

Convective Heat and Mass Transfer
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Module No. # 01
Lecture No. # 39

CONV M T Reynolds Flow Model -1

In the previous lecture, we saw how Couette flow model can be applied to discover effect of property variations through the ratio of molecular weight of the mixture in the infinity and w states, in which as the result we obtained was approximate, it did show the correct tendency that g over g star of the variable property varies directly with m divided by m infinity and raised to some power.

Today, we are going to look at Reynolds flow model and its applications. In fact, in the next two lectures, I will be dealing with the Reynolds flow model, the first of which, today, we will deal exclusively with air water wave persistent which requires use of psychrometry and steam tables.

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**LECTURE-39 CONV M T -
REYNOLDS FLOW MODEL - 1**

- Wet Bulb Thermometer
- Measurement of RH - Effect of Radiation
- Evaporation from a Porous Surface
- Evaporation from a Lake
- Humidification - Internal Flow

All problems require use of Psychrometry and/or steam tables

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The problems that I shall consider are shown here in this slide: the wet bulb thermometer; the measurement of relative humidity in a ducted flow where, we shall estimate the true vapor concentration or the RH allowing for effect of radiation and without effect of radiation; then, we will consider evaporation from a porous surface; then evaporation from a lake; then whereas, all these problems are of external flow, we will turn to internal flow, in which we will try to humidify air which is entering into a tube whose walls are maintained wet. So all these problems will require use of psychrometry and or steam tables as we shall go on to see.

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Wet Bulb Thermometer - L39($\frac{1}{16}$)
Prob: A wet bulb thermometer records 15°C when the dry bulb temperature is 27°C. Calculate (a) RH of air and (b) compare with Carrier's correlation. Assume Le = 1 and take $c_{p,v} = 1.88$ kJ/kg-K and $c_{p,a} = 1.005$

Soln: Here, $T_w = 15$ and $T_\infty = T_{db} = 27$. Since Le = 1,

$$B_m = \frac{\omega_{v,\infty} - \omega_{v,w}}{\omega_{v,w} - \omega_{v,T}} = B_h = \frac{h_{m,\infty} - h_{m,w}}{h_{m,w} - h_{TL} + q_l/N_w} \rightarrow q_l = 0$$

Taking $T_{ref} = 0^\circ\text{C}$, $\lambda_{ref} = 2503$ kJ/kg, and from Steam Tables, $\omega_{v,w} = 0.01068$, we have

$$h_{m,\infty} = 1.005 \times 27 + \{(1.88 - 1.05) \times 27 + 2503\} \omega_{v,\infty}$$

$$h_{m,w} = 15.075 + \{0.875 \times 15 + 2503\} 0.01068 = 41.95$$

$$h_{TL} = c_{p,l} \times T_w = 4.187 \times 15 = 62.805$$

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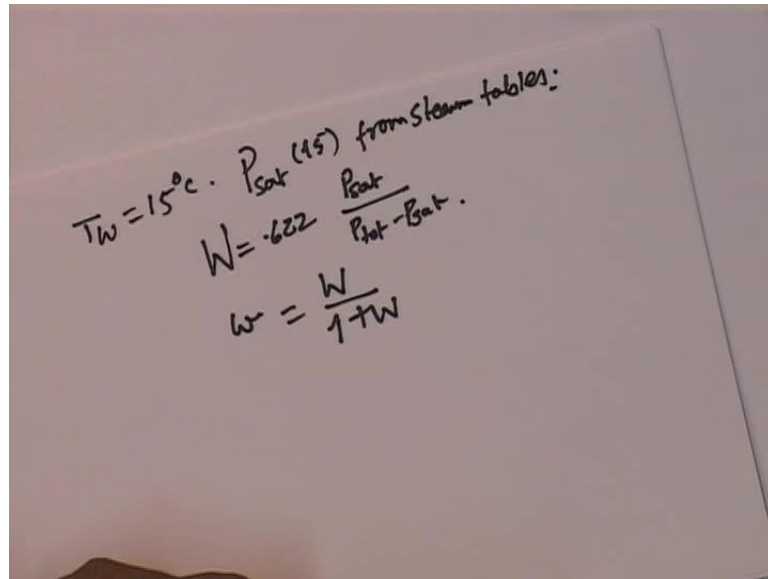
Let us look at the first problem. Here is a problem: a wet bulb thermometer records 15 degree centigrade when the dry bulb temperature is 27 degree centigrade. Calculate (a) the relative humidity of air and (b) compare with Carrier's correlation.

Assume Lewis number equal to 1 and take C_p vapor equal to 1.8 and c_p air equal to 1.005. Now, this problem can be solved using psychrometric tables. It can be used to solve by using **what is well known** Carrier's correlation, but we shall apply our Reynolds flow model.

So, here T_w is 15 degree centigrade and T_∞ is T_{db} equal to 27 degree centigrade and we have said Lewis number equal to 1. Therefore, B_m which is $\omega_{v,\infty} - \omega_{v,w}$ over $\omega_{v,w} - \omega_{v,T}$ which is 1 will equal Spalding

number calculated from mixture enthalpies as given here. Now, wet bulb records this temperature when the heat conduction to it is 0 and therefore, q_l in our formula will be set to 0. If we take T_{ref} equal to 0 degree centigrade, then λ_{ref} - the latent heat, is 2503 kilo Joules per kg. From steam tables, we would readily discover from partial pressure, the value of ω vs w . The way to do this is the following.

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$T_w = 15^\circ\text{C}$. $P_{sat}(15)$ from steam tables:
 $w = 0.622 \frac{P_{sat}}{P_{tot} - P_{sat}}$
 $\omega = \frac{w}{1 + w}$

So, for example, since you would know T_w is equal to 15 degree centigrade, you will read P_{sat} 15 from steam tables and then you will evaluate the specific humidity, which is 0.622 times P_{sat} divided by P_{total} minus P_{sat} . Then, you will evaluate ω as w over 1 plus w .

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Wet Bulb Thermometer - L39($\frac{1}{16}$)
Prob: A wet bulb thermometer records 15°C when the dry bulb temperature is 27°C. Calculate (a) RH of air and (b) compare with Carrier's correlation . Assume Le = and take $c_{p,v} = 1.88$ kJ/kg-K and $c_{p,a} = 1.005$

Soln: Here, $T_w = 15$ and $T_\infty = T_{db} = 27$. Since Le = 1,

$$B_m = \frac{\omega_{v,\infty} - \omega_{v,w}}{\omega_{v,w} - \omega_{v,T}} = B_h = \frac{h_{m,\infty} - h_{m,w}}{h_{m,w} - h_{TL} + q_l/N_w} \rightarrow q_l = 0$$

Taking $T_{ref} = 0^\circ\text{C}$, $\lambda_{ref} = 2503$ kJ/kg, and from Steam Tables, $\omega_{v,w} = 0.01068$, we have

$$h_{m,\infty} = 1.005 \times 27 + \{(1.88 - 1.05) \times 27 + 2503\} \omega_{v,\infty}$$
$$h_{m,w} = 15.075 + \{0.875 \times 15 + 2503\} 0.01068 = 41.95$$
$$h_{TL} = c_{p,l} \times T_w = 4.187 \times 15 = 62.805$$

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So, if we carry out these calculations, you will see omega v w turns out to be 0.01068, as I have shown on the slide here. So, if we calculate now the mixture enthalpy in the infinity state, it will be 1.0025 into 27 plus 1.88 minus 1.005 into 27 plus 2503 into omega v infinity. h m w will be likewise 1.005 into 15 plus 0.875 into 15 plus 2503 into the omega v w which is 0.01068, and that will give you 41.95.

Here, we did not know omega v infinity and therefore, it is left as it is. h TL is the enthalpy of the transfer substance which is water in the L state; that is just inside the W w surface. That would be C p of the liquid multiplied by T w. So, 4.187 multiplied by 15 gives you 62.805.

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Soln (Contd) - L39($\frac{2}{16}$)

Therefore

$$\frac{\omega_{v,\infty} - 0.01068}{0.01068 - 1} = \frac{27.135 + 2526.2 \omega_{v,\infty} - 41.95}{41.95 - 62.805}$$

Solving, $\omega_{v,\infty} = 0.005936$. Hence, $W_{\infty} = \omega / (1 - \omega) = 0.00594$ at 27°C. From psychrometric chart, this value corresponds to RH = 27 % (Ans a). Also, $B_m = B_h = 0.00479$ (Very small).

Carrier's correlation is

$$p_{v,\infty} = p_{sat,w} - \frac{(p_{tot} - p_{sat,w})(T_{wb} - T_{db})}{1555 - T_{wb}} \rightarrow (T^{\circ}C)$$

where $p_{tot} = 1$ bar, $W_w = 0.01068 / (1 - 0.01068) = 0.0108$ and $p_{sat,w} = W_w \times p_{tot} / 0.622 = 0.01736$ bar.
Hence, substitution gives $p_{v,\infty} = 0.0097$. Therefore

$$\omega_{v,\infty} = \frac{p_{v,\infty}}{1.61 \times p_{tot} - 0.61 \times p_{v,\infty}} = 0.00604 \quad (\text{ Ans b })$$

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So, therefore, equating B m with B h, we will have omega v infinity equal to minus 0.0168 divided by 0.0168 minus 1 equal to all this - 27.135 plus 2562.2 omega v infinity minus 41.95 divided by 41.95 minus 62.805. If we solve now for omega v infinity, we would get omega v infinity equal to 0.005936, which gives us specific humidity equal to omega over 1 minus omega equal to 0.00594 at 27 degree C.

Now, from psychrometric chart, this value corresponds to relative humidity of 27 percent which is our answer a; also B m is B h and very small. If I substitute this value here (Refer Slide Time: 06:48), then you will see B m and B h would be equal to 0.00479 which is very small.

Now, if we use Carrier's correlation which you have used in your undergraduate work, it gives p v infinity equal to p sat w minus p tot minus p sat w T wet bulb minus T dry bulb 155 minus T wet bulb where T is I given in degree centigrade.

Now, in our case p tot is 1 bar; W w is 0.01068 minus this value which is 0.0108; p sat w is 0.01736 bar which can be read from the steam tables as well, then substitution gives p v infinity equal to 0.0097. Therefore, I can evaluate omega v infinity as p v infinity into 1.61 into p tot minus 0.61 p v infinity equal to 0.00604 which is answer b.

Now, you will see this value of 0.00604 is very close to the value 0.005936 that we applied obtained from Reynolds flow model. Therefore, we can say that the Reynolds flow model very well captures the psychrometric relationships.

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Measurement of RH - L39($\frac{3}{16}$)

Prob: Moist air flows through a duct whose walls are maintained at 50°C. Dry and wet bulb thermometers placed in the duct record 70°C and 25°C respectively. Between thermometer bulb and air, $h_{cof} = 17.5 \text{ W / m}^2\text{-K}$. Calculate RH of air (a) without radiation and (b) with radiation .

Soln: Part (a) Here, again $W_w = 0.02$, $\omega_{v,w} = 0.0196$, $h_{m,w} = 74.61$, $h_{TL} = 105.7$ and $h_{m,\infty} = 70.35 + 2564.25 \omega_{v,\infty}$. Therefore, with $q_l = 0$, equating B_m and B_h ,

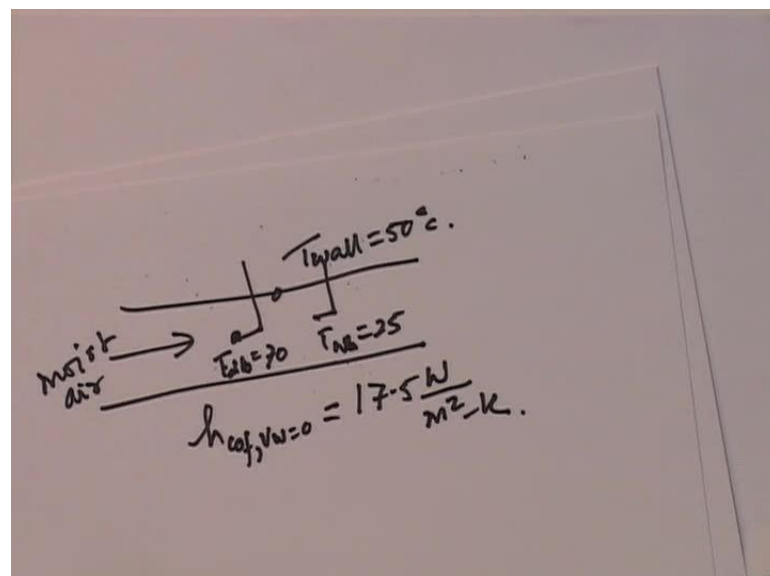
$$\frac{\omega_{v,\infty} - 0.0196}{0.0196 - 1} = \frac{70.35 + 2564.25 \omega_{v,\infty} - 74.61}{74.61 - 104.675}$$

Solving $\omega_{v,\infty} = 0.001446$ ($B = 0.0185$). Therefore, $p_{v,\infty} = 0.002328$ bar. But, from steam tables, $p_{v,sat}(T_\infty = 70^\circ\text{C}) = 0.3119$ bar.
Hence, $\text{RH} = 0.002328 / 0.3119 = 0.746\%$ (Ans) .

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Let us now turn to the second problem. It is given like this: Moist air flows through a duct whose walls are maintained at 50 degree centigrade and wet bulb thermometer placed in the duct records the dry and wet bulb thermometers located in the duct show 70 degree centigrade and 25 degree centigrade respectively.

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So, we have a duct whose wall is at 50 degree centigrade and moist air is flowing through it. We have the dry bulb equal to 70 and we have the wet bulb equal to 25. Now, it is also given that the heat transfer coefficient is $h_{cof} = 17.5 \text{ W / m}^2\text{-K}$, for this situation of the bulb is measured as 17.5 Watts per meter square Kelvin.

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Measurement of RH - L39($\frac{3}{16}$)
Prob: Moist air flows through a duct whose walls are maintained at 50°C. Dry and wet bulb thermometers placed in the duct record 70°C and 25°C respectively. Between thermometer bulb and air, $h_{cof} = 17.5 \text{ W / m}^2\text{-K}$. Calculate RH of air (a) without radiation and (b) with radiation .

Soln: Part (a) Here, again $W_w = 0.02$, $\omega_{v,w} = 0.0196$, $h_{m,w} = 74.61$, $h_{TL} = 105.7$ and $h_{m,\infty} = 70.35 + 2564.25 \omega_{v,\infty}$. Therefore, with $q_l = 0$, equating B_m and B_h ,

$$\frac{\omega_{v,\infty} - 0.0196}{0.0196 - 1} = \frac{70.35 + 2564.25 \omega_{v,\infty} - 74.61}{74.61 - 104.675}$$

Solving $\omega_{v,\infty} = 0.001446$ ($B = 0.0185$). Therefore, $p_{v,\infty} = 0.002328$ bar. But, from steam tables, $p_{v,sat} (T_{\infty} = 70^\circ\text{C}) = 0.3119$ bar.
Hence, $RH = 0.002328 / 0.3119 = 0.746 \%$ (Ans)

So, under this situation, you are asked to calculate relative humidity of air without radiation and (b) with radiation.

So, in the first part, we assume that, whatever the wet bulb and dry bulb thermometers are reading are correct values and they are not influenced by radiation between the tube wall temperature and the bulb temperatures. So, straight away, we can see that in this case, corresponding to 25 degree centigrade, we will get W_w - the specific humidity as 0.02. Therefore, the mass fraction at the surface of the wet bulb thermometer would be 0.0196, which gives us mixture enthalpy as 74.61; h_{TL} will be 4.186 into 25 giving 105.7; h_{∞} would be 70.35 to 2564.25 into $\omega_{v,\infty}$.

Therefore, equating B_m with B_h , we would simply get that relationship from which we get solving. We would get $\omega_{v,\infty}$ equal to 0.001446 and B itself would be 0.0185. Since $\omega_{v,\infty}$ is 0.00146, we can readily recover $p_{v,\infty}$ from the relationship I have shown on the previous slide as 0.002328 bar.

From the steam tables, p_v sat corresponding to T infinity equal to 70 degree centigrade which is the dry bulb temperature, would be 3119 bar. Therefore, relative humidity will be the actual value - partial pressure divided by the saturation pressure is partial pressure which gives us 0.746 percent as the relative humidity when radiation was neglected. So, that means the moist air has moisture content of 0.746 percent, but now, we wish to allow for radiation. Does that make any difference to the estimate of relative humidity? Let us look at that in the next slide.

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Soln (Contd) - 1 - L39($\frac{4}{16}$)
Soln: Part (b) Here, $q_l = 0$ and $h_{TL} = h_T + q_{rad}/N_w$.

$$B_{mh} = \frac{\omega_{v,\infty} - \omega_{v,w}}{\omega_{v,w} - \omega_{v,T}} = \frac{h_{m,\infty} - h_{m,w}}{h_{m,w} - h_T} = \frac{h_{m,\infty} - h_{m,w}}{h_{m,w} - h_{TL} + q_{rad}/N_w}$$

Now, we determine **true air temperature ($T_{a,true}$)** allowing for radiation effects. Thus

$$h_{cof} (T_{a,true} - T_{db}) = \sigma (T_{db}^4 - T_w^4)$$

$$17.5 (T_{a,true} - 343) = 5.67 \times 10^{-8} (343^4 - 323^4) \quad \text{or}$$

$$T_{a,true} = 352.6K = 79.6^\circ C \quad \text{and}$$

$$q_{rad} = \sigma (T_w^4 - T_{wb}^4)$$

$$= 5.67 \times 10^{-8} (323^4 - 298^4) = 170 \frac{W}{m^2}$$

Now, B_m and B_h are freshly evaluated (next slide)

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In this path at the wet bulb, as usual q_l will be 0 and h_{TL} will be h_T plus q_{rad} by N_w . You will recall we account for q_{rad} in the neighboring phase, and therefore, h_T will be q_{rad} divided by N_w . Therefore, B_m would be still $\omega_{v,\infty}$ by $\omega_{v,w}$ minus $\omega_{v,T}$ equal to $h_{m,\infty}$ minus $h_{m,w}$ over $h_{m,w}$ minus h_T which would transform to $h_{m,\infty}$ minus $h_{m,w}$ over $h_{m,w}$ minus h_{TL} plus q_{rad} by N_w . This relationship you already know. We have derived this when discussing the Reynolds flow model.

Now, the first thing we must do is we must determine the true air temperature that is the dry bulb temperature of the air because the dry bulb temperature itself is influenced by the effect of radiation. So, we will say the heat transfer by convection to the dry bulb thermometer which is h_{cof} into $T_{a,true}$ minus $T_{dry\ bulb}$ will equal radiation from the

thermometer bulb to the wall. So, sigma into T w T db raise to 4 minus T w raise to 4. We have assumed the emissivity of the bulb to be 1.

Now, h cof has been given to you as 17.5. So, T a true minus 343 which was 70 degree centigrade equal to 5.67 into 10 raise to minus 8 which is the Stefan Boltzmann constant into 343 raise to 4 minus the wall temperature, which is 50 degrees or 323 raise to 4, which gives us T a true equal to 352.6K equal to 79.6 degree centigrade. So, remember, in this case, the dry bulb temperature itself was not reading correctly and the true air temperature is in fact 79.6 degree centigrade.

So, **what would be the** correspondingly, then what could be the q rad? q rad will be sigma in the q rad at the wet bulb thermometer will be T w raise to 4 minus T wet bulb raise to T4. That would be equal to 5.67 into 10 raise to minus 8 into 323 raise to 4 minus 298 raise to 4 which is a wet bulb temperature of 35 and that will give you 170 Watts per meter square. So, now we must evaluate B m and B h freshly because now we are going to introduce this q rad in here (Refer Slide Time: 14:28). Now, we already know that B is going to be very small. So, I can replace N w equal to g B. That is what I have done on the next slide.

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Soln (Contd) - 2 - L39($\frac{5}{16}$)
 Here, we take $T_{ref} = T_{wb} = 25$ so that $\lambda_{ref} = 2442.3$ kJ/kg.

$$h_{m,\infty} = c_{p,a}(T_{a,true} - T_{ref}) + [(c_{p,v} - c_{p,a})(T_{a,true} - T_{ref}) + \lambda_{ref}] \omega_{v,\infty} \text{ kJ/kg}$$

$$= 54.52 + 2490 \omega_{v,\infty}$$

$$h_{m,w} = \omega_{v,w} \lambda_{ref} = 0.0196 \times 2442.3 = 47.87 \text{ kJ/kg}$$

$$h_{TL} = c_{p,l}(T_{wb} - T_{ref}) = 0 \text{ kJ/kg}$$

Also $\omega_{mean} \approx 0.5(0.0196 + 0.001446) = 0.0105$.
 Therefore, $c_{p,m} = 1.041$ kJ/kg-K.
 Hence, $g^* = h_{cof} / c_{p,m} = 17.5 / 1.041 = 0.01725$ kg/m²-s.
 Now, since B is expected to be small,
 we take $(q_{rad} / N_w) = q_{rad} / (g^* B)$, so that

$$B = \frac{\omega_{v,\infty} - \omega_{v,w}}{\omega_{v,w} - \omega_{v,T}} = \frac{h_{m,\infty} - h_{m,w} - (q_{rad} / g^*)}{h_{m,w} - h_{TL}}$$

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So, if we take T ref equal to T wet bulb equal to 25 so that lambda ref will be 2442.3, then h m infinity will be c p a into T a true minus T ref c p v minus c p a into T a true

minus T_{ref} plus $\lambda_{ref} \omega_{v, \infty}$. That would be $54.52 + 2490 \omega_{v, \infty}$ minus 47.87 . $h_m w$, of course, because these specific heat sensible heat terms go out. You will have $\omega_{v, w}$ into λ_{ref} equal to $0.0196 \times 2442.3 = 47.87$, and the liquid enthalpy would be $c_p l$ into $T_{wet\ bulb} - T_{ref}$ equal to 0 because we have taken the reference temperature at the wet bulb itself. Now, taking ω_{mean} equal to $0.0196 + 0.001446$, remember, we had evaluated $\omega_{v, \infty}$ equal to 0.001446 (Refer Slide Time: 15:50).

So, we assume that will still remain the same as far as ω_{mean} calculation is concerned so that c_{pm} is not too seriously affected by this choice, all though we do not know this value. This is simply to avoid iterative solution because c_{pm} will not be influenced too much by the value of $\omega_{v, \infty}$ in the infinity state. So, ω_{mean} is taken as 0.0105 , and therefore, c_{pm} is taken as 1.041 ; g^* would be h_{cof} divided by c_{pm} which is 17.5 divided by 1.041 equal to 0.01725 kg per meter square second.

So, now, since B is expected to be small, q_{rad} by N_w will be q_{rad} by $g^* B$ so that we get B equal to $\omega_{v, \infty} - \omega_{v, w} - \omega_{v, T}$ which is 1, equal to $h_m \infty - h_m w - q_{rad}$ by g^* over $h_m w - h_{TL}$. Substitution will show on the next slide.

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Soln (Contd) - 3 - L39($\frac{6}{16}$)

Substitutions give $(q_{rad}/g^*) = 0.17 / 0.01725 = 9.885$ kJ/kg.
Hence

$$B = \frac{\omega_{v, \infty} - 0.0196}{0.0196 - 1} = \frac{54.52 + 2490 \omega_{v, \infty} - 47.87 - 9.885}{47.85 - 0}$$

This gives $\omega_{v, \infty} = 0.001693$ and $B = 0.01826$.
So, our assumption of small B is verified.
Thus, $W_{\infty} = 0.001648$ and $p_{v, \infty} = 0.00264$ bar.
But, $p_{v, sat}(T_{\infty} = 79.58^{\circ}\text{C}) = 0.4739$ bar.
Hence $RH = 0.00264 / 0.4739 = 0.557\%$ (Ans) .

This shows that true RH is lower than that predicted by neglecting effect of radiation

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q rad by g start is 0.17 as we calculated here (Refer Slide Time: 17:09) 170 Watts per meter square; so, in kilowatts meter square it would be 0.17 divided by 0.01725 equal to 9.885 kilo Joules per kg. Therefore, our balance equation would read like that. If you solve this equation you will get ωv infinity equal to 0.001693 instead of the previous value of 0.001446.

So, as I said our value of $c p m$ is not going to be seriously affected, which gives B equal to 0.01826. So, our assumption of small B is verified from this value of ωv infinity. We calculate specific humidity as that, and therefore, $p v$ infinity equal to 0.00264 bar, but p sat at 79.58 is 0.4739 bar. Therefore, the relative humidity now is 0.557 percent instead of the earlier value of 0.746 percent as you can see here (Refer Slide Time: 18:15).

So, the effect of radiation is actually that this moist air is in fact drier than what ignoring radiation would predict. So, the true RH is lower than that predicted by neglecting effect of radiation.

Now, these kinds of calculations are very important because often when the so called dry air is supplied, it is never really completely dry. There are very small fractions of moisture and if one wants to know what that small fraction is, you can see by allowing for effect of radiation. We have shown that is in fact significant effect of radiation from 0.746 percent down to 0.557 percent which would be predicted where radiation was accounted.

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Evaporation - High B - L39($\frac{7}{16}$)

Prob: Air at 1 bar, 27°C and 90 % RH flows over a porous flat surface which is kept wet by supplying water. The plate temperature is maintained at 82.5°C by supplying heat to it from the rear side. Calculate (a) Evaporation rate and (b) Heat flux to be supplied . Given: $U_\infty = 3.0$ m/s, Length $L = 0.33$ m and Width $W = 1$ m. Take $k = 0.025 \frac{W}{m-K}$, $\rho_m = 1.37 \frac{kg}{m^3}$, $D = 0.162 \frac{m^2}{hr}$, $\alpha = 0.153 \frac{m^2}{hr}$ and $\nu = 0.103 \frac{m^2}{hr}$

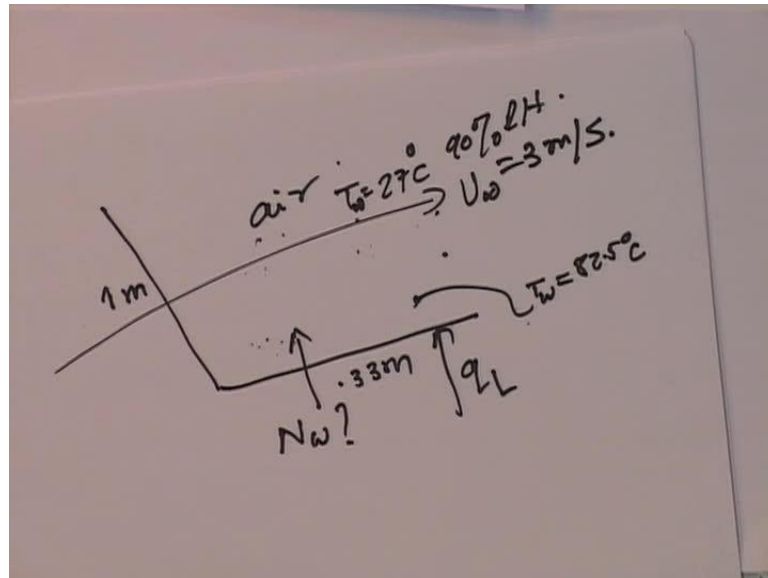
Soln: In this case,
 $Re_L = 3 \times 0.33 / (0.103 / 3600) = 34601.9 < 3 \times 10^5$.
Thus, we have a laminar BL. Also, $Pr = 0.103 / 0.153 = 0.673$
and $Sc = 0.103 / 0.162 = 0.636$.
Hence, $St = 0.664 \times Re_L^{-0.5} \times Pr^{-0.67} = 0.00465$.
Therefore, $g^* = h_{cot} / cp_m = \rho_m U_\infty St = 1.37 \times 3.0 \times 0.00465 = 0.0191$ kg/m²-s .

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So, with this, I turn now to the next problem. Here it is. This is of evaporation and a case of high B as we shall see: Air at 1 bar and 27 degree centigrade and 90 percent RH flows over a porous flat surface which is kept wet by supplying water. The plate temperature is to be maintained at 82.5 degree centigrade. Now, when you supply water, the cooling would take place and if you want to maintain the temperature of the plate at 82.5 degree centigrade, then we may have to some heat from the rear side of the plate.

So, we want to calculate the evaporation rate and heat flux to be supplied. Now, problems of this type arise in food processing, typically where some material processing where you want to maintain the surface at a given temperature, for whatever the subsequent processing is.

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So, we are given here in this problem a plate whose length is 0.33 meters and whose width is 1 meter; the air is flowing over this plate and it is at 27 degree centigrade; 90 percent RH and the wall temperature T_w is to be 82.5 degree centigrade. We want to find out what should be the q_L to be supplied and what will be the evaporation rate N_w from this plate. So, that is really the problem.

Now, the U_∞ is given as 3 meters per second; the plate is porous and we shall calculate now. First of all, we must ensure whether the boundary layer is going to be laminar or turbulent or transitional, and so on, so forth.

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Evaporation - High B - L39($\frac{7}{16}$)

Prob: Air at 1 bar, 27°C and 90 % RH flows over a porous flat surface which is kept wet by supplying water. The plate temperature is maintained at 82.5°C by supplying heat to it from the rear side. Calculate (a) Evaporation rate and (b) Heat flux to be supplied . Given: $U_\infty = 3.0$ m/s, Length $L = 0.33$ m and Width $W = 1$ m. Take $k = 0.025 \frac{W}{m-K}$, $\rho_m = 1.37 \frac{kg}{m^3}$, $D = 0.162 \frac{m^2}{hr}$, $\alpha = 0.153 \frac{m^2}{hr}$ and $\nu = 0.103 \frac{m^2}{hr}$

Soln: In this case,
 $Re_L = 3 \times 0.33 / (0.103 / 3600) = 34601.9 < 3 \times 10^5$.
Thus, we have a laminar BL. Also, $Pr = 0.103 / 0.153 = 0.673$
and $Sc = 0.103 / 0.162 = 0.636$.
Hence, $St = 0.664 \times Re_L^{-0.5} \times Pr^{-0.67} = 0.00465$.
Therefore, $g^* = h_{cot} / cp_m = \rho_m U_\infty St = 1.37 \times 3.0 \times 0.00465 = 0.0191 \text{ kg/m}^2\text{-s}$.

So first of all we calculate the plate Reynolds number from the properties that are given here. 3 into the plate length divided by the kinematic viscosity 0.103 divided by 3600 nu is given as 0.103, which will give 34600 Reynolds number, which is less than 3 lakhs. Therefore, we expect this to be a laminar boundary layer with Prandtl number equal to 0.103 divided by 0.153 equal to 0.673. The Schmidt number will be 0.636.

Therefore, the Stanton number relationship for a laminar boundary layer is 0.664 Re L to minus 0.5 Prandtl to minus 0.67. You will recall, this was the solution we develop from similarity solutions and that would be equal to 0.00465. Since Prandtl number is very close to Schmidt number, we can readily say g star will be h cof divided by cp m which is rho m U infinity into Stanton number and that rho m is 1.37. In this case, U infinity is 0.00465 is the Stanton number and that gives us 0.0191 kilo grams per meter square second. So, this is the value of the g star which is operating at the surface.

(Refer Slide Time: 22:47)

Soln (Contd.) - 1 - L39($\frac{8}{16}$)

Now, $p_{v,\infty} = 0.9 \times p_{sat} (27^\circ C) = 0.9 \times 0.03567 = 0.0321$ bar
 and $\omega_{v,\infty} = 0.0321 / (1.61 \times 1 - 0.61 \times 0.0321) = 0.02018$.
 Similarly, $p_{sat} (82.5^\circ C) = 0.5261$ bar and $\omega_{v,w} = 0.4081$.
 Thus, $B = (0.02018 - 0.4081) / (0.4081 - 1) = 0.6554$, and

$$\frac{g}{g^*} = \frac{\ln(1+B)}{B} \times \left(\frac{Pr}{Sc}\right)^{0.33} \times \left(\frac{M_{mix,w}}{M_{mix,\infty}}\right)^{0.667}$$

where $M_{mix,w} = 24.51$ and $M_{mix,\infty} = 28.76$. Therefore,
 substitution gives $g = g^* \times 0.7039 = 0.01344$ kg/m²-s .
 Hence, evaporation rate is
 $\dot{m} = g \times B \times A_{plate} = 0.01344 \times 0.6554 \times 0.33 \times 1$
 $= 0.0029$ kg/s, or 10.46 kg/hr (Ans a) .

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So, now, in order to calculate the mass transfer rate, we must calculate the value of B. So, from the data given, p v infinity will be 0.9 into p sat at 27 degree centigrade which is 0.9 into 0.03567 bar. That would be equal to 0.03 to 1 bar. Omega v infinity, therefore, can be readily calculated from our formula. That would be equal to 0.02018. Similarly, a p sat 82.5 degree centigrade which is at the wall 0.5261 bar, and therefore, omega v w will be 0.4081. Therefore, B will be simply omega v infinity, which is 0.02018 minus 0.4081 divided by 0.4081 minus 1 equal to 0.6554. So, this is a considerably large B in an evaporation problem.

Normally evaporation problems have very small b. Therefore, we now follow the Reynolds flow model in which we are advised that g over g star ln 1 plus B should be multiplied by Prandtl by Schmidt raise to 0.33 and also the property correction M mix w divided by M mix infinity raise to 0.667.

Now, here, M mix w at the wall state knowing omega v w we can evaluate M mix w as 24.51; M mix infinity would be evaluated from this value of 0.02018 as 28.76. Therefore, substitution would give g equal to g star into 0.7039 equal to 0.01344 kg per meter square second. Hence, the evaporation rate from the plate will be g times B into area of the plate which is 0.01344 into 0.6554 into 0.33 into 1 which is the width, equal to 0.0029 kg per second or 10.46 kg per hour, would be the rate of evaporation from this porous surface.

The next thing is how we keep the surface at 82.5 degree centigrade. To do that, we will proceed to the next slide.

(Refer Slide Time: 25:06)

Soln (Contd.) - 2 - L39($\frac{9}{16}$)

In this case, since $Le \simeq 1$, we have

$$B = \frac{h_{m,\infty} - h_{m,w}}{h_{m,w} - h_{TL} + q_l/N_w} = 0.6554 \quad \text{Taking } T_{ref} = 0$$

$$h_{m,\infty} = 1.005 \times 27 + (0.875 \times 27 + 3503) \times 0.02018$$

$$= 78.12 \text{ kJ/kg}$$

$$h_{m,w} = 1.005 \times 82.5 + (0.875 \times 82.5 + 3503) \times 0.4081$$

$$= 1133.85 \text{ kJ/kg}$$

$$h_{TL} = 4.187 \times 82.5 = 345.4 \text{ kJ/kg}$$

Hence $(q_l/N_w) = -2400 \text{ kJ/kg}$. Negative sign indicates that heat is to be supplied. Thus,

$$Q_{supp} = 2400 \times 0.0029 = 6.98 \text{ kW (Ans b) .}$$

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Now, in this case, Lewis number is equal to 1. Therefore, B calculated from enthalpy will also be equal to 0.6554. Therefore, we have $h_{m,\infty} - h_{m,w}$ over $h_{m,w} - h_{TL} + q_l/N_w$ equal to 0.6554. If I take T_{ref} equal to 0, then $h_{m,\infty}$ would be simply that 78.12 because I know $h_{m,\infty}$ as 0.02018. I can calculate $h_{m,w}$ also as 1133.85 because I know $\omega_{v,w}$ and h_{TL} will be 4.187 into 82.5 equal to 345.4.

Incidentally, this 3503 should actually read as 2503; there is an error here (Refer Slide Time: 26:05). So, it should read as 2503. Hence, q_l/N_w , we will evaluate to minus 2400 kilo joules per kg. The negative sign here clearly indicates that heat must be supplied because we take q_l away from the plate into the transferred substance phase is positive. Therefore, negative sign implies that q_l is to be supplied to the plate. Therefore, q_{supp} will be simply m_w which was 0.0029 into 2400 will give me 6.98 kilowatts.

So, for this plate of 0.33 meters by 1 meter, we will need to supply about 7 kilowatts of heat to maintain it at 82.5 degree centigrade, which is the requirement - the process requirement.

(Refer Slide Time: 27:04)

Evaporation from a Lake - L39 $\left(\frac{10}{16}\right)$

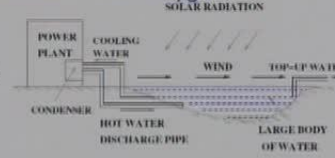
Prob: A 10 kmph breeze (at 40°C and 20 % RH) blows over a lake (at 30°C). The lake receives $q_{solar} = 500 \frac{W}{m^2}$. Calculate the time required for the water level to drop by 1 cm . Assume turbulent boundary layer. Then, assuming $Le \approx 1$. Take $\nu = 15 \times 10^{-6} m^2/s$. $Sc = 0.61$ and $Pr = 0.7$.

Soln: Here
 $p_{v,\infty} = 0.2 \times p_{sat} (40^\circ C)$
 $= 0.2 \times 0.07384$
 $= 0.01477$ bar. Therefore,
 $\omega_{v,\infty} = \frac{0.01477}{1.61 - 0.61 \times 0.01477}$
 $= 0.00919$.

Then, assuming $Le \approx 1$

$$B = \frac{\omega_{v,\infty} - \omega_{v,w}}{\omega_{v,w} - 1} = \frac{h_{m,\infty} - h_{m,w}}{h_{m,w} - h_{TL} + q_l / N_w}$$

where $\omega_{v,w}$, $h_{m,w}$ and q_l are not known.



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Let us now turn to the next problem. 10 kilometers per hour breeze at 40 degree centigrade and 20 percent RH, a situation something like the middle of India say Nagpur or somewhere, blows over a lake whose water temperature is at 30 degree centigrade. The lake also receives solar radiation of average 500 Watts per meter square average for the day. So, calculate the time required for the water level in the lake to drop by 1 centimeter. You are told that as the wind blows over the lake, assume it to be a turbulent boundary layer throughout.

You can see that this is a typical problem: You have a power plant close to the lake and you are drawing water from the lake for cooling in the condenser and the hot water again comes back here, which mixes with the lake, but the lake water is continuously dropping. Therefore, you need to do topping up of this water from some other source nearby and typically the ground water source from nearby so that the lake level does not drop too much and your power plant cooling operations are not hampered.

So, you are told to assume turbulent boundary layer throughout and take kinematic viscosity to 15 raise into 10 raise to minus 6 Schmidt number equal to 0.67 and Prandtl equal to 0.7.

So, in this case, corresponding to 40 degree centigrade and 20 percent RH, we would evaluate $p_{v,\infty}$ equal to 0.2 into saturation pressure which is 0.07384 bar, and that

will give us 0.01477 bar. Therefore, ω_v infinity would evaluate to 0.00919. Therefore, taking Lewis number as equal to 1 which is fair assumption to start with, and if b is small, then it does not really matter too much. Then ω_v infinity divided by ω_v w minus 1 h_m infinity minus h_m w h_m w minus h_T plus q_l by N_w .

Now, here, we do not know ω_v w that is the mass fraction at the surface of the lake. Therefore, we also do not know the mixture enthalpy at the surface of the lake, nor do we know q_l . So, we have 3 unknowns.

(Refer Slide Time: 29:32)

Soln (Contd.) - 1 - L39($\frac{11}{16}$)
 But, $N_w h_T + q_{rad} + q_l = N_w h_{TL}$. Hence,

$$B_h = \frac{h_{m,\infty} - h_{m,w}}{h_{m,w} - h_T - q_{rad}/N_w}$$
 Now, $N_w = g \times B_h$. Hence

$$B_h = \frac{h_{m,\infty} - h_{m,w} + q_{rad}/g}{h_{m,w} - h_T} = \frac{\omega_{v,\infty} - \omega_{v,w}}{\omega_{v,w} - 1}$$
 where taking $T_{ref} = 0$

$$h_{m,\infty} = 1.005 \times 40 + (0.875 \times 40 + 2503) \times 0.00919$$

$$= 63.3 \text{ kJ/kg}$$

$$h_{m,w} = 1.005 T_w + (0.875 T_w + 2503) \omega_{v,w}$$

$$h_T = 4.187 \times 30 = 125.4 \text{ kJ/kg}$$

So, $N_w h_T$ plus q_{rad} plus q_l is equal to $N_w h_{TL}$. This would be the balance in the neighboring phase as usual. Therefore, B_h would be h_m infinity minus h_m w over h_m w minus h_T divided by q_{rad} by N_w . N_w will be simply g into B_h , and therefore, B_h would be h_m infinity minus h_m w q_{rad} by g divided by h_m infinity by h_T equal to this value (Refer Slide Time: 30:06). What I have done here is I have replaced N_w equal to $g B_h$ and then multiplied through so that $B_h B_h$ gets cancel and q_{rad} appears in the numerator.

So, if I now take T_{ref} equal to 0, then h_m infinity would be 1.005 into 40 plus 0.875 into 40 plus 2503 into that ω_v infinity value which is 63.3. I do not know the surface of the lake temperature. Therefore, I just keep it 1.005 T_w plus 0.875 T_w plus

2503 $\omega_{v,w}$ will be simply 4.187 into 30 which is the temperature deep inside the lake and that is 125.4.

(Refer Slide Time: 31:02)

Soln (Contd.) - 2 - L39 $\left(\frac{12}{16}\right)$
 Here, $U_\infty = 10 \text{ kmph} = 2.78 \text{ m/s}$. Hence,
 $Re_L = 2.78 \times 100/15 \times 10^{-6} = 18533.3$.
 If we assume that $B \rightarrow 0$, then $g \rightarrow g^*$ and for a turbulent BL,

$$\frac{g^*}{\rho_m U_\infty} = 0.0365 Re_L^{-0.2} Pr^{-0.67} = 0.001617$$

where $\rho_m = \rho_v + \rho_a = \rho_a (1 + W_{mean})$ and
 $g^* = 0.00449 \rho_m$ or $\frac{q_{rad}}{g^*} = \frac{0.5}{(0.00449 \rho_m)} = \left(\frac{111.36}{\rho_m}\right) \text{ kJ/kg}$.
 Therefore, substitution gives

$$B = \frac{\omega_{v,\infty} - \omega_{v,w}}{\omega_{v,w} - 1}$$

$$= \frac{63.3 - [1.005 T_w + (0.875 T_w + 2503) \omega_{v,w}] + \left(\frac{111.36}{\rho_m}\right)}{[1.005 T_w + (0.875 T_w + 2503) \omega_{v,w}] - 125.4}$$

where $\omega_{v,w}$ corresponds to T_w at saturation and ρ_m is evaluated from $W_{mean} = 0.5 (W_w + W_\infty)$.

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Now, here U_∞ is 10 kilometers per hour that is been given which is essentially mean 2.78 meter per second. Therefore, the Reynolds number would work out to be 18533. Therefore, we calculate for a 100 meter lake. We assume B tending to 0 and g tending to g^* and then for a turbulent boundary layer the Stanton number would be $0.0365 Re_L^{-0.2} Pr^{-0.67}$ and that would be equal to 0.001617.

Now, ρ_m - the mixture density, will be equal to ρ of the vapor plus ρ of the air, which would be ρ_a into 1 plus specific imitative mean which is the kg of dry air divided by kg of the mixture. g^* would be therefore, 0.00449 into ρ_m . So, I have multiplied here by U_∞ which is 2.78. So, g^* becomes 0.00449 into ρ_m and q_{rad} over g^* is 0.5 divided by 0.00449. This q_{rad} is 500 watts which is 0.5 kilo watts divided by ρ_m . Therefore, this will be 111.36 divided by ρ_m kilo Joules per kg.

Therefore, on substitution, we will get B which is from the mass fraction would be this and from the mixture enthalpies, it would work out with this value (Refer Slide Time: 32:29). So, we have to solve this equation iteratively where, $\omega_{v,w}$ corresponds to T_w at saturation and ρ_m is evaluated from W_{mean} equal to W_w by plus W_∞ .

Now, what you do is you assume T_w and calculate W_w , from which you calculate W_{mean} . Therefore, you can calculate ρ_{mean} . Knowing W_w , you can also calculate $\omega_{v,w}$ and then check whether the left hand side equals the right hand side or not. So, iterative calculations are required. I have already given you a correlation for evaluating $\omega_{v,w}$ from T_w .

Alternatively, you can use steam tables or a psychrometric chart; anything will do to evaluate because we are only discovering and try to discover the saturation condition at the wall temperature. So, these two left hand side and right hand side must be balanced by iteration.

(Refer Slide Time: 33:34)

Soln (Contd.) - 3 - L39⁽¹³⁾₁₆

We need trial-and-error procedure as follows.

- ➊ Assume T_w . Hence, determine $p_{v,w}$ from steam tables and evaluate $W_w = 0.622 \times p_{v,w} / p_a$. Hence, evaluate W_{mean} and T_{mean} . Use gas law to evaluate, ρ_a at T_{mean} and, hence $\rho_m = \rho_a (1 + W_{mean})$
- ➋ Substitute in the 2 eqns for B. If LHS = RHS, accept T_w and $\omega_{v,w}$.

In the present case, $T_w \simeq 39.5^\circ\text{C}$, $\omega_{v,w} = 0.043$ and $\rho_m = 1.16$ giving $B = 0.0354$ and $N_w = g^* B = 2.035 \times 10^{-4} \text{ kg / m}^2\text{-s}$, or $0.7324 \text{ kg / m}^2\text{-hr}$.

Now, for $\Delta h = 1 \text{ cm}$ drop in water level, we use mass balance $\rho_{water} \times (\Delta h / \Delta t) = N_w$.

This gives $\Delta t = 13.65 \text{ hrs (Ans)}$.

Note that the lake surface temperature is very close to the free stream air temperature.

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So, this is the trial and error procedure and I have already explained to you how the trial and error is to be performed. So, in the present case, it works out to be T_w equal to 39.5 degree centigrade which is very close to the T_∞ in fact and $\omega_{v,w}$ turns out to be 0.043 ; ρ_m turns out to be 1.16 giving B equal to 0.0354 . So, our assumption of using N_w equal to $g^* B$ here is not too bad because B itself is very small.

So, N_w will be simply g^* into B is equal to $2.035 \times 10^{-4} \text{ kg / m}^2\text{-s}$ and g^* we have already calculated. Therefore, it is 2.035 into 10 to minus $4 \text{ kg / m}^2\text{-s}$ or $0.7324 \text{ kg / m}^2\text{-hr}$.

(Refer Slide Time: 34:37)

The image shows handwritten mathematical work on a piece of paper. The equations are as follows:

$$\rho_w \cdot \frac{\Delta h A_{lake}}{\Delta t} = \dot{m}_w$$

$$\rho_w \cdot \frac{\Delta h}{\Delta t} = \frac{\dot{m}_w}{A_{lake}} = N_w$$

$$1000 \times \frac{0.01}{\Delta t} = 0.7324$$

$$\Delta t = \frac{10}{0.7324}$$

$$\Delta t = \frac{1000 \times 0.01}{0.7324} = 13.65 \text{ hrs}$$

So, therefore, the lake water will drop. In order to calculate the lake water drop, what we do is rho water into delta h into surface area, A surface area; this will be the divided by delta t will be equal to m dot w, which is the surface area of the lake; that will give us rho w into delta h by delta t equal to m dot w divided by A lake; that will be equal to N w.

So, we can now estimate since we know what the N w is. N w, we have evaluated as 0.7324. We take rho w equal to 1000 water delta h which is 1 centimeter divided by delta t. So, if I evaluate that delta t will be equal to 0.7324 divided by 10 or 0.07324. That is what you get. **or in terms hours sorry and** here it will be 0.7324 is on the right hand side; delta h is 0.01 into 1000; therefore, delta t will be 1000 into 0.01 divided by 0.7324, which gives you 13.65 hours.

So, what we have done here is that, for the whole day, over the entire period of this time we have said the solar radiation is constant, which is not the case; the solar radiation varies during the day, but we have taken an average value. We have found that time required for 1 centimeter drop of the lake would be about 13.65, which is little. Assuming sunshine hours of about 10 hours, we can say it is about 1 and half day would be required to reduce the lake water by 1 centimeter.

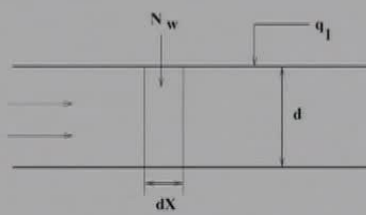
Now, such estimates are very useful for estimating the top of water required in the lake so as to maintain its level constant and not affect in anyway the cooling water provision for the power plant.

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Humidification - L39 ⁽¹⁴⁾/₁₆

Prob: Moist air ($W_m = 0.003$ at 24°C) enters a tube (2.5 cm dia, 75 cm long) at the rate of 9 kg/hr. The tube-wall is maintained wet at 24°C . Calculate (a) W_{exit} and (b) Heat to be supplied to the tube wall. Take $D = 0.09 \text{ m}^2/\text{hr}$ and $Sc = 0.6$ Assume Temp remains constant.

Soln: Part (a) In this case $\omega_{v,w}$ is constant with x.



We define $B_x = (\omega_{v,x} - \omega_{v,w}) / (\omega_{v,w} - 1)$. Then, $d\omega_{v,x} = (\omega_{v,w} - 1) dB_x$. For the differential element dx $N_{w,x} dx = (\dot{m} / \pi d) \times d\omega_{v,x}$ or $g B_x dx = (\dot{m} / \pi d) (\omega_{v,w} - 1) dB_x$ or, (next slide)

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We now look at the final problem. This time it is an internal duct flow problem. So, moist air at inlet specific humidity of 0.003 at 24 degree centigrade enters a tube which is 2.5 centimeter diameters and 0.75 meters long; I beg your pardon is 0.75 centimeter, but it is actually 0.75 meters long. At the rate of 9 kg per hour. The tube wall is maintained at 24 degree centigrade. So it is the same temperature as this. Calculate the specific humidity at exit from the tube and the heat supplied to the tube wall, to maintain its temperature at 24 degree centigrade.

Take D equal to 0.09 meter square per hour diffusivity and Schmidt number equal to 0.6; assume temperature remains constant throughout. So, even at exit, the temperature is going to be 24. So, in this case, $\omega_{v,w}$ is constant with x because T w is constant with x. Therefore, if we define B x as I said in internal flow, $\omega_{v,x}$ will be the bulk value of the mass fraction minus $\omega_{v,w}$ divided by $\omega_{v,w} - 1$.

So, if I take a differential, $d\omega_{v,x}$ will be equal to $\omega_{v,w} - 1$ into dB_x . If I now look at this small differential element of length dx , then mass transfer N_w multiplied by the perimeter which I take as πd and area multiply by

dx will be the area. So, $N w dx$ will be equal to \dot{m} into change in mass fraction, but the perimeter value here is taken as πd in the denominator here. So, $N w dx$ into \dot{m} by $\pi d d \omega_{v,w} x$; if I substitute this expression for $\omega_{v,w} x$, I get an $N w$ equal to $g B_x$. Then I get $g B_x dx$ equal to \dot{m} divided by $\pi d \omega_{v,w} w - 1 dB$.

(Refer Slide Time: 39:14)

Soln (Contd.) - 1 - L39(¹⁵/₁₆)

$$\frac{dB_x}{B_x} = \left\{ \frac{\pi d g}{\dot{m}(\omega_{v,w} - 1)} \right\} dx \rightarrow \frac{B_{out}}{B_{in}} = \exp \left\{ \frac{\pi d g L}{\dot{m}(\omega_{v,w} - 1)} \right\}$$

where L = tube length. In this problem, corresponding to 24°C, $\omega_{v,w} = 0.01875$ ($W_w = 0.01632$) and corresponding to $W_{in} = 0.003$, $\omega_{v,in} = 0.002991$. Therefore $B_{in} = 0.01606$.

Determination of g: At inlet, $W_m = 0.5 (W_w + W_{in}) = 0.01088$ and at 24°C, $\rho_a = 1.189$. Therefore $\rho_m = \rho_a (1 + W_m) = 1.202$. Further, $\nu_m = Sc \times D = 0.054 \text{ m}^2/\text{hr} = 1.5 \times 10^{-5} \text{ m}^2/\text{s}$. Also, $\dot{m} = u_{in} \times (\pi/4) d^2$ or $u_{in} = 5.09 \text{ m/s}$. Hence, $Re = u_{in} d / \nu_m = 8488.3$. Now, taking $Le = 1$, $Sh = g d / (\rho_m D) = 0.023 Re^{0.8} Sc^{0.4} = 26.06$ or, $g = 112.76 \text{ kg/m}^2\text{-hr}$. Substitution gives $B_{out} = 0.00757$.

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In the next slide, you see dB_x by B_x would be equal to $\pi dg \dot{m} w \omega_{v,w} - 1$ into dx . If I integrate this equation from inlet to outlet, then this will give me logarithm of B_{out} over B_{in} equal to that expression into L . So, that is simply becomes $\pi dgL \dot{m} \omega_{v,w} - 1$ with an exponential sign. Since $\omega_{v,w}$ is going to be less than 1, this quantity is negative. Therefore, exponential of a negative is a fraction. Therefore, B_{out} will be less than B_{in} and L is the length of the tube.

So, in this problem, corresponding to 24 degree centigrade $\omega_{v,w}$ is 0.01875; W_w is 0.01632 and corresponding to W_{in} equal to 0.003, $\omega_{v,in}$ will be 0.002991, and therefore, B_{in} is 0.01606.

We now determine g . Now, at the inlet, the mean value of the specific humidity will work out to be 0.01088 because we know W_w and we know W_{in} . At 24 degree centigrade, ρ_a is 1.189; therefore, ρ_m will be one ρ_a into 1 plus W_m equal to 1.202; ν_m will be Schmidt number into D which is 0.054 meter square per hour, or 1.5 into 10 raise to minus 5 meter square per second.

Therefore, now the mass flow rate is \dot{m} into u in into π by $4 d$ square. Therefore, u in will be 5.09 meters per second. Therefore, I evaluate Reynolds number as 8488.

Now, if I take Lewis number equal to almost 1, then I can evaluate Sherwood number from the Dittus Boelter relationship as $0.023 \text{ Reynolds}^{0.8} \text{ Schmidt}^{0.4}$ as 26.06, which gives me g equal to 112.76. Therefore, the substitution in here; Now, I know π I know the diameter of the tube I know g now which I have evaluated at 112.76 kg per meter square hour; \dot{m} is 9 L is 0.75 meters and $\omega_{v,w}$ is already known as 0.01875, which gives me B_{out} equal to 0.00757. We had evaluated B_{in} equal to 0.01606. So, I can see that B_{out} is actually smaller than B_{in} .

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Soln (Contd.) - 2 - L39(16/16)

Hence, $\omega_{v,out} = \omega_{v,w} + B_{out} (\omega_{v,w} - 1) = 0.01137$
 or $W_{out} = 0.01145 \text{ kg / kg of dry air (Ans)}$.
 Now Energy balance over element dx gives
 $\dot{m} \times dh_{m,x} = (N_{w,x} h_{TL} - q_{l,x}) \times \pi d dx$. Integration gives

$$-\bar{q}_l = -\frac{1}{L} \int_0^L q_{l,x} dx$$

$$= \left(\frac{\dot{m}}{\pi d} \right) \int_0^L \frac{d h_m}{dx} dx - g h_{TL} \int_0^L B_x dx$$

$$= \frac{\dot{m}}{\pi d} [h_{m,out} - h_{m,in} - h_{TL} (\omega_{v,out} - \omega_{v,in})]$$

where taking $T_{ref} = 0$, $h_{m,out} = 52.82$, $h_{m,in} = 31.664$
 and $h_{TL} = 4.187 \times 24 = 100.49 \text{ kJ/kg}$.
 Hence $\bar{q}_l = -0.646 \text{ kW/m}^2 \text{ (Ans)}$

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Now, $\omega_{v,out}$ will be $\omega_{v,w}$ plus B_{out} $\omega_{v,w}$ minus 1 from the definition of B_{out} and it will be 0.01137. Therefore, the specific humidity at the outlet would be 0.01145 because knowing $\omega_{v,out}$, I can evaluate W_{out} . So, you can see whereas W_{in} was 0.003. As you can see the specific humidity at inlet was 0.03. But now, the air has been humidified and the specific humidity at outlet is 0.01145. So, air has indeed picked up moisture from the surface.

Now, if I do energy balance, then you will see \dot{m} into change in enthalpy would be equal to the mass transfer from the surface h_{TL} minus q_l which is the conduction heat

transfer or the heat away from the wall into $\pi d dx$ which is the perimeter of the tube, into dx .

So, if I integrate this expression for $q_l x$, then I will get $-\bar{q} l$ equal to $-\int_0^L \frac{1}{L} q_l x dx$; it will be $m \dot{m} \pi d \int_0^L h_m dx$ minus $g h_{TL} \int_0^L B x dx$; it will give me $m \dot{m} \pi d \int_0^L h_m dx$ minus $h_{m in} - h_{m TL} \omega v_{out} - \omega v_{in}$.

Therefore, you will see, taking T_{ref} equal to 0 $h_{m out}$ will work out to 52.82 because we know T_{out} is also 24 degree centigrade. We already know ωv_{out} . So, we can work out $\omega h_{m out}$; we can work out $h_{m in}$ which is the inlet condition and h_{TL} is 4.187 into 24 . Therefore, 100.49 and that gives me $\bar{q} l$ equal to $-\text{minus } 0.646$ kilo Watts per meter square.

Negative sign indicates that the heat must be supplied to the tube. So, this is the second answer to our problem. So,, we do find that in the tube the specific humidity is increased and the temperature has been given as constant; there is no change in temperature of the air, in which case you will have to supply heat to the walls so that it is maintained at 24 degree centigrade and the value has been found.

So, you can see, we have used psychrometry in variety of applications like: wet bulb thermometer; mass transfer from a porous surface which is to be kept at a specified temperature; we have looked at flow humidification of air and tube which has to be kept at a specified wall temperature; we also looked at amount of water depletion that would take place due to evaporation, convective evaporation due to wind as in the presence of solar energy.

So, a variety of problems can be solved by using Reynolds flow model and you can see it is a very convenient tool. We have not solved any differential equation; we have simply made use of the available correlations to get the appropriate value of g_{star} in each case.