

**Cryogenic Engineering**  
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**Module No. # 01**  
**Lecture No. # 10**  
**Gas Liquefaction and Refrigeration Systems**

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**CRYOGENIC ENGINEERING**

**Earlier Lecture**

- For a real gas, J - T coefficient  $\left(\frac{\partial T}{\partial p}\right)_h$  depends on  $T_{INV}$ .
- The isentropic expansion of a gas always results in cooling irrespective of its initial state.
- J - T expansion is normally used where phase changes are required, while an isentropic expansion is used for single phase fluids.
- The isentropic expansion coefficient is  $\left(\frac{\partial T}{\partial p}\right)_s$ .

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Welcome to the 10th lecture of cryogenic engineering, under the NPTEL program. Take an overview of what we learnt in the last class - we found that for liquefaction of real gas, J - T coefficient, that is,  $\frac{\partial T}{\partial p}$  at constant enthalpy, depends on inversion temperature. So, we learnt the concept of inversion temperature, which is nothing, but  $\frac{\partial T}{\partial p}$  at constant enthalpy.

We found that, in order to get J - T equaling effect for a real gas, the temperature in the initial state of the gas should be less than T inversion temperature. Then we went away from isenthalpic process, to isentropic process, using reciprocating expansion engine, or Turbo-expanders. We understood that, the isentropic expansion of a gas always results in cooling, irrespective of its initial state. So, as it was in J - T expansion, which depends on the initial state, the isentropic expansion does not depend on the initial state.

The J - T expansion is normally used where phase changes are required. This is a very important thing that, during J - T expansion, normally there is phase change, while in an

isentropic expansion it is used for single phase fluids. So, the inlet is gas; the outlet also should be gas, while it is not true with J - T expansion. In J - T expansion, inlet could be gas, or a two phase mixture, and outlet will be liquid, or a two phase mixture.

The isentropic expansion coefficient is nothing but,  $\Delta T$  by  $\Delta p$ , at constant enthalpy. So,  $\Delta T$  by  $\Delta p$  at constant  $s$ . This was  $\Delta T$  by  $\Delta p$  at constant  $h$ , for J - T expansion.

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**Earlier Lecture**

- The gases like Air,  $N_2$ , show J - T cooling when expanded at room temperature while He,  $H_2$ , Neon are required to be pre-cooled to result in J - T cooling.
- In a thermodynamic ideal system, all the gas that is compressed gets liquefied.
- Using the ideal thermodynamic cycle, one can calculate the ideal work requirement for liquefaction of unit mass of a given gas.
- This Ideal Work requirement depends on the initial condition of the gas ( $p$  and  $T$ ).

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We also studied that, the gases like air, and nitrogen show J - T cooling, when expanded at room temperature, while other gasses like helium, hydrogen, and neon require to be pre-cooled to result in J - T cooling. This is going to the fact that, the inversion temperature of helium, hydrogen, and neon is below ambient temperature, and therefore, if you want to have J - T cooling for these gases, the initial state of the gas should be brought down below its inversion temperature; and therefore, they need to be pre-cooled, and only then they can result in J - T cooling.

We studied thermodynamic ideal system for gas liquefaction where, in a thermodynamic ideal system, all the gas that is compressed gets liquefied. Using the ideal thermodynamic cycle, one can calculate the ideal work requirement for liquefaction of unit mass for a given gas. So, the ideal thermodynamic cycle basically teaches us, or basically gives us, the ideal work requirement to liquefy the unit mass of a given gas.

This ideal work requirement depends on the initial condition of the gas, which is nothing but the initial pressure, and temperature condition of the given gas.

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**Outline of the Lecture**

**Topic : Gas Liquefaction and Refrigeration Systems (contd)**

- Parameters of Gas Liquefaction systems
- Linde - Hampson system
  - Liquid yield
  - Work requirement
  - Optimization of liquid yield

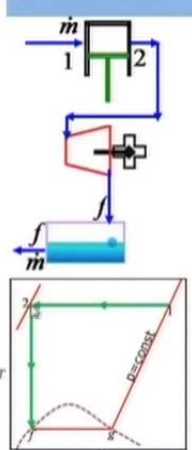
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Extending whatever we have learnt in the earlier lecture, the outline for today's lecture again holds around the topic of 'gas liquefaction and refrigeration systems'; and what we study under this today are parameters of gas liquefaction systems, and then we come to the first practical system, which is Linde - Hampson system, where we talk about - liquid yield, work requirement, and optimization of liquid yield. All these are calculations; and based firmly on their derivation, we will try to understand how Linde - Hampson system behaves.

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**CRYOGENIC ENGINEERING**

### Introduction



- As seen earlier, the schematic of an Ideal system and its T - s diagram are as shown.
- The processes of compression and expansion are from **1→2** and **2→f** respectively.
- The initial condition **1** of the gas determines the position of point **f**. The point **2** determines the final state of the gas after the compression process.

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This is an ideal system. As seen earlier, the schematic diagram of an ideal system, and its T - s diagram is given here. This is the ideal system, wherein you have got a gas input, you have got a compressor, and you have got an expander. A corresponding T - s diagram is given here; where, 1 to 2 is the compression process, and 2 to f is an isentropic expansion process. This is the ideal thermodynamic cycle. The process of compression and expansion are from, 1 to 2, and 2 to f, respectively. So, 1 to 2 is the compression; 2 to f is the expansion process.

The initial condition of gas at point 1, point 1 is the initial condition, and it could be ambient. 300 Kelvin, and 1 bar could be the standard operating condition, for the initial condition, and this determines the position of f. As you can see that, the point 1 determines the location of point f because, this is a 1 bar line, and this is corresponding to the boiling point of the gas at 1 bar. For nitrogen it will be - 77 Kelvin.

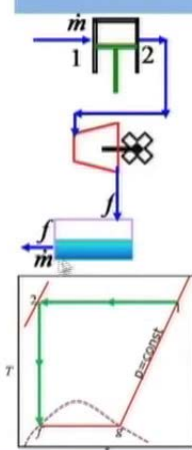
As soon as your point 1 gets determined, your point f is always known. Therefore, we say, the initial condition 1 of the gas, determines the position of point f. The point 2 determines the final state of the gas after the compression process. So, point 2 comes over here, which is nothing but a point, which is perpendicular to this line. So, this is an isentropic expansion, the entropy remains constant. So, if you extend this line further up, and if you draw a horizontal line, temperature remaining constant, the intersection of

these two lines will be nothing but the point 2, which is the final state, after compression process from 1 to 2.

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### Introduction



- Lets us take an example of  $N_2$  and the initial condition at point **1** be ambient (1 bar (0.9869 atm), 300 K).
- The required pressure at point **2** to follow an Ideal cycle is more than 70000 bar (690846.3 atm).
- Such high pressures are impractical and hence there is a need to modify the system to lower the maximum pressures.

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Let us take an example of nitrogen. The initial condition at point 1, which is 1 bar, and 300 Kelvin, we can find out now, as soon as your 1 bar and 300 Kelvin is determined, we know the point f; and if you draw a vertical line here, and if I draw a horizontal one at this point, the intersection at this point will give the final condition after the compression process - 1 to 2. Here, we can understand the required pressure at point 2 to follow an ideal cycle, is more than 70,000 bar. This is a very, very high pressure; absolutely impractical to attempt in practice. So, what you want to conclude from here is that, such high pressures are impractical; and hence, there is a need to modify the system to lower, maximum pressures.

What we understand from here is, if you want to have ideal thermodynamic cycle for gas liquefaction, you have to liquefy all the gas that is getting compressed, and if you were to do this, the point 2 lies at a very high pressure, and it is as high as 70,000 bar, which is absolutely impractical to attain; and therefore, what we say as ideal thermodynamic cycle for liquefaction is impractical, and hence, we have to modify this system. So, we lower the maximum pressures, which are normally attainable using the compressors available.

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### Introduction

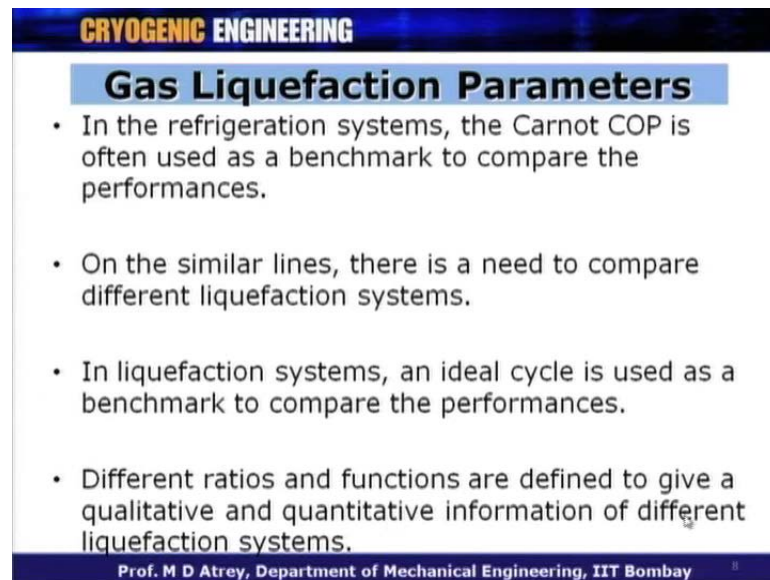
- As stated earlier, devices like heat exchangers, J - T valve, turbo expanders can be used to modify the systems.
- The heat exchangers are used to conserve cold and J - T devices are used to achieve lower temperatures.
- The following slides explain various cycles that are used for gas liquefaction.

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In the first lecture, I had talked about heat exchangers like, joule Thomson valve, and turbo expanders, all these systems could be used to modify the system, and now that we talked about changing the ideal thermodynamic cycle to get a relatively practical cycle, we will have to use all these components as a two-way heat exchanger - one gas coming in, and one gas coming in from this direction. These are the turbo expanders, and other is the J - T valve.

The heat exchangers are used to conserve cold and J - T devices are used to achieve lower temperatures. This is what all of you know. Now, the following slides in this lecture will explain various cycles, that are used for gas liquefaction.

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### Gas Liquefaction Parameters

- In the refrigeration systems, the Carnot COP is often used as a benchmark to compare the performances.
- On the similar lines, there is a need to compare different liquefaction systems.
- In liquefaction systems, an ideal cycle is used as a benchmark to compare the performances.
- Different ratios and functions are defined to give a qualitative and quantitative information of different liquefaction systems.

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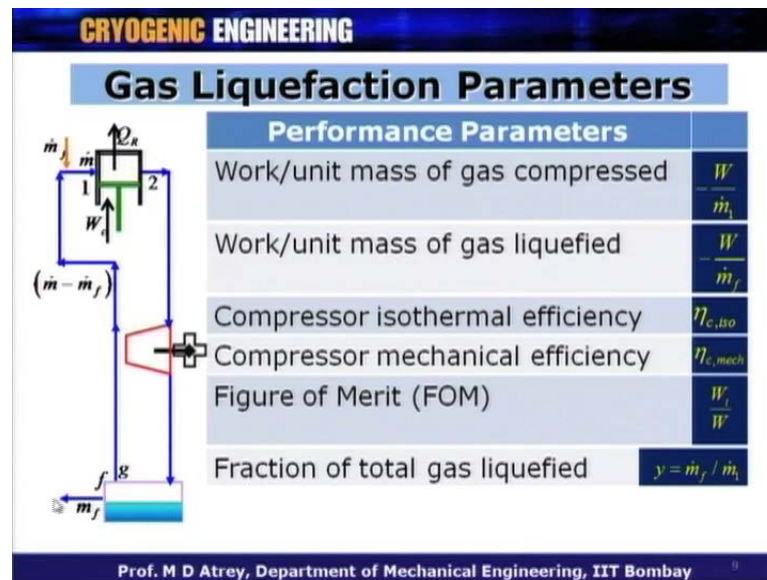
So now, we will come to the imported cycles. Before we go into various liquefaction cycles, let us find out what the different gas liquefaction parameters are. So, we have got different cycles, and if I were to compare to this different cycles, I will have to compare different parameters. What are these parameters? In the refrigeration system, for example, the Carnot COP is often used as a benchmark to compare the performances.

On a similar line, there is a need to compare different liquefaction systems. There are different cycles, and therefore, we will have some parameters so that, we can compare the performance of cycle A to cycle B, to cycle C. In liquefaction systems, an ideal cycle is used as a benchmark to compare the performances. So, the way of comparison is to compare a cycle's performance with an ideal cycle.

Different ratios and functions are defined to give qualitative and quantitative information on different liquefaction cycles.



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What are these different ratios and functions? If you take a typical example of this cycle, you have got a compressor and an expander, the gas  $\dot{m}$  is compressed at this point, out of which,  $\dot{m}_f$  gets liquefied, and therefore, the gas which returns to the cycle is  $\dot{m}$  minus  $\dot{m}_f$ . This gas goes in here, and replenishment is done at this point, which is  $\dot{m}_f$ . So, whatever mass goes out is replenished over here, and the cycle continues. So, from here,  $\dot{m}$  gets compressed, and  $\dot{m}_f$  gets liquefied. So, what are the different parameters?

At the same time, what you can see here, the power input to the compressor is  $W_C$ , and the heat ejected to the atmosphere is  $Q_R$ , or the heat of compression. The first parameter is work required per unit mass of gas compressed. So, the work required is  $W_C$ , and the gas which is compressed  $\dot{m}_1$ . So,  $W_C$  upon  $\dot{m}_1$  is the work per unit mass of gas compressed. The next is, work per unit mass of gas liquefied. So, the gas liquefied is  $\dot{m}_f$ , and therefore, the work required is  $W_C$  upon  $\dot{m}_f$ . The minus stand is given for work done on the system, which is in this case, the work done on the compressor, and therefore, we are calculating the work done per unit mass of gas liquefied, and work done for a mass of gas compressed.

The compressor isothermal efficiency, we are assuming that the compressor is isothermal, however, in the actual case the compressor may not show isothermal



compression, and therefore, we have an efficiency, which is called 'compressor isothermal efficiency'.

Similarly, we have got compressor mechanical efficiency, which amounts to all the irreversibility or the frictional losses in a system and it is called as 'eta c mechanical' or 'efficiency mechanical'. Then we define one more parameter called 'Figure of Merit' or FOM. This is the ratio of ideal work input, based on the ideal thermodynamics cycle, divided by actual work input of the system. So naturally, the ideal work input is going to be less than the actual work input of the system. So, this ratio is going to be less than 1, and this is called as the 'Figure of Merit' of any cycle.

Then is the fraction of total gas liquefied. So, you can see here that,  $y$  represents the ratio of  $m \dot{f}$  divided by  $m \dot{}$ . When the  $m \dot{}$  gas is getting compressed, the amount of gas, which is liquefied, is only  $m \dot{f}$ . So,  $m \dot{f}$  upon  $m \dot{}$  is nothing but, the fraction of the total gas liquefied.

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## Fundamentals

**Sign Convention**

- The work done by the system is taken as positive.
- The heat transferred to the system is taken as positive.

**Pressure Measurement**

- Bar or Pascal is the S.I. unit. The conversion table is as follows.

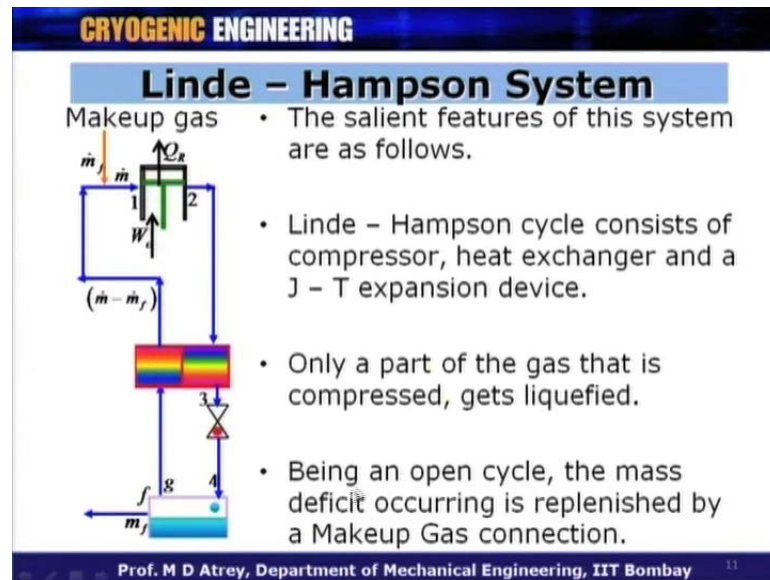
Pressure
1 Pa = 1 N/m <sup>2</sup>
1 bar = 10 <sup>5</sup> Pa
1 atm = 1.01325 bar

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With these parameters, now let us see some fundamentals, which would help us to finalize the liquefaction cycle. The sign convention that I just talked about is that, the work done by the system is taken as positive. Similarly, the work done on the system will be taken as negative. The heat transferred to the system is taken as positive, and the heat leaving the system will be taken as negative.

In all these cases, the pressures we are using is bar or Pascal; the conversion table is - 1 Pascal is equal to 1 Newton per meter square, and 1 bar is equal to 10 to the power 5 Pascal, and 1 atmosphere is nothing but - 1.01325 bar. In actual cases, I will assume that 1 atmosphere is equal to 1 bar, as you can see from this approximate expression.

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With this background, let us come to the Linde - Hampson cycle, which is the first practical cycle taking us away from the ideal thermodynamics cycle. This is the cycle; this is the schematic representation of the Linde - Hampson cycle. What it has is, you can see over here that it has got a compressor; it has got a heat exchanger, it has got a J - T wall, and it has got a container. The gas goes back, and the gas is replenished. So, in the sense, if you see the thermodynamics cycle to this cycle, what we have added is a heat exchanger. In the ideal cycle, all the gas which was getting compressed had got liquefied, while here, that is not happening; only some of the gases are getting liquefied, when the remaining gas, which is  $m - m_f$  dot is coming back to the compressor, and  $m$  dot  $f$  gas is replenished, or the makeup gas is added here.

With this background let us see the salient features of this system. The Linde - Hampson cycle consists of a compressor, a heat exchanger, and a J - T expansion wall. This is what we have seen. Only a part of the gas that is compressed gets liquefied; only  $m$  dot  $f$  is getting liquefied out of  $m$  dot which is compressed. Being an open cycle, the mass deficit occurring is replenished by a makeup gas connection.

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### Linde – Hampson System

- All the processes are assumed to be ideal in nature and there are no irreversible pressure drops in the system.
- Compression process is isothermal while the J - T expansion is isenthalpic.
- The system incorporates a Two-Fluid heat exchanger which is assumed to be 100% effective.

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We have just seen that all the processes are assumed to be ideal in nature, and there are no irreversible pressure drops in the system. This is the assumption which will be used for analyzing this system.

We assume that the compression process is isothermal, while J - T expansion is isenthalpic, and we also assume that the system incorporates a two-fluid heat exchanger, which is assumed to be 100% effective. So, it is a perfectly ideal system; however, the heat exchanger here also is assumed to be 100% effective.

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### Linde – Hampson System

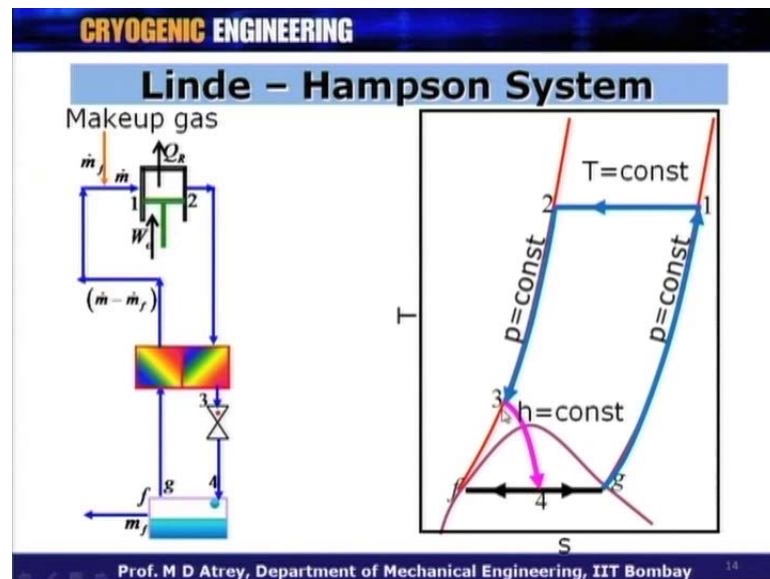
- The heat exchange process is an isobaric process and it is used to conserve cold in the system.
- That is, the stream of gas (**2→3**) is cooled by the stream of gas (**g→1**).
- The J - T expansion device is used for phase change of gas stream to liquid stream by lowering the temperature.

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The heat exchanger process is an isobaric process, and it is used to conserve cold in the system; that means, when both the fluids are travelling through the heat exchanger, there is no pressure drop across the heat exchanger. That is, the stream 2 to 3, and g to 1- these points are the direction of the fluids flowing through here, the stream of the gas 2 to 3 is cooled by the stream of the gas going back.

The J - T expansion device is used for phase change of gas stream to liquid stream, by lowering the temperature. So, once you get gas at this point, what you get at this point is a two-phase, or the liquid gets collected and gas goes back at this point.

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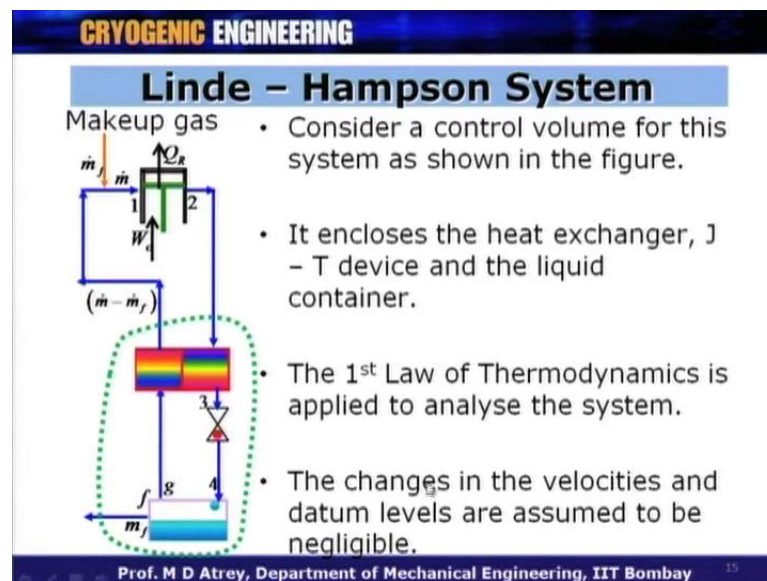


This is the most important thing, and what you have to learn from here is, how to draw a corresponding temperature entropy diagram, or a T - s diagram, from a given cycle.

So, what you can see from here is, the 1 to 2 process is an isothermal compression, and 2 to 3 is an isobaric heat exchanger process. The pressure remains constant, and the temperature gets lowered, because of the return gas stream, which is from g to 1. So, g to 1, because of this low temperature gas, which is going back to the compressor, the temperature gets down from 2 to 3. At the point 3, the gas gets expanded from 3 to 4; and as soon as the expansion happens, the state of the gas lies in the dome. This is what we say that, the state of the gas at this point 4 is a two-phase gas; that is, you got a liquid and some gas. The liquid is collected at point f of which, some of the liquid is drawn out, or the remaining gas will go back, and this will pre-cool the incoming gas from 2 to 3.

At point g, the gas would go back from g to 1, pre-cooling the stream from 2 to 3. These two processes - 2 to 3 and g to 1 are basically heat exchange processes. In the heat exchanger, while the process of 1 to 2 is a compression process, 3 to 4 is an expansion process; and 4 to g is a place where we get vapor at this point, or gas at this point. The liquid is collected depending on where the point 4 is. Depending on where the point 4 is, amount of liquid will be decided, and the amount of gas which has got converted to liquid will be decided, and this is what we will calculate now.

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This should be very, very clear to all of you on how to draw the T - s diagram for various cycles. Now, having done this, let us have a control volume over here in this region. So, we want to basically now analyze this cycle, in order to understand what the work input to the system is, or what the fraction of gas that has got into liquid region is, as a fraction of  $\dot{m}$  how much  $\dot{m}_f$  has come in a liquid form. So, consider a control volume for this system as shown in the figure. It encloses the heat exchangers; the control basically encloses the heat exchanger, J - T wall, and the liquid container. As you can see from this, let us apply the first law of thermodynamics to this system. The changes in the velocities and the datum levels are assumed to be negligible, as we had done in the last lecture also, for ideal cycle.

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### Linde – Hampson System

Makeup gas

- The quantities entering and leaving this control volume are as given below.

IN	OUT
$m \text{ @ } 2$	$(m - \dot{m}_f) \text{ @ } 1$
	$\dot{m}_f \text{ @ } f$

- Using 1<sup>st</sup> Law , we get

$$E_m = E_{out}$$

$$\dot{m}h_2 = (\dot{m} - \dot{m}_f)h_1 + \dot{m}_f h_f$$

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The quantities entering and leaving the control volume are as given below. What is entering is only the gas which is coming at  $\dot{m}$  dot 2, the mass of gas which is coming at point 2, while, what is leaving the system is  $\dot{m}$  minus  $\dot{m}_f$  dot, at this point. At the same time, what is leaving the system is the  $\dot{m}_f$  dot figure.

Using the first law, the energy in is equal to the energy out. Therefore, what you get is  $\dot{m}$  dot at this point, multiplied by enthalpy at this point. So,  $\dot{m}$  dot is equal to  $\dot{m}$  dot minus  $\dot{m}_f$  dot, and corresponding to this point is nothing but point 1. So,  $\dot{m}$  dot minus  $\dot{m}_f$  dot into  $h_1$ , which is enthalpy at point 1 plus, whatever is leaving. This control volume is  $\dot{m}_f$  dot into  $h_f$ .



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### Linde – Hampson System

- Rearranging the terms, we have
 
$$\frac{\dot{m}_f}{\dot{m}} = \left( \frac{h_1 - h_2}{h_1 - h_f} \right)$$
- The fraction of gas liquefied or liquid yield is defined as
 
$$\frac{\dot{m}_f}{\dot{m}} = y = \left( \frac{h_1 - h_2}{h_1 - h_f} \right)$$
- **y** depends on the initial conditions and the compression pressure.

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With this background if I get a ratio, it is,  $\dot{m}_f / \dot{m}$  is equal to  $(h_1 - h_2) / (h_1 - h_f)$ . So,  $(h_1 - h_2)$  gives you the enthalpy difference across the compressor, divided by  $(h_1 - h_f)$ . Here, the fraction of gas liquefied, or the liquid yield, is defined as  $\dot{m}_f / \dot{m}$  equal to  $y$ . This is nothing but,  $y$  is equal to  $(h_1 - h_2) / (h_1 - h_f)$ . It means that,  $y$  depends on the initial condition, and the compression pressure. So,  $h_1$ ,  $h_f$  depend on the initial condition, while the point  $h_2$  depends on the compression pressure.

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### Linde – Hampson System

$$y = \left( \frac{h_1 - h_2}{h_1 - h_f} \right)$$

- The values of  $h_1$  and  $h_f$  are governed by the initial conditions, which are often ambient.
- In order to maximize **y**, the value of  $h_2$  should be as small as possible.
- To have a minimum  $h_2$ , the change in enthalpy for a given change in pressure should be zero at temperature  $T_1$ .

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Here we can see that the values  $h_1$  and  $h_f$ , are governed by the initial conditions, which are often ambient condition. So,  $h_1$  is corresponding to 1 bar, and 300 K; corresponding to 1 bar,  $h_f$  will be at 77 Kelvin, which is the boiling point of liquid nitrogen, at 1 bar pressure.

In order to maximize  $y$ , the value of  $h_2$  should be as small as possible, or  $h_1 - h_2$  should be as high as possible. So, if you want to get maximum yield, or  $\dot{m}_f$  upon  $\dot{m}$ , the numerator should be as high as possible, while the denominator is fixed because,  $h_1 - h_f$  is immediately fixed, once your initial conditions are fixed. In order to maximize  $y$ ,  $h_1 - h_2$  should be as high as possible, or  $h_2$  should be as small as possible. What is  $h_2$ ? It is enthalpy at a point after the compressor. So, it depends on what the pressure of the gas after the compression is, and the corresponding enthalpy at that particular point.

To have a minimum  $h_2$ , the change in enthalpy for a given change in pressure, should be zero, at temperature  $T_1$ .  $h_1 - h_2$  is a compression process, or the process 1 to 2 is a compression process. It is happening at constant temperature, so it is an isothermal compression process. Now, in order that, the value of  $h_2$  should be minimum, the change in enthalpy, or  $\frac{dh_2}{dp_2}$ , change in enthalpy for a given change in pressure should be zero, at a given temperature  $T_1$ . This can be better understood later. What I want is,  $\frac{dh_2}{dp_2}$ . If I go on changing the pressure  $p_2$ , at a certain point, or a certain pressure, the value of  $h_2$  is going to be minimum. It also has to take into consideration that, during this process, the temperature remains constant.

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### Linde – Hampson System

- Mathematically,  $\left(\frac{\partial h}{\partial p}\right)_{T_1=T_2} = 0$
- Using calculus, for variables enthalpy (h), pressure (p) and temperature (T), we have seen earlier that  $\left(\frac{\partial h}{\partial p}\right)_T \left(\frac{\partial p}{\partial T}\right)_h \left(\frac{\partial T}{\partial h}\right)_p = -1$
- Substituting the **J - T coefficient** and the  $C_p$  for the second and third terms respectively, we have  $\mu_{JT} C_p = 0$

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Mathematically I will say that,  $\frac{\partial h}{\partial p}$  at temperature  $T_1$ , is equal to  $T_2$ , at point 1 is equal to point 2, this should be equal to 0. So, I get the minimum value of  $h_2$  here, or I get maximum value of  $h_1 - h_2$ , in that case.

Using calculus h, p, and T, we have seen earlier that,  $\frac{\partial h}{\partial p}$  at constant temperature, into  $\frac{\partial p}{\partial T}$  at constant enthalpy, into  $\frac{\partial T}{\partial h}$  at constant pressure, is equal to minus 1. So, what is this?  $\frac{\partial T}{\partial p}$  is nothing but  $\frac{1}{\mu_{JT}}$ , and  $\frac{\partial T}{\partial h}$  is nothing but  $\frac{1}{C_p}$ . If I take these points on this side, what is this point,  $\frac{\partial h}{\partial p}$  is equal to 0; it means that,  $\mu_{JT} C_p$  is equal to 0.

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## Linde – Hampson System

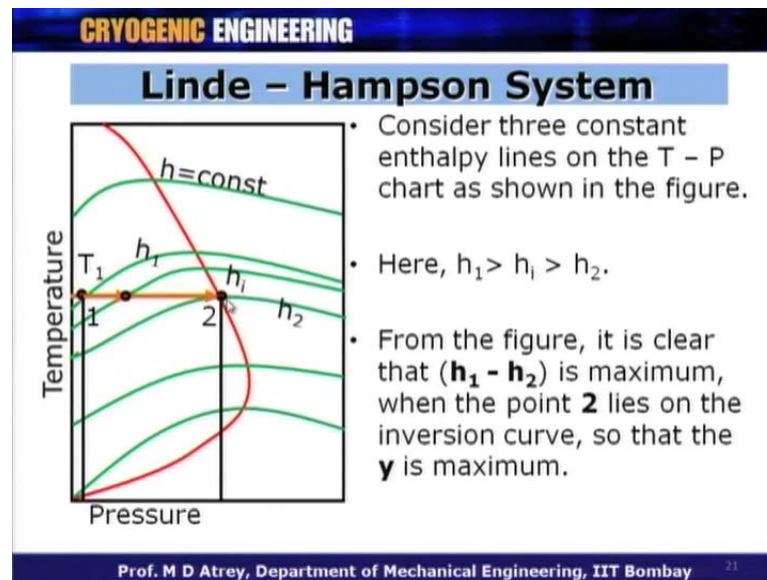
$$\mu_{JT} C_p = 0$$

- $C_p$  is a positive quantity and hence cannot be zero.
- Therefore,  $\mu_{JT} = 0$
- It implies that, in order to maximize  $y$ , the state **2** should lie on the inversion curve for a particular gas at the temperature of compression process.

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See, if I say  $\mu_{JT} C_p = 0$ , the quantity  $C_p$  is not zero, because  $C_p$  is always a finite quantity. Therefore, what you learn from this is,  $\mu_{JT}$  is equal to 0. Point 2 is in such a way that  $\mu_{JT}$  should be equal to 0. In that case, for that particular temperature, it implies that, in order to maximize  $y$ , the state 2 should lie on the inversion curve. As you remember that, on inversion curve,  $\mu_{JT}$  is equal to 0. So, point 2 should lie in such a way that, the point 2 lies on the inversion curve for a particular gas, at a temperature of compression. The temperature remains constant, and if you were to get  $h_2$  as a minimum value, in that case, the point should lie on the inversion curve. This is the most important requirement that one can understand, in order to maximize the value of  $y$ , or in order to minimize the value of  $h_2$ .

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What do you understand from this? If you see at (( )) curve, which is the temperature, pressure diagram, and this is your inversion curve, and there are different enthalpy lines. If I want to compress the gas from point 1 to point 2 isothermally, that means, the temperature remaining constant, you can understand from here that, I can have various infinite values of different enthalpies, but my  $h_1 - h_2$  is going to be maximum only when the point 2 lies on the inversion curve.

All other points in between will have enthalpy more than point  $h_2$ . Therefore,  $h_1 - h_2$  will not be at maximum in that case. The  $h_1 - h_2$  is at maximum, only when point 2 lies on the inversion curve. One can easily understand from this curve. So, consider 3 constant enthalpy lines here as,  $h_1$ , a intermediate values over here, and  $h_2$ . What you can see from here is,  $h_1$  is more than any intermediate value over here, and more than  $h_2$ .

Intermediate value lies between  $h_1$  and  $h_2$ . From the figure it is clear that,  $h_1 - h_2$  is at maximum when the point 2 lies on the inversion curve, so that,  $y$  is maximum. This is very clear from this figure that, if you want to have  $y$  as maximum,  $h_1 - h_2$  has to be maximum; and then,  $h_2$  has to lie, or the point 2 has to lie on the inversion curve.

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**CRYOGENIC ENGINEERING**

### Linde - Hampson System

- The work requirement for a Linde - Hampson system can be derived by considering a control volume enclosing the compressor.
- The quantities entering and leaving this control volume are as given below.

IN	OUT
m @ 1	m @ 2
$-W_c$	$-Q_R$

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Let us find out the work requirement for this. The work requirement for a Linde - Hampson cycle can be derived by considering a control volume, enclosing the compressor. Let us have a compressor over here, and again do the energy balance at this point. The quantities entering, and leaving this control volume are as given below. So, what is entering is mass at point 1, and what is also entering is  $W_c$ , or the work done on the system, which is written as negative minus  $W_c$ . What is leaving the system is mass at m 2, and also the  $q_r$ , because it is leaving the system, it is given as minus  $q_r$ .

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**CRYOGENIC ENGINEERING**

### Linde - Hampson System

- Using 1<sup>st</sup> Law for the following table, we get

IN	OUT
m @ 1	m @ 2
$-W_c$	$-Q_R$

$$E_{in} = E_{out}$$

$$\dot{m}h_1 - W_c = \dot{m}h_2 - Q_R$$

- Rearranging the terms, we have

$$Q_R - W_c = \dot{m}(h_2 - h_1)$$

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Using the first law, for the following tables, whatever is coming in, is equal to whatever is going out; as E in is equal to E out. We can find out that, therefore,  $\dot{m} h_1 - \dot{W}_c$  is equal to  $\dot{m} h_2 - \dot{Q}_r$ . If we rearrange these terms, what we get is  $\dot{Q}_r - \dot{W}_c$ , is equal to  $\dot{m} (h_2 - h_1)$ .

We are taking energy balance across the compressor. What you get is  $\dot{Q}_r - \dot{W}_c$  is equal to  $\dot{m} (h_2 - h_1)$ .

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**CRYOGENIC ENGINEERING**

**Linde – Hampson System**

$$\dot{Q}_R - \dot{W}_c = \dot{m}(h_2 - h_1)$$

- The expression for  $\dot{Q}_R$  can be obtained by using 2<sup>nd</sup> Law for an isothermal compression. It is given by,

$$\dot{Q}_R = \dot{m} T_1 (s_2 - s_1)$$

- Combining the above equations, the work required for a unit mass of gas compressed is

$$-\frac{\dot{W}_c}{\dot{m}} = T_1 (s_1 - s_2) - (h_1 - h_2)$$

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The expression for  $\dot{Q}_r$  can be obtained by using the second law of isothermal compression, and it is given by  $\dot{Q}_r = \dot{m} T_1 (s_2 - s_1)$ . So,  $\dot{Q}_r$  is equal to  $\dot{m} T_1$ , into change of entropy across the process, which is  $s_2 - s_1$ . If you put this value over here, and rearrange the terms, combining the above equations, the work required for a unit mass of gas compressed is  $-\frac{\dot{W}_c}{\dot{m}} = T_1 (s_1 - s_2) - (h_1 - h_2)$ . This is the work of compression per mass of gas, which is compressed;  $\dot{m}$  is nothing but, per mass of gas which is compressed. This is the expression for that, which one as to calculate if you want to calculate the work of compression, if the gases compressed from point 1 to point 2 isothermally, at temperature  $T_1$ .

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**CRYOGENIC ENGINEERING**

### Linde – Hampson System

Makeup gas

$\dot{m}_f$

$\dot{m}$

$(\dot{m} - \dot{m}_f)$

$W_c$

$f$   $g$

$m_f$

$1$   $2$   $3$   $4$

- The liquid yield  $y$  is given by
 
$$y = \frac{\dot{m}_f}{\dot{m}} = \frac{h_1 - h_2}{h_1 - h_f}$$
- Combining the above equations the work required for a unit mass of gas liquefied is
 
$$-\frac{W_c}{\dot{m}_f} = \frac{-\frac{W_c}{\dot{m}}}{y} = -\frac{W_c}{y\dot{m}}$$

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The liquid yield  $y$ , which is already calculated, is given by  $\dot{m}_f / \dot{m}$  is equal to  $h_1 - h_2$  upon  $h_1 - h_f$ . Combining the above equations, the work required for a unit mass of gas liquefied, if I want to calculate now  $W_c$  by  $\dot{m}_f$ ,  $W_c$  by  $\dot{m}_f$  is nothing but  $W_c$  upon  $\dot{m}$ , divided by  $y$ , which is nothing but,  $W_c$  upon  $y \dot{m}$ . So, if I know what my  $W_c$  upon  $\dot{m}$  is, I just divide by  $y$ , and what you get therefore, is  $W_c$  upon  $\dot{m}_f$ , or work of compression per unit mass of gas, which is liquefied.

One has to be very sure about what is the question that is being asked. Is it the work done per unit mass of gas compressed? In which case, we have to use this expression. If the question is, what is the work done per mass of gas, which is liquefied? Then, whatever you have calculated here should be divided by  $y$ . Therefore, what you get is work of compression for unit mass of gas, which is liquefied.

With this background, now I would like to solve a few problems, so that, whatever we have learnt till now could be first used, and you will understand how to do this calculation, and importantly, how to interpret the results.



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**CRYOGENIC ENGINEERING**

### Tutorial - 1

- Determine the liquid yield, the work per unit mass compressed and work for unit mass liquefied for a Linde - Hampson cycle with air as working fluid. The system is operated between 1.013 bar (1 atm) and 202.6 bar (200 atm) at 300 K.

• **Step 1**

- The T - s diagram for a Linde - Hampson Cycle is as shown.

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Let us have a small tutorial of one or two lectures. Under the first Tutorial now, the problem statement is, 'determine the liquid yield, which is nothing but  $y$ , the work per unit mass of gas compressed, which is nothing but  $W_c$  upon  $m \dot{}$ , and the work for unit mass of gas liquefied, which is nothing but  $w$  upon  $m \dot{}$   $f$ , for a Linde - Hampson cycle with air as working fluid'. It is very important to understand what has been asked in the problem.

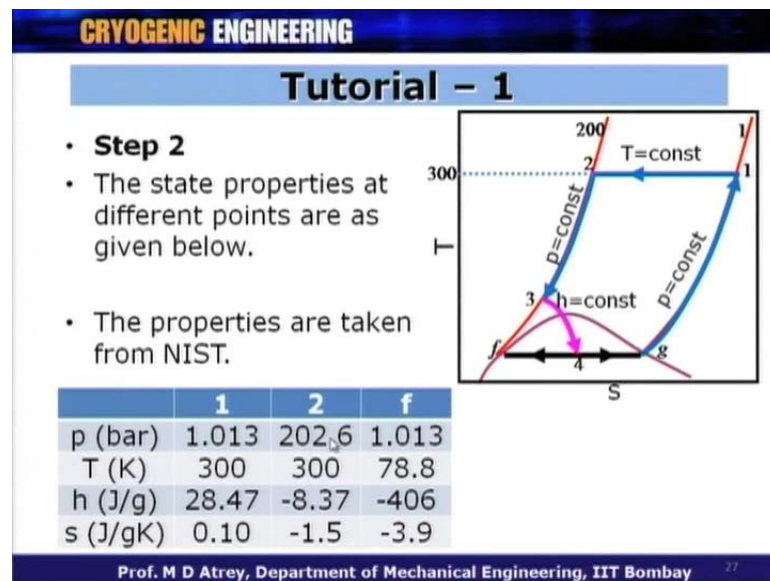
Sometimes, the problem statements here are very, very large; and you have to really split those into small sub-problems, and then solve the problem. So, here you are to understand first what has been asked. So, what is asked is  $y$ , which is  $m \dot{}$   $f$  upon  $m \dot{}$ ; work per unit mass of gas compressed, which is  $w$  upon  $m \dot{}$ ; and work per unit mass of gas which is liquefied, which is  $w$  upon  $m \dot{}$   $f$  for a Linde - Hampson cycle, and your working fluid is air. Therefore, what you should have the T - s diagram for air, that is the basic requirement that, you should have a temperature entropy diagram of air. This can be obtained from NIST website, or various books.

The system is operated between 1.013 bar, which is nothing but 1 atmosphere. This is the point 1, and 200 atmospheres, or 202.6 bar at 300 Kelvin. So, your point 1 is going to be one atmosphere and 300 Kelvin, pressure and temperature respectively. The point 2 is equal to 200 atmosphere, and 300 K, respectively.

So, what is step 1? Step 1 is basically the T - s diagram for Linde - Hamson cycle as shown. So, first plot T - s diagram, and get the different enthalpy and entropy values, associated with those points from the chart, which is available with you. So, here I am drawing a small schematic representation, and you should also draw a small schematic representation showing the cycle 1, 2, 3, and 4, or the Linde - Hamson cycle as given over here. The point 1 is at 300 Kelvin and 1 bar. So, locate the pressure line at 1 bar; locate the temperature at 300 Kelvin, and locate the point 1.

The process 1 to 2 is an isothermal compression, and the point 2 should be located in such a way that, this pressure is 200 atmospheres, or 200 bar, at this point, temperature remaining constant because, 1 to 2 is an isothermal compression process. Now, as soon as your point 1 is fixed, we know where the point f is. What we need to know is the state 1, and state 2. You may not know state 3 immediately, which may not be required for solving this problem for the data, which has to be calculated over here.

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What you understand from here is, the state properties at different points are as shown over here. What are the state points? These properties are taken from NIST. I have talked about NIST in the first lecture - National Institute for Standards and Technology. If you go to this site, you will have all the T - s diagrams for various gases, which could be downloaded, and could be used; or, there are other books on cryogenics which give these T - s diagrams. So, if you see the format which we want to use now, what properties does

it require? The property at point 1, the properties at point 2, and the properties at point f, is what is required.

We have got a small table, and I will request you to also, while solving such problems to make a small table of all the points, and get all the property data, like, pressure, temperature, enthalpy, and entropy at those points. Why only these 3 points? It is because, only the data of these 3 points is required to solve all the problems associated with whatever question has been asked in the problem.

So, we have got a point 1, which is the pressure, around 1 bar, temperature is 300 Kelvin. The corresponding enthalpy and entropy are this. If I go to point 2, the pressure is 202 bar, the temperature remains constant which is 300 Kelvin, corresponding enthalpy and entropy values are taken over here. Similarly, at point f, which is this, pressure is 1 bar or atmospheric pressure. The temperature at this point is the boiling point of air, associated with this pressure. So, what is the boiling point of air at 1 bar? It is 78.1 Kelvin. Corresponding enthalpy and entropy values are over here, and this is the most important thing. If you make mistakes in finding out these properties, it will go wrong, and you will get very absurd values, which will be very difficult for you to explain or interpret later.

Therefore, you should get this point two or three times. Some error is always allowed, because you are reading the chart, and therefore, some mistakes can happen. However, you should see that 28.47T should not become 35 or 40; it can be 30 or 26, or 27, on this side. Always 10% error could be allowed in that case.

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**CRYOGENIC ENGINEERING**

**Tutorial - 1**

• **Liquid yield**

$$y = \frac{h_1 - h_2}{h_1 - h_f}$$

	1	2	f
p (bar)	1.013	202.6	1.013
T (K)	300	300	78.8
h (J/g)	28.47	-8.37	-406
s (J/gK)	0.10	-1.5	-3.9

$$y = \frac{h_1 - h_2}{h_1 - h_f} = \frac{28.47 + 8.37}{28.47 + 406}$$

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Now, the first problem is to solve the liquid yield. The formula for this is, y is equal to  $h_1 - h_2$  upon  $h_1 - h_f$ . So, what do you understand from here? The properties are required from point 1, point 2 and f. We know all the enthalpy values, which have been given in the earlier table. This is the table; I have just produced it for you, you can read now - what is  $h_1$ , what  $h_2$  is, and what  $h_f$  is. This is the third row of this table; y is equal to  $h_1 - h_2$  upon  $h_1 - h_f$ . If I put these values - 28.47 minus of 8.37, so, I will get plus 8.37 over here.

Similarly,  $h_1 - h_f$ , again minus of 406, which is going to be kept over here, if you solve this, what you get is, 36.84 divided by 434.47, which is equal 0.085. What does it mean that, y is equal to 0.085? What does it mean? It means that,  $m \cdot f$  upon  $m \cdot i$  is equal to 0.085, which also means that  $m \cdot f$  is only 8.5% of whatever  $m \cdot i$  is compressed. This is a very important thing that, whatever amount of gas which is getting compressed in the compressor at point 1, only 8.5% of that gas gets liquefied. That is what the product is. The liquefaction product is only 8.5% of whatever is getting compressed at point 1.

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**CRYOGENIC ENGINEERING**

**Tutorial - 1**

• **Work/unit mass of gas compressed**

$$-\frac{W_c}{\dot{m}} = T_1 (s_1 - s_2) - (h_1 - h_2)$$

	1	2	f
p (bar)	1.013	202.6	1.013
T (K)	300	300	78.8
h (J/g)	28.47	-8.37	-406
s (J/gK)	0.10	-1.5	-3.9

$$-\frac{W_c}{\dot{m}} = 300(0.1 + 1.5) - (28.47 + 8.37) = 443.16 \text{ J/g}$$

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Now, let us calculate the work per unit mass of gas, which is compressed. Again understand that, this is  $w$  upon  $m$  dot. So, apply the formula which is minus  $W_c$  upon  $m$  dot is equal to  $T_1$  into  $s_1$  minus  $s_2$ , minus,  $h_1$  minus  $h_2$ . Again take the property data from this table. Put the value of  $s_1$ , and  $s_2$  which is this;  $s_1$  and  $s_2$  and  $h_1$  minus  $h_2$ . So, if I put those values,  $W_c$  by  $m$  dot is equal to 300, which is nothing but,  $T_1$  into  $s_1$  minus  $s_2$ , which is this. Point 1 minus of minus 1.5, minus,  $h_1$  minus  $h_2$ ;  $h_1$  minus  $h_2$  over here, which is nothing but 443.16 joules per gram.

The amount of compressor power required per unit mass of gas compressed is 443.16 joule per gram.

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**CRYOGENIC ENGINEERING**

**Tutorial - 1**

• **Work/unit mass of gas liquefied**

$-\frac{W_c}{\dot{m}} = 443.16$

$y = 0.085$

$$-\frac{W_c}{\dot{m}_f} = -\frac{W_c}{y\dot{m}} = \frac{443.16}{0.085} = 5213.64 \text{ J/g}$$

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Now, the third requirement of the problem is to calculate the work required per unit mass of the gas, which is liquefied. What we calculated earlier, is work per unit mass of gas which is compressed. So, what we have earlier found is minus  $w_c$  upon  $\dot{m}$  is equal to 443.16 joule per gram.

We know  $y$  is equal to 0.085. Choosing this, we can find out what is work done per unit mass of gas, which is liquefied, which is nothing but  $W_c$  upon  $\dot{m}_f$ , which is nothing but equal to minus  $W_c$  upon  $\dot{m}$ , divided by  $y$ , or minus  $w_c$  upon  $y$  into  $\dot{m}$ . So, what is  $w_c$  upon  $\dot{m}$ ? It is 443. If you divide that by  $y$ , what you get is  $w_c$  upon  $\dot{m}_f$ .

So, work required per unit mass of gas which liquefied is 5213.64 joule per gram. This is way higher as compared to the work required per unit mass of gas, which is compressed. This is quite understandable because,  $\dot{m}_f$  value is very, very small; therefore, work done per unit of mass, which is liquefied, is going to be very, very high. It is just a way of representing the values. This is the work done per unit mass of gas, which is compressed, and this is the work done per unit mass of gas, which is liquefied. Therefore,  $\dot{m}_f$  being a very small quantity, this quantity is always going to be much higher as compared to the work done per unit mass of gas, which is compressed.

Now, this was the problem wherein, we have got all the three values. We have understood now how to calculate these values, and now I will take you to the next

tutorial, where we will understand the effects of pressure of compression, and also the temperature at which, the gas compression happens.

Let us see the next tutorial. Again, as I said, it is very important to understand what the problem is, because, the problems are normally statements which are very, very big; and therefore, you have to understand them in parts.

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**CRYOGENIC ENGINEERING**

**Tutorial - 2**

- Determine the liquid yield for a Linde - Hampson cycle with Nitrogen as working fluid for the following operating conditions. Comment on the results.

	Point 1	Point 2
<b>I</b>	300 K, 1 bar	300 K, 50 bar
<b>II</b>	200 K, 1 bar	200 K, 50 bar
<b>III</b>	300 K, 1 bar	300 K, 100 bar
<b>IV</b>	200 K, 1 bar	200 K, 100 bar

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In this case now, determined the liquid yield for a Linde - Hampson cycle, with nitrogen as the working fluid, for the following operating conditions; and ultimately what is important is to comment on the results, or interpret the results. So, again we use the Linde - Hampson cycle, with the working fluid in this case is nitrogen, and not air. This is the schematic representation of the Linde - Hampson cycle, and these are the different operating conditions. What do you understand from this?

There are 4 cases - case 1, case 2, case 3, and case 4. In the first case, the point 1, and point 2 are at same temperature, where the temperature is 300 Kelvin, and the pressures are at point 1 is 1 bar. So, 1 bar, and 300 Kelvin is the condition at point 1 over here, while the point 2 condition is 300 Kelvin, and 50 bar. The gas is getting compressed from 1 bar to 40 bar.

In the case 2, the temperature is now not 300 Kelvin, but it is at 200 Kelvin. So now, the compression is happening at 200 Kelvin, which is way below the ambient temperature,



while the pressure is same as 1 bar compressed to 50 bar; pressure after the compression is 50 bar.

In the case three, here, the pressure is now changed from 50 bar to 100 again. The cases are taken at 300 Kelvin, and 200 Kelvin respectively. In both the cases, the compression is happening from 1 bar to 100 bar. The objective of this tutorial is to understand what happens, if you have a compression at 300 Kelvin, or at 200 Kelvin. This is the isothermal compression from point 1 to point 2 here. This can happen at 300 Kelvin, which is the room temperature, or we can have the compression at reduced temperature, which is at 200 Kelvin, and you will have to understand how it affects the yield, or work done, etcetera.

Similarly now, instead of compressing the gas up to 50 bar, if I went ahead, and if I compress it up to 100 bar, what are the corresponding changes? We will like to understand these changes even when the temperature of compression is reduced from 300 to 200 Kelvin. So, this is a very important, and interesting problem, where you understand the effect of compression pressure, as well as, you can understand the effect of compression temperature as thermal compression temperature. Let us study the effect of this parameter in this problem.

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**CRYOGENIC ENGINEERING**

**Tutorial - 2**

Step : 1

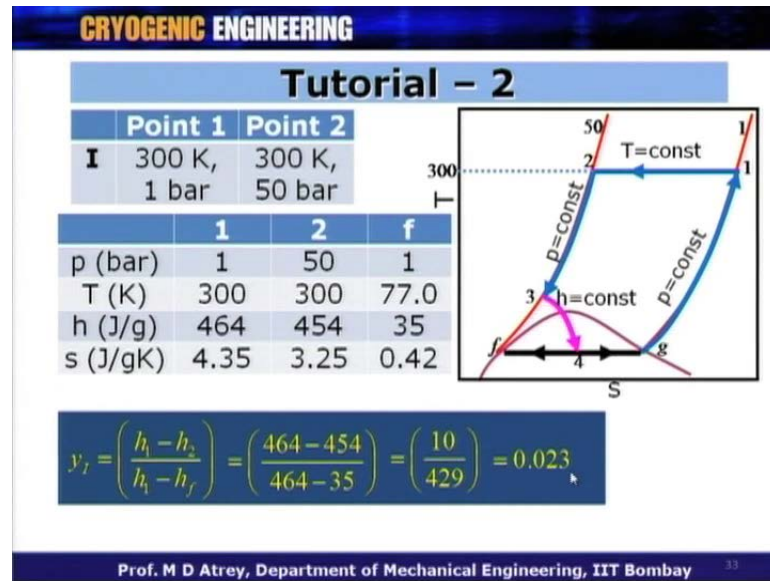
- Assuming the heat exchange process to 100% effective, the T - s diagram for a Linde - Hampson Cycle is as shown.
- In this tutorial , we assume that 1 atm = 1 bar.

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So, step one, as I said, the assumption is that, the heat exchanger is effective; 100% effectiveness. The T - s diagram from Linde - Hampson cycle is as shown. What is

important? What I wanted to say was that, first of all, get hold of the T - s diagram of nitrogen. Again, it could be downloaded from various places. In this tutorial, we assume 1 atmosphere is equal to 1 bar. I mean, I do not want to write every time that, 1 atmosphere is 1.013 bar. We can fairly assume that 1 atmosphere is 1 bar, and take the values accordingly.

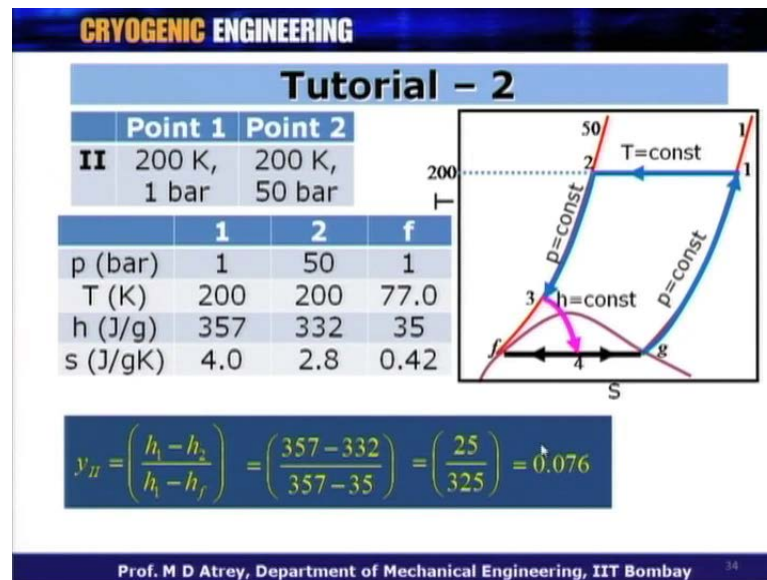
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The point one in the first case, the temperature at 300 Kelvin, and the compression is happening from 1 bar to 50 bar. The T - s diagram is here. 1 bar to 50 bar, 1 to 2 the compression pressure happening at 300 Kelvin. The properties at 1, 2, and f are as given over here. As soon as I say 1 bar nitrogen, the point f will be at 77 Kelvin. So, the point 1 is 1 bar, 300 Kelvin, and corresponding enthalpy and entropy is given over here.

The point 2 is 50 bar, and 300 Kelvin, corresponding enthalpy and entropy are given here. The point f, which is again at 1 bar, 77 Kelvin, and enthalpy and entropy values are given over here. So, I can straight away start calculations now, which is y for the first case, is equal to h 1 minus h 2 upon h 1 minus h f. Put all the values over here, what you get ultimately is 0.023.

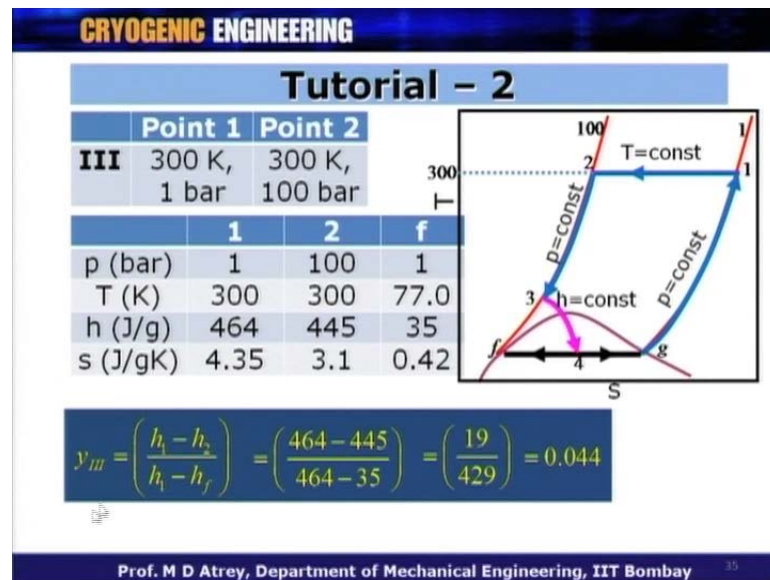
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So, for the first case I am getting yield as 0.023. Now I go to the second case, which is the compression process happening at 200 Kelvin. The pressure remains the same, that is, 1 bar to 50 bar compression. What I am doing is now, instead of 300 Kelvin I am carrying out the process of compression at 200 Kelvin. This is schematically shown. 1 to 50 bar, at 200 Kelvin now. Again I go up to T - s diagram, and I get values. The point 1 is at 1 bar, 200 Kelvin, corresponding enthalpy and entropy values are here.

The point 2 is 50 bar, 200 Kelvin, corresponding enthalpy and entropy over here. The point f is at 1 bar, and 77 Kelvin; this will not change. If I calculate now y, for case number 2, same formula, putting the values of enthalpies, at this point, I get 0.076. So, as compared to 0.023, what we got in case one, I am now getting is 0.076. What is the reason? The reason is now the process of compression is carried out at a lower temperature. At 1 to 2 is happening at 200 Kelvin, while the earlier process was happening at 300 Kelvin, and you see correspondingly, I have got an increase in the value of y in this case.

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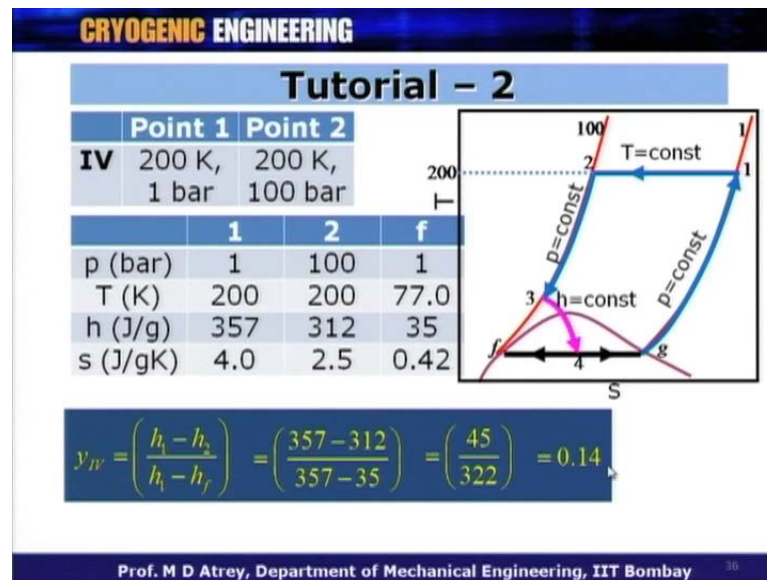


Let us go to case number 3, which is happening at 300K. The process of compression is happening at 300K only; but the pressure at the end of compression is now 100 bar, and not 50 bar anymore. Correspondingly, now I have got point 1 at 1 bar, and point 2 is at 100 bar, and 300 Kelvin.

Again go back to T - s diagram, and get the properties. The point 1 is now at 1 bar, and 300 Kelvin; the point 2 is now 100 bar, and 300 Kelvin; the point f is remains at the same condition. If I put these values, y for case 3 is going to be, putting all the respective values of the enthalpy in this formula, what you get here is 0.044.

If you again compare it with the first value, which was at 50 bar, this value is going to be higher as compared to what it was for 50 bar, and 300 Kelvin compression temperature. As soon as your pressure after the compression at this point has increased, what you get is a higher yield, or higher 'y' value.

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I have got case 4, where the same compression, instead of doing it at 300 Kelvin, now from 1 bar to 100 bar, I am doing it at 200 Kelvin. Again I will go back to the T - s diagram, again I will get points 1, 2 and f from this chart. Here, my point 1 is at 1 bar, and 200 Kelvin; point 2 is at 100 bar, and 200 Kelvin. So, the process of compression is happening at a lower temperature. We can say it is a pre-cooled cycle, which is happening at 200 Kelvin, and not at 300 Kelvin.

Again I will take the enthalpy and entropy values from the T - s diagram, and calculate y. So, y in the 4th case by this formula is going to be 0.14, which is very, very high as compared to what it was for earlier cases. What does it mean? It means that, if you carry out compression at lower temperature, you get higher yield. At the same time, if you compress the gas at higher pressure, you get higher yield. This is what we understood from this. So now, let us summarize the result.

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**CRYOGENIC ENGINEERING**

**Tutorial - 2**

	Point 1	Point 2	$\gamma$
<b>I</b>	300 K, 1 bar	300 K, 50 bar	0.023
<b>II</b>	200 K, 1 bar	200 K, 50 bar	0.076
<b>III</b>	300 K, 1 bar	300 K, 100 bar	0.044
<b>IV</b>	200 K, 1 bar	200 K, 100 bar	0.14

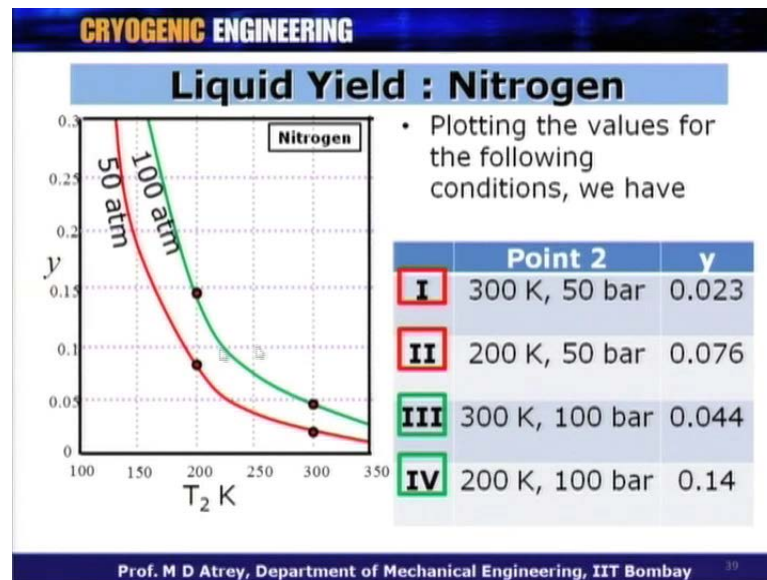
- As the compression temperature decreases, the liquid yield  $\gamma$  increases at a given compression pressure.

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Here are the 4 cases - case number 1, 2, 3, and 4. Case number 1 was compressed from 1 to 50 bar at 300 Kelvin, and it gave us 0.023  $\gamma$ , while I carried out this compression not at 300 Kelvin, but at 200 Kelvin, the yield increased from 0.023 to 0.076. When I did the compression not up to 50 bar, but up to 100 bar, the yield increased from here to here. Again, 0.023 to 0.044 at 300 Kelvin compression temperature, while when I reduced the temperature of compression from 1 to 100 bar at 300 Kelvin to 200 Kelvin, the yield increased drastically, from 0.044 to 0.14.

This can be graphically represented in the slides over here. So, understand from here that, as the compression process increases, the liquid yield increases at a given compression temperature; and as the compression temperature decreases, the liquid yield increases at a given compression pressure. This is what we learnt from this.

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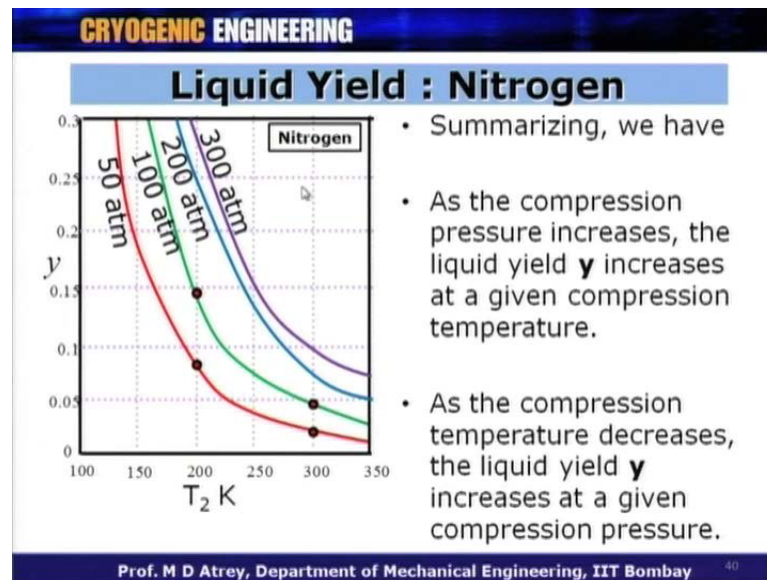


If I want to plot this now on  $y$  as a function of  $T_2$  that means, the compression temperature, or the isothermal compression temperature you can see is, point number 1. This point 2, if I plot at 200 Kelvin and 50 bar pressure is this; and if I join this point and extend this point further, this is the line of 50 atmospheres, and you can see as you go on loading the temperature of the compression, the yield is going to increase in this fashion.

If I take into consideration of point 3, and point 4, or case 3 and case 4. This is the case 3, which is 0.044 which will come at this point at 300K, and 100 bar. This 200K and 100 bar; if I join these 2 points, this is the 100 bar case, and you can see now. You can see how much yield has increased, if we went from 50 bar to 100 bar; or, you can also see how much yield has increased when the process of compression was carried instead of at 300 Kelvin, I carried it at 200 Kelvin.



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If I now extend this further, for different pressures, what you get is this at 200 atmospheres; and this at 300 atmospheres. It is fair to say that I should go for as high pressure as possible, and as low temperature as possible for the compression process.

Summarizing, the compression pressure increases, the liquid yield  $y$  increases at a given compression temperature; as the compression temperature decreases, the liquid yield  $y$  increases at a given compression pressure. This is what has been summarized from this.

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**CRYOGENIC ENGINEERING**

### Assignment

1. Consider a Linde – Hampson cycle with Nitrogen as working fluid. The system is operated between 1.013 bar (1 atm) and 101.3 bar (100 atm) at 300 K. Determine
  1. Liquid yield
  2. Work per unit mass compressed
  3. Work for unit mass liquefied.
2. Calculate the above parameters if the same cycle is operated with air as working fluid.

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This is very important to understand what happens when the pressure changes, and what happens when the temperature changes. Best of this, we have got different assignments. Please calculate the liquid yield work done compression, and work done for unit mass liquefied. For the above parameters, again as air as the working fluid, and nitrogen as the working fluid, please do this assignments, and similar to what we have learnt I have got a different assignment for you. Please carryout these assignments.

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**CRYOGENIC ENGINEERING**

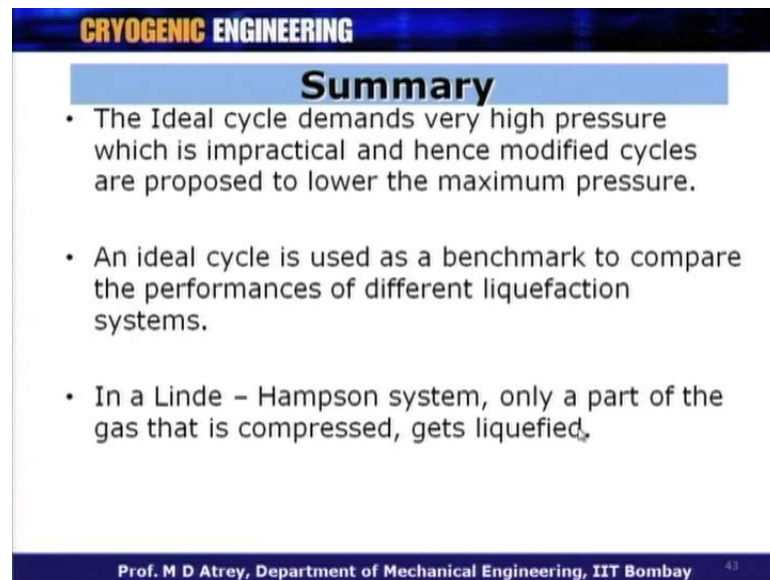
### Summary

- The Ideal cycle demands very high pressure which is impractical and hence modified cycles are proposed to lower the maximum pressure.
- An ideal cycle is used as a benchmark to compare the performances of different liquefaction systems.
- In a Linde - Hampson system, only a part of the gas that is compressed, gets liquefied.

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To summarize today's lecture, the ideal cycle demands very high pressure, which is impractical, and hence modified cycles are proposed to lower the maximum pressure. An ideal cycle is used as a benchmark to compare the performances of different liquefaction cycles. In a Linde - Hampson system, only a part of the gas that is compressed gets liquefied, that is why you got  $m \cdot$  or  $m \cdot f$ , which is much less than  $m \cdot$  and therefore, a fraction comes which is called  $y$ .

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**CRYOGENIC ENGINEERING**

### Summary

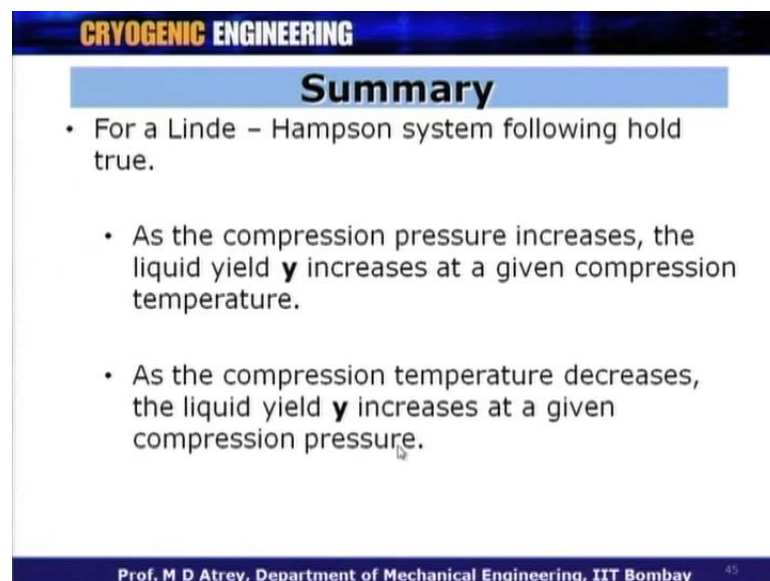
- The Ideal cycle demands very high pressure which is impractical and hence modified cycles are proposed to lower the maximum pressure.
- An ideal cycle is used as a benchmark to compare the performances of different liquefaction systems.
- In a Linde - Hampson system, only a part of the gas that is compressed, gets liquefied.

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A heat exchanger is used in a Linde - Hampson system to conserve cold in the system. This process is an isobaric process, and is assumed to be 100% effective.

In order to maximize  $y$ , or  $m \dot{f}$  upon  $m \dot{}$ , for a Linde - Hampson system, the state 2 should lie on the inversion curve, at the temperature of the compression process. This is made fairly clear in this lecture.

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**CRYOGENIC ENGINEERING**

### Summary

- For a Linde - Hampson system following hold true.
  - As the compression pressure increases, the liquid yield  $y$  increases at a given compression temperature.
  - As the compression temperature decreases, the liquid yield  $y$  increases at a given compression pressure.

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The work required for a unit mass of gas compressed for a Linde - Hampson cycle is given by this formula. We have seen the derivation of this formula also. For a Linde -

Hampson cycle, the following holds true - as the compression pressure increases, the liquid yield  $y$  increases at a given compression temperature. This is fairly clear from the examples also, which we have taken. At the same time, as the compression temperature decreases, the liquid yield  $y$  increases at a given compression pressure, this is what we have understood from the last problem.

Thank you very much.