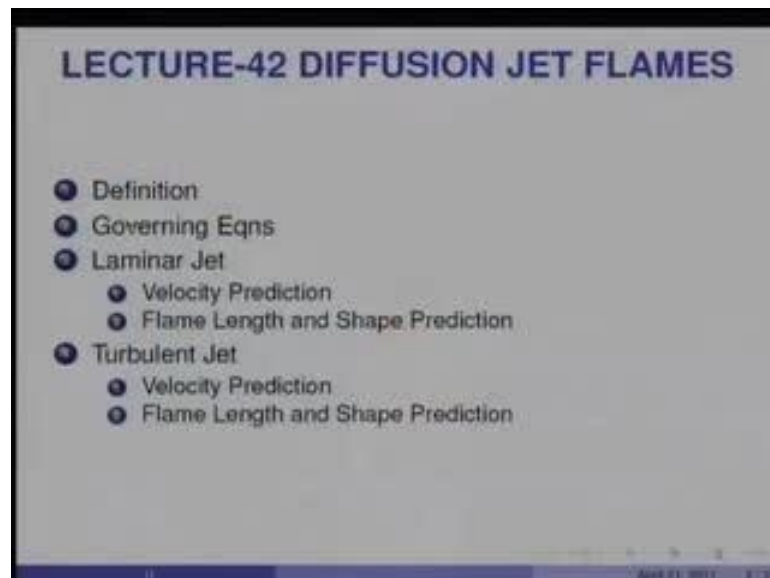


Convection Heat and Mass Transfer
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Module No. # 01
Lecture No. # 42
Diffusion Jet Flames

In all previous lectures, our interest was to consider heat and mass transfer close to a wall, whether it is force convection or natural convection. In this last lecture of this course, I am now going to consider a flow with heat mass transfer and chemical reaction in which, no wall is present. In everyday language, it is nothing but the combustion flame. My task is to see how we can apply the knowledge that we have gained so far, to predict the length and thickness of a diffusion flame.

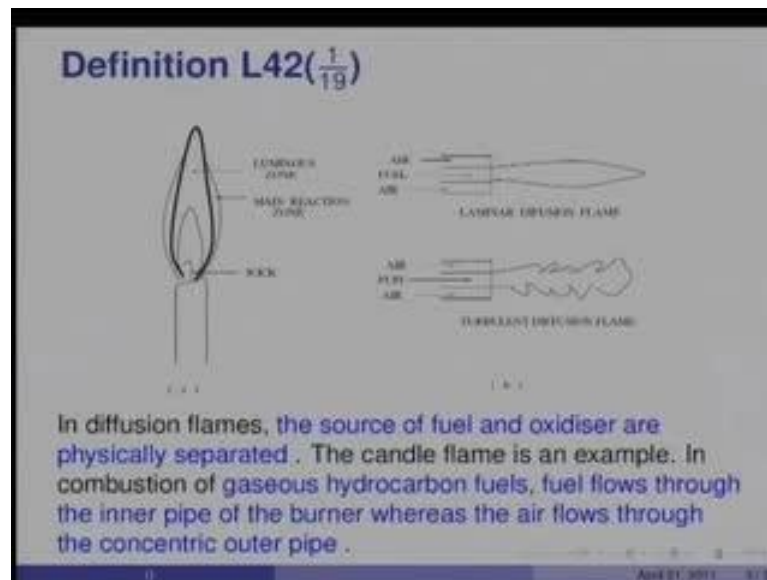
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I will proceed as shown here. First of all, I will define a flame and then, I will set up the governing equations. First, consider laminar jet, whose velocity would be predicted. The flame length and shape of the laminar jet flame would be predicted. Likewise, we will do

the same for the turbulent jet flame. Remember, the word jet implies that there is no wall present. It is sometimes called as the free shear flow.

(Refer Slide Time: 02:04)



Let us look at the definition of a flame. Here is the definition and the most commonly encountered diffusion flame is the candle flame that you can see here. In diffusion flame, the source of the fuel and the oxidizer are physically separated. So, the candle flame is an example, in which there is a combustion of gaseous hydrocarbon fuels. Sorry, the candle flame is an example, in which the melted wax evaporates and so the fuel alone arrives into the flame. Whereas, the oxygen required is gathered from the surrounding. So, fuel itself does not have any oxygen in it, but the oxygen required for burning is obtained from the surroundings by the process of diffusion.

Gaseous hydrocarbon fuels are often burnt in the same manner. So, here is a typical burner in the fuel. It is carried through an inner pipe and air is carried through an annular pipe. Sometimes, this air may also be swirled to enhance the rate of mixing. Here, the air entrains inside the burning zone and a flame is formed. So, this is a typical laminar diffusion flame, where the velocities are low. If the velocities were high, you will get what is called as turbulent diffusion flame. It is with very jagged edges, unlike the laminar flame, which has a nice smooth edge.

So, gaseous hydrocarbon fuels fuel flows through the inner pipe of the burner, whereas the air flows through a concentric pipe. This is the situation at hand and essentially, you have a free shear flow; there is no wall present across the thickness of the flow.

(Refer Slide Time: 03:59)

Main Objective L42($\frac{2}{19}$)

The main objective is to predict

- 1. Flame Length L_f
- 2. Flame shape or $r_f(x)$

in stagnant surroundings assuming SCR:
 $1 \text{ kg of fuel} + R_{ox} \text{ kg of oxidant air} = (1 + R_{ox}) \text{ kg of product.}$

What is the main objective? Here, I am defining the fuel, which is coming in through a diameter pipe of diameter D . The jet spreads along the dotted line, whereas the flame radius varies with x . It is highest at the beginning and goes on decreasing, when the radius goes to 0. You essentially say that is the flame length L_f . We are assuming stagnant surroundings completely and this is essentially the burning zone of the jet.

The temperature profiles across any cross section would look like this. So, here is the radius (Refer Slide Time: 04:42) and you can see that the value of the velocity is 0 at the edge somewhere. Of course, the exact location where it will be 0 is not known and that is what we wish to find out. At the edge of the flame, velocity would be 0. In the center, it would be high. So that is the velocity profile and the oxygen profile would go like that. The temperature profile would go something like that and the fuel profile would be as shown here. It will be highest and the edge of the flame is shown somewhere here. So, the fuel would be high at the axis. The oxygen would be high at the edge of the layer. The temperature would be low in the environment, but would increase to a peak value somewhere and decrease a little in the core region.

The main reaction actually takes place at the edge and therefore, you get the temperature there. So, the main objective is to predict the flame length L_f and flame shape, which means function r_f flame as a function of x . In order to make life simple, we again use the simple chemical reaction as the combustion model. In stagnant surroundings, assuming simple chemical reaction, I can write 1 kilograms of fuel plus R_{st} kilograms of air oxidant air. R_{st} would be the air fuel ratio. It gives me 1 plus R_{st} kg of product. The postulated chemical mechanism is simple chemical mechanism. Our interest is to predict L_f and r_f as a function of x . This is an axis symmetric case; it is a round jet and therefore, this is an axis symmetric case.

(Refer Slide Time: 06:37)

Governing Eqns L42(3/19)

$$\frac{\partial}{\partial x}(\rho_m u r) + \frac{\partial}{\partial r}(\rho_m v r) = 0 \quad \left(\frac{dp}{dx} = 0\right)$$

$$\frac{\partial}{\partial x}(\rho_m u r u) + \frac{\partial}{\partial r}(\rho_m v r u) = \frac{\partial}{\partial r} \left[\mu_{eff} r \frac{\partial u}{\partial r} \right]$$

$$\frac{\partial}{\partial x}(\rho_m u r \omega_{fu}) + \frac{\partial}{\partial r}(\rho_m v r \omega_{fu}) = \frac{\partial}{\partial r} \left[\rho_m D_{eff} r \frac{\partial \omega_{fu}}{\partial r} \right] - r |R_{fu}|$$

$$\frac{\partial}{\partial x}(\rho_m u r \omega_{ox}) + \frac{\partial}{\partial r}(\rho_m v r \omega_{ox}) = \frac{\partial}{\partial r} \left[\rho_m D_{eff} r \frac{\partial \omega_{ox}}{\partial r} \right] - r |R_{ox}|$$

$$\frac{\partial}{\partial x}(\rho_m u r h_m) + \frac{\partial}{\partial r}(\rho_m v r h_m) = \frac{\partial}{\partial r} \left[\frac{k_{eff}}{c_{pm}} r \frac{\partial h_m}{\partial r} \right] + r |R_{fu}| \Delta H_c$$

The continued equation without constant properties would look like $\frac{d}{dx}$ equal to $\rho_m u r$ plus $\frac{d}{dr} \rho_m v r$ equal to 0. In a free jet, the pressure gradient term is 0. So, in the momentum equations, you do not see any pressure gradient term. You have simply convection terms and a diffusion term according to the boundary layer approximation.

This is the equation for the fuel; this is the equation for the oxidant and this is the equation for the energy or the enthalpy h_m . Source term is r times R_{fu} multiplied by heat of combustion. So, h_m would now simply represent the sensible enthalpy h_m equal to $c_{pm} t$ minus t_f and that is what h_m would represent.

(Refer Slide Time: 07:39)

Laminar Vel Prediction - 1 - L42($\frac{4}{19}$)

Main assumption: Properties are uniform

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial r} + \frac{v}{r} = 0 \text{ and}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} = \frac{\nu_m}{r} \frac{\partial}{\partial r} \left[r \frac{\partial u}{\partial r} \right]$$

$$\psi(x, \eta) \equiv \nu_m x f(\eta) \text{ and } \eta \equiv C \frac{r}{x}$$

$$u \equiv \frac{1}{r} \frac{\partial \psi}{\partial r} = \frac{C^2 \nu_m}{x} \left[\frac{f}{\eta} \right]$$

$$v \equiv -\frac{1}{r} \frac{\partial \psi}{\partial x} = \frac{C \nu_m}{x} \left[f' - \frac{f}{\eta} \right]$$

Boundary conditions are: $f(0) = f'(0) = 0$ and $f'(\infty) = 0$.

These are the equations to be solved for this problem. First of all, let us solve the velocity problem and we are going to make a very drastic assumption. We are going to say that the properties of the fuel, properties of the mixture inside the flame zone are constants. So, properties are uniform.

(Refer Slide Time: 08:01)

Governing Eqns L42($\frac{3}{19}$)

$$\frac{\partial}{\partial x} (\rho_m u r) + \frac{\partial}{\partial r} (\rho_m v r) = 0 \quad \left(\frac{dp}{dx} = 0 \right)$$

$$\frac{\partial}{\partial x} (\rho_m u r u) + \frac{\partial}{\partial r} (\rho_m v r u) = \frac{\partial}{\partial r} \left[\mu_{eff} r \frac{\partial u}{\partial r} \right]$$

$$\frac{\partial}{\partial x} (\rho_m u r \omega_{H_2}) + \frac{\partial}{\partial r} (\rho_m v r \omega_{H_2}) = \frac{\partial}{\partial r} \left[\rho_m D_{eff} r \frac{\partial \omega_{H_2}}{\partial r} \right] - r |R_{H_2}|$$

$$\frac{\partial}{\partial x} (\rho_m u r \omega_{O_2}) + \frac{\partial}{\partial r} (\rho_m v r \omega_{O_2}) = \frac{\partial}{\partial r} \left[\rho_m D_{eff} r \frac{\partial \omega_{O_2}}{\partial r} \right] - r |R_{O_2}|$$

$$\frac{\partial}{\partial x} (\rho_m u r h_m) + \frac{\partial}{\partial r} (\rho_m v r h_m) = \frac{\partial}{\partial r} \left[\frac{k_{eff}}{C_{pm}} r \frac{\partial h_m}{\partial r} \right] + r |R_{H_2}| \Delta H_c$$

You will see that all these rho m's will come out and u would come out here. I am now taking laminar flows. Therefore, you have mu m by r d by dr r du by dr as the diffusion

term. This is the convection term (Refer Slide Time: 08:20) and the continuity equation would look like this.

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Laminar Vel Prediction - 1 - L42($\frac{4}{19}$)

Main assumption: Properties are uniform

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial r} + \frac{v}{r} = 0 \text{ and}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} = \frac{\nu_m}{r} \frac{\partial}{\partial r} \left[r \frac{\partial u}{\partial r} \right]$$

$$\psi(x, \eta) \equiv \nu_m x f(\eta) \text{ and } \eta \equiv C \frac{r}{x}$$

$$u \equiv \frac{1}{r} \frac{\partial \psi}{\partial r} = \frac{C^2 \nu_m}{x} \left[\frac{f'}{\eta} \right]$$

$$v \equiv -\frac{1}{r} \frac{\partial \psi}{\partial x} = \frac{C \nu_m}{x} \left[f - \frac{f'}{\eta} \right]$$

Boundary conditions are: $f(0) = f'(0) = 0$ and $f'(\infty) = 0$.

Now, we can solve these two equations by similarity method, where we define stream function $x \eta$ ψ $x \eta$ equal to ν multiplied by $f \eta$ and η equal to C into r by x . U equal to $\frac{1}{r} \frac{d \psi}{dr}$ would become $C^2 \nu_m$ by $x f'$ by η v equal to $-\frac{1}{r} \frac{d \psi}{dx} = C \nu_m$ by $x f - f'$ by η . Now, the boundary conditions are at the axis and there is no, v equal to 0. You have f as 0 and also f' 0 is 0 and f' infinity is equal to 0. So, these are boundary conditions for the similarity variable f . The substitution gives similarity equation, if we make substitution for u and calculate $d \psi$ by dy and so on.

(Refer Slide Time: 09:19)

Laminar Vel Prediction - 2 - L42($\frac{5}{19}$)

Substitutions give similarity Eqn

$$\frac{f f'}{\eta^2} - \frac{f f''}{\eta} - \frac{f'^2}{\eta} = \frac{d}{d\eta} \left[f' - \frac{f}{\eta} \right] \quad \text{or}$$

$$\frac{d}{d\eta} \left[f' - \frac{f}{\eta} + \frac{f f'}{\eta} \right] = 0$$

Integrating from $\eta = 0$ to η and noting BCs $f(0) = f'(0) = 0$, we get $f f' = f - \eta f''$. The soln is

$$f = \frac{\eta^2}{1 + \eta^2/4}, \quad f' = \frac{2\eta}{(1 + \eta^2/4)^2}, \quad f'' = \frac{2(1 - 3\eta^2/4)}{(1 + \eta^2/4)^3} \quad \text{or}$$

$$u = \frac{C^2 \nu_m}{x} \left[\frac{2}{(1 + \eta^2/4)^2} \right], \quad v = \frac{C \nu_m}{x} \left[\frac{\eta - \eta^3/4}{(1 + \eta^2/4)^2} \right]$$

If you make this substitution here, then the two equations give us the following transformations, in terms of similarity variable $f f' / \eta^2 - f f'' / \eta - f'^2 / \eta = d/d\eta [f' - f/\eta]$. It can be represented as $d/d\eta = d/d\eta [f' - f/\eta + f f' / \eta]$. Combining these two, it can also be written as $d/d\eta [f' - f/\eta + f f' / \eta] = 0$.

If I integrate this from 0 to η , which is going from axis of the jet to some radius and the boundary conditions are $f(0)$ and $f'(0)$ are 0, we get simply this. It will transform to $f f' = f - \eta f''$. The solution is $f = \eta^2 / (1 + \eta^2/4)$, which is of interest. This is the velocity in the jet $f' = 2\eta / (1 + \eta^2/4)^2$ and $f'' = 2(1 - 3\eta^2/4) / (1 + \eta^2/4)^3$ and so forth.

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Laminar Vel Prediction - 1 - L42($\frac{4}{19}$)

Main assumption: Properties are uniform

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial r} + \frac{v}{r} = 0 \text{ and}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} = \frac{\nu_m}{r} \frac{\partial}{\partial r} \left[r \frac{\partial u}{\partial r} \right]$$

$$\psi(x, \eta) \equiv \nu_m x f(\eta) \text{ and } \eta \equiv C \frac{r}{x}$$

$$u \equiv \frac{1}{r} \frac{\partial \psi}{\partial r} = \frac{C^2 \nu_m}{x} \left[\frac{f}{\eta} \right]$$

$$v \equiv -\frac{1}{r} \frac{\partial \psi}{\partial x} = \frac{C \nu_m}{x} \left[f' - \frac{f}{\eta} \right]$$

Boundary conditions are: $f(0) = f'(0) = 0$ and $f'(\infty) = 0$.

Therefore, we can interpret u, which requires f dash and v. It requires f dash minus f by eta in the following manner.

(Refer Slide Time: 09:19)

Laminar Vel Prediction - 2 - L42($\frac{5}{19}$)

Substitutions give similarity Eqn

$$\frac{f f'}{\eta^2} - \frac{f f'}{\eta} - \frac{f'^2}{\eta} = \frac{d}{d\eta} \left[f' - \frac{f}{\eta} \right] \text{ or}$$

$$\frac{d}{d\eta} \left[f' - \frac{f}{\eta} + \frac{f f'}{\eta} \right] = 0$$

Integrating from $\eta = 0$ to η and noting BCs $f(0) = f'(0) = 0$, we get $f f' = f' - \frac{f}{\eta} f''$. The soln is

$$f = \frac{\eta^2}{1 + \eta^2/4}, \quad f' = \frac{2\eta}{(1 + \eta^2/4)^2}, \quad f'' = \frac{2(1 - 3\eta^2/4)}{(1 + \eta^2/4)^3} \text{ or}$$

$$u = \frac{C^2 \nu_m}{x} \left[\frac{2}{(1 + \eta^2/4)^2} \right], \quad v = \frac{C \nu_m}{x} \left[\frac{\eta - \eta^3/4}{(1 + \eta^2/4)^2} \right]$$

Here, u will be C square mu m divided by x in to 2 over 1 over eta square by 4 square. V would be given by this expression as eta minus eta cube by 4 divided by 1 plus eta square by 4 square into C nu m by x.

(Refer Slide Time: 11:18)

Determination of C - L42($\frac{6}{19}$)
 Multiply momentum eqn by r and integrate from $r = 0$ to $r = \infty$.
 Then

$$\frac{\partial}{\partial x} \left[\int_0^{\infty} \rho_m u^2 r dr \right] + \{ \rho_m v u r |_{\infty} - \rho_m v u r |_0 \}$$

$$= \left\{ \mu_m r \frac{\partial u}{\partial r} |_{\infty} - \mu_m r \frac{\partial u}{\partial r} |_0 \right\}$$

From BCs, terms in curly brackets are zero.
 Hence, substituting for u , we have

$$J_{mom} = 2\pi \int_0^{\infty} \rho_m u^2 r dr$$

$$= \frac{16}{3} \pi \rho_m \nu_m^2 C^2 = \rho_0 U_0^2 \left(\frac{\pi}{4} D^2 \right) = \text{constant}$$

$$C = \frac{\sqrt{3}}{8} Re \left(\frac{\rho_0}{\rho_m} \right)^{0.5} = \frac{1}{4 \nu_m} \sqrt{\frac{3 J_{mom}}{\pi \rho_m}} \rightarrow Re = \frac{U_0 D}{\nu_m}$$

Our next task is to estimate what is C. To do that we multiply momentum equation by r and integrate for r equal to 0 to r equal to infinity.

(Refer Slide Time: 11:30)

Laminar Vel Prediction - 1 - L42($\frac{4}{19}$)
 Main assumption: Properties are uniform

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial r} + \frac{v}{r} = 0 \text{ and}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} = \frac{\nu_m}{r} \frac{\partial}{\partial r} \left[r \frac{\partial u}{\partial r} \right]$$

$$\psi(x, \eta) \equiv \nu_m x f(\eta) \text{ and } \eta \equiv C \frac{r}{x}$$

$$u \equiv \frac{1}{r} \frac{\partial \psi}{\partial r} = \frac{C^2 \nu_m}{x} \left[\frac{f'}{\eta} \right]$$

$$v \equiv -\frac{1}{r} \frac{\partial \psi}{\partial x} = \frac{C \nu_m}{x} \left[f - \frac{f'}{\eta} \right]$$

Boundary conditions are: $f(0) = f'(0) = 0$ and $f'(\infty) = 0$.

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Determination of C - L42($\frac{6}{19}$)
 Multiply momentum eqn by r and integrate from $r = 0$ to $r = \infty$.
 Then

$$\frac{\partial}{\partial x} \left[\int_0^{\infty} \rho_m u^2 r dr \right] + \left\{ \rho_m v u r \Big|_{\infty} - \rho_m v u r \Big|_0 \right\} = \left\{ \mu_m r \frac{\partial u}{\partial r} \Big|_{\infty} - \mu_m r \frac{\partial u}{\partial r} \Big|_0 \right\}$$

From BCs, terms in curly brackets are zero.
 Hence, substituting for u, we have

$$J_{mom} = 2\pi \int_0^{\infty} \rho_m u^2 r dr$$

$$= \frac{16}{3} \pi \rho_m U_0^2 C^2 = \rho_0 U_0^2 \left(\frac{\pi}{4} D^2 \right) = \text{constant}$$

$$C = \frac{\sqrt{3}}{8} Re \left(\frac{\rho_0}{\rho_m} \right)^{0.5} = \frac{1}{4} \frac{U_0 D}{\nu_m} \sqrt{\frac{3 J_{mom}}{\pi \rho_m}} \rightarrow Re = \frac{U_0 D}{\nu_m}$$

You will see that the momentum equation that we have is multiplied by r. It is written in conservative form and then integrated from 0 to r. You will get d by dx of 0 to infinity $\rho_m u^2 r dr$ equal to $\rho_m v u r$ infinity minus $\rho_m v u r$ 0 and likewise, the gradient terms. Now, you will see these terms are absolutely 0 because at r equal to infinity, u is 0. So that is 0 and at r equal to 0, v is 0. That is 0 at r equal to infinity and $\frac{du}{dr}$ is 0 because u itself is 0. At the axis symmetry, $\frac{du}{dr}$ will be 0 and both this term as well as this term vanishes. As a result, all we get is 0 to infinity $\rho_m u^2 r dr$ and it should be a constant.

If I now multiply this by 2 pi, then you will see this is nothing but the jet momentum $\rho_m u^2 r dr$ into 2 pi r and integration. So, substituting for u, which is this expression here, I can show that the integration would give me $\frac{16}{3} \pi \rho_m U_0^2 C^2$ and that would be equal to constant. With x, it will amount to $\rho_0 U_0^2 \left(\frac{\pi}{4} D^2 \right)$. Here, ρ_0 and U_0 are the density and velocity at the entrance to the jet. This is a constant and therefore, I can now determine C in terms of jet momentum. I can also determine it, in terms of Reynolds number. So, C is equal to $\frac{\sqrt{3}}{8} Re \left(\frac{\rho_0}{\rho_m} \right)^{0.5}$ or in terms of jet momentum, where Reynolds number is $\frac{U_0 D}{\nu_m}$.

(Refer Slide Time: 13:35)

Final Soln - L42($\frac{7}{19}$)

$$\eta = \frac{\sqrt{3}}{8} \left(\frac{\rho_0}{\rho_m}\right)^{0.5} Re \frac{r}{x}$$

$$u^* = \frac{u x}{\nu_m} = \frac{3}{32} Re^2 [1 + \eta^2/4]^{-2} \left(\frac{\rho_0}{\rho_m}\right)$$

$$\frac{u}{U_0} = \frac{3}{32} \left(\frac{D}{x}\right) Re [1 + \eta^2/4]^{-2} \left(\frac{\rho_0}{\rho_m}\right)$$

$$\frac{u}{u_{max}} = \frac{1}{(1 + \eta^2/4)^2} \rightarrow \frac{u}{u_{max}} = \frac{1}{2} \text{ at } \eta_{1/2} = 1.287$$

Because at any x , $u \rightarrow 0$ as $y \rightarrow \infty$, it is difficult to identify the jet-width exactly. Hence, by convention, $\eta_{1/2}$ characterises the *jet half-width* ($r_{1/2}$). Thus,

$$\frac{r_{1/2}}{x} = \frac{\eta_{1/2}}{C} = 1.287 \times \frac{8}{\sqrt{3} Re} \left(\frac{\rho_0}{\rho_m}\right)^{-0.5} = \frac{5.945}{Re} \left(\frac{\rho_0}{\rho_m}\right)^{-0.5} = \tan \alpha$$

where α is the jet-spread angle.

Remember, what was eta? Eta is C into r by x (Refer Slide Time: 13:37). If I substitute for C, I would get eta equal to that definition u star, which is an \dots I can now write u also as u star equal to u x by nu m in this manner - in terms of Reynolds number and u over U naught would be 3 by 32 D by x Re 1 plus eta square by 4 raise to minus 2 rho naught divided by rho m. This is u divided by U at inlet. It will be function of x and it will be function of \dots . You can see, as x increases, u is decreasing. At the same time, radius y increases, u decreases. It will be maximum, when eta is equal to 0. So, u over u max would simply be 1 over 1 plus eta square by 4 whole square.

Since, we do not know where the edge of the jet will be, it is customary to define what is called half jet-width. So, where u over u max will be equal to half eta, it would assume a value of 1.287. So, you can see 1.287 square divided 4 plus 1 whole square would give you 1 by 2 and therefore, eta half equal to 1.287 is dimensionless and it is jet half width. So, this is by convention and we say eta half characterizes the jet-width r half. So, r half by x will be eta half divided by C equal to 1.287 8 by 3 Reynolds rho naught by rho m raised to minus 0.5 equal to 5.945 divided by Reynolds rho naught by rho m raise to minus 0.5. It is nothing but tan alpha, which is what we call as the jet spread angle.

(Refer Slide Time: 15:55)

Main Objective L42($\frac{2}{19}$)

(a) LAMINAR BURNING BY (b) PROBLEM

The main objective is to predict

- Flame Length L_f
- Flame shape or $r_f(x)$

in stagnant surroundings assuming SCR:
 $1 \text{ kg of fuel} + R_{ox} \text{ kg of oxidant air} = (1 + R_{ox}) \text{ kg of product.}$

April 21, 2011 4/21

You will recall that I had shown you the jet spread. So, r half divided by x and corresponding x will simply give you the angle of the jet.

(Refer Slide Time: 13:35)

Final Soln - L42($\frac{7}{19}$)

$$\eta = \frac{\sqrt{3}}{8} \left(\frac{\rho_0}{\rho_m}\right)^{0.5} Re \frac{r}{x}$$

$$u' = \frac{u x}{\nu_m} = \frac{3}{32} Re^2 [1 + \eta^2/4]^{-2} \left(\frac{\rho_0}{\rho_m}\right)$$

$$\frac{u}{U_0} = \frac{3}{32} \left(\frac{D}{x}\right) Re [1 + \eta^2/4]^{-2} \left(\frac{\rho_0}{\rho_m}\right)$$

$$\frac{u}{U_{max}} = \frac{1}{(1 + \eta^2/4)^2} \rightarrow \frac{u}{U_{max}} = \frac{1}{2} \text{ at } \eta_{1/2} = 1.287$$

Because at any x , $u \rightarrow 0$ as $y \rightarrow \infty$, it is difficult to identify the jet-width exactly. Hence, by convention, $\eta_{1/2}$ characterises the *jet half-width* ($r_{1/2}$). Thus,

$$\frac{r_{1/2}}{x} = \frac{\eta_{1/2}}{C} = 1.287 \times \frac{8}{\sqrt{3} Re} \left(\frac{\rho_0}{\rho_m}\right)^{-0.5} = \frac{5.945}{Re} \left(\frac{\rho_0}{\rho_m}\right)^{-0.5} = \tan \alpha$$

where α is the jet-spread angle.

April 21, 2011 4/21

It is obvious that bigger the Reynolds number, smaller would be the jet spread angle.

(Refer Slide Time: 16:15)

Prediction of L_f and $r_f(x) - 1 - L42(\frac{8}{19})$
 Assuming $Le = 1$ and SCR, all eqns can be rendered in conserved-property form

$$\frac{\partial}{\partial x} (\rho_m u r \Phi) + \frac{\partial}{\partial r} (\rho_m v r \Phi) = \frac{\partial}{\partial r} \left[\Gamma_m r \frac{\partial \Phi}{\partial r} \right]$$

$$\Phi = \omega_{fu} - \frac{\omega_{ox}}{R_{st}} = h_m + \Delta H_c \omega_{fu} = \frac{u}{U_0} \rightarrow R_{st} = \frac{A}{F}$$

To locate the flame, we define conserved scalar $\Phi = f$

$$f \equiv \frac{\Phi - \Phi_A}{\Phi_F - \Phi_A} = \frac{(\omega_{fu} - \omega_{ox}/R_{st}) - (\omega_{fu} - \omega_{ox}/R_{st})_A}{(\omega_{fu} - \omega_{ox}/R_{st})_F - (\omega_{fu} - \omega_{ox}/R_{st})_A}$$

where subscripts A and F refer to Air and Fuel Streams .
 Hence,

$$f = \frac{(\omega_{fu} - \omega_{ox}/R_{st}) + 1/R_{st}}{1 + 1/R_{st}} \text{ because } \omega_{ox,ox} = \omega_{fu,F} = 1$$

Now, we want to turn to the prediction of length and r_f . Assuming Louise number equal to 1 and making simple chemical reaction, all equations can be rendered in conservative form of this type. We have done this many times and ϕ will be simply be ω_{fu} minus ω_{ox} divided by R_{st} equal to h_m in plus $\Delta H_c \omega_{fu}$. It will also be equal to u over U_{naught} because our momentum equation itself is in conserved property form. It does not have a pressure gradient and so ϕ can now represent both the mass fractions. It can represent enthalpy and it can represent velocity.

Now, to locate the flame, we define what is called as the conserved scalar ϕ equal to f . Here, f is defined as ϕ minus ϕ_A divided by ϕ_F minus ϕ_A equal to ω_{fu} minus ω_{ox} . Now, suffix A implies the air stream and suffix F implies the fuel stream. So, ω_{fu} minus ω_{ox} R_{st} divided by ω_{fu} minus ω_{ox} R_{st} in the air stream. The same quantity is in the F stream, the fuel stream and the air stream

Now, you can see ω_{fu} in the air stream would be 0, whereas ω_{ox} in the air stream would be 1 because ω_{ox} represents air. So that would be 1 and this will be 1 divided by R_{st} ω_{ox} . Fuel in the fuel stream is 1, whereas ω_{ox} in the fuel stream is 0. So, this will be simply 1, as you can see here and this would again be 0 and this would be 1, so you get 1 by R_{st} .

Essentially, f is often called as the mixture fraction. It can be given as $\omega_{fu} - \omega_{ox}$ divided by $R_{st} + 1$ over R_{st} divided by $1 + 1/R_{st}$. This is by taking ϕ equal to $\omega_{fu} - \omega_{ox} R_{st}$ and f is given by that.

(Refer Slide Time: 18:34)

Prediction of L_f and $r_f(x)$ - 2 - L42($\frac{9}{19}$)

The flame is located where $(\omega_{fu} - \omega_{ox}/R_{st}) = 0$. Hence,

$$f = f_{stoich} = 1/(1 + R_{st}) \quad (\text{flame edge})$$

$$f = f_{stoich} + \omega_{fu}/(1 + 1/R_{st}) \quad (\text{inside})$$

$$f = f_{stoich} - (\omega_{ox}/R_{st})/(1 + 1/R_{st}) \quad (\text{outside})$$

(a) FLAME EDGE
(b) FLAME STRUCTURE

April 21, 2011 11:21

Now, the flame is located where $\omega_{fu} - \omega_{ox}$ divided by R_{st} is 0, where the fuel and oxygen are in stoichiometric proportion. As we have seen, stoichiometric proportion means ω_{fu} will be equal to ω_{ox} by R_{st} . So, f equal to f_{stoich} will be equal to $1 / (1 + R_{st})$ and that is the edge of the flame f . It will be equal to $f_{stoich} + \omega_{fu} / (1 + 1/R_{st})$ inside the flame. Now, f will be equal to $f_{stoich} - \omega_{ox} / R_{st} / (1 + 1/R_{st})$ and this follows from this.

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Prediction of L_f and $r_f(x)$ - 1 - L42($\frac{8}{19}$)
 Assuming $Le = 1$ and SCR, all eqns can be rendered in conserved-property form

$$\frac{\partial}{\partial x} (\rho_m u r \Phi) + \frac{\partial}{\partial r} (\rho_m v r \Phi) = \frac{\partial}{\partial r} \left[\Gamma_m r \frac{\partial \Phi}{\partial r} \right]$$

$$\Phi = \omega_{fu} - \frac{\omega_{ox}}{R_{st}} = h_m + \Delta H_c \omega_{fu} = \frac{u}{U_b} \rightarrow R_{st} = \frac{A}{F}$$

To locate the flame, we define conserved scalar $\Phi = f$

$$f \equiv \frac{\Phi - \Phi_A}{\Phi_F - \Phi_A} = \frac{(\omega_{fu} - \omega_{ox}/R_{st}) - (\omega_{fu} - \omega_{ox}/R_{st})_A}{(\omega_{fu} - \omega_{ox}/R_{st})_F - (\omega_{fu} - \omega_{ox}/R_{st})_A}$$

where subscripts A and F refer to Air and Fuel Streams .
 Hence,

$$f = \frac{(\omega_{fu} - \omega_{ox}/R_{st}) + 1/R_{st}}{1 + 1/R_{st}} \text{ because } \omega_{ox,ox} = \omega_{fu,F} = 1$$

The flame edge would correspond to simply $\omega_{fu} - \omega_{ox} = 0$.

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Prediction of L_f and $r_f(x)$ - 2 - L42($\frac{9}{19}$)
 The flame is located where $(\omega_{fu} - \omega_{ox}/R_{st}) = 0$. Hence,

$$f = f_{stoich} = 1/(1 + R_{st}) \text{ (flame edge)}$$

$$f = f_{stoich} + \omega_{fu}/(1 + 1/R_{st}) \text{ (inside)}$$

$$f = f_{stoich} - (\omega_{ox}/R_{st})/(1 + 1/R_{st}) \text{ (outside)}$$

It would be equal $1 / (1 + R_{st})$. This shows outside of the flame, where f lies between 0 and f_{stoich} and whereas inside the flame, f lies between 0 and 1. To see this graphically, we draw a graph of f equal to 0 to f equal to 1. I have shown f_{stoich} because R_{st} for a given fuel is known that is oxi air to fuel ratio is known. Therefore, f_{stoich} can always be plotted on this f . It is dimensionless fraction and it can only go from 0 to 1.

You will see that on the outside of the flame, this is the variation of oxygen up to f stoich. At the flame front, the oxygen disappears. So, outside the flame, there is oxygen, but it will disappear at f stoich. The fuel is inside and it would go on decreasing from central line to the edge of the flame. The product would go on increasing up to the flame edge and then would decrease the temperature. It would be T infinity in the outside; it will increase to T at the f stoich and will again decrease.

Now, ω_{fu} minus ω_{Rst} need not necessarily be a single value. There can be a range of values of ω_{fu} and ω_{ox} , where the difference ω_{fu} minus ω_{ox} divided by Rst is equal to 0. In fact that is what is often found and that is what I have shown here. The fuel fraction would also appear little bit on the outside, very close there to f stoich. Oxygen would appear little bit on the inside and this zone is the flame thickness zone; the flame edge thickness, which you often see. For all practical purpose, we are going to say that the flame edge is a very sharp surface and f equal to f stoich is a very sharp surface.

(Refer Slide Time: 21:48)

Prediction of L_f and $r_f(x)$ - 3 - L42($\frac{10}{19}$)

If we take $\Phi = h^* = h_m + \Delta H_c \omega_{fu}$ then

$$f = h^* = \frac{(h_m + \Delta H_c \omega_{fu}) - (h_m + \Delta H_c \omega_{fu})_A}{(h_m + \Delta H_c \omega_{fu})_F - (h_m + \Delta H_c \omega_{fu})_A}$$

$$= \frac{c p_m (T - T_\infty) + \Delta H_c \omega_{fu}}{c p_m (T_0 - T_\infty) + \Delta H_c}$$

Thus, noting that r_f corresponds to $r_{fj} = C (r_f/x)$ and f_{stoich} , and $r_f = 0$ at $x = L_f$

$$\Phi = \frac{u}{U_0} = f = h^* = \frac{3}{32} \left(\frac{D}{x} \right) \left(\frac{\rho_0}{\rho_m} \right) Re (1 + r^2/4)^{-2}$$

$$\frac{r_f}{x} = \frac{16}{3^{0.5} Re} \left(\frac{\rho_m}{\rho_0} \right)^{0.5} \left[\left\{ \frac{3}{32} \left(\frac{D}{x} \right) \left(\frac{\rho_0}{\rho_m} \right) \frac{Re}{f_{stoich}} \right\}^{0.5} - 1 \right]^{0.5}$$

$$\frac{L_f}{D} = \frac{3}{32} \left(\frac{\rho_0}{\rho_m} \right) \frac{Re}{f_{stoich}} = \frac{3}{32} Re (1 + R_{st}) \left(\frac{\rho_0}{\rho_m} \right)$$

April 21, 2011 18/21

Now, if we take ϕ equal to h^* equal to $h_m \Delta H_c \omega_{fu}$, where f is equal to h^* . You will see that I will get h_m minus $f \Delta H_c \omega_{fu}$. It is the same quantity in the air stream, the same quantity in the F stream and same quantity in the air stream. So, this would simply be $c p_m T$ minus T infinity $\Delta H_c \omega_{fu}$ because, remember there is no fu in the air stream. It is $c p_m T$ naught minus T infinity because in the air

stream, the temperature is T_{∞} and in the fuel stream, f_u is equal to 1. So, you get ΔH_c here, whereas there is no f_u in the air stream. So, ΔH_c will be 0 and h_m in the fuel stream minus h_m in the air stream is $c_p m T_{\infty} - T_{\infty}$.

Thus noting that r_f corresponds to $\eta_f C r_f$ by x and f_{stoich} . Here, r_f is equal to 0 at x equal to L_f . We can say that ϕ is equal to u over $U_{\infty} f_{h^*}$. It is equal to 3 by 32 , which is the solution that we had written earlier. The solution to ϕ would be simply equal to u over U_{∞} for all variables ϕ . I have written (Refer Slide Time: 23:10) that as ϕ equal to u over U_{∞} and all that sort of thing here.

At r_f , if I replace η by η_f , then I can recover r_f by x equal to 16 by 3 raised to 0.5 Re into this density into that expression. Here, f_{stoich} is of course 1 over $1 + R_{st}$ and setting r_f equal to 0. If I set that to 0, which is essentially means this quantity is equal to 1, then x equal to L_f . I will get length of the flame divided by diameter equal to 3 by 32 ρ_m naught by $\rho_m Re f_{stoich}$, when f_{stoich} is 1 over $1 + R_{st}$. What this shows is that L_f would increase with Reynolds number of the jet. The higher the velocity of the jet, longer will be the flame.

(Refer Slide Time: 24:16)

Turbulent Jet Flame - L42($\frac{11}{19}$)

- 1 In Laminar flames, L_f increases with U_0 .
- 2 In Turbulent flames, $L_f \approx \text{const.}$ and
- 3 Radial distribution of u is nearly uniform over greater part of δ .

Eqs of slide 3 apply with

$$\rho_m D_{eff} \approx \frac{\mu_{eff}}{Sc_f}$$

$$\rho_m \alpha_{eff} \approx \frac{\mu_{eff}}{Pr_f}$$

with $Sc_f = Pr_f = 0.9$

Now, turning to turbulent jet flame, let us look at what happens with increasing flame length and increasing flame velocity. Experimentally, for the first time, a scientist called Hutton documented it. He showed that in the laminar flames, the flame length increase

almost linearly with nozzle velocity. As we have shown on the previous slide, we had shown here that L_f would almost linearly increase with velocity U_{naught} . After a certain velocity, the flame length actually decreases. When the flow is completely turbulent, the flame is actually independent of the nozzle velocity. In turbulent flows, turbulent jets flames - L_f is almost constant that is the length of the flame is almost constant.

We have something to think about here, how we can predict the turbulent jet flame? The radial distribution of u is nearly uniform over greater part of the length. Experimentally, it is very difficult to identify the flame length because the turbulent flame is never steady; it is unsteady and the edges are very jagged. So, you get the flame, which is oscillating in the actual direction. You can only take photographs of that to measure what sort of time average and flame length, if you like to observe from photographs.

The previous equations that we had used for laminar jet flame applies for turbulent flow also. The only thing we have to use is effective values of viscosity or kinematic viscosity, thermal diffusivity and mass diffusivity. As I show here, the equations of the slide 3 will apply. The only thing is the effective values will be used as $\mu_{\text{effective}}$ divided by Schmidt number. Effective thermal diffusivity will be $\mu_{\text{effective}}$ divided by Prandtl number. In gases, Schmidt number and Prandtl number under turbulent range is point about 0.9 and we have seen that in turbulence modeling part of the course.

(Refer Slide Time: 26:47)

Velocity Prediction - 1 - L42($\frac{12}{19}$)

- The simplest formula¹ is $\mu_{\text{eff}} = 0.01 \times \rho_m |u_{\infty} - u_{ax}| \delta$
- For stagnant surroundings $u_{\infty} = 0$ and $u_{ax} = u_{\text{max}}$. Also, from experiments, $(\delta/r_{1/2}) \approx 2.5$. Hence, $\mu_{\text{eff}} = 0.0256 \rho_m u_{\text{max}} r_{1/2} = F(r)$
- Thus, all laminar solns apply with μ changed to μ_{eff} . Hence

$$u_{\text{max}}^* = \frac{u_{\text{max}} x}{\nu_m} = \left(\frac{3}{32}\right) Re_{\text{turb}}^2 \left(\frac{\rho_0}{\rho_m}\right)$$

$$\frac{r_{1/2}}{x} = \frac{5.945}{Re_{\text{turb}}} \left(\frac{\rho_0}{\rho_m}\right)^{-0.5}$$

$$= 5.945 \times \left[\frac{0.0256 \rho_m u_{\text{max}} r_{1/2}}{\rho_m U_0 D} \right] \left(\frac{\rho_0}{\rho_m}\right)^{-0.5} \text{ Hence}$$

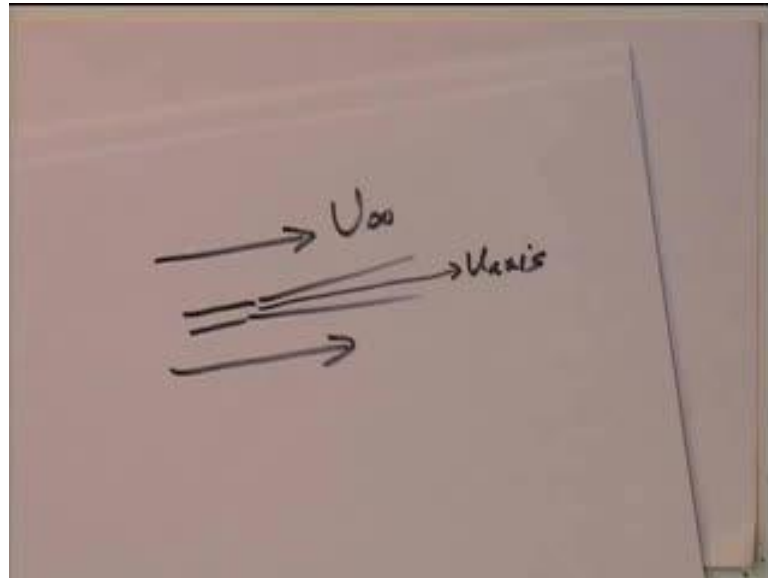
$$\frac{u_{\text{max}}}{U_0} = 6.57 \left(\frac{D}{x}\right) \left(\frac{\rho_0}{\rho_m}\right)^{0.5} \text{ or, Combining with } u_{\text{max}}^*$$

¹Spalding D. B. Combustion and Mass Transfer, Pergamon Press, Oxford (1979)

April 21, 2011 14:21

We now need to define μ effectively. Now, the simplest formula for μ effective is $0.01 \rho_m u_\infty$ minus u_{axis} . Here, u_{axis} is the velocity of the jet at the axis and u_∞ is a co flowing jet velocities.

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For example, I may have a jet, which is like that and there is a parallel stream at u_∞ . Sometimes, you do get the co flowing streams. We define u_{axis} , u_x and u_∞ .

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Velocity Prediction - 1 - L42($\frac{12}{19}$)

- The simplest formula¹ is $\mu_{\text{eff}} = 0.01 \times \rho_m |u_\infty - u_{\text{axis}}| \delta$
- For stagnant surroundings $u_\infty = 0$ and $u_{\text{ax}} = u_{\text{max}}$. Also, from experiments, $(\delta/r_{1/2}) \approx 2.5$. Hence, $\mu_{\text{eff}} = 0.0256 \rho_m u_{\text{max}} r_{1/2} F(r)$
- Thus, all laminar solns apply with μ changed to μ_{eff} . Hence

$$u_{\text{max}}^* = \frac{u_{\text{max}} x}{\nu_m} = \left(\frac{3}{32}\right) Re_{\text{lamb}}^2 \left(\frac{\rho_0}{\rho_m}\right)$$

$$\frac{r_{1/2}}{x} = \frac{5.945}{Re_{\text{lamb}} \nu_m} \left(\frac{\rho_0}{\rho_m}\right)^{-0.5}$$

$$= 5.945 \times \left[\frac{0.0256 \rho_m u_{\text{max}} r_{1/2}}{\rho_m U_0 D}\right] \left(\frac{\rho_0}{\rho_m}\right)^{-0.5} \text{ Hence}$$

$$\frac{u_{\text{max}}}{U_0} = 6.57 \left(\frac{D}{x}\right) \left(\frac{\rho_0}{\rho_m}\right)^{0.5} \text{ or, Combining with } u_{\text{max}}^*$$

¹Spalding D. B. Combustion and Mass Transfer, Pergamon Press, Oxford (1979)

April 21, 2011 14:21

The turbulent viscosity would be given as $0.01 \rho_m u_{\infty} \delta$. This is a simplest form of the turbulent viscosity specification. It was proposed by Spalding in the book - Combustion and Mass Transfer, Pergamon Press at Oxford in 1979. For stagnant surrounding, u_{∞} is 0 and u_{ax} will be u_{max} . From experiments, it is found that the jet width δ divided by $r_{1/2}$ the jet half width is about 2.5 in turbulent jets. Therefore, if I substitute δ for $r_{1/2}$ and set u_{∞} equal to 0, then you will see I get $\mu_{effective}$ equal to $0.0256 \rho_m u_{max} r_{1/2}$. As you can see, $r_{1/2}$ can only be a function of x u_{max} . Likewise, it can be a function of x and therefore, this is not a function of r .

Essentially, what we have said is that $\mu_{effective}$ is constant across the width of the jet. It may vary with x and we will see whether it does or not from all our solution. We will apply, if we assume constant properties. Since, $\mu_{effective}$ is also nearly constant, we will simply say change μ to $\mu_{effective}$. You will get $u_{star\ max}$ equal to u_{max} by μ_m equal to $Re_{turbulent}$ square $r_{1/2}$ by x . It will be again given by that where Re is replaced by $Re_{turbulent}$. What is $Re_{turbulent}$? It will be $\rho_m U_{naught}$ into D divided by $\mu_{effective}$, which is $0.0256 \rho_m$ into $u_{max} r_{1/2}$. From this, I can get u_{max} over U_{naught} . It will be equal to $c\ 6.57$. Remember, $r_{1/2}$ will get cancelled and $6.57 D$ by x ρ_m is raised to 0.5.

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Velocity Prediction - 2 - L42($\frac{13}{19}$)

$$\left(\frac{u_{max}}{U_0}\right)^2 = 3.662 \left(\frac{D^2}{r_{1/2} x}\right) \left(\frac{\rho_0}{\rho_m}\right)$$

$$\frac{r_{1/2}}{x} = \frac{3.662}{(6.57)^2} = 0.0848 \text{ constant}$$

This result agrees very well with Expt data for $(x/D) > 6.5$.
Replacing $r_{1/2}$ and u_{max} , we have

$$\mu_{eff} = 0.0256 \rho_m \times 6.57 U_0 \left(\frac{D}{x}\right) \left(\frac{\rho_0}{\rho_m}\right)^{0.5} \times (0.0848 x)$$

$$= 0.01426 (\rho_0 \rho_m)^{0.5} U_0 D \approx \text{constant}$$

Also, since $\eta_{1/2} = 1.287$,

$$\eta = 1.287 \left(\frac{r}{r_{1/2}}\right) \rightarrow \frac{u}{u_{max}} = \left[1 + 0.414 \left(\frac{r}{r_{1/2}}\right)^2\right]^{-2}$$

April 21, 2011 18:21

Now, combining this with u_{max} , I will get u_{max} over U_{naught} square equal to that or r_{half} by x equal to 3.662 divided by 6.57 square is equal to 0.0848. So, r_{half} is definitely a function of x . In fact, it increases linearly with x as r_{half} divided by x 0.0848. In our effective viscosity formula, r_{half} will vary with x . Now, let us see how u_{max} will vary.

(Refer Slide Time: 26:47)

Velocity Prediction - 1 - L42(12/19)

- The simplest formula¹ is $\mu_{eff} = 0.01 \times \rho_m |U_{\infty} - U_{ax}| \delta$
- For stagnant surroundings $U_{\infty} = 0$ and $U_{ax} = U_{max}$. Also, from experiments, $(\delta / r_{1/2}) \approx 2.5$. Hence, $\mu_{eff} = 0.0256 \rho_m U_{max} r_{1/2} = F(r)$
- Thus, all laminar solns apply with μ changed to μ_{eff} . Hence

$$u_{max}^* = \frac{U_{max} x}{\nu_m} = \left(\frac{3}{32}\right) Re_{lamb}^2 \left(\frac{\rho_0}{\rho_m}\right)$$

$$\frac{r_{1/2}}{x} = \frac{5.945}{Re_{turb} \nu_m} \left(\frac{\rho_0}{\rho_m}\right)^{-0.5}$$

$$= 5.945 \times \left[\frac{0.0256 \rho_m U_{max} r_{1/2}}{\rho_m U_0 D} \right] \left(\frac{\rho_0}{\rho_m}\right)^{-0.5} \text{ Hence}$$

$$\frac{U_{max}}{U_0} = 6.57 \left(\frac{D}{x}\right) \left(\frac{\rho_0}{\rho_m}\right)^{0.5} \text{ or, Combining with } u_{max}^*$$

¹Spalding D. B. Combustion and Mass Transfer, Pergamon Press, Oxford (1979)

The results agree very well. This particular result agrees very well with experimental data, when x by D is greater than 6.5 because the initial range from outside the mouth of the jet, r_{half} is largely governed by ellipticity, whereas we have used parabolic assumption. Replacing r_{half} and u_{max} , it would be $\mu_{effective}$ equal to 0.0256. Remember, I am replacing u_{max} in terms of $U_{naught} D$ by x . In fact, you can see u_{max} is inversely proportional to x . We just showed that r_{half} is directly proportional to x . Therefore, $\mu_{effective}$ is not at all a function of x and in fact, it is absolute constant.

We just showed that r_{half} is directly proportional to x (Refer Slide Time: 31:37) Therefore, $\mu_{effective}$ is not at all a function of x and in fact, it is absolute constant. That is why we our replacement of μ to $\mu_{effective}$ is perfectly varied. All we are saying is - a turbulent jet is simply a laminar jet with a much more augmented viscosity, which is constant. $\mu_{effective}$ then becomes 0.01426. I have simply replaced here, u_{max} equal to 6.57 U_{naught} and so on. Here, r_{half} is 0.0848 x . So, x and x gets cancel and we get u_{mu} infinity equal to an absolute constant as 0.01426 $U_{naught} D$ multiplied

by ρ naught ρ m raised to half. Since η half is 1.287, we can say that η will be $1.287 r$ over r half and u over u max will be $1 - 0.414 r$ over r half, whole square is raised to minus 2. So, this is the velocity profile of a turbulent jet u over u max equal to all that. η would be $1.287 r$ over r half and r half is simply 0.0848 into x .

(Refer Slide Time: 32:39)

Prediction of L_f and $r_f(x)$ - 1 - L42($\frac{14}{19}$)

- A turbulent flame is essentially unsteady and its edges are jagged. Fragments of gas intermittently detach from the main body of the flame and flare, diminishing in size. Turbulence affects not only L_f , but also the entire reaction zone near the edge of the flame. Compared with a laminar flame, this zone is also much thicker.
- This implies that if the time-averaged values $\bar{\omega}_{fu}$ and $\bar{\omega}_{ox}$ are plotted with radius r , then the two profiles show considerable overlap around the crossover point $f = f_{stoich}$.
- Unlike the overlap in a laminar flame, which is caused by finite chemical kinetic rates, however, in turbulent diffusion flames, the overlap is caused by turbulence.
- In the presence of turbulence, R_{fu} actually experienced is not as high as that estimated from $\bar{R}_{fu} \propto \bar{\omega}_{fu}^x \bar{\omega}_{ox}^y$. This is because the fuel and oxidant at a point are present at different times - must allow for probability.

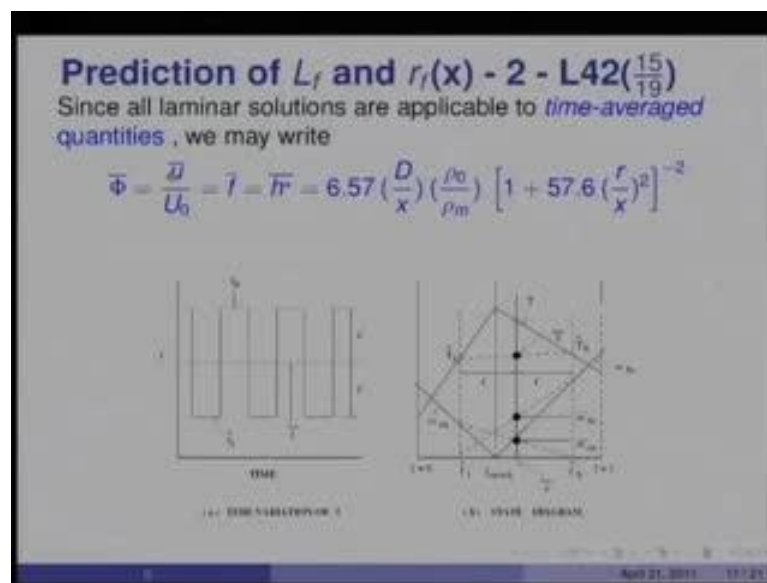
April 21, 2011 18:25

Now, you turn to L_f and r_f prediction. A turbulent flame is essentially unsteady and its edges are jagged. As I said earlier, fragments of gas intermittently detach from the main body of the flame and flare outside is diminishing in size. Turbulence of x is not only L_f , but also the entire reaction zone near the edge of the flame. Compared with the laminar flame, this zone is much thicker. We had identified that in laminar flame, there is a slight overlap, where f equal is equal to f stoich.

In turbulent jets, this zone is little bit thicker and it is called as flame zone or the edge zone. It is somewhat thicker. This implies that if the time average values of ω_{fu} and ω_{ox} are plotted with radius r , then the two profile show considerable overlap around a cross over point f equal to f stoich, unlike the overlap in laminar flame. It is caused by finite chemical kinetic rays, however in turbulent diffusion flames, the overlap is caused by turbulence. In the presence of turbulence R_{fu} actually experienced is not as high as that estimated from R_{fu} proportional to ω_{fu} raise to x ω_{ox} raise to y .

This is because the fuel and oxidant are present at different times. Although the average values of ω_{fu} and ω_{ox} may be high, the actual reaction rates R and \dot{m}_{fu} is found to be somewhat less. Therefore, how much it is less depends on the probability of ω_{fu} and ω_{ox} meeting each other in the right proportions to cause a chemical reaction. So, the effective rates of burning are actually smaller. It would be calculated as ω_{fu} raised to x and ω_{fu} raised to y , which is the typical formula for burning the rate of a fuel. Therefore, we must allow for the probability of turbulence.

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Since, all laminar solutions are applicable to time average quantities, we may write $\bar{\Phi} = \bar{u} / U_0 = \bar{f} = \bar{r} = h^* = \dots$ and the solution is repeated. Now, let us see what actually happens. Let us consider mixture fraction f and let this be the time average value.

In reality, the mixture fraction f would fluctuate between f_{high} with a cap on top and f_{low} with a cap on top. These are the instantaneous values. So, what we are saying is that at the edge of the flame, in the presence of turbulence or because of the turbulent, the mixture fraction would jump from a low value to a high value and high value to a low value almost instantly. When it reaches a high value, it will spend some time in the high value. When it is suddenly fluctuates to a low value, again it spends a little time there. This is called the square wave or the rectangular wave.

This is an assumed variation of f . So, f is equal to f mean plus f dash on the positive side and f dash on the negative side. So, f dash can be both positive and negative around f bar and this is what I have shown here. Although, f stoich of the fuel is 1 over 1 plus R st, it resides somewhere here on the f equal to zero and f equal to 5. So, the instantaneous high value may be here. The average value will be in between the two, but that may not equal f stoich. The average value of f may not equal the f stoich value. What are the deductions that we can draw for ω_{ox} average, ω_{fuel} average and T average.

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Prediction of L_f and $r_f(x)$ - 3 - L42($\frac{16}{19}$)

- ① With reference to the figure, suppose that the value of \hat{f} truly fluctuates between a low value \hat{f}_l and a high value \hat{f}_h .
- ② Let us assume that the fluid spends *equal* times at the two extremes and sharply moves from one extreme to the other
- ③ Then, $\bar{f} = \frac{1}{2}(\hat{f}_h + \hat{f}_l)$ and $f' = \frac{1}{2}(\hat{f}_h - \hat{f}_l)$
- ④ Thus, $\bar{\omega}_{fu} = 0.5(\omega_{fu,l} + \omega_{fu,h})$ (filled circle) $>$ ω_{fu} (open circle) corresponding to $\bar{f} > f_{stoich}$.
- ⑤ Likewise, $\bar{\omega}_{ox} = 0.5(\omega_{ox,l} + \omega_{ox,h}) >$ ω_{ox} (which is zero) corresponding to $\bar{f} > f_{stoich}$.
- ⑥ $\bar{T} = 0.5(\hat{T}_l + \hat{T}_h) <$ T corresponding to $\bar{f} > f_{stoich}$.
- ⑦ The above observations will also apply when $\bar{f} < f_{stoich}$. Thus, in general, finite amounts of fuel and oxidant are found when $\bar{f} = f_{stoich}$
- ⑧ If f_{stoich} does not lie between \hat{f}_l and \hat{f}_h then \bar{T} , $\bar{\omega}_{ox}$ and $\bar{\omega}_{fu}$ will of course correspond to \bar{f} .

April 21, 2011 18 / 21

With reference to the figure, the value of f truly fluctuates between a low value f and high value f . Let us assume that the fluids spend equal time at the two extremes and sharply moves from one extreme to the other. So, f bar - the average time and average value f will be half of high and low values instantaneous values. The fluctuation - f dash will be half of difference between high and low values. Thus, ω_{fu} bar would be 0.5 ω_{fu} . Instantaneous value of low and high mass fractions of fuel are shown by the field circle.

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Prediction of L_f and $r_f(x)$ - 2 - L42($\frac{15}{19}$)
 Since all laminar solutions are applicable to *time-averaged* quantities, we may write

$$\bar{\Phi} = \frac{\bar{D}}{U_0} = \bar{T} = \bar{T} = 6.57 \left(\frac{D}{x}\right) \left(\frac{\rho_0}{\rho_m}\right) \left[1 + 57.6 \left(\frac{r}{x}\right)^2\right]^{-2}$$

(a) TIME AVERAGE (b) STATE AVERAGE

April 21, 2011 17:57

You can see that this is the value shown by the field circle. It would be greater than omega fu, which is the value corresponding to the local value. The time average value is over there; it is shown by the field value. It is greater corresponding to f bar, greater than f stoich. Now, we are taking the case of f bar greater than f stoich, but the story can be repeated, even when f bar is less than f stoich.

(Refer Slide Time: 38:52)

Prediction of L_f and $r_f(x)$ - 3 - L42($\frac{16}{19}$)

- With reference to the figure, suppose that the value of \bar{T} truly fluctuates between a low value \bar{T}_l and a high value \bar{T}_h .
- Let us assume that the fluid spends *equal* times at the two extremes and sharply moves from one extreme to the other
- Then, $\bar{T} = \frac{1}{2}(\bar{T}_h + \bar{T}_l)$ and $\bar{T} = \frac{1}{2}(\bar{T}_h - \bar{T}_l)$
- Thus, $\bar{\omega}_{fu} = 0.5(\bar{\omega}_{fu,l} + \bar{\omega}_{fu,h})$ (filled circle) $> \omega_{fu}$ (open circle) corresponding to $\bar{T} > f_{stoich}$.
- Likewise, $\bar{\omega}_{ox} = 0.5(\bar{\omega}_{ox,l} + \bar{\omega}_{ox,h}) > \omega_{ox}$ (which is zero) corresponding to $\bar{T} > f_{stoich}$.
- $\bar{T} = 0.5(\bar{T}_l + \bar{T}_h) < T$ corresponding to $\bar{T} > f_{stoich}$.
- The above observations will also apply when $\bar{T} < f_{stoich}$. Thus, in general, finite amounts of fuel and oxidant are found when $\bar{T} = f_{stoich}$.
- If f_{stoich} does not lie between \bar{T}_l and \bar{T}_h then \bar{T} , $\bar{\omega}_{ox}$ and $\bar{\omega}_{fu}$ will of course correspond to \bar{T} .

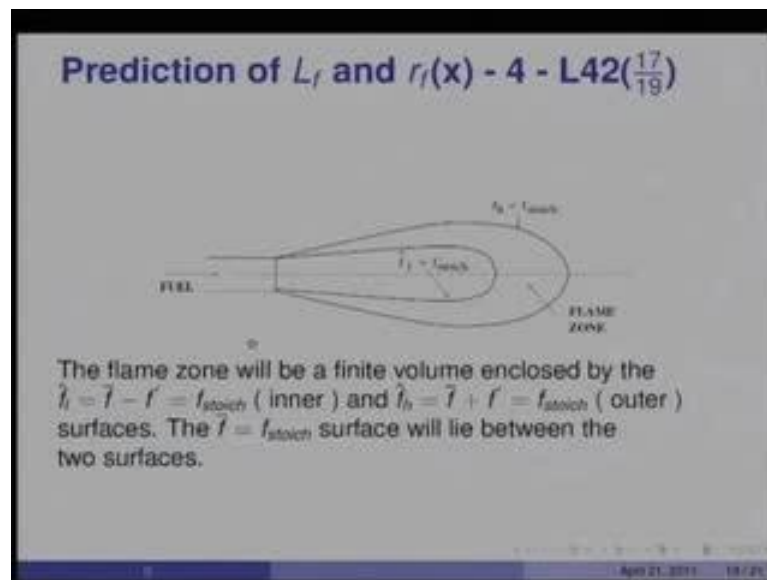
April 21, 2011 18:24

Likewise, omega ox is also greater than omega ox, which is 0 for bar greater than f stoich. You will see that the oxygen has already been consumed. So, the local value is 0,

but the time average value is somewhere there as ω_{ox} time average. For \bar{f} greater than f_{stoich} , time average is greater than ω_{ox} . \bar{T} , $0.5 T$ instantaneous low plus T instantaneous high is less than T . You can see (Refer Slide Time: 39:07) that \bar{T} is less than T , the local value T corresponding to \bar{f} the time average \bar{f} .

The above observations reveal we will also apply when \bar{f} is less than f_{stoich} thus in general finite amounts of fuel and oxygen are found when \bar{f} is equal to f_{stoich} when \bar{f} is equal to f_{stoich} you will get finite amounts of ω_{fu} and ω_{ox} at \bar{f} equal to f_{stoich} and therefore we get a little overlap so if f_{stoich} does not lie between f_l instantaneous in f_h instantaneous then $\bar{\omega}_{fu}$ and $\bar{\omega}_{ox}$ will of course correspond to \bar{f} values

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The flame zone will be a finite volume enclosed by f_l equal to \bar{f} minus f dash when that quantity is equal to f_{stoich} which is the inner edge of the flame flame edge rather and f_h equal to \bar{f} plus dash would equal to f_{stoich} would represent the outside of the flame and thus \bar{f} equal to f_{stoich} surface will lie somewhere between the two surfaces so in this case the thickness of the flame edge is now being analyze

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Prediction of L_f and $r_f(x)$ - 5 - L42($\frac{18}{19}$)

Thus

$$\frac{r_{f,out}}{x} = \left[\frac{1}{57.6} \left\{ \sqrt{\left(\frac{6.57}{f_{stoich} - f'} \right) \left(\frac{D}{x} \right) \left(\frac{\rho_0}{\rho_m} \right) - 1} \right\} \right]^{0.5}$$

$$\frac{r_{f,in}}{x} = \left[\frac{1}{57.6} \left\{ \sqrt{\left(\frac{6.57}{f_{stoich} + f'} \right) \left(\frac{D}{x} \right) \left(\frac{\rho_0}{\rho_m} \right) - 1} \right\} \right]^{0.5}$$

$$\frac{r_{f,stoich}}{x} = \left[\frac{1}{57.6} \left\{ \sqrt{\left(\frac{6.57}{f_{stoich}} \right) \left(\frac{D}{x} \right) \left(\frac{\rho_0}{\rho_m} \right) - 1} \right\} \right]^{0.5}$$

where f' is estimated from

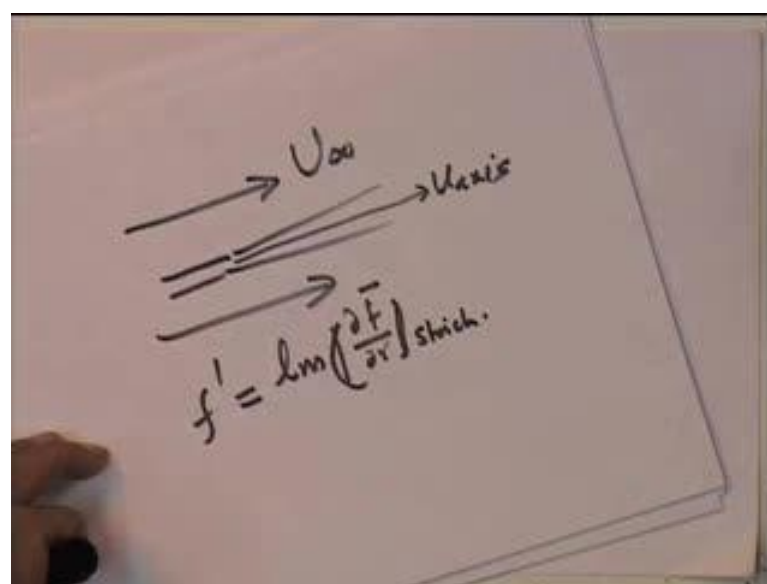
$$f' \approx l_m \times \left| \frac{\partial \bar{f}}{\partial r} \right|_{stoich} = (0.1875 \times r_{f,2}) \times \left| \frac{\partial \bar{f}}{\partial r} \right|_{stoich}$$

$$= 24.07 \left(\frac{\rho_0}{\rho_m} \right) \left(\frac{D}{x} \right) \left(\frac{r_{f,stoich}}{x} \right) \left[1 + 57.6 \left(\frac{r_{f,stoich}}{x} \right)^2 \right]^{-3}$$

April 21, 2011 26/27

How do we estimate f' and that is the issue. From our results of laminar flow, we can say that $r_{f,out} - x$ will be $f_{stoich} - f'$ because that is the value of you can see for the outside. Now, f_{stoich} will be equal to $f' + \bar{f}$ and therefore, \bar{f} would be equal to $f_{stoich} - f'$ for inner surface. Here, $r_{f,in}$ would be $f_{stoich} + f'$ into all this. For the stoichiometric case, when \bar{f} will be equal to f_{stoich} , you will get $r_{f,stoich}$ given by that.

(Refer Slide Time: 41:27)



Now, we have to determine f dash. It is determined from a mixing length formula and Spalding recommends that f dash is evaluated as mixing length, l_m into df bar by dr under stoichiometric conditions.

(Refer Slide Time: 41:44)

Prediction of L_f and $r_f(x)$ - 2 - L42($\frac{15}{19}$)
 Since all laminar solutions are applicable to *time-averaged* quantities, we may write

$$\bar{\Phi} = \frac{\bar{v}}{U_0} = \bar{f} = \bar{f} = 6.57 \left(\frac{D}{x}\right) \left(\frac{\rho_0}{\rho_m}\right) \left[1 + 57.6 \left(\frac{r}{x}\right)^2\right]^{-2}$$

So, f bar solution is already known to you; this is the solution to f bar. You take a derivative of this with respect to r and l_m

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$f' = \lim \left(\frac{df}{dx}\right) \text{ stich.}$
 $\lim = 0.1535 r^{1/2}$
 turbulenz strom jet.

Now, l_m for turbulent round jet is simply l_m . It is equal to 0.01875 into r half for a turbulent round jet. This is found to fit the experimental data quite well.

(Refer Slide Time: 42:16)

Prediction of L_f and $r_f(x)$ - 5 - L42($\frac{18}{19}$)

Thus

$$\frac{r_{f, out}}{x} = \left[\frac{1}{57.6} \left\{ \sqrt{\left(\frac{6.57}{l_{stoich} - f} \right) \left(\frac{D}{x} \right) \left(\frac{\rho_0}{\rho_m} \right) - 1} \right\} \right]^{0.5}$$

$$\frac{r_{f, in}}{x} = \left[\frac{1}{57.6} \left\{ \sqrt{\left(\frac{6.57}{l_{stoich} + f} \right) \left(\frac{D}{x} \right) \left(\frac{\rho_0}{\rho_m} \right) - 1} \right\} \right]^{0.5}$$

$$\frac{r_{f, stoich}}{x} = \left[\frac{1}{57.6} \left\{ \sqrt{\left(\frac{6.57}{l_{stoich}} \right) \left(\frac{D}{x} \right) \left(\frac{\rho_0}{\rho_m} \right) - 1} \right\} \right]^{0.5}$$

where f is estimated from

$$f = l_m \times \left. \frac{\partial \bar{T}}{\partial r} \right|_{stoich} = (0.1875 \times r_{f/2}) \times \left. \frac{\partial \bar{T}}{\partial r} \right|_{stoich}$$

$$= 24.07 \left(\frac{\rho_0}{\rho_m} \right) \left(\frac{D}{x} \right) \left(\frac{r_{f, stoich}}{x} \right) \left[1 + 57.6 \left(\frac{r_{f, stoich}}{x} \right)^2 \right]^{-3}$$

There is no distance from the axis term here because there is no presence of the wall and therefore, the mixing length becomes 0.1875 into r half is essentially a constant. If we substitute for r half, then we will get f dash equal to l_m . It would essentially become like that. At each x , we can predict f dash r f stoich by x whole square. Now, if I said r f in each case and if I said that equal to 0 ; that equal to 0 and that equal to 0 , then I will get L f out L f in and L f stoich.

(Refer Slide Time: 42:47)

Prediction of L_f and $r_f(x)$ - 6 - L42($\frac{19}{19}$)
 Setting $r_{f,in,out,stoich} = 0$, $x = L_f$ can be estimated from

$$\frac{L_{f,out}}{D} = \frac{6.57}{l_{stoich} - f} \left(\frac{\rho_0}{\rho_m} \right)$$

$$\frac{L_{f,in}}{D} = \frac{6.57}{l_{stoich} + f} \left(\frac{\rho_0}{\rho_m} \right)$$

$$\frac{L_{f,stoich}}{D} = \frac{6.57}{l_{stoich}} \left(\frac{\rho_0}{\rho_m} \right) = 6.57 (1 + R_{st}) \left(\frac{\rho_0}{\rho_m} \right)$$

Thus, if $L_{f,stoich}$ is regarded as the mean flame length then, knowing $l_{stoich} = (1 + R_{st})^{-1}$, the flame length can be estimated for any fuel. Although the above relations are only approximate, they do embody the form of the experimentally determined empirical correlations

$$L_{f,exp} = F(D, R_{st}, \frac{\rho_0}{\rho_\infty}, \frac{\rho_0}{\rho_m}) \rightarrow \frac{\rho_0}{\rho_\infty} \approx \frac{\rho_{fu}}{\rho_\infty} \text{ in most cases}$$

Apr 21, 2011 21:21

That is what I have done here, so L_f out divided by D would be $6.57 f_{stoich} \text{ minus } f \text{ dash } \rho_0 \text{ by } \rho_m$. L_f in would be $6.57 f_{stoich} \text{ minus } f \text{ dash}$. Since, this is subtracted; you will see L_f out will be longer and L_f in will be smaller. The stoichiometric case, where the in between length would be $6.57 f_{stoich} \rho_0 \text{ by } m$ equal to $6.57 (1 + R_{st}) \rho_0 \text{ by } m$. If L_f stoich is regarded as the mean flame length, then knowing $f_{stoich} \text{ equal to } (1 + R_{st})^{-1}$.

The flame length can be estimated for any fuel. Remember, air fuel ratio would vary; stoichiometric air fuel ratio would vary for the fuel under consideration. Therefore, we can say that we can readily predict the L_f stoich. The most important thing is you can see this relationship does not show effect of Reynolds number at all. It as was observed by Hutton experimentally that in turbulent flames, the length of the flame remains constant and that has been shown. So, out of a very simple analysis, we have recovered the most important result.

Although, the above relations are only approximate, they do embody the form of the experimentally determined empirical correlations. What do they look like? The experimentally determined correlations for L_f show that F will be function of D . As you seen here, it will be function of R_{st} . It will be function of $\rho_0 \text{ by } \rho_m$ and $\rho_0 \text{ by } \rho_\infty$. In some cases, the experimental correlation and $\rho_0 \text{ by } \rho_\infty$ is essentially the density of the fuel divided by density of the surrounding gas ρ_∞ .

Rho naught by rho infinity is the density ratio, which can be different for different types of fuel in diffusion flame. Except for that ratio, we have not been able to identify rho m and rho infinity because we assumed constant property ratio. With this, I conclude the lecture on flames and I also conclude the entire course on convective heat and mass transfer.

In the first 20 lectures, I covered laminar flows, both external flows and internal flows with and without suction and blowing in the presence of pressure gradients and wall temperature variations. We also considered laminar internal flows, both in simple ducts as well as complex ducts of non-circular cross section. We were able to calculate Nusselt number, both in the presence of circumferentially varying boundary conditions or actually varying boundary conditions.

We moved to the next 10 lectures, where we saw turbulent flows and the formal aspects as well as turbulence modeling and also phenomenological arguments of flow near the wall. It gave us the universal laws of velocity distribution and that of temperature distribution close to a wall, which enable us to calculate the Nusselt number and friction factor for a turbulent boundary layer as well as turbulent ducted flow. We moved to the convective mass transfer problems, in which we first of all postulated and considered the full boundary layer flow equation. We said there are simplified forms, which are good proxies for mass transfer problem that can be derived.

The Reynolds flow model was found to be a very good proxy for the full boundary layer flow model. Using that model we solved several problems, but of course, diffusion mass transfer is as important as convective mass transfer. Diffusion mass transfer is simply a special case of a boundary layer flow model, which we modeled as the Stefan flow model and found that the results from there. It gave us the logarithmic form of the connection between mass transfer rate and spalding number B. In between these two - Reynolds flow model and Stefan flow model, we also invoked the quake flow model, which showed us why property variations and large mass transfer rates are required. The model was successful in showing us the trends of property correction that should be applied.

In the last two lectures, I considered natural convection heat and mass transfer. In the present lecture, I considered the case of heat and mass transfer in a free jet and that is the

case of a jet diffusion flame. I hope you enjoyed these lectures and I also hope that this will promote you to take up a career in the field of convective heat and mass transfer. Thank you very much and all the very best wishes to you.