper value of y for the liquefaction for Linde - Hampson cycle.

Furthermore, as we can see from this that the effectiveness should be much more than 85 percent in order to have liquid yield. So, this tutorial basically, shows how important is the effectiveness of heat exchanger is in order to implement Linde - Hampson cycle for gas liquefaction. It is not a very simple process to have an epsilon of 85 percent and more to realize in practice. It is not very simple and therefore, one has to have a very very important task to design a heat exchanger to have higher values of epsilon and to have a very higher values close to 95 percent epsilon values are to be realized in practice. It is not a very simple thing and therefore, in cryogenics what we have is very special kind of heat exchangers of which we will talk about in the coming lectures. So, this tutorial one basically shows how important the heat exchanger effectiveness is from gas liquefaction point of view.

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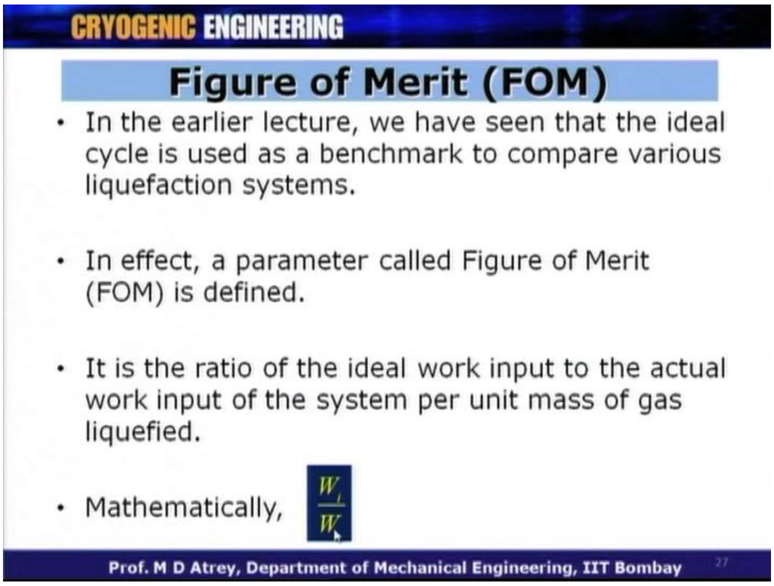

CRYOGENIC ENGINEERING

Figure of Merit (FOM)

- In the earlier lecture, we have seen that the ideal cycle is used as a benchmark to compare various liquefaction systems.
- In effect, a parameter called Figure of Merit (FOM) is defined.
- It is the ratio of the ideal work input to the actual work input of the system per unit mass of gas liquefied.
- Mathematically, $\frac{W_i}{W}$

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The second term now, I want to introduce here is figure of merit or FOM and we then solve some more tutorials on this. So, what is figure of merit? In the earlier lecture, we have seen that the ideal cycle is used as a benchmark to compare a various liquefaction cycle. We talked about that the ideal thermodynamic cycle is used as a benchmark to compare various liquefaction cycle.

In effect, a parameter called Figure of Merit is defined and what is it? It is the ratio of the ideal work input to the actual wok input to the system per unit mass of gas which is liquefied. So, what is the work input per unit mass of gas which is liquefied ideally and in actual case. So, ideal work is going to be always less than that actual case and therefore mathematically, we can write figure of merit is equal to W I that is ideal work input divided by w, which is actual work input system.

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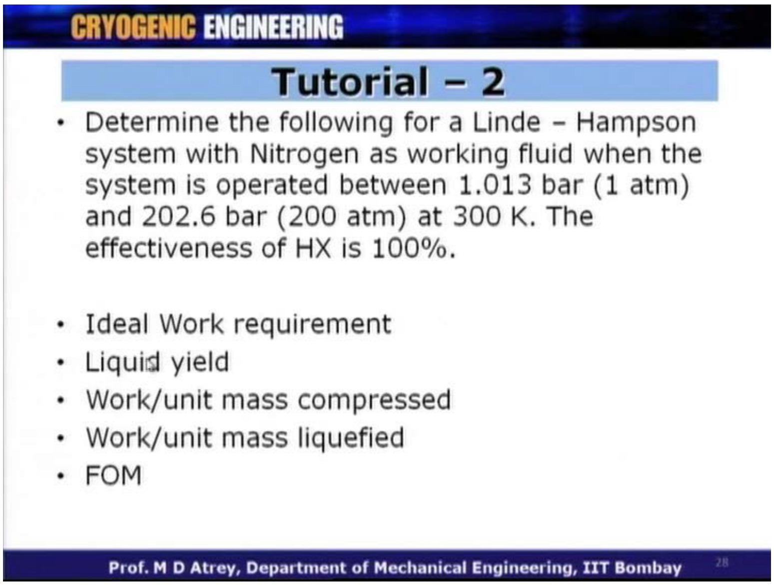

CRYOGENIC ENGINEERING

Tutorial – 2

- Determine the following for a Linde – Hampson system with Nitrogen as working fluid when the system is operated between 1.013 bar (1 atm) and 202.6 bar (200 atm) at 300 K. The effectiveness of HX is 100%.

- Ideal Work requirement
- Liquid yield
- Work/unit mass compressed
- Work/unit mass liquefied
- FOM

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And based on this now, we got a second tutorial which is this determine the following for a Linde - Hampson system with nitrogen as a working fluid when the system is operated between again 1 bar atmosphere to 200 atmosphere at 300 Kelvin. The effectiveness of heat exchanger is 100 percent; this is my tutorial number 2. So, working fluid is nitrogen, again the state 1 and state 2 are at one atmosphere to 200 atmospheres and the compression temperature is 300 Kelvin.

The heat exchanger effectiveness for this particular purpose is 100 percent what you want to compare? Or what you want to get is? What is the ideal work requirement? What is the liquid yield? What is the work per unit mass of gas which is compressed? What is the work per unit mass of gas which is liquefied? And finally, what is the figure of merit for this particular case? So, 1, 2, 3, 4 and 5 are required to be answered for this particular tutorial using nitrogen as a working fluid. So, as I said the first thing to be done is to get a TS diagram for nitrogen.

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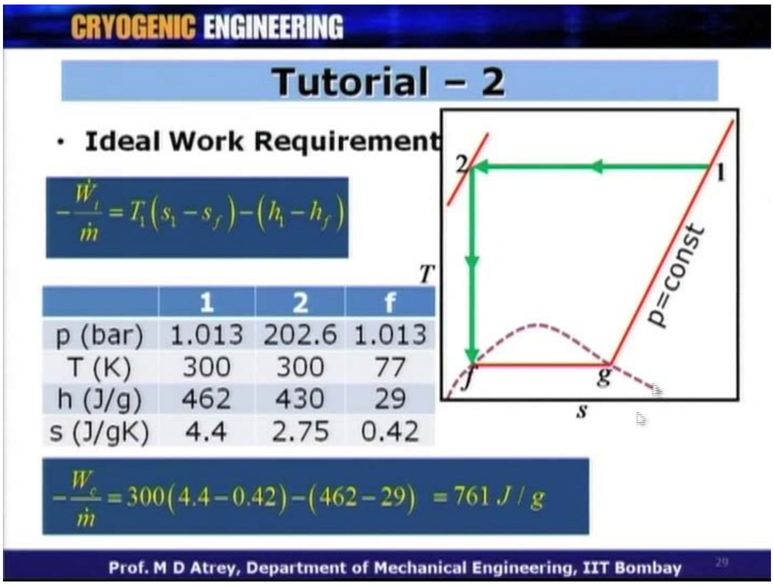

CRYOGENIC ENGINEERING

Tutorial – 2

- **Ideal Work Requirement**

$$-\frac{\dot{W}_i}{\dot{m}} = T_1(s_1 - s_f) - (h_1 - h_f)$$

	1	2	f
p (bar)	1.013	202.6	1.013
T (K)	300	300	77
h (J/g)	462	430	29
s (J/gK)	4.4	2.75	0.42

$$-\frac{W_c}{\dot{m}} = 300(4.4 - 0.42) - (462 - 29) \ = 761\ J/g$$

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Now, when we want to calculate the ideal work requirement, we have got this cycle. In the ideal cycle, we say whatever is compressed gets liquefied and therefore, your point one only determines what the point f is? All right. So, we need to know only the state 1 or the starting point to get the ideal work requirement and we have done the previous calculation. The ideal work requirement is in W I upon m dot is equal to T 1 into s 1 minus s f minus h 1 minus h f. So, now, there are 2 points, which is 1 and f. So, if I get a value of entropy at 1 and f so, if I get value of entropy at 1 and f and if I get value of enthalpy at 1 and f. I can calculate the ideal work requirement per unit mass of gas which is compressed.

So, again as I said go to the TS diagram, temperature entropy diagram of nitrogen and gets point 1 and f in this particular case for ideal cycle, point 2 is actually not required in this case. So, the point 1 is 1 bar and 300 Kelvin, point f is 1 bar and 77 Kelvin, which is the boiling point of nitrogen get corresponding enthalpy and entropy for this. Apply the formula, this formula and put the values T 1 is equal to 300 Kelvin s 1 minus s f this bracket h 1 minus h f this bracket and if you calculate this the ideal work is 761 joule per gram. So, the ideal work requirement for nitrogen for this particular condition is 761 joule per gram when it is compressed at 300 Kelvin from 1 bar.

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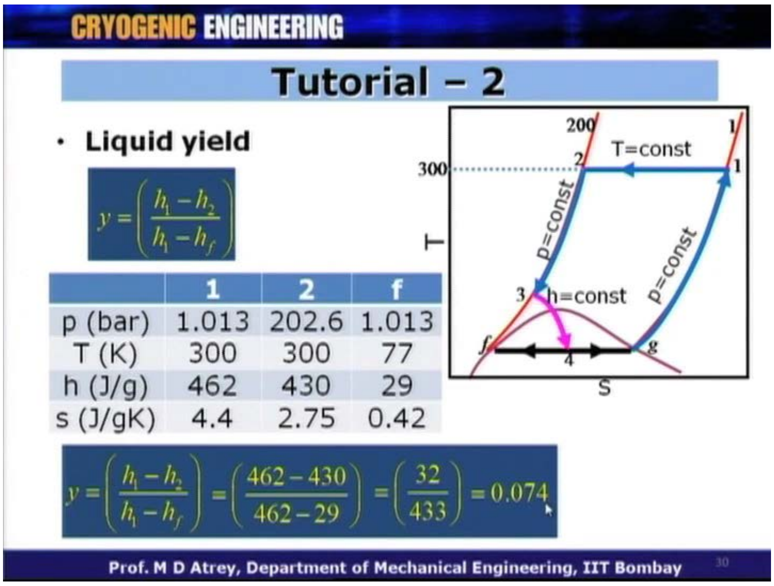

Tutorial – 2

- **Liquid yield**

$$y=\left(\frac{h_1-h_2}{h_1-h_f}\right)$$

	1	2	f
p (bar)	1.013	202.6	1.013
T (K)	300	300	77
h (J/g)	462	430	29
s (J/gK)	4.4	2.75	0.42

$$y=\left(\frac{h_1-h_2}{h_1-h_f}\right)=\left(\frac{462-430}{462-29}\right)=\left(\frac{32}{433}\right)=0.074$$

The liquid yield is h 1 minus h 2 upon h 1 minus h f; this is for the actual cycle. Now, the state 2 is at this point, which is 200 bar or 200 atmosphere again get data for 1, 2 and f. This is now, Linde - Hampson cycle. So, get all the values associated with this and calculate the value of y which is nothing, but m dot f upon m dot, which is the liquid yield.So, putting this value of h 1 minus h 2 upon h 1 minus h f is equal to this, what you get is 0.074. So, liquid yield is y which is 0.074 in this case.

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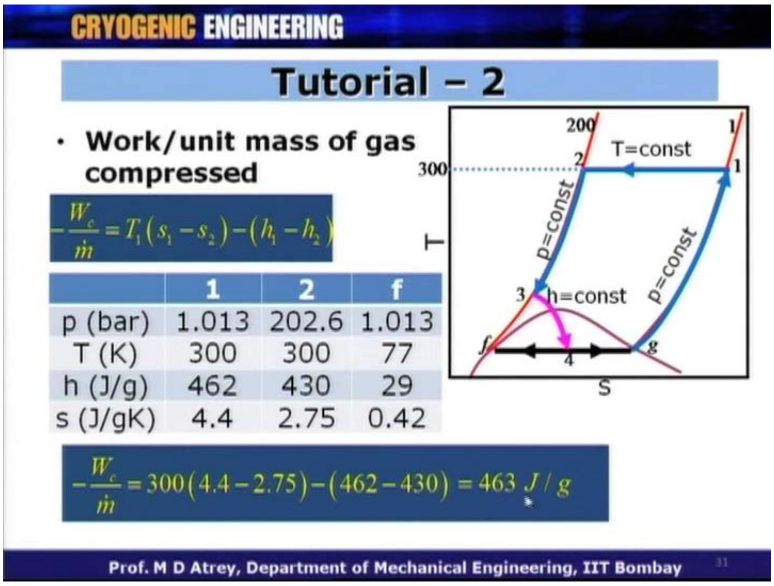

Tutorial – 2

- **Work/unit mass of gas compressed**

$$-\frac{W_c}{\dot{m}}=T_1(s_1-s_2)-(h_1-h_2)$$

	1	2	f
p (bar)	1.013	202.6	1.013
T (K)	300	300	77
h (J/g)	462	430	29
s (J/gK)	4.4	2.75	0.42

$$-\frac{W_c}{\dot{m}}=300(4.4-2.75)-(462-430)=463\ J/g$$

So, work per unit mass of gas which is compressed is now, T 1 into s 1 minus h 2 minus h 1 minus h 2 here not s 1 minus s f and not h 1 minus h f, which was the case for the ideal cycle. So, if I get the state 1 and state 2 and if I put this property in that particular case, I can calculate the value and my actual work of compression per unit mass of gas, which is compressed, is 463 joule per gram.

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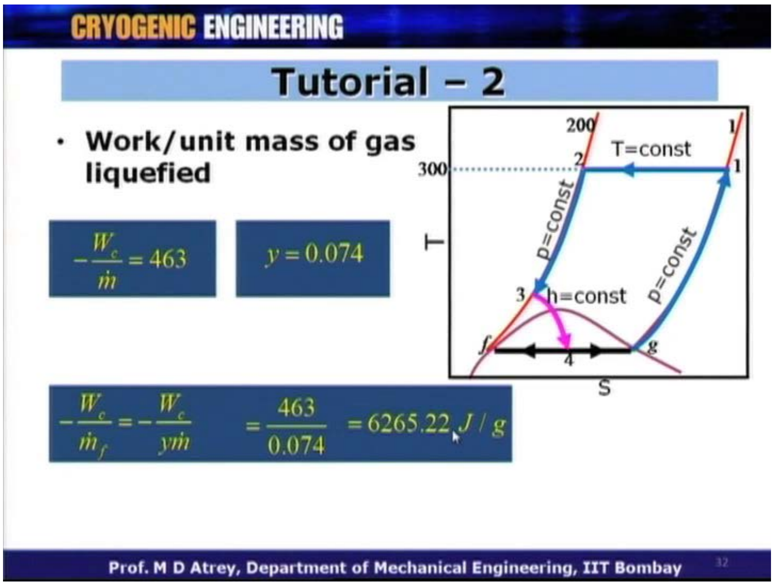

CRYOGENIC ENGINEERING

Tutorial – 2

- **Work/unit mass of gas liquefied**

$-\frac{W_c}{\dot m} = 463$

$y = 0.074$

$-\frac{W_c}{\dot m_f} = -\frac{W_c}{y\dot m} = \frac{463}{0.074} = 6265.22\ J/g$

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Now, I want to calculate work per unit mass of gas which is liquefied and not compressed, what we obtained earlier is work per unit mass of gas which is compressed. So, W c by m dot is 463, which is work per unit mass of gas which is compressed, y value is 0.074 and if I want to calculate. Now, work per unit mass of gas which is liquefied which is W c upon m dot f what I am going to do is, to divide W c by m dot by y, if you put the value of y here what you get is W C upon m dot f. So, 463 upon 0.074, gives you 6265.22 joule per gram all right.

So, this is rather high value as compare to what you got ideally. In ideal case work per unit mass of gas liquefied is equal to the work per unit mass of gas which is compressed. So, what we had calculated there is also work per unit mass of gas liquefied, what you are getting here also is. Now, work per unit mass of gas which is liquefied in this case.

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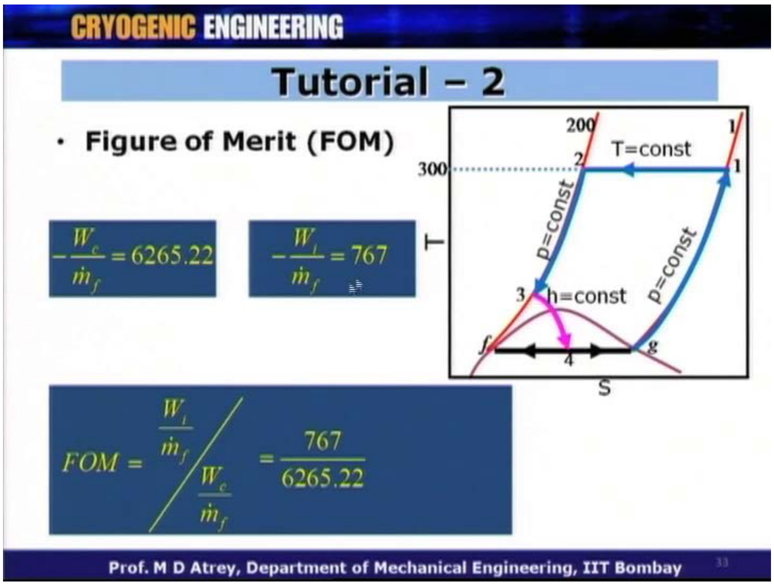

CRYOGENIC ENGINEERING

Tutorial – 2

- **Figure of Merit (FOM)**

$-\frac{W_c}{\dot{m}_f} = 6265.22$

$-\frac{W_i}{\dot{m}_f} = 767$

$$FOM = \frac{W_i}{\dot{m}_f} \bigg/ \frac{W_c}{\dot{m}_f} = \frac{767}{6265.22}$$

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So, if I want to calculate figure of merit or FOM which is I should get W c upon m dot f for actual case and ideal case is W I upon m dot f which is 767.So, the figure of merit is that the ideal work of compression per unit mass of gas which is liquefied, divided by the actual work per unit mass of gas which is liquefied. So, if I put that I will get 767 upon 6 to 65 and 0.1225. This is my figure of merit in this particular case all right. So, we have solved all the four required things for the tutorial number 2.

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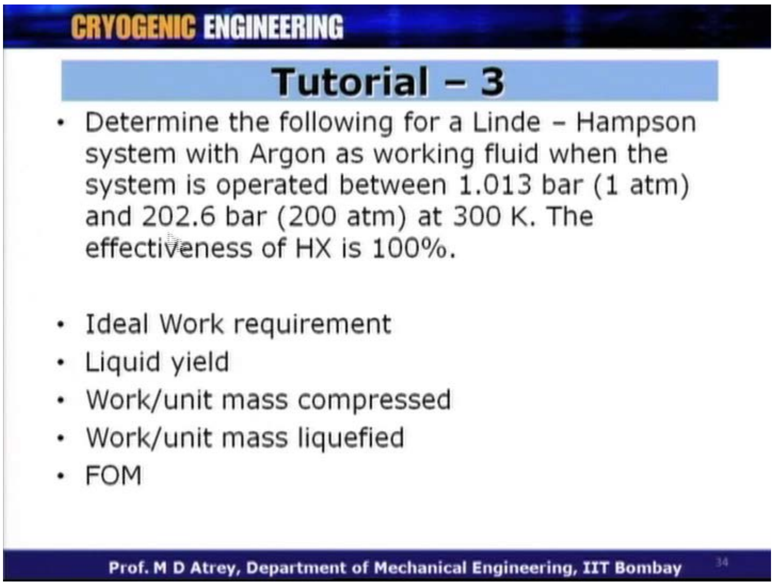

CRYOGENIC ENGINEERING

Tutorial – 3

- Determine the following for a Linde – Hampson system with Argon as working fluid when the system is operated between 1.013 bar (1 atm) and 202.6 bar (200 atm) at 300 K. The effectiveness of HX is 100%.

- Ideal Work requirement
- Liquid yield
- Work/unit mass compressed
- Work/unit mass liquefied
- FOM

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Now, I go to tutorial number 3, which is the same thing, but now, here I am changing the gas from nitrogen to Argon. So, determine the following for Linde - Hampson system with Argon as working fluid, when the system is operated from again 1 atmosphere to 200 atmospheres at 300 Kelvin. The effectiveness of heat exchanger is 100 percent. The tutorial is solved just to show the difference what happens, when I go from A gas to B gas all right. So, it is just repetition of the earlier tutorial just to get you habituated with how to read TS diagram for argon, TS diagram for nitrogen and repeat of the methodologies involved.

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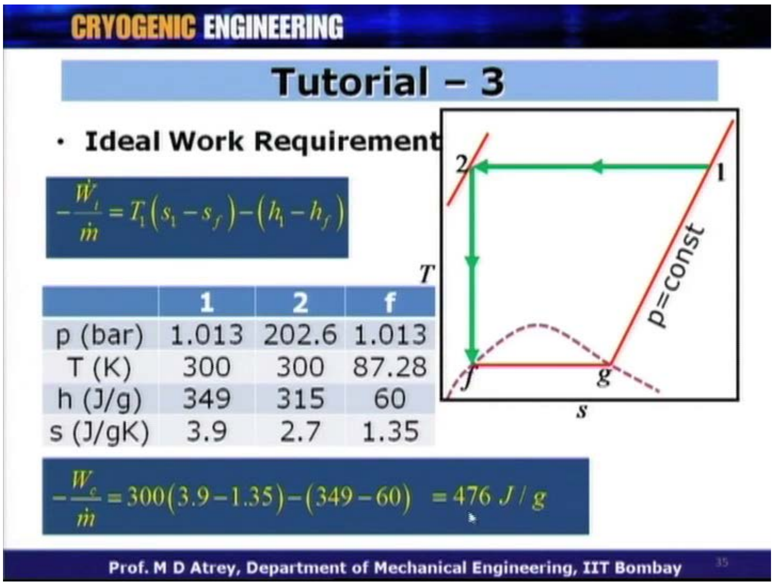

CRYOGENIC ENGINEERING

Tutorial – 3

- **Ideal Work Requirement**

$$-\frac{\dot{W}_i}{\dot{m}} = T_1\left(s_1 - s_f\right) - \left(h_1 - h_f\right)$$

	1	2	f
p (bar)	1.013	202.6	1.013
T (K)	300	300	87.28
h (J/g)	349	315	60
s (J/gK)	3.9	2.7	1.35

$$-\frac{\dot{W}_c}{\dot{m}} = 300(3.9 - 1.35) - (349 - 60) \quad = 476\ J/g$$

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So, again I will calculate ideal work of requirement, which is W I upon m dot. Now, I will go for a argon TS chart, the moment I go for argon TS chart I got different enthalpy and entropy value at point 1, 2 and f. As far as the ideal work is concerned I do not want to depend on this state 2, I have to go for 1 and f only while for actual case I will have to go for points 1, 2 and f. So, if I will get these values 1 and f and put in this equation, I will get the ideal work requirement as 476 joule per gram for Argon.

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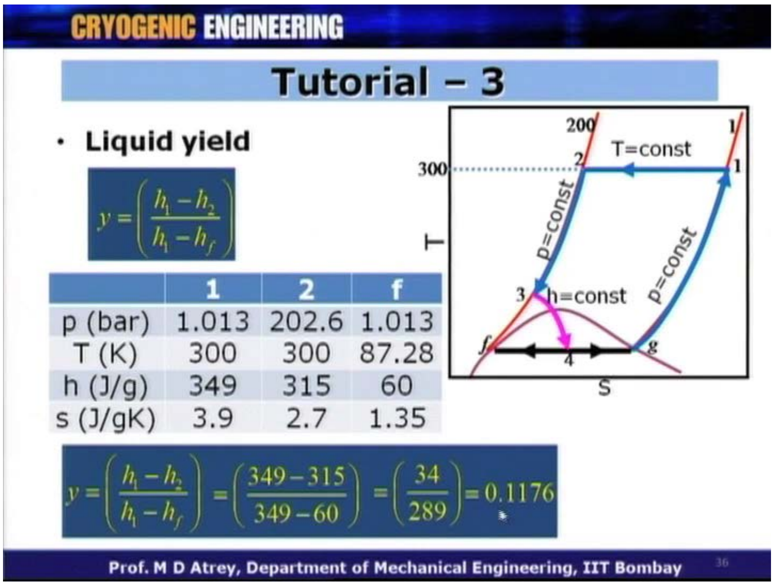

CRYOGENIC ENGINEERING

Tutorial – 3

- **Liquid yield**

$$y = \left(\frac{h_1 - h_2}{h_1 - h_f}\right)$$

	1	2	f
p (bar)	1.013	202.6	1.013
T (K)	300	300	87.28
h (J/g)	349	315	60
s (J/gK)	3.9	2.7	1.35

$$y = \left(\frac{h_1 - h_2}{h_1 - h_f}\right) = \left(\frac{349-315}{349-60}\right) = \left(\frac{34}{289}\right) = 0.1176$$

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The liquid yield is depending on the state 1 and state 2 h 1 minus h 2 upon h 1 minus h f again get a table and I can calculate the value of liquid in this case is 0.1176.

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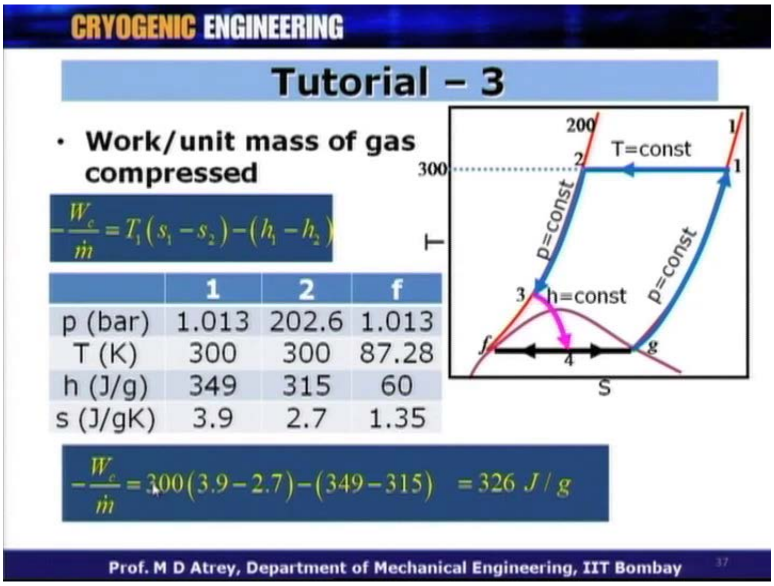

CRYOGENIC ENGINEERING

Tutorial – 3

- **Work/unit mass of gas compressed**

$$-\frac{W_c}{\dot{m}} = T_1(s_1 - s_2) - (h_1 - h_2)$$

	1	2	f
p (bar)	1.013	202.6	1.013
T (K)	300	300	87.28
h (J/g)	349	315	60
s (J/gK)	3.9	2.7	1.35

$$-\frac{W_c}{\dot{m}} = 300(3.9-2.7) - (349-315) \quad = 326\ J/g$$

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So, if I want to calculate work per unit mass of gas which is compressed my formula is W c upon m dot is T 1 into s 1 minus s 2 minus h 1 minus h 2, 1 and 2 are the state point for the compression processes. Again put the value of enthalpy and entropy in this equation and what you get is. Now, 326 joule per gram, this is the work per unit mass of gas compressed, if I want to calculate the work per unit mass of gas liquefied I will divide this value by y or the liquid yield.

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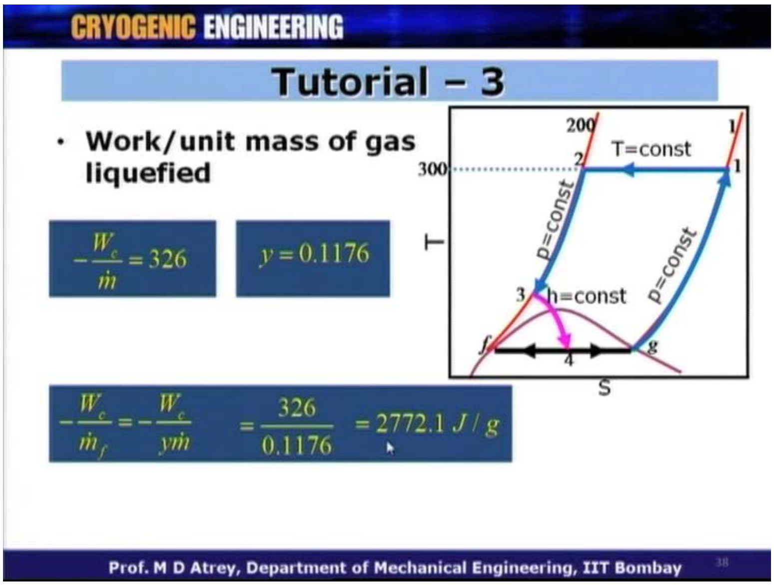

CRYOGENIC ENGINEERING

Tutorial – 3

- Work/unit mass of gas liquefied

$-\frac{W_c}{\dot{m}} = 326$

$y = 0.1176$

$$-\frac{W_c}{\dot{m}_f} = -\frac{W_c}{y\dot{m}} = \frac{326}{0.1176} = 2772.1\ J/g$$

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So, work per unit mass of gas liquefied is going to come from this value and corresponding y. So, W c upon m dot f is equal to W c upon m dot divided by y and now, work per unit mass of gas liquefied comes to be 2772.1.

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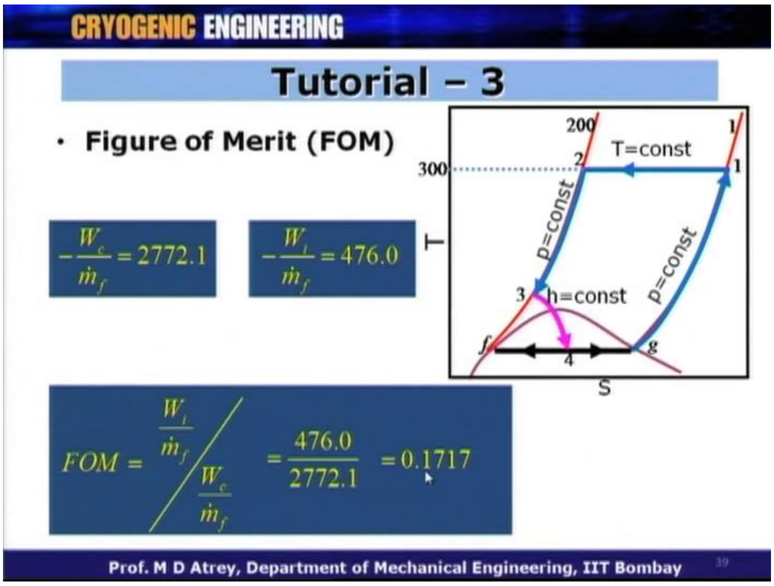

CRYOGENIC ENGINEERING

Tutorial – 3

- Figure of Merit (FOM)

$-\frac{W_c}{\dot{m}_f} = 2772.1$

$-\frac{W_i}{\dot{m}_f} = 476.0$

$$FOM = \frac{W_i / \dot{m}_f}{W_c / \dot{m}_f} = \frac{476.0}{2772.1} = 0.1717$$

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And finally, when I want to calculate the Figure of Merit W c upon m dot f is equal to 2772.1, W I upon m dot f is 476. This is the ideal work input per mass of gas which is liquefied, actual work input per mass of gas which is liquefied and this divided by this is nothing, but figure of merit.

So, figure of merit is ideal work input, actual work input per unit of gas which is liquefied putting this value over here. The figure of merit comes out to be 0.1717 for Argon, when compressed form 1 bar to 2 hundred bar at 300 Kelvin.

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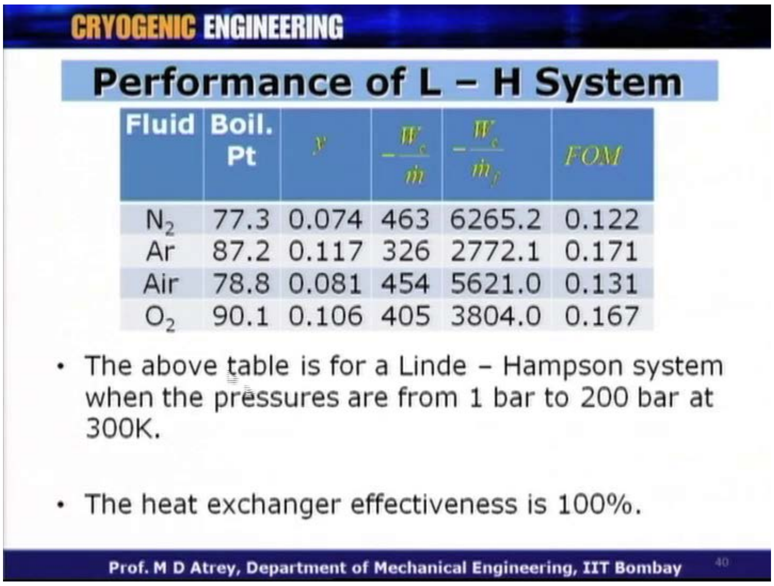

CRYOGENIC ENGINEERING

Performance of L – H System

Fluid	Boil. Pt	y	$-\frac{W_c}{\dot{m}}$	$-\frac{W_c}{\dot{m}_f}$	FOM
N_2	77.3	0.074	463	6265.2	0.122
Ar	87.2	0.117	326	2772.1	0.171
Air	78.8	0.081	454	5621.0	0.131
O_2	90.1	0.106	405	3804.0	0.167

- The above table is for a Linde – Hampson system when the pressures are from 1 bar to 200 bar at 300K.
- The heat exchanger effectiveness is 100%.

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We just solved the problem for nitrogen and argon and we found that the figure of merit in this case was around 0.12 and 0.17 corresponding boiling points are given, corresponding y values are given, liquid yield are given.

What is the work of compression per unit mass of gas which is compressed, what is the work of compression per unit mass of gas which is liquefied and then finally, what what you can do the figure of merit? We have also given the values for air and oxygen for which we have done calculations, but these 2 values I am going to give you for assignments at the end of this lecture. Please solve those assignments and compare your values with the values of Figure of Merit or y or these values given in this table, which will see about in the later part of this lecture.

The above table is for a Linde - Hampson cycle when the pressures are from 1 bar to 200 bar at 300 Kelvin. The heat exchanger in all these cases is to be 100 percent.

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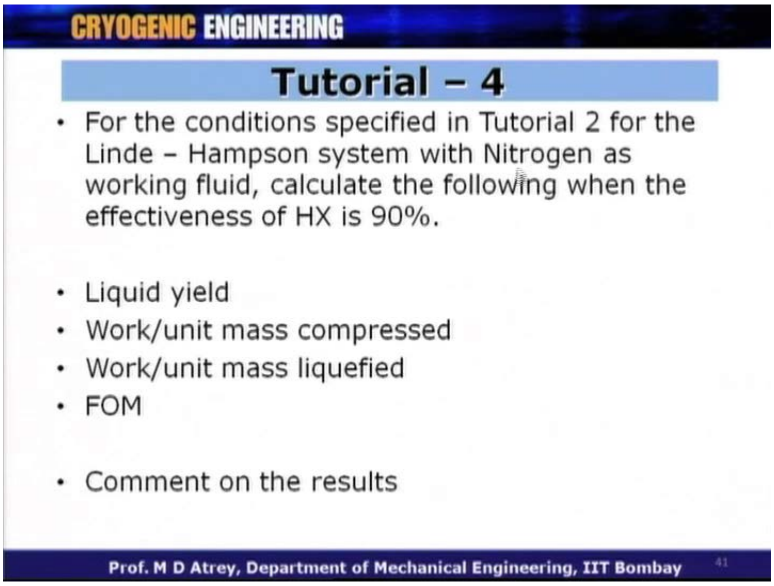

CRYOGENIC ENGINEERING

Tutorial – 4

- For the conditions specified in Tutorial 2 for the Linde – Hampson system with Nitrogen as working fluid, calculate the following when the effectiveness of HX is 90%.

- Liquid yield
- Work/unit mass compressed
- Work/unit mass liquefied
- FOM

- Comment on the results

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I am now, solving a tutorial number 4, which is basically made to understand again effectiveness of heat exchangers on all these parameters. So, for the conditions specified in the tutorial number 2 for Linde - Hampson system with nitrogen as working fluid, calculate the following when the effectiveness of heat exchanger is 90 percent. So, get a liquid yield work per unit mass compressed, work per unit mass of gas liquefied and Figure of Merit and ultimately what is most important is comment on the result. So, what you got earlier was for effectiveness of heat exchanger equal to 100 percent and what you calculate now, in this problem is going to be for the epsilon value equal to the 90 percent.

So, get the results for this problem and compare all this values with the tutorial number 1 or 2, which you got for nitrogen as working fluid all right and comment on the result is the most important thing, because what has happened. Because the effectiveness came down from 100 to 90 percent is very important to understand.

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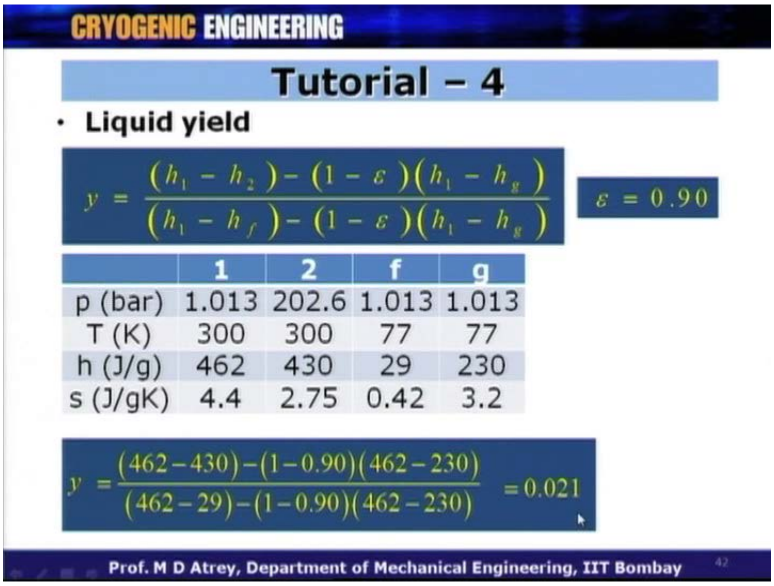

CRYOGENIC ENGINEERING

Tutorial – 4

- **Liquid yield**

$$y = \frac{(h_1 - h_2) - (1-\varepsilon)(h_1 - h_g)}{(h_1 - h_f) - (1-\varepsilon)(h_1 - h_g)}$$

$\varepsilon = 0.90$

	1	2	f	g
p (bar)	1.013	202.6	1.013	1.013
T (K)	300	300	77	77
h (J/g)	462	430	29	230
s (J/gK)	4.4	2.75	0.42	3.2

$$y = \frac{(462-430)-(1-0.90)(462-230)}{(462-29)-(1-0.90)(462-230)} = 0.021$$

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So, the process is same and the formula changes now, because epsilon comes into picture. So, this is the formula I am going to use now, in this case. So, epsilon is equal to 90 percent and this table now, 1, 2, f and g what you get is. y is equal to in this case 0. 021. So, y has suddenly decreased now, to 0.021.

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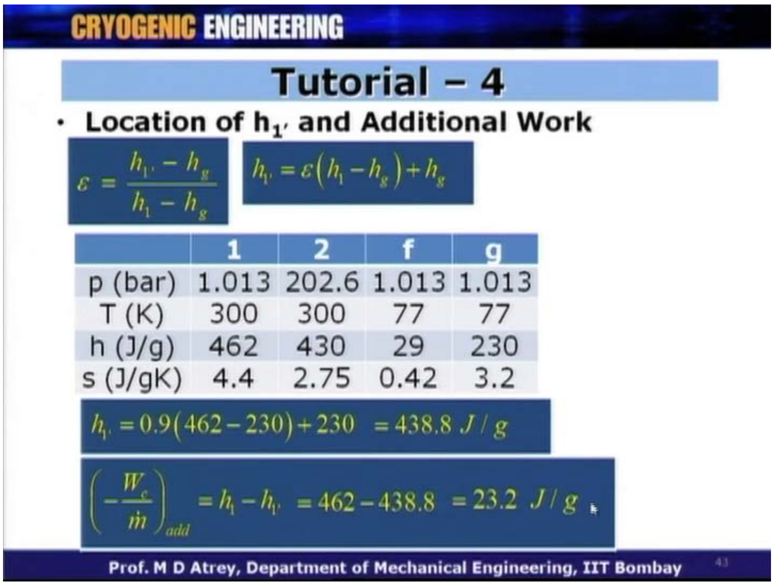

CRYOGENIC ENGINEERING

Tutorial – 4

- **Location of $h_{1'}$ and Additional Work**

$$\varepsilon = \frac{h_{1'} - h_g}{h_1 - h_g}$$

$$h_{1'} = \varepsilon(h_1 - h_g) + h_g$$

	1	2	f	g
p (bar)	1.013	202.6	1.013	1.013
T (K)	300	300	77	77
h (J/g)	462	430	29	230
s (J/gK)	4.4	2.75	0.42	3.2

$$h_{1'} = 0.9(462-230)+230 = 438.8\ J/g$$

$$\left(-\frac{W_c}{\dot{m}}\right)_{add} = h_1 - h_{1'} = 462 - 438.8 = 23.2\ J/g$$

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In order to calculate, the additional work that is to be done we have to calculate what is h 1 dash, if you remember from earlier discussion h 1 dash is equal to epsilon h 1 minus h g plus h g putting those values epsilon value is 90 percent h 1 dash is equal 438.8. So, if I want to calculate now, the additional work of the compressor does which is equal to h 1 minus h 1 dash is equal to 23.2 joule per grams.

So, the compressor does this much additional work in 462, which is almost 5 percent of what it was earlier. So, 5 percent more work has to be done in order to, get the enthalpy from h 1 dash to h 1 and then the compress the gas from h 1 to h 2 in this case when the effectiveness of the heat exchanger is 90 percent.

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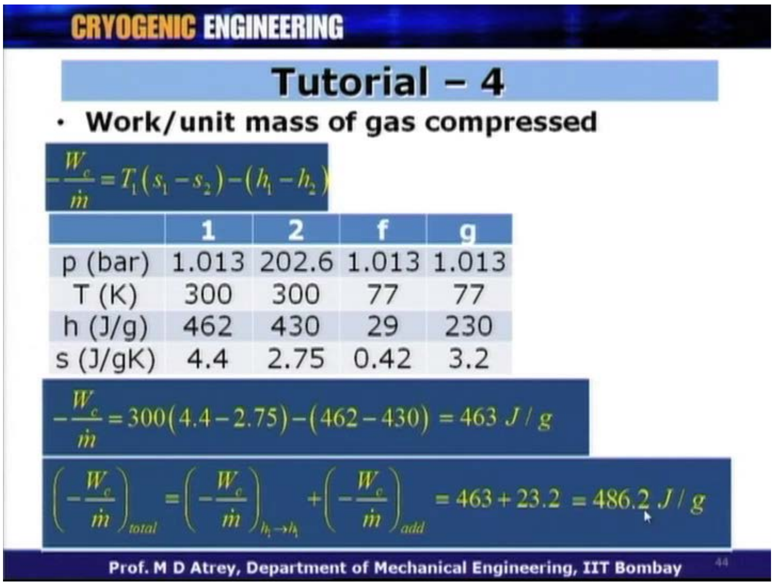

CRYOGENIC ENGINEERING

Tutorial – 4

- **Work/unit mass of gas compressed**

$$-\frac{W_c}{\dot{m}} = T_1(s_1 - s_2) - (h_1 - h_2)$$

	1	2	f	g
p (bar)	1.013	202.6	1.013	1.013
T (K)	300	300	77	77
h (J/g)	462	430	29	230
s (J/gK)	4.4	2.75	0.42	3.2

$$-\frac{W_c}{\dot{m}} = 300(4.4 - 2.75) - (462 - 430) = 463\ J/g$$

$$\left(-\frac{W_c}{\dot{m}}\right)_{total} = \left(-\frac{W_c}{\dot{m}}\right)_{h_1 \rightarrow h_1} + \left(-\frac{W_c}{\dot{m}}\right)_{add} = 463 + 23.2 = 486.2\ J/g$$

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So, work per unit mass of gas compressed is the same formula, I will get these values and calculate this and what you get now, is 463 over here. So, if I want to get total work done is equal to this is the work done from point 1 to 2. So, point 1 to 2 plus additional work which is going to be done is going to give me actual work, but the total work of compression per unit mass of gas which is compressed which is now, 486.2. So, earlier case it was 463 now, it is 486.2.

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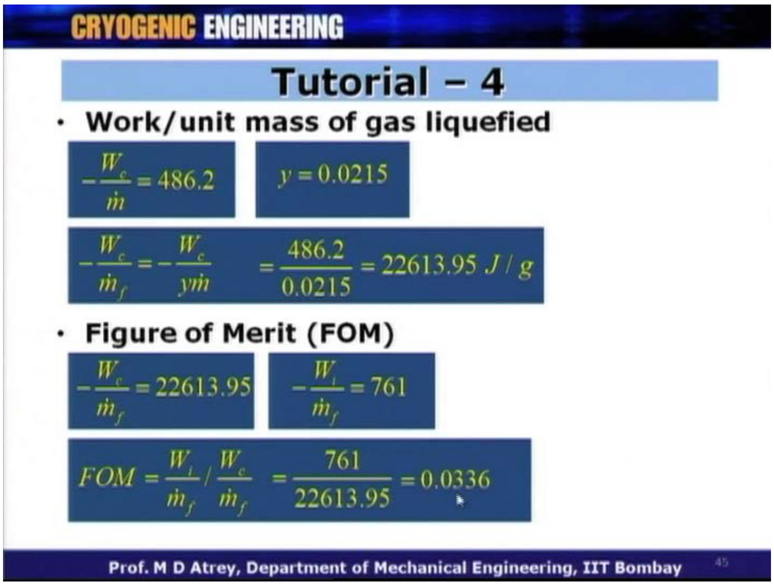

CRYOGENIC ENGINEERING

Tutorial – 4

- **Work/unit mass of gas liquefied**

$-\frac{W_c}{\dot{m}} = 486.2$ $\quad y = 0.0215$

$$-\frac{W_c}{\dot{m}_f} = -\frac{W_c}{y\dot{m}} = \frac{486.2}{0.0215} = 22613.95\ J/g$$

- **Figure of Merit (FOM)**

$-\frac{W_c}{\dot{m}_f} = 22613.95$ $\quad -\frac{W_i}{\dot{m}_f} = 761$

$$FOM = \frac{W_i}{\dot{m}_f} / \frac{W_c}{\dot{m}_f} = \frac{761}{22613.95} = 0.0336$$

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Work per unit mass of gas which is liquefied, it is this divided by y which is very small 0.021. So, I get now, W c upon m dot f is equal to 22613.95. This is rather high as compared to what it was earlier. And the corresponding figure of merit now, here is W I upon m f dot is equal to 761, W c upon m f dot is this if I divide this therefore, FOM comes to be 0.03.

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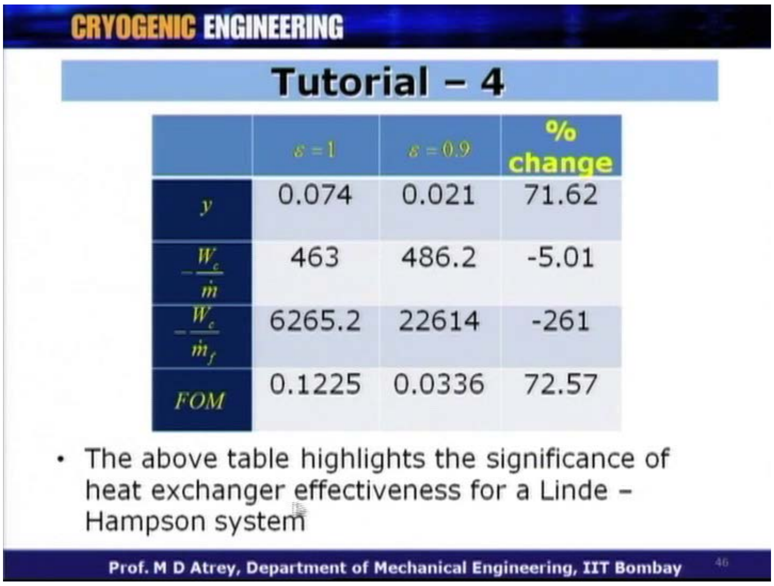

CRYOGENIC ENGINEERING

Tutorial – 4

	$\varepsilon = 1$	$\varepsilon = 0.9$	% change
y	0.074	0.021	71.62
$-\frac{W_c}{\dot{m}}$	463	486.2	-5.01
$-\frac{W_c}{\dot{m}_f}$	6265.2	22614	-261
FOM	0.1225	0.0336	72.57

- The above table highlights the significance of heat exchanger effectiveness for a Linde – Hampson system

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So, you can see if I want to summarize, what you can see is. These are the 2 columns when the epsilon is 100 percent, when the epsilon is 90 percent in this case. So, you can compare the y value, the y value for 100 percent effectiveness was 0.074 and it has come to 0.021, the percentage decrease has been 71 percent. If I see the compressor work per mass of gas which is compressed, it has increased from 463 to 486 and the 4 percent changes minus 5 percent. If I see the compression work done per unit mass of gas which is liquefied, it has been rather high 6265 it has increased to 22614; that means, there is an increase of 261 percent.

This is happening, because y value had decreased drastically by 71 percent and finally, the figure of merit has decreased by almost 72 percent. So, this example shows that as soon as you reduce down from 100 percent to 90 percent. There are so many changes in drastic changes are happening in the yield value as well as figure of merit and therefore, one has to be very careful in the effectiveness calculation and heat exchanger design. These are very very important design aspects of any liquefaction system. So, this example highlights the importance of epsilon value for any cycle in cryogenic liquefaction of gases all right. So, above table is a significance of heat exchanger effectiveness for a Linde - Hampson system.

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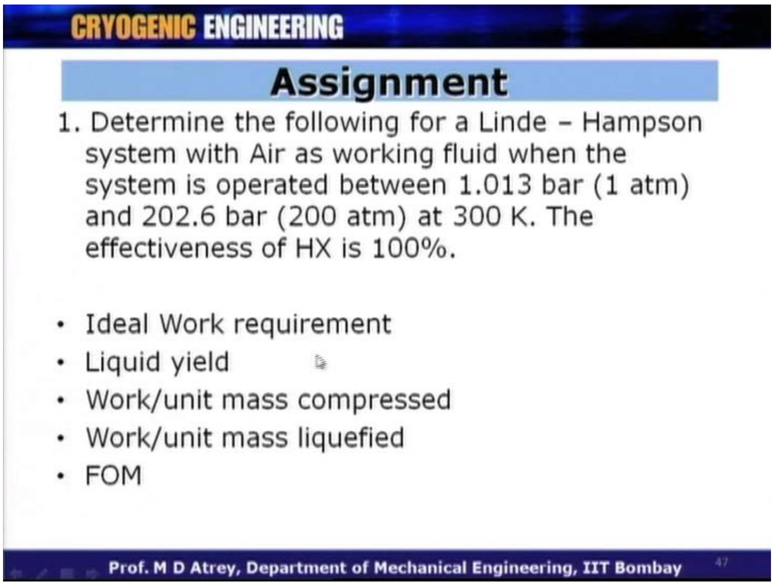

CRYOGENIC ENGINEERING

Assignment

1. Determine the following for a Linde – Hampson system with Air as working fluid when the system is operated between 1.013 bar (1 atm) and 202.6 bar (200 atm) at 300 K. The effectiveness of HX is 100%.

- Ideal Work requirement
- Liquid yield
- Work/unit mass compressed
- Work/unit mass liquefied
- FOM

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And we have got assignments here, based on what we have done with air as a working fluid calculate this, we have done for nitrogen and argon calculate all these things. Please do it for air and oxygen.

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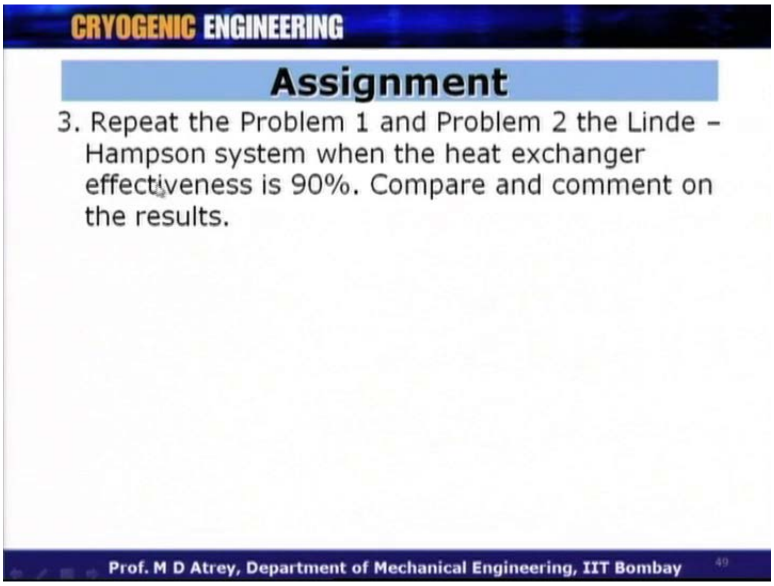

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Assignment

3. Repeat the Problem 1 and Problem 2 the Linde – Hampson system when the heat exchanger effectiveness is 90%. Compare and comment on the results.

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And also repeat the same problem when the effectiveness is 90 percent and compare and comment on the results. Please do these 3 assignments. This will really give you a feel for the values for these 2 particular gases. Finally, to summarize what we have learned today.

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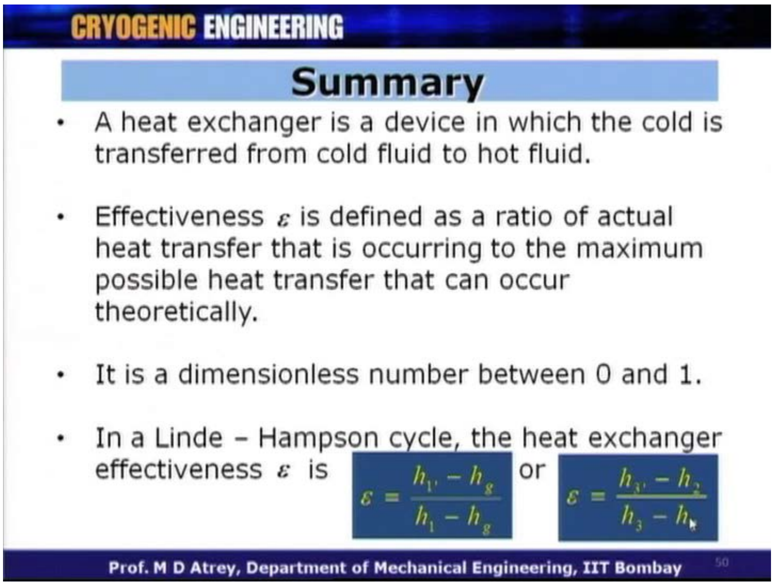

CRYOGENIC ENGINEERING

Summary

- A heat exchanger is a device in which the cold is transferred from cold fluid to hot fluid.
- Effectiveness ε is defined as a ratio of actual heat transfer that is occurring to the maximum possible heat transfer that can occur theoretically.
- It is a dimensionless number between 0 and 1.
- In a Linde – Hampson cycle, the heat exchanger effectiveness ε is $\varepsilon = \frac{h_{1'} - h_g}{h_1 - h_g}$ or $\varepsilon = \frac{h_{3'} - h_2}{h_3 - h}$

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A heat exchanger is a device in which the cold is transferred from cold fluid to hot fluid. Everybody knows this; the effectiveness epsilon is defined as ratio of actual heat transfer that is occurring to the maximum possible heat transfer that can occur theoretically in a heat exchanger. It is a dimensionless number between 0 and 1.

In a Linde - Hampson cycle, the heat exchanger effectiveness is epsilon is equal to h 1 dash upon h g divided by h 1 minus h g or on a high pressure side, which is h 3 dash minus h 2 divided by h 3 minus h 2.

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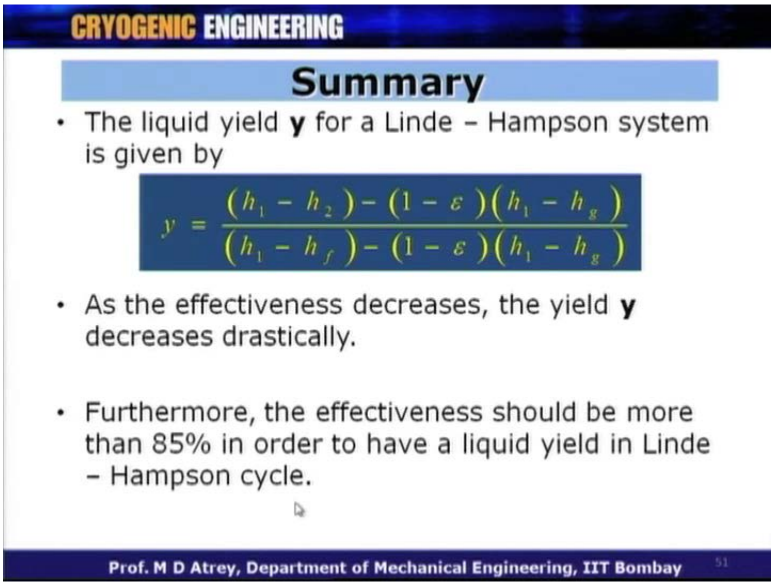

CRYOGENIC ENGINEERING

Summary

- The liquid yield **y** for a Linde – Hampson system is given by

$$y = \frac{(h_1 - h_2) - (1 - \varepsilon)(h_1 - h_g)}{(h_1 - h_f) - (1 - \varepsilon)(h_1 - h_g)}$$

- As the effectiveness decreases, the yield **y** decreases drastically.
- Furthermore, the effectiveness should be more than 85% in order to have a liquid yield in Linde – Hampson cycle.

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The liquid yield y of a Linde - Hampson cycle is given by this expression where epsilon plays a very important role as we have just found. As the effectiveness decreases, the yield y decreases drastically. Furthermore, the effectiveness should be more than 95 percent in order to have liquid yield in Linde - Hampson cycle.

Thank you very much.