Convective Heat and Mass Transfer

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Module No. # 01
Lecture No. # 35
Boundary Layer Flow Model

In the last three lectures, we looked at the proxies of the boundary layer flow model namely, the Stephen flow model, the Kuwait flow model and the Reynolds flow model, the last one being the algebraic model. Whereas, the Stephen and Kuwait flow models were one dimensional models.

Now, we look at the complete two dimensional boundary layer flow model. So as to recover some ideas, as to why we expect the simpler models to mimic the boundary layer flow model.

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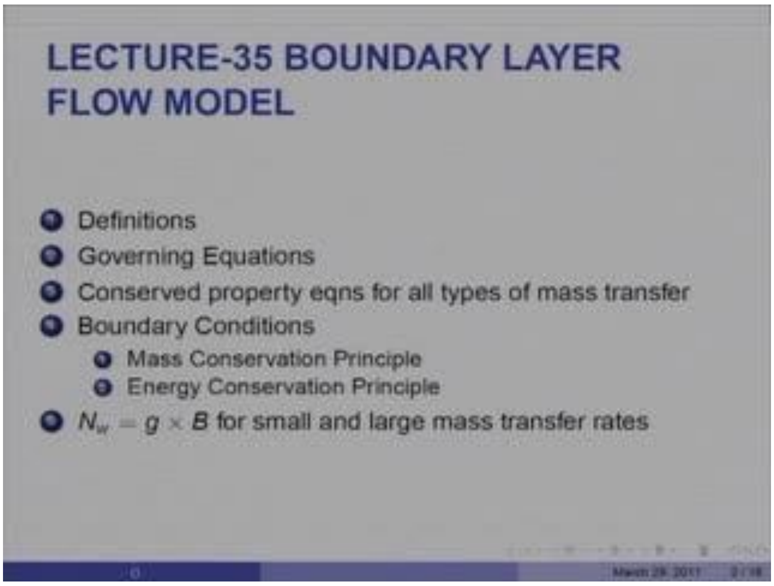

LECTURE-35 BOUNDARY LAYER FLOW MODEL

- Definitions
- Governing Equations
- Conserved property eqns for all types of mass transfer
- Boundary Conditions
  - Mass Conservation Principle
  - Energy Conservation Principle
- $N_w = g \times B$ for small and large mass transfer rates

In this lecture, I will introduce some definitions and then of course, look at the governing equations. Of course, we will convert them to conserved probably forms for all types of mass transfer problems. Then the main important thing is we will recover the boundary conditions from mass conservation principle and the energy conservation principle. Ultimately, we will show that N w is equal to g B for small and large mass transfer rates.

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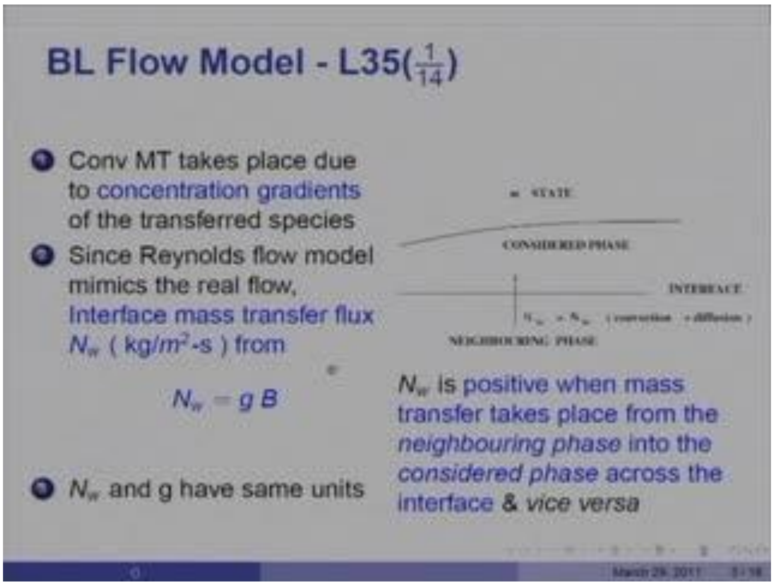

BL Flow Model - L35($\frac{1}{14}$)

- Conv MT takes place due to concentration gradients of the transferred species
- Since Reynolds flow model mimics the real flow, Interface mass transfer flux N_w (kg/m^2-s) from

$$N_w = g\,B$$

- N_w and g have same units

N_w is positive when mass transfer takes place from the *neighbouring phase* into the *considered phase* across the interface & *vice versa*

The definition of the boundary layer flow model is as follows, there is a surface and this is the considered phase and this is the neighboring phase as you know, this is the interface (Refer Slide Time: 01:43). In the considered phase, you have there may be presence of chemical reactions, there may be presence of turbulence, concentration gradients, temperature gradients and so on so forth. Infinity state is one, where all the gradients vanish.

The total mass transfer that is both convective plus diffusive mass transfer takes place at the interface from the neighboring phase. In deep inside the neighboring phase we defined a T-state, which of course, where the concentration gradients and temperature gradients as again like infinity state. Here again, in the T-state they are 0 or we say that the fluid properties are uniform.

Convective mass transfer takes place due to concentration gradients in the considered phase. Since, the Reynolds flow model mimics the real flow, the interface mass flux is given as N w equal to g B and N w and g have the same units.

N w is required as a boundary condition, because mass transfer equation as well as the energy equation and as well as the momentum equation because N w brings with it a certain velocity at which the mass comes in.

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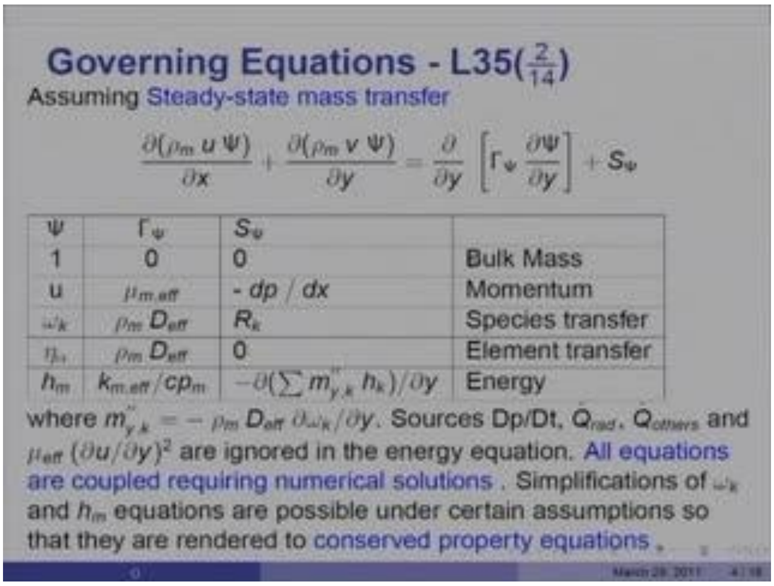

Governing Equations - L35($\frac{2}{14}$)

Assuming Steady-state mass transfer

$$\frac{\partial(\rho_m u \Psi)}{\partial x} + \frac{\partial(\rho_m v \Psi)}{\partial y} = \frac{\partial}{\partial y}\left[\Gamma_\Psi \frac{\partial \Psi}{\partial y}\right] + S_\Psi$$

Ψ	Γ_Ψ	S_Ψ	
1	0	0	Bulk Mass
u	$\mu_{m,eff}$	$- dp / dx$	Momentum
ω_k	$\rho_m D_{eff}$	R_k	Species transfer
η_α	$\rho_m D_{eff}$	0	Element transfer
h_m	$k_{m,eff} / cp_m$	$-\partial(\sum m''_{y,k} h_k)/\partial y$	Energy

where $m''_{y,k} = -\rho_m D_{eff} \partial\omega_k/\partial y$. Sources Dp/Dt, $\dot{Q}_{rad}$, $\dot{Q}_{others}$ and $\mu_{eff} (\partial u/\partial y)^2$ are ignored in the energy equation. All equations are coupled requiring numerical solutions . Simplifications of ω_k and h_m equations are possible under certain assumptions so that they are rendered to conserved property equations .

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Assuming steady state mass transfer, now you have d by dx of rho m u psi plus d by dy of rho m v psi plus d by dy of gamma psi d psi by dy plus a source term, where psi is any property like when it is 1 we will get bulk mass equation, because that term is 0 and S w is specified as 0 and that being 1 you simply get d rho m u by dx plus d rho m v by dy equal to 0. There is bulk mass equation under steady state.

If I say psi is equal to u then gamma psi would be mu m effective assuming that is turbulent flow. S psi would be usually minus dp by dx, which is the pressure gradient. If psi is equal to omega k then gamma psi as you now, would be rho m D effective plus equal to and source would be R k, which is the species transfer equation. Then there would be the element transfer equation with the source is always 0, because element alpha is a conserved property. The h m is the mixture enthalpy that would be has a source term compressing mass transfer due to diffusion flux m double dash y k.

The source, there can be other sources like Dp Dt Q rad then many other sources, these are all ignored, we also ignored the viscous dissipation here. All equations are coupled requiring numerical solutions. Simplifications of omega k and h m equations are possible under certain assumptions, so that they are rendered to a conserved property equation.

We have gone over all these processes of converting omega k and h m equations to conserved property forms in variety of mass transfer problems.

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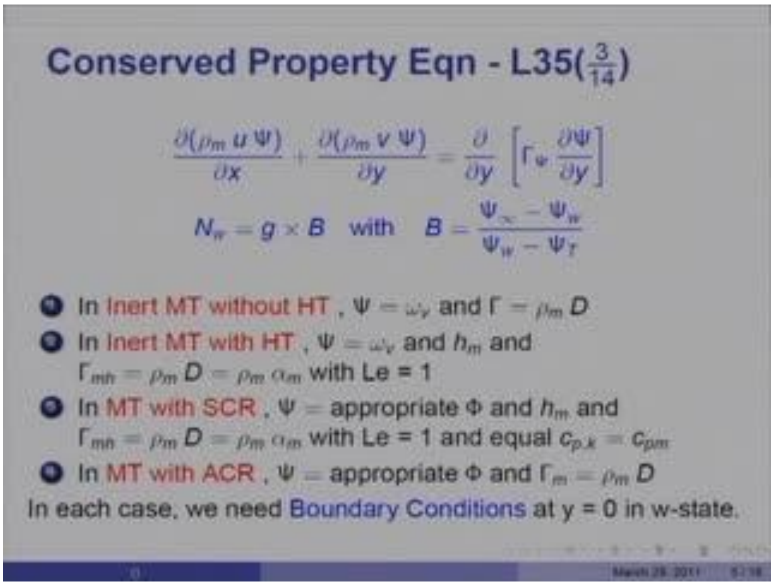

Conserved Property Eqn - L35($\frac{3}{14}$)

$$\frac{\partial(\rho_m u \Psi)}{\partial x} + \frac{\partial(\rho_m v \Psi)}{\partial y} = \frac{\partial}{\partial y}\left[\Gamma_\Psi \frac{\partial \Psi}{\partial y}\right]$$

$$N_w = g \times B \quad \text{with} \quad B = \frac{\Psi_\infty - \Psi_w}{\Psi_w - \Psi_T}$$

- In Inert MT without HT , $\Psi = \omega_v$ and $\Gamma = \rho_m D$
- In Inert MT with HT , $\Psi = \omega_v$ and h_m and $\Gamma_{mh} = \rho_m D = \rho_m \alpha_m$ with Le = 1
- In MT with SCR , $\Psi =$ appropriate Φ and h_m and $\Gamma_{mh} = \rho_m D = \rho_m \alpha_m$ with Le = 1 and equal $c_{p,k} = c_{pm}$
- In MT with ACR , $\Psi =$ appropriate Φ and $\Gamma_m = \rho_m D$

In each case, we need Boundary Conditions at y = 0 in w-state.

If I write in conserved property form the equation is simply this with S pi equal to 0 N w equal to g B; B is equal to psi infinity minus psi w psi w minus psi t (Refer Slide Time: 05:23).

Now, in inert mass transfer without heat transfer as you will recall, psi is simply omega of the vapor and gamma is rho m D the defensibility. In inert mass transfer with heat transfer psi equal to omega v as and h m and it is energy equation. We make use of the assumption of Lewis number equal to 1 that is gamma m h equal to rho m D equal to rho m alpha m. In mass transfer with simple chemical reaction means, we choose psi appropriately as a combination of fuel and oxygen, or fuel and product and so on, so forth and h m.

Again, make the Lewis number equal to 1 assumption and also say that the specific heats are equal that is what renders the energy equation in the conserved property form. In mass transfer with arbitrary chemical reaction for psi is again form from appropriate combinations of eta, alpha and gamma m is rho m D. In each case we need boundary conditions at y equal to 0, which will come from that form of the Reynolds flow model.

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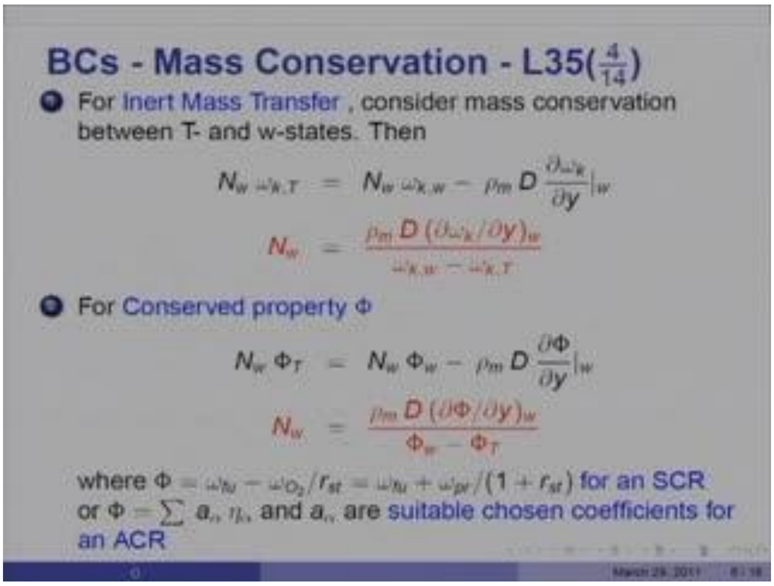

BCs - Mass Conservation - L35($\frac{4}{14}$)

- For Inert Mass Transfer , consider mass conservation between T- and w-states. Then

$$N_w\, \omega_{k,T} = N_w\, \omega_{k,w} - \rho_m D \frac{\partial \omega_k}{\partial y}\Big|_w$$

$$N_w = \frac{\rho_m D\, (\partial \omega_k / \partial y)_w}{\omega_{k,w} - \omega_{k,T}}$$

- For Conserved property Φ

$$N_w\, \Phi_T = N_w\, \Phi_w - \rho_m D \frac{\partial \Phi}{\partial y}\Big|_w$$

$$N_w = \frac{\rho_m D\, (\partial \Phi / \partial y)_w}{\Phi_w - \Phi_T}$$

where $\Phi = \omega_{fu} - \omega_{O_2}/r_{st} = \omega_{fu} + \omega_{pr}/(1 + r_{st})$ for an SCR or $\Phi = \sum a_{\alpha}\, \eta_{\alpha}$, and a_{α} are suitable chosen coefficients for an ACR

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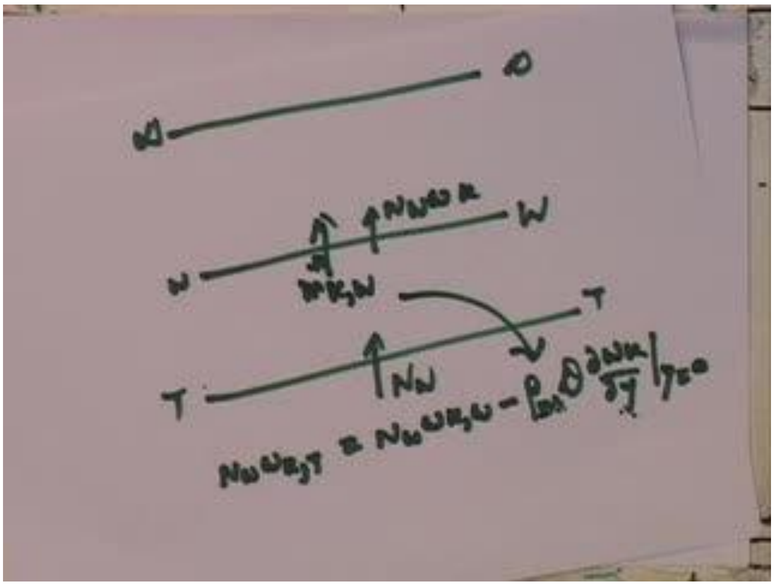

For the inert mass transfer, consider mass conservation between T and w-states. As we recall, this is our w w-state, this is T-state and this is the infinity state. We specify N w here, N w here, and then you will see that the mass which is coming in here is N w omega k at T. What is going out is N w times omega k at w minus the diffusion mass transfer, which is rho m diffusivity omega k by dy at y equal to 0 (Refer Slide Time: 07:03). This would be the mass balance, because this multiplied by omega k will be the convective transfer and diffusion transfer will be m dot k at w, which is nothing but, from fix law this is rho m diffusivity into omega k dy at y equal to 0.

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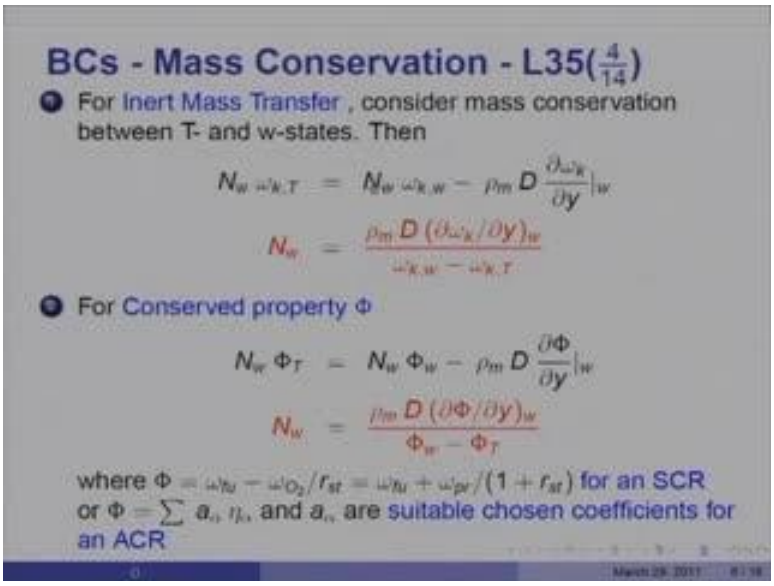

BCs - Mass Conservation - L35($\frac{4}{14}$)

- For Inert Mass Transfer , consider mass conservation between T- and w-states. Then

$$N_w\,\omega_{k,T} = N_w\,\omega_{k,w} - \rho_m D \frac{\partial \omega_k}{\partial y}|_w$$

$$N_w = \frac{\rho_m D\,(\partial \omega_k/\partial y)_w}{\omega_{k,w} - \omega_{k,T}}$$

- For Conserved property Φ

$$N_w\,\Phi_T = N_w\,\Phi_w - \rho_m D \frac{\partial \Phi}{\partial y}|_w$$

$$N_w = \frac{\rho_m D\,(\partial \Phi/\partial y)_w}{\Phi_w - \Phi_T}$$

where $\Phi = \omega_{fu} - \omega_{O_2}/r_{st} = \omega_{fu} + \omega_{pr}/(1 + r_{st})$ for an SCR or $\Phi = \sum a_\alpha\,\eta_\alpha$ and a_α are suitable chosen coefficients for an ACR

If I rearrange this equation take N w common out here, then I get the form N w is equal to rho m D omega k dy at wall divided by omega k w minus omega k at T. Likewise, for any other conserved property phi I have the same equation, as you can see here (Refer Slide Time: 08:37). N w will be rho m D omega phi d phi by dy at w phi w minus phi T, where phi would be as you will recall, omega f u minus omega o 2 divide by r st equal to omega fu plus omega product by 1 plus r st for a simple chemical reaction and the phi can be any linear combination of eta alpha. You choose alpha, it is suitable for a given problem as we indicated last time.

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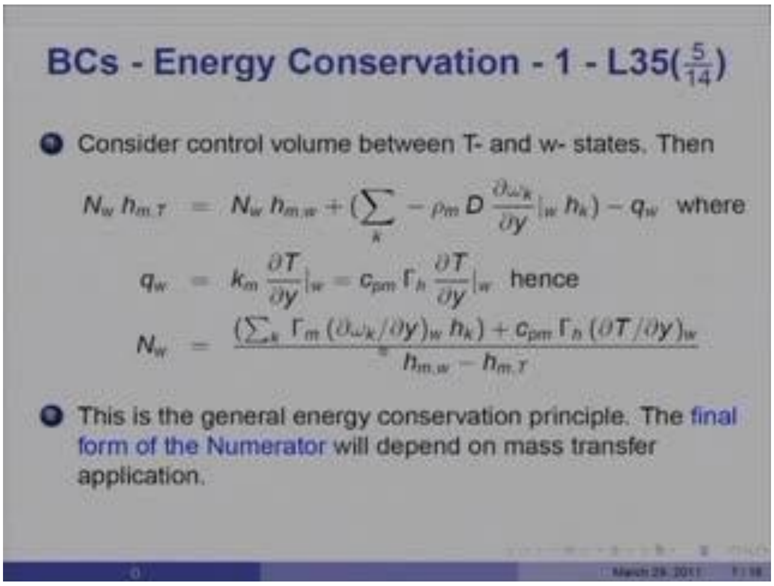

BCs - Energy Conservation - 1 - L35($\frac{5}{14}$)

- Consider control volume between T- and w- states. Then

$$N_w\,h_{m,T} = N_w\,h_{m,w} + \Big(\sum_k - \rho_m D \frac{\partial \omega_k}{\partial y}|_w\,h_k\Big) - q_w \quad \text{where}$$

$$q_w = k_m \frac{\partial T}{\partial y}|_w = c_{pm}\,\Gamma_h \frac{\partial T}{\partial y}|_w \quad \text{hence}$$

$$N_w = \frac{\left(\sum_k \Gamma_m\,(\partial \omega_k/\partial y)_w\,h_k\right) + c_{pm}\,\Gamma_h\,(\partial T/\partial y)_w}{h_{m,w} - h_{m,T}}$$

- This is the general energy conservation principle. The final form of the Numerator will depend on mass transfer application.

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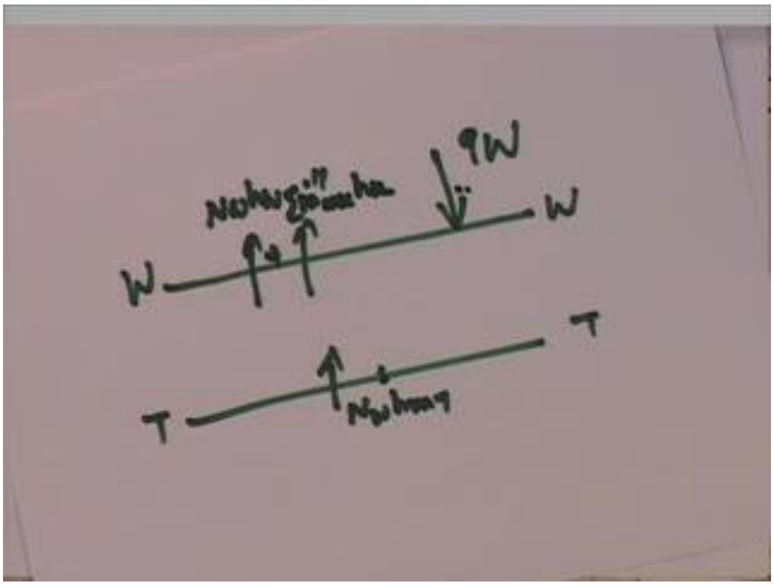

These are the boundary conditions derived from mass conservation principle (Refer Slide Time: 09:35). Likewise, we can do for the energy equation. For example, in a energy equation again, if I consider this to be w w and this to be T T-state, then N w times h mT will be the flux coming in. As well as, q w will be another flux coming in whereas, what is going out is the convective flux N w h w plus m dot w diffusion of the wk h k sigma.

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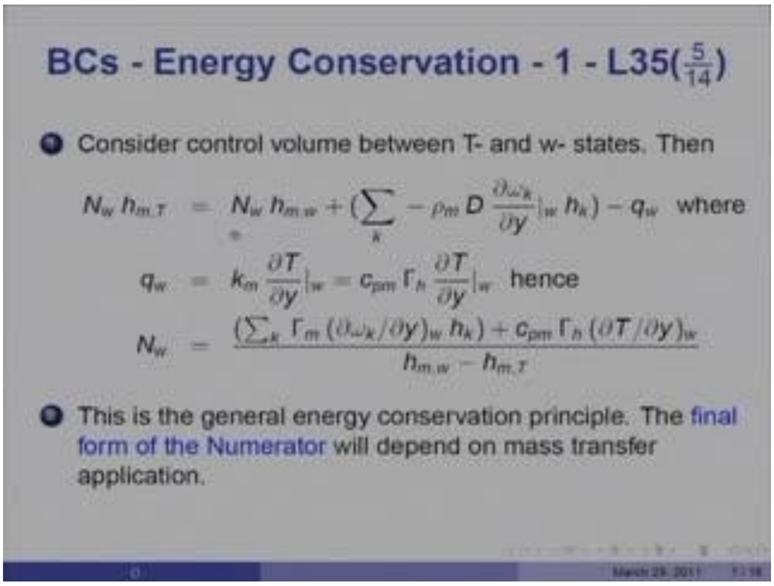

BCs - Energy Conservation - 1 - L35($\frac{5}{14}$)

- Consider control volume between T- and w- states. Then

$$
\begin{aligned}
N_w\, h_{m,T} &= N_w\, h_{m,w} + \Big(\sum_k -\rho_m D \frac{\partial \omega_k}{\partial y}\Big|_w h_k\Big) - q_w \quad \text{where} \\
q_w &= k_m \frac{\partial T}{\partial y}\Big|_w = c_{pm}\, \Gamma_h \frac{\partial T}{\partial y}\Big|_w \quad \text{hence} \\
N_w &= \frac{(\sum_k \Gamma_m\, (\partial \omega_k/\partial y)_w\, h_k) + c_{pm}\, \Gamma_h\, (\partial T/\partial y)_w}{h_{m,w} - h_{m,T}}
\end{aligned}
$$

- This is the general energy conservation principle. The final form of the Numerator will depend on mass transfer application.

If I equate the in fluxes with outgoing fluxes I would get the form, which I have showed here (Refer Slide Time: 10:10). This is the incoming flux, q w is also the incoming flux, which should be on this side. N w is equal to h m w and this is the diffusion flux of species k multiplied by the enthalpy, let goes with it of the species k.

Now, q w is equal to k m dT dy, because q w is shown inwards and that would be equal to c pm gamma h, where gamma h is rho m alpha m dT by dy at the w. Hence, Nw would written as sigma k; a gamma m d omega k by dy at w h k plus c pm gamma h dt by dy at w h m w minus h m t. This is the general energy conservation principle. The final form of the numerator will depend on the mass transfer application. Let us see, what are the different forms? That this numerator will take in different types of mash transfer problems.

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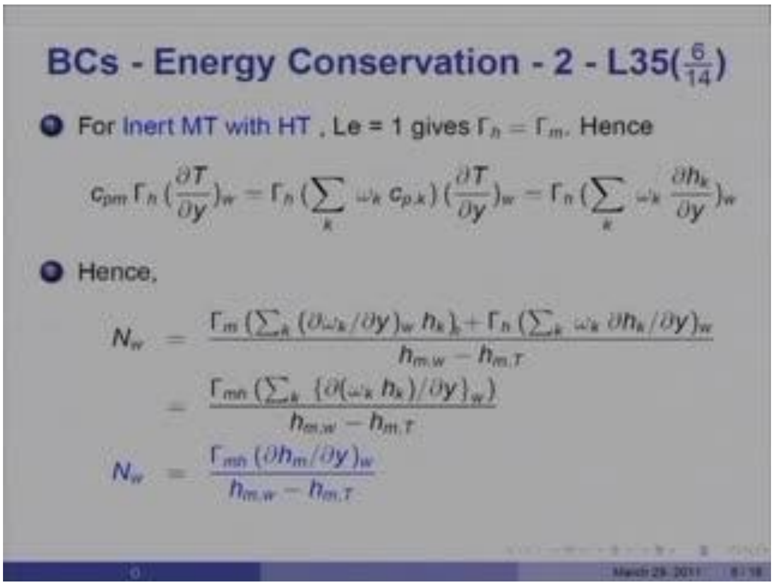

BCs - Energy Conservation - 2 - L35($\frac{6}{14}$)

- For Inert MT with HT , Le = 1 gives $\Gamma_h = \Gamma_m$. Hence

$$c_{pm}\,\Gamma_h\,(\frac{\partial T}{\partial y})_w = \Gamma_h\,(\sum_k \omega_k\, c_{p,k})\,(\frac{\partial T}{\partial y})_w = \Gamma_h\,(\sum_k \omega_k\,\frac{\partial h_k}{\partial y})_w$$

- Hence,

$$N_w = \frac{\Gamma_m\,(\sum_k (\partial\omega_k/\partial y)_w\, h_k) + \Gamma_h\,(\sum_k \omega_k\,\partial h_k/\partial y)_w}{h_{m,w} - h_{m,T}}$$

$$= \frac{\Gamma_{mh}\,(\sum_k \{\partial(\omega_k\, h_k)/\partial y\}_w)}{h_{m,w} - h_{m,T}}$$

$$N_w = \frac{\Gamma_{mh}\,(\partial h_m/\partial y)_w}{h_{m,w} - h_{m,T}}$$

In inert mass transfer with heat transfer and if I make the assumption of Lewis number 1 as we usually do, then gamma h will be simply equal to gamma m. Hence, c pm d gamma h dt by dy would be gamma h into c pm is simply sigma omega k c p k into dt by dy.

If I now, absorb c p k inside this derivative then I get a gamma h into sigma omega k dh k by dy at w. Hence, you will see that the numerator now reads as gamma m into sigma k d omega k by dy, where w h k plus gamma h into sigma k of omega k dh k by dy at w.

Since, we have assumed gamma h is equal to gamma m, these two terms are simply differentials of the product omega k hk dy at w summation of omega k hk is simply gamma mh dh m by dy, where w h m w minus h m T.

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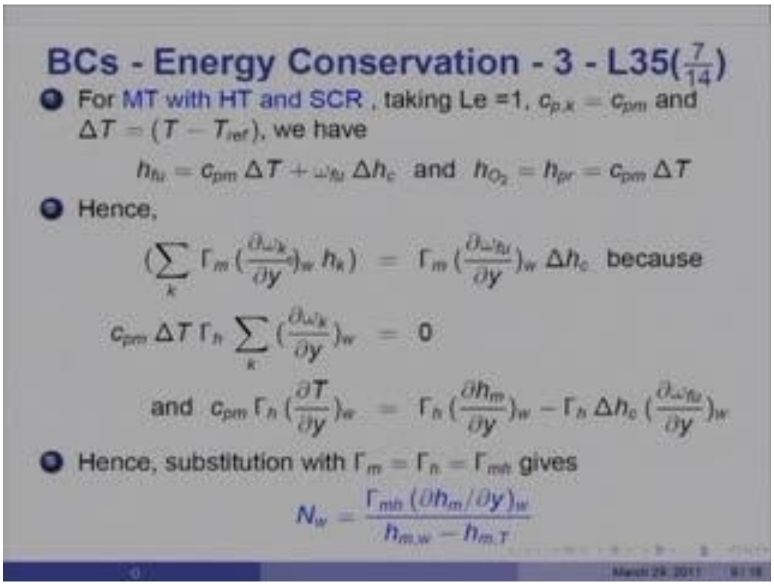

BCs - Energy Conservation - 3 - L35($\frac{7}{14}$)

- For MT with HT and SCR , taking Le =1, $c_{p,k} = c_{pm}$ and $\Delta T = (T - T_{ref})$, we have

$$h_{fu} = c_{pm}\,\Delta T + \omega_{fu}\,\Delta h_c \quad \text{and} \quad h_{O_2} = h_{pr} = c_{pm}\,\Delta T$$

- Hence,

$$\left(\sum_k \Gamma_m \left(\frac{\partial \omega_k}{\partial y}\right)_w h_k\right) = \Gamma_m \left(\frac{\partial \omega_{fu}}{\partial y}\right)_w \Delta h_c \quad \text{because}$$

$$c_{pm}\,\Delta T\,\Gamma_h \sum_k \left(\frac{\partial \omega_k}{\partial y}\right)_w = 0$$

$$\text{and} \quad c_{pm}\,\Gamma_h \left(\frac{\partial T}{\partial y}\right)_w = \Gamma_h \left(\frac{\partial h_m}{\partial y}\right)_w - \Gamma_h\,\Delta h_c \left(\frac{\partial \omega_{fu}}{\partial y}\right)_w$$

- Hence, substitution with $\Gamma_m = \Gamma_h = \Gamma_{mh}$ gives

$$N_w = \frac{\Gamma_{mh}\,(\partial h_m/\partial y)_w}{h_{m,w} - h_{m,T}}$$

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So, N w takes the same form as derived from the mass conservation principle but, with the variable enthalpy h m. Let us see, the numerator when mass transfer with heat transfer and simple chemical reaction. Here, we take Lewis number equal to 1 and also say that c p k will be equal to c pm. For the moment let us say, delta T stands for T minus T ref, then we have enthalpy of the fuel would be c p m into delta T plus omega f u into delta h c that is I have associated the heat of combustion with fuel. Therefore, h o 2 and h product will be simply c pm delta T.

Hence, summation of sigma gamma m d omega k by dy h k would simply amount to gamma m d fu by dy at the wall multiplied by dh c, because the other quantity c pm delta T if I substitute c pm delta T for all the three spaces, it would simply give me c pm delta T gamma h into summation of d omega k by dy at w equal to 0, because summation of sigma omega k is 1. Likewise, c pm gamma h dT by dy would be gamma h dh m by dy at w from this relationship minus gamma h d hc by d fu by dy.

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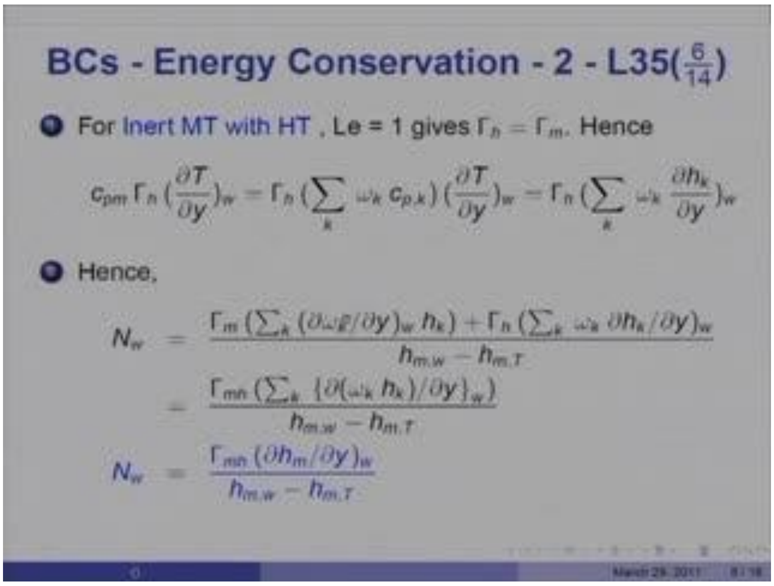

BCs - Energy Conservation - 2 - L35($\frac{6}{14}$)

- For Inert MT with HT , Le = 1 gives $\Gamma_h = \Gamma_m$. Hence

$$c_{pm}\,\Gamma_h\,(\frac{\partial T}{\partial y})_w = \Gamma_h\,(\sum_k \omega_k\,c_{p,k})\,(\frac{\partial T}{\partial y})_w = \Gamma_h\,(\sum_k \omega_k\,\frac{\partial h_k}{\partial y})_w$$

- Hence,

$$\begin{aligned} N_w &= \frac{\Gamma_m\,(\sum_k (\partial\omega_k/\partial y)_w\,h_k) + \Gamma_h\,(\sum_k \omega_k\,\partial h_k/\partial y)_w}{h_{m,w} - h_{m,T}} \\ &= \frac{\Gamma_{mh}\,(\sum_k \{\partial(\omega_k\,h_k)/\partial y\}_w)}{h_{m,w} - h_{m,T}} \\ N_w &= \frac{\Gamma_{mh}\,(\partial h_m/\partial y)_w}{h_{m,w} - h_{m,T}} \end{aligned}$$

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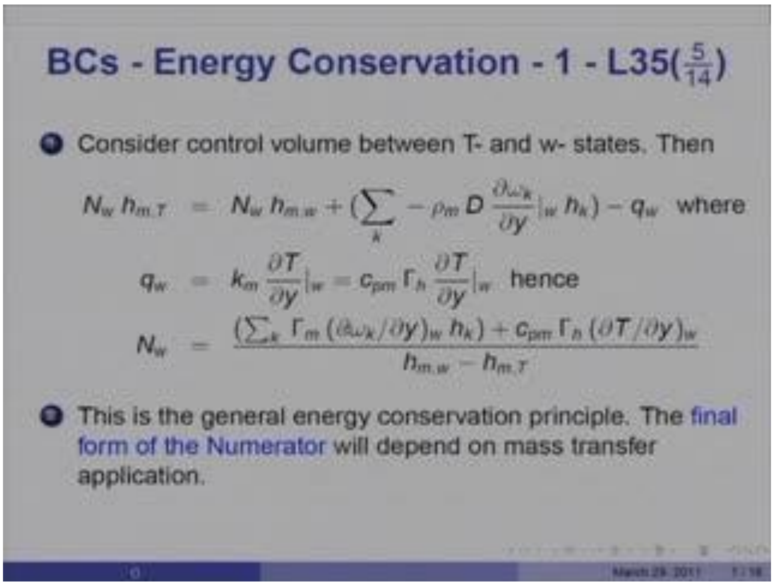

BCs - Energy Conservation - 1 - L35($\frac{5}{14}$)

- Consider control volume between T- and w- states. Then

$$\begin{aligned} N_w\,h_{m,T} &= N_w\,h_{m,w} + (\sum_k -\rho_m\,D\,\frac{\partial\omega_k}{\partial y}|_w\,h_k) - q_w \quad \text{where} \\ q_w &= k_m\,\frac{\partial T}{\partial y}|_w = c_{pm}\,\Gamma_h\,\frac{\partial T}{\partial y}|_w \quad \text{hence} \\ N_w &= \frac{(\sum_k \Gamma_m\,(\partial\omega_k/\partial y)_w\,h_k) + c_{pm}\,\Gamma_h\,(\partial T/\partial y)_w}{h_{m,w} - h_{m,T}} \end{aligned}$$

- This is the general energy conservation principle. The final form of the Numerator will depend on mass transfer application.

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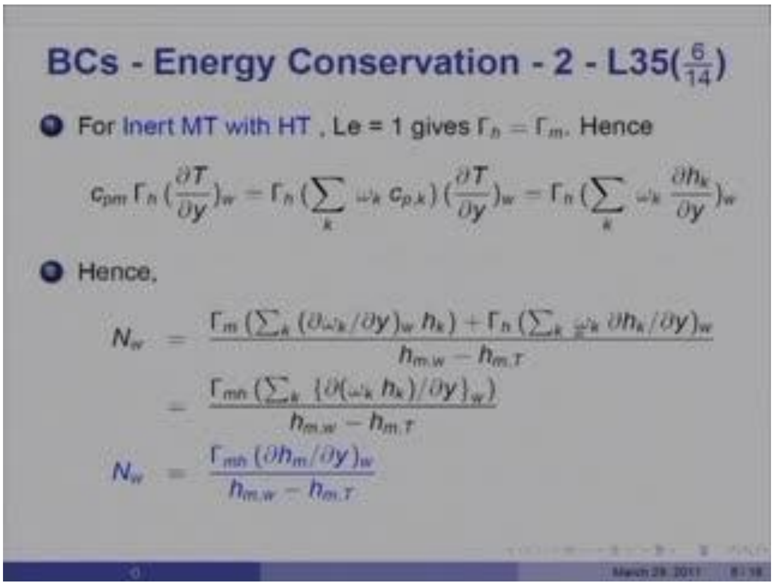

BCs - Energy Conservation - 2 - L35($\frac{6}{14}$)

- For Inert MT with HT , Le = 1 gives $\Gamma_h = \Gamma_m$. Hence

$$c_{pm}\,\Gamma_h\,(\frac{\partial T}{\partial y})_w = \Gamma_h\,(\sum_k \omega_k\,c_{p,k})\,(\frac{\partial T}{\partial y})_w = \Gamma_h\,(\sum_k \omega_k\,\frac{\partial h_k}{\partial y})_w$$

- Hence,

$$N_w = \frac{\Gamma_m\,(\sum_k (\partial\omega_k/\partial y)_w\,h_k) + \Gamma_h\,(\sum_k \omega_k\,\partial h_k/\partial y)_w}{h_{m,w} - h_{m,T}}$$

$$= \frac{\Gamma_{mh}\,(\sum_k \{\partial(\omega_k\,h_k)/\partial y\}_w)}{h_{m,w} - h_{m,T}}$$

$$N_w = \frac{\Gamma_{mh}\,(\partial h_m/\partial y)_w}{h_{m,w} - h_{m,T}}$$

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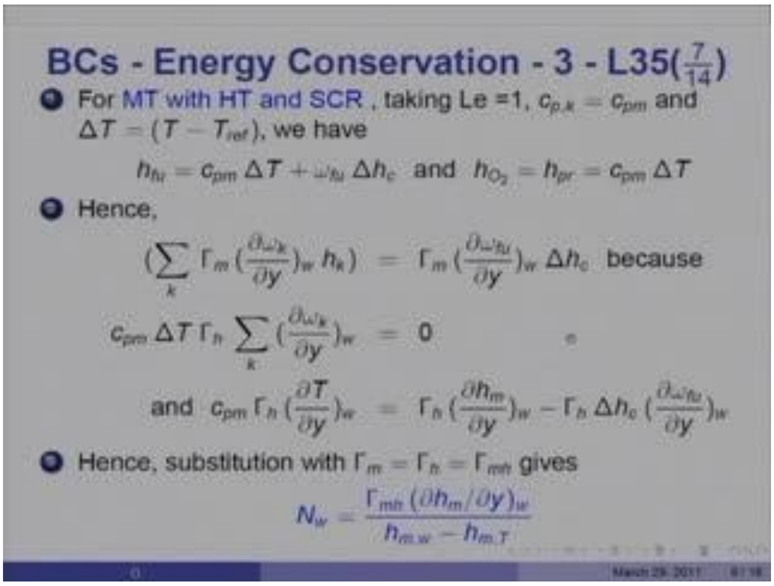

BCs - Energy Conservation - 3 - L35($\frac{7}{14}$)

- For MT with HT and SCR , taking Le =1, $c_{p,k} = c_{pm}$ and $\Delta T = (T - T_{ref})$, we have

$$h_{fu} = c_{pm}\,\Delta T + \omega_{fu}\,\Delta h_c \quad \text{and} \quad h_{O_2} = h_{pr} = c_{pm}\,\Delta T$$

- Hence,

$$(\sum_k \Gamma_m\,(\frac{\partial\omega_k}{\partial y})_w\,h_k) = \Gamma_m\,(\frac{\partial\omega_{fu}}{\partial y})_w\,\Delta h_c \quad \text{because}$$

$$c_{pm}\,\Delta T\,\Gamma_h\,\sum_k (\frac{\partial\omega_k}{\partial y})_w = 0$$

$$\text{and} \quad c_{pm}\,\Gamma_h\,(\frac{\partial T}{\partial y})_w = \Gamma_h\,(\frac{\partial h_m}{\partial y})_w - \Gamma_h\,\Delta h_c\,(\frac{\partial\omega_{fu}}{\partial y})_w$$

- Hence, substitution with $\Gamma_m = \Gamma_h = \Gamma_{mh}$ gives

$$N_w = \frac{\Gamma_{mh}\,(\partial h_m/\partial y)_w}{h_{m,w} - h_{m,T}}$$

Again, you see that the addition of this term with the summation of these two terms, this one, this term and this term, would amount to this term we will cancel with this term, because gamma m is equal to gamma h (Refer Slide Time: 14:35). Therefore, we get N w again is equal to gamma mh dh m by dy at w divided by h m w minus h m T.

Even in simple chemical reaction with these assumptions of Lewis number equal to 1 and equal specific heats, we recover the same form as we had done earlier with inert mass transfer with heat transfer.

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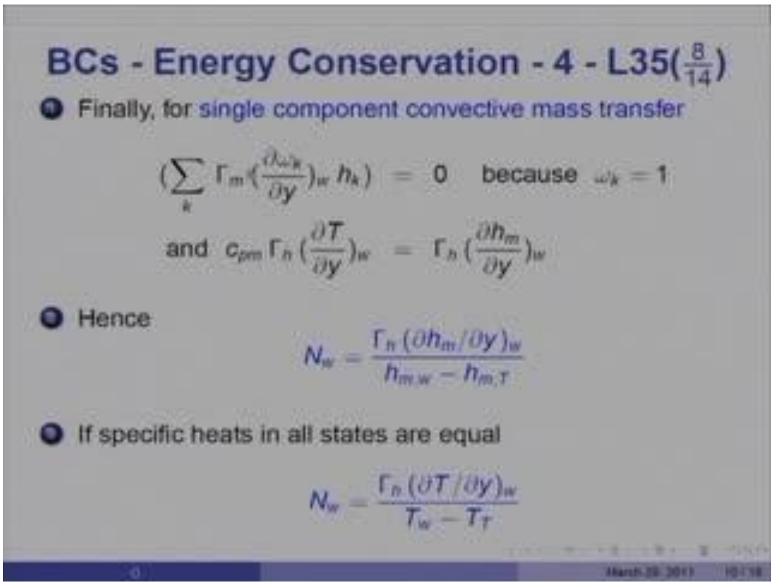

BCs - Energy Conservation - 4 - L35($\frac{8}{14}$)

- Finally, for single component convective mass transfer

$$\left(\sum_k \Gamma_m \left(\frac{\partial \omega_k}{\partial y}\right)_w h_k\right) = 0 \quad \text{because } \omega_k = 1$$

$$\text{and } \; c_{pm} \Gamma_h \left(\frac{\partial T}{\partial y}\right)_w = \Gamma_h \left(\frac{\partial h_m}{\partial y}\right)_w$$

- Hence

$$N_w = \frac{\Gamma_h (\partial h_m / \partial y)_w}{h_{m,w} - h_{m,T}}$$

- If specific heats in all states are equal

$$N_w = \frac{\Gamma_h (\partial T / \partial y)_w}{T_w - T_T}$$

Finally, let us look at single component convective mass transfer. Now, in this case, omega k is equal to 1 therefore, this part of the numerator will be 0 and c pm gamma h dt by dy would be simply gamma h, because I can multiply c pm inside this and therefore, that will give me h m by dy at w. Hence, I get N w equal to d h m by dy w h m w minus h m T, which is again, what we had earlier.

Single component convective heat transfer means supposing I have a boundary layer with air, I am injecting in air itself from the wall at a given rate, so the energy equation would take the form that I have shown here. Now, if I further make the assumption that this specific heats do not vary between w T and infinity states. Then of course, I can show that N w will be equal to gamma h into dT by dy w divided by T w minus T T.

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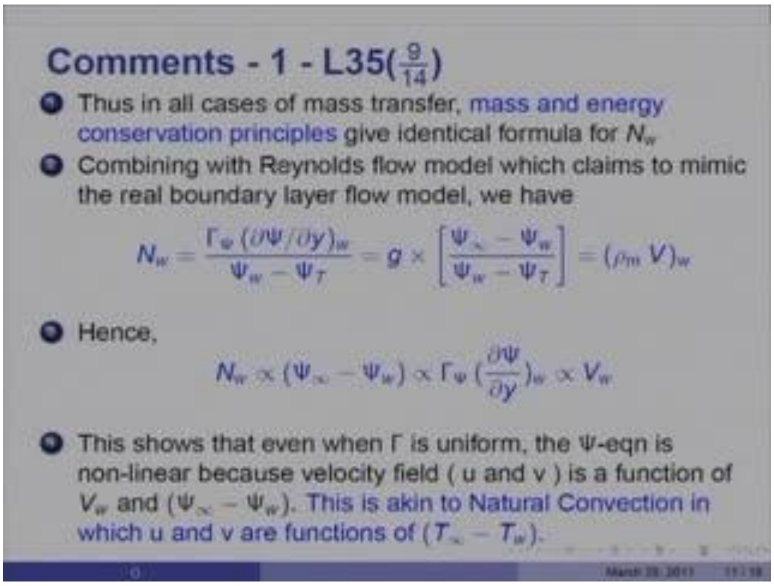

Comments - 1 - L35($\frac{9}{14}$)

- Thus in all cases of mass transfer, mass and energy conservation principles give identical formula for N_w
- Combining with Reynolds flow model which claims to mimic the real boundary layer flow model, we have

$$N_w = \frac{\Gamma_\psi (\partial\Psi/\partial y)_w}{\Psi_w - \Psi_T} = g \times \left[\frac{\Psi_\infty - \Psi_w}{\Psi_w - \Psi_T}\right] = (\rho_m V)_w$$

- Hence,

$$N_w \propto (\Psi_\infty - \Psi_w) \propto \Gamma_\psi \left(\frac{\partial\Psi}{\partial y}\right)_w \propto V_w$$

- This shows that even when Γ is uniform, the Ψ-eqn is non-linear because velocity field (u and v) is a function of V_w and $(\Psi_\infty - \Psi_w)$. This is akin to Natural Convection in which u and v are functions of $(T_\infty - T_w)$.

Thus, in all cases of mass transfer mass and energy conservation principles give identical formula for N w. Combining with Reynolds flow model, which claims to mimic the real boundary layer flow model. We can say that N w will be equal to gamma psi d psi by dy w over psi w minus psi T, which we are just now derived is equal to g times B and that must equal rho m V w. You can see now that the momentum equations, which require these quantities, are coupled with the mass fraction and energy equations through these quantities.

What it shows is that the rate of mass transfer would be proportional to psi infinity minus psi w. It will also be proportional to the gradient of psi at the wall and it will be proportional to V w, which is from that one. This shows that even if gamma is uniform the psi equation is non-linear that gamma is the property, so even if it was uniform through the considered phase the psi equation is non-linear, because velocity field u v is a function of V w and psi infinity minus psi w.

Now, this kind of coupling through the boundary conditions of momentum and the mass trans scalar equations is akin to the natural convection problem in which u and v are coupled to the energy equation through the source terms, we call the buoyancy term.

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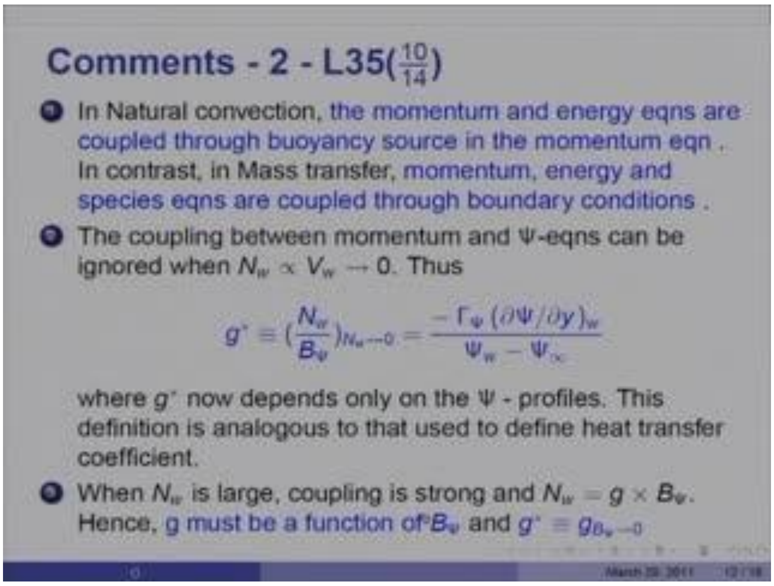

Comments - 2 - L35($\frac{10}{14}$)

- In Natural convection, the momentum and energy eqns are coupled through buoyancy source in the momentum eqn . In contrast, in Mass transfer, momentum, energy and species eqns are coupled through boundary conditions .
- The coupling between momentum and Ψ-eqns can be ignored when $N_w \propto V_w \to 0$. Thus

$$g^* \equiv \left(\frac{N_w}{B_\Psi}\right)_{N_w \to 0} = \frac{-\Gamma_\Psi \, (\partial \Psi / \partial y)_w}{\Psi_w - \Psi_\infty}$$

  where g^* now depends only on the Ψ - profiles. This definition is analogous to that used to define heat transfer coefficient.
- When N_w is large, coupling is strong and $N_w = g \times B_\Psi$. Hence, g must be a function of B_Ψ and $g^* \equiv g_{B_\Psi \to 0}$

In natural convection momentum and energy equations are coupled through buoyancy resource terms whereas, in mass transfer the coupling arises because of the boundary condition at the wall. This is the difference between a natural convection problem and a force convection mass transfer problem.

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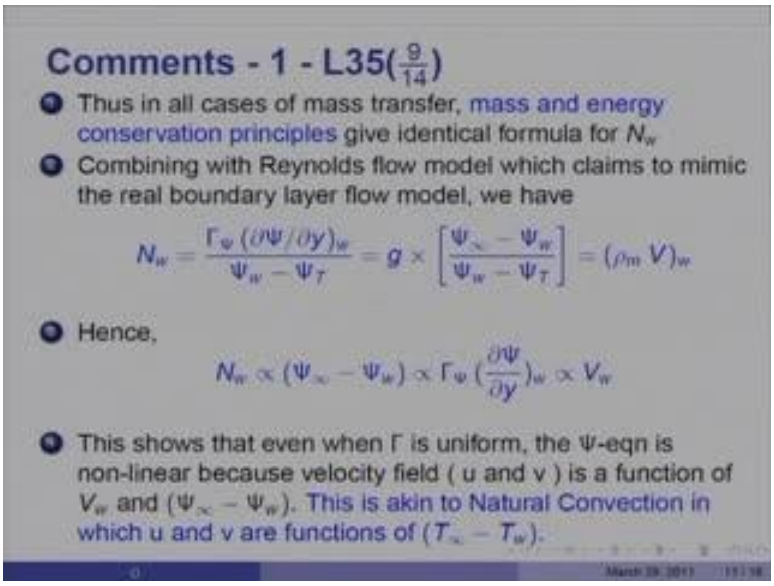

Comments - 1 - L35($\frac{9}{14}$)

- Thus in all cases of mass transfer, mass and energy conservation principles give identical formula for N_w
- Combining with Reynolds flow model which claims to mimic the real boundary layer flow model, we have

$$N_w = \frac{\Gamma_\Psi \, (\partial \Psi / \partial y)_w}{\Psi_w - \Psi_T} = g \times \left[\frac{\Psi_\infty - \Psi_w}{\Psi_w - \Psi_T}\right] = (\rho_m \, V)_w$$

- Hence,

$$N_w \propto (\Psi_\infty - \Psi_w) \propto \Gamma_\Psi \left(\frac{\partial \Psi}{\partial y}\right)_w \propto V_w$$

- This shows that even when Γ is uniform, the Ψ-eqn is non-linear because velocity field (u and v) is a function of V_w and $(\Psi_\infty - \Psi_w)$. This is akin to Natural Convection in which u and v are functions of $(T_\infty - T_w)$.

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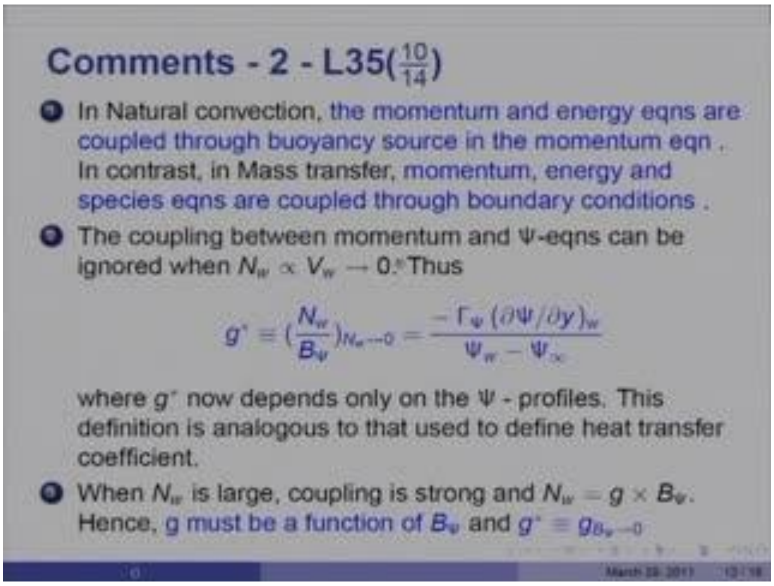

Comments - 2 - L35($\frac{10}{14}$)

- In Natural convection, the momentum and energy eqns are coupled through buoyancy source in the momentum eqn . In contrast, in Mass transfer, momentum, energy and species eqns are coupled through boundary conditions .
- The coupling between momentum and Ψ-eqns can be ignored when $N_w \propto V_w \rightarrow 0$. Thus

$$g^* \equiv (\frac{N_w}{B_\psi})_{N_w \rightarrow 0} = \frac{-\Gamma_\psi (\partial \Psi / \partial y)_w}{\Psi_w - \Psi_\infty}$$

  where g^* now depends only on the Ψ - profiles. This definition is analogous to that used to define heat transfer coefficient.
- When N_w is large, coupling is strong and $N_w = g \times B_\psi$. Hence, g must be a function of B_ψ and $g^* \equiv g_{B_\psi \rightarrow 0}$

A coupling between the momentum and psi equations can be ignored, when N w which is proportional to V w tends to 0, which means if the mass transfer rate is very small then we can say that g star, which is the value of g for small mass transfer rights. Then N w divided by B psi N w tends to 0 would be minus gamma psi d omega by dy divided by psi w minus psi infinity.

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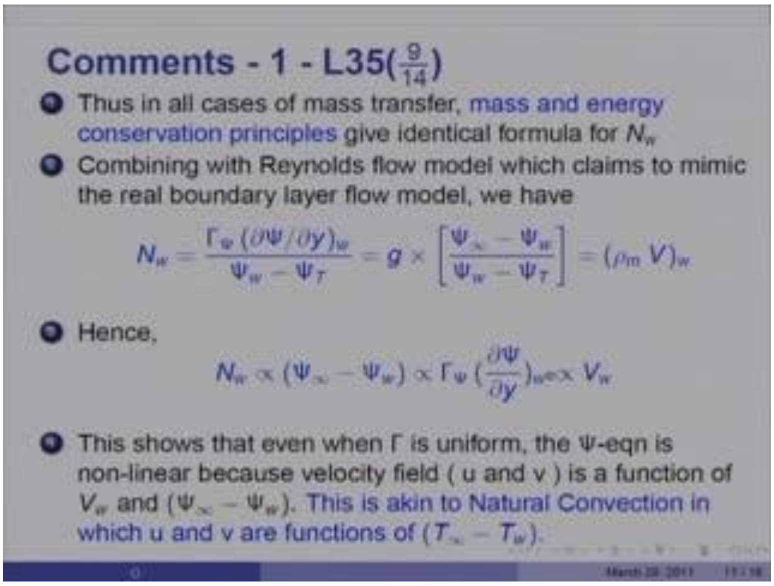

Comments - 1 - L35($\frac{9}{14}$)

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- Combining with Reynolds flow model which claims to mimic the real boundary layer flow model, we have

$$N_w = \frac{\Gamma_\psi (\partial \Psi / \partial y)_w}{\Psi_w - \Psi_T} = g \times \left[\frac{\Psi_\infty - \Psi_w}{\Psi_w - \Psi_T}\right] = (\rho_m V)_w$$

- Hence,

$$N_w \propto (\Psi_\infty - \Psi_w) \propto \Gamma_\psi (\frac{\partial \Psi}{\partial y})_w \propto V_w$$

- This shows that even when Γ is uniform, the Ψ-eqn is non-linear because velocity field (u and v) is a function of V_w and $(\Psi_\infty - \Psi_w)$. This is akin to Natural Convection in which u and v are functions of $(T_\infty - T_w)$.

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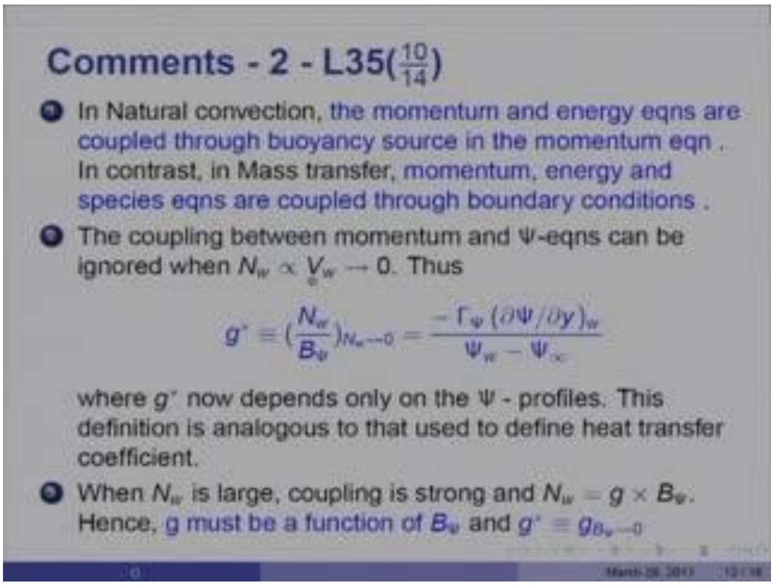

Comments - 2 - L35($\frac{10}{14}$)

- In Natural convection, the momentum and energy eqns are coupled through buoyancy source in the momentum eqn . In contrast, in Mass transfer, momentum, energy and species eqns are coupled through boundary conditions .
- The coupling between momentum and Ψ-eqns can be ignored when $N_w \propto V_w \rightarrow 0$. Thus

$$g^* \equiv (\frac{N_w}{B_\Psi})_{N_w \rightarrow 0} = \frac{-\Gamma_\Psi (\partial \Psi / \partial y)_w}{\Psi_w - \Psi_\infty}$$

  where g^* now depends only on the Ψ - profiles. This definition is analogous to that used to define heat transfer coefficient.
- When N_w is large, coupling is strong and $N_w = g \times B_\Psi$. Hence, g must be a function of B_Ψ and $g^* \equiv g_{B_\Psi \rightarrow 0}$

This follows from the previous equation which I have shown here, so I simply cancel psi w psi T psi w minus psi T. This is psi infinity minus psi w is taken to the denominator, so you get g star is equal to this quantity (Refer Slide Time: 19:05). The g star now depends only on the psi profiles and not on the boundary condition, because V w is tending to 0. This definition is analogous to that use to define the heat transfer coefficient as you will recall, we define the heat transfer coefficient as minus k dT dy T w minus T infinity.

Likewise, we say that the g star which is the mass transfer coefficient at V w equal to 0 is gamma minus gamma psi d psi by dy at w divided by psi w minus psi infinity, so the two definitions are now analogous. Now, you can see why we have been calling g as the mass transfer coefficient.

When N w is large the coupling between the momentum equations and the psi equation is strong and N w equal to g times B psi. Hence, g must be a function of B psi and g star equal to g when B psi tends to 0; g star is simply g with B psi sending to 0, but it would also g will also be itself be a function of g. Perhaps even of Reynolds number Prandtl number and so on so forth, but we shall shortly see, what are the states of affairs?

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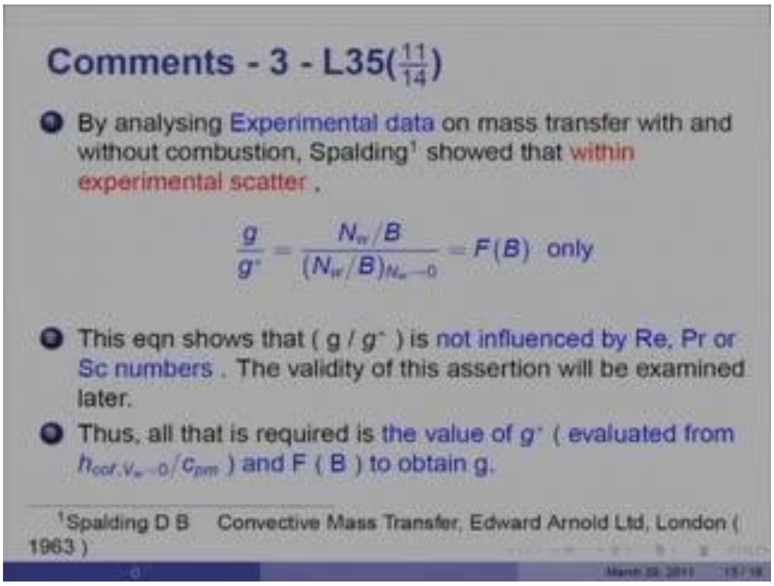

Comments - 3 - L35($\frac{11}{14}$)

- By analysing Experimental data on mass transfer with and without combustion, Spalding[1] showed that within experimental scatter ,

$$\frac{g}{g^*} = \frac{N_w/B}{(N_w/B)_{N_w \to 0}} = F(B) \text{ only}$$

- This eqn shows that (g / g*) is not influenced by Re, Pr or Sc numbers . The validity of this assertion will be examined later.
- Thus, all that is required is the value of g* (evaluated from $h_{cof,V_w=0}/c_{pm}$) and F (B) to obtain g.

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[1] Spalding D B Convective Mass Transfer, Edward Arnold Ltd, London (1963)

By analyzing experimental data on mass transfer with and without combustions, Spalding showed that within experimental scatter g by g star is equal to N w by B divided by N w by B at very small mass transfer rates, is in fact equal to F B only, this is what noting F B only.

In other words, the equation shows that g over g star is not influenced by Reynolds number, Prandtl number or Schmidt numbers. This is what the experimental data shows. In internal flows as well as external flows; flow over cylinder, flow over flat plates and flow through the tubes and so on so forth. It may be a problem of evaporation, it may be a problem of condensation and it may be a problem of combustion, all these problems for which experimental data were available.

Spalding showed that g over g star turns within experimental scatter say about plus minus 10 to 15 percent g by g star is a function of B only, which is a remarkable result that this ratio should not be influenced by any other quantity other than the driving force B. So, it is independent of Reynolds Prandtl and Schmidt numbers.

All that we would require now is the value of g star, evaluate it from h c of V w equal to 0 divided by c pm and F B to obtain g. What are the forms of F B? That is what we want to find out. Suddenly, this book Spalding D B Convective Mass Transfer published in 1963 is a pioneering book on mass transfer and particularly of great relevance to engineering community.

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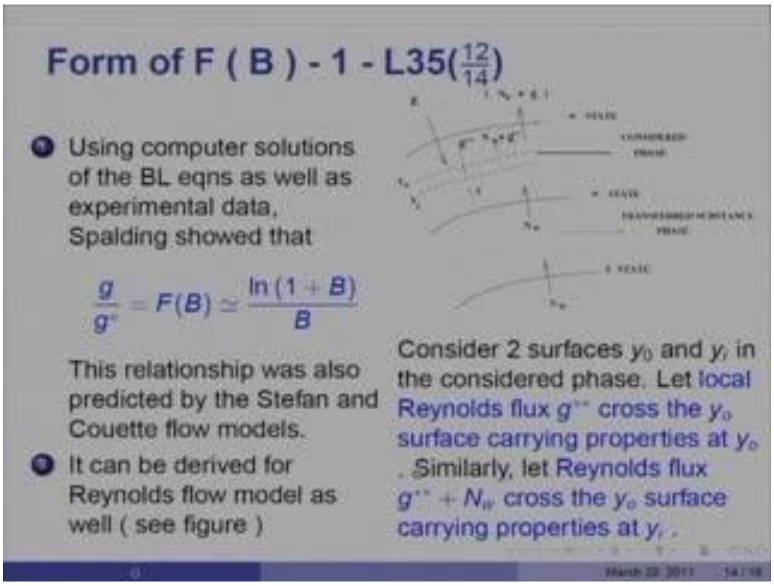

Form of F (B) - 1 - L35($\frac{12}{14}$)

- Using computer solutions of the BL eqns as well as experimental data, Spalding showed that

$$\frac{g}{g^*} = F(B) \simeq \frac{\ln(1+B)}{B}$$

This relationship was also predicted by the Stefan and Couette flow models.

- It can be derived for Reynolds flow model as well (see figure)

Consider 2 surfaces y_0 and y_i in the considered phase. Let local Reynolds flux g^{**} cross the y_o surface carrying properties at y_o . Similarly, let Reynolds flux $g^{**} + N_w$ cross the y_o surface carrying properties at y_i .

Now, let us see, what the form of F B should be. Using computer simulations of the boundary layer equation as well as, experimental data, Spalding showed that g over g star is equal to F B and that function F B is nothing but, ln 1 plus B by B.

Now, if you recall, this is the relationship, which we had also predicted using Stefan flow model as well as the Couette flow models. Of course, Stefan flow model was for diffusion mass transfer only whereas, the Couette flow model was for included convection in its flow. Therefore, this relationship is unique it does not contain any Reynolds number, Prandtl number or anything like that.

We take up the issues as together this relationship can actually be predicted can also be derived from the Reynolds flow model. Just as we had shown it mathematically that the Stefan and Couette flow models yield that relationship.

In order to do that let us reconsider about T-state, w-state and the infinity state with the considered phase shown here, between w and infinity state (Refer Slide Time: 24:05). Here, what I have done is let us consider an elemental strip here of thickness delta y the outer edge of that strip is y o the inner edge of the strip is y i. Let us postulate g star star as a flux at the y o surface, which is coming in and bringing with it the properties of the y o surface. Likewise, N w plus g star star is an outgoing flux, which brings the properties of the y i state.

So, this is what I have said here g star star crosses the y o surface carrying properties of the y o surface. Similarly, the Reynolds flux n g star star plus N w crosses y o surface carrying with it properties of the y i surface.

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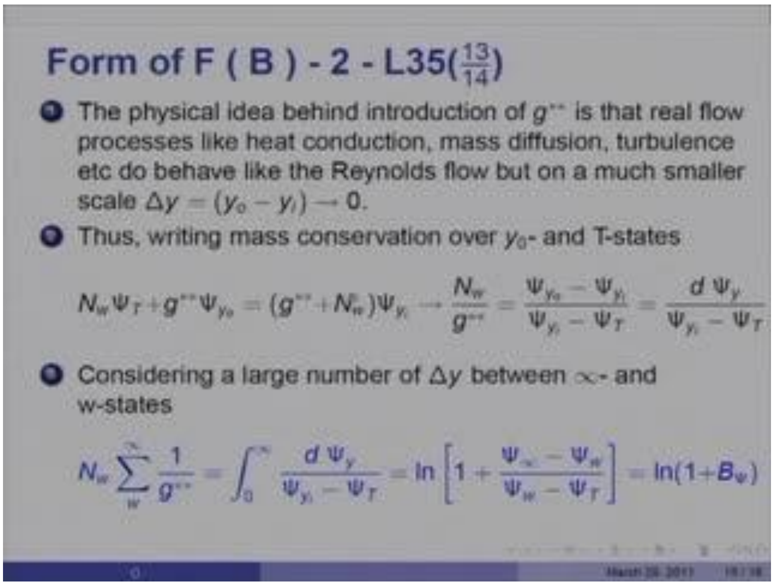

Form of F (B) - 2 - L35($\frac{13}{14}$)

- The physical idea behind introduction of g^{**} is that real flow processes like heat conduction, mass diffusion, turbulence etc do behave like the Reynolds flow but on a much smaller scale $\Delta y = (y_o - y_i) \rightarrow 0$.
- Thus, writing mass conservation over y_o- and T-states

$$N_w \Psi_T + g^{**} \Psi_{y_o} = (g^{**} + N_w) \Psi_{y_i} \rightarrow \frac{N_w}{g^{**}} = \frac{\Psi_{y_o} - \Psi_{y_i}}{\Psi_{y_i} - \Psi_T} = \frac{d\,\Psi_y}{\Psi_{y_i} - \Psi_T}$$

- Considering a large number of Δy between ∞- and w-states

$$N_w \sum_{w}^{\infty} \frac{1}{g^{**}} = \int_0^{\infty} \frac{d\,\Psi_y}{\Psi_{y_i} - \Psi_T} = \ln\left[1 + \frac{\Psi_\infty - \Psi_w}{\Psi_w - \Psi_T}\right] = \ln(1 + B_\psi)$$

The physical idea behind introduction of g star star is that the real flow processes like heat conduction mass diffusion turbulence etcetera, do behave like the Reynolds flow but on a much smaller scale delta y y o minus y i tending to 0.

If I now write the mass conservation over y o and T-states, then N w psi T would be the incoming flux plus g star star into psi times y o would be the incoming flux from y o surface and that would equal g star star plus N w into psi y i. Therefore, rearrangement of this would give me N w divided by g star star is equal to psi y o minus psi y i divided by psi y i minus psi T or this difference is nothing but, d psi y and this is psi y minus psi T (Refer Slide Time: 26:04).

If we consider large numbers of delta y between 0 and delta of the considered phase called between infinity and w-states, then simply N w into summation of 1 over g star star would amount to integration of 0 to infinity d psi y divided by psi y minus psi T. The integration would naturally yield ln 1 plus psi y infinity minus psi w divided by psi w minus psi T, which as you know is nothing but, B psi ln 1 plus B psi.

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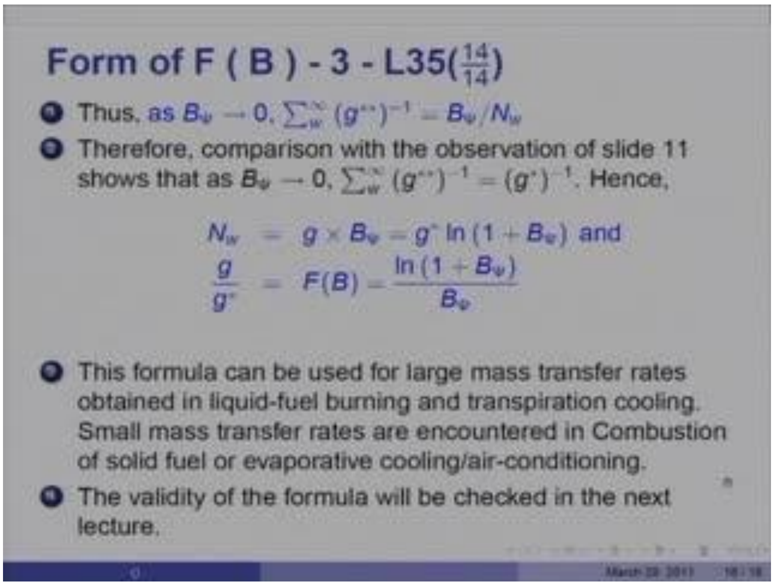


Form of F (B) - 3 - L35($\frac{14}{14}$)

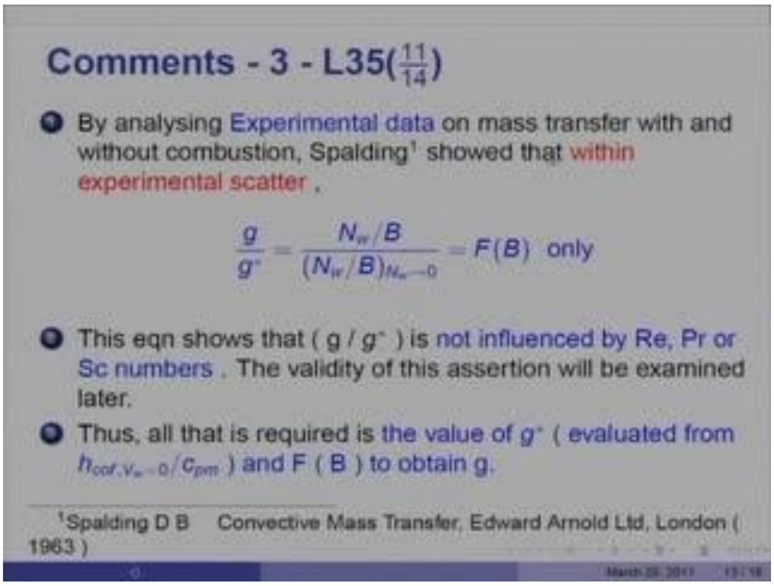
- Thus, as $B_\psi \rightarrow 0$, $\sum_w^\infty (g^{**})^{-1} = B_\psi / N_w$
- Therefore, comparison with the observation of slide 11 shows that as $B_\psi \rightarrow 0$, $\sum_w^\infty (g^{**})^{-1} = (g^*)^{-1}$. Hence,

$$\begin{aligned} N_w &= g \times B_\psi = g^* \ln(1 + B_\psi) \text{ and} \\ \frac{g}{g^*} &= F(B) = \frac{\ln(1 + B_\psi)}{B_\psi} \end{aligned}$$

- This formula can be used for large mass transfer rates obtained in liquid-fuel burning and transpiration cooling. Small mass transfer rates are encountered in Combustion of solid fuel or evaporative cooling/air-conditioning.
- The validity of the formula will be checked in the next lecture.

If I take this result further then as B psi tends to 0 N w w into infinity 1 over g star star would simply tend to B psi itself. So, as B psi tends to 0 w to infinity is g star star raise to minus 1 would stand to B psi by N w.

(Refer Slide Time: 27:18)



Comments - 3 - L35($\frac{11}{14}$)

- By analysing Experimental data on mass transfer with and without combustion, Spalding[1] showed that within experimental scatter ,

$$\frac{g}{g^*} = \frac{N_w / B}{(N_w / B)_{N_w \rightarrow 0}} = F(B) \text{ only}$$

- This eqn shows that (g / g^*) is not influenced by Re, Pr or Sc numbers . The validity of this assertion will be examined later.
- Thus, all that is required is the value of g^* (evaluated from $h_{cof, V_w \rightarrow 0} / c_{pm}$) and F (B) to obtain g.

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[1] Spalding D B Convective Mass Transfer, Edward Arnold Ltd, London (1963)

Therefore, comparison of with the observation of slide 11, what is the observation we made on slide 11? The g over g star is equal to N w divided by B over N w by B N w 0 is equal to F B.

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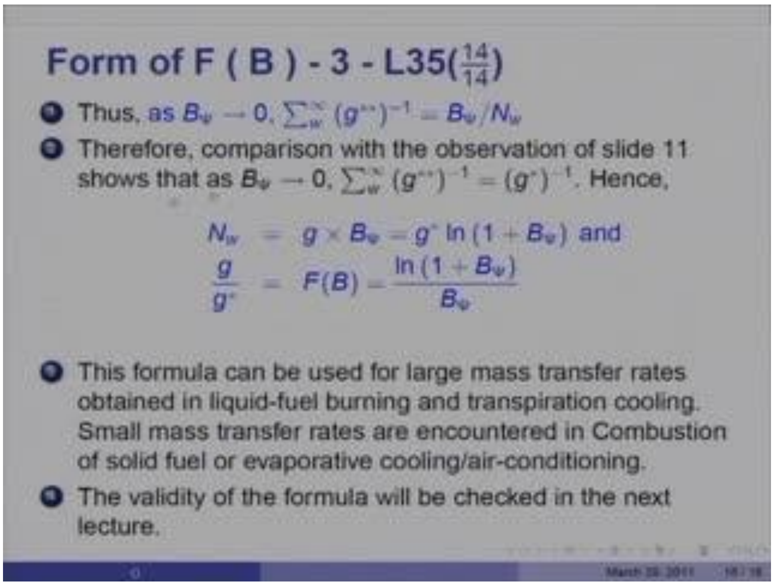


Form of F (B) - 3 - L35($\frac{14}{14}$)

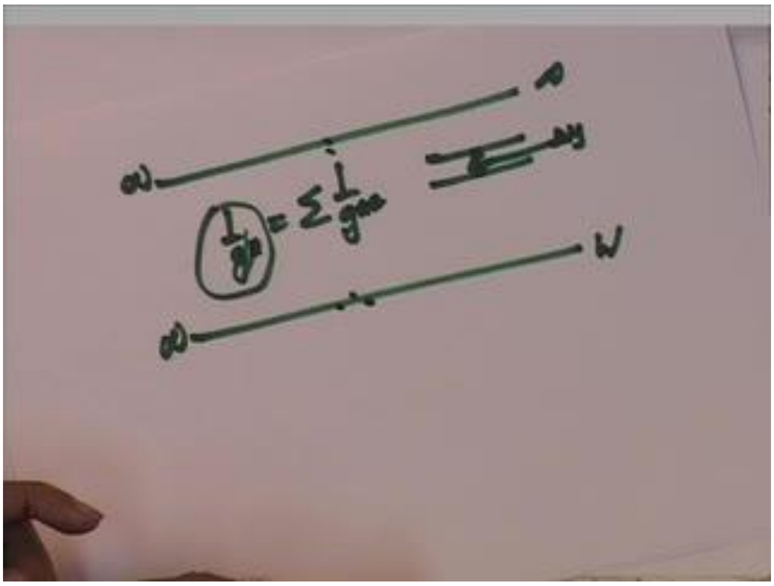
- Thus, as $B_\psi \rightarrow 0$, $\sum_w^\infty (g^{**})^{-1} = B_\psi / N_w$
- Therefore, comparison with the observation of slide 11 shows that as $B_\psi \rightarrow 0$, $\sum_w^\infty (g^{**})^{-1} = (g^*)^{-1}$. Hence,

$$\begin{aligned} N_w &= g \times B_\psi = g^* \ln(1 + B_\psi) \text{ and} \\ \frac{g}{g^*} &= F(B) = \frac{\ln(1 + B_\psi)}{B_\psi} \end{aligned}$$

- This formula can be used for large mass transfer rates obtained in liquid-fuel burning and transpiration cooling. Small mass transfer rates are encountered in Combustion of solid fuel or evaporative cooling/air-conditioning.
- The validity of the formula will be checked in the next lecture.

Comparison with that will show that as that as B psi tends to 0, the sum of g star star raise to minus 1 is nothing but, the g star raise to minus 1. Hence, N w equal to g B psi is equal to g star ln 1 plus B psi and g over g star is equal to F B and that is equal to ln 1 plus B psi over B psi. This is a very interesting result (Refer Slide Time: 27:56). Remember, g is conductance and therefore, 1 over g is resistance.

(Refer Slide Time: 28:11)



The total resistance between w and infinity states, which we say is 1 over g star is nothing but, sigma 1 over g star star, where g star star is the resistance of the small element g bar delta y over delta y. We can interpret now that the 1 over g star the resistance is the sum of the resistances to mass transfer between w and infinity states. The inverse of that is the conductance of mass transfer.

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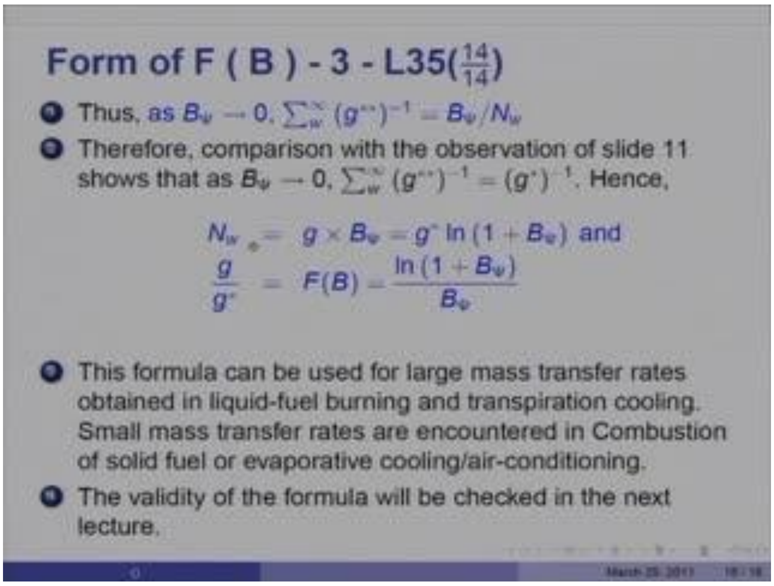


Form of F (B) - 3 - L35($\frac{14}{14}$)

- Thus, as $B_\psi \to 0$, $\sum_w^\infty (g^{**})^{-1} = B_\psi / N_w$
- Therefore, comparison with the observation of slide 11 shows that as $B_\psi \to 0$, $\sum_w^\infty (g^{**})^{-1} = (g^*)^{-1}$. Hence,

$$N_{w,*} = g \times B_\psi = g^* \ln(1 + B_\psi) \text{ and}$$
$$\frac{g}{g^*} = F(B) = \frac{\ln(1 + B_\psi)}{B_\psi}$$

- This formula can be used for large mass transfer rates obtained in liquid-fuel burning and transpiration cooling. Small mass transfer rates are encountered in Combustion of solid fuel or evaporative cooling/air-conditioning.
- The validity of the formula will be checked in the next lecture.

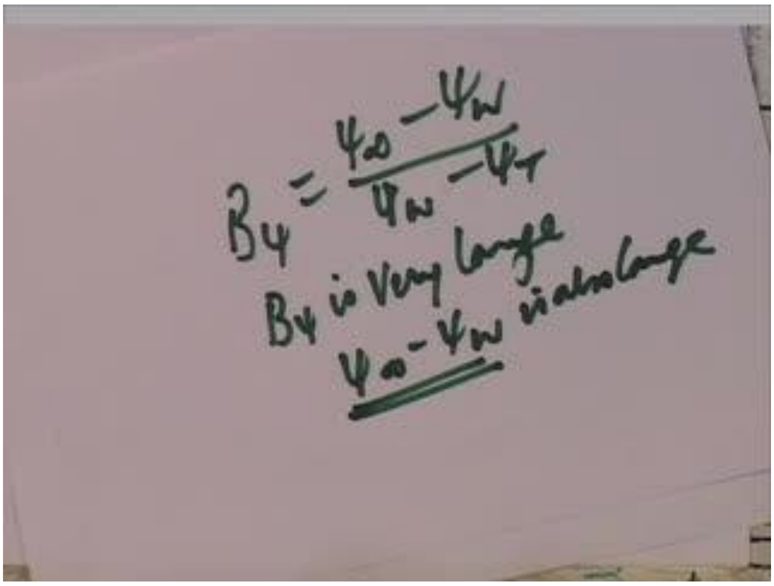
In other words, the ratio of the conductance at large mass transfer rates divided by conductance at small mass transfer rates or negligible mass transfer rates is simply a function of ln 1 plus B psi by B psi. This formula can be used for large mass transfer rates obtained in liquid-fuel burning and in transpiration cooling as we shall see in later lectures, where I will be considering problems. A small B psi usually occurs in combustion of solid fuels and in evaporative cooling or air conditioning and so on so forth.

But, in these applications, B psi would be a less than 0.1 in these applications combustion of solid fuel, evaporative cooling and air in fact it will be of the order of 0.02 0.03, whereas in transpiration cooling and liquid fuel burning value of B psi can vary between 0.5 to nearly 10. So, B psi can be really large in liquid fuel burning and in transpiration cooling.

We shall check how good is this formula from the analytical solution we have derived so far for laminar flows as well as, in turbulent flows. Let us see, how well this formula is, so that we can confidently use it at different mass transfer rate.

If the mass transfer rate is very low then of course, there is no problem because g by g star will be simply 1 and you would have straight away, there is a no difficultly. If g by g star is moderate then of course, this formula will work.

(Refer Slide Time: 30:50)



$$B_\psi = \frac{\psi_\infty - \psi_w}{\psi_w - \psi_T}$$

B_ψ is Very large

$\underline{\psi_\infty - \psi_w}$ is also large

At very large mass transfer rate remember the definition of B psi says that B psi is psi infinity minus psi w divided by psi w minus psi T. When B psi is very large psi infinity minus psi w is also large and this means that when the property in the infinity state and the w-state is large we would expect the property is to vary between the w and infinity state.

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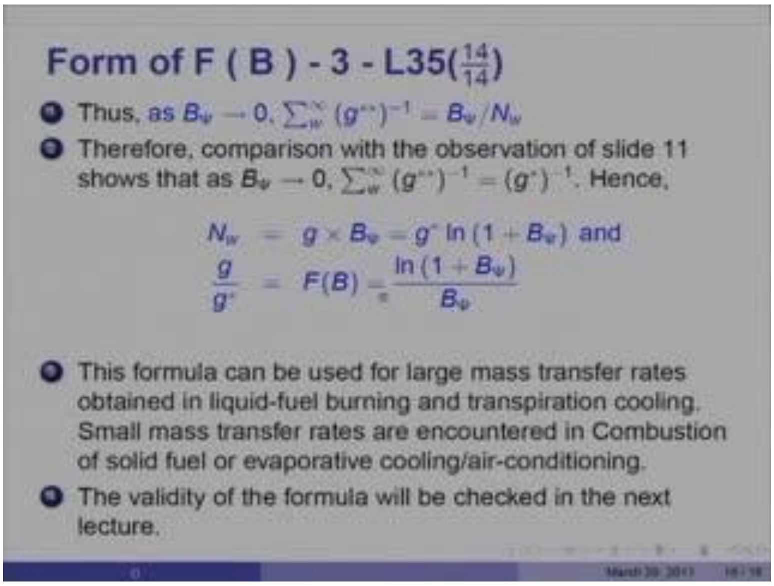

Form of F (B) - 3 - L35($\frac{14}{14}$)

- Thus, as $B_\psi \to 0$, $\sum_w^\infty (g^{**})^{-1} = B_\psi / N_w$
- Therefore, comparison with the observation of slide 11 shows that as $B_\psi \to 0$, $\sum_w^\infty (g^{**})^{-1} = (g^*)^{-1}$. Hence,

$$N_w = g \times B_\psi = g^* \ln(1 + B_\psi) \text{ and}$$
$$\frac{g}{g^*} = F(B) = \frac{\ln(1 + B_\psi)}{B_\psi}$$

- This formula can be used for large mass transfer rates obtained in liquid-fuel burning and transpiration cooling. Small mass transfer rates are encountered in Combustion of solid fuel or evaporative cooling/air-conditioning.
- The validity of the formula will be checked in the next lecture.

Any departure from this formula is found in experiments would be largely due to property variations. Therefore, this formula is usually corrected for property variation and that is the matter for discussion in the next lecture.