Convective Heat and Mass Transfer

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Module No.# 01

Lecture No. # 41

Natural Convection BLs

So far we have considered heat and mass transfer from a wall, essentially, in force convection situations. The purpose of today's lecture is to consider heat and mass transfer from a wall under natural convection conditions. Natural convection is of importance both in practical equipments as well as in the environment, and therefore, student of convective heat and mass transfer should also know something about natural convection heat transfer as well as mass transfer.

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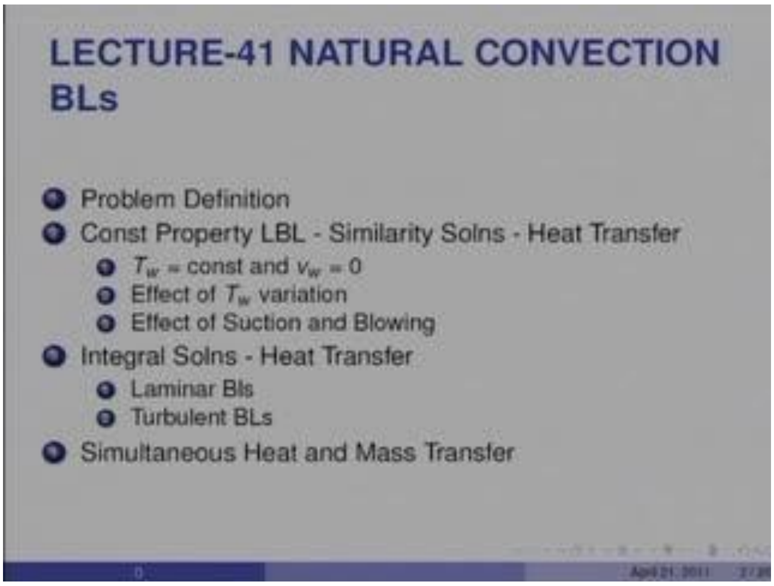

So, the topics for today are, first of all I will define the problem of natural convection passed a vertical wall, then I will develop constant property laminar boundary layer solutions using similarity method, in which initially we shall assume that the vertical wall is at constant wall temperature and there is no suction or blowing. But then, we shall allow for variation of wall temperature as well as, allow for study the effect of suction and blowing.

Likewise, we will also develop integral solutions of laminar boundary layers and also of turbulent boundary layers. And finally, we shall consider simultaneous heat and mass transfer in a natural convection boundary layer, because natural convection can arise both due to temperature differences between the wall and the surroundings, as well as concentration differences between the wall and the surroundings. These differences setup forces which are opposed by gravity, and therefore, both buoyancy due to concentration difference and buoyancy due to a temperature difference are of importance.

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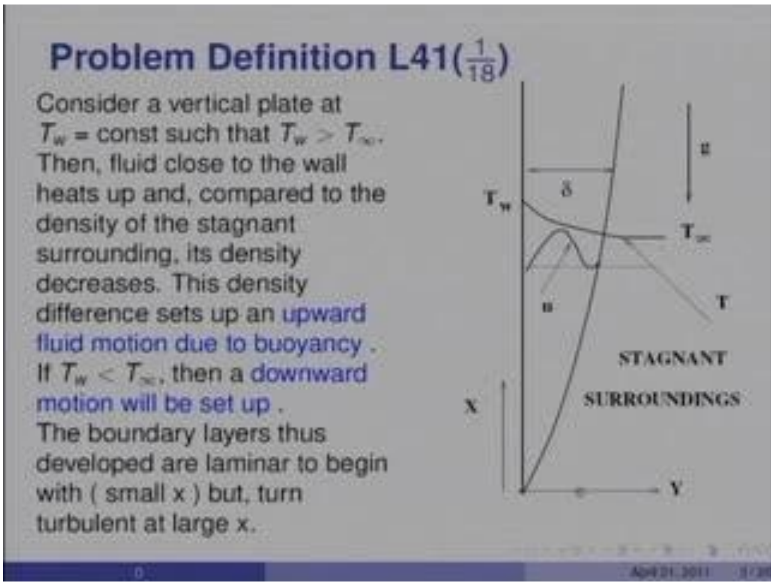

So, let us go to the problem definition, so as I have shown here, consider a vertical wall or a vertical plate which is at a constant wall temperature T w, and let us for the time being, suppose, that T w is greater than T infinity, where T infinity is the temperature of the stagnant surroundings. As a result, the fluid close to the wall will have a smaller density because it heats up, and compare to density of the stagnant surrounding, the density in this part would be smaller. And as long as the temperature difference persists between T and T infinity, there would be a density difference and this density difference would drive the flow up against the gravity force.

The density difference sets up an upward flow, so the flow would come in here, and at x equal to 0, there would be 0 boundary layer thickness. But as 1 moves up, the boundary layer would grow of course, assuming a velocity profile which is something of this type. Notice the difference between a force convection boundary layer and a natural convection boundary layer, in both the velocity is 0 of course, at the wall, but in natural convection boundary layers, the velocity peaks somewhere between the edge of the layer and the wall, and the velocity again falls down to 0 at the edge of the boundary layer. And that is because, there are no among or any temperature differences beyond that point, and therefore, there is no driving force for the velocity and the velocity assumes the value of the stagnant surroundings which is 0.

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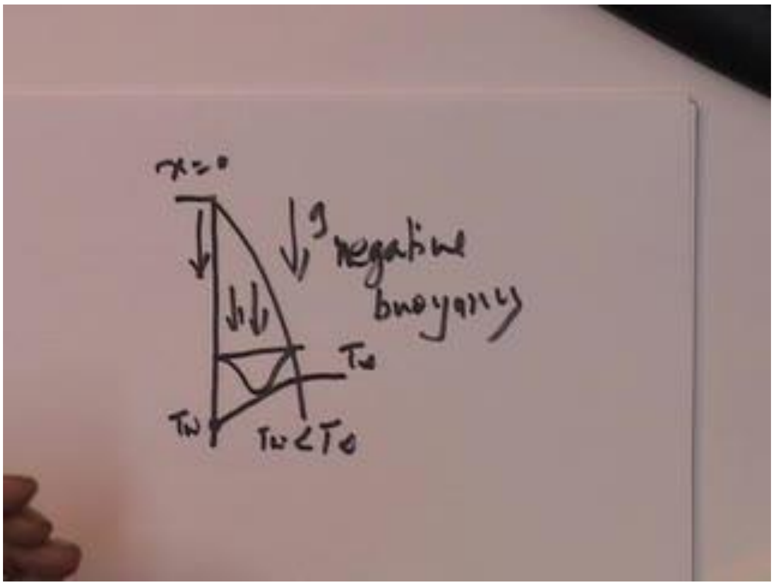

If I have a T w was less than T infinity, then natural convection would be as I am showing here now, the natural convection flow will be like that and the velocity profile would be like so, and the temperature profile would go something like that T w being less than T infinity. And then, we would measure x equal to 0 here and downwards, so this is the negative buoyancy if you like, with g acting in that direction.

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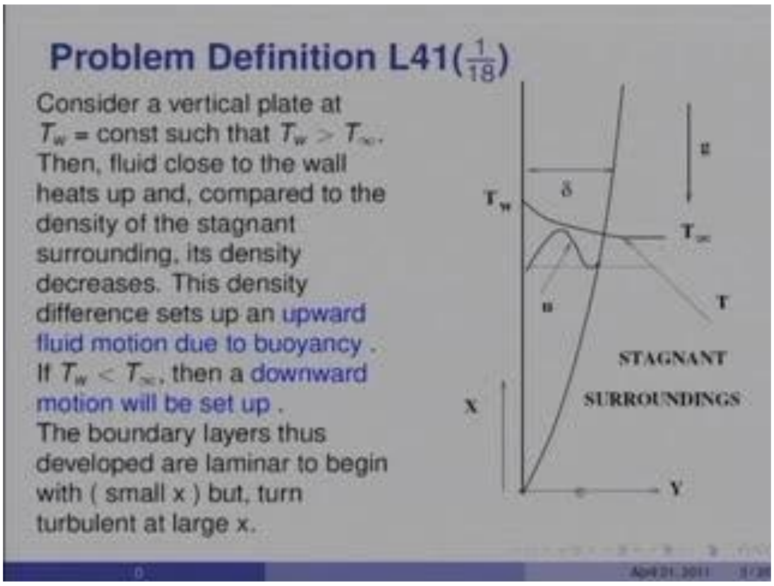

The boundary layers thus develop a laminar to begin with, but beyond the certain length the boundary layer becomes unstable and turns turbulence. So, there is actually a transition somewhere, and then, the boundary layer becomes turbulent.

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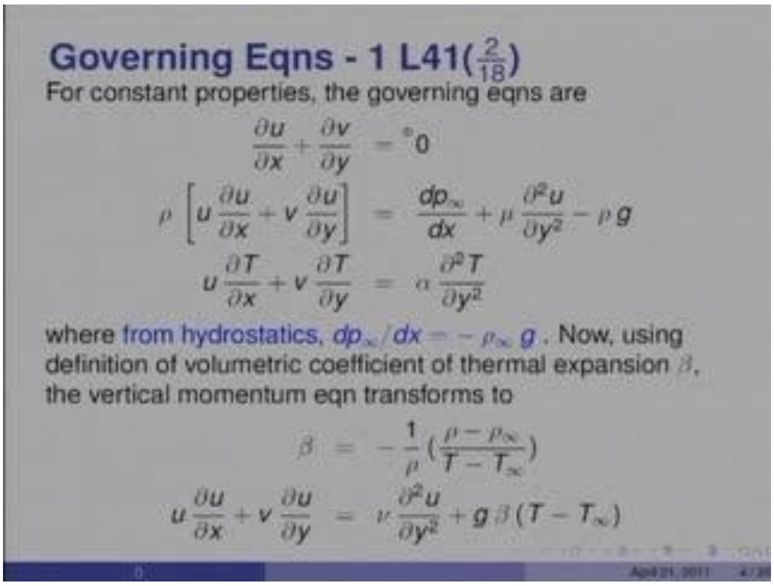

So, to begin with let us consider only the laminar boundary layer. So, for a constant properties the governing equations would be the mass conservation or the continuity equation d u by d x plus d v by d y equal to 0, rho into u d u by d x plus v d u by d y which are the convection terms. There would be pressure gradient mu d 2 u d y square, which is the boundary layer approximation, we are saying that there is no diffusion in the axial direction. And minus rho g is the body force due to gravity, likewise there would be temperature equation or the energy equation in its familiar boundary layer form.

Now, from hydrostatics d p infinity by d x would be minus rho infinity into g, because a pressure gradient would be simply balance by the gravity.

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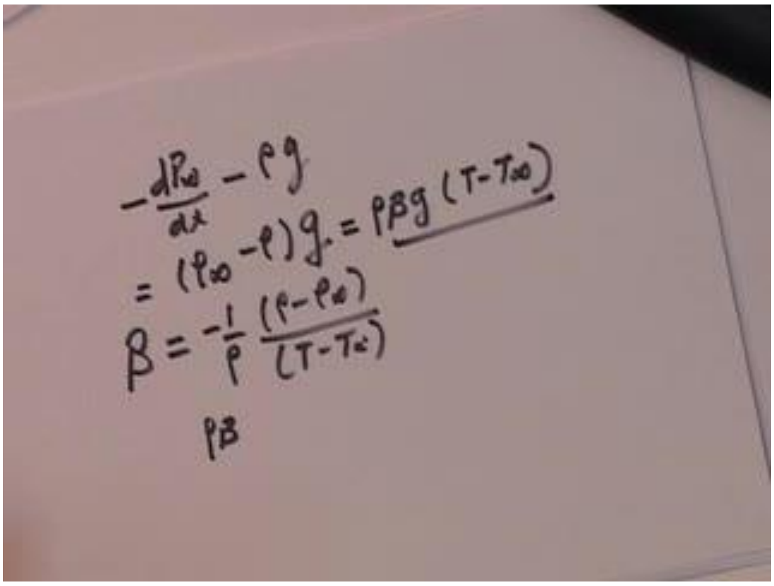

So, therefore, d p infinity minus rho g will become equal to, sorry, this should be minus d p infinity by d x minus rho g, and therefore, this will become rho infinity minus rho into g.

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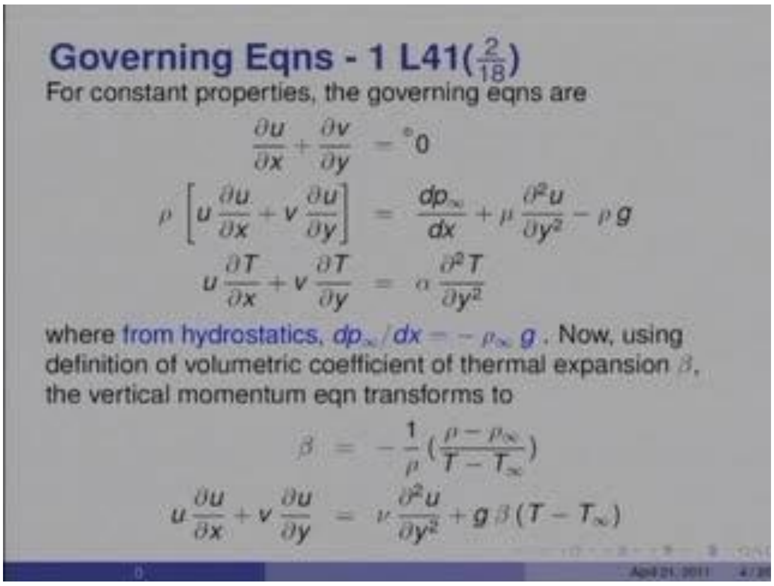

Now, if I defined the beta - the coefficient of thermal expansion - as 1 over rho into rho minus rho infinity divided by T minus T infinity, then you can see that this term rho infinity minus rho g can be written as rho times beta, this will becomes rho times beta g T minus T infinity, and that is what I have written here. So, notice the small error here that the pressure gradient should be minus d pd p infinity by d x.

So, if I define beta equal to minus 1 over rho rho minus rho infinity over T minus T infinity, then these 2 terms would transform to g beta and divide through by density, then you will have the momentum equation with nu d 2 u by d y square plus g beta T minus T infinity, this is called the buoyancy force term. So, our equations now are the continuity equation, the energy equation and the momentum equation with this. And you can see there is a clear coupling between energy equation and the momentum equation through the buoyancy term.

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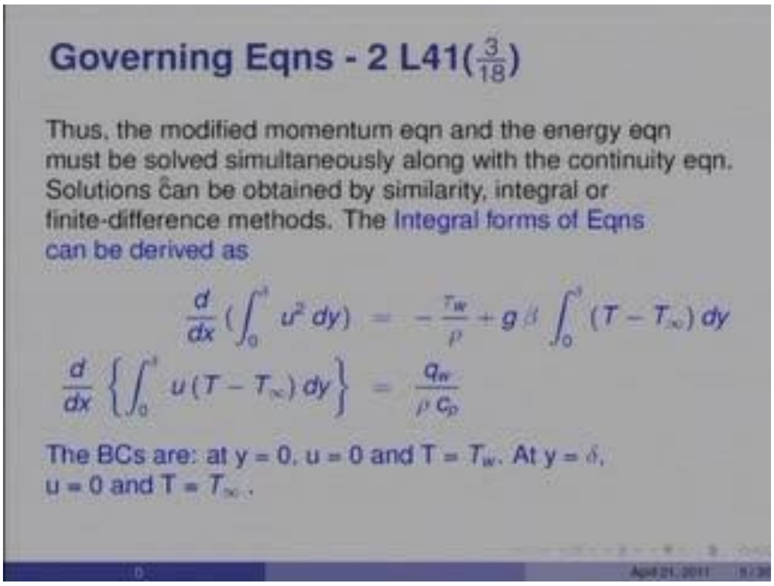

Thus, the modified form of the momentum equation and the energy equation must be solved simultaneously along with the continuity equation. Solutions can be obtained by similarity integral or finite difference methods as in force convection, now the integral form of the equations can be derived as follows.

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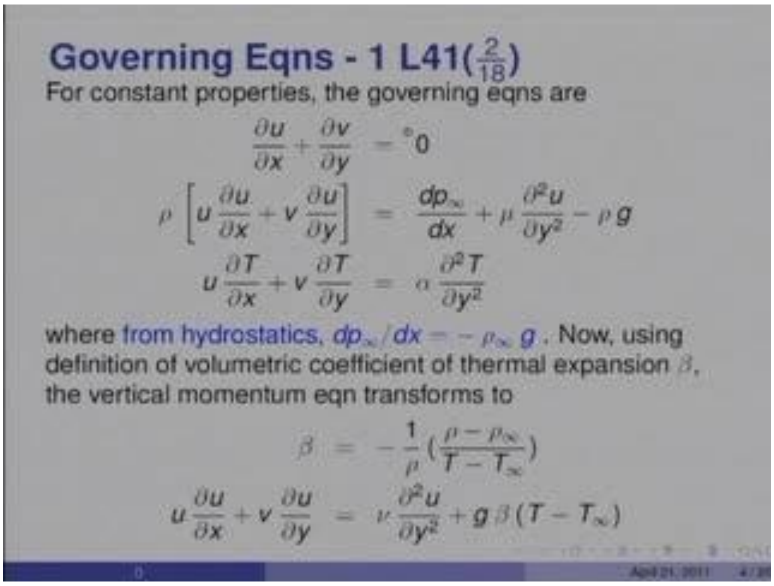

For example, I can write the left hand side of this in conserved property form as d u square by d x plus d v u by d y equal to all that. And if I integrate each term from 0 to delta, then you will see that you will get d by d x 0 to delta u square d y, which will be the which will follow from the first term. This term which will be d v u by d y would of course, go to 0, both at y equal to delta, where u is equal to 0 as well as at the wall where u is equal to 0, and therefore, the term would simply vanish.

This term would be nu d 2 d u by d y at y equal to delta minus nu d u d y at y equal to 0, and that would be since d u d y at y equal to delta is 0, you will simply get this as minus tau wall by rho plus g beta into 0 to delta T minus T infinity d y.

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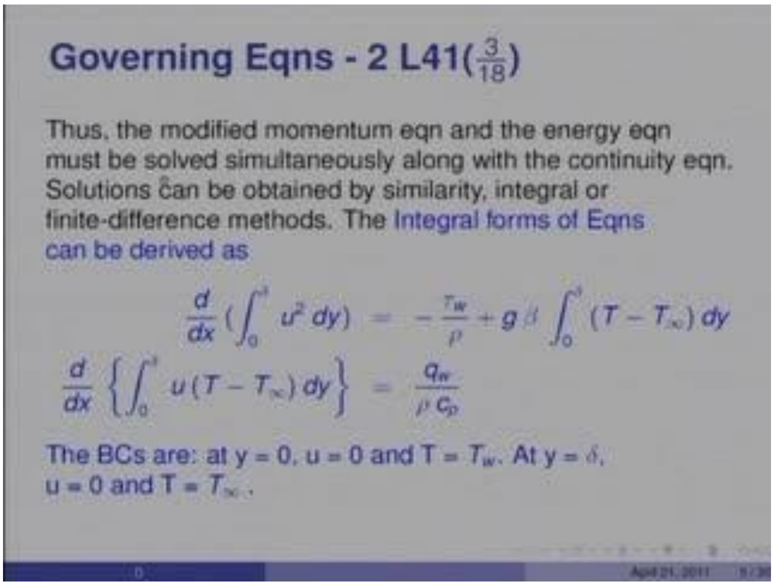

Likewise, if I write this left hand side in conserved property form as d u t by d x plus d v t by d y and integrate from 0 to delta, then I will get d by d x equal to 0 to delta u into T minus T infinity d y equal to q wall over rho c p. And the boundary conditions are at y equal to 0, u is equal to 0 and T is equal to T wall at y equal to delta, u is equal to again 0 and T is equal to T infinity.

So, this would be the integral forms of equations, we shall take up these equations when we come to solution, but presently let us turn to solving the equations of this slide by similarity method.

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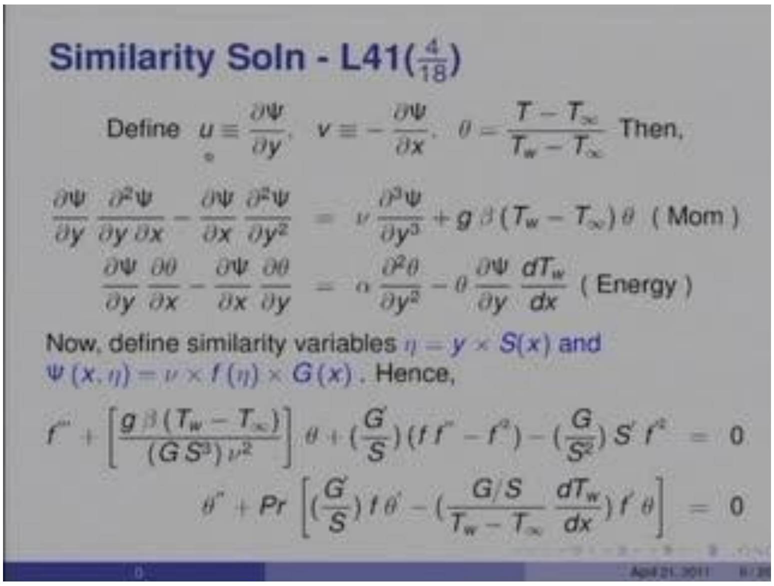

So, as usual we defined stream function u equal to d psi by d y, and v equal to minus d psi by d x, and we define a dimensionless temperature T minus T infinity divided by Twall minus T infinity, the denominator being constant given constant.

And then, the momentum equation simply would become, this is u d psi by d y into d u by d x which is d 2 psi by d y d x minus d psi by d x, which is v, into d u by d y, which is d 2 psi by d y square, plus nu times d 2 u by d y square which will be d 3 psi be d y cube plus g beta T w minus T infinity into theta and that could be the momentum equation. And likewise, this would be the energy equation, d psi by d y d theta by d x minus d psi by d y d theta by d y equal to alpha d 2 theta by d y square minus theta d psi by d y d Twall by d x.

Now, this term arises out of the u d t by d x term when T w minus T infinity, I have retained this term at the movement, because later on we are going to consider the case in which Twall itself would vary with x, but presently if T w is 0 then of course - i mean T w is constant then of course, that term would be 0.

We shall define as usual similarity variables eta equal to y multiplied by stretching function S x and stream function psi x eta equal to nu times f eta into G x another function of x. Then, if we substitute in this, then you will get f triple prime g beta T w minus T infinity G s cube nu square theta plus G dash by s f f double prime minus f dash square minus G by S square S dash f dash square equal to 0. Now, of course, f dash is here are all derivatives with respect to eta and G dash and S dash are derivatives with respect to x.

Similarly, the energy equation would read in like this theta double prime plus prandtl into G dash by S f theta prime minus G by S T w minus T infinity over d T w minus d x into f dash theta equal to 0. Again the two equations are coupled because theta appears in the momentum equation and likewise f appears in the energy equation. These equations can be solved by similarity method, if all these factors which I have shown are absolute constant.

Remember, G is a function of x, S is also a function of x T w minus T infinity can also be a function of x, but the group as a whole must be a constant. Likewise, G dash divided by S must also be a constant and G by S square into S dash must also be a constant for similarity solutions to exist, likewise G by S T wall minus T infinity d T w by d x must also be an absolute constant.

Only when all these factors are absolute constants, we would get two perfect ordinary differential equations which is the requirement for obtaining similarity solution. So, let us see, what do these conditions of constant c imply, which would give us these as a function of x as well as g as a function of x.

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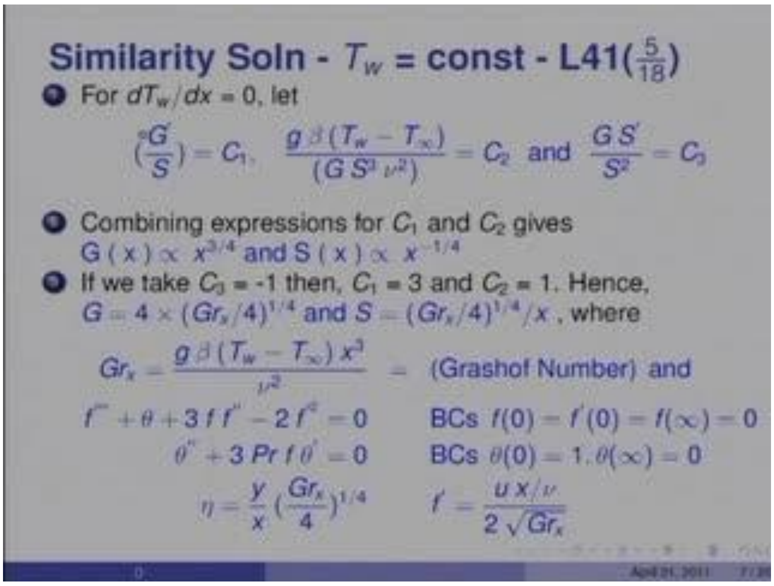

So, let us first consider d T w by d x equal to 0, and let G dash by s equal to C 1 g beta T w minus T infinity divided by G S cube nu square equal to C 2 and G S dash by S square is equal to C 3.

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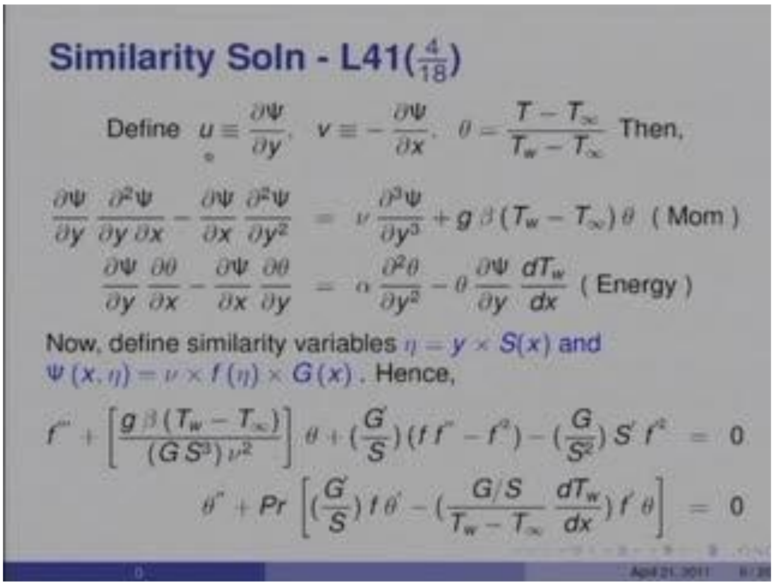

In other words ,what I have d1 is, I have said this is C 1, this is C 2 and this is C 3, this whole term is C 3 d T w by d x is 0, and therefore, that whole term is 0 in the present case.

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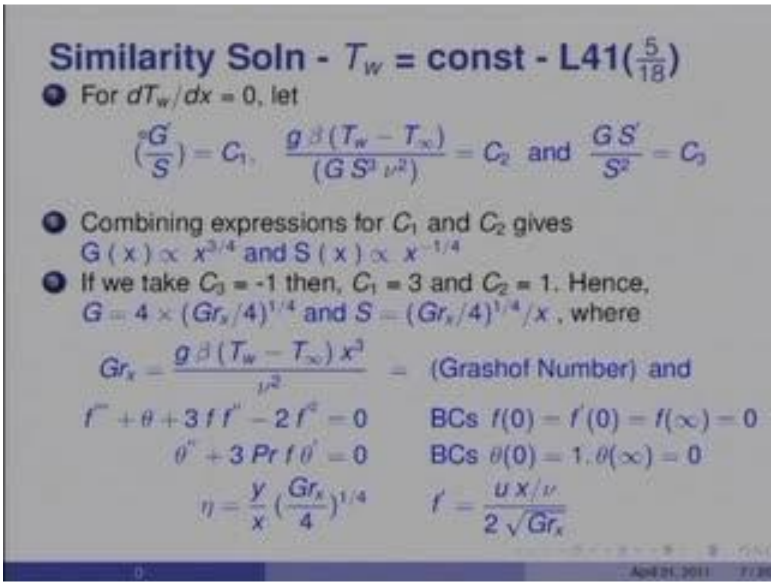

Now, combining expressions for C 1 and C 2 gives that G x should be proportional to x raise to 3 by 4 and S x should be proportional to x raise to minus 1 by 4. So, therefore, if we said quite arbitrarily, C 3 equal to minus 1, then it would follow that C 1equal to 3 and C 2 equal to 1. Now, there is a little algebra here of over 1 page to show all this. And hence, G function would be 4 times grash of number based on x divided by 4 raise to a quarter. And S will be again grash of number divided by 4 raised quarter divided by x, where the grash of number as you know is defined in this fashion g beta T w minus T infinity x cube nu square.

Remember, here T w minus T infinity is constant, and then the momentum equation would read as f triple prime plus theta plus 3 f f double prime minus 2 f dash square equal to 0 theta double prime plus 3 prandtl f theta dash equal to 0, this will be the energy equation. And the boundary conditions would be f 0 equal to f dash 0 which is the velocity, f dash 0 is velocity, f dash infinity would be 0. So, all these are 0's and f 0 equal to 0 implies that v at the wall is also 0. And the boundary conditions of theta would be from its definition theta 0 would be 1 and theta infinity equal to 0.

The similarity variable eta which was y into s x would be y divided by x as you can see here into grash of x divided by 1 raise to 1 by 4 and f dash will give us the velocity u x divided by nu divided by 2 root G r x. Now, how are these equations to be solved? One can, these are coupled equations mind you, so they have to be solve simultaneously.

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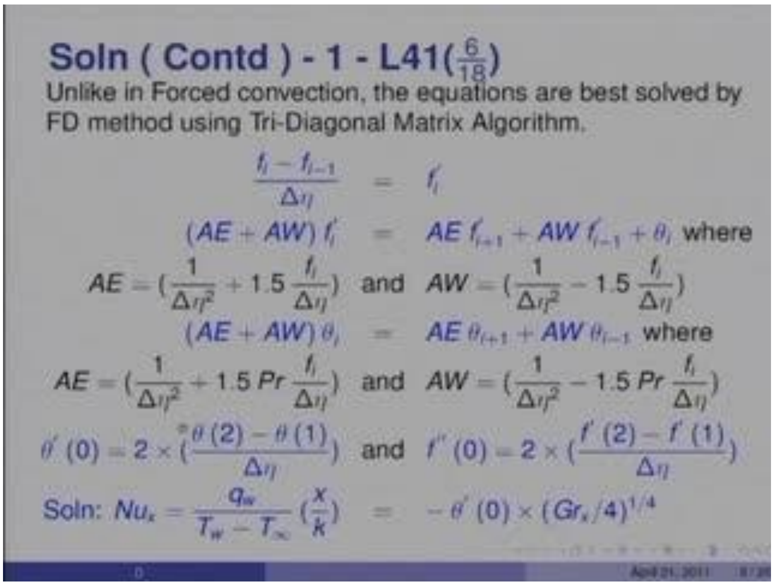

Now, of course, like in force convection, we solved equations of this type by what was called shooting method, but it turns out to be little bit unstable when we consider natural convection. And therefore, unlike in force convection the equations are best solved by finite difference method using what is called as a tri-diagonal matrix algorithm, which you must have studied in your numerical analysis class.

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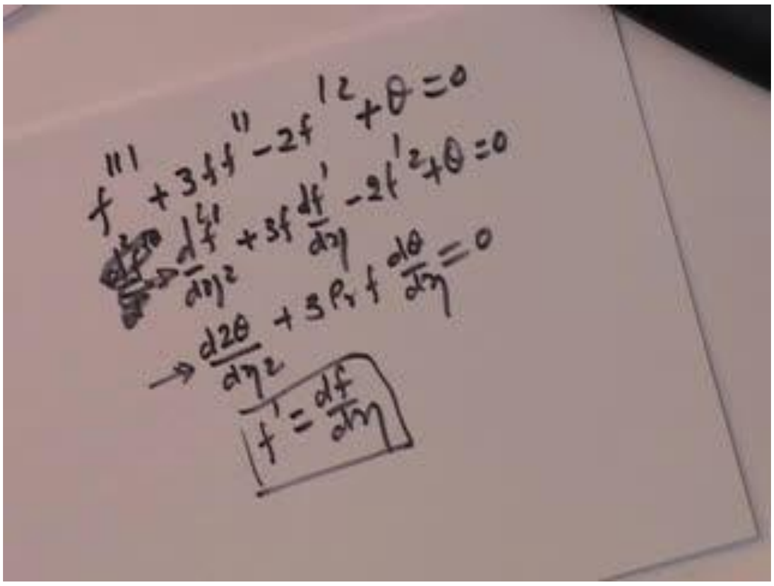

So, very simply what 1 does is that the momentum equation which is f triple prime plus 3 ff double prime minus 2 f dash square plus theta equal to 0. So, I can write this as d f double prime by d, sorry, d by d eta square of f prime plus 3 into f of d f dash by d eta minus 2 f dash square plus theta equal to 0.

And likewise, I can say the energy equation is d 2 theta by d eta square plus 3 times prandtl f into d theta by d eta equal to 0 and f dash is nothing but d f by d eta. So, what I do is, I solve for f, from which I get f dash and then, I solve this equation - this second order equation - in f dash by finite difference method and this again by finite difference method.

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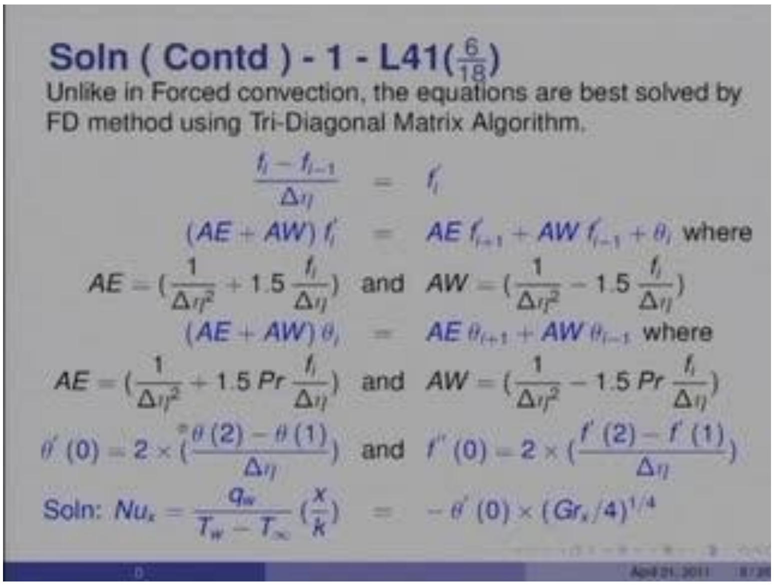

So, that is what I have shown on this slide, so this is the equation d f by d eta equal to f dash which will simply give, and we do d f by d eta is been represent by forward difference f i minus f i minus 1 over delta eta equal to f i dash.

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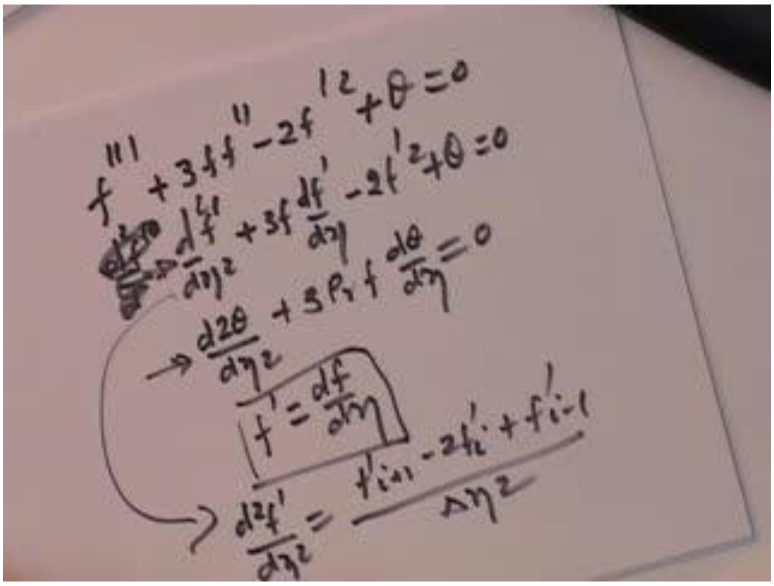

Then, if I turn to this equation for the for then, d 2 f dash d 2 f dash by d eta square will be simply f dash i plus 1 minus 2 f dash i plus f dash i minus 1 by delta eta square, and likewise d f dash by d eta and so on and so forth.

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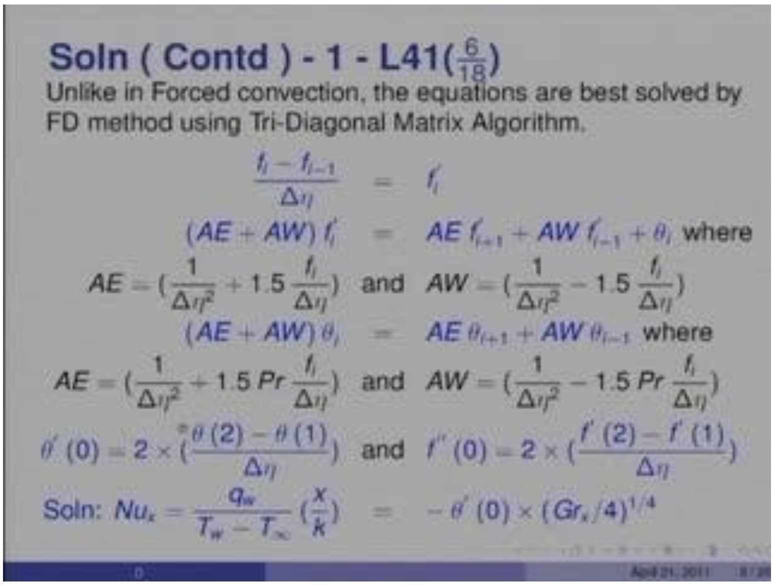

So, if you do this finite difference representation of all the derivatives, we can derive an equation for f dash A E plus A W f dash i is equal to A E into f dash i plus 1 plus A W i f dash i minus 1 plus theta i, where A E is equal to 1 over delta eta square plus 1.5 f i by delta eta and A W would be 1 over delta eta square minus 1.5 f i by delta eta. And likewise similarly, energy equation would be A E plus A W theta i equal to A E theta i plus 1 plus A E theta i minus 1 and in this case A E and A W would take those expression.

Now, this sort of equations that you see here, are perfectly suited for applying tri-diagonal matrix algorithm, because the diagonal element would always dominate over the neighboring elements - neighbouring coefficients A E and A W, whereas the diagonal element of the matrix would be A E plus A W.

And therefore, one can get very, very, fast solutions; once the solutions are obtained our interest is to be obtained theta prime 0, which is the temperature gradient at the wall, and that would be 2 times into theta 2 minus theta 1 over delta eta at eta equal to 0. And likewise, if we want to get the resistance at the wall - frictional resistance at the wall – then, it will be f double prime 0 equal to 2 into f dash 2 minus f dash 1 divided by delta eta.

Now, the nusselt number of course, is q wall divided by T wall minus T infinity x by k the usual definition. And the q wall over T w minus T infinity would be simply minus theta prime 0 grash of x by 4 raise to a quarter. So, once we have solved the temperature and f dash and f distributions, we can readily obtain our nusselt number as well as the skin friction, we should be interested. Although in natural convection in generally, one is not really interested in skin friction at all, because there is no pumping power require in natural convection, and therefore, it is of only academic interest, because this simply represents the velocity gradient at the wall.

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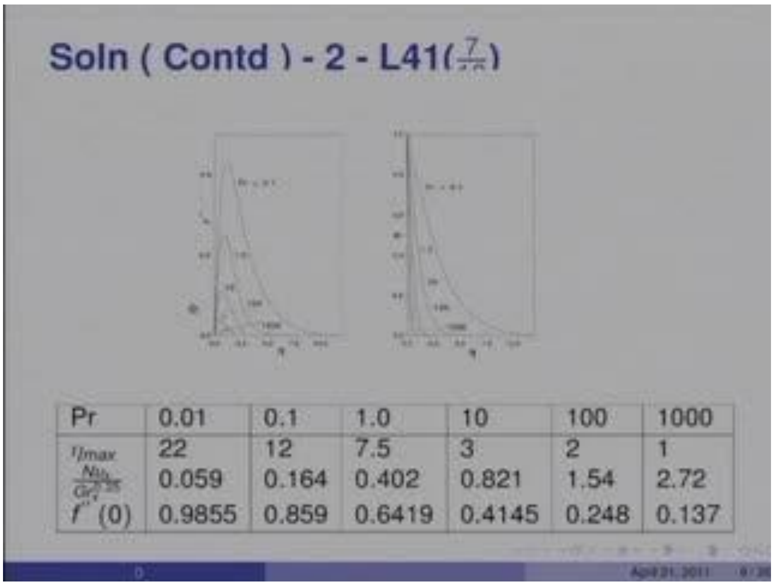

So, here are the solutions obtained for T wall equal to constant and I have allowed for a range of prandtl numbers going from as low as 0.01 which is for liquid metals to as high as 1000 which is for viscous oils. Around 1 is the gases, and you can see the eta max required is about 7.5 as the prandtl number increases beyond 1, you can see that the boundary layer is thinner because, the temperature gradients occurs much more rapidly near the wall.

And therefore, you need eta max as to be shorten through about 1 at 1000 whereas, you know in the liquid metal side, you get a thicker boundary layer and eta is of the order of 22. And here are the temperature profiles. For example, you can see that this is the distribution of velocity f dash and this is for prandtl equal to 0.1, 1, 10, 100 and 1000. In dimensionless form f dash eta, whereas this is theta versus eta, and you can see for 0.1

you are going right up to almost eta of nearly 12 whereas, as the prandtl number increases the boundary layer thickness reduces as you can see.

The nusselt number on the other hand or nusselt number divided by grash of raise to 0.25 is 0.059 at 0.1, 0.164 at 0.1, 0.402 for 1 and 0.821 at 10 and 1.54 at 100 and 2.72 at 1000. F double prime 0 of course, goes on reducing as you increase the prandtl number. But as I said earlier, our main interest is not in f double prime 0, it is of academic interest, the main thing is the nusselt number, the manner in which nusselt number varies.

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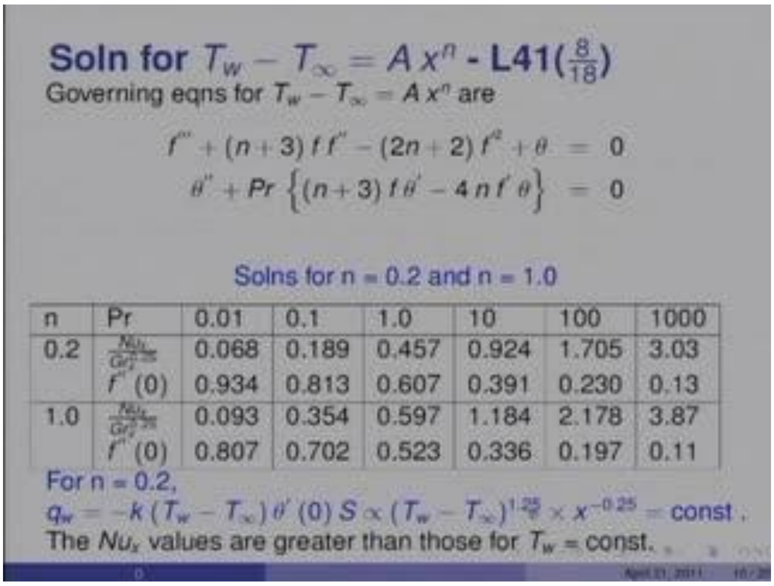

Now, similarity solutions are also possible when T w minus T infinity varies as A X to the power of n.

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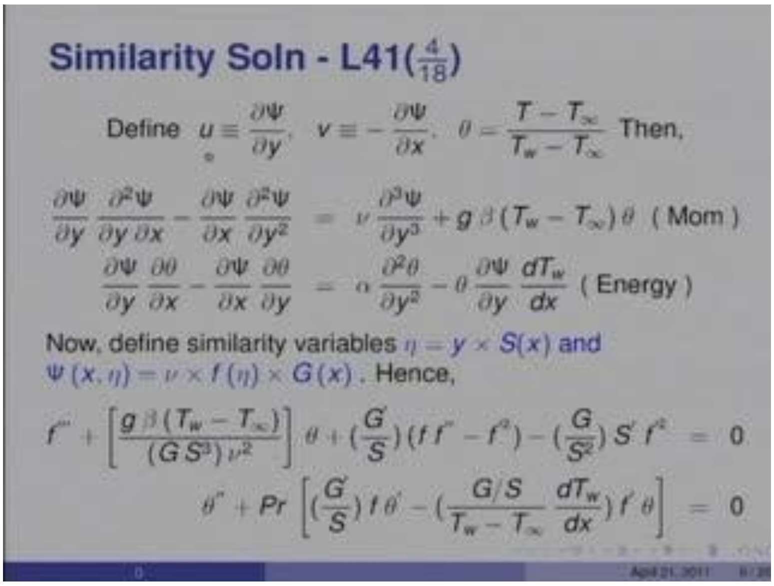

Remember, now this term which was set to 0 would be finite and if T w minus T infinity varies as A X to the power of n, then this term would also be constant.

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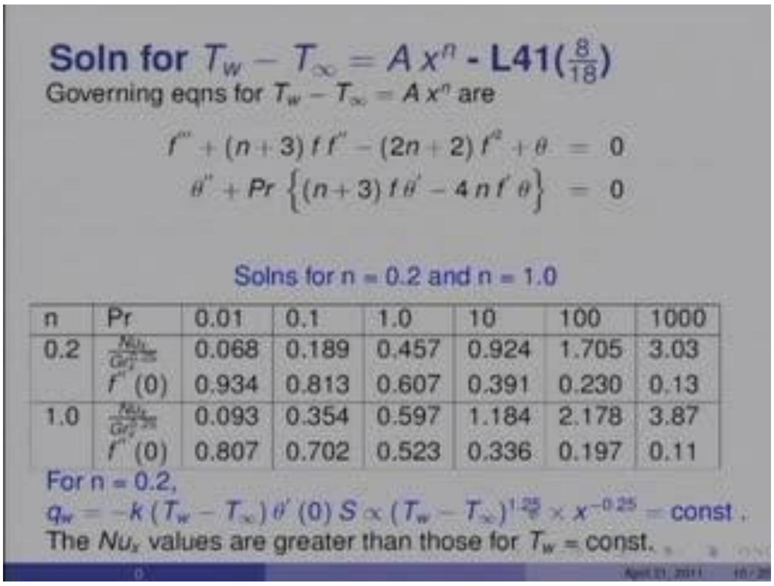

And the governing equations would then look like this, f triple prime plus n plus 3 f f double prime minus 2 n plus 2 f prime square plus theta equal to 0. And theta double prime plus prandtl n plus 3 f theta prime minus 4 n f dash theta equal to 0. Now, you can see, if I set n equal to 0 which would be the constant temperature case, then the previous equations are readily recovered.

I am considering two values of n, n equal to 2 and n equal to 1; n equal to 1 of course, implies linear variation of temperature, but n equal to 2 has a very special significance. As you can see, for n equal to 2, if I write q wall will be equal to minus k T w minus T infinity theta prime 0 into S, and S itself is proportional to x to the power of 0.25, as you will recall from here.

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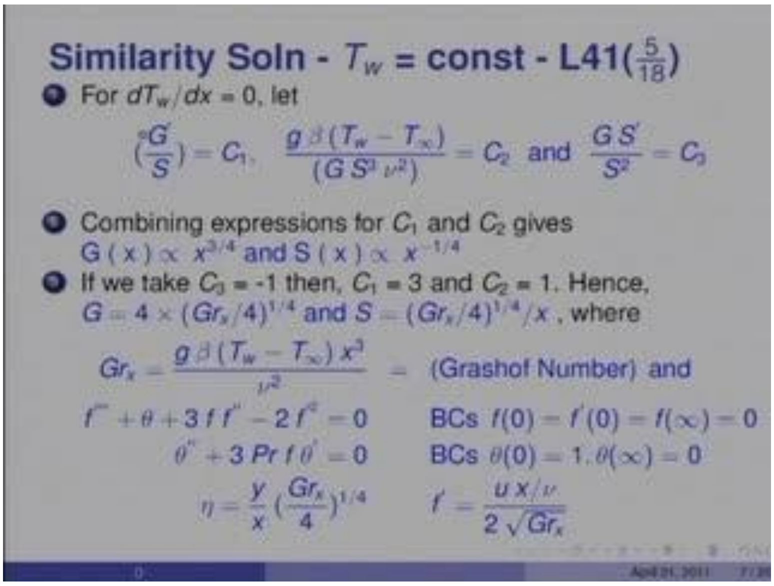

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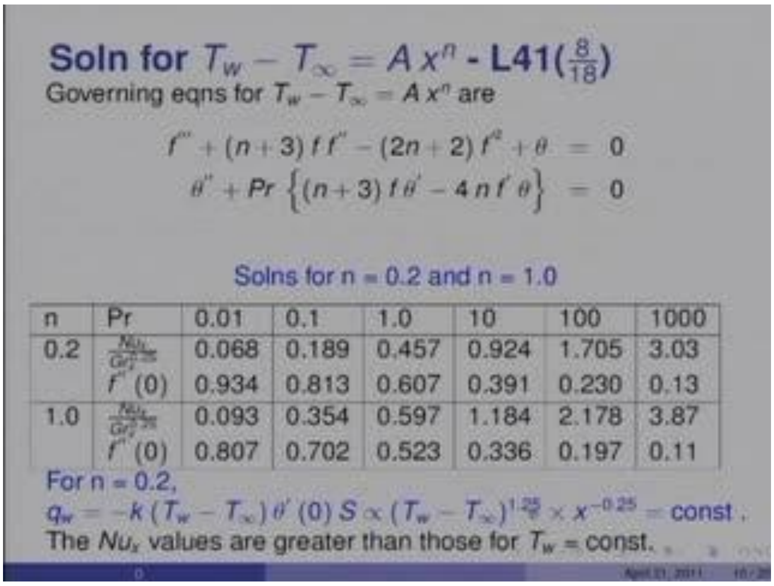

As we said s x proportional to the minus 0.25, and here, if T w minus T infinity itself varies as 0.2 x to the power of 0.2, then further raise to 1.25 will give 0.25 x to the power of 0.25 into x to the power of minus 0.25 and therefore, this would result into constant. And therefore, n equal to 0.2 represents the constant wall heat flux case, and therefore it is of interest. So, again, you will see the values of N u x divide by grash of raise to 0.245 or 0.068, 0.189, 0.457, 0.924, 1.705 and 3.03.

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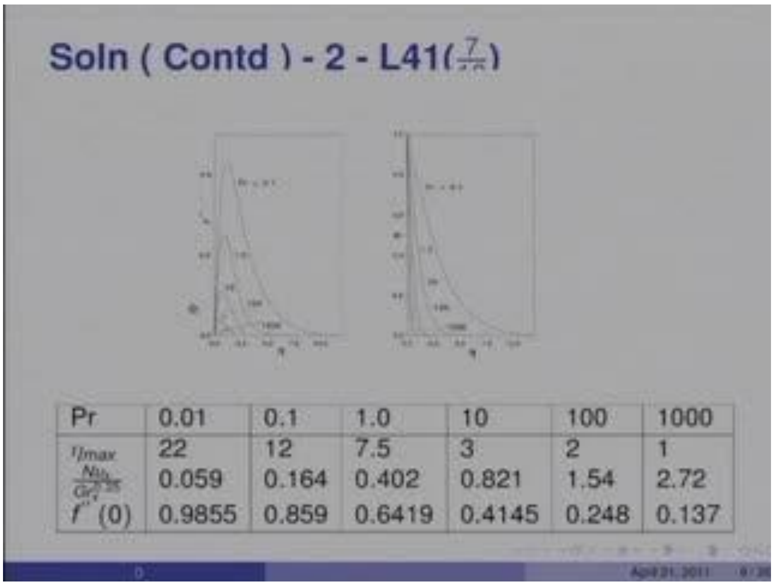

You will see, compare to T wall equal to constant case at all prandtl numbers q wall equal to constant gives you a higher nusselt number - a higher nusselt rate.

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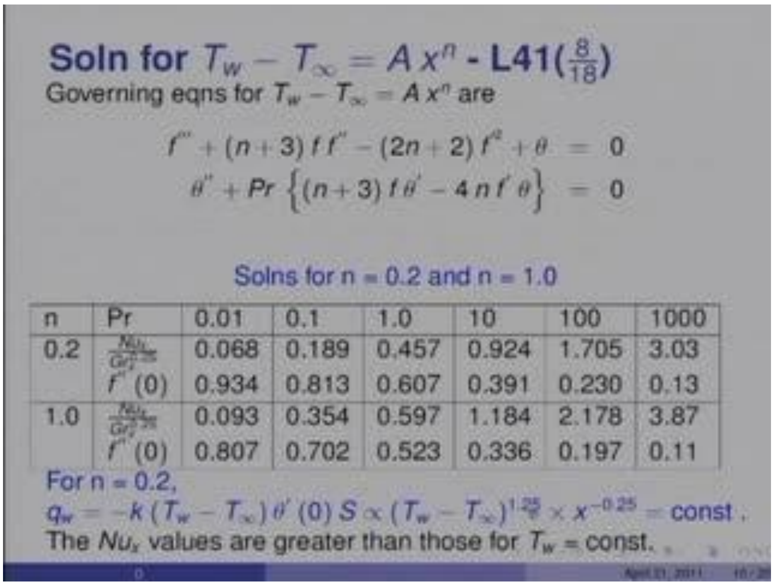

This is very much like what we had observed in force convection laminar boundary layers as well. For linear case - linear temperature variation case - the nusselt number variation is again higher then n equal to 0.2, 0.093, 0.354, 0.597, 1.1842, 2.178 and 3.87. So, you can get very good solutions and these have usually been found to be vary in good agreement with experimental data.

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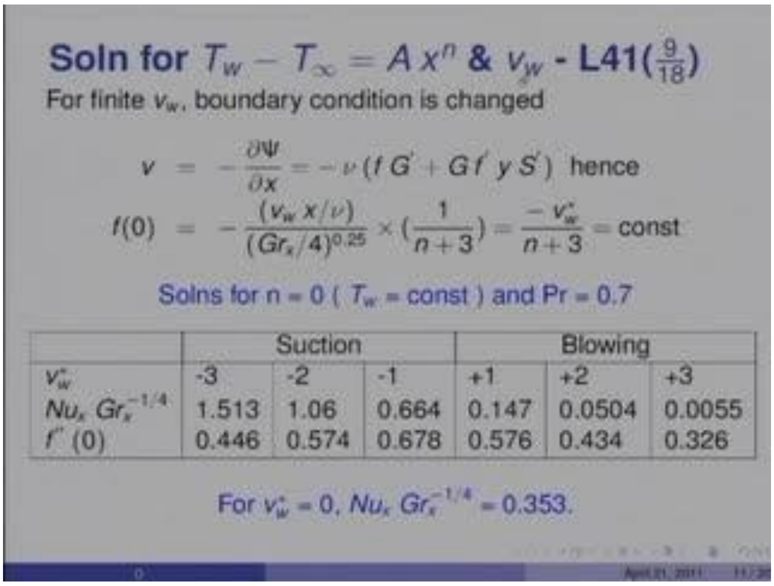

Now, let us consider V w the effect of suction and blowing again in the presence of T w minus T infinity varying as A X to the power of n. Now, v is minus d psi by d x and that would transform to minus nu times f G dash plus G f dash y into S dash. And therefore, at y equal to 0, I would get the value of V w and this would be f 0. So, f 0 would be simply V w divided by nu G dash and substituting for G dash from over G x relationship, you will see f 0 is nothing but minus V w x divided by nu divided by G r x by 4 raise to 0.25 into 1 over n plus 3 or simply minus V w star, this is like a reynolds number, so I can make it into minus V w star divided by n plus 3 and that could be a constant f w.

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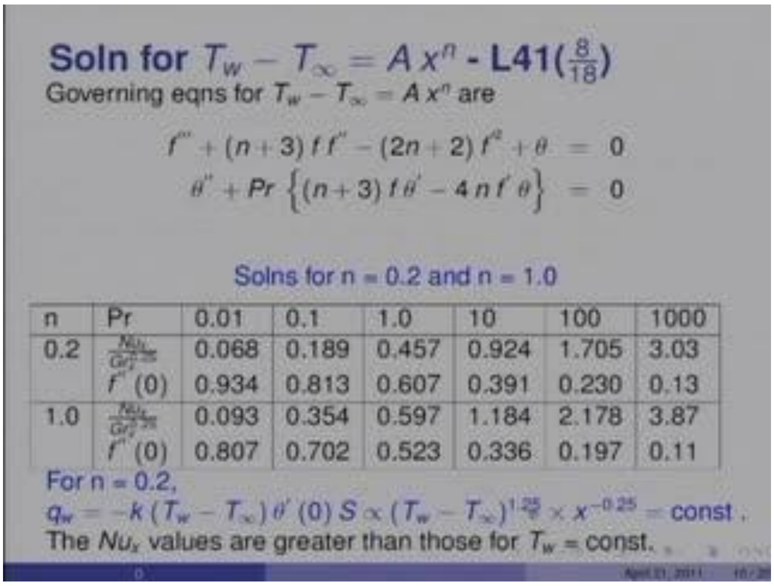

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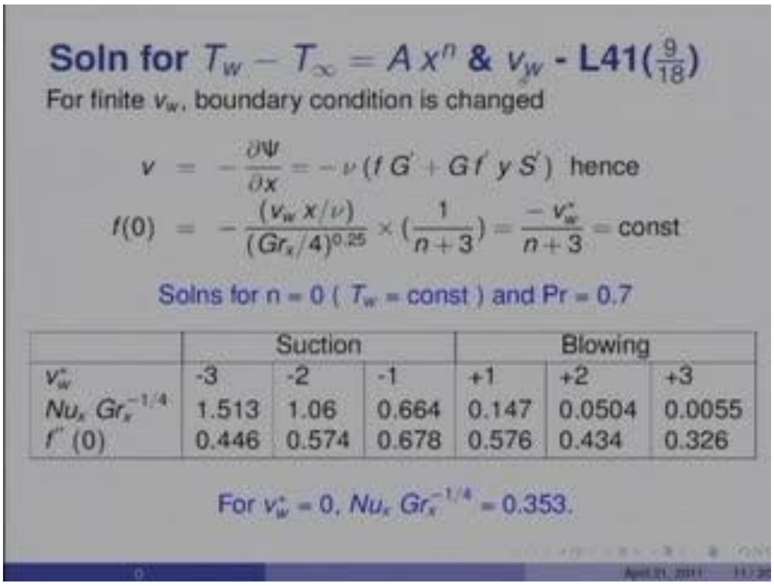

So, you solve the equations of the previous slide, you solve these equations with f 0 equal to minus V w star over n n plus 3, where n is specified as well as V w star is specified. Now here, I am giving solutions for n equal to 0 which is the constant wall temperature case and taking prandtl equal to 0.7, only the two values. And you can see on the suction side V w star would be negative, so V w star very high suction rate goes on reducing, reducing, and this blowing rate V w star positive 1, 2 and 3.

Then, you can see nusselt number goes on reducing as suction is reduce, and it reduces still further as blowing is increase, and this is very much like what we had observed in case of force convection. So, the effect of suction is to enhance heat transfer and for reference, I have given for V w star equal to 0 and prandtl equal to 0.7 N u x G r x to the power of minus 4 is 0.353. So, you can see when V w star is minus 1, it is 0.664, when it is minus 0.2 is 3 times, and when it is minus 3 it is almost 4 and a half times the nusselt number, so suction enhances a heat transfer and blowing reduces heat transfer.

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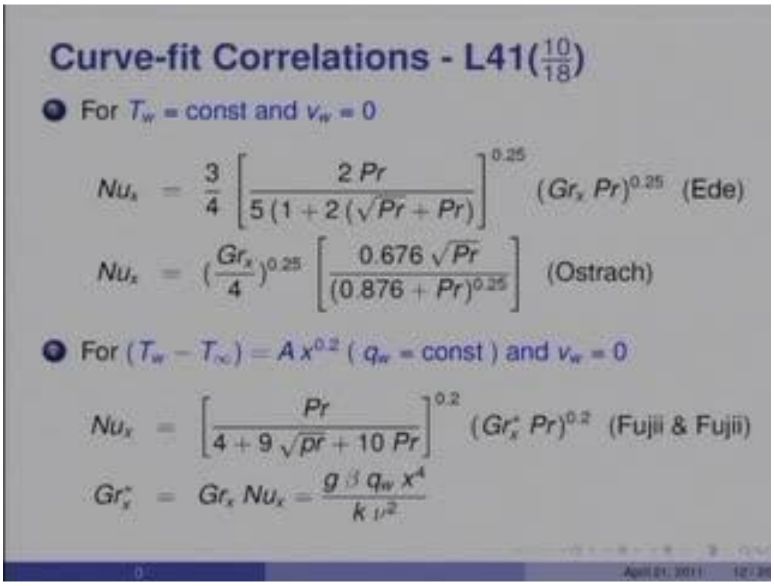

Now, for T w equal to constant and V w equal to 0, there are curve fit correlations have been obtained. So, for example, a correlation due to Ede is given here, the correlation due to Ede in Britten is given here that due to ostrich is given here. This ostrich solution is often found in under graduate text books, for T w minus T infinity A X raise to 0.2 which implies constant wall heat flux and V w equal to 0. Here is the correlation for the data by obtained by fujii and fujii, where G r star is now based on qw x raise to 4 k nu square. So, these are simply for your reference that solutions presented on the previous slides can be correlated in terms of functions of prandtl number.

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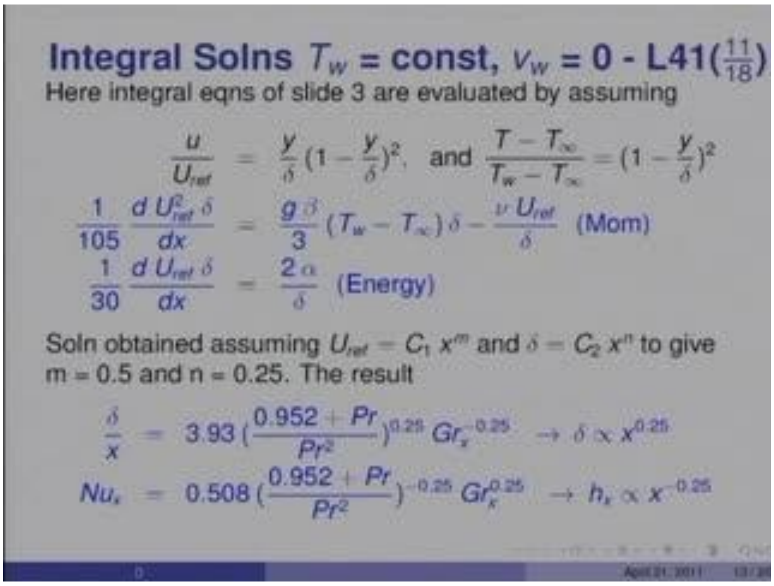

Now, we turned to integral solutions of the laminar boundary layer, and as you know integral solutions always require specification of a velocity and temperature profile. So, here I am going to defined a quantity called u ref, some reference velocity, I do not know what it is we will say, we shall find out in a moment what that is.

So, let us say, the velocity profile in the boundary layer will be u over U ref y over delta 1 minus y by delta whole square, you can see it satisfied the condition that u is equal to 0 at y equal to 0, as well as u equal to 0 at y equal to delta. So, it satisfy both conditions and it also satisfy the idea that the profile would be would would peak somewhere between 0 and delta. Likewise, T minus T infinity divided by T w minus T infinity is taken as 1 minus y by delta whole square.

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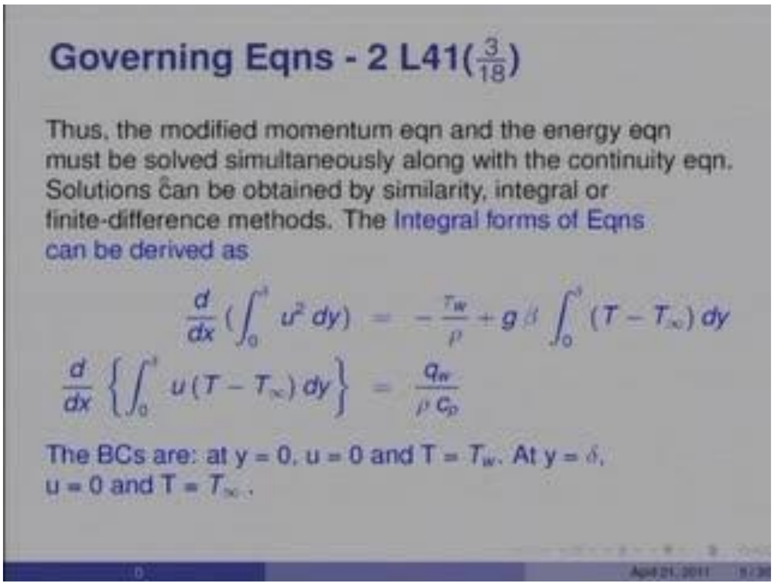

So, you substitute these temperature profiles and use our integral momentum equations which I have given; so, substitute for u here, T here as well as T here, and carry out the integrations.

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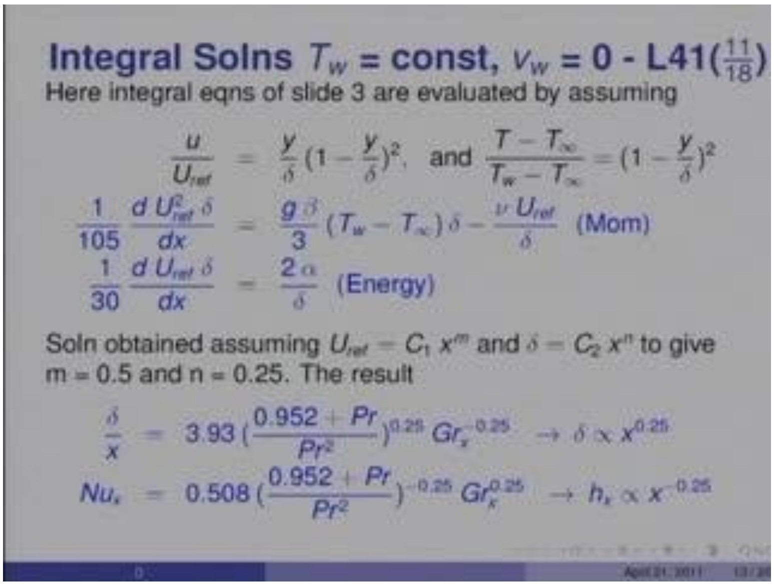

Then, you will see that the momentum equation transforms to 1 over 105 d U ref square delta divided by d x into g beta by 3 T w minus T infinity delta minus nu U ref by delta this will be the momentum equation, and 1 over 30 d U ref by delta d U ref delta by d x equal to 2 alpha by delta, where alpha is the thermal diffusivity. So, in order to obtained, these two equations can be solved by assuming, let U ref b equal to C 1 x raise to m, and delta be equal to C 2 x raise to n. And if you substitute, then algebra will show that m equal to 0.5 and n equal to 0.25 satisfies these equation.

Now, again, this is about one, one and half page of algebra which I am not presenting here, but it is a fairly straightforward algebra to do, to give m equal to 0.5 and n equal to 0.25. And therefore, you get the result, that the result is, once you substitute for U ref equal to C 1 x raise to power of m and C 2 x raise to power of n. With these values you can show that delta by x would be 3.93 into 0.952 plus prandtl divided by prandtl square raise to 0.25 grashof x to the power of minus 0.25, and delta would be proportional to x to the power of 0.25 - delta increases with x.

N u x is 0.508, 0.952 plus prandtl divided by prandtl square raise to minus 0.25 grashof x to the power of 0.25, and therefore, h x decreases with x - the local heat transfer coefficient decreases with x. These are important results, they agree fairly well, although they look very different from the forms of the curve fit correlations that I have given you in the previous page, but they agree fairly well with the correlations that I have shown you.

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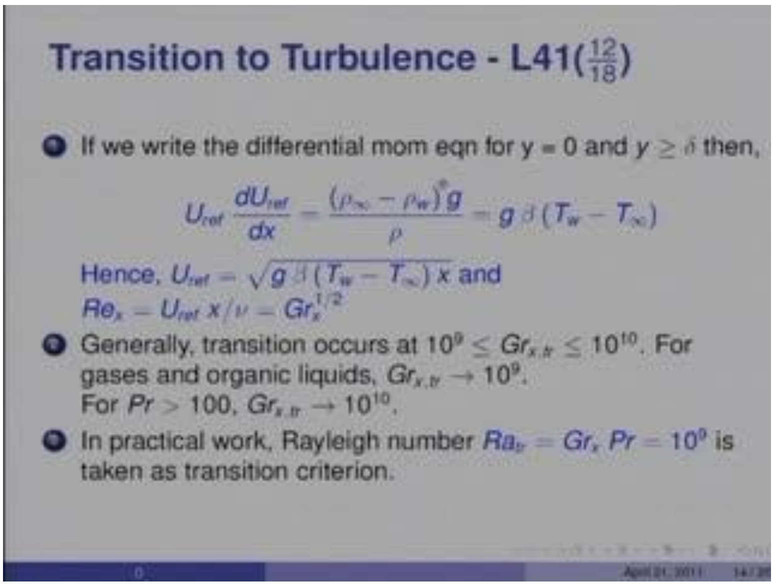

Now, if we write the differential momentum equation, now we are looking at the idea of transitional to turbulence, before we move on to turbulent.

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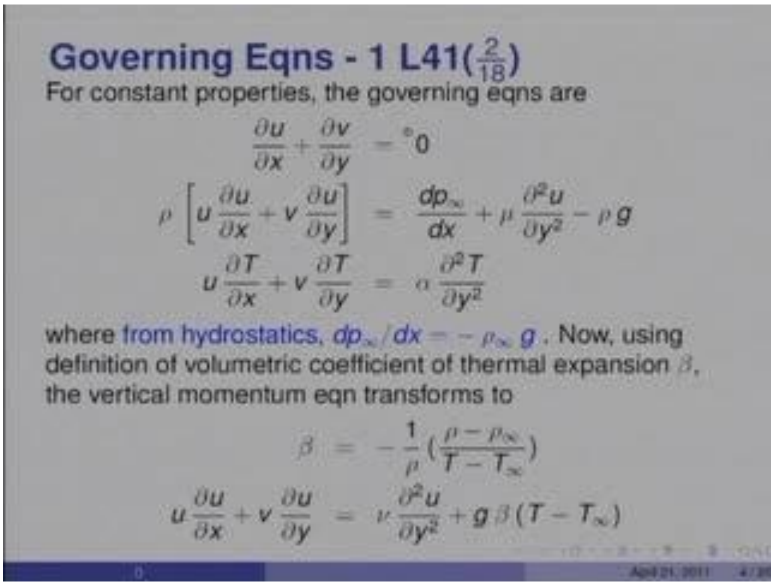

If we rewrite the momentum equation of the differential form, equation for y equal 0 and y greater than delta, that is, momentum equation that you see here. You write it for u equal to 0 and you write it for y equal to 0 and for y greater than delta, then in both cases convection terms will be absent, at y equal to 0 d p infinity by d x when it replace by d p w by d x, and this would be rho minus rho g.

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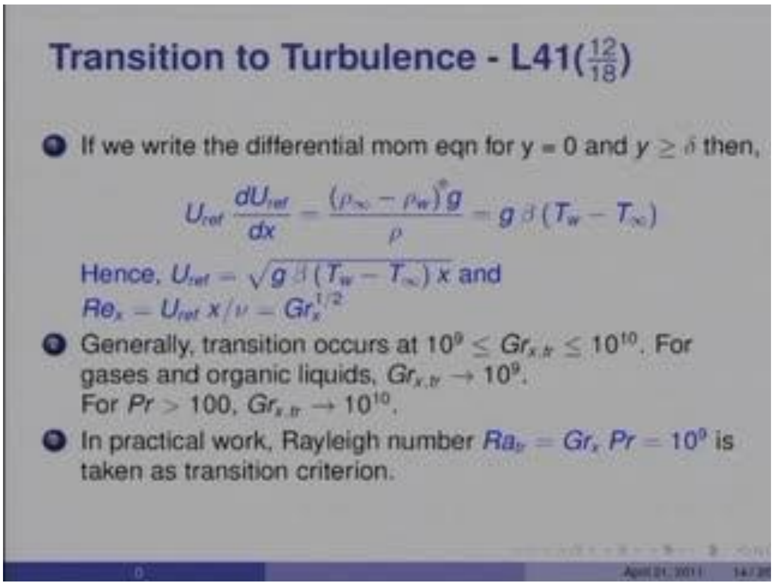

So, you will get, then U ref d U ref by d x would be equal to minus rho u infinity minus rho u w divided by rho into g and that would be equal to g beta T w minus T infinity.

So, U ref essentially is nothing but remember, this U ref into d U ref by d x will be d U ref square by d x divided by 1 by 2, and therefore, u will be equal to U ref would be simply equal to under root of g beta T w minus T infinity x - there should be a factor 2 here inside the root.

And reynolds x would be equal to U ref x divided by nu, so correspondingly from this U ref I can imagine a reynolds number, although it is not usually defined and that would equal grashof x to the power of half. So, essentially, if we say that the reynolds number of 10 raise to 5 or so, represents transitional then grashof number of 10 raise to 10 should roughly give you the transition criteria.

Now, generally, the transition occurs at 10 raise to 9 to 10 raise to 10 grashof numbers in that range. For gases and organic liquids, the lower range is found, and for prandtl number greater than 1 that is viscous fluids, the transition occurs little bit much later. So, in practical work, we define a rayleigh number which is grashof which is grashof into prandtl. And usually when rayleigh number assumes a value of 10 raise to 9, we say transition has occurred, so this is taken as the transitional criterion.

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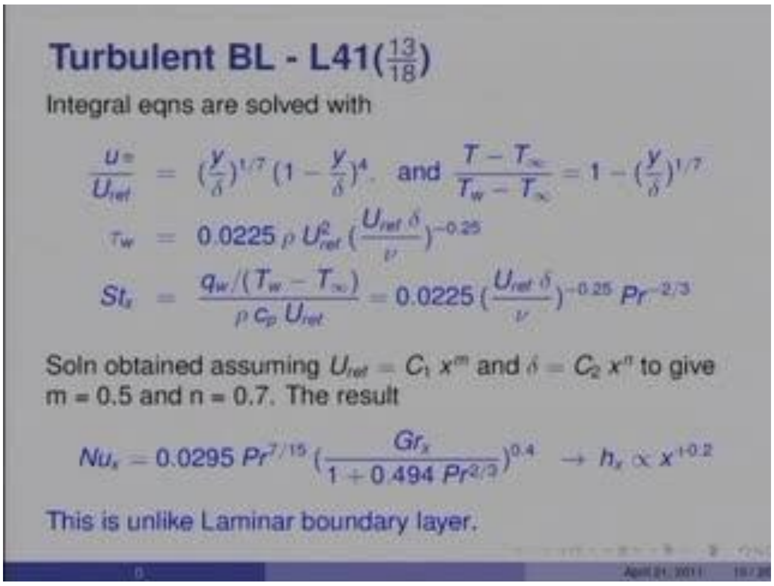

To solve the temperature boundary layer then, again we need velocity profiles and this slide shows what the velocity profiles in turbulent boundary layer are u over U ref equal to y by delta 1 raise to 1 by 7 into 1 minus y by delta raise to 4 again u is equal to 0 at y equal to 0 and u is equal to 0 at y equal to delta and T minus T infinity over T w minus T infinity is equal to 1 minus y by delta raise to 1 by 7.

Now, of course, as usual in turbulent boundary layers, we cannot depend on these velocity profiles to give us the shear stress, because it will simply tend to infinity at y equal to 0. And therefore, we use this form which was also used in laminar boundary layers, force convection boundary layers 0.0225 rho U ref square, instead of u infinity we are now replacing by U ref, U ref delta by nu raise to minus 0.25 and stanton x likewise is 0.0225 U ref delta by nu minus 0.25 prandtl raise to minus 2 by 3.

So, both tau wall and q wall which are required on the right hand side of the momentum and energy equations are represented in terms of delta. So, if you make the if you assume now u 1 equal to U ref equal to C 1 x raise to m and delta equal C 2 x raise to n and substitute in the integral momentum equations, then you find that the two equations are satisfied if m equal to 0.5 which was also the case of laminar boundary layer - laminar natural convection boundary layer. But n is now different, n is 0.7, instead of the value we had earlier n value was 0.25 in laminar boundary layer, but in turbulent boundary layer, we find that n should be about 0.7.

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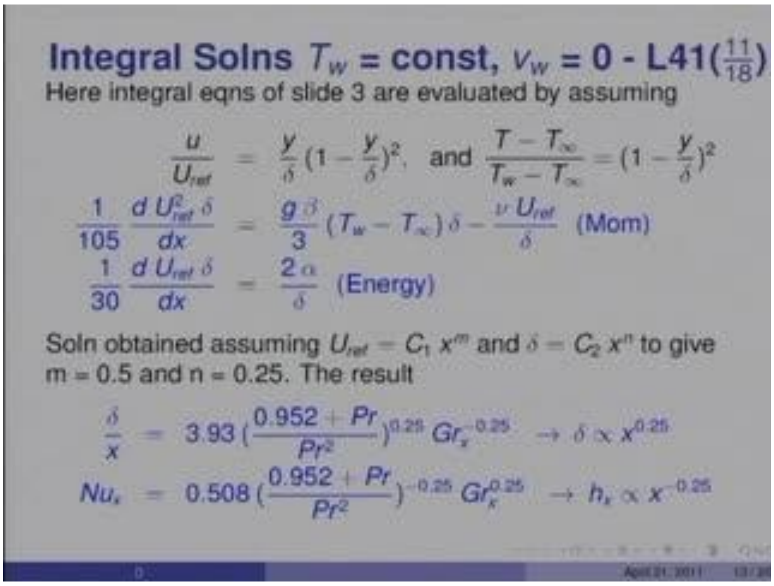

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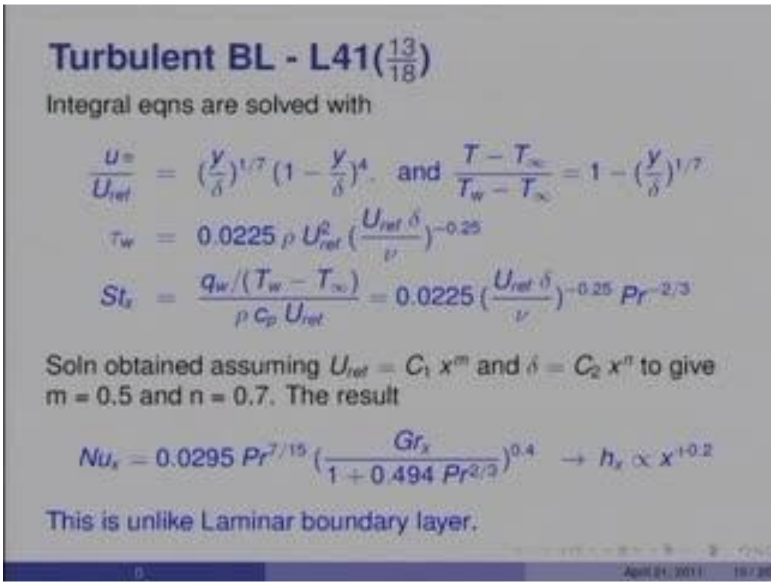

And with these two choices, the equations momentum and energy equations are satisfied, and as a result if you follow the same algebra that we did for laminar boundary layer, we find that the in turbulent boundary layers the nusselt x is 0.0295 prandtl raise to 7 by 15 into grashof x divided by 1 plus 0.494 prandtl raise to 2 by 3 raise to 4.

Now, in this case h x, this is h into x divided by k, so h x here turns out be proportional to x raise to plus 0.2. In other words, in turbulent boundary layer the heat transfer local heat transfer coefficient increases with x, whereas in laminar boundary layer the heat transfer coefficient was decreasing with x. Now, this is a very special feature of turbulent boundary layers - natural convection turbulent boundary layer - that the local heat transfer coefficient actually increases with x as shown here, so we note this difference and that is of considerably importance.

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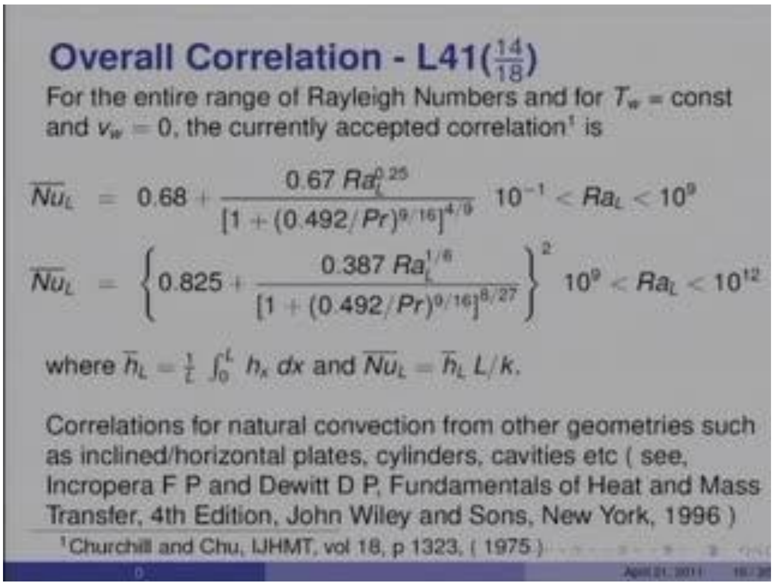

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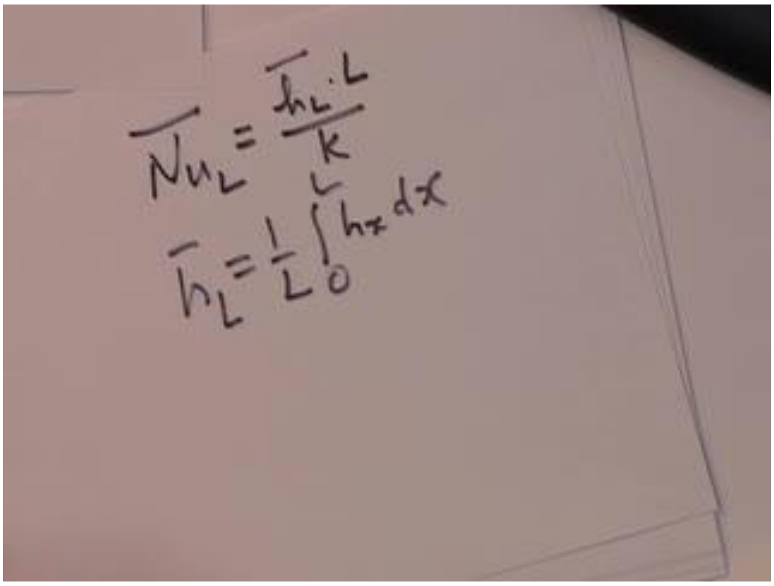

Now, for the entire range of rayleigh numbers and for T w constant and V w equal to 0, the currently accepted correlations is like this. So, over all correlations, these are the integral value, that is, N u L is actually h L into L by k and h bar L is 1 over L integral h x d x 0 to L, where L is the length of the plate.

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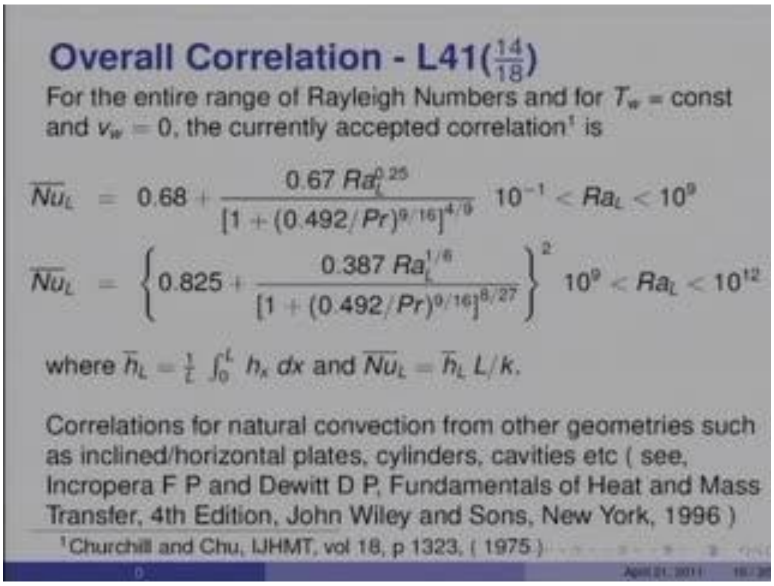

So, the currently accepted correlations are N U L equal to 0.68 rayleigh number based on L, and rayleigh number as you know is the product of grashof number and prandtl number. And then, these are the correlations given by churchill and chu international journal of heat and mass transfer 1975. For the lower range of reynolds number, this would be the laminar range, as we said, transition occurs at around 10 raise to 9, and this would be, if the turbulent range obtained, that is, if the plate grashof number was 10 raise to 9 and above then, we would use that correlation.

So, correlations for natural convection from other geometries such as inclined or horizontal plates, cylinders, cavities, etcetera, are readily available and you might have already used - I mean – in your undergraduate work. I am giving here one reference Incropera and Dewitt, Fundamentals of Heat and Mass transfer John Wiley in 1996, which gives a number of correlations for natural convection in different situations.

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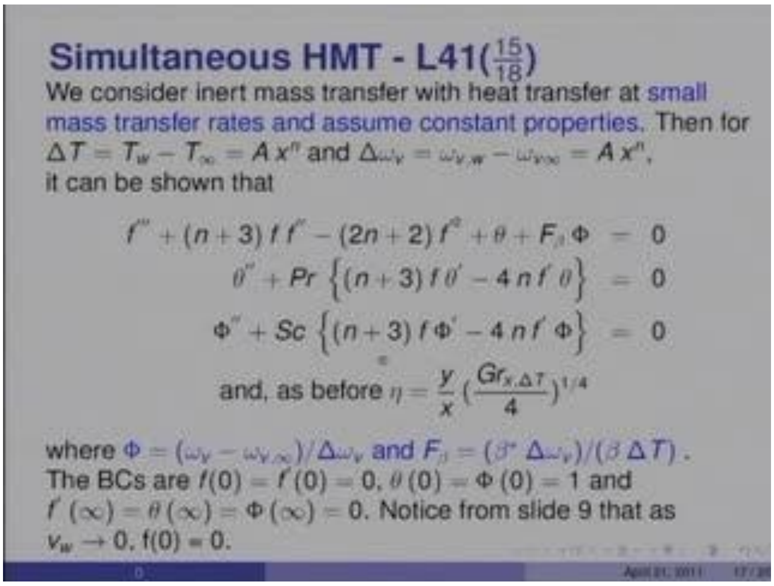

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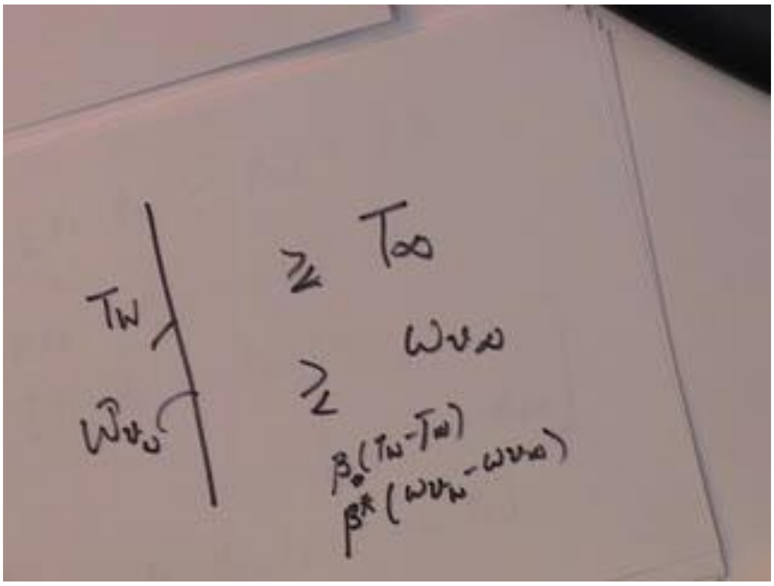

Now, we turn to the case of simultaneous heat and mass transfer, there is, we are assuming that T w, say, this is T w and there is omega V w is greater than or less than omega v infinity and T w is greater than or less than T infinity. So, main driving forces are a beta times T w minus T infinity, which I will call beta t if you like, or simply beta t which is thermal cubical expansion, but we can also define beta star for mass transfer into omega V w minus omega v infinity, so the definition of beta and beta star is very, very, parallel to each other.

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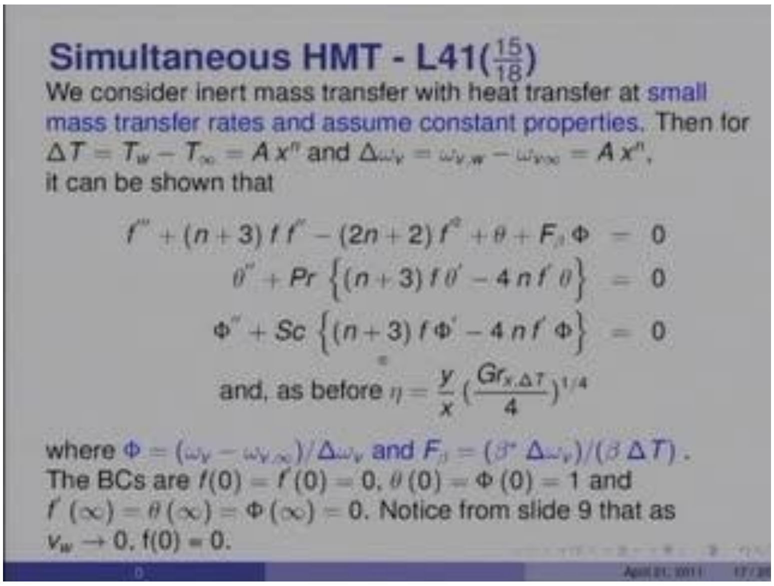

So, again, we can show that similarity solutions are possible, if T w minus T infinity varies as A X to the power of n and also delta omega v which is omega V w minus omega v infinity in equal to A X to the power of n both should vary as x to the power of n. Then, similarity solutions are possible and you get the momentum equation as f triple prime n plus 3 f f double prime minus 2 n plus 2 f f dash square plus theta plus, there the buoyancy also due to the concentration difference.

And the energy equation would read like that, and the species transfer the omega v equation would read in this fashion. I have define phi as omega v minus omega v infinity divided by delta omega V w minus omega v infinity f beta is simply the ratio of the buoyancies due to concentration and temperature differences. The boundary conditions would be f 0 f dash 0 theta 0 phi 0, sorry, theta 0 and phi 0 will be 1 and f dash infinity theta infinity and phi infinity will be 0.

Now, notice from slide 9 that as V w tends to 0 f 0 tends to 0, so we are assuming at the moment that we are dealing with a very small mass transfer right from. Now eta would be still taken in the as] as defined as G r x based on temperature difference rather than based on mass transfer difference - i mean - mass fraction difference.

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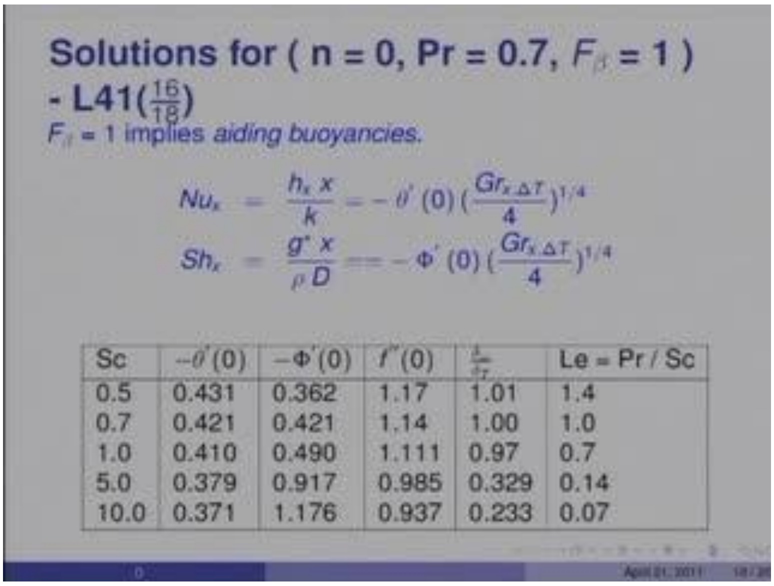

So, these are the set of equations, again, we can solve them by finite difference method and here is the solutions. So, I am taking the case of constant wall temperature, the prandtl number is taken as 0.7 and f beta is equal to 1, now f beta equal to 1 implies that we have aiding that there is the concentration buoyancy and the thermal buoyancy are aiding each other.

And in which case the nusselt number would get defined in this fashion and sherwood number which is the dimensionless mass transfer coefficient, would get defined in this fashion. And this is the case of limiting very, very, small mass transfer at the wall. Then, you will get for different values of S c you get 0.5, 0.7, 1, 5, 10, I have calculate minus theta 0 minus phi dash 0, so you can get nusselt numbers out of this, you get sherwood numbers out of this.

And notice that I have also printed out here, the boundary layer thickness of concentration and boundary layer thickness of temperature. So, when schmidt number is close to equal to prandtl number, the ratio is 1; when schmidt number is lower than prandtl number, then the ratio is 1.01 very very close, but the lewis number now is 1.4. And then, when schmidt number exceeds prandtl number then, you find that the concentration boundary layer thickness is smaller than the temperature boundary layer thickness. Now this is to be expected, but it is good to notice it from the results as well.

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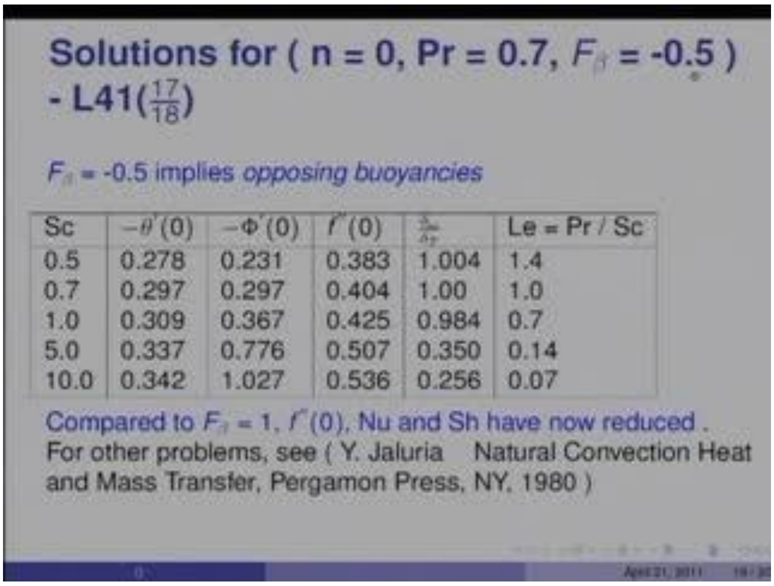

Sc	$-\theta'(0)$	$-\Phi'(0)$	$f''(0)$	$\frac{\delta_m}{\delta_T}$	Le = Pr / Sc
0.5	0.278	0.231	0.383	1.004	1.4
0.7	0.297	0.297	0.404	1.00	1.0
1.0	0.309	0.367	0.425	0.984	0.7
5.0	0.337	0.776	0.507	0.350	0.14
10.0	0.342	1.027	0.536	0.256	0.07

Similarly, the results for f beta equal to minus 0.5, now you find that whereas, the thermal buoyancy is trying to take the fluid up, mass transfer buoyancy is trying to take it down. So, you have opposing effects due to opposing buoyancies. And you can see that compare to f beta equal to 1 with the aiding buoyancies, you find the nusselt number and sherwood numbers are all smaller irrespective of the schmidt number that you use.

Now, there are many other variations of, now in all this cases, I have use simply constant wall temperature, but there are variety of cases of finite n and so on so forth, which have been handle in a very specialize book by Y Jaluria and it is called Natural Convection Heat and Mass transfer, Pergamon Press and published in 1980, you can refer to that. But the important thing is, when aiding compare to aiding situation, the opposing situation lowers the nusselt number as well as the sherwood number.

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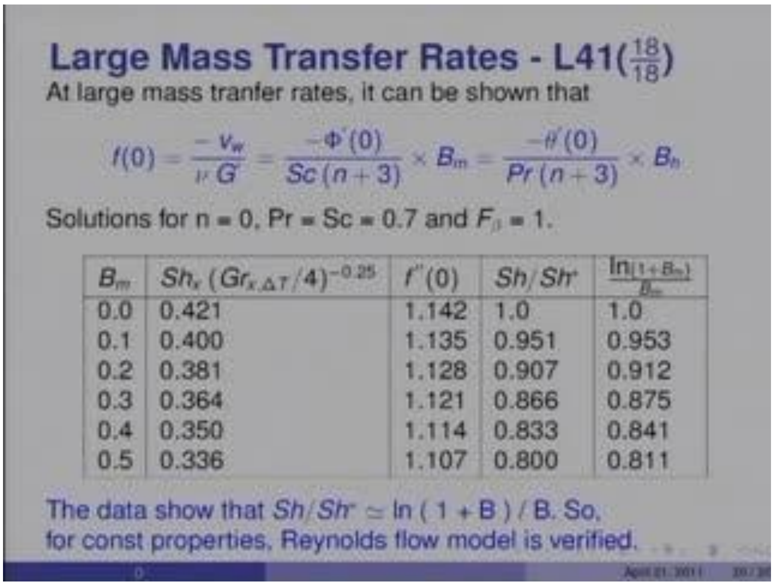

Finally, those solutions were obtained at low vanishing mass transferring. Now, suppose, we take f 0 equal to minus V w nu nu G dash, then that will become minus phi dash 0 s c n plus 3 into b m and minus theta prime 0 prandtl n plus 3 b h. So, if I use this as the boundary conditions for the momentum equation, I can now deal with large mass transfer rates also and for n equal to 0 which is constant wall temperature and I have taken deliberately the case of prandtl and schmidt equal and f beta equal to 1 that is aiding buoyancies. Then, you find that for different values of b m from 0 to 0.5, the sherwood number goes from 0.421, 0.4, 0.381, 0.364, 0.35, so has b increases the sherwood number goes on decreasing.

But if I calculate g over g star that is sherwood number or any b divided by sherwood number for b m equal to 0, then I get 1.951 and so on so forth. Now, if I also calculate l n 1 plus b m divided by b m, then I get those values and you can see those are very, very, close to the sherwood versus star which have obtained from exact solution to the boundary layer equation. And therefore, these data show that sherwood over sherwood star equal to l n 1 plus b by b is also verified in natural convection boundary layers as long as properties are constant.

And this was the result we use extensively in force convection in reynolds flow modeling, but it is also applicable to the case of natural convection boundary layers and other property corrections can also be applied in the same way that we had applied in reynolds flow model.

So with this, I conclude the lecture on Natural Convection Heat and Mass Transfer.