Convective Heat and Mass Transfer

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Lecture No. # 38

CONV – MT Couette Flow Model

We shall take up the application of the Couette flow model. Couette flow model is a proxy for boundary layer flow model, but with a very drastic assumption, but which, nonetheless helps us to assess, what is the effect of property variations on mass transfer.

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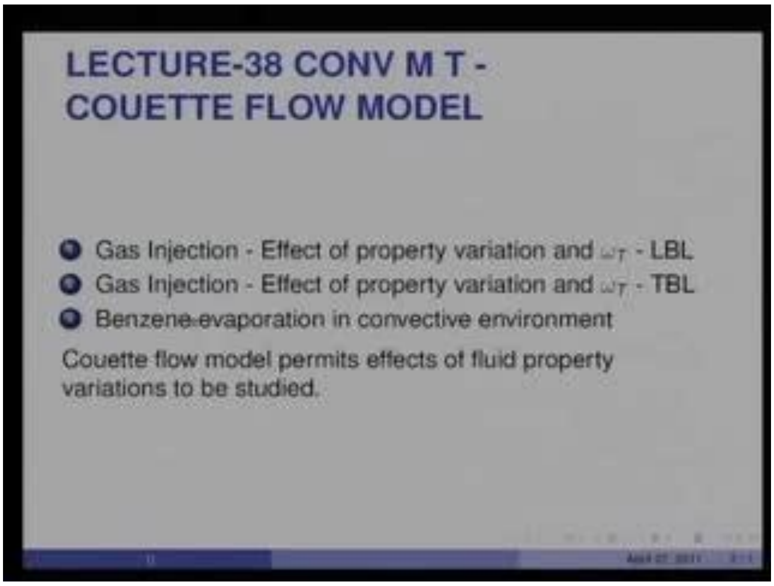

LECTURE-38 CONV M T - COUETTE FLOW MODEL

- Gas Injection - Effect of property variation and ω_T - LBL
- Gas Injection - Effect of property variation and ω_T - TBL
- Benzene evaporation in convective environment

Couette flow model permits effects of fluid property variations to be studied.

So, in effect you recall that, we said that the recommended practice for property correction is to take the ratio of the molecular weight at the w state and an infinite state and raise it to a power. Today, we are going to look at that proposition whether there is any theoretical justification for that, so what we will do is, we will consider a mass transfer situation in which a certain gas is being injected into the laminar boundary layer of air, and in the t state we have the gas, the gas may be pure in which case omega t will be 1, or it may be a mixture of air and the gas in which case omega t will be diffraction; we do the same thing for a turbulent boundary layer.

And in both these I shall show that the analytically derived result will actually show that, the mass transfer coefficient g is in fact influenced by the molecular weight at infinity divided by the molecular weight at w state, and then finally, we will solve a practical problem in which there is a benzene evaporation in a convective environment. So, the main objective of application of the convective of the Couette flow model to convective mass transfer is really to study effect of fluid property variations.

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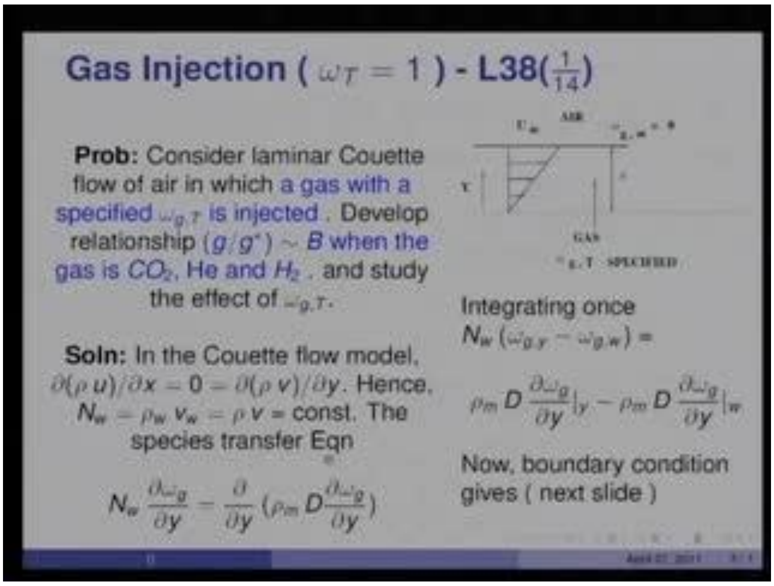

Gas Injection ($\omega_T = 1$) - L38($\frac{1}{14}$)

Prob: Consider laminar Couette flow of air in which a gas with a specified $\omega_{g,T}$ is injected . Develop relationship $(g/g^*) \sim B$ when the gas is CO_2, He and H_2 , and study the effect of $\omega_{g,T}$.

Soln: In the Couette flow model, $\partial(\rho\, u)/\partial x = 0 = \partial(\rho\, v)/\partial y$. Hence, $N_w = \rho_w\, v_w = \rho\, v$ = const. The species transfer Eqn

$$N_w \frac{\partial \omega_g}{\partial y} = \frac{\partial}{\partial y}(\rho_m D \frac{\partial \omega_g}{\partial y})$$

Integrating once
$N_w\,(\omega_{g,y} - \omega_{g,w})$ =

$$\rho_m D \frac{\partial \omega_g}{\partial y}|_y - \rho_m D \frac{\partial \omega_g}{\partial y}|_w$$

Now, boundary condition gives (next slide)

So, let us consider the first case of a…, let us say laminar Couette flow of air as it shown here, and in Couette flow as you know the velocity profile is assume linear with velocity profile in u infinity at a distance delta from the wall, there is a gas being injected here at the wall.

So, consider a laminar Couette flow of air in which a gas with a specified omega gas t here, omega t is injected, develop the relationship g by g star as a function of b when the gas is either c o 2 or helium or hydrogen. Now, why we are chosen this is that the three gases is that, molecular weight of c o 2 is, of course, 44, whereas molecular weight of air is 29, molecular weight of helium is 4, and that of hydrogen is 2.

So, you can we have gases whose molecular weight is both higher or lower as well as lower than that of air, so this is this will be a good case to see how the how do this property variations affect g over g star. So, in the Couette flow model all derivatives with respect to x are 0, so the mass conservation equation will give d rho u by d x equal to 0, which implies that d rho v by d y equal to 0, and which means that rho v is a constant with respect to y rho into v is constant with respect to y.

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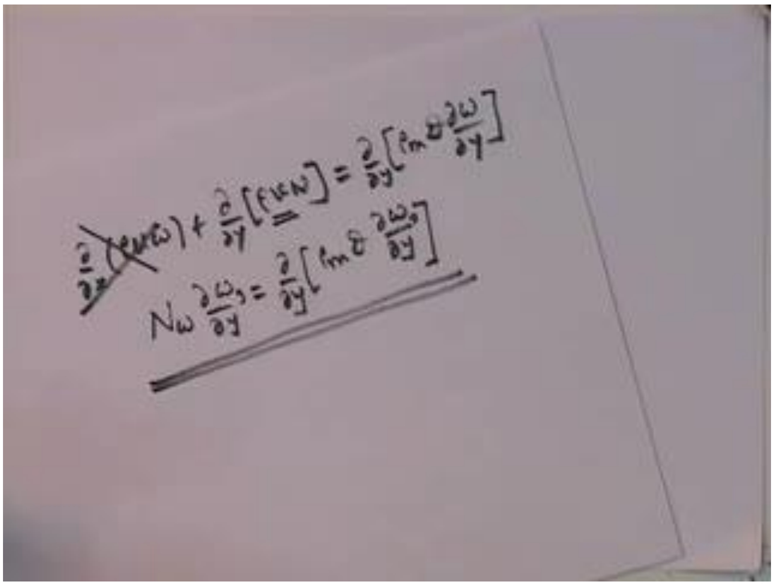

$$\cancel{\frac{\partial}{\partial x}(\rho u \omega)} + \frac{\partial}{\partial y}\left[\underline{\underline{\rho v}}\,\omega\right] = \frac{\partial}{\partial y}\left[\rho_m D \frac{\partial \omega}{\partial y}\right]$$

$$N_w \frac{\partial \omega_g}{\partial y} = \frac{\partial}{\partial y}\left[\rho_m D \frac{\partial \omega_g}{\partial y}\right]$$

And therefore, n w the mass flux at the wall will be rho w v w equal to rho v equal to constant; and similarly, the species transfer equation which as you now is d by d x of rho u omega plus d by d y of rho v omega equal to d by d y of rho m d d omega by d y, and since there is no chemical reaction, there is no r term here, but since all the gradients here are neglected in the x direction, therefore, d by d x term vanishes, and rho v is a constant equal to n w; and therefore, we will have n w equal to d omega by d y equal to d by d y of rho m d ah d omega by d y that would be the scalar transport equation for the species omega g, the gas species being injected.

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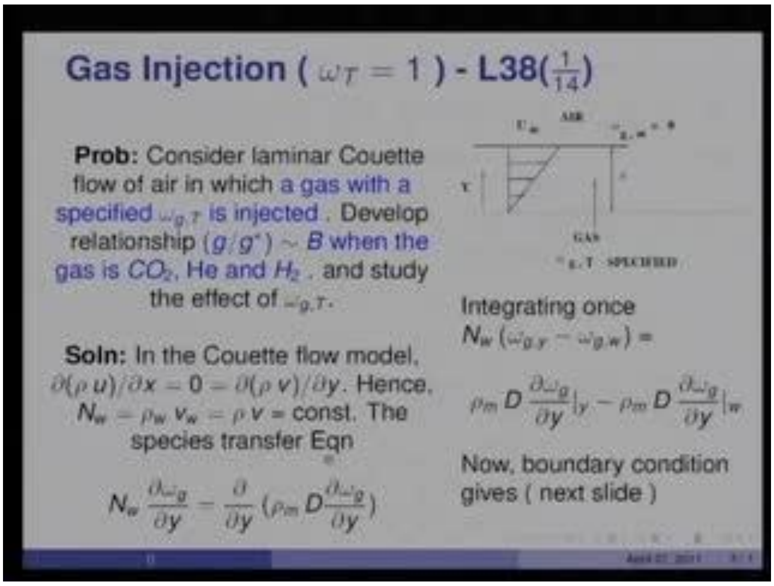

Gas Injection ($\omega_T = 1$) - L38($\frac{1}{14}$)

Prob: Consider laminar Couette flow of air in which a gas with a specified $\omega_{g,T}$ is injected . Develop relationship $(g/g^*) \sim B$ when the gas is CO_2, He and H_2 , and study the effect of $\omega_{g,T}$.

Soln: In the Couette flow model, $\partial(\rho\, u)/\partial x = 0 = \partial(\rho\, v)/\partial y$. Hence, $N_w = \rho_w\, v_w = \rho\, v =$ const. The species transfer Eqn

$$N_w \frac{\partial \omega_g}{\partial y} = \frac{\partial}{\partial y} (\rho_m D \frac{\partial \omega_g}{\partial y})$$

Integrating once
$N_w\, (\omega_{g,y} - \omega_{g,w}) =$

$$\rho_m D \frac{\partial \omega_g}{\partial y}|_y - \rho_m D \frac{\partial \omega_g}{\partial y}|_w$$

Now, boundary condition gives (next slide)

N w d omega g by d y equal to d by d y rho m d d omega g by d y, should I integrate that from 0 to y, then you will see the left hand side will simply integrate to n w into omega g y minus omega g at w, because n w is constant. And this term will integrate to rho m d omega g by d y at y minus rho m d omega g by d y at the wall w; now, we will replace this from the boundary condition at the wall, and that is what is shown on the next slide. So, we know that n w is rho m d omega g d omega g by d y at w divided by omega g w minus omega g t, and therefore, that quantity is simply n w into omega g w minus omega g t.

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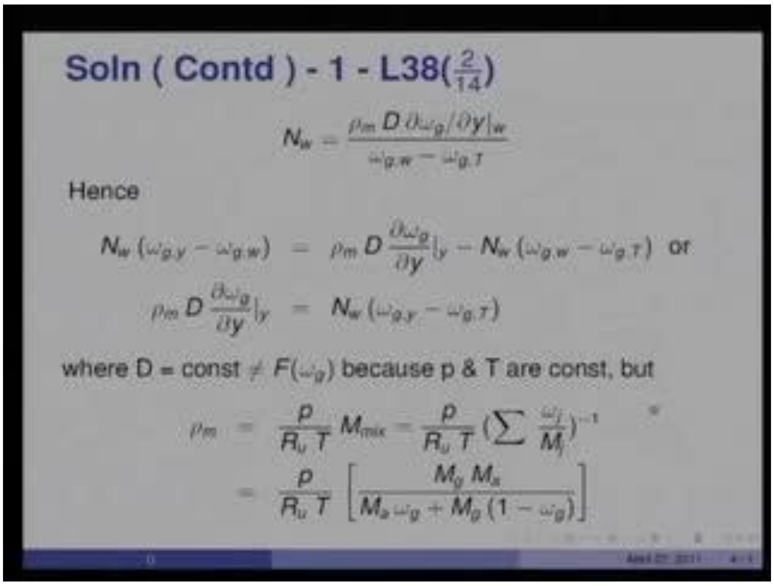

Soln (Contd) - 1 - L38($\frac{2}{14}$)

$$N_w = \frac{\rho_m D\, \partial \omega_g / \partial y|_w}{\omega_{g,w} - \omega_{g,T}}$$

Hence

$$N_w\,(\omega_{g,y} - \omega_{g,w}) = \rho_m D \frac{\partial \omega_g}{\partial y}|_y - N_w\,(\omega_{g,w} - \omega_{g,T}) \quad \text{or}$$

$$\rho_m D \frac{\partial \omega_g}{\partial y}|_y = N_w\,(\omega_{g,y} - \omega_{g,T})$$

where D = const $\neq F(\omega_g)$ because p & T are const, but

$$\rho_m = \frac{p}{R_u\,T} M_{mix} = \frac{p}{R_u\,T} (\sum \frac{\omega_j}{M_j})^{-1}$$

$$= \frac{p}{R_u\,T} \left[\frac{M_g\,M_a}{M_a\,\omega_g + M_g\,(1 - \omega_g)}\right]$$

So, if I substitute that in this expression here for that, then you will see i will get n w into omega g y minus omega g w equal to the gradient diffusion here at any y minus this quantity which is n w into that denominator. So, you will see n omega g w gets cancelled on both sides, and therefore I have rho m d d omega g by d y at any y equal to n w into omega g y minus omega g t.

Now, here d is constant, we assume that the diffusivity is not going to be function of omega g, also the pressure and temperatures are absolutely constant, and therefore there is no which means this is a case of a pure injection of a gas, there is no variation of temperature or pressure in the y direction, and therefore d will be constant. But the density will be function of the mass fractions, because rho m is equal to p by r u t into molecular weight of the mixture, and the molecular weight of the mixture is given by omega g over m g raise to minus 1.

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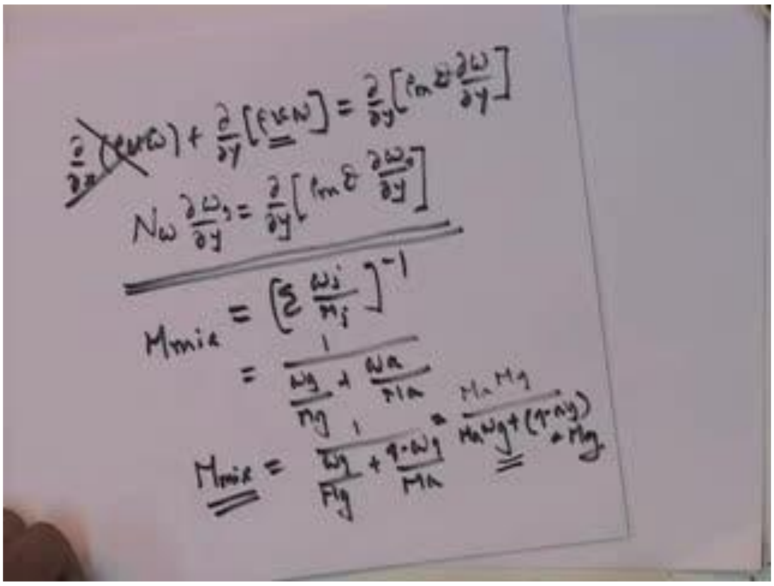

$$\cancel{\frac{\partial}{\partial x}(\rho u \omega)} + \frac{\partial}{\partial y}[\rho v \omega] = \frac{\partial}{\partial y}\left[\rho_m D \frac{\partial \omega}{\partial y}\right]$$

$$N_w \frac{\partial \omega_g}{\partial y} = \frac{\partial}{\partial y}\left[\rho_m D \frac{\partial \omega_g}{\partial y}\right]$$

$$M_{mix} = \left[\sum \frac{\omega_j}{M_j}\right]^{-1} = \frac{1}{\frac{\omega_g}{M_g} + \frac{\omega_a}{M_a}}$$

$$M_{mix} = \frac{1}{\frac{\omega_g}{M_g} + \frac{1-\omega_g}{M_a}} = \frac{M_a M_g}{M_a \omega_g + (1-\omega_g) M_g}$$

Basically what I am I am saying is M mix sigma omega j m j raise to minus 1, and therefore that will be equal to 1 over [om/omega] omega g divided by m g plus omega air divided by m air, and that would be nothing but omega g over m g plus omega 1 minus omega g over m a, which will transfer to m a m g divided by m a omega g plus 1 minus omega g into m g.

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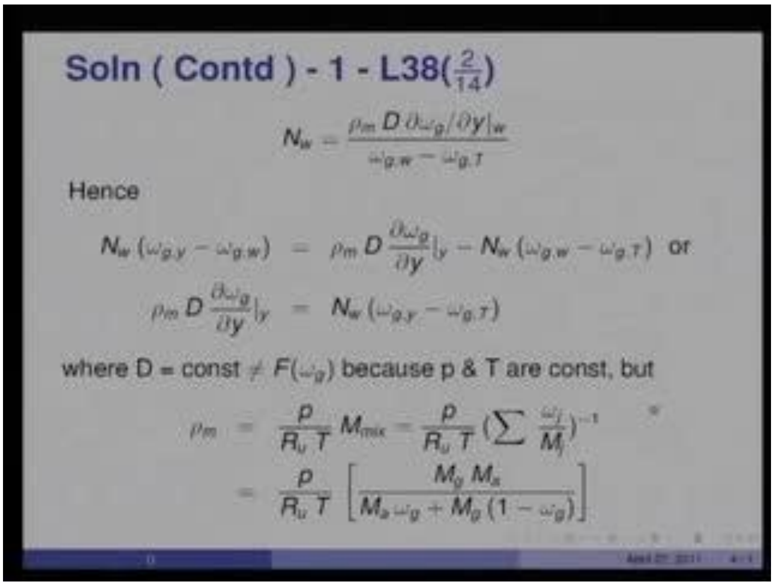

Soln (Contd) - 1 - L38$(\frac{2}{14})$

$$N_w = \frac{\rho_m D\, \partial \omega_g / \partial y|_w}{\omega_{g,w} - \omega_{g,T}}$$

Hence

$$N_w\,(\omega_{g,y} - \omega_{g,w}) = \rho_m D \frac{\partial \omega_g}{\partial y}|_y - N_w\,(\omega_{g,w} - \omega_{g,T}) \quad \text{or}$$

$$\rho_m D \frac{\partial \omega_g}{\partial y}|_y = N_w\,(\omega_{g,y} - \omega_{g,T})$$

where D = const $\neq F(\omega_g)$ because p & T are const, but

$$\rho_m = \frac{p}{R_u T} M_{mix} = \frac{p}{R_u T} \left(\sum \frac{\omega_j}{M_j}\right)^{-1}$$

$$= \frac{p}{R_u T} \left[\frac{M_g M_a}{M_a \omega_g + M_g (1 - \omega_g)}\right]$$

So, that will be the mixture molecular weight which will vary with omega g in this manner, p by r u t remains as it is p and t are constant, and m a m g divided by m a omega g plus m g 1 minus omega g; so, should I substitute for this rho m here and then integrate, then you will see this is what would happen.

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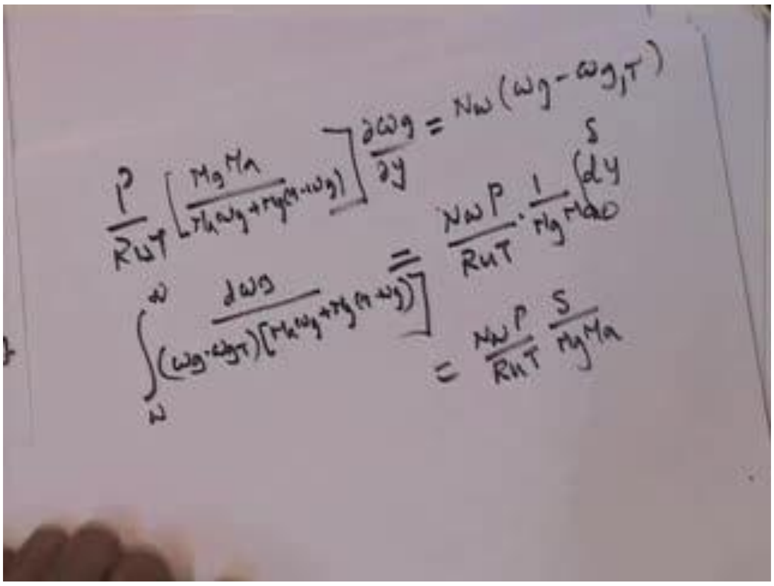

$$\frac{P}{R_u T}\left[\frac{M_g M_a}{M_a\omega_g + M_g(1-\omega_g)}\right]\frac{\partial \omega_g}{\partial y} = N_w(\omega_g - \omega_{g,T})$$

$$\int_{w}^{\infty} \frac{d\omega_g}{(\omega_g-\omega_{g,T})\left[M_a\omega_g + M_g(1-\omega_g)\right]} = \frac{N_w P}{R_u T}\cdot\frac{1}{M_g M_a}\int_0^\delta dy$$

$$= \frac{N_w P}{R_u T}\frac{\delta}{M_g M_a}$$

Basically, you will see that, what I have got here is into m g m a divided by m a omega g plus m g 1 minus omega g into d omega g by d y equal to n w into omega g minus omega g t; and therefore, should I reorganize this equation you will see I will get d omega g divided by omega g minus omega g t into m a omega g plus m g 1 minus omega g equal to n w p by r u t into 1 over m g m a.

So, that is what I have got. Now, I am going to integrate this from w to infinity state, and likewise this one from w to well d y here from 0 to delta, the integration will go from 0 to delta, and this will then become n w p r u t delta divided by m g m a, so that is what you will see on the slide here.

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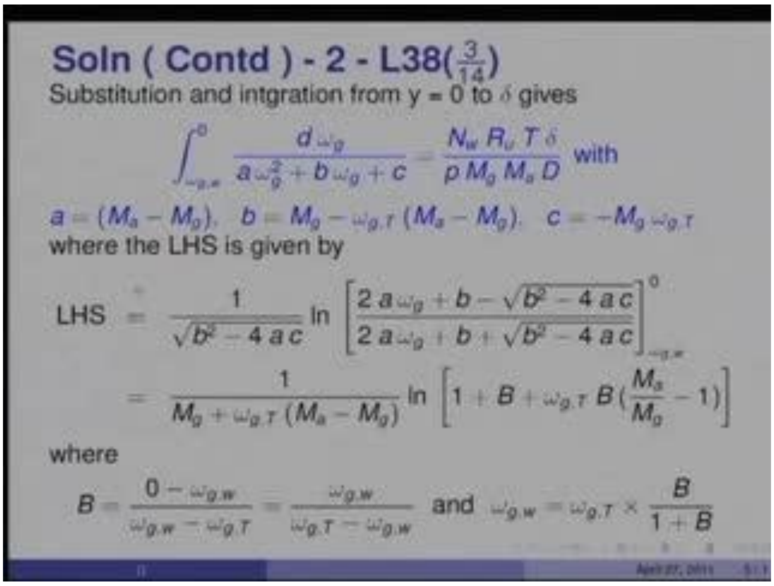

Soln (Contd) - 2 - L38($\frac{3}{14}$)

Substitution and intgration from y = 0 to δ gives

$$\int_{\omega_{g,w}}^{0} \frac{d\,\omega_g}{a\,\omega_g^2 + b\,\omega_g + c} = \frac{N_w\,R_u\,T\,\delta}{p\,M_g\,M_a\,D} \text{ with}$$

$a = (M_a - M_g)$, $b = M_g - \omega_{g,T}\,(M_a - M_g)$, $c = -M_g\,\omega_{g,T}$

where the LHS is given by

$$\text{LHS} = \frac{1}{\sqrt{b^2 - 4\,a\,c}} \ln\left[\frac{2\,a\,\omega_g + b - \sqrt{b^2 - 4\,a\,c}}{2\,a\,\omega_g + b + \sqrt{b^2 - 4\,a\,c}}\right]_{\omega_{g,w}}^{0}$$

$$= \frac{1}{M_g + \omega_{g,T}\,(M_a - M_g)} \ln\left[1 + B + \omega_{g,T}\,B\,(\frac{M_a}{M_g} - 1)\right]$$

where

$$B = \frac{0 - \omega_{g,w}}{\omega_{g,w} - \omega_{g,T}} = \frac{\omega_{g,w}}{\omega_{g,T} - \omega_{g,w}} \text{ and } \omega_{g,w} = \omega_{g,T} \times \frac{B}{1+B}$$

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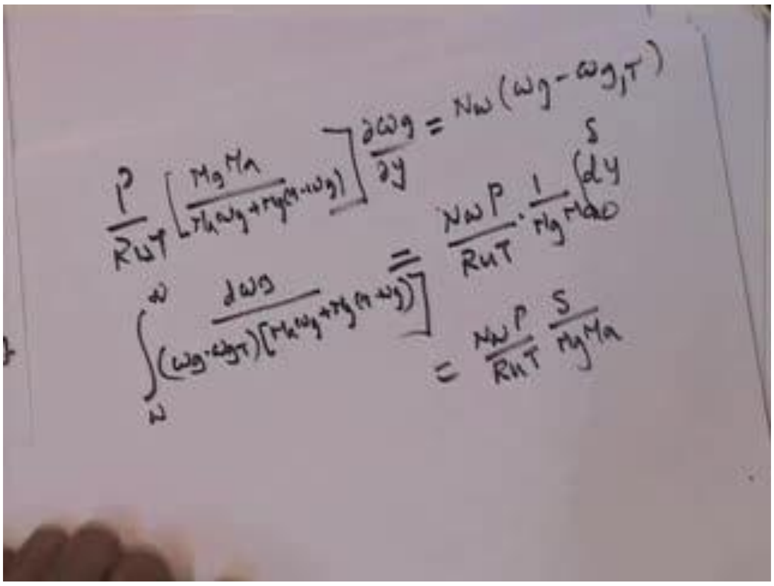

$$\frac{p}{R_u T}\left[\frac{M_g M_a}{M_a \omega_g + M_g(1-\omega_g)}\right]\frac{\partial \omega_g}{\partial y} = N_w(\omega_g - \omega_{g,T})$$

$$\int_{w}^{w} \frac{d\omega_g}{(\omega_g - \omega_{gT})\left[M_a\omega_g + M_g(1-\omega_g)\right]} = \frac{N_w P}{R_u T}\cdot\frac{1}{M_g M_a}\int^{\delta} dy$$

$$= \frac{N_w P}{R_u T}\frac{\delta}{M_g M_a}$$

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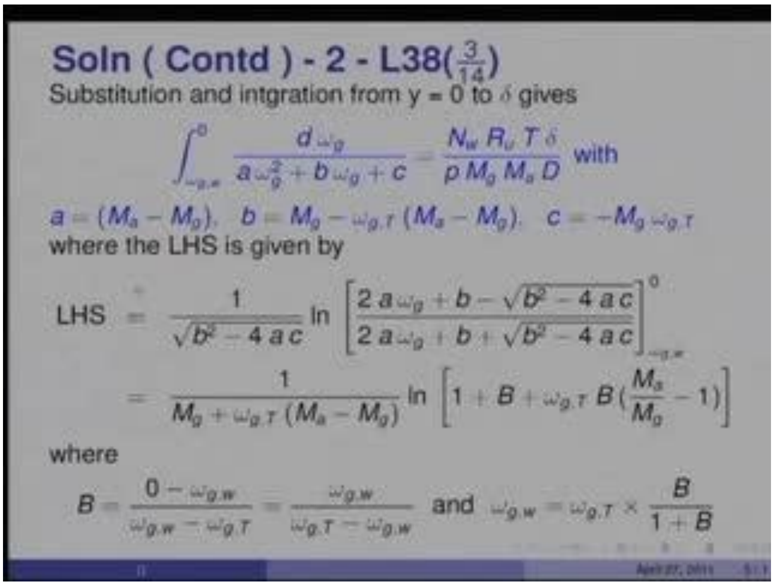

Soln (Contd) - 2 - L38($\frac{3}{14}$)

Substitution and intgration from y = 0 to δ gives

$$\int_{\omega_{g,w}}^{0} \frac{d\,\omega_g}{a\,\omega_g^2 + b\,\omega_g + c} = \frac{N_w\,R_u\,T\,\delta}{p\,M_g\,M_a\,D} \quad \text{with}$$

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where the LHS is given by

$$\text{LHS} = \frac{1}{\sqrt{b^2 - 4\,a\,c}} \ln \left[\frac{2\,a\,\omega_g + b - \sqrt{b^2 - 4\,a\,c}}{2\,a\,\omega_g + b + \sqrt{b^2 - 4\,a\,c}} \right]_{\omega_{g,w}}^{0}$$

$$= \frac{1}{M_g + \omega_{g,T}\,(M_a - M_g)} \ln \left[1 + B + \omega_{g,T}\,B\,(\frac{M_a}{M_g} - 1) \right]$$

where

$$B = \frac{0 - \omega_{g,w}}{\omega_{g,w} - \omega_{g,T}} = \frac{\omega_{g,w}}{\omega_{g,T} - \omega_{g,w}} \quad \text{and} \quad \omega_{g,w} = \omega_{g,T} \times \frac{B}{1 + B}$$

Now, the left hand side which I have shown here on in writing, this would transform to a time, this will be a quadratic like denominator a times omega g square plus b times omega g plus c, that is what it will be, and that is what i have written here; so, you can see here it will be n w r u t delta divided p m g m a d and a omega j square plus b omega g plus c, where a is equal to m a minus m g, b is equal to m g minus omega g t which is given into m a minus m g plus c is equal to minus m g omega g t.

So, the left hand side integration which is from wall state to infinity state, and in the infinity state there is no gas there is pure air, and therefore that is 0; and the left hand side is integrates to one over under root b square minus a c l n of 2 a omega g plus b minus b square minus 4 a c, and likewise here with plus sign.

Now omega g w to 0, and therefore, you get one over m g omega g t into m a minus m g l n 1 plus b plus omega g t into b m a minus m g minus 1 takes a little is…, simply substituting a b and c, and here, and what would be b in the present case omega g infinity, which is 0 minus omega g wall divided by omega g wall minus omega g t as usual, which would be nothing but omega g w divided by omega g t minus omega g wall, and what would be omega g, therefore, omega g wall itself can be given as omega g t into b over 1 plus b; we will make use of this relationship in the next slide.

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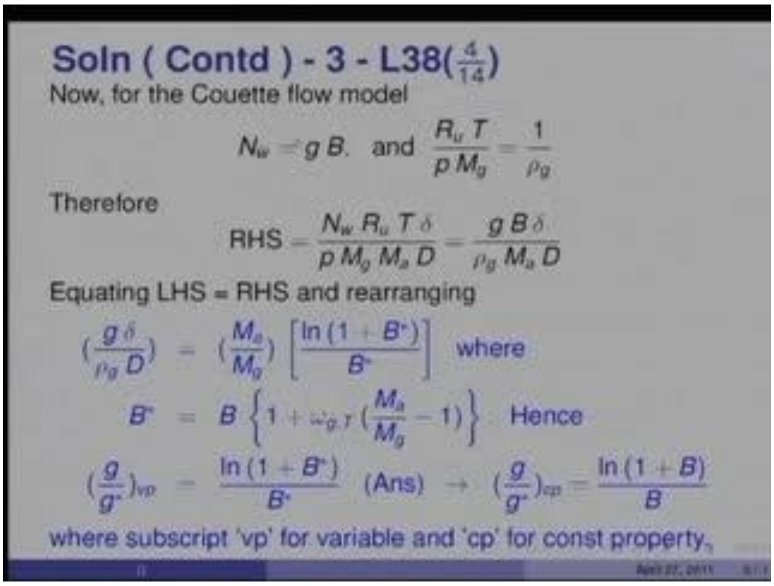

Soln (Contd) - 3 - L38($\frac{4}{14}$)

Now, for the Couette flow model

$$N_w = g\,B. \quad \text{and} \quad \frac{R_u\,T}{p\,M_g} = \frac{1}{\rho_g}$$

Therefore

$$\text{RHS} = \frac{N_w\,R_u\,T\,\delta}{p\,M_g\,M_a\,D} = \frac{g\,B\,\delta}{\rho_g\,M_a\,D}$$

Equating LHS = RHS and rearranging

$$\left(\frac{g\,\delta}{\rho_g\,D}\right) = \left(\frac{M_a}{M_g}\right)\left[\frac{\ln(1+B^*)}{B^*}\right] \quad \text{where}$$

$$B^* = B\left\{1+\omega_{g,T}\left(\frac{M_a}{M_g}-1\right)\right\} \quad \text{Hence}$$

$$\left(\frac{g}{g^*}\right)_{vp} = \frac{\ln(1+B^*)}{B^*} \quad \text{(Ans)} \rightarrow \left(\frac{g}{g^*}\right)_{cp} = \frac{\ln(1+B)}{B}$$

where subscript 'vp' for variable and 'cp' for const property.

So, if c so from the Couette flow model, we know that n w is equal to g b, and I can replace r u t by p into m g as simply 1 over rho g well known, and therefore the right hand side will simply become g b delta divided by rho g m a d rho g m a d the m g part goes out r u t goes out, and p goes out, and therefore you get and d is the diffusion coefficient which is which you said is constant.

So, if I now integrate the left hand side and right sides, then you will see I will get…, I can form reformulate this as g delta divided by rho g into diffusivity is equal to m a by m g into l n 1 plus b star over b star, where b star is actually given by b into 1 plus omega g t m a by m g minus 1, of course there is a little algebra here in rearrangement of about 5 or 6 line, and this is the modified driving force b star which is in…; if the properties were constant and we did not worry about molecular weight difference, then m a by m g would be 1 and this term would simply cancel out.

Also it shows that if omega g t was very very small, let us say very very small fraction, let us say I will explain their point one or something like that, then b star would simply equal b, so the modified b star very much depends on the concentration of the gas being injected from the t state, and therefore g over g star would simply become l n 1 plus b star over b star.

G over g star of the variable property solution would be b over b star, I have written this equation for g when v is finite and g star as you know is when v is tending to 0 or b star is tending to 0, and therefore g over g star ah of the variable property you have case would be l n 1 plus b star over b star is the answer to our problem.

Now, g over g star of the constant property case would, of course, be l n 1 plus b by b as you will recall, we have done this several times before, so here I will use v p and c p as subscripts to represent that this is a variable property solution, whereas this is a constant property solution. Now, let us see what does it imply.

So, if we take the case in which omega g t is equal to 1, then what would be the g over g star verses b relationship; so, if b is equal to 0, then in the constant property case g over g star would be 1, and so it would be in the variable property case irrespective of the gas; remember, b star is a function of the omega g t, and the molecular weight of a and the gas.

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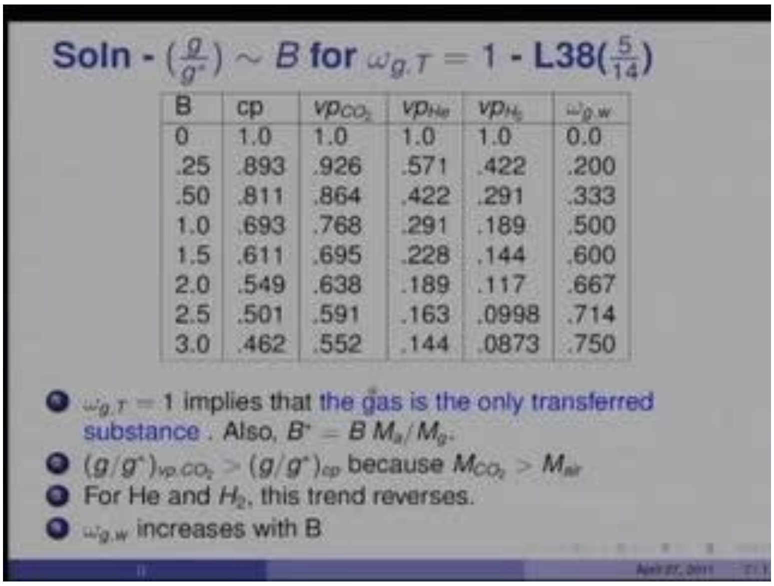

Soln - $(\frac{g}{g^*}) \sim B$ for $\omega_{g,T} = 1$ - L38$(\frac{5}{14})$

B	cp	vp_{CO_2}	vp_{He}	vp_{H_2}	$\omega_{g,w}$
0	1.0	1.0	1.0	1.0	0.0
.25	.893	.926	.571	.422	.200
.50	.811	.864	.422	.291	.333
1.0	.693	.768	.291	.189	.500
1.5	.611	.695	.228	.144	.600
2.0	.549	.638	.189	.117	.667
2.5	.501	.591	.163	.0998	.714
3.0	.462	.552	.144	.0873	.750

- $\omega_{g,T} = 1$ implies that the gas is the only transferred substance . Also, $B^* = B\, M_a/M_g$.
- $(g/g^*)_{vp,CO_2} > (g/g^*)_{cp}$ because $M_{CO_2} > M_{air}$
- For He and H_2, this trend reverses.
- $\omega_{g,w}$ increases with B

So, here I am considering the variable property solutions for carbon dioxide, helium, and hydrogen; and here I am considering the case of constant property, where of course the molecular weights do not matter. So, if b is equal to 0.25, then the constant property solution is go like that, simply l n 1 plus b by b, whereas the variable property solutions go like that, so you can see for c o 2 g over g star of the variable property is greater than g over g star of constant property through out through out.

Converse is the case when molecular weights are small that is helium has a molecular weight of 4, and you can see that g over g star on the other variable property case is less, than the values for constant property case, and that is the reason why we worry about, and you can see the difference increases as b increases; you can see it is .4144 here for helium, whereas it is .462 in factor of nearly 3, and the factor becomes even bigger when we go to hydrogen; each of this case suppose I can what is implied here is omega g w; so, when omega g t is one and b is very very small then, of course, omega g w is small, but it goes on increasing as b increases.

So, the value of omega g at the wall goes on increasing by omega g t is equal to 1; so, you will see omega g t equal to 1 implies that the gas is the only transfer substance, and and b star would then be b into m a by m g, because you will see that if omega g t is equal to 1, you will get a 1 minus 1 cancels out, and therefore you will simply get b into m a by m g, and that is what I have said here. So, b is b star is simply augmented by m a by m g, g over g star variable property of c o 2 is greater than g o g star, because m c o 2 is greater than m air, whereas for helium and hydrogen this trend reverses and omega g w increases with p, so we have shown that g over g star in a laminar boundary layer will be affected by m a by m g.

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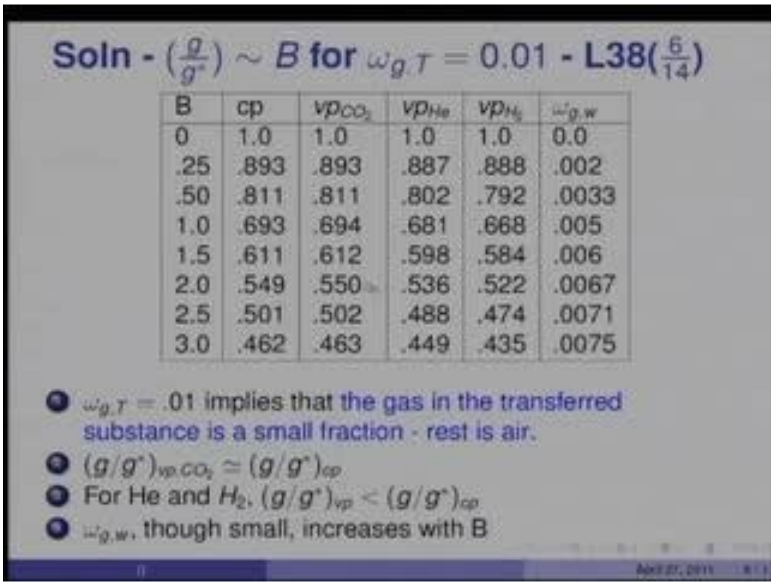

Soln - $\left(\frac{g}{g^*}\right) \sim B$ for $\omega_{g,T} = 0.01$ - L38$\left(\frac{6}{14}\right)$

B	cp	vp_{CO_2}	vp_{He}	vp_{H_2}	$\omega_{g,w}$
0	1.0	1.0	1.0	1.0	0.0
.25	.893	.893	.887	.888	.002
.50	.811	.811	.802	.792	.0033
1.0	.693	.694	.681	.668	.005
1.5	.611	.612	.598	.584	.006
2.0	.549	.550	.536	.522	.0067
2.5	.501	.502	.488	.474	.0071
3.0	.462	.463	.449	.435	.0075

- $\omega_{g,T} = .01$ implies that the gas in the transferred substance is a small fraction - rest is air.
- $(g/g^*)_{vp,CO_2} \simeq (g/g^*)_{cp}$
- For He and H_2, $(g/g^*)_{vp} < (g/g^*)_{cp}$
- $\omega_{g,w}$, though small, increases with B

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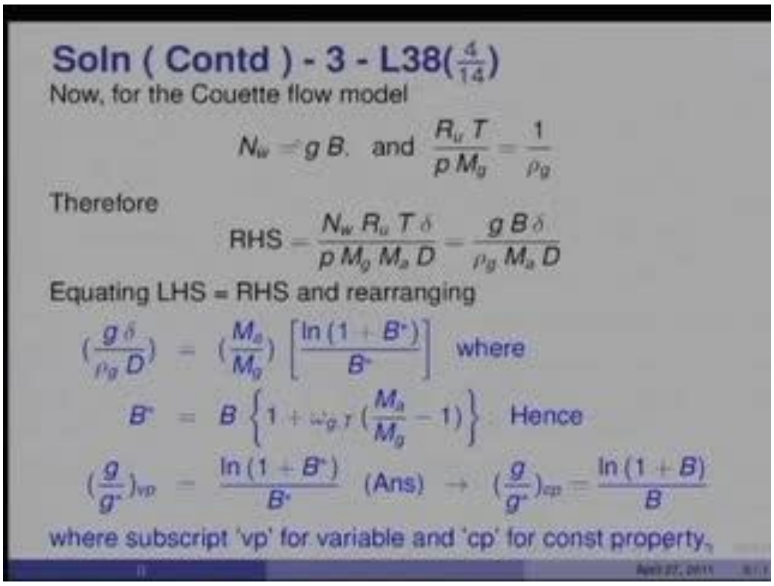

Soln (Contd) - 3 - L38($\frac{4}{14}$)

Now, for the Couette flow model

$$N_w = g\,B, \quad \text{and} \quad \frac{R_u\,T}{p\,M_g} = \frac{1}{\rho_g}$$

Therefore

$$\text{RHS} = \frac{N_w\,R_u\,T\,\delta}{p\,M_g\,M_a\,D} = \frac{g\,B\,\delta}{\rho_g\,M_a\,D}$$

Equating LHS = RHS and rearranging

$$\left(\frac{g\,\delta}{\rho_g\,D}\right) = \left(\frac{M_a}{M_g}\right)\left[\frac{\ln\left(1 + B^*\right)}{B^*}\right] \quad \text{where}$$

$$B^* = B\left\{1 + \omega_{g,T}\left(\frac{M_a}{M_g} - 1\right)\right\} \quad \text{Hence}$$

$$\left(\frac{g}{g^*}\right)_{vp} = \frac{\ln\left(1 + B^*\right)}{B^*} \quad \text{(Ans)} \rightarrow \left(\frac{g}{g^*}\right)_{cp} = \frac{\ln\left(1 + B\right)}{B}$$

where subscript 'vp' for variable and 'cp' for const property.

Now, how is m a by m g related to m infinity by m w, and that is what our interest is; but before we do that let us look at the value of omega g t being very very small, so in this case b star almost equals b, because as you will see here this is .01 very very small value, and therefore, it will try to annihilate the effect of m a by m g to some extent, and that is what we will see here.

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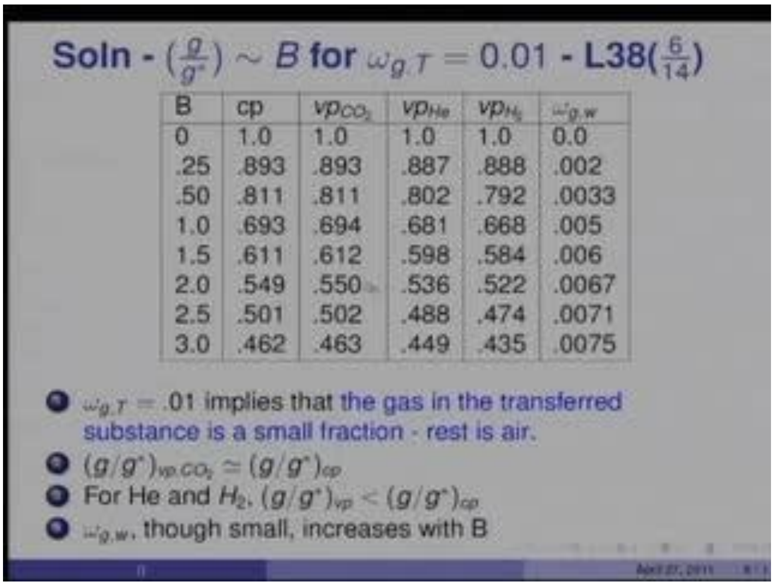

Soln - $\left(\frac{g}{g^*}\right) \sim B$ for $\omega_{g,T} = 0.01$ - L38($\frac{6}{14}$)

B	cp	vp_{CO_2}	vp_{He}	vp_{H_2}	$\omega_{g,w}$
0	1.0	1.0	1.0	1.0	0.0
.25	.893	.893	.887	.888	.002
.50	.811	.811	.802	.792	.0033
1.0	.693	.694	.681	.668	.005
1.5	.611	.612	.598	.584	.006
2.0	.549	.550	.536	.522	.0067
2.5	.501	.502	.488	.474	.0071
3.0	.462	.463	.449	.435	.0075

- $\omega_{g,T} = .01$ implies that the gas in the transferred substance is a small fraction - rest is air.
- $(g/g^*)_{vp,CO_2} \simeq (g/g^*)_{cp}$
- For He and H_2, $(g/g^*)_{vp} < (g/g^*)_{cp}$
- $\omega_{g,w}$, though small, increases with B

So, for example, for c o 2 you will see that the constant property and variable property solutions go almost same, very little difference, and so is the case with variable hydrogen and helium.

So, when omega g t is very very small, apparently it does not matter whether we allow for variable properties or not, so this is a very important deduction that we have got; just a reminder that omega g t equal to .01 implies that the gas is in the transfer substance is a very very small fraction and the rest is air.

So, all are very close to what the constant property solution will be; so, if you have a very weak concentration of the gas in the transferred substance states, then constant property solutions are quite valid only when the transferred gas is the only substance being transferred, then you have to worry about correcting property corrections alright next slide.

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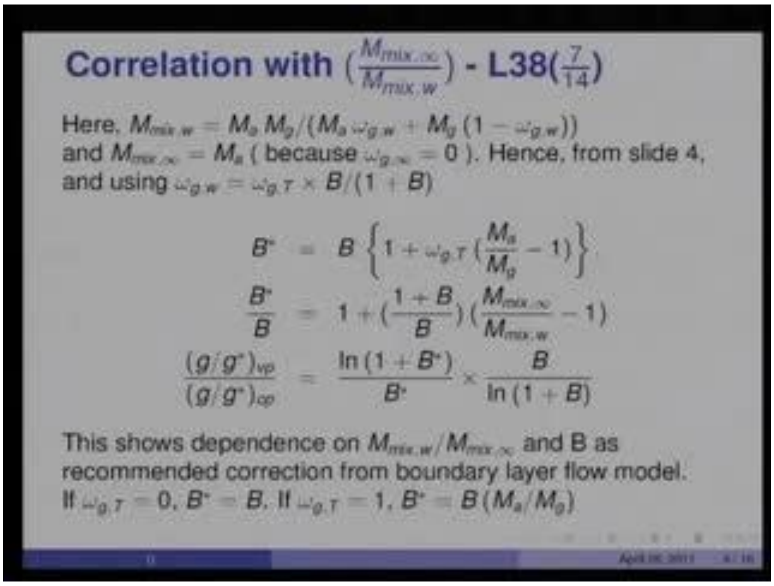

Correlation with $(\frac{M_{mix,\infty}}{M_{mix,w}})$ - L38$(\frac{7}{14})$

Here, $M_{mix,w} = M_a M_g/(M_a \omega_{g,w} + M_g(1-\omega_{g,w}))$ and $M_{mix,\infty} = M_a$ (because $\omega_{g,\infty} = 0$). Hence, from slide 4, and using $\omega_{g,w} = \omega_{g,T} \times B/(1+B)$

$$B^* = B\left\{1 + \omega_{g,T}\left(\frac{M_a}{M_g} - 1\right)\right\}$$

$$\frac{B^*}{B} = 1 + \left(\frac{1+B}{B}\right)\left(\frac{M_{mix,\infty}}{M_{mix,w}} - 1\right)$$

$$\frac{(g/g^*)_{vp}}{(g/g^*)_{cp}} = \frac{\ln(1+B^*)}{B^*} \times \frac{B}{\ln(1+B)}$$

This shows dependence on $M_{mix,w}/M_{mix,\infty}$ and B as recommended correction from boundary layer flow model. If $\omega_{g,T} = 0$, $B^* = B$. If $\omega_{g,T} = 1$, $B^* = B(M_a/M_g)$

Now, let us try and interpret m a by m g in terms of m mix infinity by m w, now what will be m mix in the w state, it will be simply as you will see m g into omega g w into m a into 1 minus omega g w, and m mix infinity will be m a, because omega g infinity is 0; and therefore, the from slide four we will get b star is equal to 1 plus omega g t m a by m g minus 1. and b star over b would be given by all this algebra, I have replaced m a and m g in terms of m mix w and m mix infinity.

And therefore, g by g star v p divided by g star g by g star c p will be l n 1 plus b star over b star into b over l n 1 plus b, and this clearly shows the influence that of m mix w divided by m mix infinity on the value of g over g star c p, b star over b would be given by that, and the rest is this.

Now, if omega g t is equal to 0 or tending to 0 then, of course, b star is equal to b, but if omega g t is 1, then as you know b star is equal to b m a by m g. Now, here what I have trying to do is to simply show that how does m mix w and m mix infinity influences arise; for specific cases we have to find a proper property correction; and therefore, in the Reynolds flow model we had recommended that the constant property solutions be corrected by m mix w by m mix infinity.

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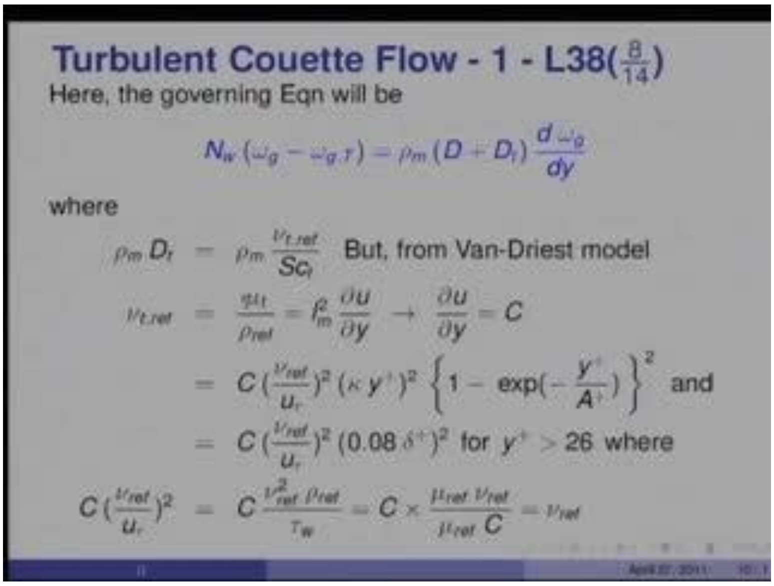


Turbulent Couette Flow - 1 - L38($\frac{8}{14}$)

Here, the governing Eqn will be

$$N_w\,(\omega_g - \omega_{g,T}) = \rho_m\,(D + D_t)\,\frac{d\,\omega_g}{dy}$$

where

$$\rho_m D_t = \rho_m \frac{\nu_{t,ref}}{Sc_t} \quad \text{But, from Van-Driest model}$$

$$\nu_{t,ref} = \frac{\mu_t}{\rho_{ref}} = l_m^2 \frac{\partial u}{\partial y} \rightarrow \frac{\partial u}{\partial y} = C$$

$$= C\,(\frac{\nu_{ref}}{u_\tau})^2\,(\kappa\, y^+)^2 \left\{1 - \exp(-\frac{y^+}{A^+})\right\}^2 \quad \text{and}$$

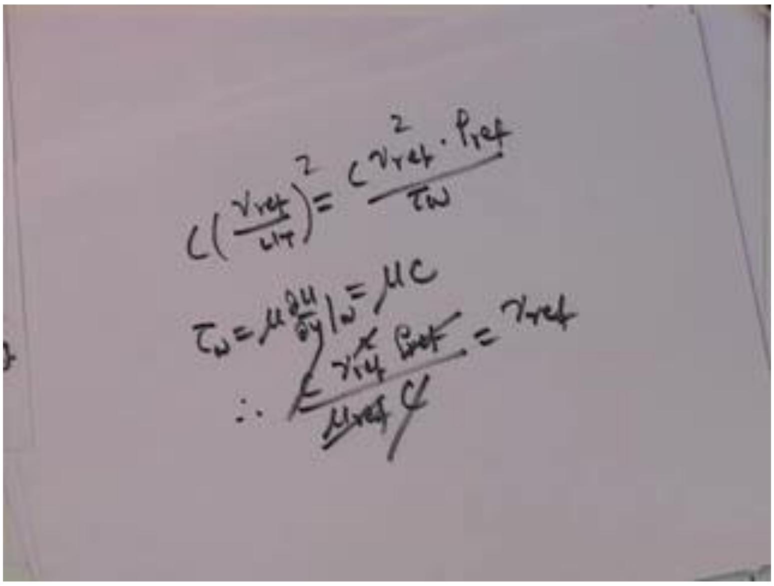
$$= C\,(\frac{\nu_{ref}}{u_\tau})^2\,(0.08\,\delta^+)^2 \quad \text{for } y^+ > 26 \text{ where}$$

$$C\,(\frac{\nu_{ref}}{u_\tau})^2 = C\,\frac{\nu_{ref}^2\,\rho_{ref}}{\tau_w} = C \times \frac{\mu_{ref}\,\nu_{ref}}{\mu_{ref}\,C} = \nu_{ref}$$

So, in laminar boundary layers we have shown that the ratio is indeed important. What about turbulent Couette flow; now, in the turbulent Couette flow the scalar transport equation will be n w omega g minus omega g t equal to rho m into effective diffusivity into d omega g by d y, and where rho m into d turbulent would be rho m into nu t ref divided by Schmidt number in the turbulent case, this is the effective the turbulent Schmidt number.

But now, from van driest model what will be nu t reference, nu t will be mu t by ref into l m square equal to l m square into d u by d y; and in our case d u by d y is a constant, because we are using a Couette flow model; if I were to interpret l m square substitute for l m from van driest model, it will be nu t whole square divided by u tau into kappa y square into 1 minus exponential y plus by a plus whole square multiplied by c, which is d u by d y, and this will become c times u ref square; this would be the case the first expression is for when y plus is less than 26; the second expression where the mixing length becomes constant with respect to y, y plus is greater than 26.

(Refer Slide Time: 23:48)



$$c\left(\frac{\nu_{ref}}{u_\tau}\right)^2 = \frac{c\,\nu_{ref}^2\cdot\rho_{ref}}{\tau_w}$$

$$\tau_w = \mu\frac{\partial u}{\partial y}\Big|_w = \mu c$$

$$\therefore \frac{\cancel{c}\,\nu_{ref}^{\cancel{2}}\,\cancel{\rho_{ref}}}{\cancel{\mu_{ref}}\,\cancel{c}} = \nu_{ref}$$

Now, what is this factor nu c times nu ref divided by u tau whole square, c into new ref divided by u tau square will be c times nu ref divided by u tau whole square will be c times nu ref square into u tau divided by u tau square is simply rho ref divided by tau, and as you know tau wall will be mu times d u by d y at the wall, but that is equal to mu times c, and therefore you will see c times nu ref square rho ref will divided by mu ref divided by c, so c and c gets cancel, mu ref and rho ref gets cancel with 1 nu nu ref, and that will be equal to nu ref itself.

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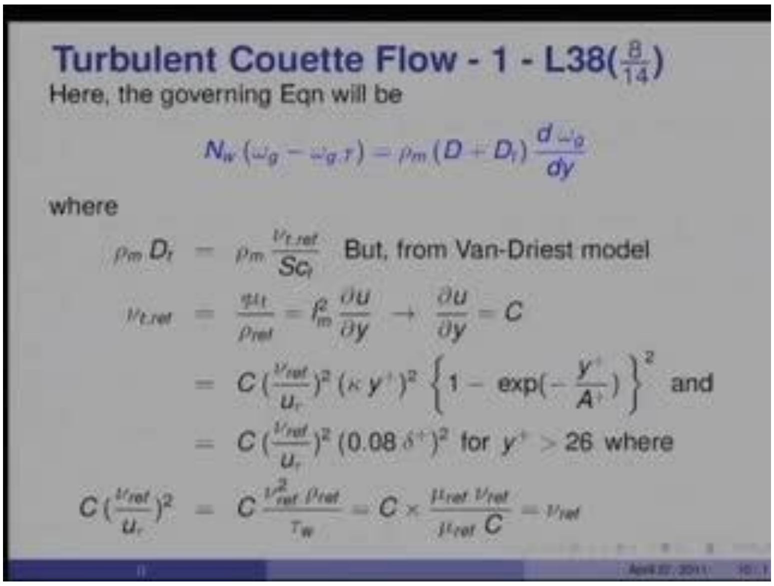

Turbulent Couette Flow - 1 - L38($\frac{8}{14}$)

Here, the governing Eqn will be

$$N_w\,(\omega_g - \omega_{g,T}) = \rho_m\,(D + D_t)\,\frac{d\,\omega_g}{dy}$$

where

$$\begin{aligned}
\rho_m\,D_t &= \rho_m\,\frac{\nu_{t,ref}}{Sc_t} \quad \text{But, from Van-Driest model} \\
\nu_{t,ref} &= \frac{\mu_t}{\rho_{ref}} = \ell_m^2\,\frac{\partial u}{\partial y} \rightarrow \frac{\partial u}{\partial y} = C \\
&= C\,(\frac{\nu_{ref}}{u_\tau})^2\,(\kappa\,y^+)^2\left\{1 - \exp(-\frac{y^+}{A^+})\right\}^2 \quad \text{and} \\
&= C\,(\frac{\nu_{ref}}{u_\tau})^2\,(0.08\,\delta^+)^2 \quad \text{for } y^+ > 26 \text{ where} \\
C\,(\frac{\nu_{ref}}{u_\tau})^2 &= C\,\frac{\nu_{ref}^2\,\rho_{ref}}{\tau_w} = C \times \frac{\mu_{ref}\,\nu_{ref}}{\mu_{ref}\,C} = \nu_{ref}
\end{aligned}$$

So, that is what I have shown here at the bottom of the slide that this whole factor c nu ref whole square by nu tau is nothing but nu ref itself; so, nu t ref the turbulent viscosity divided by density would be equal to the laminar viscosity divided by the density into these functions of y by plus for y plus greater than 26 it will be simply a constant.

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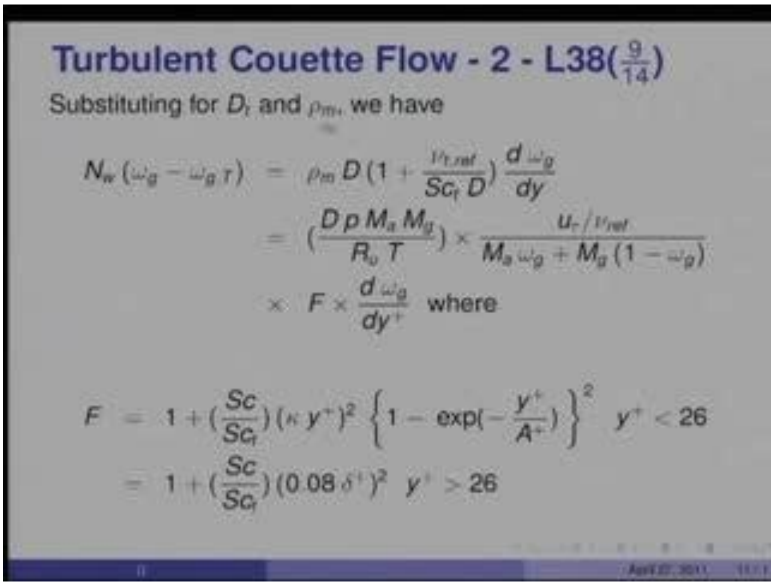

Turbulent Couette Flow - 2 - L38($\frac{9}{14}$)

Substituting for D_t and ρ_m, we have

$$\begin{aligned}
N_w\,(\omega_g - \omega_{g,T}) &= \rho_m\,D\,(1 + \frac{\nu_{t,ref}}{Sc_t\,D})\,\frac{d\,\omega_g}{dy} \\
&= (\frac{D\,p\,M_a\,M_g}{R_u\,T}) \times \frac{u_\tau/\nu_{ref}}{M_a\,\omega_g + M_g\,(1-\omega_g)} \\
&\times F \times \frac{d\,\omega_g}{dy^+} \quad \text{where}
\end{aligned}$$

$$\begin{aligned}
F &= 1 + (\frac{Sc}{Sc_t})\,(\kappa\,y^+)^2\left\{1 - \exp(-\frac{y^+}{A^+})\right\}^2 \quad y^+ < 26 \\
&= 1 + (\frac{Sc}{Sc_t})\,(0.08\,\delta^+)^2 \quad y^+ > 26
\end{aligned}$$

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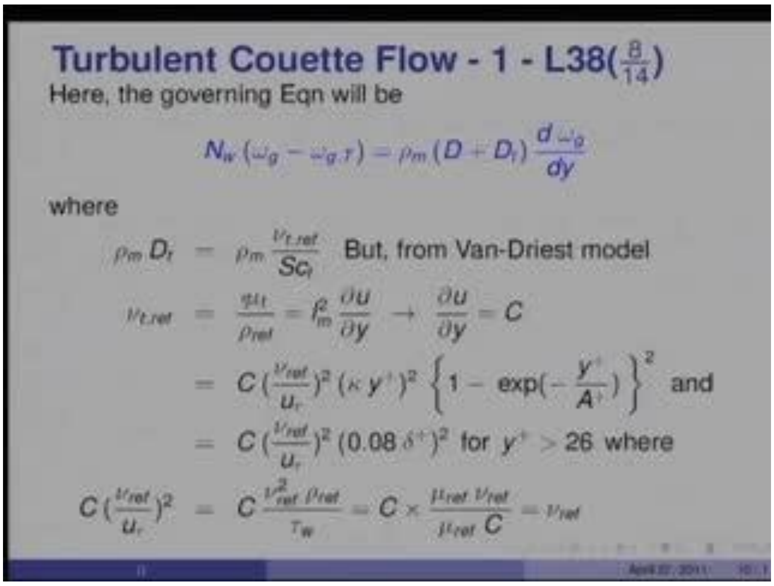

Turbulent Couette Flow - 1 - L38($\frac{8}{14}$)

Here, the governing Eqn will be

$$N_w\,(\omega_g - \omega_{g,T}) = \rho_m\,(D + D_t)\,\frac{d\,\omega_g}{dy}$$

where

$$\begin{aligned}
\rho_m\,D_t &= \rho_m\,\frac{\nu_{t,ref}}{Sc_t} \quad \text{But, from Van-Driest model} \\
\nu_{t,ref} &= \frac{\mu_t}{\rho_{ref}} = l_m^2\,\frac{\partial u}{\partial y} \rightarrow \frac{\partial u}{\partial y} = C \\
&= C\,(\frac{\nu_{ref}}{u_\tau})^2\,(\kappa\,y^+)^2\,\left\{1 - \exp(-\frac{y^+}{A^+})\right\}^2 \quad \text{and} \\
&= C\,(\frac{\nu_{ref}}{u_\tau})^2\,(0.08\,\delta^+)^2 \quad \text{for } y^+ > 26 \text{ where} \\
C\,(\frac{\nu_{ref}}{u_\tau})^2 &= C\,\frac{\nu_{ref}^2\,\rho_{ref}}{\tau_w} = C \times \frac{\mu_{ref}\,\nu_{ref}}{\mu_{ref}\,C} = \nu_{ref}
\end{aligned}$$

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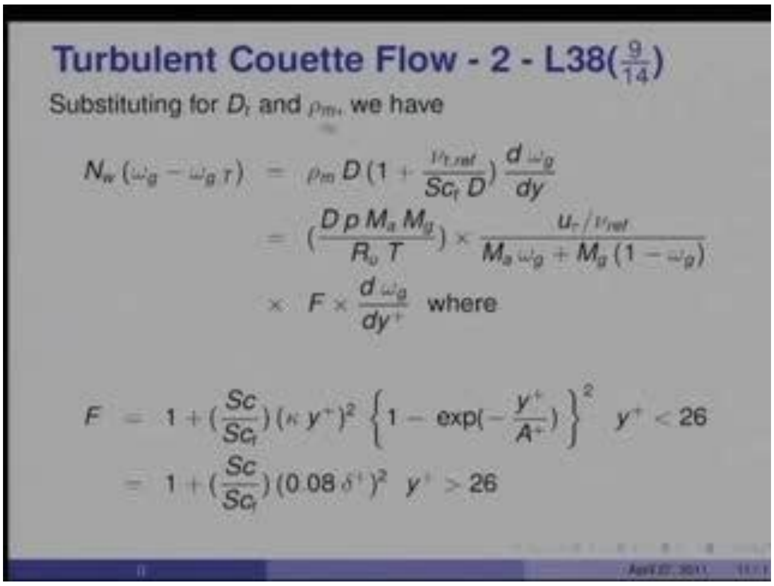

Turbulent Couette Flow - 2 - L38($\frac{9}{14}$)

Substituting for D_t and ρ_m, we have

$$\begin{aligned}
N_w\,(\omega_g - \omega_{g,T}) &= \rho_m\,D\,(1 + \frac{\nu_{t,ref}}{Sc_t\,D})\,\frac{d\,\omega_g}{dy} \\
&= (\frac{D\,p\,M_a\,M_g}{R_u\,T}) \times \frac{u_\tau/\nu_{ref}}{M_a\,\omega_g + M_g\,(1 - \omega_g)} \\
&\times\ F \times \frac{d\,\omega_g}{dy^+} \quad \text{where}
\end{aligned}$$

$$\begin{aligned}
F &= 1 + (\frac{Sc}{Sc_t})\,(\kappa\,y^+)^2\,\left\{1 - \exp(-\frac{y^+}{A^+})\right\}^2 \quad y^+ < 26 \\
&= 1 + (\frac{Sc}{Sc_t})\,(0.08\,\delta^+)^2 \quad y^+ > 26
\end{aligned}$$

And therefore, substituting for d t and rho m which is a function of omega g, and therefore, omega y we will have n w equal to omega g minus omega g t, in these expression I am now replacing d t, I will have…, and if i take rho m d common then I will get one plus nu t ref divided by Schmidt turbulent Schmidt number divided by diffusion coefficient into omega d omega g by d y.

And if I have to substitute for nu t ref and also rho m first of all I will get d p m a by m g divided by r u t u tau by nu ref m a by omega g plus m g into 1 minus omega g into f into d omega g by d y plus, and this big factor f is simply 1 plus s c by s t kappa y plus square, this is a little algebra that is easy to perform. So, you will see that the f function will vary with y only for y plus less than 26, but would remain constant for y plus greater than 26.

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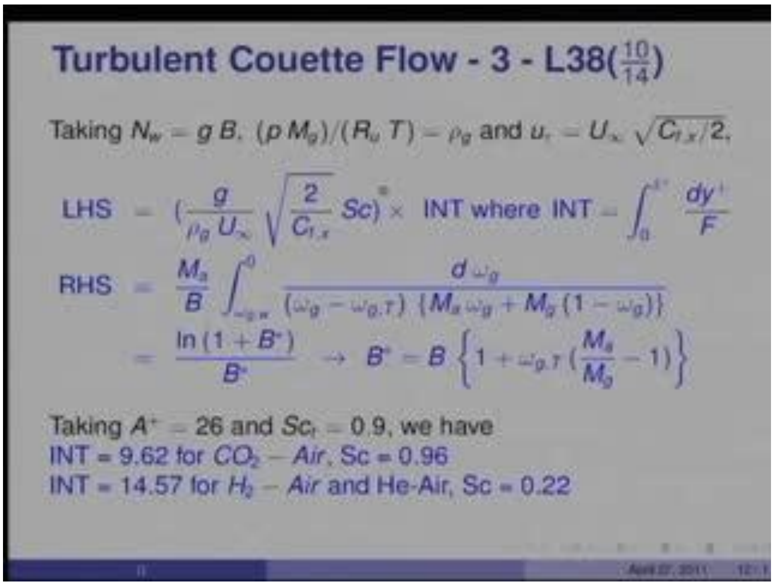

Turbulent Couette Flow - 3 - L38($\frac{10}{14}$)

Taking $N_w = g\,B.\ (p\,M_g)/(R_u\,T) = \rho_g$ and $u_\tau = U_\infty\,\sqrt{C_{f,x}/2}$.

$$\text{LHS} = \left(\frac{g}{\rho_g\,U_\infty}\sqrt{\frac{2}{C_{f,x}}}\,Sc\right)^{*} \times \ \text{INT where INT} = \int_0^{\delta^+} \frac{dy^+}{F}$$

$$\text{RHS} = \frac{M_a}{B}\int_{\omega_{g,w}}^{0} \frac{d\,\omega_g}{(\omega_g - \omega_{g,T})\ \{M_a\,\omega_g + M_g\,(1-\omega_g)\}}$$

$$= \frac{\ln\,(1+B^*)}{B^*} \ \rightarrow\ B^* = B\left\{1 + \omega_{g,T}\,\left(\frac{M_a}{M_g} - 1\right)\right\}$$

Taking $A^+ = 26$ and $Sc_t = 0.9$, we have
INT = 9.62 for CO_2 – Air, Sc = 0.96
INT = 14.57 for H_2 – Air and He-Air, Sc = 0.22

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So, now, if I transfer, if I carry out the integration in the in just as i did in the previous case in which I bring omega g minus omega g t on this side, and put it under d omega g by d y and integrate from 0 to delta plus, then you will see if I take n w equal to g b then p by m g r u t equal to rho g again, and u tau equal to u infinity under root c f x by 2, then the left hand side would be g over rho u infinity under root 2 by c f x Schmidt number into int, where int is 0 to delta plus into d y plus by f, which is means sorry which means I have simply transferred f on the left hand side with a multiplication d y plus. And the right hand side would be m a by g into integration of all this and we have carried out this integration before for laminar boundary layer, and it would remain the same it will be l n 1 plus b star over b star, and b star would b into 1 plus omega g t m a by m g minus 1.

Now, taking a plus equal to 26, that is assuming a smooth wall, and the turbulent Schmidt number equal to .9, this in this factor integrating factor can be integrated once for all, because as you will see f is simply a function of y plus; so, one can integrate that depending on what the molecular weight is, so the int is equal to 9.62 because remember, the Schmidt number would be a function of molecular weight of the gas being injected, you already know that I have given you the values of diffusion coefficient for c o 2 and air, so one can readily workout the Schmidt number; so, when Schmidt number is .96 which is for c o 2, the int factor is 9.62, but for hydrogen and air it is 14.57, and for helium and air well almost 14 point for both of them, because the Schmidt number for both of them is .22.

So, you can see that the factor int makes a significant contribution to the left hand side, because it depressed, it reduces, I mean it is 9.62 or for c o 2, and 14.57 for helium or and hydrogen.

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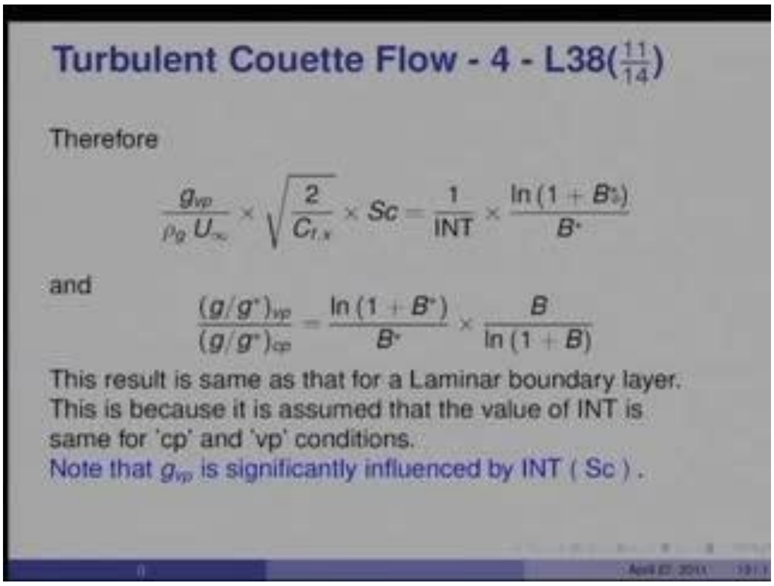


Turbulent Couette Flow - 4 - L38($\frac{11}{14}$)

Therefore

$$\frac{g_{vp}}{\rho_g\, U_\infty} \times \sqrt{\frac{2}{C_{f,x}}} \times Sc = \frac{1}{\mathrm{INT}} \times \frac{\ln(1 + B^*)}{B^*}$$

and

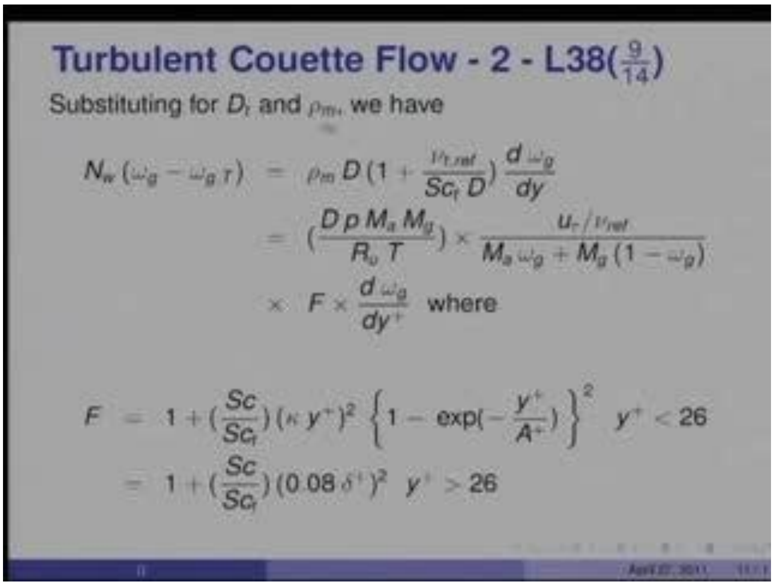
$$\frac{(g/g^*)_{vp}}{(g/g^*)_{cp}} = \frac{\ln(1 + B^*)}{B^*} \times \frac{B}{\ln(1 + B)}$$

This result is same as that for a Laminar boundary layer. This is because it is assumed that the value of INT is same for 'cp' and 'vp' conditions.

Note that g_{vp} is significantly influenced by INT (Sc) .

So, as a result we can show now that g v p divided by rho g u infinity, which is a kind of a mass transfer Stanton number if you like into under root 2 by c f x into Schmidt number will be 1 over int into l n 1 plus b star by b star or I can say that g v p by g star variable property will divide by g over g star constant property will again be equal to l n 1 plus b star by b star into b over.

(Refer Slide Time: 25:06)



Turbulent Couette Flow - 2 - L38($\frac{9}{14}$)

Substituting for D_t and ρ_m, we have

$$N_w\,(\omega_g - \omega_{g,T}) = \rho_m D\,(1 + \frac{\nu_{t,ref}}{Sc_t\,D})\,\frac{d\,\omega_g}{dy}$$

$$= (\frac{D\,p\,M_a\,M_g}{R_u\,T}) \times \frac{u_\tau / \nu_{ref}}{M_a\,\omega_g + M_g\,(1 - \omega_g)}$$

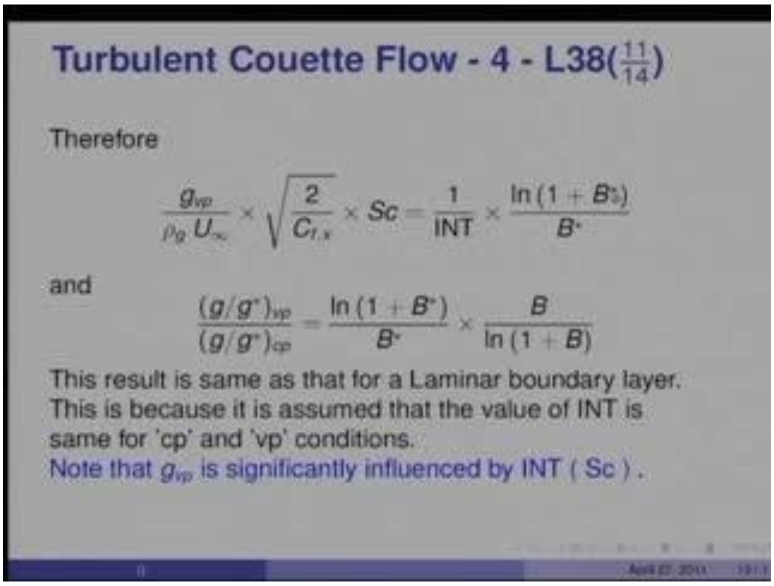
$$\times\ F \times \frac{d\,\omega_g}{dy^+} \quad \text{where}$$

$$F = 1 + (\frac{Sc}{Sc_t})\,(\kappa\,y^+)^2\left\{1 - \exp(-\frac{y^+}{A^+})\right\}^2 \quad y^+ < 26$$

$$= 1 + (\frac{Sc}{Sc_t})\,(0.08\,\delta^+)^2 \quad y^+ > 26$$

So, this result is again same as that for a laminar boundary layer, this is because it is assume that the value of int is the same for both constant property and variable property conditions, this is an assumption we have been making, that these the mixing length distributions are not influenced by whether there is a mass transfer, whether there is a property variation or there is int any.

(Refer Slide Time: 29:02)



Turbulent Couette Flow - 4 - L38($\frac{11}{14}$)

Therefore

$$\frac{g_{vp}}{\rho_g\,U_\infty} \times \sqrt{\frac{2}{C_{f,x}}} \times Sc = \frac{1}{\text{INT}} \times \frac{\ln\,(1 + B^*_{s})}{B^*}$$

and

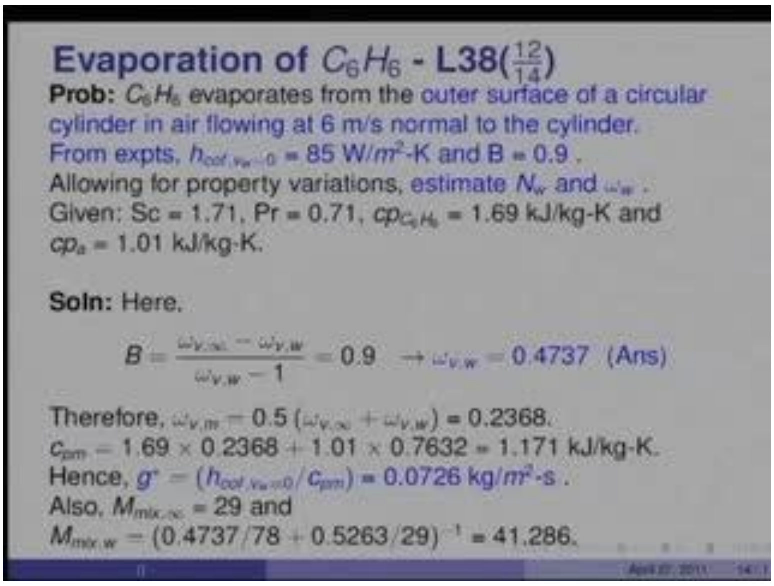
$$\frac{(g/g^*)_{vp}}{(g/g^*)_{cp}} = \frac{\ln\,(1 + B^*)}{B^*} \times \frac{B}{\ln\,(1 + B)}$$

This result is same as that for a Laminar boundary layer. This is because it is assumed that the value of INT is same for 'cp' and 'vp' conditions.

Note that g_{vp} is significantly influenced by INT (Sc) .

The int factors remain the same both the under variable and constant property conditions, the int factor simply give you resistance to mass transfer due to turbulence. So, noting that g v p is significantly influenced by a integrating factor which is function of Schmidt number Schmidt number, so the actual value of g over g star v p will be influenced by the Schmidt number.

(Refer Slide Time: 30:33)



Evaporation of C_6H_6 - L38($\frac{12}{14}$)

Prob: C_6H_6 evaporates from the outer surface of a circular cylinder in air flowing at 6 m/s normal to the cylinder. From expts, $h_{cof,V_w=0}$ = 85 W/m^2-K and B = 0.9 . Allowing for property variations, estimate N_w and ω_w . Given: Sc = 1.71, Pr = 0.71, $cp_{C_6H_6}$ = 1.69 kJ/kg-K and cp_a = 1.01 kJ/kg-K.

Soln: Here,

$$B = \frac{\omega_{v,\infty} - \omega_{v,w}}{\omega_{v,w} - 1} = 0.9 \rightarrow \omega_{v,w} = 0.4737 \text{ (Ans)}$$

Therefore, $\omega_{v,m} = 0.5\,(\omega_{v,\infty} + \omega_{v,w}) = 0.2368$.
$c_{pm} = 1.69 \times 0.2368 + 1.01 \times 0.7632 = 1.171$ kJ/kg-K.
Hence, $g^* = (h_{cof,V_w=0}/c_{pm}) = 0.0726$ kg/m^2-s .
Also, $M_{mix,\infty} = 29$ and
$M_{mix,w} = (0.4737/78 + 0.5263/29)^{-1} = 41.286$.

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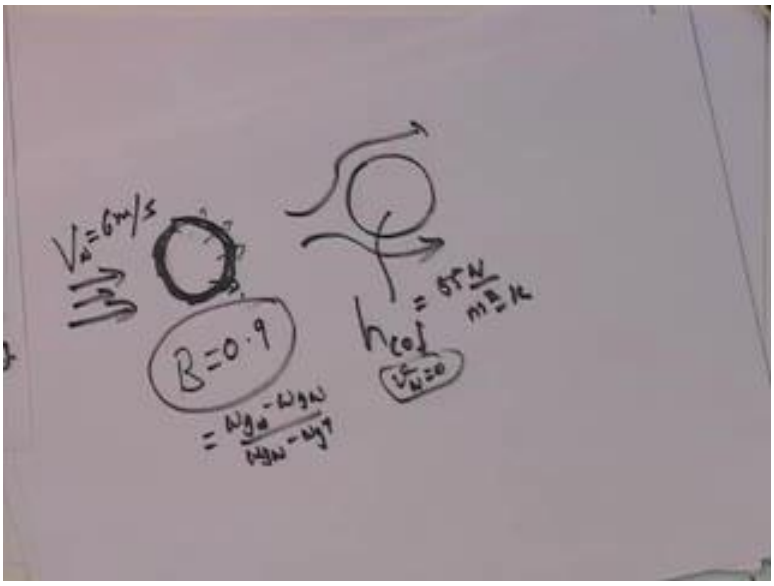

We now take up a problem, benzene evaporates from the outer surface of a circular cylinder, so let us say I have a circular cylinder like this on which a benzene has been put, and there is a cross flow across the across the cylinder, and therefore the benzene evaporates, the approach velocity v infinity is 6 meters per second.

Now, from experiments for the same case of flow over the cylinder, heat transfer coefficient h cof has been determine to be equal to 85 watts per meter square kelvin, and this is when there is v w is equal to 0, that mean, there is no mass no mass transfer from the wall. But in the present case because of the evaporation there will be mass transfer, but on and there is experiment being conducted without mass transfer gave h cof of 85 watts per meter square kelvin. And here the b has been found to be .9, that is this omega g infinity minus omega g w over omega g w minus omega g t has been found to be, or in this case actually it is 1, simply is 0.9.

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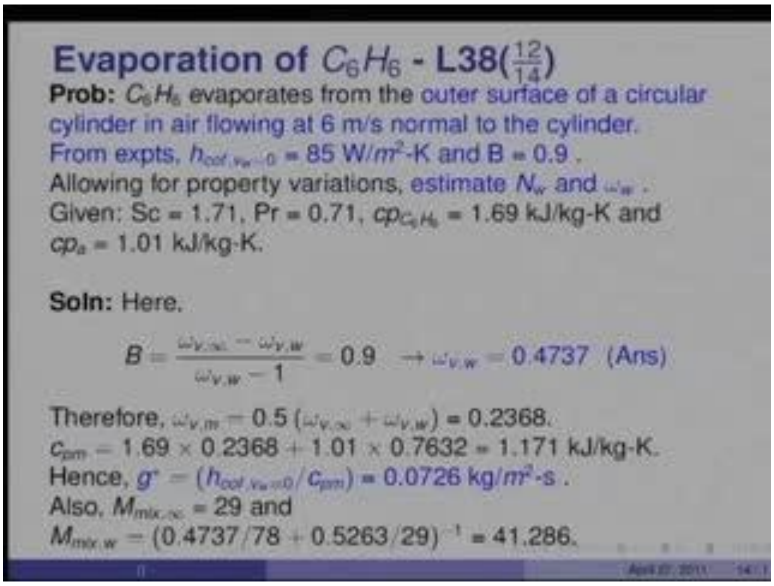


Evaporation of C_6H_6 - L38($\frac{12}{14}$)

Prob: C_6H_6 evaporates from the outer surface of a circular cylinder in air flowing at 6 m/s normal to the cylinder. From expts, $h_{cof,v_w=0} = 85$ W/m^2-K and B = 0.9 . Allowing for property variations, estimate N_w and ω_w . Given: Sc = 1.71, Pr = 0.71, $cp_{C_6H_6} = 1.69$ kJ/kg-K and $cp_a = 1.01$ kJ/kg-K.

Soln: Here,

$$B = \frac{\omega_{v,\infty} - \omega_{v,w}}{\omega_{v,w} - 1} = 0.9 \rightarrow \omega_{v,w} = 0.4737 \text{ (Ans)}$$

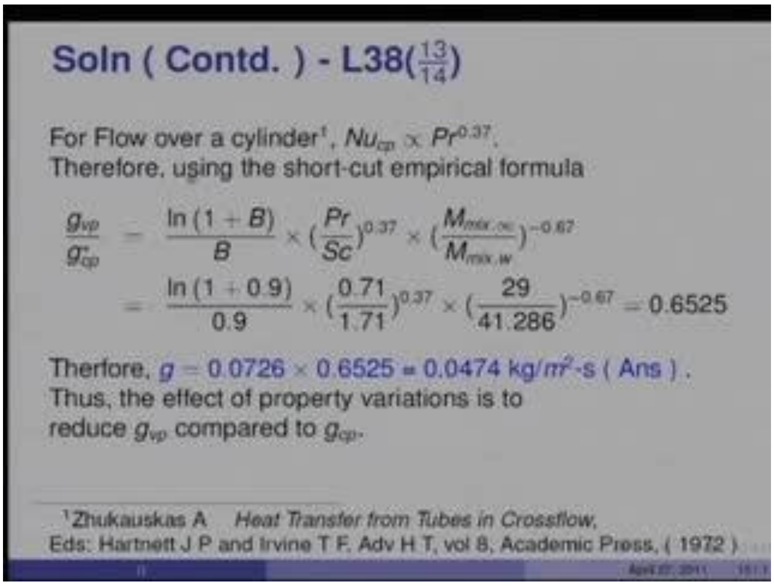
Therefore, $\omega_{v,m} = 0.5\,(\omega_{v,\infty} + \omega_{v,w}) = 0.2368$.
$c_{pm} = 1.69 \times 0.2368 + 1.01 \times 0.7632 = 1.171$ kJ/kg-K.
Hence, $g^* = (h_{cof,v_w=0}/c_{pm}) = 0.0726$ kg/m^2-s .
Also, $M_{mix,\infty} = 29$ and
$M_{mix,w} = (0.4737/78 + 0.5263/29)^{-1} = 41.286$.

So allowing for property variation estimate n w and value of omega w, so remember omega g infinity here is is 0, omega v w, omega v w, and omega g t being the only gas is minus 1, so that is equal to .9, that is what is been given to you. And that gives you omega v w equal to .4737 as the first transfer, that is what we want to get for omega w; and therefore, we take the mean concentration would be .2368, because remember omega v infinity omega g infinity is 0.

And therefore, c p m would be 1.69 times mean specific heat, that is been given to you plus 1.01 into .7632 which is the, which is the specific heat of air, which gives you 1.171 kilo joules per kilo grams kelvin.

And hence the g star for this case is h cof v w equal to 0 divided by c p m would be .076 k g per meter square second; now, m mix infinity is 29, whereas m mix w will be .4737 divided by 78 plus .4263 divided by 29 raise to minus 1 or 41.28, so this is what we get as the mixture molecular weight in the w state.

(Refer Slide Time: 34:13)



Soln (Contd.) - L38($\frac{13}{14}$)

For Flow over a cylinder[1], $Nu_{cp} \propto Pr^{0.37}$.
Therefore, using the short-cut empirical formula

$$\frac{g_{vp}}{g^*_{cp}} = \frac{\ln(1+B)}{B} \times \left(\frac{Pr}{Sc}\right)^{0.37} \times \left(\frac{M_{mix,\infty}}{M_{mix,w}}\right)^{-0.67}$$

$$= \frac{\ln(1+0.9)}{0.9} \times \left(\frac{0.71}{1.71}\right)^{0.37} \times \left(\frac{29}{41.286}\right)^{-0.67} = 0.6525$$

Therfore, $g = 0.0726 \times 0.6525 = 0.0474$ kg/m²-s (Ans) .
Thus, the effect of property variations is to reduce g_{vp} compared to g_{cp}.

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[1] Zhukauskas A *Heat Transfer from Tubes in Crossflow,* Eds: Hartnett J P and Irvine T F. Adv H T, vol 8, Academic Press, (1972)

If we were to apply our Reynolds flow model with property corrections, then you will see…, on this slide as you will see g over g v p over g star over g g star c p would be l n 1 plus b divided by b into prandtl by Schmidt number raise to .37, because the nusselt number for pure heat transfer without mass without any suction are blowing where is as prandtl raise to .37, and this is a very well-known book by a well-known work of zhukauskas extensive work on flow over cylinders variety of prandtl numbers with and without property variations.

And the concerned property correlation that zhukauskas has developed is n u c p being proportional to .37; and therefore, the firstly we must allow for prandtl and Schmidt number variations which are .71 and 1.71 as we calculated raise to .37; therefore, and then we must also allow for m mix infinity divided by m mix w were raise to minus .67

So, you get l n 1 plus .9 divided by .9 into .71 divided by 1.71, .37 and m mix infinity is 29, and this value we just calculated. So, you get .6525, and as a result gs v p will be g star which was calculated as h cofv w 0 divided c p as .0726 multiplied by 6525 equal to

.0474 kg per meter square second, so this is what Reynolds flow model with property correction gives us. So, the effect of property variation is to reduce g v p compare to g c p; so, that now let us see what Couette flow theory advices us, and that is the calculation I have done on the next slide.

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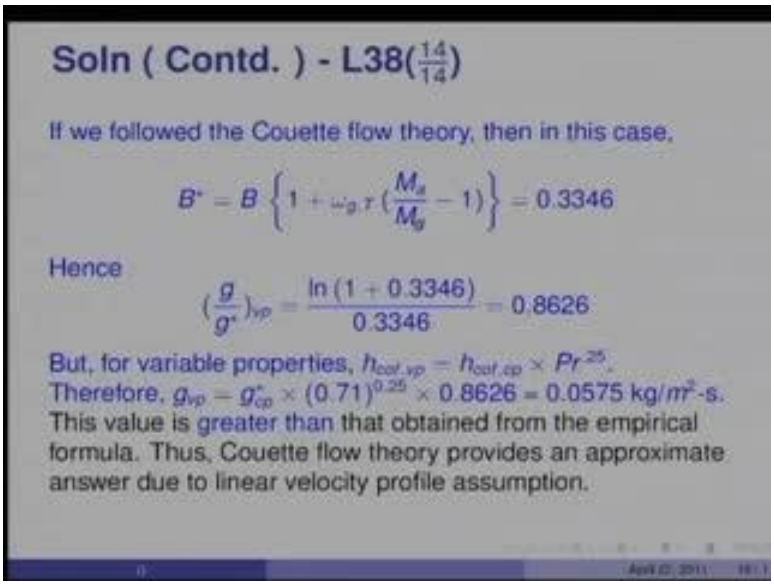

Soln (Contd.) - L38($\frac{14}{14}$)

If we followed the Couette flow theory, then in this case,

$$B^* = B\left\{1 + \omega_{g,T}\left(\frac{M_a}{M_g} - 1\right)\right\} = 0.3346$$

Hence

$$\left(\frac{g}{g^*}\right)_{vp} = \frac{\ln(1 + 0.3346)}{0.3346} = 0.8626$$

But, for variable properties, $h_{cof,vp} = h_{cof,cp} \times Pr^{.25}$.
Therefore, $g_{vp} = g^*_{cp} \times (0.71)^{0.25} \times 0.8626 = 0.0575$ kg/m^2-s.
This value is greater than that obtained from the empirical formula. Thus, Couette flow theory provides an approximate answer due to linear velocity profile assumption.

So, if we follow the Couette flow theory, which as you know is an approximate theory, then in this case b star would be omega g t is equal to 1 m a by m g as you know as a 29; and for benzene the molecular weight is the molecular weight for benzene is 78; so, you take 29 by 78 and put here one, and calculate b star, then you get .3346, .3346.

And therefore, g over g star v p will be l n 1 plus b star over b star which is .8626; now, under a variable property conditions n with v w equal to 0, zhukauskas says that h cof v p will be h cof c p into prandtl raise to .25, and therefore, g v p would be g star c p into .71 into 25 into .8626 equal to .0575.

Now, this value is not the same as .474, but nonetheless it shows you this is greater than that, but in it nonetheless shows you that our model is able to show that g v p will be smaller than g c p, which is you know was only g star c p which is .0726 into 1 plus b by b only that factor would have come in; and therefore, our solution thus show that Couette flow model predicts the correct trend that g v p should be depressed compare to g c p compare to g c p.

So, that is that is precisely what this result also shows, to that extent Couette flow model has justified the property correction recommended in the Reynolds flow model, which of course, which based on solution of boundary layer equations, the complete boundary layer equations under variety of conditions; and therefore, we do not expect the correction derived from Couette flow model to give us the same amount of reduction, but nonetheless it does show that there is a molecular weight effect or the effect of property variation is definitely to disgrace the variable property value of g. So, with this I conclude discussion of Couette flow. In the next two lectures I will consider the application of Reynolds flow model to various problems.