Convection Heat and Mass Transfer

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Module No. # 01

Lecture No. # 36

Evaluation of g and Nw

In the previous lecture, we saw how the Reynolds flow model provides the boundary conditions for the boundary layer flow model. And as you recall that, the Reynolds flow model is nothing but N w equal to g times b. So, today, our interest is to find out how to evaluate g and b so that we can safely assume apply Reynolds flow model for practical problem solving.

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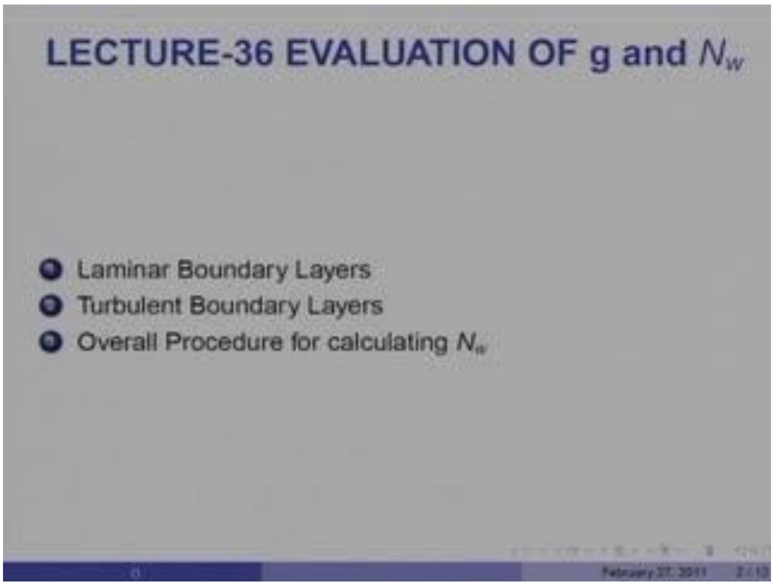

LECTURE-36 EVALUATION OF g and N_w

- Laminar Boundary Layers
- Turbulent Boundary Layers
- Overall Procedure for calculating N_w

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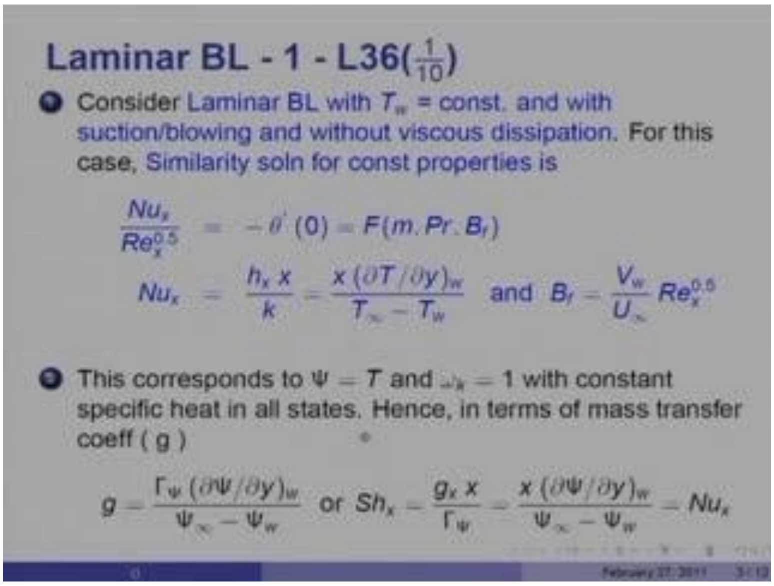

Laminar BL - 1 - L36($\frac{1}{10}$)

- Consider Laminar BL with T_w = const. and with suction/blowing and without viscous dissipation. For this case, Similarity soln for const properties is

$$\frac{Nu_x}{Re_x^{0.5}} = -\theta'(0) = F(m, Pr, B_f)$$

$$Nu_x = \frac{h_x\,x}{k} = \frac{x\,(\partial T/\partial y)_w}{T_\infty - T_w} \quad \text{and} \quad B_f = \frac{V_w}{U_\infty} Re_x^{0.5}$$

- This corresponds to $\Psi = T$ and $\omega_k = 1$ with constant specific heat in all states. Hence, in terms of mass transfer coeff (g)

$$g = \frac{\Gamma_\Psi\,(\partial \Psi/\partial y)_w}{\Psi_\infty - \Psi_w} \quad \text{or} \quad Sh_x = \frac{g_x\,x}{\Gamma_\Psi} = \frac{x\,(\partial \Psi/\partial y)_w}{\Psi_\infty - \Psi_w} = Nu_x$$

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So, I will first consider the laminar boundary layers, then turbulent boundary layers, and then, give the overall procedure for calculating the mass transfer rate Nw. Now, consider laminar boundary layer with T w constant, it is a constant wall temperature. And with suction and blowing, but without viscous dissipation.

Now, for this case, you will recall from our lectures on laminar boundary layers, that the similarity solution for constant properties is N u x divided by R e x to the half equal to minus theta prime 0 as a function of m, the pressure gradient parameter, prandtl number and the suction and blowing parameter b f, and b f as you will recall is nothing but V w by U infinity R e x to the half. And the Nusselt number is given h x X by k is equal to x d T by d y at the whole divided by T infinity minus T w.

This corresponds to In our general mass transfer problem, this correspond to psi equal to T and because this is a single phase fluid, same fluid being blown or suck through the boundary layer, we would have omega k equal to 1. In other words, there are there is no mass transfer driven by a concentration gradient. And if you assume constant specific heats in all states, then this simply represents g gamma psi by d psi by d y at the whole psi infinity minus psi whole.

And therefore, the Sherwood number g x X by gamma psi would be equal to x d psi by d y at the whole psi divide by psi wall equal to N U x. So, these are there is a clear correspondence between between heat transfer and the corresponding mass transfer, where presently the variable T is interpreted in terms of mass transfer g mass transfer coefficient g.

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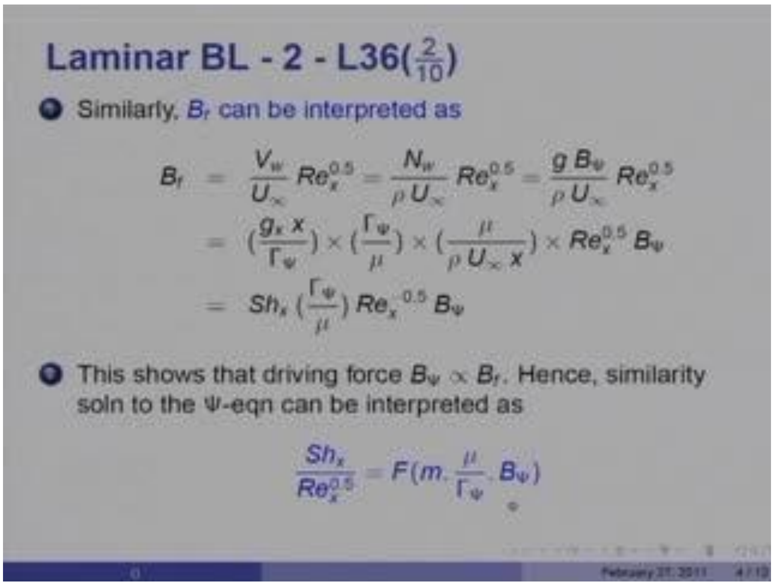

Laminar BL - 2 - L36($\frac{2}{10}$)

- Similarly, B_f can be interpreted as

$$\begin{aligned} B_f &= \frac{V_w}{U_\infty} Re_x^{0.5} = \frac{N_w}{\rho U_\infty} Re_x^{0.5} = \frac{g\, B_\psi}{\rho U_\infty} Re_x^{0.5} \\ &= (\frac{g_x\, x}{\Gamma_\psi}) \times (\frac{\Gamma_\psi}{\mu}) \times (\frac{\mu}{\rho U_\infty x}) \times Re_x^{0.5}\, B_\psi \\ &= Sh_x\, (\frac{\Gamma_\psi}{\mu})\, Re_x^{-0.5}\, B_\psi \end{aligned}$$

- This shows that driving force $B_\psi \propto B_f$. Hence, similarity soln to the Ψ-eqn can be interpreted as

$$\frac{Sh_x}{Re_x^{0.5}} = F(m, \frac{\mu}{\Gamma_\psi}, B_\psi)$$

Now, this will enable us to convert all the data that we had computed for the heat transfer case to the Sherwood converted the Nusselt numbers to Sherwood numbers and also convert B f to the corresponding b psi for the general mass transfer variable. So, we can intepret B f for example, which is V w divided by U infinity R e x to the half; V w is nothing but N w divided by rho and therefore, N w by rho infinity R e x to the half, and N w is g times B psi divided by rho U infinity R e x to the half.

I am going to now multiply this with x and divide by x as well as multiply by gamma psi and divided by gamma psi, then you will see, this is will became g x by gamma psi into gamma psi divided by mu into mu by rho U infinity x into R e x to the half b psi, but rho U infinity x divided by mu is one over Reynolds number and therefore, this will become equal to… this is Sherwood number; this is the gamma psi by mu R e x to the minus half into B psi. So, B f is equivalent to B psi with this multiplier this multiplier.

And this shows that the driving force B psi is proportional to B f and hence the similarity solution to the psi equation can be interpreted as a Sherwood number divided by R e x to the half as a function of m mu by gamma psi which may be taken as Schmidt number and B psi the driving force based on a conserved property psi.

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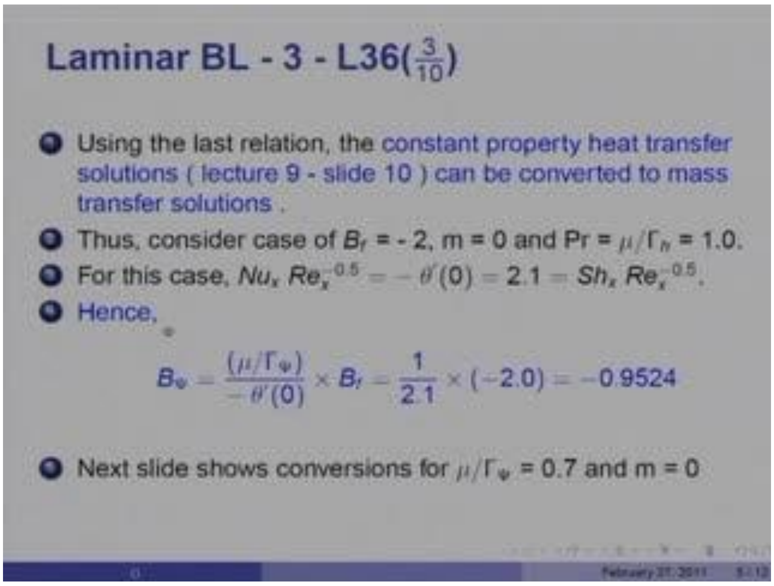

Laminar BL - 3 - L36($\frac{3}{10}$)

- Using the last relation, the constant property heat transfer solutions (lecture 9 - slide 10) can be converted to mass transfer solutions .
- Thus, consider case of B_f = - 2, m = 0 and Pr = μ/Γ_h = 1.0.
- For this case, $Nu_x\, Re_x^{-0.5} = -\theta'(0) = 2.1 = Sh_x\, Re_x^{-0.5}$.
- Hence,

$$B_\psi = \frac{(\mu/\Gamma_\psi)}{-\theta'(0)} \times B_f = \frac{1}{2.1} \times (-2.0) = -0.9524$$

- Next slide shows conversions for μ/Γ_ψ = 0.7 and m = 0

M again here is the pressure gradient parameter of course, but the Prandtl number is replaced by mu by gamma psi which is the Schmidt number and the B f is replaced by B psi, which is the driving force for conserved property psi. So, using the last relation, the constant property heat transfer solutions can be converted to mass transfer solution.

So, consider for example, the case of B f equal to minus two m equal to zero and Prandtl equal to mu by gamma h equal to 1, these are given in lecture number nine and in slide ten and for this particular case, N u x R e x to the minus half equal to minus theta prime 0 was read as was calculated as 2.1. And that would equal Sherwood into R e x to the half.

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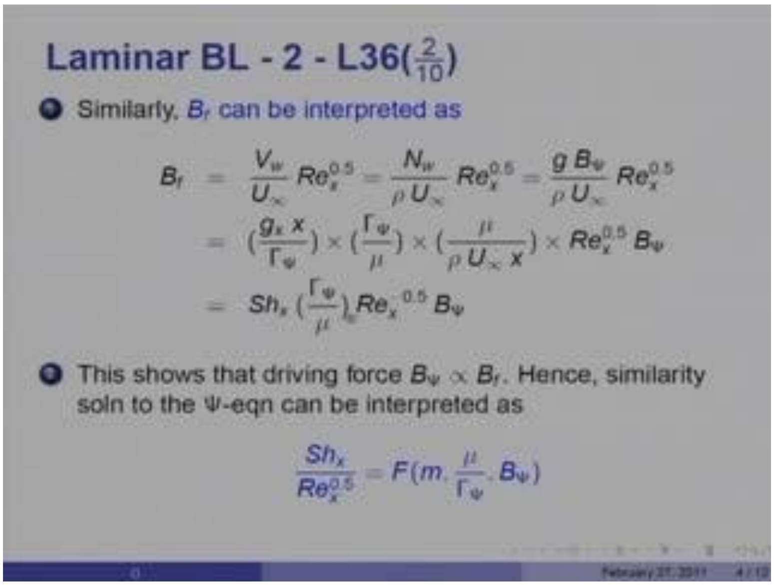

Laminar BL - 2 - L36($\frac{2}{10}$)

- Similarly, B_f can be interpreted as

$$\begin{aligned} B_f &= \frac{V_w}{U_\infty} Re_x^{0.5} = \frac{N_w}{\rho U_\infty} Re_x^{0.5} = \frac{g B_\psi}{\rho U_\infty} Re_x^{0.5} \\ &= (\frac{g_x x}{\Gamma_\psi}) \times (\frac{\Gamma_\psi}{\mu}) \times (\frac{\mu}{\rho U_\infty x}) \times Re_x^{0.5} B_\psi \\ &= Sh_x (\frac{\Gamma_\psi}{\mu}) Re_x^{-0.5} B_\psi \end{aligned}$$

- This shows that driving force $B_\psi \propto B_f$. Hence, similarity soln to the Ψ-eqn can be interpreted as

$$\frac{Sh_x}{Re_x^{0.5}} = F(m, \frac{\mu}{\Gamma_\psi}, B_\psi)$$

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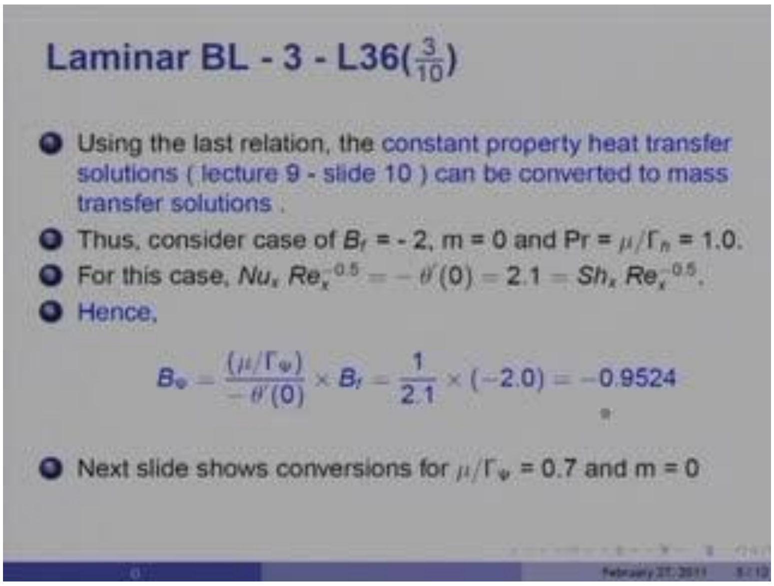

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- Thus, consider case of B_f = - 2, m = 0 and Pr = μ/Γ_h = 1.0.
- For this case, $Nu_x Re_x^{-0.5} = -\theta'(0) = 2.1 = Sh_x Re_x^{-0.5}$.
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- Next slide shows conversions for μ/Γ_ψ = 0.7 and m = 0

So, what will be B psi? As you can see from the previous slide, B psi would be B f divided by S h x R e x to the half into mu by psi and that is what I have done.

So, mu by psi minus theta prime 0 into B f and this is 1over 2.1 into B f which is minus 2.0; so this will become… mu by gamma psi here is 1, so that will be minus 0.9524. So I have this would be the exact conversion when mu by gamma h will be equal to mu by gamma psi.

But if for example, if I now take different Schmidt numbers, so next slide shows convergence for mu over gamma psi equal to 0.7and m equal to 0. So, that is what I am going to do now. Remember here mu by gamma psi is same as mu by gamma h 1, so b psi would be…, whereas B f is minus 2; B psi is minus 0.9524.

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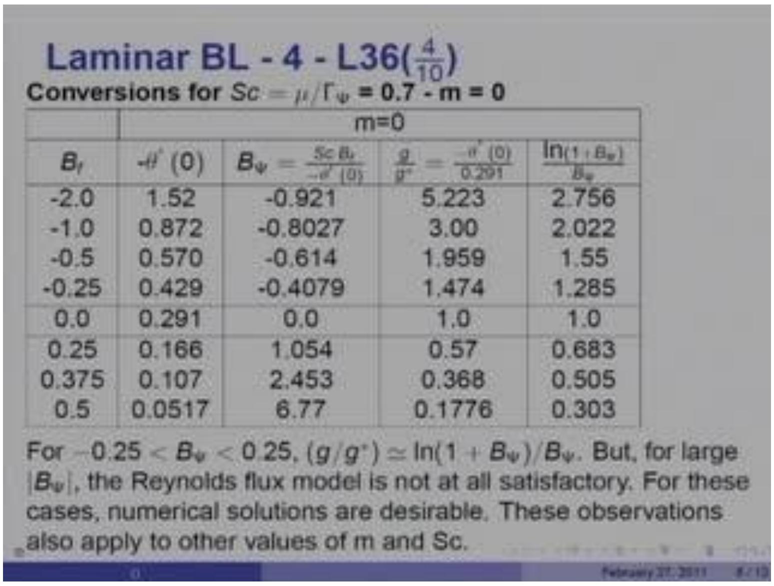

Laminar BL - 4 - L36($\frac{4}{10}$)

Conversions for $Sc = \mu/\Gamma_\psi$ = 0.7 - m = 0

	m=0			
B_f	$-\theta'(0)$	$B_\psi = \frac{Sc\, B_f}{-\theta'(0)}$	$\frac{g}{g^*} = \frac{-\theta'(0)}{0.291}$	$\frac{\ln(1+B_\psi)}{B_\psi}$
-2.0	1.52	-0.921	5.223	2.756
-1.0	0.872	-0.8027	3.00	2.022
-0.5	0.570	-0.614	1.959	1.55
-0.25	0.429	-0.4079	1.474	1.285
0.0	0.291	0.0	1.0	1.0
0.25	0.166	1.054	0.57	0.683
0.375	0.107	2.453	0.368	0.505
0.5	0.0517	6.77	0.1776	0.303

For $-0.25 < B_\psi < 0.25$, $(g/g^*) \simeq \ln(1+B_\psi)/B_\psi$. But, for large $|B_\psi|$, the Reynolds flux model is not at all satisfactory. For these cases, numerical solutions are desirable. These observations also apply to other values of m and Sc.

So, let us see what the convergences are on the next slide. So, here are the convergence B f values are given; here these represent the suction negative values represent suction; the positive values represent blowing 0 of course, means no suction or blowing and these where the values computed in our lecture nine, minus theta prime 0 equal to 1.52,0.872,0.570 and so on and so forth.

The higher the suction, the greater were the Nusselt number as you can recall. Now, correspondingly B psi which is equal to s c Schmidt number into B f divided by minus theta prime 0 would become minus 0.921 and I am now taking mu by gamma psi equal to 0.7, then B psi would assume all these values; notice that for B f is equal to 0.5, B psi actually becomes equal to 6.77, because theta prime 0 is very very small.

We can now calculate g over g star, essentially g star is 0.921, so g over g star would be minus theta prime 0 divided by 0.921. So, this 5.223 simply 1.52 divided by 0.29; 1.872 divided by 0.291 is 3; 0.57 by 0.291 is 1.959, and 0.429 divided by 0.291 is 1.474 and then likewise here, 0.57, 0.368 and so on so forth.

Most important thing to note is that, when B f is very large both on the negative side or positive side and likewise B psi is very large, g over g star does not at all equate to l n 1 plus B psi by B psi. As you can see, this is 5.223, whereas this is 2.754 large variation; same thing at minus 1, same thing at minus 0.5, but for minus 2.5 and bigger values between say 0 and perhaps these two values will come much more close to each other.

Similarly, at on the blowing side, 0.57 and 0.683 are reasonably close, but 0.368 and 0.5 are widely apart and 0.1776 and 0.303 are again very widely apart. So, we can tentatively conclude that for minus 0.25, twenty five B psi g over g star is approximately equal to l n 1 plus B psi by B psi, but for large mass transfer rates, that is, for large driving forces, the Reynolds flux model is not at all satisfactory and for this condition cases, numerical solutions are desirable.

These observations also applied to other values of m and s c. So, by enlarge the whole idea of relating g over g star to l n 1 plus B psi by B psi, this type of convergence can be safely done if B psi values are small, let us say smaller than 0.2 or even 0.1 to be exact, but for any in larger values, we must seek alternative forms; we must seeks numerical solution, but there is a shortcut method and I will represent that to you very shortly.

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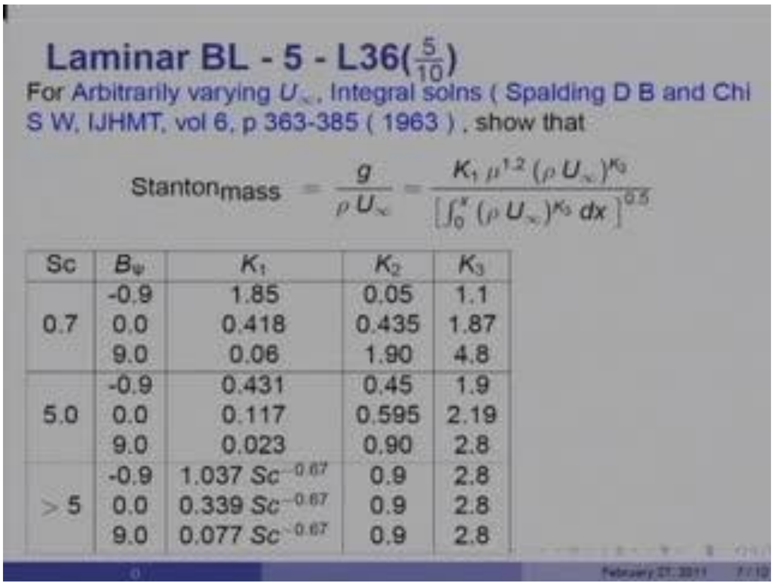

Laminar BL - 5 - L36($\frac{5}{10}$)

For Arbitrarily varying U_∞, Integral solns (Spalding D B and Chi S W, IJHMT, vol 6, p 363-385 (1963) , show that

$$\text{Stanton}_{\text{mass}} = \frac{g}{\rho U_\infty} = \frac{K_1 \mu^{1.2} (\rho U_\infty)^{K_2}}{\left[\int_0^x (\rho U_\infty)^{K_3} dx\right]^{0.5}}$$

Sc	B_ψ	K_1	K_2	K_3
0.7	-0.9	1.85	0.05	1.1
	0.0	0.418	0.435	1.87
	9.0	0.06	1.90	4.8
5.0	-0.9	0.431	0.45	1.9
	0.0	0.117	0.595	2.19
	9.0	0.023	0.90	2.8
> 5	-0.9	$1.037\ Sc^{-0.67}$	0.9	2.8
	0.0	$0.339\ Sc^{-0.67}$	0.9	2.8
	9.0	$0.077\ Sc^{-0.67}$	0.9	2.8

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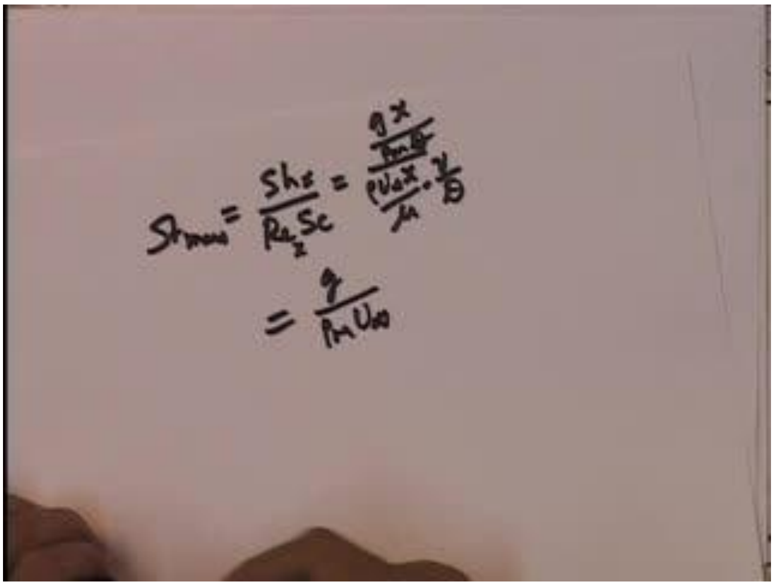

$$St_{mass} = \frac{Sh_x}{Re_x Sc} = \frac{\frac{g x}{\rho_m D}}{\frac{\rho U_\infty x}{\mu} \cdot \frac{\nu}{D}}$$

$$= \frac{g}{\rho_m U_\infty}$$

Now, that was for m equal to zero and therefore, U infinity constant, what about if arbitrary variation of U infinity is there, what would happen? Then the Stanton number based on mass transfer, Stanton number as you know is Nusselt number divided by Reynolds Prandtl, Stanton number for mass transfer will be Sherwood number divided by Reynolds number into Schmidt number and that would equal rho U infinity x by mu, this is Reynolds x; this is x; this will be g x divided by rho m times diffusivity into Schmidt number which is nu times diffusivity.

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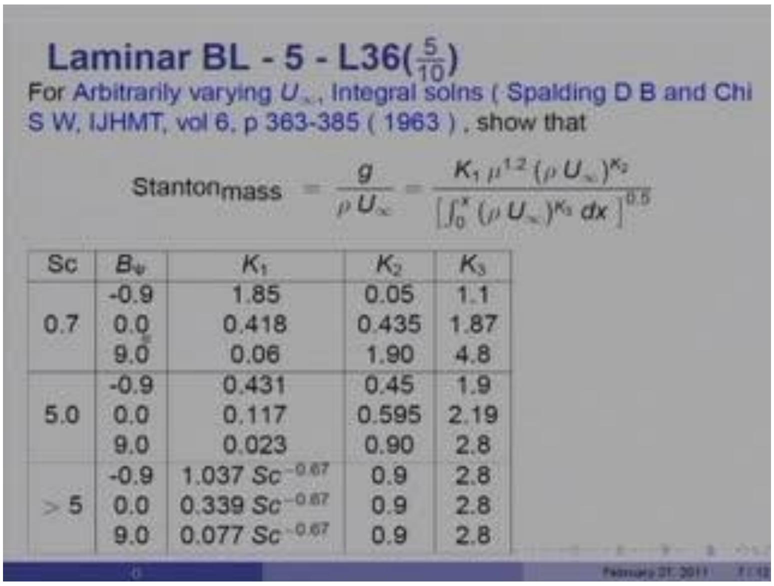

Laminar BL - 5 - L36($\frac{5}{10}$)

For Arbitrarily varying U_∞, Integral solns (Spalding D B and Chi S W, IJHMT, vol 6, p 363-385 (1963) , show that

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Sc	B_ψ	K_1	K_2	K_3
0.7	-0.9	1.85	0.05	1.1
	0.0	0.418	0.435	1.87
	9.0	0.06	1.90	4.8
5.0	-0.9	0.431	0.45	1.9
	0.0	0.117	0.595	2.19
	9.0	0.023	0.90	2.8
> 5	-0.9	$1.037\ Sc^{-0.67}$	0.9	2.8
	0.0	$0.339\ Sc^{-0.67}$	0.9	2.8
	9.0	$0.077\ Sc^{-0.67}$	0.9	2.8

So, you can see that the resultant thing will be g divided by rho m U infinity that is the mass transfer Stanton number. And Spalding and Chi have calculated this problem for arbitrary free stream variation and you will recall this formula, which we had developed for the case of arbitrary variation of U infinity using integral solutions and the same thing has been done for mass transfer by Spalding and Chi.

For three values of Schmidt number 0.7, 5, greater than 5 in each case B psi equal to 0 of course, means no mass transfer B psi equal to minus 0.9 B, psi equal to plus 9 and so on so forth and the k 1, k 2, k 3 are given here; these are simply to be used if needed for calculating the mass transfer coefficient g, but the b value must be converted to B psi first as we demonstrated in the previous slide.

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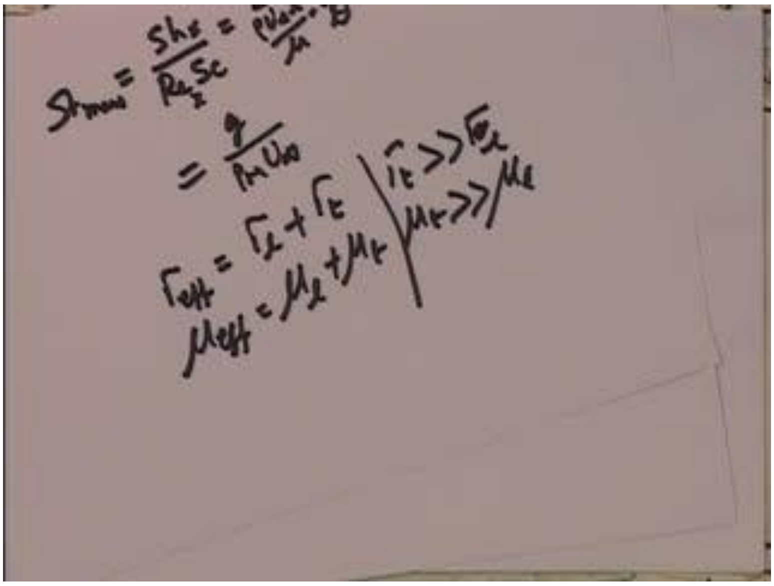

$$St_{mass} = \frac{Sh_x}{Re_x Sc} = \frac{\rho U_\infty x}{\mu} \cdot B$$

$$= \frac{g}{\rho_m U_\infty}$$

$$\Gamma_{eff} = \Gamma_l + \Gamma_t \qquad \Gamma_t >> \Gamma_l$$

$$\mu_{eff} = \mu_l + \mu_t \qquad \mu_t >> \mu_l$$

Now, let us turn to the turbulent boundary layers. Now, in turbulent boundary layers, we have gamma effective equal to gamma laminar plus gamma t, and gamma t is much much greater than gamma sorry gamma gamma l. And we have mu effective equal to mu l plus mu t, and mu t is much much greater than mu l as we have noticed all most of the order hundred in most problems of turbulent flow and therefore, the analogy between momentum and heat and mass transfers actually holds very well for much better in fact for turbulent boundary layers.

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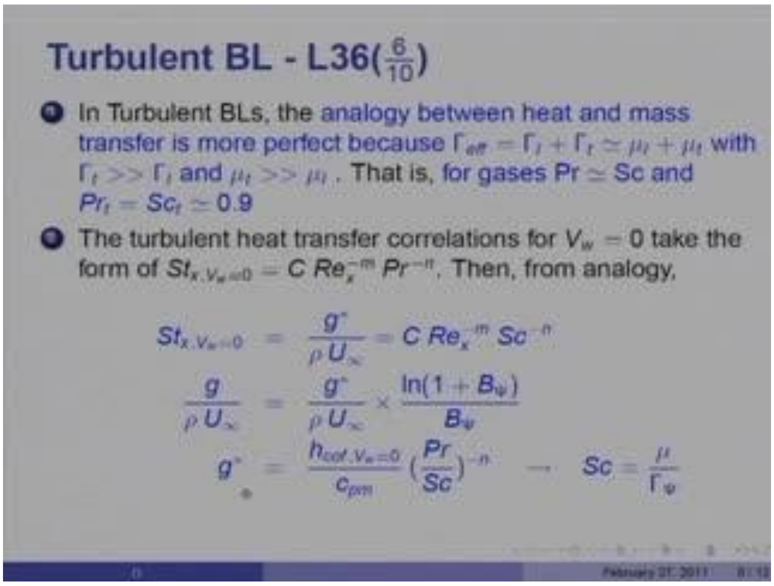

Turbulent BL - L36($\frac{6}{10}$)

- In Turbulent BLs, the analogy between heat and mass transfer is more perfect because $\Gamma_{eff} = \Gamma_l + \Gamma_t \simeq \mu_l + \mu_t$ with $\Gamma_t >> \Gamma_l$ and $\mu_t >> \mu_l$. That is, for gases Pr $\simeq$ Sc and $Pr_t = Sc_t \simeq 0.9$
- The turbulent heat transfer correlations for $V_w = 0$ take the form of $St_{x,V_w=0} = C\, Re_x^{-m}\, Pr^{-n}$. Then, from analogy,

$$St_{x,V_w=0} = \frac{g^*}{\rho U_\infty} = C\, Re_x^{-m}\, Sc^{-n}$$

$$\frac{g}{\rho U_\infty} = \frac{g^*}{\rho U_\infty} \times \frac{\ln(1+B_\psi)}{B_\psi}$$

$$g^* = \frac{h_{cof,V_w=0}}{c_{pm}} \left(\frac{Pr}{Sc}\right)^{-n} \rightarrow Sc = \frac{\mu}{\Gamma_\psi}$$

The turbulent heat transfer correlations for V w equal to 0 take the form of Stanton x V w equal to 0 equal to C Reynolds x to the power of minus m and Prandtl to the minus n this you remember, then from analogy, Stanton x V w equal to 0 would be g star over rho U infinity equal to C R e x to the power of minus m and Schmidt number to the power of minus n.

And g over rho U infinity therefore, will be g star rho u infinity into l n 1 plus B psi by B psi and g star would be simply a heat transfer coefficient that V w equal to 0 divided by C p m into Prandtl divided by Schmidt raise to minus m. So, you can see now, how we can calculate and Schmidt is nothing but mu by gamma psi.

So, we can actually get the effective value of g star if Prandtl number was equal to Schmidt number, then of course, g star would be exactly equal to h cof V w equal to 0 by C p m and Prandtl equal to Schmidt essential means [lewis/louis] number equal to 0 and this part of the result we had already shown in an earlier lecture.

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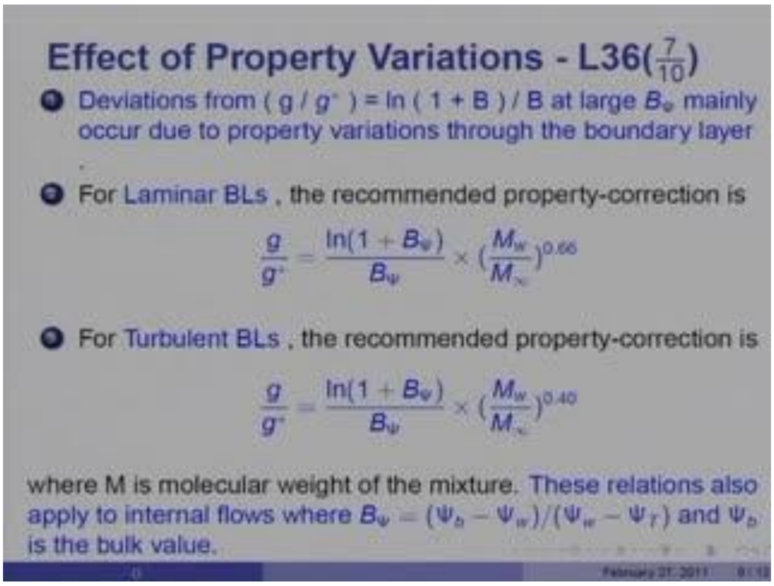

Effect of Property Variations - L36($\frac{7}{10}$)

- Deviations from (g / g*) = ln (1 + B) / B at large B_ψ mainly occur due to property variations through the boundary layer
- For Laminar BLs , the recommended property-correction is

$$\frac{g}{g^*} = \frac{\ln(1+B_\psi)}{B_\psi} \times \left(\frac{M_w}{M_\infty}\right)^{0.66}$$

- For Turbulent BLs , the recommended property-correction is

$$\frac{g}{g^*} = \frac{\ln(1+B_\psi)}{B_\psi} \times \left(\frac{M_w}{M_\infty}\right)^{0.40}$$

where M is molecular weight of the mixture. These relations also apply to internal flows where $B_\psi = (\Psi_b - \Psi_w)/(\Psi_w - \Psi_T)$ and Ψ_b is the bulk value.

So, what this shows is that, in turbulent boundary layers, you must account for Prandtl number and Schmidt number effects to the power of minus n. Of course, in gases Prandtl and Schmidt are very very close, but in other cases, they may be different. So, this is the way one obtains g star in a turbulent boundary layer from a correlation. So, deviations from g over g star equal to l n 1 plus B divided by B at large B w mainly occur due to variations through the layer.

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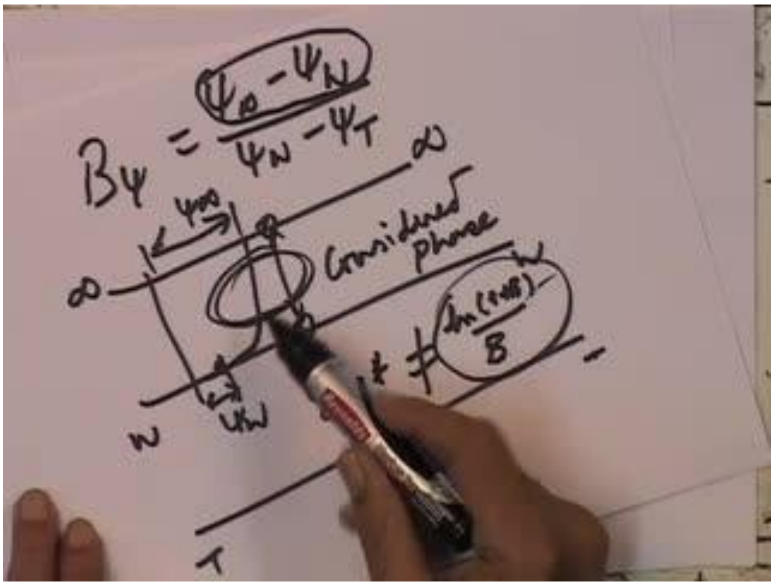

In order to appreciate that, let us see what is B psi. B psi as you will recall is psi value in the infinity state minus psi value in the wall state divided by psi value in the wall state minus psi value in the t state. So, if I have this as w w, this as infinity state and this as t state, then a large B psi means that psi infinity minus psi w is very very large.

In other words, this value and this value are separated significantly in the considered phase and therefore, it is quite in effect you can get a profile like that with the value of psi w here and value of psi infinity there. So, if these values are widely separated, it is quite likely that there would be property variations in the considered phase.

So, the reason why g over g star is not equal to l n one plus B divided by B is simply because this relation was develop for constant property solutions as we saw earlier and therefore, it only applies to very small mass transfer rates, but the moment b become very large, you must account for property variations.

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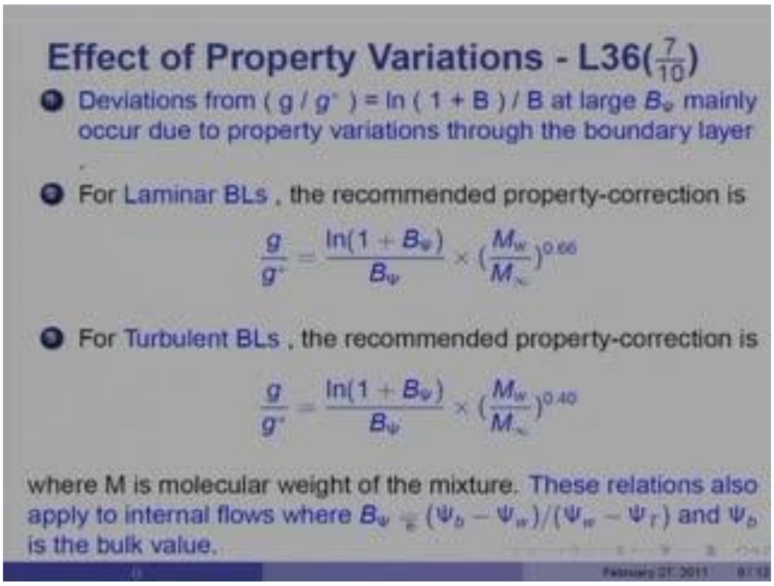

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where M is molecular weight of the mixture. These relations also apply to internal flows where $B_\psi = (\Psi_b - \Psi_w)/(\Psi_w - \Psi_T)$ and Ψ_b is the bulk value.

Now, in order to avoid this solution of the complete set of boundary layer equations, here is an advice. A laminar boundary layers one could easily take, write g over g star equal to l n 1 plus B w by B psi multiplied by molecular weight of the mixture in the w state divided by molecular weight of the mixture in the infinity state raise to 0.66.

This is from this this correction is derived from numerical solution of the differential complete differential equations for large b and allowing for molecular property variations. So, there is no need now again to solve the differential equations; we can simply take this as a correction for laminar boundary layer. Likewise, in the turbulent boundary layer, the recommended property correction is same thing M w over n divided by M infinity raise to 0.4

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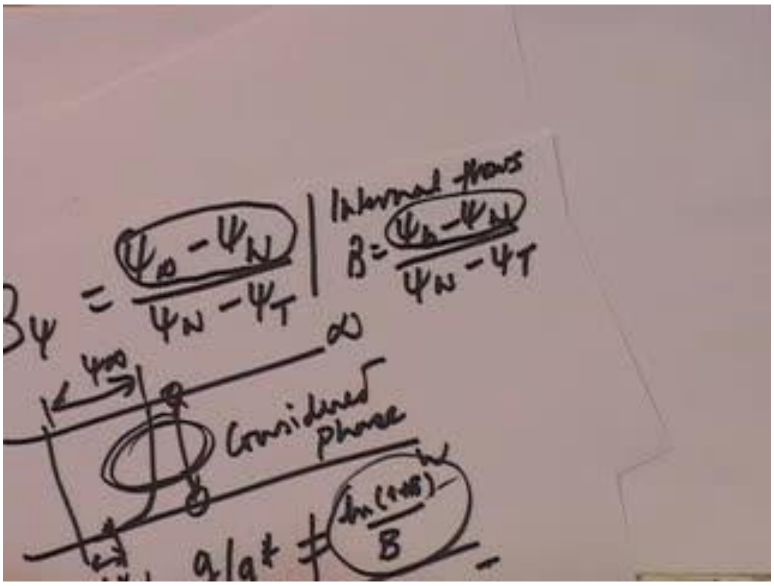

Now, these relations these sorts of corrections that we have indicated also apply to internal flows, the only difference is instead of psi infinity divided by psi w, so in internal flows instead of psi infinity, we simply take B equal to psi of the bulk value minus psi wall divided by psi wall minus psi t. So that is the only difference, quite analogs to what we do in heat transfer, where we define heat transfer based on the bulk value and wall value in internal flows, whereas we define it based on infinity and wall value in external boundary layers.

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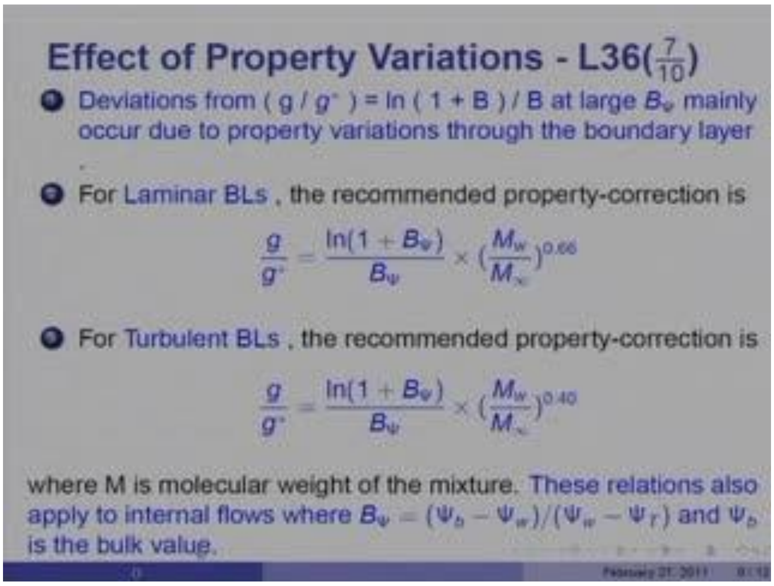

Effect of Property Variations - L36($\frac{7}{10}$)

- Deviations from (g / g*) = ln (1 + B) / B at large B_ψ mainly occur due to property variations through the boundary layer
- For Laminar BLs , the recommended property-correction is

$$\frac{g}{g^*} = \frac{\ln(1 + B_\psi)}{B_\psi} \times (\frac{M_w}{M_\infty})^{0.66}$$

- For Turbulent BLs , the recommended property-correction is

$$\frac{g}{g^*} = \frac{\ln(1 + B_\psi)}{B_\psi} \times (\frac{M_w}{M_\infty})^{0.40}$$

where M is molecular weight of the mixture. These relations also apply to internal flows where $B_\psi = (\Psi_b - \Psi_w)/(\Psi_w - \Psi_T)$ and Ψ_b is the bulk value.

So, with that change you simply interpret B psi equal to psi b minus psi wall divided by psi wall minus psi t, and psi B is the bulk value calculated as usual from integration of the actual velocity profile and psi profile.

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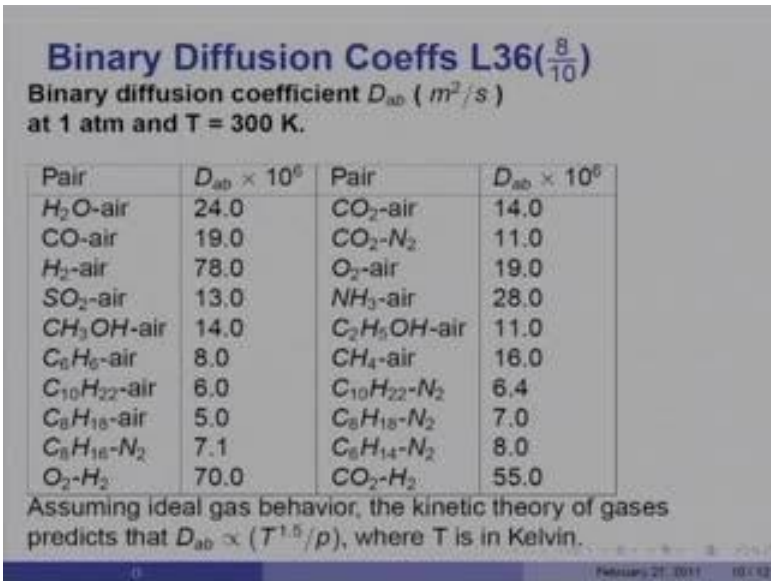

Binary Diffusion Coeffs L36($\frac{8}{10}$)

Binary diffusion coefficient D_{ab} (m^2/s) at 1 atm and T = 300 K.

Pair	$D_{ab} \times 10^6$	Pair	$D_{ab} \times 10^6$
H_2O-air	24.0	CO_2-air	14.0
CO-air	19.0	CO_2-N_2	11.0
H_2-air	78.0	O_2-air	19.0
SO_2-air	13.0	NH_3-air	28.0
CH_3OH-air	14.0	C_2H_5OH-air	11.0
C_6H_6-air	8.0	CH_4-air	16.0
$C_{10}H_{22}$-air	6.0	$C_{10}H_{22}$-N_2	6.4
C_8H_{18}-air	5.0	C_8H_{18}-N_2	7.0
C_8H_{18}-N_2	7.1	C_6H_{14}-N_2	8.0
O_2-H_2	70.0	CO_2-H_2	55.0

Assuming ideal gas behavior, the kinetic theory of gases predicts that $D_{ab} \propto (T^{1.5}/p)$, where T is in Kelvin.

Now, in order to help computations what I have done here is, to give you the values of binary diffusion coefficient D a b so that you can evaluate Schmidt numbers and other quantities for any problem that we shall be encountering in the subsequent lectures.

So, for example, D a b is the diffusion coefficient; it is always defined for a pair of gases a and b. So, here I am defining it for example, water vapor and air 24 into 10 raise to minus 6; CO in air 19 into 10 raise to minus 6 and so on and so forth.

So, this is for your ready reference which you can use; the only thing to remember though is that, these values are at temperature of 300 K and 1 atmosphere, but many times we have higher pressure particularly in combustion problems and temperature would also be higher in which case you simply make a correction.

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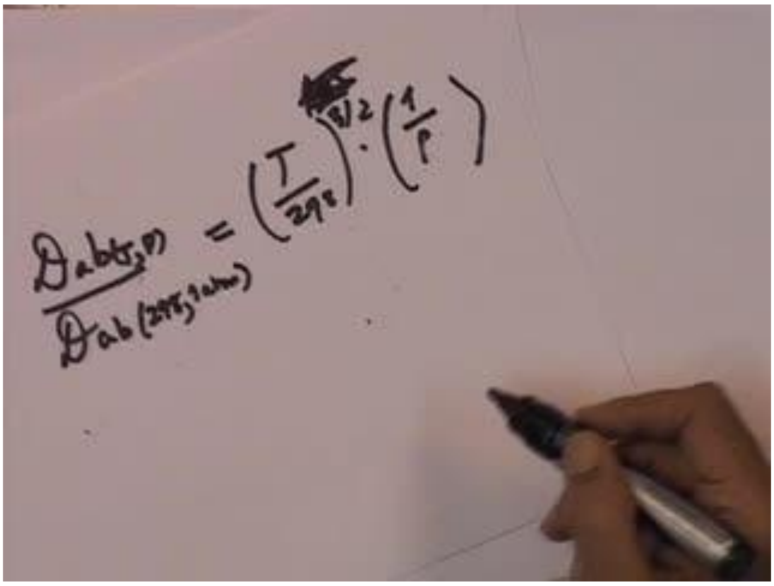

$$\frac{D_{ab(T,P)}}{D_{ab(298,1atm)}} = \left(\frac{T}{298}\right)^{3/2} \cdot \left(\frac{1}{P}\right)$$

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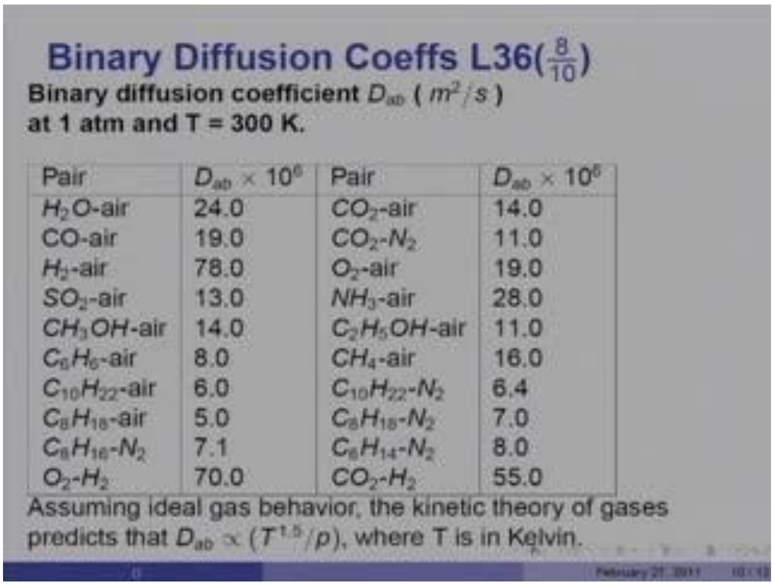

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Assuming ideal gas behavior, the kinetic theory of gases predicts that $D_{ab} \propto (T^{1.5}/p)$, where T is in Kelvin.

So, D a b at any temperature and pressure divided by D a b at 298 and 1 atmosphere is simply T raise to T divided by 298 raise to 1.5 or 3/2rather divided by 1 over p, so that is how one calculates. So, the higher the pressure, lower is the value of D a b, but D a b is proportional to t to the half so and temperature is to be evaluated in kelvin that is to be remember. So, this is just for your ready reference.

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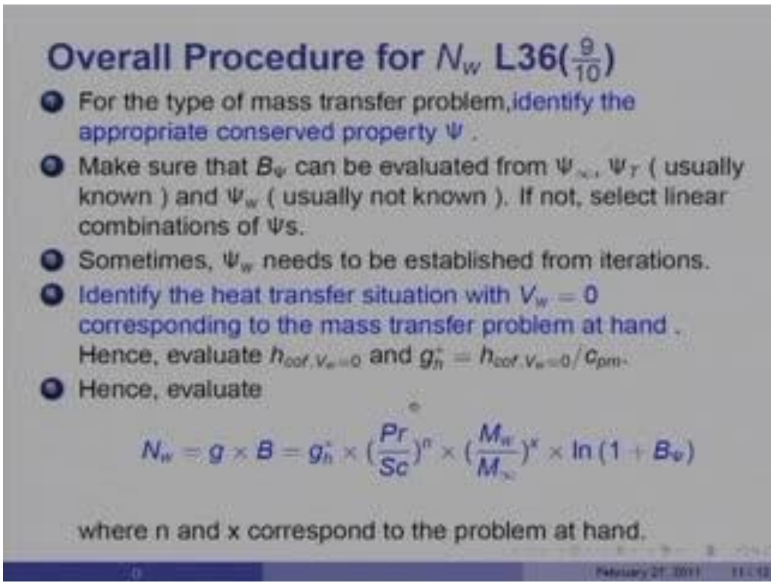

Overall Procedure for N_w L36($\frac{9}{10}$)

- For the type of mass transfer problem,identify the appropriate conserved property Ψ .
- Make sure that B_Ψ can be evaluated from Ψ_∞, Ψ_T (usually known) and Ψ_w (usually not known). If not, select linear combinations of Ψs.
- Sometimes, Ψ_w needs to be established from iterations.
- Identify the heat transfer situation with $V_w = 0$ corresponding to the mass transfer problem at hand . Hence, evaluate $h_{cof, V_w = 0}$ and $g_h^* = h_{cof, V_w = 0} / c_{pm}$.
- Hence, evaluate

$$N_w = g \times B = g_h^* \times (\frac{Pr}{Sc})^n \times (\frac{M_w}{M_\infty})^x \times \ln(1 + B_\Psi)$$

where n and x correspond to the problem at hand.

So, then for overall procedure now for calculating N w will be for all types of mass transfer problems that we encountered. Identify first the appropriate conserved property psi and we have shown how to do this in inert mass transfer, inert mass transfer with heat transfer, mass transfer with simple chemical reaction and mass transfer with arbitrary chemical reaction.

So, we know to how to evaluate psi. Make sure that B psi can be evaluated from psi infinity psi, psi t and psi w. Now, usually psi infinity and psi t are known, but psi w is usually not known and therefore, it has to be determined. There are various ways to avoid knowing psi w, for example, psi w can be rendered 0, simply by taking t ref as t w.

But many a times that is not possible, and then, we must select such combinations of psi that the composite variable would not require value of psi w; we will we will shown in practical problem of this nature a little later.

But many a times psi w needs be established from iterations and I have given you already one example of this type, but we will see many more as we go into the next lectures. Then for the mass transfer problem at hand, identify the corresponding heat transfer situation with V w equal to 0 for which the correlation is available and hence evaluate the value of h cof V w equal to 0.

So, as to evaluate g star h is equal to h cof V w equal to 0 divided by C p m the specific heat of the mixture and hence, then evaluate N w equal to g B which is equal to g star h Prandtl divided by Schmidt raise to n which will come from the correlation itself. Because Prandtl raise to n and Schmidt raise to n are identical, because Nusselt x equal to Sherwood x at constant properties, and then, allow for variable property correction M w by M infinity raise to some power x into l n 1 plus B psi.

So, n and x are to be carefully chosen from the correlation itself and x value only take in laminar flow 0.66; in turbulent flow, 0.4. So, this can be easily… So, that is the overall procedure, then for calculating N w using Reynolds flow model, it will require no solution of any differential equation, only thing is the psi must be carefully chosen so that B can be cal[culated]- B psi can be calculated, and then, appropriate co relation must be chosen with appropriate values of n and x again depending on whether its laminar or turbulent boundary layer.

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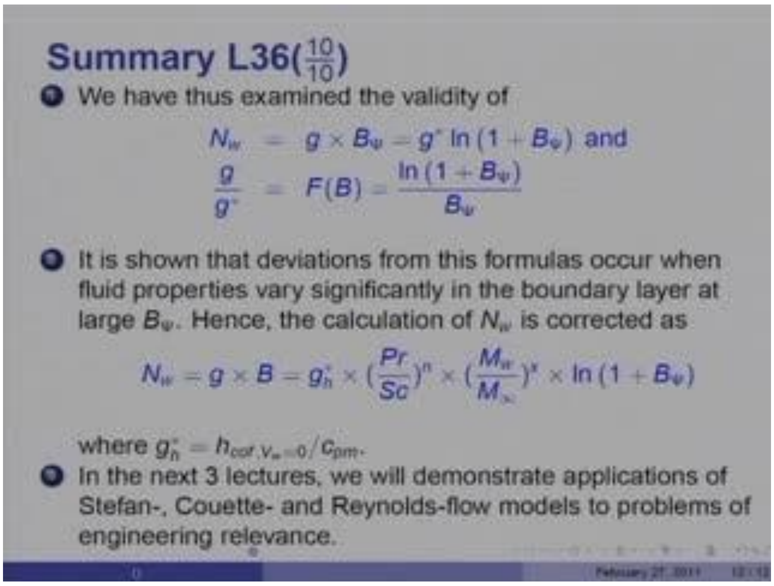

Summary L36($\frac{10}{10}$)

- We have thus examined the validity of

$$N_w = g \times B_\psi = g^* \ln(1 + B_\psi) \text{ and}$$
$$\frac{g}{g^*} = F(B) = \frac{\ln(1 + B_\psi)}{B_\psi}$$

- It is shown that deviations from this formulas occur when fluid properties vary significantly in the boundary layer at large B_ψ. Hence, the calculation of N_w is corrected as

$$N_w = g \times B = g_h^* \times \left(\frac{Pr}{Sc}\right)^n \times \left(\frac{M_w}{M_\infty}\right)^x \times \ln(1 + B_\psi)$$

where $g_h^* = h_{cof,V_w=0}/c_{pm}$.

- In the next 3 lectures, we will demonstrate applications of Stefan-, Couette- and Reynolds-flow models to problems of engineering relevance.

So, then in summary I would say that we have thus examine the validity of N w equal to g B psi equal to g star l n 1 plus B psi, and g over g star is in fact equal to F B is equal to l n 1 plus B psi by B psi. But this applies only to very very small mass transfer rates b psi, when b psi is very very small, this relationship we have verified even from by looking at laminar flow data, boundary layer data that we had computed earlier and also the turbulent flow data. So, this is valid only for very very small mass transfer rates and constant properties.

It is shown that the deviations from this formula occur when the fluid properties vary significantly in the boundary layer at large B psi and hence, the calculation of N w is corrected as what I have showed on the previous slide, where g star h is equal to V w 0 divided by C p m.

So, with this, we are now ready to demonstrate applications of Stefan, Couette and Reynolds flow models to problems of engineering relevance. And we shall actually calculate numerical problems in which first we will consider the Stefan flow model; for diffusion problems, we will consider Couetee flow model to show that this kind of property variation correction based on molecular weights of the infinity and w stage is in fact required, and then, we shall move to Reynolds flow model and solve several problems about at least eight to ten problems so that we can you can appreciate how to apply these methods for calculating practical convective mass transfer problems.