Convective Heat and Mass transfer

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Module No. # 01
Lecture No. # 37

Diffusion Mass Transfer Problems

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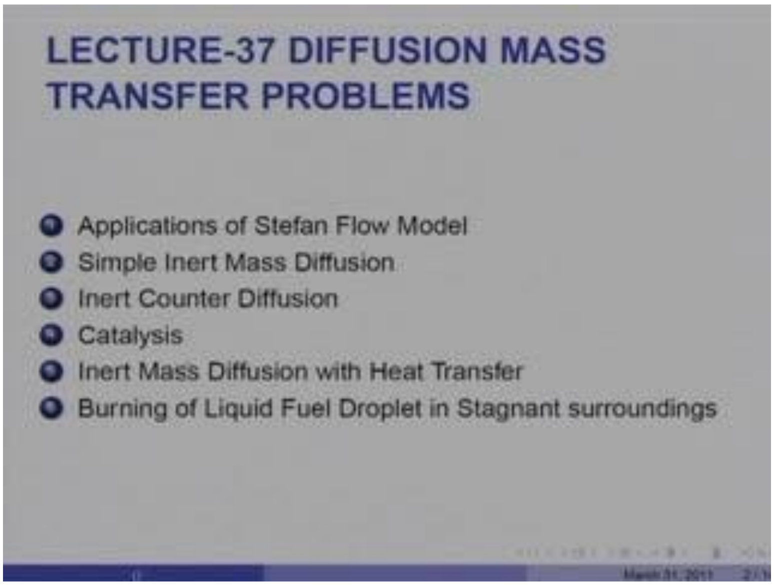

LECTURE-37 DIFFUSION MASS TRANSFER PROBLEMS

- Applications of Stefan Flow Model
- Simple Inert Mass Diffusion
- Inert Counter Diffusion
- Catalysis
- Inert Mass Diffusion with Heat Transfer
- Burning of Liquid Fuel Droplet in Stagnant surroundings

So, we now know how to calculate mass transfer flux in Stefan flow models, Couette flow model as well as in Reynolds flow model and we are ready to solve practical problems. So, what I am going to do now is to begin with the Stefan flow model in this lecture. Stefan flow model essentially deals with diffusion problems, because the in the considered phase, the velocity is zero.

So, we shall look at simple inert mass diffusion, inert counter diffusion; we will take a problem a very simplified problem of catalysis; we look at inert mass diffusion with heat transfer, and then, finally consider actually chemical reaction in which burning of liquid fuel droplet in stagnant surroundings.

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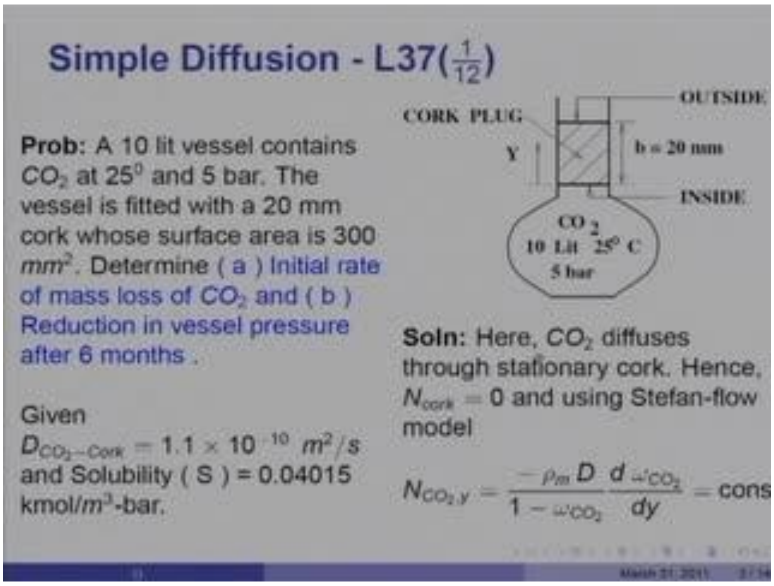

Simple Diffusion - L37($\frac{1}{12}$)

Prob: A 10 lit vessel contains CO_2 at 25^0 and 5 bar. The vessel is fitted with a 20 mm cork whose surface area is 300 mm^2. Determine (a) Initial rate of mass loss of CO_2 and (b) Reduction in vessel pressure after 6 months .

Given
$D_{CO_2-Cork} = 1.1 \times 10^{-10}\ m^2/s$
and Solubility (S) = 0.04015 kmol/m^3-bar.

Soln: Here, CO_2 diffuses through stationary cork. Hence, $N_{cork} = 0$ and using Stefan-flow model

$$N_{CO_2,y} = \frac{-\rho_m D}{1-\omega_{CO_2}} \frac{d\,\omega_{CO_2}}{dy} = \text{cons}$$

So, quite a variety of problems to help you with the understanding of this of the variety of applications to which Stefan flow model can be put. So, here is a problem, first problem - a 10 liter vessel contains carbon dioxide at 25 degree c and 5 bar as I have shown here, 10 liter vessel 25 degree c and 5 bar.

The vessel is fitted with the 20 millimeter cork 20 millimeter thick cork whose surface area is 300 millimeter square; so the surface area here is 300 millimeter square; the height of the cork is 20 millimeters, determine the initial rate of mass loss of CO 2 through the cork, remember cork because this is at high pressure, CO 2 would diffuse out of the cork.

So, what is the initial rate of mass loss of CO 2? When the mass loss takes place of course, the pressure in the vessel will reduce. So, we want to find out what the reduction in vessel pressure will be after, let us say 6 months if this was left unused for 6 months.

You have been given a property data, the d diffusivity of CO 2 cork is as 1.1 into 10 raise to minus 10 meter square per second very very small value and there is another quantity called solubility.

See, when CO 2 is in contact with cork surface, then the concentration of CO 2 at this surface, the inner surface assumes a certain value not the same as one, but a certain value and that value is usually quoted in terms of solubility as 0.04015 kilo moles per meter cube bar. We shall use this value of S to determine what the inside mass fraction of CO 2 is, the outside mass fraction of course is zero.

So, let us begin our solution; so here CO 2 diffuses through stationary cork and hence, there is no mass transfer flux of cork and using Stefan flow model therefore, we will have N CO 2 at any distance y as minus rho m diffusivity 1 minus omega CO 2 d omega CO 2 by d y equal to constant.

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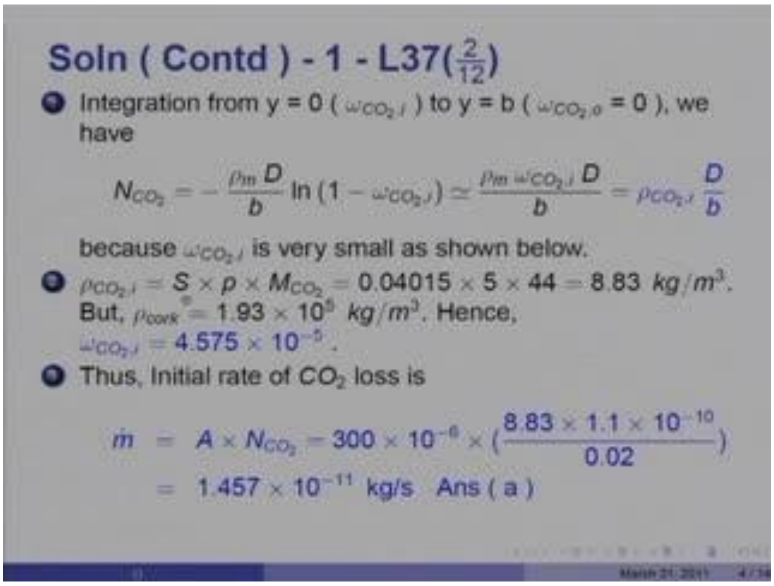

Soln (Contd) - 1 - L37($\frac{2}{12}$)

- Integration from y = 0 ($\omega_{CO_2,i}$) to y = b ($\omega_{CO_2,o} = 0$), we have

$$N_{CO_2} = -\frac{\rho_m D}{b} \ln(1 - \omega_{CO_2,i}) \simeq \frac{\rho_m \omega_{CO_2,i} D}{b} = \rho_{CO_2,i} \frac{D}{b}$$

because $\omega_{CO_2,i}$ is very small as shown below.

- $\rho_{CO_2,i} = S \times p \times M_{CO_2} = 0.04015 \times 5 \times 44 = 8.83\ kg/m^3$. But, $\rho_{cork} = 1.93 \times 10^5\ kg/m^3$. Hence, $\omega_{CO_2,i} = 4.575 \times 10^{-5}$.
- Thus, Initial rate of CO_2 loss is

$$\begin{aligned} \dot{m} &= A \times N_{CO_2} = 300 \times 10^{-6} \times (\frac{8.83 \times 1.1 \times 10^{-10}}{0.02}) \\ &= 1.457 \times 10^{-11}\ kg/s \quad \text{Ans (a)} \end{aligned}$$

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In other words, N CO 2 the mass flux of CO 2 will remains constant and since a is constant, even the mass transfer rate would remain constant throughout the throughout the surface. So, how do we solve this problem further? If we integrate this equation from y equal to 0, where omega CO 2 i to y equal to b omega CO 2 O which is 0, then we have N CO 2 equal to minus rho m D divided by b l n 1 minus omega CO 2 i.

And we will shortly discover that omega CO 2 i is in fact very very small and therefore, l n 1 minus omega CO 2 i can be taken as a minus omega CO 2 i that is what I have done here approximately equal to rho m omega CO 2 D by b i and what is omega CO 2 i? That is rho density of omega CO 2 divided by density of the mixture of cork and CO 2. And therefore, this will be rho CO 2 i D by b.

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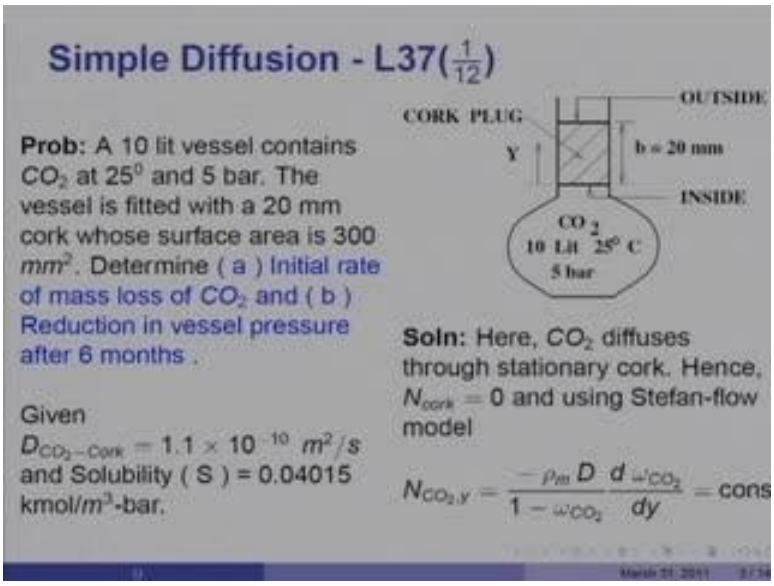

Simple Diffusion - L37($\frac{1}{12}$)

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Given $D_{CO_2-Cork} = 1.1 \times 10^{-10}\ m^2/s$ and Solubility (S) = 0.04015 kmol/m^3-bar.

Soln: Here, CO_2 diffuses through stationary cork. Hence, $N_{cork} = 0$ and using Stefan-flow model

$$N_{CO_2,y} = \frac{-\rho_m D}{1-\omega_{CO_2}} \frac{d\,\omega_{CO_2}}{dy} = \text{cons}$$

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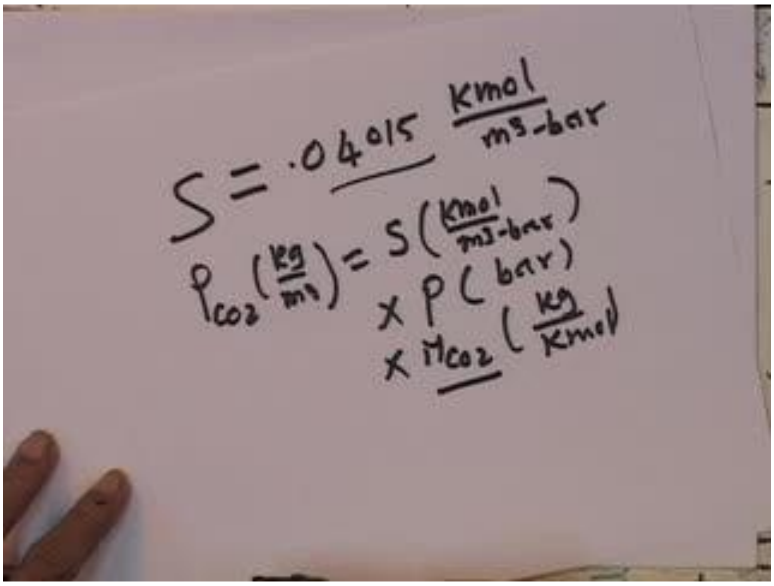

$$S = .04015 \ \frac{Kmol}{m^3\text{-bar}}$$

$$\rho_{CO_2}\left(\frac{kg}{m^3}\right) = S\left(\frac{Kmol}{m^3\text{-bar}}\right) \times P\,(bar) \times M_{CO_2}\left(\frac{kg}{Kmol}\right)$$

Now, this is where we use solubility as density of carbon dioxide at the inside surface is equal to solubility; remember solubility is given as solubility is given as S equal to 0.4015 kilo moles per meter cube bar.

And therefore, on the next slide, coupled rho CO 2 which is kilograms per meter cube would be given as S kilo moles per meter cube bar multiplied by P which is in bars multiplied by molecular weight of CO 2 which is kilograms per kilo mole. So, this will give me straightaway of course, value of P must be used in bars, S is solubility given here and this is the molecular weight of CO 2 which is 44.

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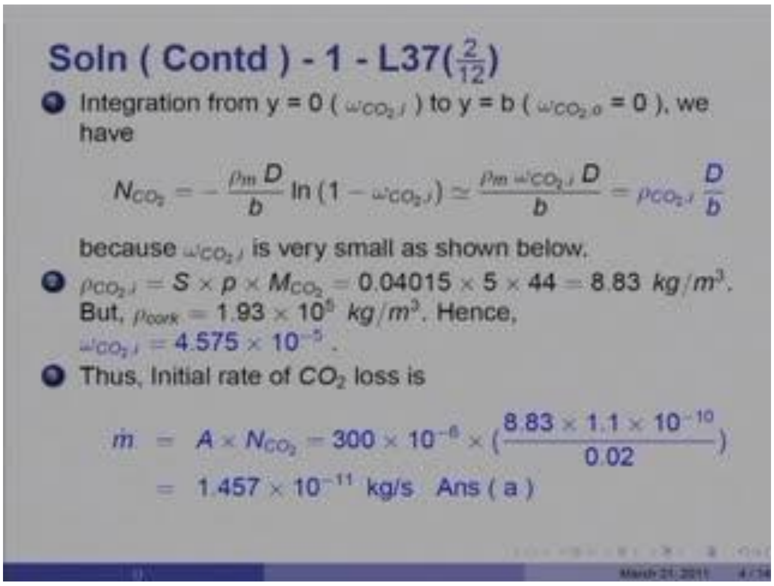

Soln (Contd) - 1 - L37($\frac{2}{12}$)

- Integration from y = 0 ($\omega_{CO_2,i}$) to y = b ($\omega_{CO_2,o} = 0$), we have

$$N_{CO_2} = -\frac{\rho_m D}{b} \ln(1 - \omega_{CO_2,i}) \simeq \frac{\rho_m \omega_{CO_2,i} D}{b} = \rho_{CO_2,i} \frac{D}{b}$$

  because $\omega_{CO_2,i}$ is very small as shown below.
- $\rho_{CO_2,i} = S \times p \times M_{CO_2} = 0.04015 \times 5 \times 44 = 8.83\ kg/m^3$. But, $\rho_{cork} = 1.93 \times 10^5\ kg/m^3$. Hence, $\omega_{CO_2,i} = 4.575 \times 10^{-5}$.
- Thus, Initial rate of CO_2 loss is

$$\begin{aligned} \dot{m} &= A \times N_{CO_2} = 300 \times 10^{-6} \times \left(\frac{8.83 \times 1.1 \times 10^{-10}}{0.02}\right) \\ &= 1.457 \times 10^{-11}\ \text{kg/s} \quad \text{Ans (a)} \end{aligned}$$

So, if we use this, then you get density of CO 2 and the inside surface would be 8.83 kilograms per meter cube. So, what will be the mass fraction? It will be the rho cork is 1.93 into 10 raise to 5.

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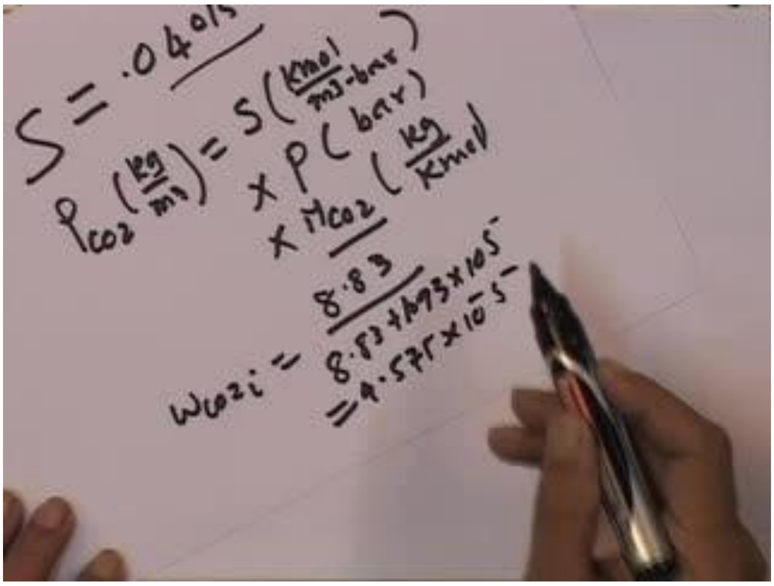

$S = .04015$

$\rho_{co2}\left(\frac{kg}{m^3}\right) = S\left(\frac{kmol}{m^3\text{-}bar}\right) \times P(bar) \times M_{co2}\left(\frac{kg}{kmol}\right)$

$w_{co2i} = \frac{8.83}{8.83 + 1.93 \times 10^5} = 4.575 \times 10^{-5}$

So, essentially it will be 8.83 divided by a very large quantity plus 8.83. So, you get essentially omega CO 2 i equal to omega CO 2 i will be 8.83 divided by 8.83 plus 1.93 into 10 raise to 5, so that gives you equal to 4.575 into 10 raise to minus 5.

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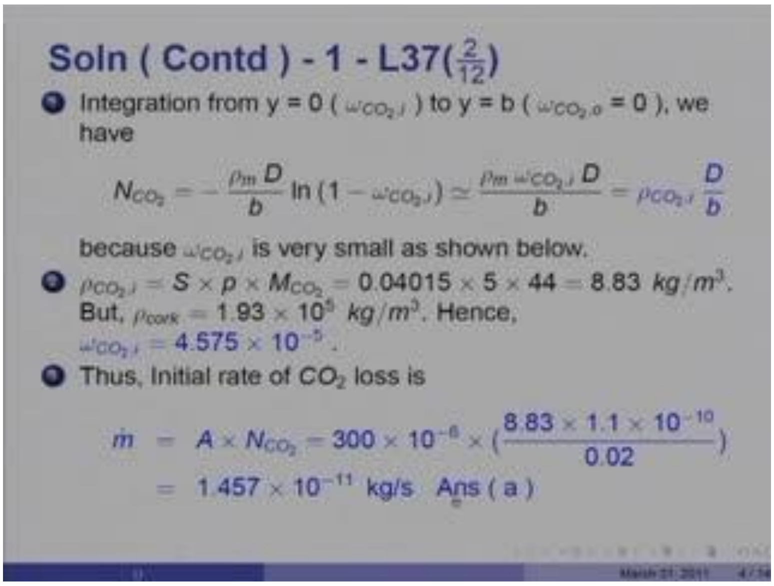

Soln (Contd) - 1 - L37$(\frac{2}{12})$

- Integration from y = 0 ($\omega_{CO_2,i}$) to y = b ($\omega_{CO_2,o} = 0$), we have

$$N_{CO_2} = -\frac{\rho_m D}{b} \ln(1 - \omega_{CO_2,i}) \simeq \frac{\rho_m \omega_{CO_2,i} D}{b} = \rho_{CO_2,i} \frac{D}{b}$$

because $\omega_{CO_2,i}$ is very small as shown below.

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$$= 1.457 \times 10^{-11} \text{ kg/s} \quad \text{Ans (a)}$$

And therefore, the initial rate of loss of CO 2 m dot will be the area of the cork multiplied by N CO 2, the area of the cork is 300 millimeter square and therefore, this will be 300 into 10 raise to minus 6 meter square 8.83 is the density 1. 1 into 10 raise to minus 10 is the diffusivity divided by the height D b; this is diffusivity is 1.1 into 10 raise to minus 6 and b is 20 millimeters, so that is 0.02.

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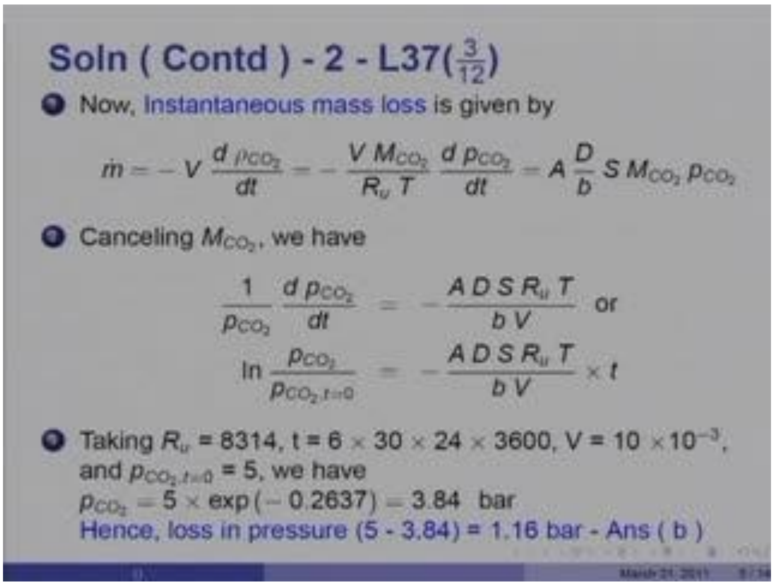

Soln (Contd) - 2 - L37($\frac{3}{12}$)

- Now, Instantaneous mass loss is given by

$$\dot{m} = -V\frac{d\,\rho_{CO_2}}{dt} = -\frac{V\,M_{CO_2}}{R_u\,T}\frac{d\,p_{CO_2}}{dt} = A\frac{D}{b}S\,M_{CO_2}\,p_{CO_2}$$

- Canceling M_{CO_2}, we have

$$\frac{1}{p_{CO_2}}\frac{d\,p_{CO_2}}{dt} = -\frac{A\,D\,S\,R_u\,T}{b\,V} \quad \text{or}$$

$$\ln\frac{p_{CO_2}}{p_{CO_2,t=0}} = -\frac{A\,D\,S\,R_u\,T}{b\,V}\times t$$

- Taking R_u = 8314, t = 6 × 30 × 24 × 3600, V = 10 × 10^{-3}, and $p_{CO_2,t=0}$ = 5, we have $p_{CO_2} = 5\times\exp(-0.2637) = 3.84$ bar
Hence, loss in pressure (5 - 3.84) = 1.16 bar - Ans (b)

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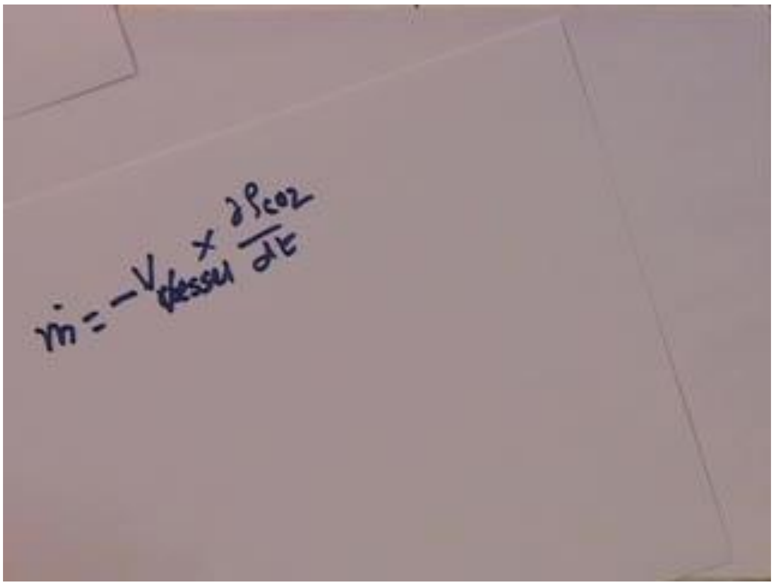

$$\dot{m} = -V_{vessel}\times\frac{\partial\rho_{co2}}{\partial t}$$

And this gives you the answer, the initial rate of mass flow will be 1.457 into 10 raise to minus 11 kilograms per second which is the answer of the path a. Now, the instantaneous mass loss can be given by, you can see here, that the mass loss m dot will be volume of the vessel volume multiplied by rate of change of density of d t, this is the density into volume is mass, but the volume remains constant so there you get that.

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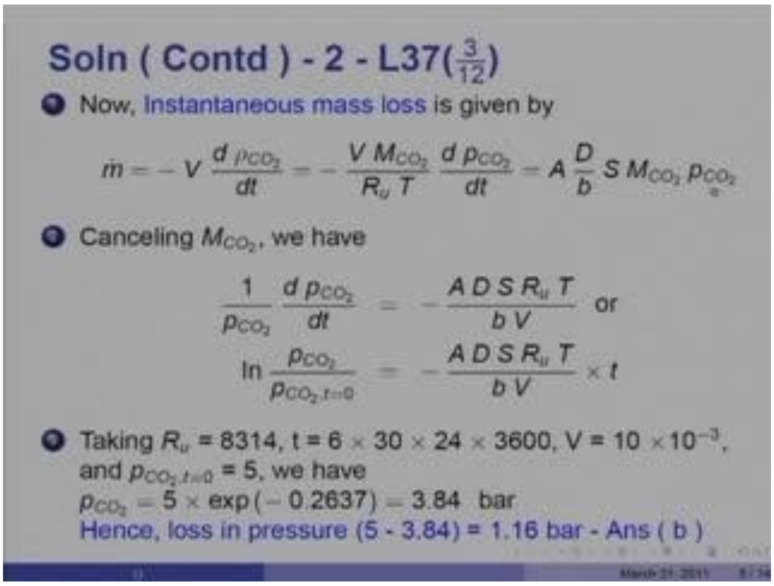

Soln (Contd) - 2 - L37($\frac{3}{12}$)

- Now, Instantaneous mass loss is given by

$$\dot{m} = -V\frac{d\rho_{CO_2}}{dt} = -\frac{V M_{CO_2}}{R_u T}\frac{d p_{CO_2}}{dt} = A\frac{D}{b} S M_{CO_2} p_{CO_2}$$

- Canceling M_{CO_2}, we have

$$\frac{1}{p_{CO_2}}\frac{d p_{CO_2}}{dt} = -\frac{A D S R_u T}{b V} \text{ or}$$

$$\ln\frac{p_{CO_2}}{p_{CO_2,t=0}} = -\frac{A D S R_u T}{b V} \times t$$

- Taking $R_u = 8314$, $t = 6 \times 30 \times 24 \times 3600$, $V = 10 \times 10^{-3}$, and $p_{CO_2,t=0} = 5$, we have
  $p_{CO_2} = 5 \times \exp(-0.2637) = 3.84$ bar
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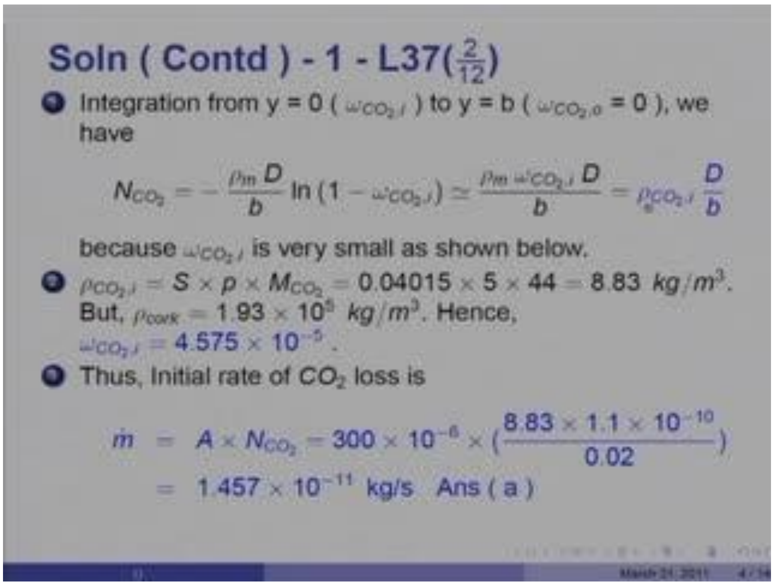

Soln (Contd) - 1 - L37($\frac{2}{12}$)

- Integration from y = 0 ($\omega_{CO_2,i}$) to y = b ($\omega_{CO_2,o} = 0$), we have

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- Thus, Initial rate of CO_2 loss is

$$\dot{m} = A \times N_{CO_2} = 300 \times 10^{-6} \times \left(\frac{8.83 \times 1.1 \times 10^{-10}}{0.02}\right)$$

$$= 1.457 \times 10^{-11} \text{ kg/s} \quad \text{Ans (a)}$$

And rho CO 2 from ideal gas law would be simply minus V M CO 2 divided by R u T p CO 2 by d t that is replacing rho CO 2 in terms of M CO 2 p CO 2 R u T d t and that would equal as we said in the previous slide A into D by b into omega 2 i S into M CO 2 p CO 2 i which is the density or density of cork, I mean, density of CO 2 on the inside surface is what we want rho CO 2 i which is going to now vary with time.

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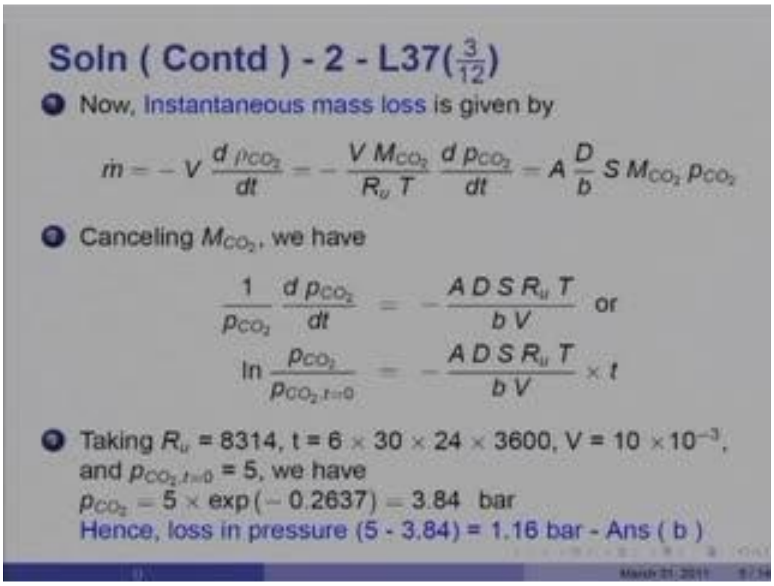

Soln (Contd) - 2 - L37($\frac{3}{12}$)

- Now, Instantaneous mass loss is given by

$$\dot{m} = -V\frac{d\rho_{CO_2}}{dt} = -\frac{V M_{CO_2}}{R_u T}\frac{d p_{CO_2}}{dt} = A\frac{D}{b} S M_{CO_2} p_{CO_2}$$

- Canceling M_{CO_2}, we have

$$\frac{1}{p_{CO_2}}\frac{d p_{CO_2}}{dt} = -\frac{A D S R_u T}{b V} \text{ or}$$

$$\ln\frac{p_{CO_2}}{p_{CO_2,t=0}} = -\frac{A D S R_u T}{b V} \times t$$

- Taking $R_u = 8314$, $t = 6 \times 30 \times 24 \times 3600$, $V = 10 \times 10^{-3}$, and $p_{CO_2,t=0} = 5$, we have
  $p_{CO_2} = 5 \times \exp(-0.2637) = 3.84$ bar
  Hence, loss in pressure (5 - 3.84) = 1.16 bar - Ans (b)

And therefore, that would also equal S into M CO 2 into p CO 2, so this is what you get. So, rearranging this dividing by CO 2 here, you get 1 over p CO 2 d p CO 2 by d t equal to minus A D S the solubility, R u into T divided by b into volume or l n p CO 2 by p CO 2 at time t equal to 0 would be minus A D S R u T divided by b V into time in seconds.

So, R u as you know the universal gas constant is 8314, the time in seconds for 6 months would be 6 into 30 into 24 hours into 3600, the volume itself being 10 liters is 10 into 10 raise to minus 3 and the initial pressure is 5 bar initial pressure is 5 bar.

And therefore, after 6 months, the pressure would be if you substitute all this value here, you will see 5 into l n of this is this therefore, p CO 2 by p CO 2 at t equal to 0 will be exponential of all these quantity is equal to 0.2637 equal to 3.84 bar.

So that would be the pressure inside the vessel after 6 months and therefore, the total loss in pressure would be 5, which was initial minus 3 point (( )) 1.1 6 bar and which is the answer. So, engineers and maintenance engineers are often interested in knowing these sorts of values so that they know that they would order only the amount that is required for their use and not leave vessels unguarded, unused and lose CO 2 thereby.

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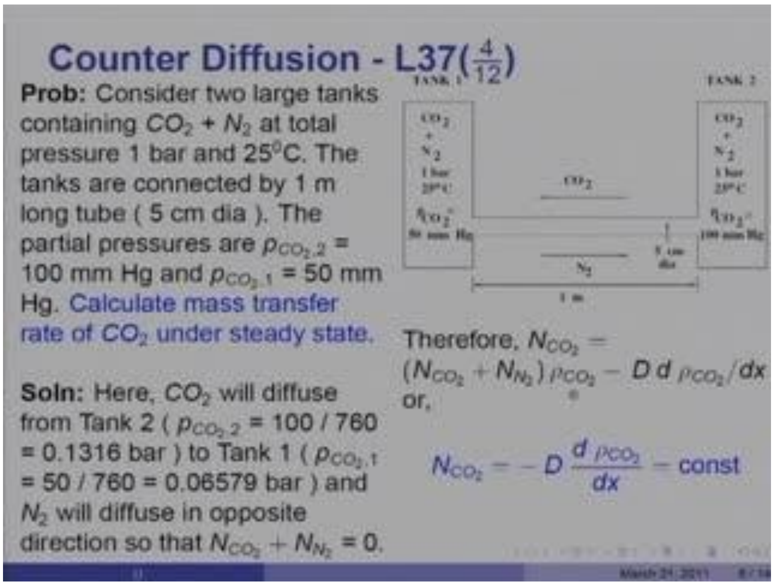

Counter Diffusion - L37($\frac{4}{12}$)

Prob: Consider two large tanks containing $CO_2 + N_2$ at total pressure 1 bar and 25°C. The tanks are connected by 1 m long tube (5 cm dia). The partial pressures are $p_{CO_2,2}$ = 100 mm Hg and $p_{CO_2,1}$ = 50 mm Hg. Calculate mass transfer rate of CO_2 under steady state.

Soln: Here, CO_2 will diffuse from Tank 2 ($p_{CO_2,2}$ = 100 / 760 = 0.1316 bar) to Tank 1 ($p_{CO_2,1}$ = 50 / 760 = 0.06579 bar) and N_2 will diffuse in opposite direction so that $N_{CO_2} + N_{N_2} = 0$.

Therefore, $N_{CO_2} = (N_{CO_2} + N_{N_2})\, p_{CO_2} - D\, d\, p_{CO_2}/dx$ or,

$$N_{CO_2} = -D \frac{d\, p_{CO_2}}{dx} = \text{const}$$

Let us take another problem, here we consider two large tanks of CO 2 which can each contains CO 2 plus N 2, this is a tank one and this is tank two, both of them contains CO 2 and N 2 at 1 bar and 25 degree centigrade; so the total pressure and total temperature of both the tanks is identical.

But they are very large, the tanks are very very large; the tanks are connected by a 1 meter long tube a 1 meter long tube of 5 centimeter diameter, although the total pressure in tank one is 1 bar, the partial pressure of CO 2 is simply is a low 50 millimeters mercury, whereas the partial pressure of CO 2 in tank two is 100 millimeters of mercury.

Now as result what would happen is, although the total pressures are equal, CO 2 would defuse to tank one from tank two and since mass must be conserved, the nitrogen here whose partial will now be bigger than in tank two, the nitrogen would defuse in the opposite direction.

So, we have been asked to calculate the mass transfer rate of CO 2 under study state. So, here CO 2 will defuse from tank two to tank one; now p CO 2 of 100 millimeters of H g, because 760 millimeters of H g equals 1 bar, p CO 2 in bars would be 100 by 760 is equal to 0.1316 bar and in tank one, it would be 50 by 760, which is 0.06579 bar and N 2 will defuse in the opposite direction.

So that N CO 2 plus N N 2 will be 0 and as you will recall from our diffusion model, N CO 2 is N total rho CO 2 minus D d rho CO 2 by d x and since this quantity is 0, we get N CO 2 would be simply minus D into d rho CO 2 by d x equal to constant.

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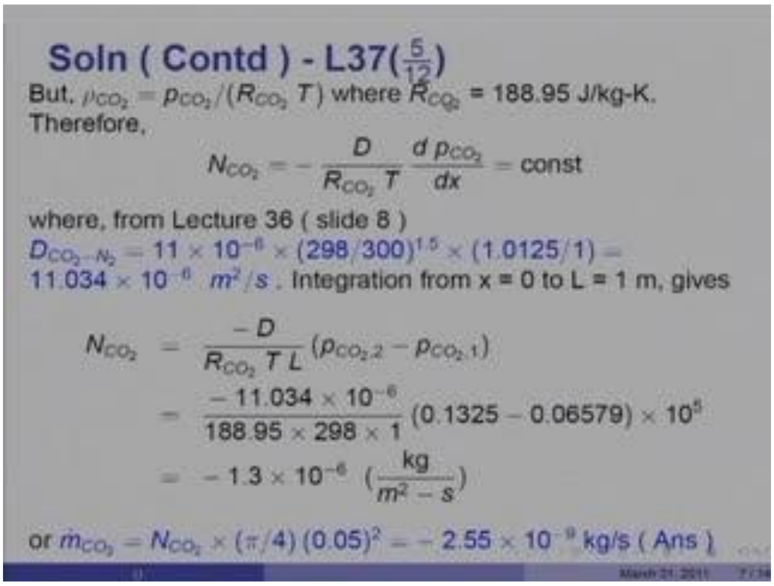

Soln (Contd) - L37($\frac{5}{12}$)

But, $\rho_{CO_2} = p_{CO_2}/(R_{CO_2}\,T)$ where R_{CO_2} = 188.95 J/kg-K. Therefore,

$$N_{CO_2} = -\frac{D}{R_{CO_2}\,T}\,\frac{d\,p_{CO_2}}{dx} = \text{const}$$

where, from Lecture 36 (slide 8)

$D_{CO_2-N_2} = 11 \times 10^{-6} \times (298/300)^{1.5} \times (1.0125/1) =$ $11.034 \times 10^{-6}\ m^2/s$. Integration from x = 0 to L = 1 m, gives

$$\begin{aligned} N_{CO_2} &= \frac{-D}{R_{CO_2}\,T\,L}\,(p_{CO_2,2} - p_{CO_2,1}) \\ &= \frac{-11.034 \times 10^{-6}}{188.95 \times 298 \times 1}\,(0.1325 - 0.06579) \times 10^5 \\ &= -1.3 \times 10^{-6}\ \left(\frac{kg}{m^2 - s}\right) \end{aligned}$$

or $\dot{m}_{CO_2} = N_{CO_2} \times (\pi/4)\,(0.05)^2 = -2.55 \times 10^{-9}$ kg/s (Ans)

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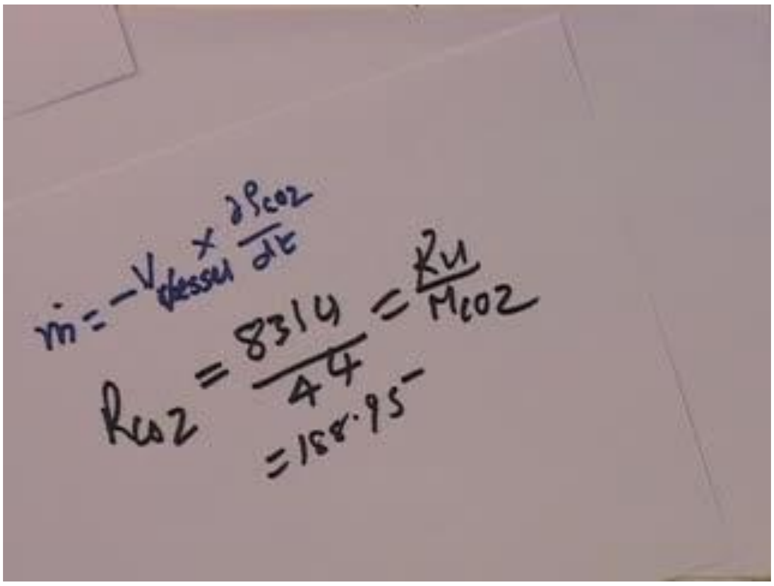

$$\dot{m} = -V_{vessel} \times \frac{d\rho_{CO_2}}{dt}$$

$$R_{CO_2} = \frac{8314}{44} = \frac{R_u}{M_{CO_2}}$$

$$= 188{\cdot}95$$

So, now, this equation must be integrated of course, to obtain our formula. So, rho CO 2 is p CO 2 divided by R CO 2 divided by T from ideal gas law, and R CO 2 would be R CO 2 will be 8314 divided by 44 and that is equal to 188.95, this is R u divided by molecular weight of CO 2. So, you get R CO 2 equal to 188.95 joules per kilograms per kelvin.

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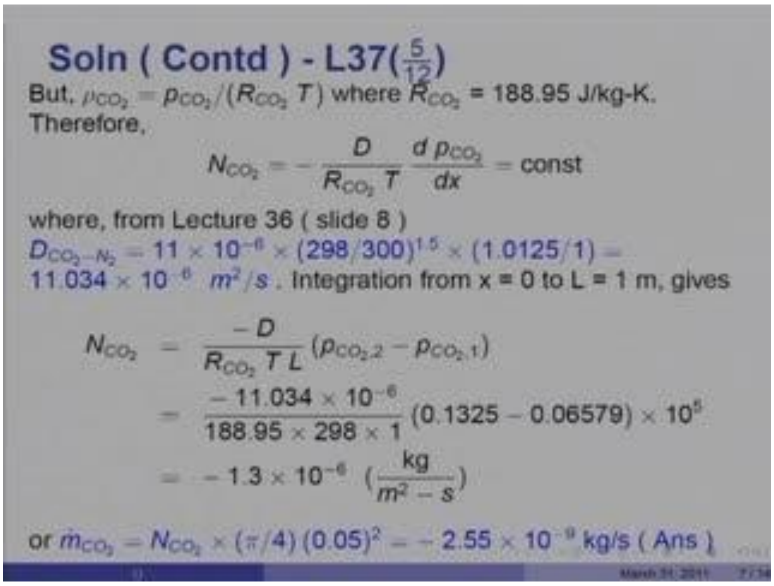

Soln (Contd) - L37($\frac{5}{12}$)

But, $\rho_{CO_2} = p_{CO_2}/(R_{CO_2}\ T)$ where R_{CO_2} = 188.95 J/kg-K.
Therefore,

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where, from Lecture 36 (slide 8)
$D_{CO_2-N_2} = 11 \times 10^{-6} \times (298/300)^{1.5} \times (1.0125/1) =$
$11.034 \times 10^{-6}\ m^2/s$. Integration from x = 0 to L = 1 m, gives

$$\begin{aligned} N_{CO_2} &= \frac{-D}{R_{CO_2}\ T\ L}(p_{CO_2,2} - p_{CO_2,1}) \\ &= \frac{-11.034 \times 10^{-6}}{188.95 \times 298 \times 1}(0.1325 - 0.06579) \times 10^5 \\ &= -1.3 \times 10^{-6}\ (\frac{kg}{m^2 - s}) \end{aligned}$$

or $\dot{m}_{CO_2} = N_{CO_2} \times (\pi/4)\,(0.05)^2 = -2.55 \times 10^{-9}$ kg/s (Ans)

And therefore, N CO 2 is equal to minus D by R CO 2 T into d p CO 2 by d x equal to constant. Now, from our previous lecture, you will you can check out that diffusivity of CO 2 in nitrogen is 11 into 10 raise to minus 6 298 divided by 300 into 1.5.

And since we have our hour pressure is in is in bar, whereas 1 .0125 1 to 1 atmosphere is equal to 1 .0125 bar and our temperature is 25 degree centigrade, so its 298 divided by 300. So, you get 11.034 into 10 raise to minus 6 meter square per second.

So, integration of this equation from x equal to 0 to x equal to l gives N CO 2 equal to minus D diffusivity R CO 2 T L which is temperature into p CO 2 2 minus p CO 2 1 and diffusivity being equal to 11.034 into 10 raise to minus 6 by 188.95 is R CO 2, temperature is 298 k and L the length of tube is 1 meter, p CO 2 is 0.1325, p CO 2 1 is that into 10 raise to 5 Newton's per meter square.

And as a result you will get N CO 2 equal to minus all these quantity, the negative side [in/indicates] indicates that the N CO 2 is flowing in the negative x direction that is from tank two to tank one. And therefore, the mass flow rate m dot CO 2 will be N CO 2 multiplied by pi by 4 into diameter of the pipe 0.05, which was 5 centimeters, 0.05

square is equal to minus 2.55 into 10 raise to minus nine kg per second. So, this is the answer to our problem.

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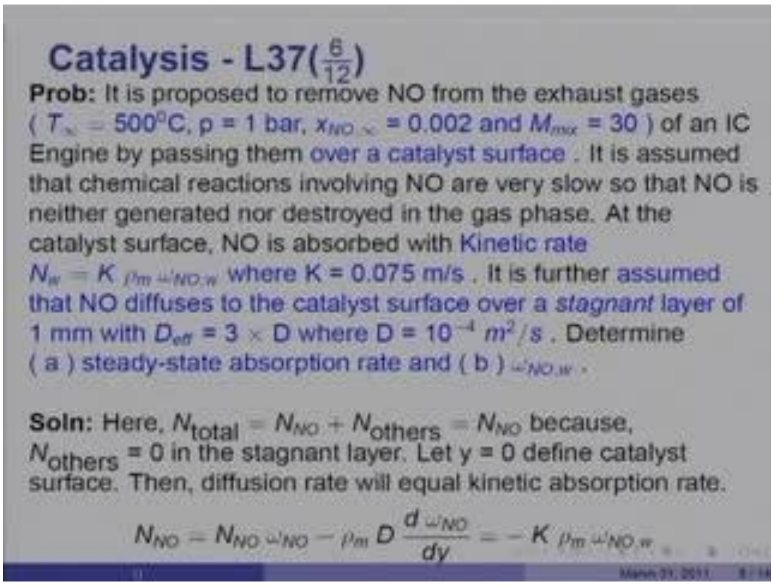

Catalysis - L37($\frac{6}{12}$)

Prob: It is proposed to remove NO from the exhaust gases ($T_\infty = 500^0$C, p = 1 bar, $x_{NO,\infty}$ = 0.002 and M_{mix} = 30) of an IC Engine by passing them over a catalyst surface . It is assumed that chemical reactions involving NO are very slow so that NO is neither generated nor destroyed in the gas phase. At the catalyst surface, NO is absorbed with Kinetic rate $N_w = K \rho_m \omega_{NO,w}$ where K = 0.075 m/s . It is further assumed that NO diffuses to the catalyst surface over a *stagnant* layer of 1 mm with D_{eff} = 3 × D where D = 10^{-4} m^2/s . Determine (a) steady-state absorption rate and (b) $\omega_{NO,w}$.

Soln: Here, $N_{total} = N_{NO} + N_{others} = N_{NO}$ because, N_{others} = 0 in the stagnant layer. Let y = 0 define catalyst surface. Then, diffusion rate will equal kinetic absorption rate.

$$N_{NO} = N_{NO}\,\omega_{NO} - \rho_m D \frac{d\,\omega_{NO}}{dy} = -K\,\rho_m\,\omega_{NO,w}$$

Now, the rate of diffusion of CO 2 in this pipe in this problem is from tank two to tank one. We now look at a third problem, you know that in exhaust gas of a let us say of an IC engine may contain nitrous NO nitric oxide and you are told that it is the exhaust gases at 500 degree centigrade 1 bar pressure and it contains the mole facts of NO in the that stream is point 0.002 and the mixture molecular weight is 30. Now, in order to extract NO out of this, you pass it over a catalyst surface.

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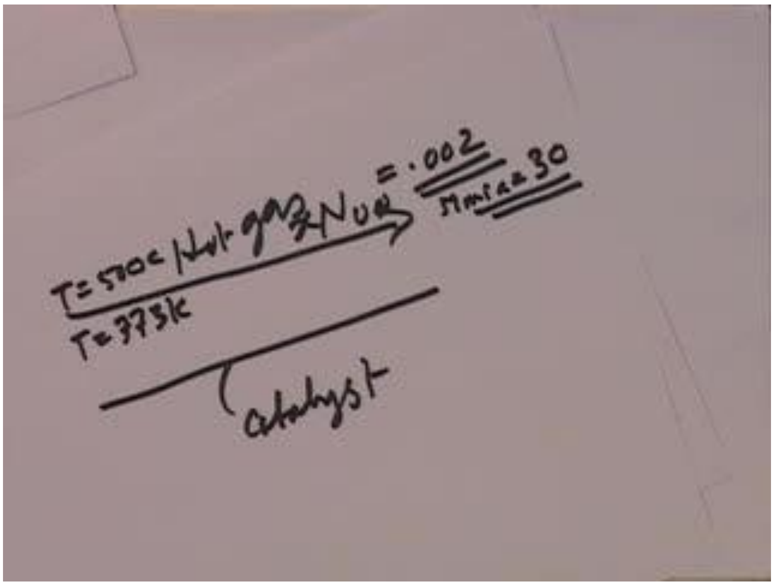

So, you have a basically you have, a let us say catalyst surface and this is the hot gas hot gas which contains n o x NO infinity equal to 0.002. The temperature here is 500 degree centigrade or t is equal to 773 kelvin and the mixture molecular weight in the infinity state is 30.

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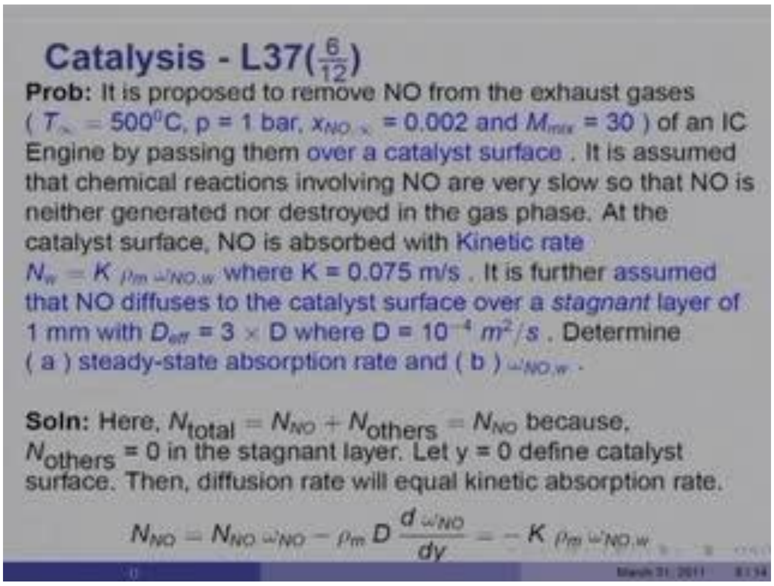

Catalysis - L37($\frac{6}{12}$)

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Soln: Here, $N_{\text{total}} = N_{NO} + N_{\text{others}} = N_{NO}$ because, $N_{\text{others}} = 0$ in the stagnant layer. Let y = 0 define catalyst surface. Then, diffusion rate will equal kinetic absorption rate.

$$N_{NO} = N_{NO}\,\omega_{NO} - \rho_m D \frac{d\,\omega_{NO}}{dy} = -K\,\rho_m\,\omega_{NO,w}$$

Now, what I am going to do is although the gases are moving, I am going to make few assumptions in order to render the problem tractable in a very simple way. So, it is assumed that chemical reactions involving NO are very very low at 500 degree centigrade.

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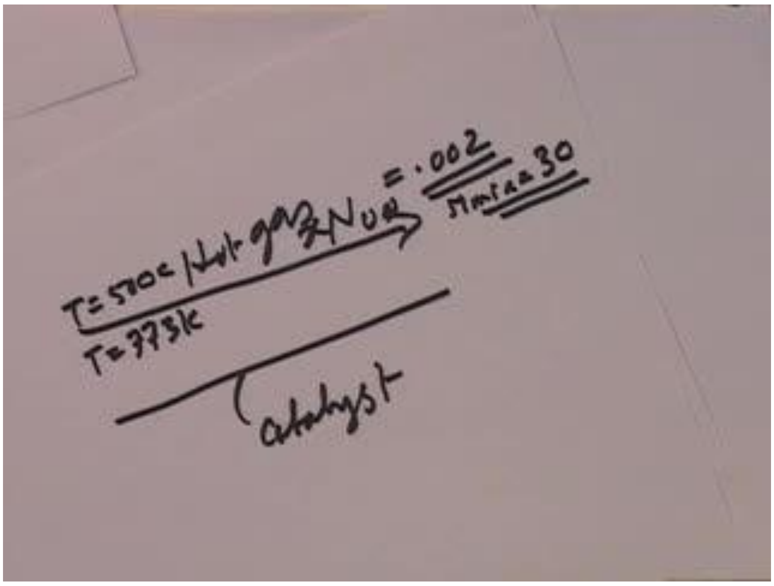

In other words, over the entire flow of the gas over this surface NO does not react with any other substance any other species in the gas and therefore, NO essentially remains constant; the concentration of NO essentially remains constant; it is neither destroyed nor generated in this over this length of the pipe of the surface.

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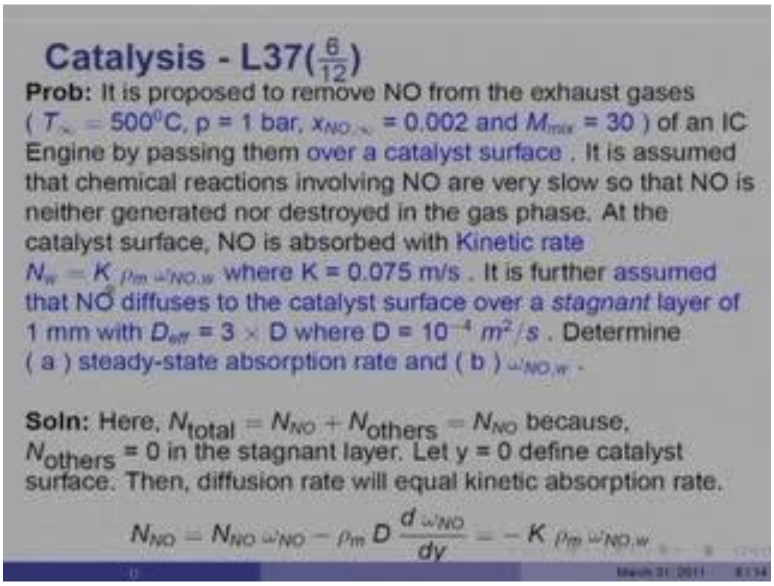

Catalysis - L37($\frac{6}{12}$)

Prob: It is proposed to remove NO from the exhaust gases ($T_\infty = 500^0$C, p = 1 bar, $x_{NO,\infty}$ = 0.002 and M_{mix} = 30) of an IC Engine by passing them over a catalyst surface . It is assumed that chemical reactions involving NO are very slow so that NO is neither generated nor destroyed in the gas phase. At the catalyst surface, NO is absorbed with Kinetic rate $N_w = K\ \rho_m\ \omega_{NO,w}$ where K = 0.075 m/s . It is further assumed that NO diffuses to the catalyst surface over a *stagnant* layer of 1 mm with $D_{eff} = 3 \times D$ where D = $10^{-4}\ m^2/s$. Determine (a) steady-state absorption rate and (b) $\omega_{NO,w}$.

Soln: Here, $N_{\text{total}} = N_{NO} + N_{\text{others}} = N_{NO}$ because, N_{others} = 0 in the stagnant layer. Let y = 0 define catalyst surface. Then, diffusion rate will equal kinetic absorption rate.

$$N_{NO} = N_{NO}\ \omega_{NO} - \rho_m\ D\ \frac{d\ \omega_{NO}}{dy} = -K\ \rho_m\ \omega_{NO,w}$$

March 31, 2011

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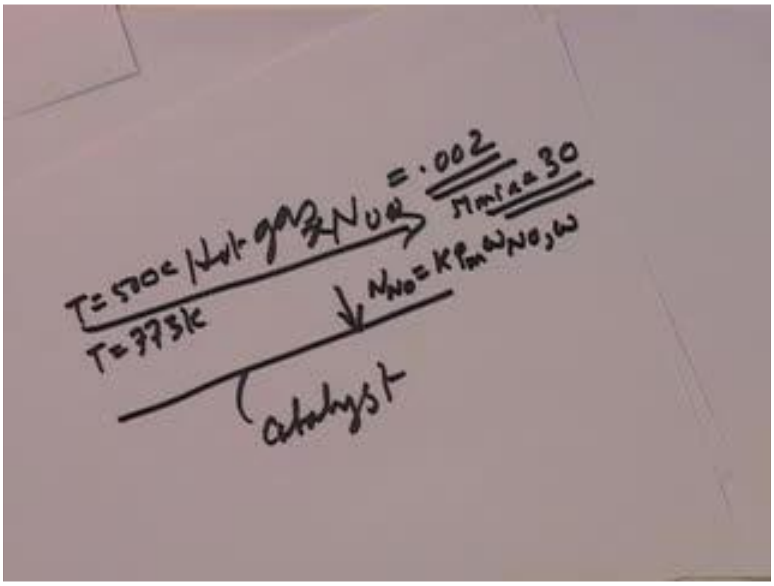

So, we can take NO equal to absolutely constant 0.002 which is the volume mass fraction of the NO as given. Further at the catalyst surface, NO is absorbed with kinetic rate n w now of course, absorbed means it is in this direction and you are told that N NO will be k times rho m into omega NO w, where K is the kinetic rate constant and its value has been given as 0.075 meters per second.

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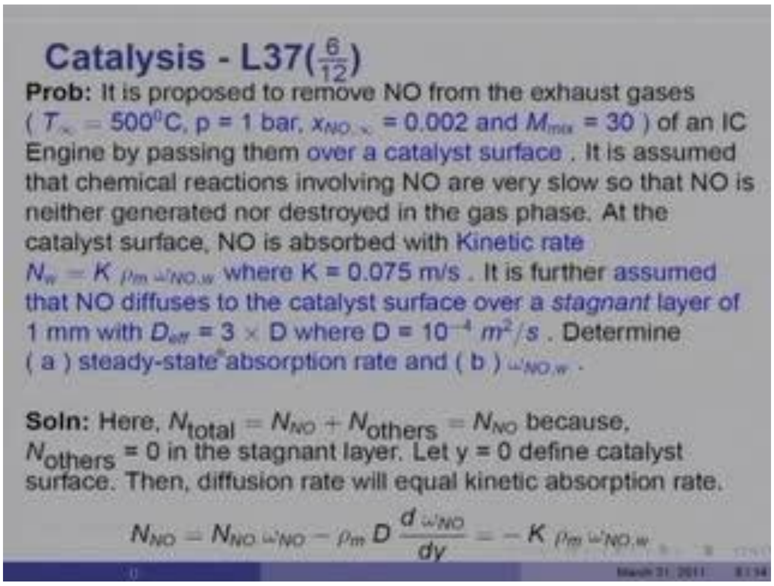

Catalysis - L37($\frac{6}{12}$)

Prob: It is proposed to remove NO from the exhaust gases ($T_\infty = 500^0$C, p = 1 bar, $x_{NO,\infty}$ = 0.002 and M_{mix} = 30) of an IC Engine by passing them over a catalyst surface . It is assumed that chemical reactions involving NO are very slow so that NO is neither generated nor destroyed in the gas phase. At the catalyst surface, NO is absorbed with Kinetic rate $N_w = K \rho_m \omega_{NO,w}$ where K = 0.075 m/s . It is further assumed that NO diffuses to the catalyst surface over a *stagnant* layer of 1 mm with $D_{eff} = 3 \times$ D where D = 10^{-4} m^2/s . Determine (a) steady-state absorption rate and (b) $\omega_{NO,w}$.

Soln: Here, $N_{total} = N_{NO} + N_{others} = N_{NO}$ because, N_{others} = 0 in the stagnant layer. Let y = 0 define catalyst surface. Then, diffusion rate will equal kinetic absorption rate.

$$N_{NO} = N_{NO}\,\omega_{NO} - \rho_m D \frac{d\,\omega_{NO}}{dy} = -K \rho_m \omega_{NO,w}$$

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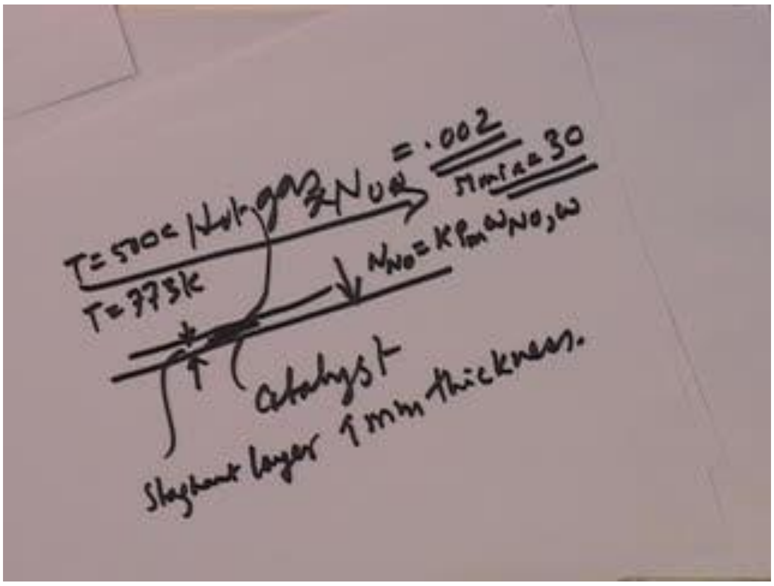

Now, it is further assumed that NO diffuses to the catalyst surface over a stagnant layer of 1 millimeter. What it means is that, near the catalyst surface, the velocity is so low, as you can see it will be like that, so the velocity is so low that we can in fact take this as a very stagnant layer in the extreme case stagnant layer of 1 millimeter thickness.

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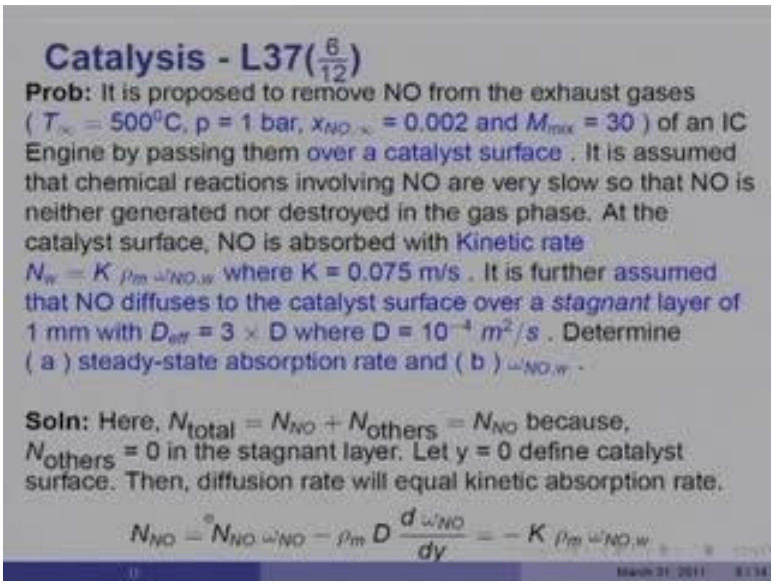

Catalysis - L37($\frac{6}{12}$)

Prob: It is proposed to remove NO from the exhaust gases (T_∞ = 500°C, p = 1 bar, $x_{NO,\infty}$ = 0.002 and M_{mix} = 30) of an IC Engine by passing them over a catalyst surface . It is assumed that chemical reactions involving NO are very slow so that NO is neither generated nor destroyed in the gas phase. At the catalyst surface, NO is absorbed with Kinetic rate $N_w = K \rho_m \omega_{NO,w}$ where K = 0.075 m/s . It is further assumed that NO diffuses to the catalyst surface over a *stagnant* layer of 1 mm with $D_{eff} = 3 \times D$ where $D = 10^{-4}\ m^2/s$. Determine (a) steady-state absorption rate and (b) $\omega_{NO,w}$.

Soln: Here, $N_{total} = N_{NO} + N_{others} = N_{NO}$ because, N_{others} = 0 in the stagnant layer. Let y = 0 define catalyst surface. Then, diffusion rate will equal kinetic absorption rate.

$$N_{NO} = \overset{\circ}{N}_{NO}\,\omega_{NO} - \rho_m D \frac{d\,\omega_{NO}}{dy} = -K\,\rho_m\,\omega_{NO,w}$$

So, that is what we will do and you have been told that take in this in this very small layer, remember there will be some effects of turbulence and therefore, although it is very very small, we have told that, take D effective equal to 3 times D the laminar diffusivity and the laminar diffusivity itself is given as 10 raise to minus 4 meter square per second. So, determine the study state absorption rate of NO and omega NO w.

So, here the total n total will be N N O plus N others, but we have not told that, in this layer, it is a stagnant layer; so N others is 0, because NO is the only one that is diffusing, whereas the other species simply do not diffuse and let y equal to 0 define the catalyst surface, then diffusion rate will equal the kinetic rate and what will be the diffusion rate? N N0 in equal to N NO omega NO which will be the convective rate and this will be the diffusion rate out.

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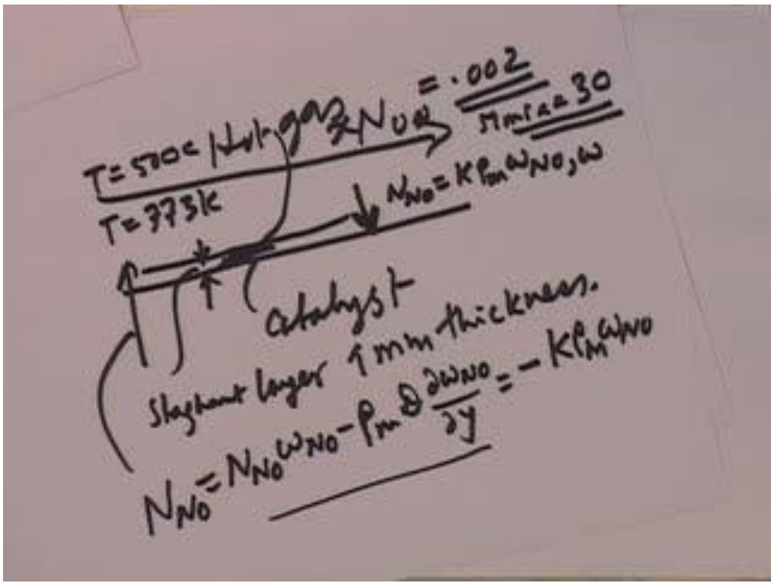

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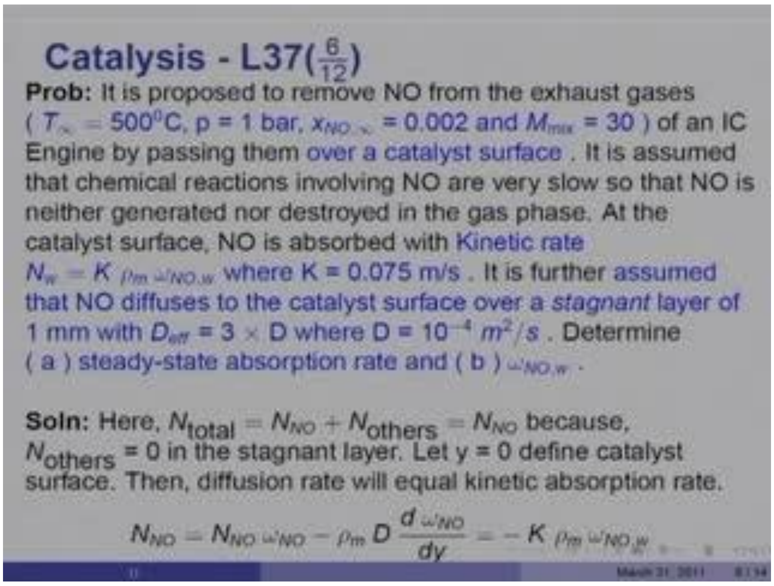

Catalysis - L37($\frac{6}{12}$)

Prob: It is proposed to remove NO from the exhaust gases ($T_\infty = 500^0$C, p = 1 bar, $x_{NO,\infty}$ = 0.002 and M_{mix} = 30) of an IC Engine by passing them over a catalyst surface . It is assumed that chemical reactions involving NO are very slow so that NO is neither generated nor destroyed in the gas phase. At the catalyst surface, NO is absorbed with Kinetic rate $N_w = K\, \rho_m\, \omega_{NO,w}$ where K = 0.075 m/s . It is further assumed that NO diffuses to the catalyst surface over a *stagnant* layer of 1 mm with $D_{eff} = 3 \times$ D where D = 10^{-4} m^2/s . Determine (a) steady-state absorption rate and (b) $\omega_{NO,w}$.

Soln: Here, $N_{\text{total}} = N_{NO} + N_{\text{others}} = N_{NO}$ because, N_{others} = 0 in the stagnant layer. Let y = 0 define catalyst surface. Then, diffusion rate will equal kinetic absorption rate.

$$N_{NO} = N_{NO}\,\omega_{NO} - \rho_m D \frac{d\,\omega_{NO}}{dy} = -K\,\rho_m\,\omega_{NO,w}$$

So, N NO is as shown here, this is the N NO equal to N NO into omega NO minus rho m diffusivity omega NO by d y and that will be equal to the opposite minus K rho m omega NO, because absorption is taking place in this way, whereas our formula is for positive N NO and this is the expression. So, that is what I have written here, N NO is equal to N NO omega NO minus rho m D omega NO by d y equal to minus K rho m omega NO at w.

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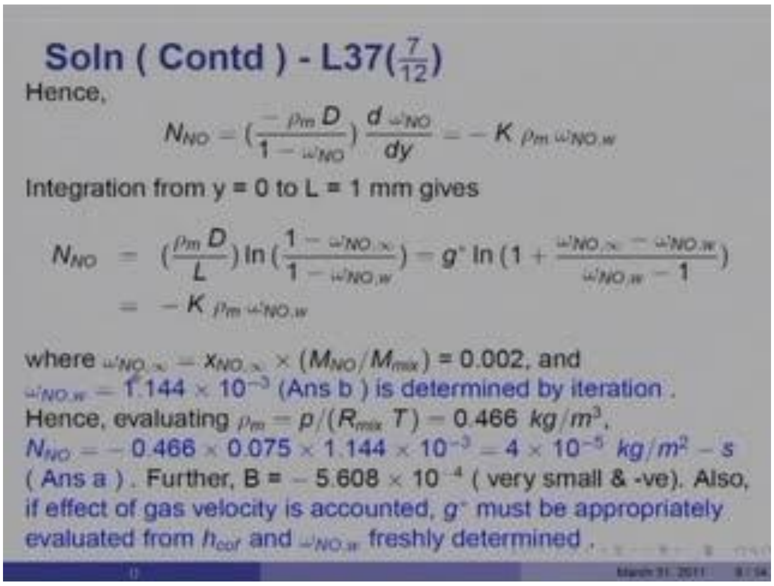

Soln (Contd) - L37($\frac{7}{12}$)

Hence,

$$N_{NO} = \left(\frac{-\rho_m D}{1-\omega_{NO}}\right)\frac{d\,\omega_{NO}}{dy} = -K\,\rho_m\,\omega_{NO,w}$$

Integration from y = 0 to L = 1 mm gives

$$\begin{aligned} N_{NO} &= \left(\frac{\rho_m D}{L}\right)\ln\left(\frac{1-\omega_{NO,\infty}}{1-\omega_{NO,w}}\right) = g^*\ln\left(1+\frac{\omega_{NO,\infty}-\omega_{NO,w}}{\omega_{NO,w}-1}\right) \\ &= -K\,\rho_m\,\omega_{NO,w} \end{aligned}$$

where $\omega_{NO,\infty} = x_{NO,\infty} \times (M_{NO}/M_{mix}) = 0.002$, and $\omega_{NO,w} = 1.144 \times 10^{-3}$ (Ans b) is determined by iteration . Hence, evaluating $\rho_m = p/(R_{mix}\,T) = 0.466\ kg/m^3$, $N_{NO} = -0.466 \times 0.075 \times 1.144 \times 10^{-3} = 4 \times 10^{-5}\ kg/m^2 - s$ (Ans a) . Further, B = -5.608×10^{-4} (very small & -ve). Also, if effect of gas velocity is accounted, g^* must be appropriately evaluated from h_{cor} and $\omega_{NO,w}$ freshly determined .

So, if I rearrange the equation then N NO will be minus rho m D 1 over omega NO d omega NO by d y equal to minus K rho m omega NO w. If I integrate this equation from y equal to 0 to L, the height of the stagnant layer, which is 1 millimeter, then I will get N NO equal to rho m D divided by L l n of 1 minus omega NO infinity divided by 1 minus omega NO w, which can be written as g star l n 1 plus omega NO infinity minus omega NO w over omega NO w minus 1.

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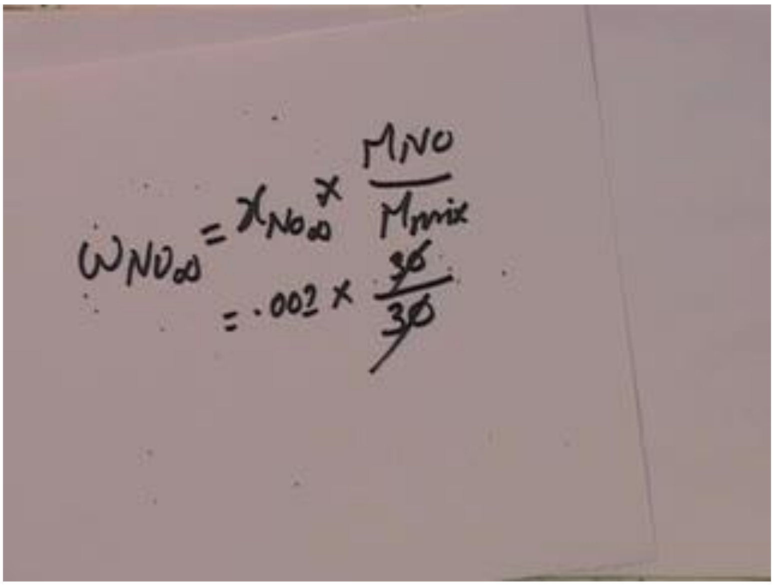

$$\omega_{NO\infty} = x_{NO\infty} \times \frac{M_{NO}}{M_{mix}}$$
$$= .002 \times \frac{\cancel{30}}{\cancel{30}}$$

And g star is nothing but rho m D by L as you know for a Stefan flow module and that would equal minus K rho m NO w. Now, you know already omega is NO infinity, because remember omega NO infinity will be x NO infinity into M NO divided by M mix is a molecular weight and that is given as point 0.002, M NO the molecular weight of NO will be 14 plus 1630 and you have already given molecular weight of mixture is also 30, so this gets cancelled.

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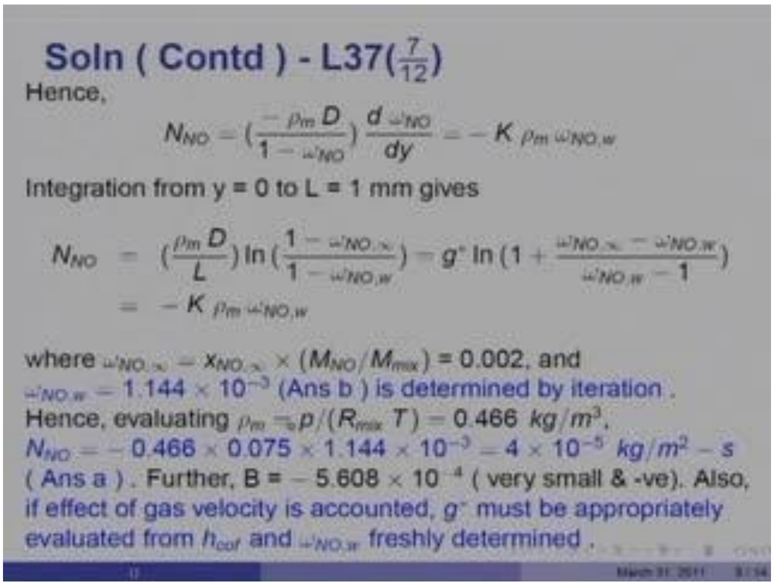

Soln (Contd) - L37($\frac{7}{12}$)

Hence,

$$N_{NO} = (\frac{-\rho_m D}{1-\omega_{NO}}) \frac{d\,\omega_{NO}}{dy} = -K\,\rho_m\,\omega_{NO,w}$$

Integration from y = 0 to L = 1 mm gives

$$N_{NO} = (\frac{\rho_m D}{L}) \ln(\frac{1-\omega_{NO,\infty}}{1-\omega_{NO,w}}) = g^* \ln(1 + \frac{\omega_{NO,\infty} - \omega_{NO,w}}{\omega_{NO,w} - 1})$$
$$= -K\,\rho_m\,\omega_{NO,w}$$

where $\omega_{NO,\infty} = x_{NO,\infty} \times (M_{NO}/M_{mix}) = 0.002$, and $\omega_{NO,w} = 1.144 \times 10^{-3}$ (Ans b) is determined by iteration . Hence, evaluating $\rho_m \simeq p/(R_{mix}\,T) = 0.466\ kg/m^3$, $N_{NO} = -0.466 \times 0.075 \times 1.144 \times 10^{-3} = 4 \times 10^{-5}\ kg/m^2 - s$ (Ans a) . Further, B = -5.608×10^{-4} (very small & -ve). Also, if effect of gas velocity is accounted, g^* must be appropriately evaluated from h_{cof} and $\omega_{NO,w}$ freshly determined .

And in other word, the mass fraction is same as the mole fraction, so 0.002 and omega NO at w will be… if we equate this and this, then by iteration you can determine 1.44 into 10 raise to minus 3, which is in fact very very small.

And therefore, 1 plus b will actually be equal to b itself in which case no iterations are required. Hence, the value of rho m p R mix by T will be 0.466 kg per meter cube. Remember R mix will be 8314 divided by M mix which is 30; in the infinity state omega, infinity is 0.002 and in the ball state, it is point 1.144 into 10 raise to minus 3.

So, first of all one evaluates the mean mass fraction and that is equal to 0.466 and therefore, N NO is equal to minus 0.466 into 0.075, which is the K value already given to you into 1.144, which is omega NO w into 10 raise to minus 3.

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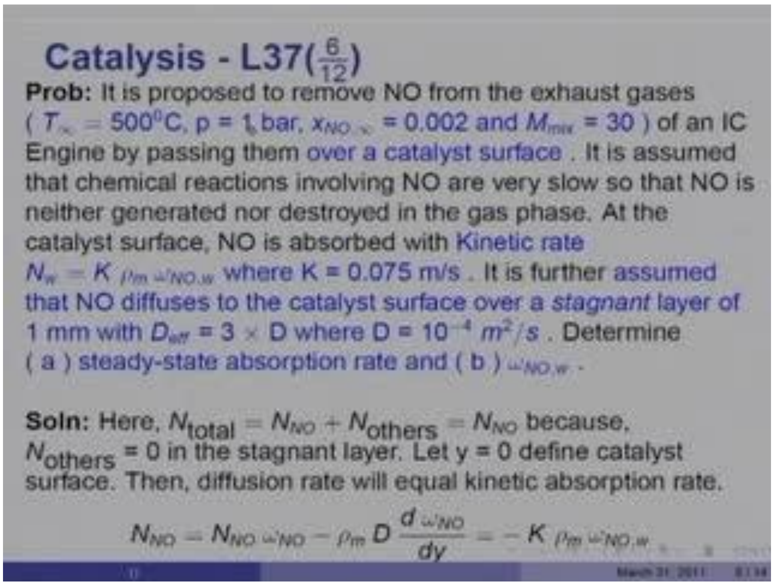

Catalysis - L37($\frac{6}{12}$)

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Soln: Here, $N_{\text{total}} = N_{NO} + N_{\text{others}} = N_{NO}$ because, $N_{\text{others}} = 0$ in the stagnant layer. Let y = 0 define catalyst surface. Then, diffusion rate will equal kinetic absorption rate.

$$N_{NO} = N_{NO}\,\omega_{NO} - \rho_m D \frac{d\,\omega_{NO}}{dy} = -K\,\rho_m\,\omega_{NO,w}$$

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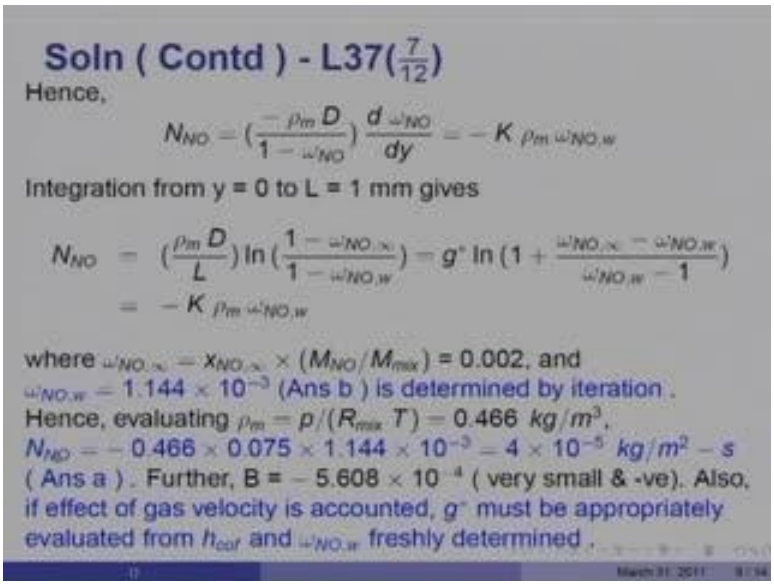

Soln (Contd) - L37($\frac{7}{12}$)

Hence,

$$N_{NO} = \left(\frac{-\rho_m D}{1-\omega_{NO}}\right)\frac{d\,\omega_{NO}}{dy} = -K\,\rho_m\,\omega_{NO,w}$$

Integration from y = 0 to L = 1 mm gives

$$\begin{aligned} N_{NO} &= \left(\frac{\rho_m D}{L}\right)\ln\left(\frac{1-\omega_{NO,\infty}}{1-\omega_{NO,w}}\right) = g^*\ln\left(1+\frac{\omega_{NO,\infty}-\omega_{NO,w}}{\omega_{NO,w}-1}\right) \\ &= -K\,\rho_m\,\omega_{NO,w} \end{aligned}$$

where $\omega_{NO,\infty} = x_{NO,\infty} \times (M_{NO}/M_{mix}) = 0.002$, and $\omega_{NO,w} = 1.144 \times 10^{-3}$ (Ans b) is determined by iteration . Hence, evaluating $\rho_m = p/(R_{mix}\,T) = 0.466\ kg/m^3$, $N_{NO} = -0.466 \times 0.075 \times 1.144 \times 10^{-3} = 4 \times 10^{-5}\ kg/m^2 - s$ (Ans a) . Further, B = -5.608×10^{-4} (very small & -ve). Also, if effect of gas velocity is accounted, g^* must be appropriately evaluated from h_{cor} and $\omega_{NO,w}$ freshly determined .

Remember this is rho m is from the property data you have been given; you take the temperature as 500 degree centigrade, p is 1 bar so you can evaluate the density as p divided by R mix T and 4 6 6 into 0.075 into 1.1 4 4 into 10 raise to minus 3 would give you 4 into 10 raise to minus 5 kilograms per meter square seconds. So that is the first part of the problem.

So, in other words, if you want to remove, this will tell you what is the amount of catalyst surface you will require to obtain given amount of removal of No from that surface, the value of B itself would be…. Because you know now the omega NO w, you now omega NO infinity and that is the value of B would evaluate to minus 5.608 to 10 raise to minus 4 and it is very small as well as notice that it is negative.

Because there is an absorption going on, also if the gas velocity now is accounted, then g star must be appropriately augmented from h cof from the heat transfer situation corresponding heat transfer situation and in which case you will need to evaluate omega NO w freshly and carry out the calculation. But as of now in assuming stagnancy, you get the mass transfer flux is known which is the answer a, and answer b omega NO w is 1.144 into 10 raise to minus 3.

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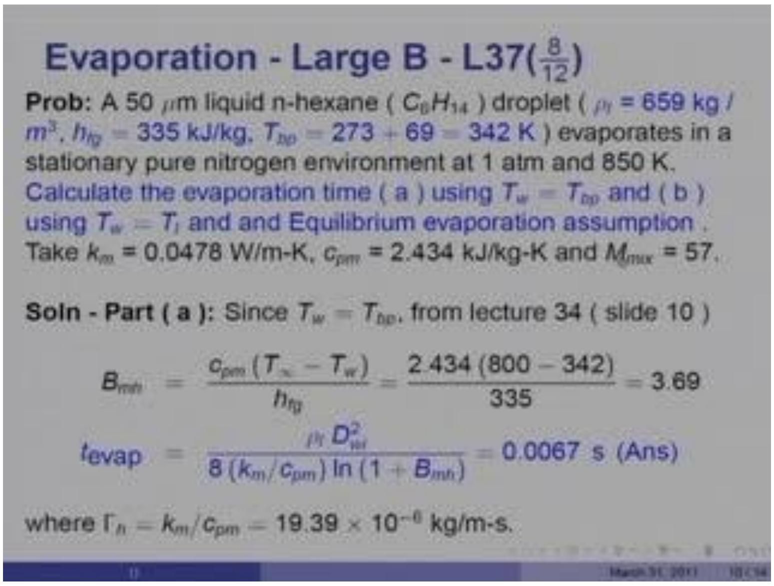

Evaporation - Large B - L37($\frac{8}{12}$)

Prob: A 50 μm liquid n-hexane (C_6H_{14}) droplet (ρ_l = 659 kg / m^3, h_{fg} = 335 kJ/kg, T_{bp} = 273 + 69 = 342 K) evaporates in a stationary pure nitrogen environment at 1 atm and 850 K. Calculate the evaporation time (a) using $T_w = T_{bp}$ and (b) using $T_w = T_l$ and and Equilibrium evaporation assumption . Take k_m = 0.0478 W/m-K, c_{pm} = 2.434 kJ/kg-K and M_{mix} = 57.

Soln - Part (a): Since $T_w = T_{bp}$, from lecture 34 (slide 10)

$$B_{mh} = \frac{c_{pm}(T_\infty - T_w)}{h_{fg}} = \frac{2.434\,(800 - 342)}{335} = 3.69$$

$$t_{evap} = \frac{\rho_l D_{wi}^2}{8\,(k_m/c_{pm})\ln(1 + B_{mh})} = 0.0067 \text{ s (Ans)}$$

where $\Gamma_h = k_m/c_{pm} = 19.39 \times 10^{-6}$ kg/m-s.

Now, we look at evaporation problem. A 50 micro meter liquid n hexane droplet whose density is 659 kg per meter cube its latent heat is 335 kilo joules per kilogram, its boiling point is 69 degree centigrade or 342 K; it evaporates in a stationary pure nitrogen environment at 1 atmosphere and 850 K. So, calculate the evaporation time using T w equal to T b p and using T w equal to T l and equilibrium evaporation assumption. In each case take k m equal to 0.0478, C p m equal to 2.434 kilo joules per kg kelvin and M mix equal to 57.

So, in the part a of the problem since the droplet is at boiling point, B m h would simply evaluate to T infinity minus T b p h f g into C p m and it will be 2.434 into 800 minus 342 divided by 335 kilo joules per kg and therefore, that would be equal to 3.69.

So, notice that when the fuel evaporates in a nitrogen environment, it is prevented from burning and therefore, 3.69 very very high driving forces compared to the problems we have seen so far. And therefore, evaporation rate would be rho l into D w i square 8 times k m by c p m into l n 1 plus B m h equal to 6 . 7 milliseconds; remember gamma h is k m by C p m and that would be 19.39 into 10 raise to minus 6.

So, it takes 6.7 milliseconds when the droplet is initially at its boiling point. Now, suppose we do not we inject it at some other temperature T l in which case we will have to find out what is the value of T w by from equilibrium evaporation assumption that is by iteration.

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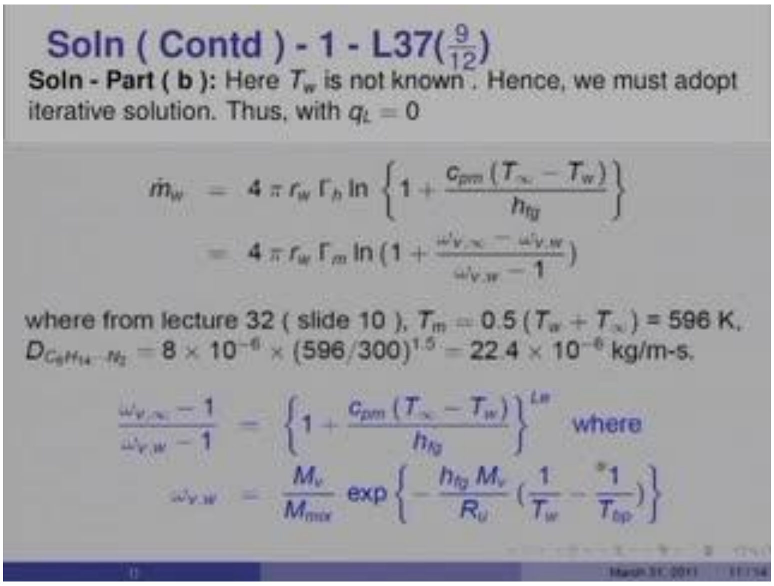

Soln (Contd) - 1 - L37($\frac{9}{12}$)

Soln - Part (b): Here T_w is not known . Hence, we must adopt iterative solution. Thus, with $q_L = 0$

$$\dot{m}_w = 4 \pi r_w \Gamma_h \ln \left\{ 1 + \frac{c_{pm} (T_\infty - T_w)}{h_{fg}} \right\}$$

$$= 4 \pi r_w \Gamma_m \ln \left(1 + \frac{\omega_{v,\infty} - \omega_{v,w}}{\omega_{v,w} - 1} \right)$$

where from lecture 32 (slide 10), $T_m = 0.5 (T_w + T_\infty) = 596$ K, $D_{C_6H_{14}-N_2} = 8 \times 10^{-6} \times (596/300)^{1.5} = 22.4 \times 10^{-6}$ kg/m-s.

$$\frac{\omega_{v,\infty} - 1}{\omega_{v,w} - 1} = \left\{ 1 + \frac{c_{pm} (T_\infty - T_w)}{h_{fg}} \right\}^{Le} \quad \text{where}$$

$$\omega_{v,w} = \frac{M_v}{M_{mix}} \exp \left\{ - \frac{h_{fg} M_v}{R_u} \left(\frac{1}{T_w} - \frac{1}{T_{bp}} \right) \right\}$$

So, T w equal to T l is not known and hence we must adopt iterative solution. Now, since inside the droplet there is no temperature variation, q L will be equal to 0 and therefore, our formula would be m dot w equal to 4 pi r w gamma h l n 1 plus C p m T infinity minus T w divided by h f g and that would also equal b m which is the mass transfer formula; from b m 4 pi r w gamma m, gamma m as you know is rho m multiplied by diffusivity, gamma h is k m divided by specific heat 1 plus omega v infinity minus omega v w omega v w minus 1. And where from lecture 32, T m if we must assume T w and evaluate T m, so at presently I am taking it as let us say 596 k and diffusivity of therefore, I will calculate from our previous lecture, where diffusivity of ethane in nitrogen is also given as 8 into 10 raise to minus 6 t divided by t naught 596 by 300 into 1.5 the pressures are equal to 1 bar, so that is that is 1 atmosphere.

And therefore, nothing to correct for pressure and therefore, diffusivity would be taken as 22.4 into 10 raise to minus 6. So, if we now equate these two equations, then I get gamma h l n 1 plus b h equal to gamma m l n 1 plus b m.

And that would essentially mean that this quantity is equal to 1 plus C p m T infinity by T w divided by h f g raise to Lewis number which is gamma m by gamma h. So, as you can see that would be the Lewis number and omega v w will be from Clausius-Clapeyron equation would be M v by M mix, which is M v being the vapor minus h f g M v R u into 1 over T w minus T b p.

So, what one does is, assume a value of T w and hence, since you know T b p; you know h f g; you know the molecular weight of vapor; you know what is M mix therefore, from T b p, you evaluate omega v w, substitute that value here and calculate the left hand side as well as the right hand side and see if the two are in agreement.

Of course in each case, you must calculate the gamma m and gamma h for each temperature, because the mean temperature would change for each choice of T w, and T infinity is 850 as you know already.

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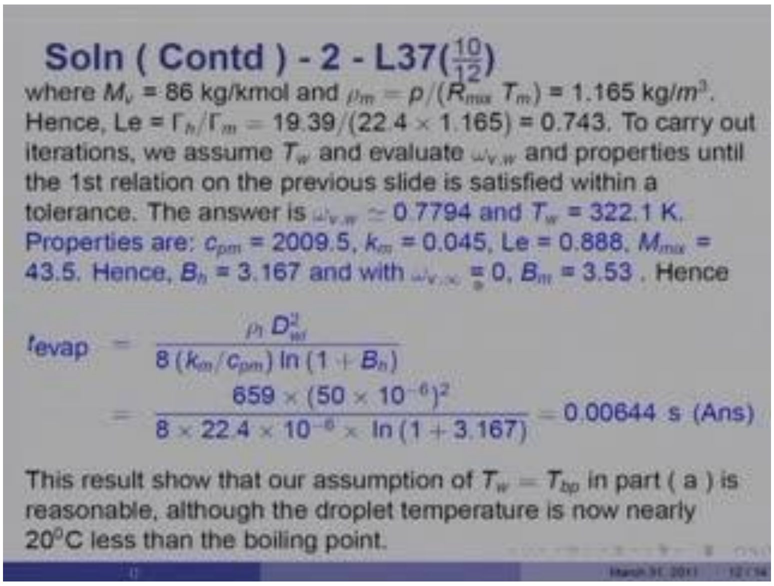

Soln (Contd) - 2 - L37($\frac{10}{12}$)

where M_v = 86 kg/kmol and $\rho_m = p/(R_{mix}\ T_m)$ = 1.165 kg/m^3. Hence, Le = Γ_h/Γ_m = 19.39/(22.4 × 1.165) = 0.743. To carry out iterations, we assume T_w and evaluate $\omega_{v,w}$ and properties until the 1st relation on the previous slide is satisfied within a tolerance. The answer is $\omega_{v,w} \simeq 0.7794$ and T_w = 322.1 K. Properties are: c_{pm} = 2009.5, k_m = 0.045, Le = 0.888, M_{mix} = 43.5. Hence, B_h = 3.167 and with $\omega_{v,\infty} \simeq 0$, B_m = 3.53 . Hence

$$t_{evap} = \frac{\rho_l\, D_{wi}^2}{8\,(k_m/c_{pm})\ \ln\,(1 + B_h)}$$
$$= \frac{659 \times (50 \times 10^{-6})^2}{8 \times 22.4 \times 10^{-6} \times \ln\,(1 + 3.167)} = 0.00644 \text{ s (Ans)}$$

This result show that our assumption of $T_w = T_{bp}$ in part (a) is reasonable, although the droplet temperature is now nearly 20°C less than the boiling point.

So, one need to iterate on these two formula, on these two equations. In our case, M v is 8 6 for n x n, rho m is 1.165 kilograms meter cube, Lewis number is gamma h by gamma m is 0.743. So, to carry out iterations, we assume T w and evaluate omega v w and properties until the first relation on the left hand side on the previous slide is satisfied within tolerance.

So after iterations, the answer is convergence is obtained when ah T w is 322.1 kelvin or 49.1 degree centigrade, this is forty nine point 1 degree centigrade for which omega v w is 0.7794 and the properties are mean properties turn out to be 2009.5 joules per kg per kelvin that is specific heat, conductivity would be 0.45; Lewis number turns out to be 0.888.

And the mixture a molecular weight would turn out to be 43.5 and B h would be 3.167, whereas B m would be 3.53, because omega b infinity is 0. So, do you do see that the effect of Lewis number would be to have b m and b h are different.

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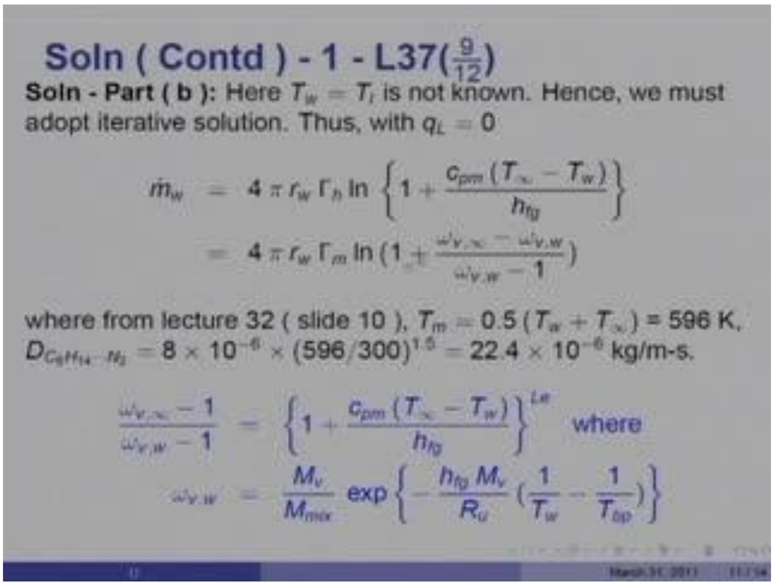

Soln (Contd) - 1 - L37($\frac{9}{12}$)

Soln - Part (b): Here $T_w = T_l$ is not known. Hence, we must adopt iterative solution. Thus, with $q_L = 0$

$$\dot{m}_w = 4\,\pi\,r_w\,\Gamma_h \ln\left\{1 + \frac{c_{pm}\,(T_\infty - T_w)}{h_{fg}}\right\}$$
$$= 4\,\pi\,r_w\,\Gamma_m \ln\left(1 + \frac{\omega_{v,\infty} - \omega_{v,w}}{\omega_{v,w} - 1}\right)$$

where from lecture 32 (slide 10), $T_m = 0.5\,(T_w + T_\infty) = 596$ K, $D_{C_6H_{14}-N_2} = 8 \times 10^{-6} \times (596/300)^{1.5} = 22.4 \times 10^{-6}$ kg/m-s.

$$\frac{\omega_{v,\infty} - 1}{\omega_{v,w} - 1} = \left\{1 + \frac{c_{pm}\,(T_\infty - T_w)}{h_{fg}}\right\}^{Le} \quad \text{where}$$
$$\omega_{v,w} = \frac{M_v}{M_{mix}} \exp\left\{-\frac{h_{fg}\,M_v}{R_u}\left(\frac{1}{T_w} - \frac{1}{T_{bp}}\right)\right\}$$

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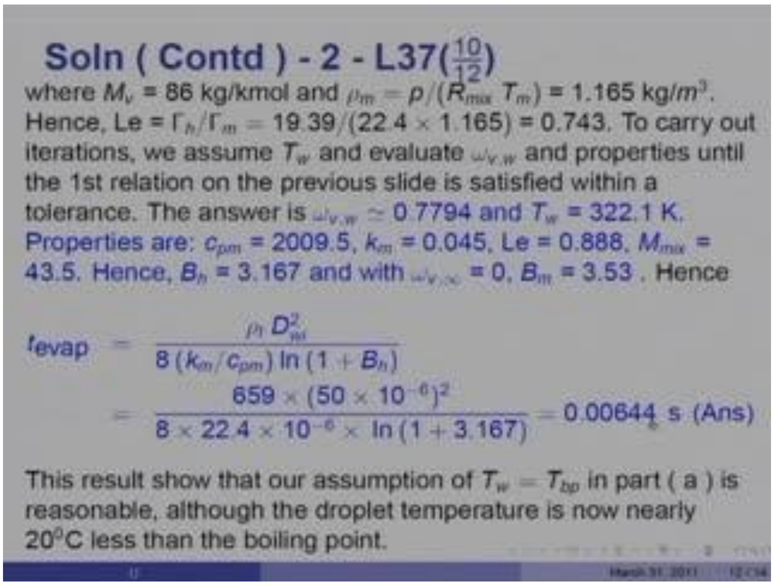


Soln (Contd) - 2 - L37($\frac{10}{12}$)

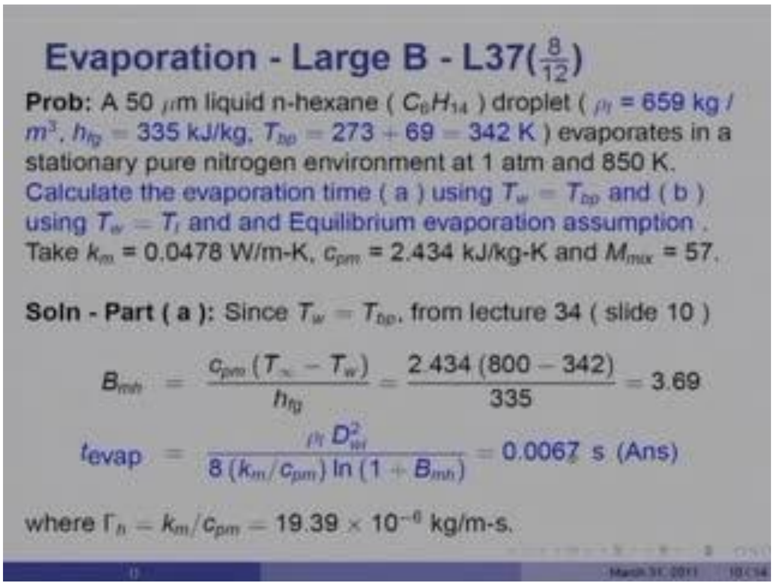
where $M_v = 86$ kg/kmol and $\rho_m = p/(R_{mix}\,T_m) = 1.165$ kg/m^3. Hence, $Le = \Gamma_h/\Gamma_m = 19.39/(22.4 \times 1.165) = 0.743$. To carry out iterations, we assume T_w and evaluate $\omega_{v,w}$ and properties until the 1st relation on the previous slide is satisfied within a tolerance. The answer is $\omega_{v,w} \simeq 0.7794$ and $T_w = 322.1$ K. Properties are: $c_{pm} = 2009.5$, $k_m = 0.045$, $Le = 0.888$, $M_{mix} = 43.5$. Hence, $B_h = 3.167$ and with $\omega_{v,\infty} = 0$, $B_m = 3.53$. Hence

$$t_{evap} = \frac{\rho_l\,D_{wi}^2}{8\,(k_m/c_{pm})\ln(1 + B_h)}$$
$$= \frac{659 \times (50 \times 10^{-6})^2}{8 \times 22.4 \times 10^{-6} \times \ln(1 + 3.167)} = 0.00644 \text{ s (Ans)}$$

This result show that our assumption of $T_w = T_{bp}$ in part (a) is reasonable, although the droplet temperature is now nearly 20°C less than the boiling point.

So, I can use any of the formulae of the previous slide, I can use this formula or I can use this formula; only thing is, I have to use either omega h and b h or omega m and b m. I am evaluating here from omega h therefore, k m by rho C p m and for these conditions of T w which is 49.1 degree centigrade, I evaluate k m by C p m equal to 22.4 into 10 raise to minus 6 and 1 plus 3.167 which is…

(Refer Slide Time: 35:50)



Evaporation - Large B - L37($\frac{8}{12}$)

Prob: A 50 μm liquid n-hexane (C_6H_{14}) droplet (ρ_l = 659 kg / m^3, h_{fg} = 335 kJ/kg, T_{bp} = 273 + 69 = 342 K) evaporates in a stationary pure nitrogen environment at 1 atm and 850 K. Calculate the evaporation time (a) using $T_w = T_{bp}$ and (b) using $T_w = T_l$ and and Equilibrium evaporation assumption . Take k_m = 0.0478 W/m-K, c_{pm} = 2.434 kJ/kg-K and M_{mix} = 57.

Soln - Part (a): Since $T_w = T_{bp}$, from lecture 34 (slide 10)

$$B_{mh} = \frac{c_{pm}\,(T_\infty - T_w)}{h_{fg}} = \frac{2.434\,(800 - 342)}{335} = 3.69$$

$$t_{evap} = \frac{\rho_l\, D_{wi}^2}{8\,(k_m/c_{pm})\ln(1 + B_{mh})} = 0.0067 \text{ s (Ans)}$$

where $\Gamma_h = k_m/c_{pm} = 19.39 \times 10^{-6}$ kg/m-s.

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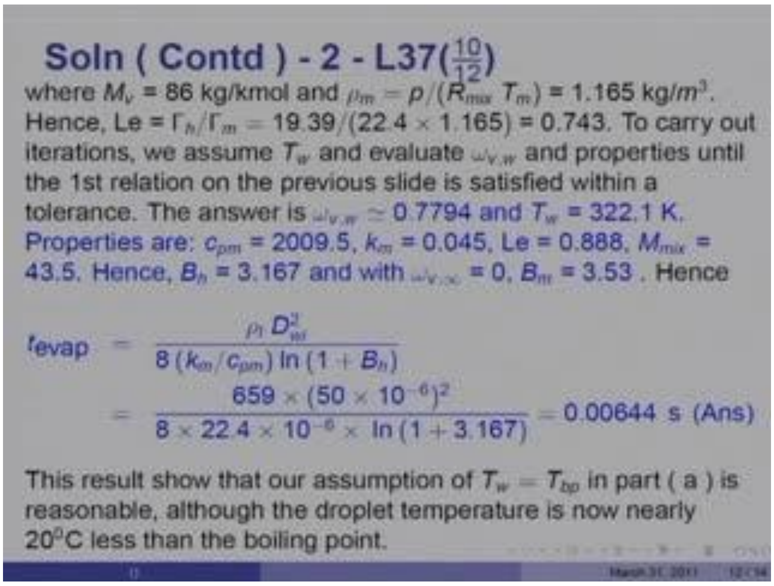

Soln (Contd) - 2 - L37($\frac{10}{12}$)

where M_v = 86 kg/kmol and $\rho_m = p/(R_{mix}\,T_m)$ = 1.165 kg/m^3. Hence, Le = Γ_h/Γ_m = 19.39/(22.4 × 1.165) = 0.743. To carry out iterations, we assume T_w and evaluate $\omega_{v,w}$ and properties until the 1st relation on the previous slide is satisfied within a tolerance. The answer is $\omega_{v,w} \simeq 0.7794$ and T_w = 322.1 K. Properties are: c_{pm} = 2009.5, k_m = 0.045, Le = 0.888, M_{mix} = 43.5. Hence, B_h = 3.167 and with $\omega_{v,\infty}$ = 0, B_m = 3.53 . Hence

$$t_{evap} = \frac{\rho_l\, D_{wi}^2}{8\,(k_m/c_{pm})\ln(1 + B_h)} = \frac{659 \times (50 \times 10^{-6})^2}{8 \times 22.4 \times 10^{-6} \times \ln(1 + 3.167)} = 0.00644 \text{ s (Ans)}$$

This result show that our assumption of $T_w = T_{bp}$ in part (a) is reasonable, although the droplet temperature is now nearly 20^{0}C less than the boiling point.

Now, you get the answer as 6.44 milliseconds 6 point forty 4 millisecond, whereas by injecting at T b p, you had 6.7 milliseconds. Now, you have 6.4 milliseconds so that our assumption of T w in equal to T b in part a is reasonable, although the droplet temperature is now nearly 20 degree centigrade less than the boiling point.

Now, this is as far as evaporation goes, but in actual engines this difference of 0.3 milliseconds can be significant. Because the total burning time itself is in milliseconds and therefore, 0.3 milliseconds for evaporation is the significant quantity for engine applications, but here in order to study evaporation of a volatile fuel what we have done is, we have taken the environment at 85 k to comprise nitrogen.

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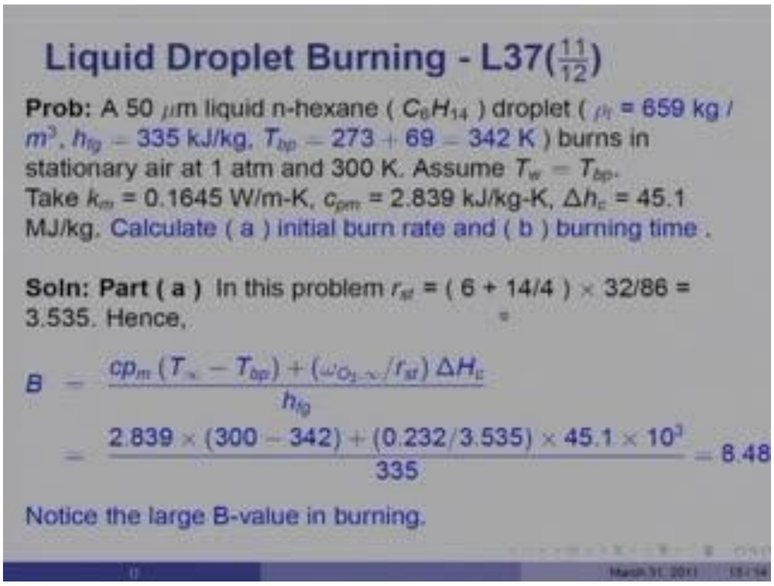

Liquid Droplet Burning - L37($\frac{11}{12}$)

Prob: A 50 μm liquid n-hexane (C_6H_{14}) droplet (ρ_l = 659 kg / m^3, h_{fg} = 335 kJ/kg, T_{bp} = 273 + 69 = 342 K) burns in stationary air at 1 atm and 300 K. Assume $T_w = T_{bp}$. Take k_m = 0.1645 W/m-K, c_{pm} = 2.839 kJ/kg-K, Δh_c = 45.1 MJ/kg. Calculate (a) initial burn rate and (b) burning time .

Soln: Part (a) In this problem r_{st} = (6 + 14/4) × 32/86 = 3.535. Hence,

$$B = \frac{cp_m\,(T_\infty - T_{bp}) + (\omega_{O_2,\infty}/r_{st})\,\Delta H_c}{h_{fg}}$$

$$= \frac{2.839 \times (300 - 342) + (0.232/3.535) \times 45.1 \times 10^3}{335} = 8.48$$

Notice the large B-value in burning.

So, now, let us take the same liquid droplet of n hexane of 50 micron diameter and allow it to burn again the properties are given and it is burning in 1 atmospheric pressure and 300 K, we will assume that the droplet is already at its boiling point, so T w is equal to T b p and the mean conductivity and C p m of the considered phase are given.

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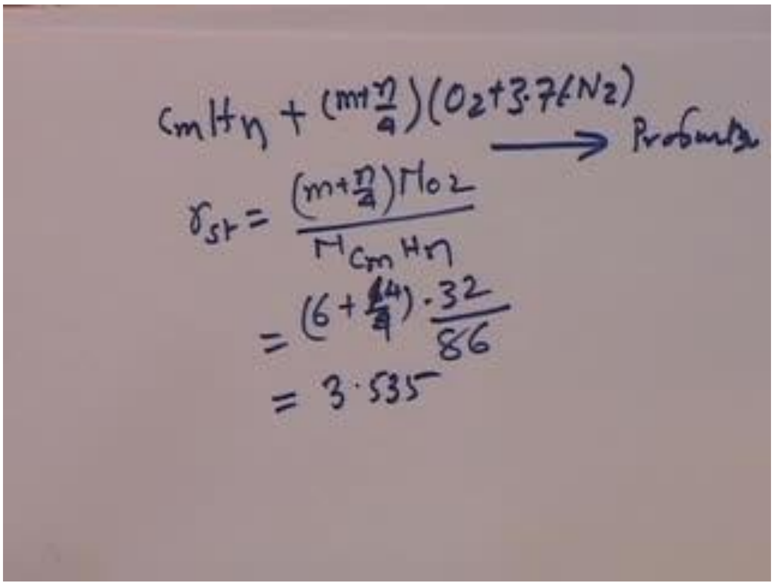


$$C_mH_n + (m+\frac{n}{4})(O_2+3.76N_2) \longrightarrow \text{Products}$$

$$r_{st} = \frac{(m+\frac{n}{4})M_{O_2}}{M_{C_mH_n}}$$

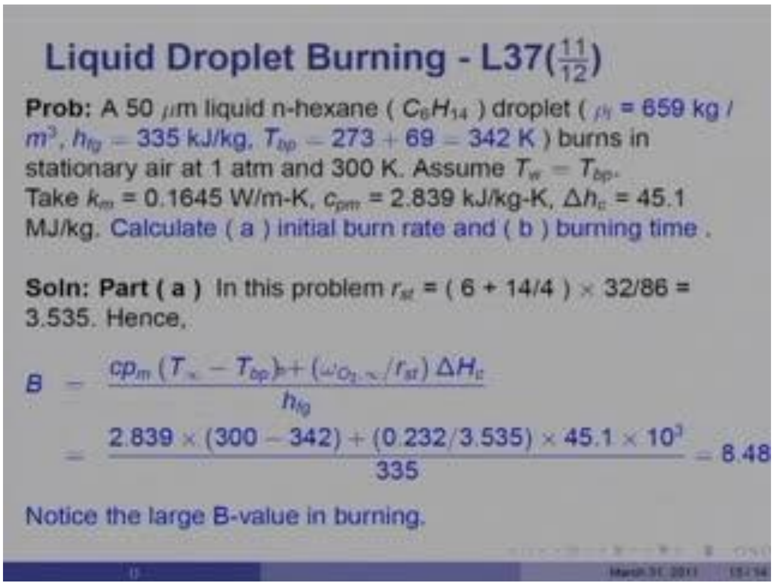
$$= (6+\frac{14}{4})\cdot\frac{32}{86}$$

$$= 3.535$$

The heat of combustion of n hexane is 45.1 mega joules per kilogram and therefore, we want to calculate the initial burning rate and the burning time. Now, for any hydro carbon reaction t m, any hydro carbon reaction C m H n into m plus n by 4 into O 2 plus 3.76 N 2 gives the products.

And therefore, our stoichiometric ratio r s t is equal to m plus n by 4 into molecular weight of O 2 divided by molecular weight of C m H n. So, in our present case, m is equal to 6; n is also equal to 6; n is equal to 14 beg a pardon divided by this is 14 divided by 4 into 3 2 divided by molecular weight of this is 86. And that is what I have done and that would give you 3.535.

(Refer Slide Time: 38:51)



Liquid Droplet Burning - L37($\frac{11}{12}$)

Prob: A 50 μm liquid n-hexane (C_6H_{14}) droplet (ρ_l = 659 kg / m^3, h_{fg} = 335 kJ/kg, T_{bp} = 273 + 69 = 342 K) burns in stationary air at 1 atm and 300 K. Assume $T_w = T_{bp}$. Take k_m = 0.1645 W/m-K, c_{pm} = 2.839 kJ/kg-K, Δh_c = 45.1 MJ/kg. Calculate (a) initial burn rate and (b) burning time .

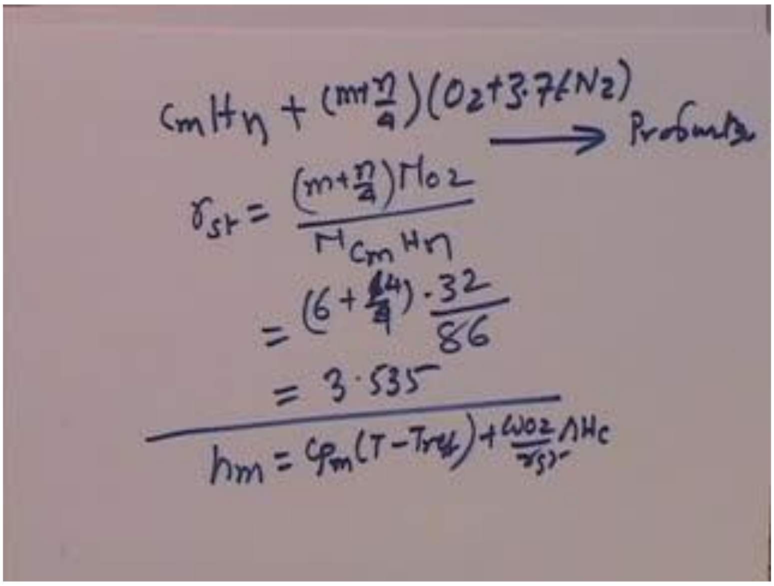
Soln: Part (a) In this problem r_{st} = (6 + 14/4) $\times$ 32/86 = 3.535. Hence,

$$B = \frac{cp_m\,(T_\infty - T_{bp}) + (\omega_{O_2,\infty}/r_{st})\,\Delta H_c}{h_{fg}}$$

$$= \frac{2.839 \times (300 - 342) + (0.232/3.535) \times 45.1 \times 10^3}{335} = 8.48$$

Notice the large B-value in burning.

(Refer Slide Time: 39:16)



$$C_mH_n + \left(m+\frac{n}{4}\right)(O_2+3.76N_2) \longrightarrow \text{Products}$$

$$r_{st} = \frac{\left(m+\frac{n}{4}\right)M_{O_2}}{M_{C_mH_n}} = \left(6+\frac{14}{4}\right)\cdot\frac{32}{86} = 3.535$$

$$h_m = c_{p_m}(T-T_{ref}) + \frac{\omega_{O_2}}{r_{st}}\Delta H_c$$

So, if you look at this, then over the B itself would be C p m into T infinity minus T boiling point, because T w is which is equal to T boiling point omega O 2 infinity r s divided by r s t del H c divided by latent heat. This formula stems from the fact that h m can in defining h m, I can say C p m into T minus T ref plus omega O 2 by r s t del H c, that means, I have associated the heat of combustion with oxygen rather than with fuel.

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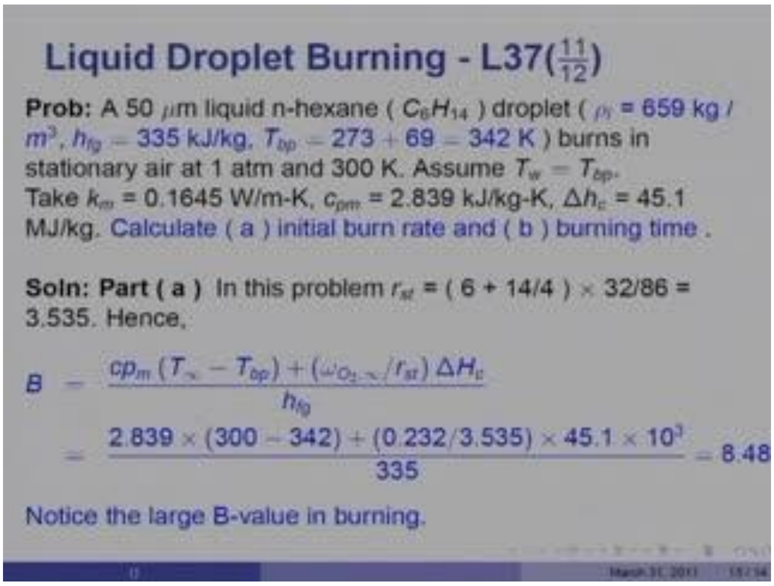

Liquid Droplet Burning - L37($\frac{11}{12}$)

Prob: A 50 μm liquid n-hexane (C_6H_{14}) droplet (ρ_l = 659 kg / m^3, h_{fg} = 335 kJ/kg, T_{bp} = 273 + 69 = 342 K) burns in stationary air at 1 atm and 300 K. Assume $T_w = T_{bp}$. Take k_m = 0.1645 W/m-K, c_{pm} = 2.839 kJ/kg-K, Δh_c = 45.1 MJ/kg. Calculate (a) initial burn rate and (b) burning time .

Soln: Part (a) In this problem r_{st} = (6 + 14/4) × 32/86 = 3.535. Hence,

$$B = \frac{cp_m\,(T_\infty - T_{bp}) + (\omega_{O_2,\infty}/r_{st})\,\Delta H_c}{h_{fg}} = \frac{2.839 \times (300 - 342) + (0.232/3.535) \times 45.1 \times 10^3}{335} = 8.48$$

Notice the large B-value in burning.

And therefore, I must divide this by r s t and that would give you the formula that C p m T infinity minus T b p omega O 2 infinity by r s t del H c divided by h f g and substituting for C p m 2.839 in kilo joules per kg 300minus 342. So, remember the sensible part is negative. But the heat of combustion part is positive 0.232 divided by 3.535 into 45.1 into 10 raise to 3 divided by the heat latent heat that gives you 8.48 8.48.

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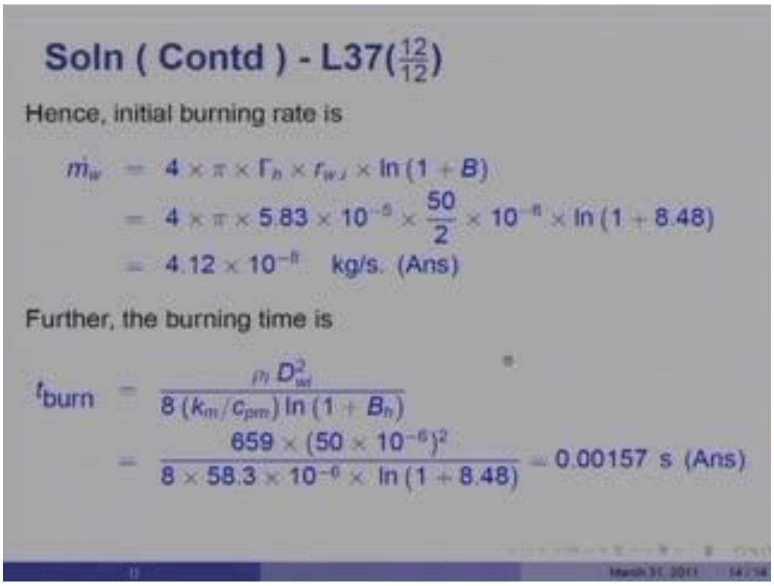

Soln (Contd) - L37($\frac{12}{12}$)

Hence, initial burning rate is

$$\begin{aligned} \dot{m}_w &= 4 \times \pi \times \Gamma_h \times r_{w,i} \times \ln(1+B) \\ &= 4 \times \pi \times 5.83 \times 10^{-5} \times \frac{50}{2} \times 10^{-6} \times \ln(1+8.48) \\ &= 4.12 \times 10^{-8} \quad \text{kg/s. (Ans)} \end{aligned}$$

Further, the burning time is

$$\begin{aligned} t_{burn} &= \frac{\rho_l D_{wi}^2}{8\,(k_m/c_{pm})\ln(1+B_h)} \\ &= \frac{659 \times (50 \times 10^{-6})^2}{8 \times 58.3 \times 10^{-6} \times \ln(1+8.48)} = 0.00157 \text{ s (Ans)} \end{aligned}$$

So, we get even a larger value in a burning problem compared to the evaporation problem, which was 3 .16 or so, here we get it as 8.48 and therefore, we expect the burning rate to be high and the burning time to be small.

So, you can see now here, I can evaluate burning rate, initial burning rate is 4.4 into pi into gamma h which is k m by C p m into r w r which is the initial radius 50 by 2 into 10 raise to minus 6 l n 1 plus 8.48 giving 4.12 into 10 raise to minus 8 kg per second.

That will be the initial burning rate and the burning time would be rho l D w i square 8 into gamma m gamma h into l n 1 plus B h. So, substituting for these values, you now see this is just 1.57 milliseconds.

So, same droplet evaporating, now it’s burning and the time has reduced quite considerably to 1.57 seconds. So, what we have learnt from these sets of problems is that, in general, in air water evaporation problems, the b e is very very small, but when fuels evaporate, b can be substantial, even bigger than 1 of the order of 2 or 3 or something of that kind.

When the when the fuel burns, it would be of the order of 8 to 10 or even little greater than 10 in variety of different liquid fuels, so but all this in stagnant surroundings all this is done in stagnant surroundings. Our interest in convective mass transfer and that we will take up as you go along.

But you will recall that even convective environment can be handled by treating it as a diffusion problem provided we multiply the multiply the effective g star of a Stefan problem by the Sherwood number that is a kind of something we had done in our lecture on Stefan flow model at an earlier time, where I showed you how evaporation of a liquid droplet in a moving surroundings can be handled.