Convective Heat and Mass Transfer

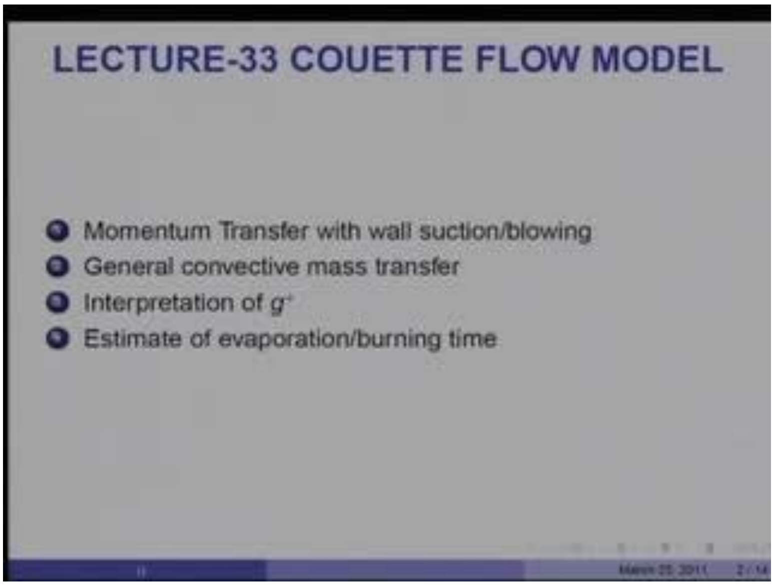
Prof. A. W. Date
Department of Mechanical Engineering
Indian Institute of Technology, Bombay

Module No. # 01
Lecture No. # 33
Couette Flow Model

In lecture 32 we looked at Stefan flow model which is essentially a diffusion model to be applied to stagnant surroundings and therefore, momentum equation was not invoked. Only the species transfer and the energy equations were invoked. We applied these equations to the 3 types of mass transfer or 4 types of mass transfer that we normally encounter. There is a one is the inert mass transfer without heat transfer, inert mass transfer with heat transfer, then mass transfer with heat transfer and simple chemical reaction and then finally, we applied it to the case of mass transfer with arbitrary chemical reactions. All these four types almost described the overall problem of mass transfer.

Today, we are going to consider the Couette flow model and therefore, we shall be invoking both the momentum equation and continuity equation along with the species transfer and the energy equation.

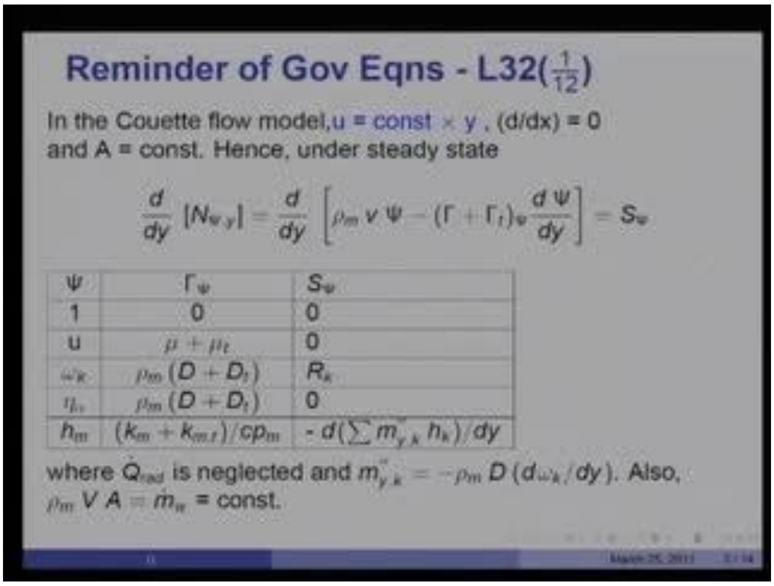
(Refer Slide Time: 01:39)



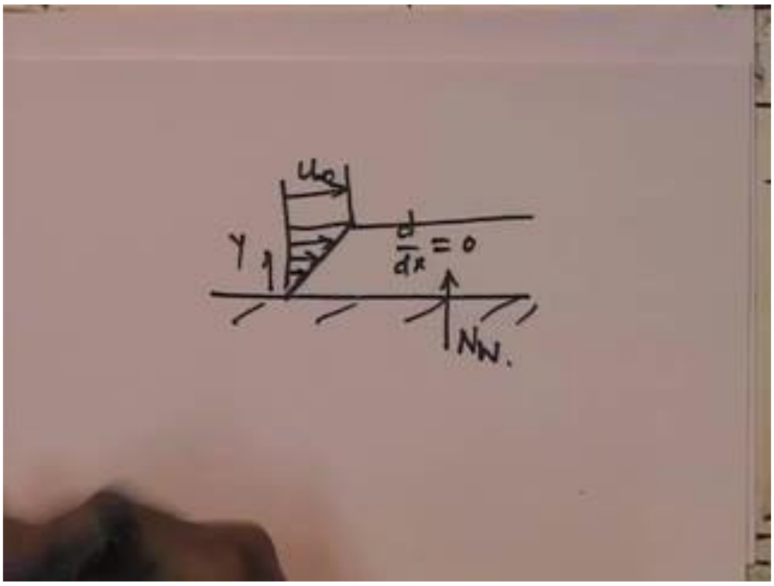
So, we shall apply the Couette flow model to mass transfer with wall suction and blowing. This is essentially a case of, let us say, air flowing over surface and air itself is being sucked or blown through the wall which would be the simple case of suction and blowing that we considered even with similarity solutions.

Then we will move to the general problem of convective mass transfer and in this, we shall interpret the effective mass transfer coefficient which we shall term as g star and by way of an example we will calculate, estimate, the evaporation slash burning time of a liquid droplet.

(Refer Slide Time: 02:34)

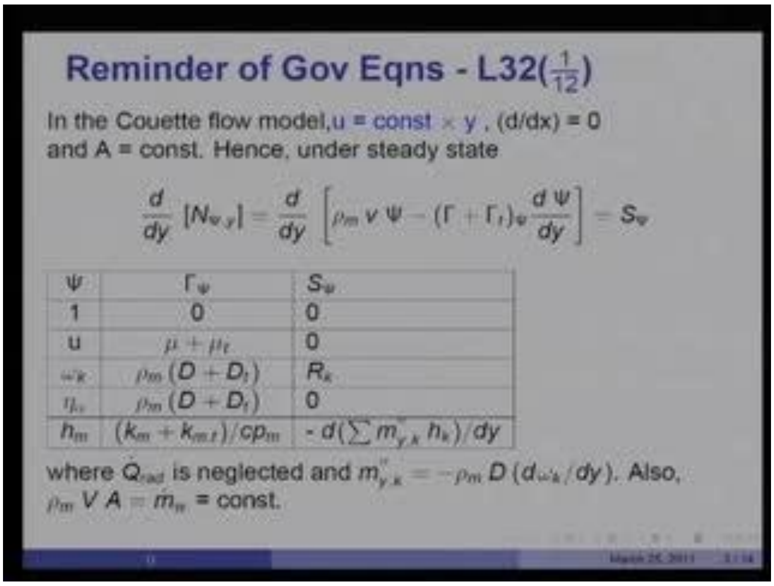


(Refer Slide Time: 03:21)



So, let us begin then with the momentum transfer with wall suction and blowing. Now, in the Couette flow model as you will recall, the velocity u is taken to be constant multiplied by y and all actual derivatives are set to 0 and therefore A is constant. Under steady state [transport/transfer] equation would read like this: d by dy of N psi y equal to d by dy of rho m v psi minus gamma plus gamma t, where gamma t is the turbulent exchange coefficient; d psi by dy equal to S psi just by way of the reminder that the Couette flow model is really let us say, this is the surface. We assume that the velocity profile will be like. So, with u infinity here all d by dx are 0 for all variables and u infinity remains constant along the plate and this is the direction y.

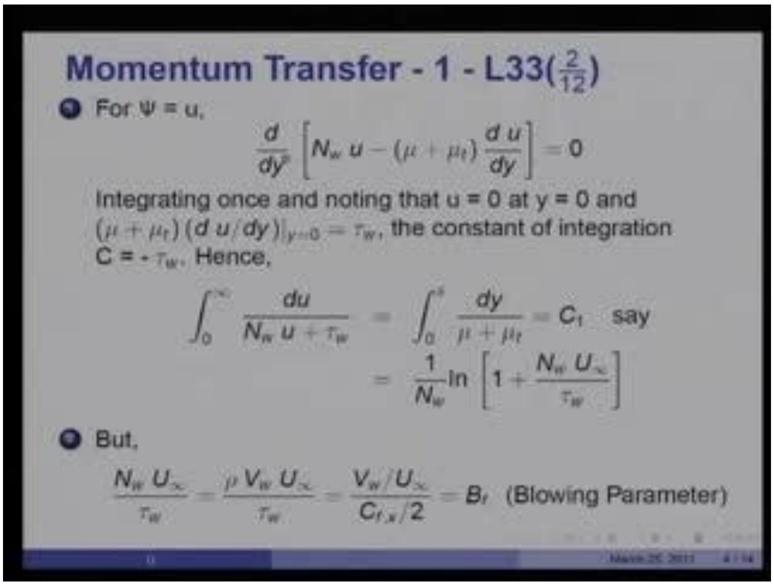
(Refer Slide Time: 03:46)



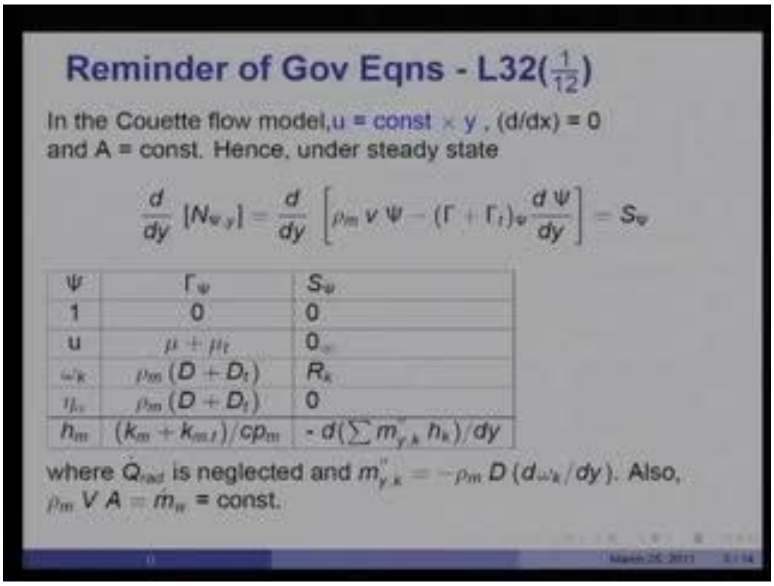
So, this is really the Couette flow model; the mass transfer would be taking place in this direction. If you look at the meanings of psi, psi equal to 1 would imply simply continuity equation or mass conservation equation, psi equal to u will imply momentum equation with gamma equal to mu and then if it is a mass transfer mass fraction omega k this would be the species transfer equation and this would be the energy equation.

We have ignored the radiation and other heat generation terms here; m double dot y k is really the Fick's law of diffusion flux given by the Fick's law and rho m V A will be m dot w is equal to constant which is a mass flux would remain constant.

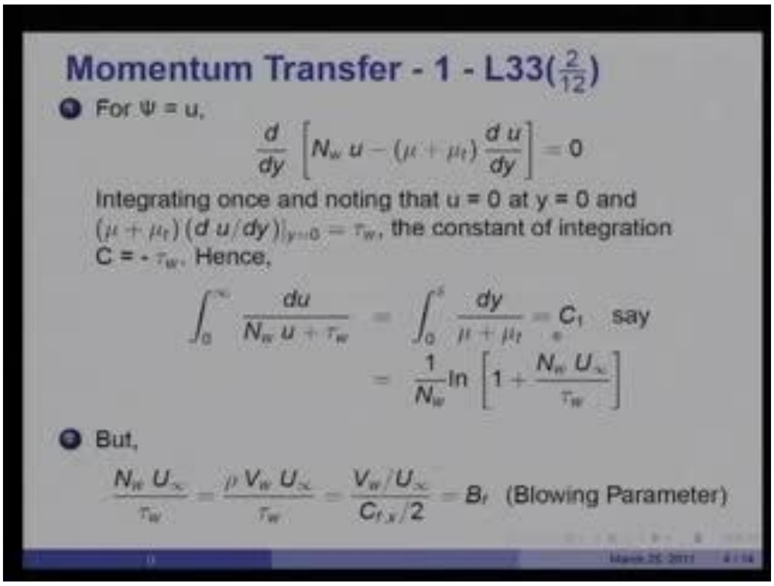
(Refer Slide Time: 04:39)



(Refer Slide Time: 04:47)



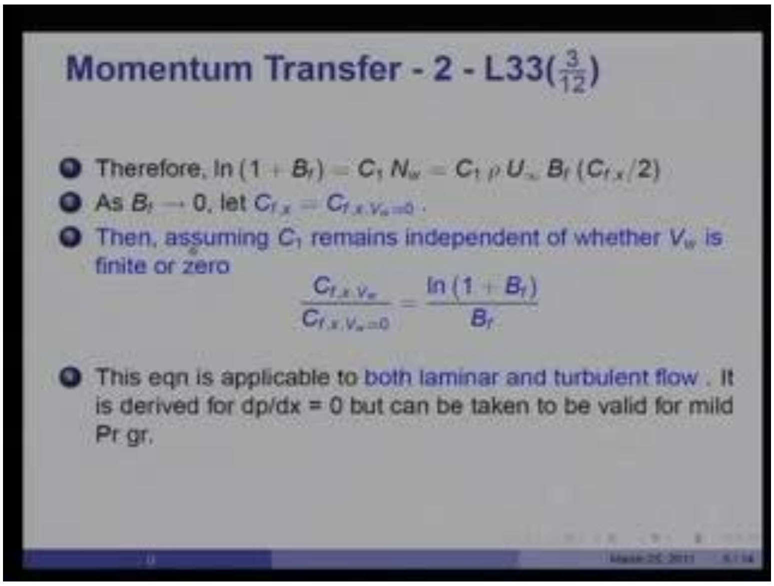
(Refer Slide Time: 04:51)



Let us consider momentum transfer. If psi is equal to u, then the governing equation remember, there is no source term here but the pressure gradient is also 0 and therefore the equation will be d by dy N w u minus mu plus mu t du by dy equal to 0. If I integrate this once and note that the boundary condition is u equal to 0 at y is equal to 0, the shear stress is given by mu times du by dy y equal to 0 of course mu t would be 0 at the wall. The constant of integration C will be simply minus tau wall and hence the integrating from 0 to infinity would give me du divide by N w u tau w equal to 0 to delta dy mu plus mu t which I am for the moment calling it as constant C 1 - some integrated value C 1 and the integration of this term would simply result in 1 over N w ln 1 plus N w U infinity by tau w.

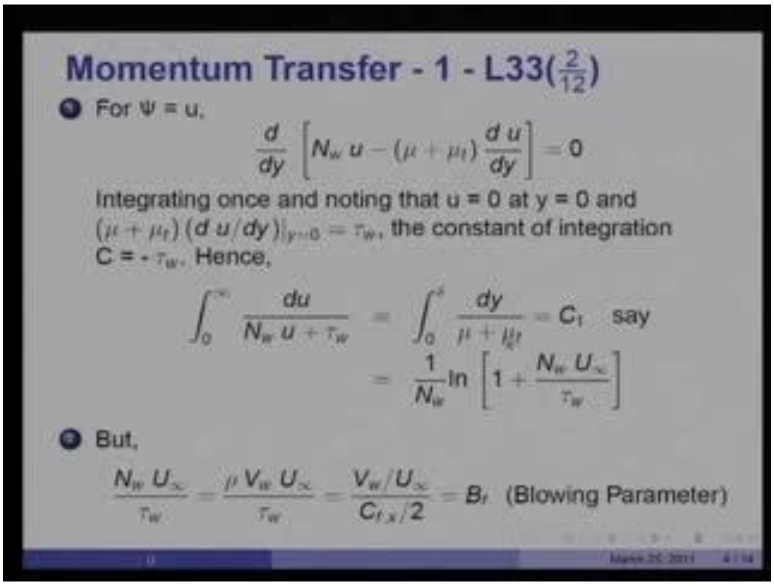
But what is N w u infinity by tau w? We can interpret that; remember, N w is rho times V w because the same air, same fluid is being blown into the boundary layer as is flowing over a plate rho V w U infinity divided by tau wall. If I multiply and divide this by U infinity then, I would get V w divide by U infinity equal to rho U infinity square by tau w which is nothing but V w by U infinity C f x by 2 and as you will recall this is nothing but the blowing parameter which we had invoked during similarity solutions and integral solutions.

(Refer Slide Time: 06:41)



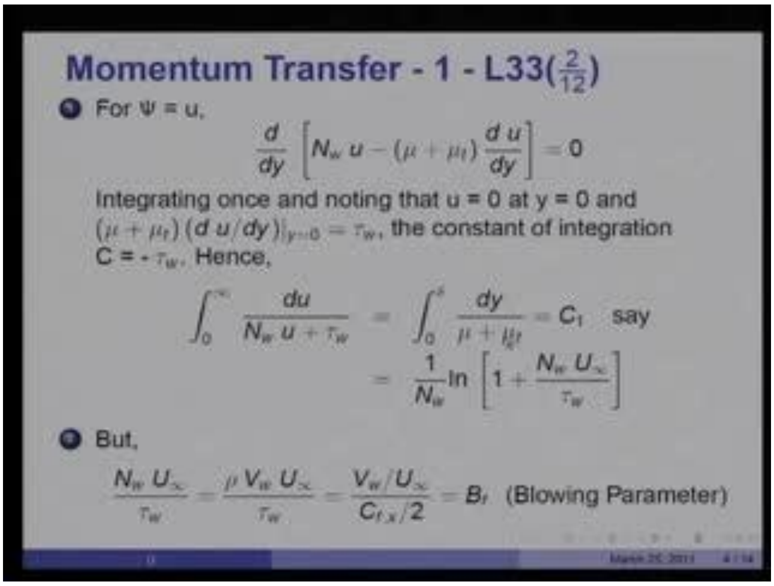
So, essentially then N w would be 1 over N w ln 1 plus B f would equal some constant C 1. That is what I have written here In 1 plus B f would equal C 1 times N w and C 1 rho U infinity B f over C f x by 2 because, N w is rho V w which can be written as rho U infinity B f C f x by 2.

(Refer Slide Time: 07:11)



Of course, as B f tends to 0, the C f x must tend to C f x at v w equal to 0 and then assuming that this integration, that is, integration 0 to delta mu plus mu t would remain the same whether there is mass transfer at the wall or no mass transfer at the wall which of course would be exactly true if it was a laminar boundary layer. But, even in turbulent boundary layer if we say that mu t essentially is a function of y and not effected by whether as you will recall from Prandtl's mixing length, there it will not be too seriously affected.

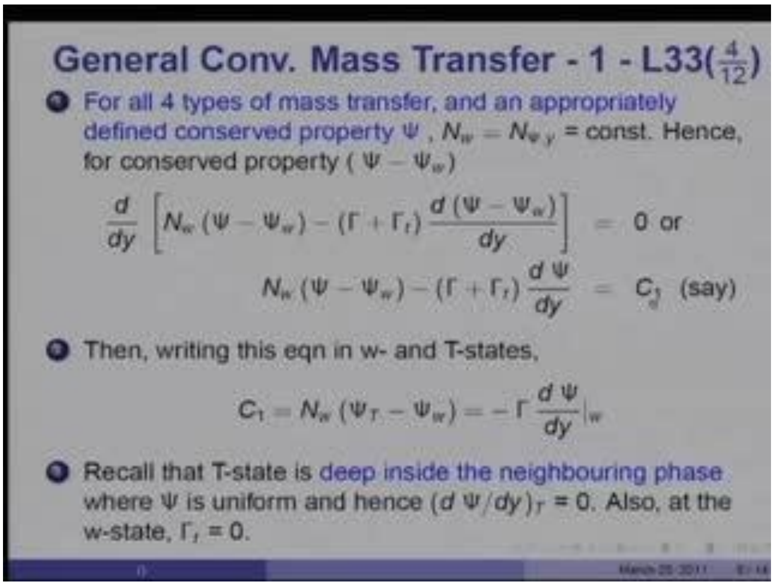
(Refer Slide Time: 07:42)



If we assume C 1 is independent of whether v w is finite or 0 we can show that it follows from this equation that C f x v w divided by C f x v w 0 would be ln 1 plus B f by B f. This equation is applicable to both laminar and turbulent flows and it is derived for dp dx equal to 0 but can be taken to be valid even for mild pressure gradients as was done during integral analysis of momentum equations in our previous analysis of fluid flow problems.

Now, for all 4 types of mass transfer I am not going to re-derive as I did in case of Stefan flow. I will simply say that in each case we simply converted the applicable mass transfer and energy transfer equation to a conserved property equation with appropriately defined conserved property psi.

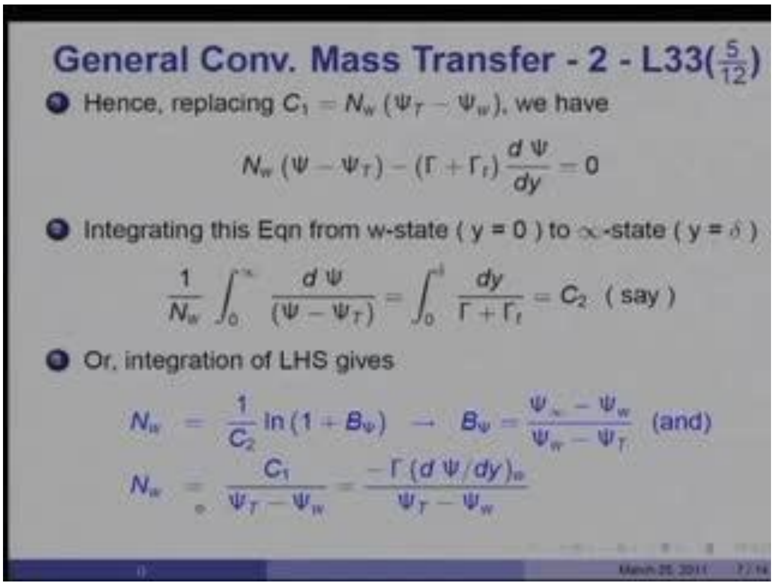
(Refer Slide Time: 08:40)



So, for all types of mass transfer and an appropriately defined conserved property, psi N w will be equal to N psi y equal to constant and hence for conserved property instead of psi, I can also take psi minus psi w as a conserved property then N w into psi minus psi w gamma plus gamma t d by dy psi minus psi w is equal to 0 or N w into psi minus psi w minus gamma plus gamma t d psi by dy equal to C 1 - some constants C 1. Now, remember d psi omega d psi w by dy is of course always 0, so that is that does not appear here but d psi by dy would certainly survive.

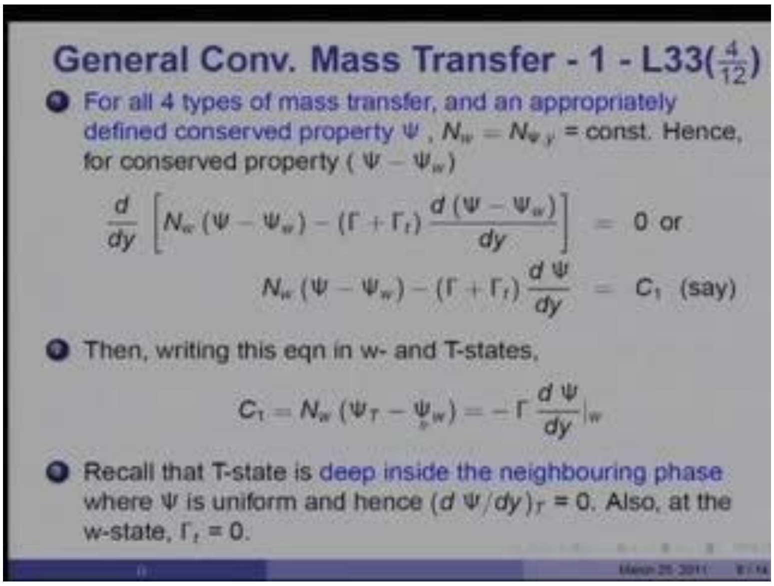
Then if I write this equation in the w state then I would get C 1 equal to gamma d psi by dy at y equal to 0 and if I write it in the T-state I will get N w psi minus psi t minus psi w and in the T-state there are no variations of psi and therefore this entire term would be 0. Again, C 1 and therefore equating the 2 equations, because they are both equal to C 1, I would get C 1 equal to N w psi t minus psi w equal to minus gamma d psi by dy at w and at w-state gamma t is equal to 0; so that is what I have stated here.

(Refer Slide Time: 10:33)

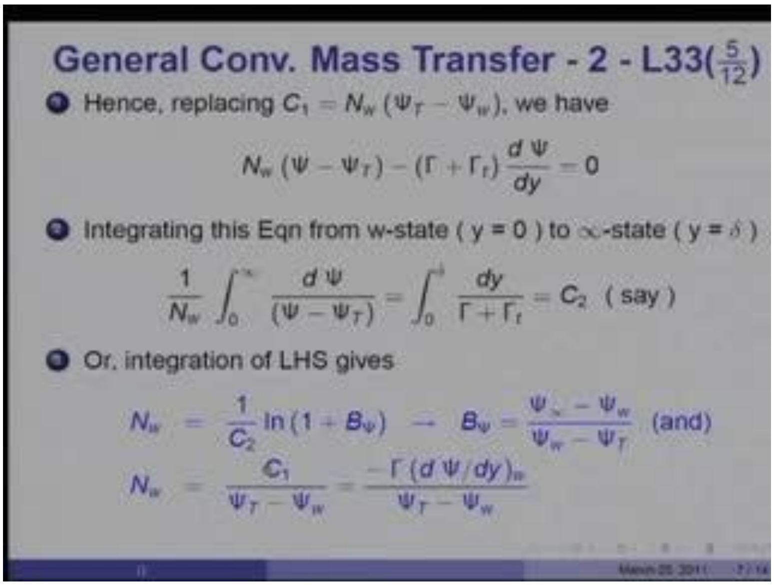


We shall make use of this equation and substitute for C 1 here. C 1 equal to N w psi t minus psi w; I will substitute that here on the next slide and you will see that therefore, I would get N w psi minus psi t minus gamma plus gamma t d psi by dy equal to 0. If I integrate this equation from w-state y equal to 0 to infinity state y equal to delta then I will get 1 over N w equal to 0 to infinity d psi by dy d psi by psi minus psi t equal to 0 to delta dy gamma plus gamma t, where gamma in case of energy gamma is k by C p in case of species transfer it is rho m times diffusivity.

(Refer Slide Time: 11:41)

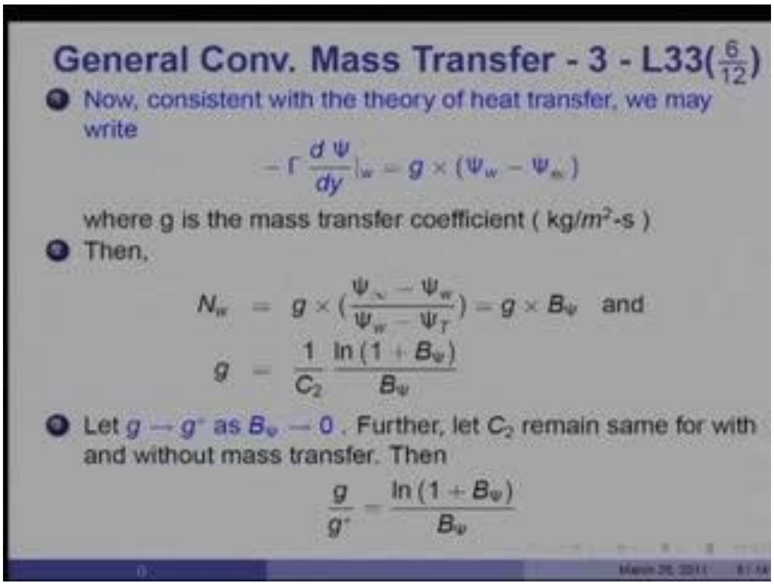


(Refer Slide Time: 11:46)



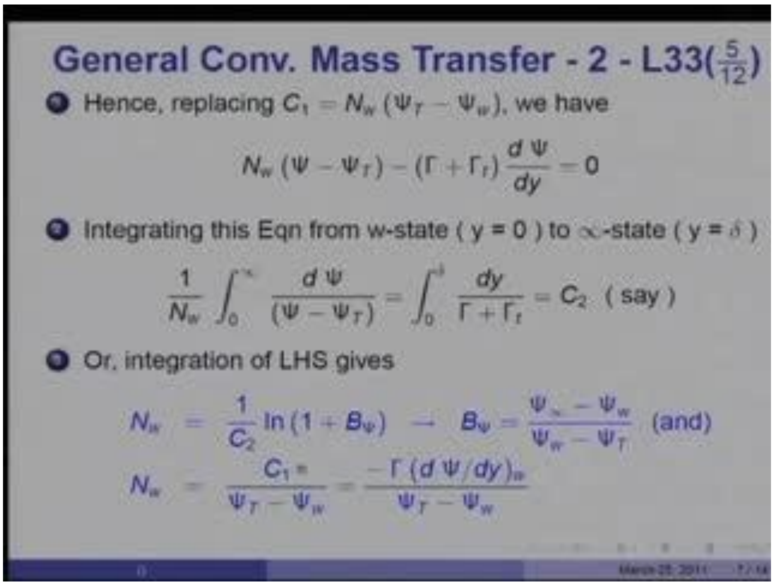
So, let us say that integral like in the previous case we will say is equal to C 2. Let us say then the integration of the left hand side would give me N w equal to 1 over C 2 ln plus B psi because, same as previous case B psi is equal to psi infinity minus psi w divided by psi w minus psi T. N w will be from the previous slide N w would be C 1 divided by psi T minus psi w and that would equal minus gamma d psi by dy at w psi T minus psi w. So, we have got 1 equation which is of this form and the other equation which is of this form the first containing C 2 and the second containing C 1.

(Refer Slide Time: 12:05)

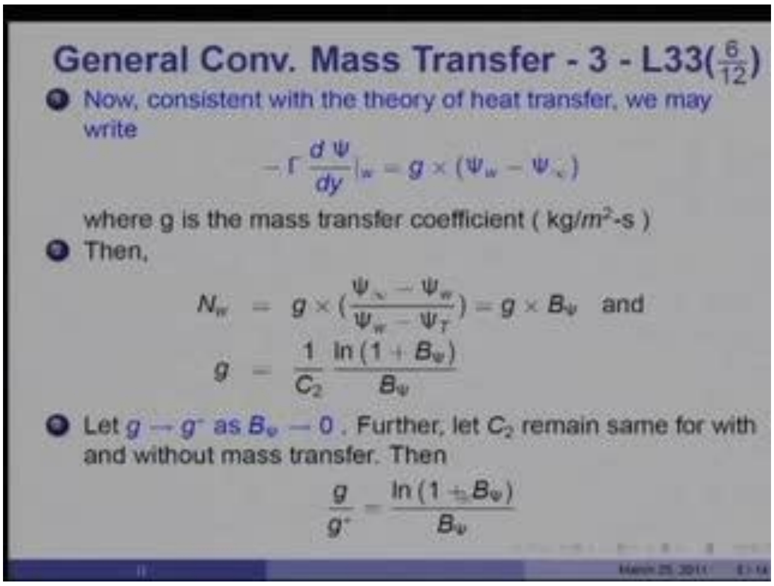


Now, consistent with the theory of heat transfer, we may write minus gamma d psi by dy at w is equal to g times psi w minus psi infinity, where g is now the mass transfer coefficient.

(Refer Slide Time: 12:29)



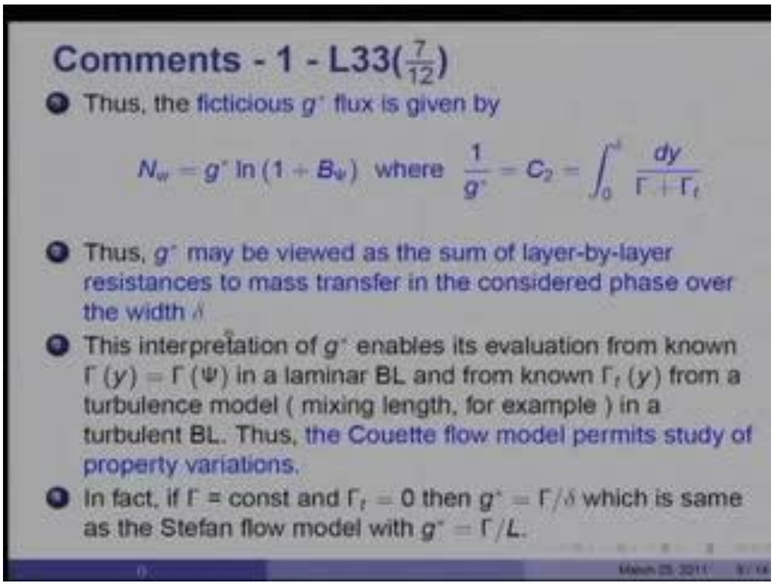
(Refer Slide Time: 12:38)



If I replace this quantity by this quantity in the previous expression here then, I can get C 1 from there in terms of g. Therefore, the final form would look like N w equal to g time psi infinity minus psi w divided by psi w minus psi T, or simply g times B psi and g itself would be 1 over C 2 times ln 1 plus B psi divided by B psi.

If I assume that as B psi tends to 0 g tends to g star, the mass transfer coefficient let us say it tends to g star corresponding to B psi tending to 0. Further, and if I say that C 2 remains constant with or without mass transfer as we said earlier, this statement is of course perfectly true for laminar boundary layer. But, even if I say it is true for turbulent boundary layer where 1 over where gamma t would be function of y then, it follows that g over g star would be simply ln 1 plus B psi by B psi.

(Refer Slide Time: 13:51)



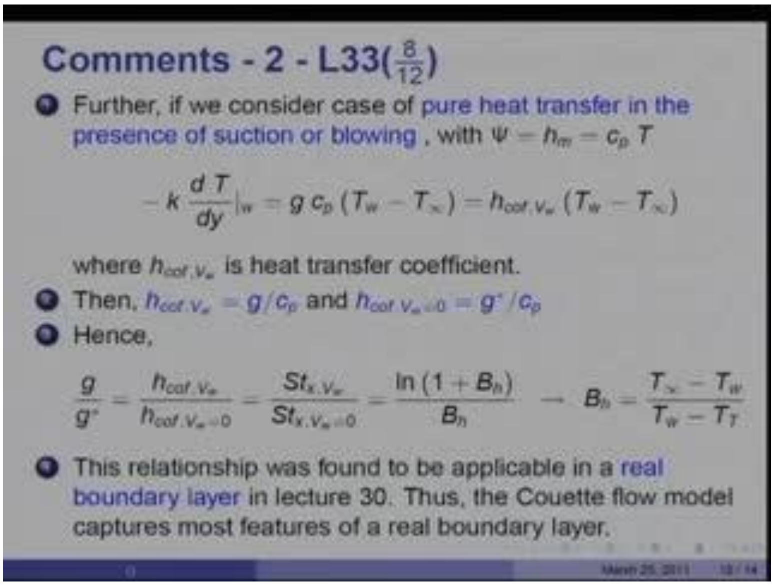
This is a very important result thus the fictitious g star flux is now given by N w g star ln 1 plus B psi, where 1 over g star is equal to is equal to C 2 which is equal to 0 to delta dy over gamma plus gamma t.

So we can now view g star itself as the sum of layer by layer from 0 to delta resistances to mass transfer in the considered phase over the width delta because, remember this is the diffusion coefficient. So, one over diffusion coefficient would simply be resistances resistance and we are simply saying that 1 over g star which itself is a kind of a resistance because g star is conductance then 1 over g star would be simply layer by layer addition of resistances to mass transfer in the considered phase.

This interpretation of g star enables its evaluation from gamma y equal to gamma psi in a laminar boundary layer and from the known gamma t y from a turbulence model like a mixing length. Or for example, in a turbulent boundary layer thus the Couette flow model permits study of the property variations. Remember, a gamma itself could be a function of psi of temperature or the mass fraction itself gamma t on the other hand would be function of the turbulence characteristics of the boundary layer. In fact, if gamma is equal to constant and gamma t is equal to 0 which is the case of a laminar diffusion problem, then g star would be simply gamma by delta which is same as the Stefan flow model in which g star was shown to be equal to gamma by L.

We have recovered most of the features of the Stefan flow model for gamma equal to constant and gamma t equal to 0 and g star would then be gamma by L ln 1 plus B psi which is what we had shown in the Stefan flow model.

(Refer Slide Time: 16:03)



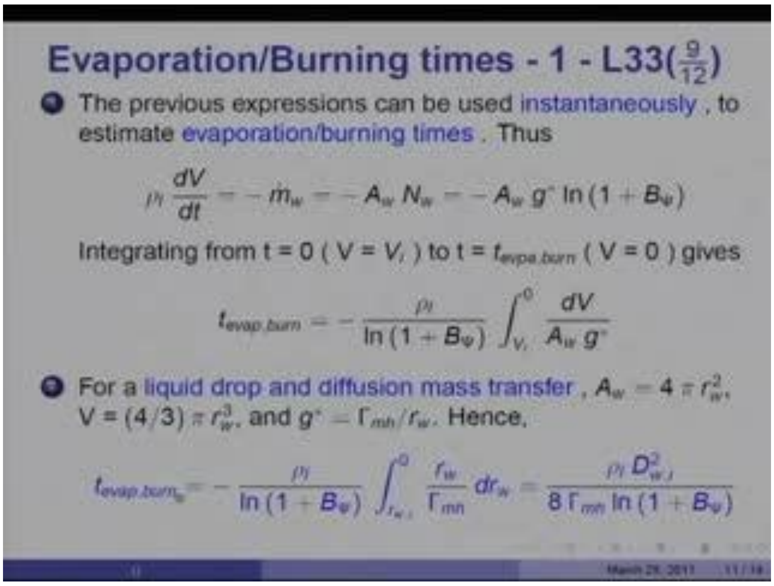
Now, if we consider the case of pure heat transfer in the presence of suction and blowing something we have solved by similarity method then, with psi is equal to h m equal to C p T because it is an inert situation, just considered phase as air flowing over it and suction and blown fluid is same as the fluid in the considered phase its temperature may be same or different, that does not matter. but Therefore, we would get minus k d T dy equal to 0 which is the heat flux and that would equal g times C p into T w minus T infinity according to our model and that would equal heat transfer coefficient for suction and blowing into T w minus T infinity.

So, that is what the heat transfer coefficient is and therefore we deduce that the heat transfer coefficient for finite v w is nothing but g by C p and likewise h cof v w equal to 0; that is in the absence of mass transfer would be equal to g star by C p because g star is a value of g when v w is equal to 0. Therefore, we deduce that g over g star would also be equal to h cof v w divided by h cof v w 0 equal to Stanton x at v w for v w divided by Stanton x for v w equal to 0. That would equal ln B h by B h where B h now is T infinity minus T w over T w minus T T because, specific heats in all states are taken to be constant at the moment.

Now, this relation was found to be applicable in real boundary layer flow in lecture 30, we will recall and therefore the Couette flow model captures also the features of a real boundary layer flow.

So the Couette flow model on the one hand when gamma t is 0 captures this Stefan flow model feature and it also captures the boundary layer flow features at least when the species are the same; it captures both the features.

(Refer Slide Time: 18:23)

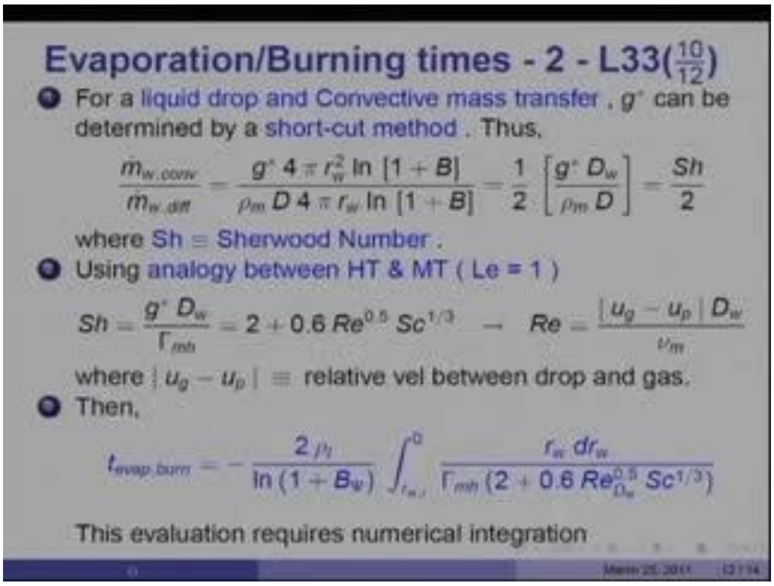


Now, we turn to the application of these two evaporation and burning in which we shall invoke the previous expression for can be used instantaneously to estimate the evaporation or burning time. Thus, let us say if I have rho l dV by dt is equal to minus m dot w which is the rate of change of mass is equal to minus m dot w k g per second and that would equal A w N w and that would equal minus A w g star ln 1 plus B psi where A w is the area of the surface through which the mass transfer is taking place.

Then, integrating this from time t equal to 0 to complete evaporation when the volume disappears gives us the relationship t evaporation or burned is equal to minus rho l ln 1 plus B psi equal to v initial to 0 dV by A w g star. Now, let us say if we are considering a liquid droplet and diffusion mass transfer that is gamma t is equal to 0 then A w will be equal to 4 pi r w square V will be 4 pi 3 pi r w square and g star will be gamma h by r w. As you will recall then, hence you will see that evaporation or burning time would be minus rho l divided by 1 over B psi r w i to 0 r w by gamma m h dr w and that will yield the so called D squared law rho l D w i square h times gamma h ln 1 plus B psi.

This expression is used extensively in designing dryers and so on because, all it says is that if you reduce, the smaller the diameter of the droplet, faster will be the drying achieved; because, if you reduce the diameter by a factor of 2 the evaporation time will reduce by factor of 4 in stagnant surrounding. This will be considered as a guidance for atomization of fuels atomization of let us say milk which is to be dried into a powder and or in a cooling tower where you send hot cooling water for cooling purposes, then in a shower, then you want to reduce the diameter as small as possible so that you get very quick drying and a smaller dryer or a cooling tower.

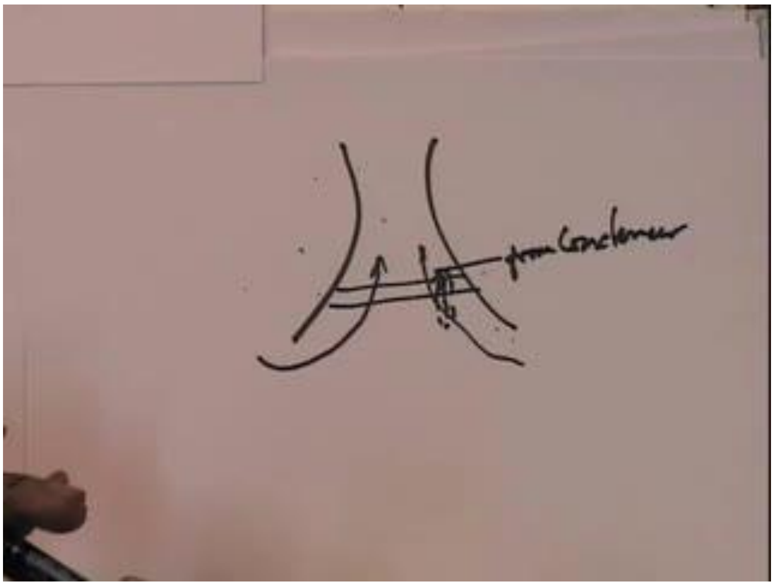
(Refer Slide Time: 21:08)



Now, let us say the liquid droplet was in a convective environment as in inside a diesel engine whereas, you know in a diesel engine the liquid droplets are injected in a atomized state and they come out like a cloud. When the piston is at d t top dead center, the temperature is already very high in the surroundings and the droplets evaporate and then burn inside the cylinder.

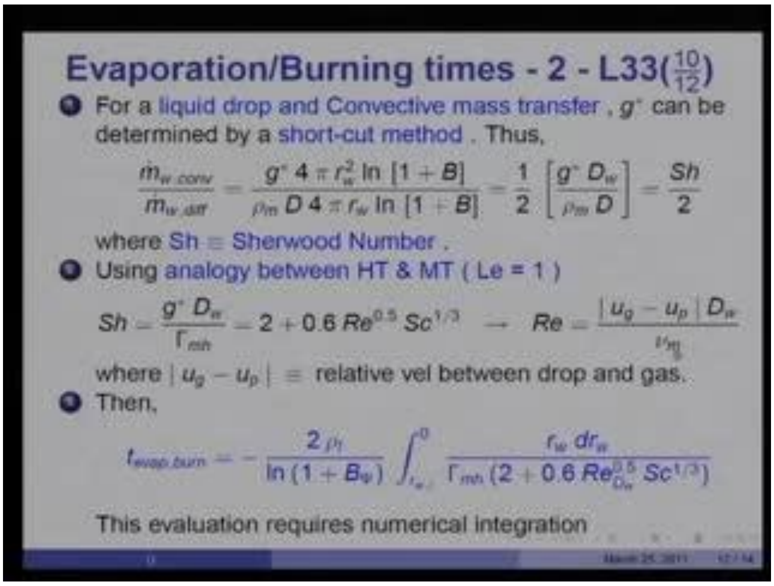
We can assess such we can evaluate the evaporation or drying times for evaporation or burning times in such situations; but, the environment there is convective because the pistons head is modeled in such a way that there is a swell inside the cylinder. So, the air movement and the particle and the droplet movement there is a relative velocity between them. So, essentially you get now convective evaporation or convective drying. Same thing happens when you have a cooling tower where the droplet is falling down through, let us say, stagnant air; but, that sets up a resultant velocity between the droplet and the surrounding air which is stagnant.

(Refer Slide Time: 22:39)



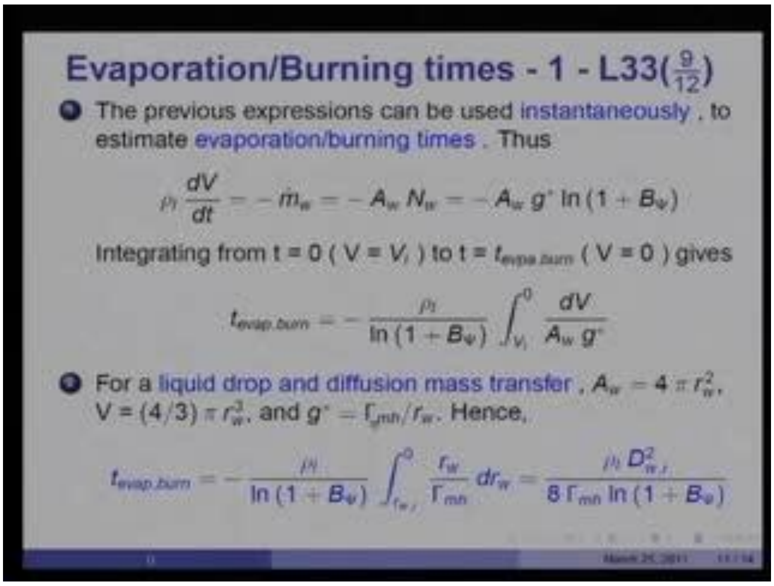
On the other hand, in thermal power stations you will recall that we have cooling towers like that and the air ingresses like this. and on Here you have showers of water where the cooling water from the condenser falls down and there is an air. So, you have a counter current of air and water and therefore the liquid droplets evaporate under counter flow. So, the relative velocity is additive of the air flow and the water flow droplet flow which is coming down step in the process that the water gets cooled and they send back to condenser. Of course, some amount of water is lost because the air moving upwards picks up some moisture which has to be topped up; of course, from time to time so that the condenser is not starved of cooled water.

(Refer Slide Time: 23:35)

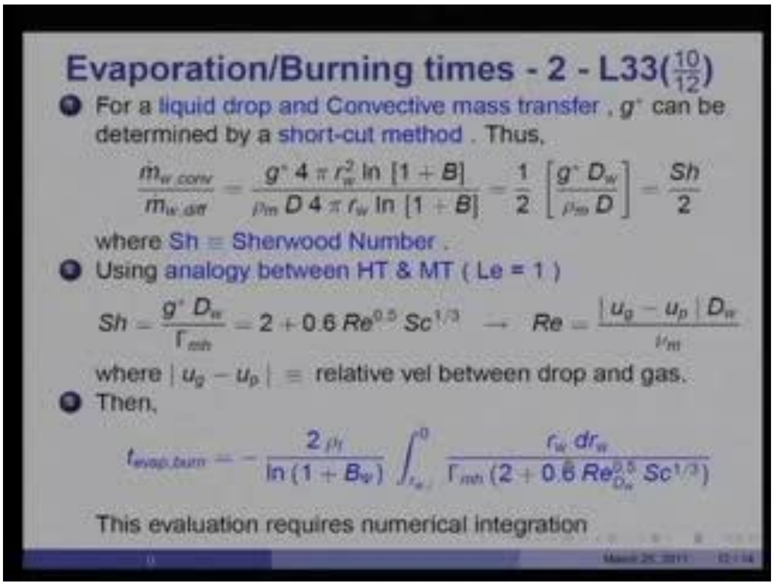


In such situation, we can use a short cut method and thus we can say that m dot w that is mass transfer rate under convection divided by mass transfer rate under diffusion would be mass transfer rate under convection. According to the Couette flow model is g star times A w which is 4 pi r w square ln 1 plus B and mass transfer rate. According to diffusion model which is a Stefan flow model is, rho m D 4 times pi r w into ln 1 plus B. Therefore, canceling the terms you will get 1 over 2 g star D w which is the diameter of the droplet divided by rho m diffusivity and this quantity like Nusselt number is called the Sherwood number in mass transfer. The analog analogous to Nusselt number, we have a Sherwood number which is essentially g star the mass transfer coefficient into the diameter divided by rho m D which is equivalent of the conductivity of the fluid.

(Refer Slide Time: 25:32)



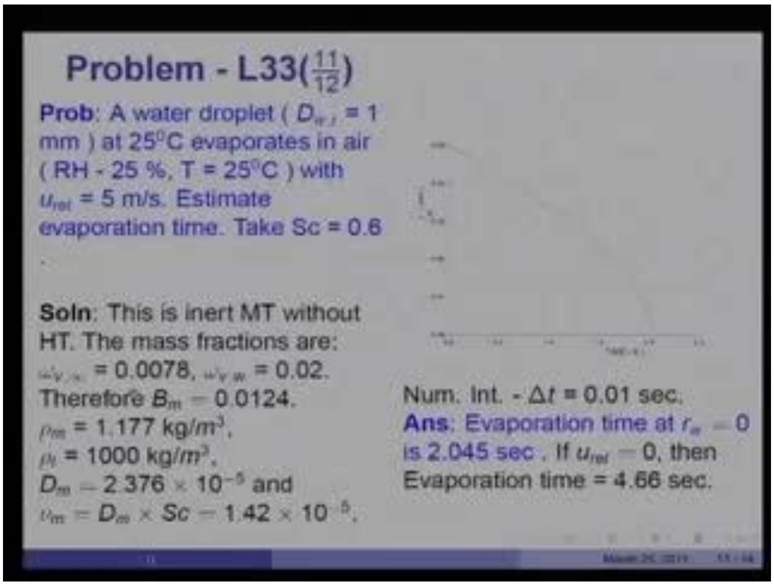
(Refer Slide Time: 25:35)



Now, if I use analogy between heat and mass transfer for Lewis number 1 then, you know that the for a flow over a sphere you know that the Nusselt number is 2 plus 0.6, Reynolds raised to 0.5, Prandtl raised to third, but I can now say Sherwood number would be 2 times 0.6 Re raise to 0.5 Schmidt number raise to one-third where Re is the is based on the relative velocity between the gas and the droplet - the diameter of the droplet and the kinematic viscosity. Or this expression for example, would now get changed to 2 times rho l ln 1 plus B psi r w i to 0 r w dr w divided by gamma h 2 plus 0.6 Re D w 0.5 Schmidt number raise to 1 by 3.

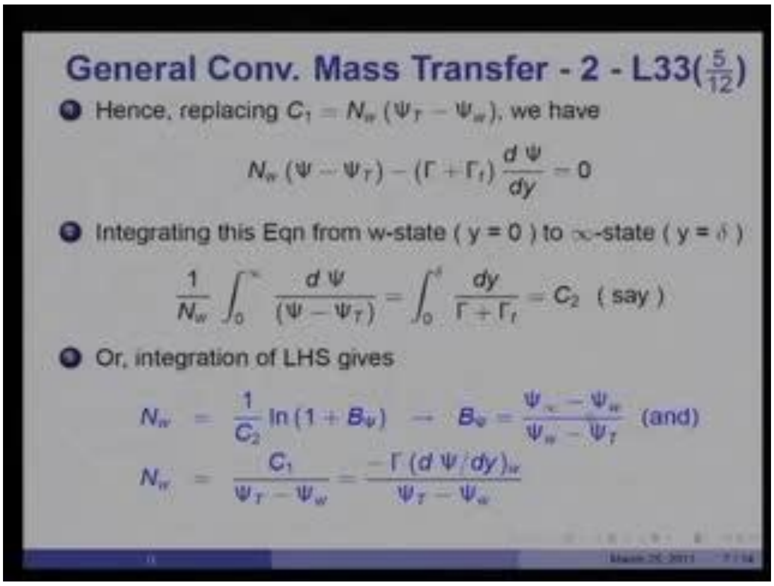
Now, since this also contains the radius r w wired because D w is 2 times r w this expression integration requires numerical integration because close form solutions are not found.

(Refer Slide Time: 26:13)



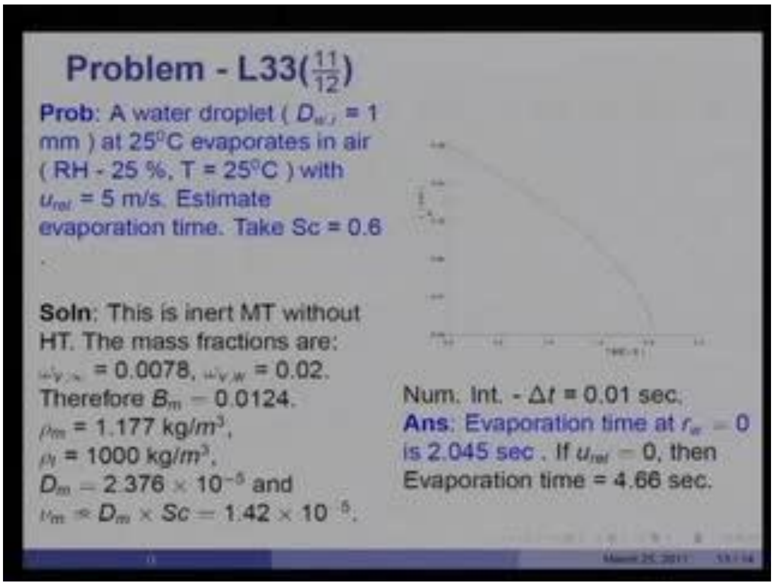
Let us take for example: if I took a problem, let us say a water droplet D w i is 1 millimeter diameter at 25 degree centigrade evaporates in air whose relative humidity is let us say 25 percent then and temperature is 25 degree centigrade, so that this is a case of mass transfer without heat transfer and no chemical reaction. Let us assume that the relative velocity between the 2 is 5 meters per second; so estimate the evaporation time and take Schmidt number equal to 0.6 which is quite typical of gaseous mixture.

(Refer Slide Time: 27:20)



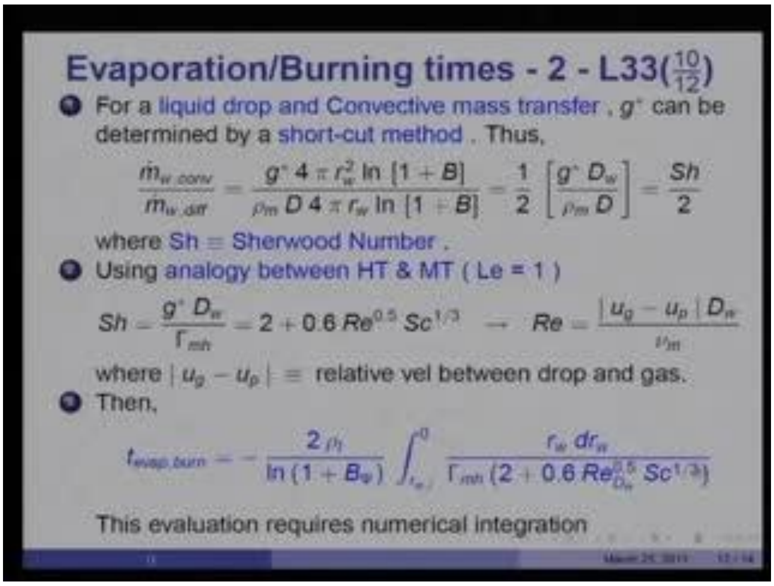
So, this is a mass inert mass transfer problem without heat transfer the mass fractions are in the infinity state corresponding to 25 percent RH and T equal to 25 will give you omega v infinity equal to 0.0078 omega v w corresponding to 100 percent relative humidity at the droplet surface. 25 degree centigrade will be 0.02 B m, would be omega vapor and the infinity state minus omega vapor in the w state and omega vapor in the w state minus in the T state which is the transfer state and that would be equal to 1.

(Refer Slide Time: 27:39)

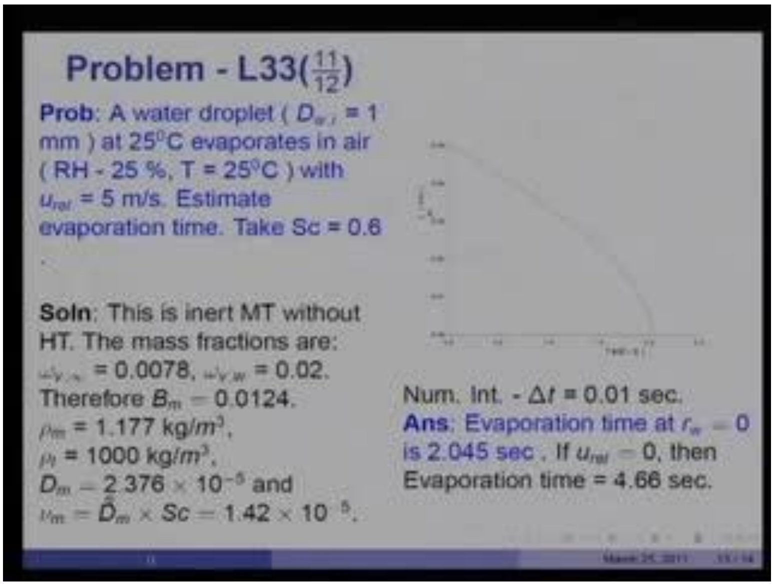


So, using this definition for omega v as we did in problems on diffusion mass transfer the B m would turn out to be 0.0124 at the mean conditions between the infinity and w states mixture density can be evaluated as 1.177 kilograms per meter cube. Liquid density would be 1000 kg per meter cube; whereas, a water diffusivity from the lectures we say that the diffusivity of water vapor through air is 2.376 into 10 raise to minus 5 and nu m will be diffusivity into Schmidt number which will be equal to 1.42 into 10 raise to minus 6.

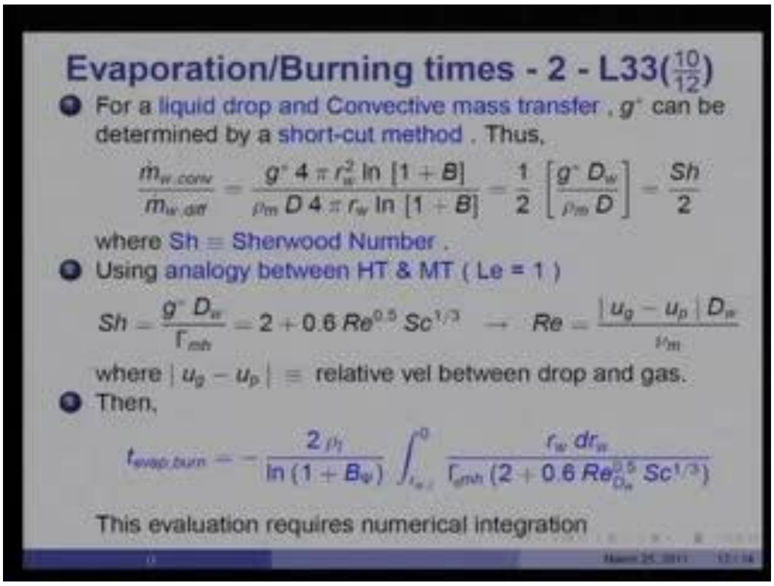
(Refer Slide Time: 28:31)



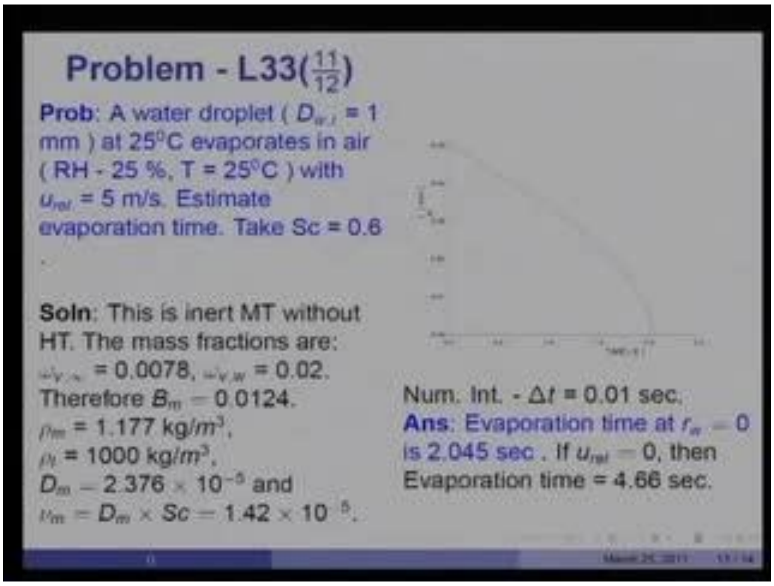
(Refer Slide Time: 28:34)



(Refer Slide Time: 28:40)

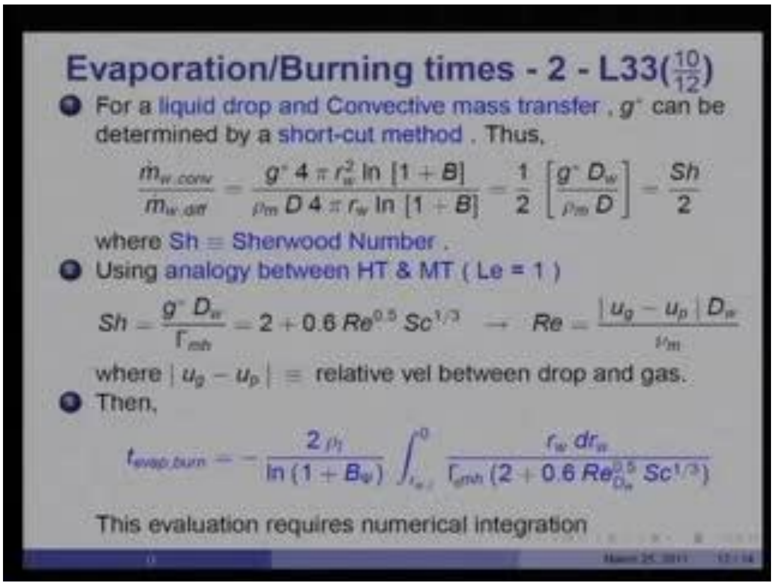


(Refer Slide Time: 28:46)

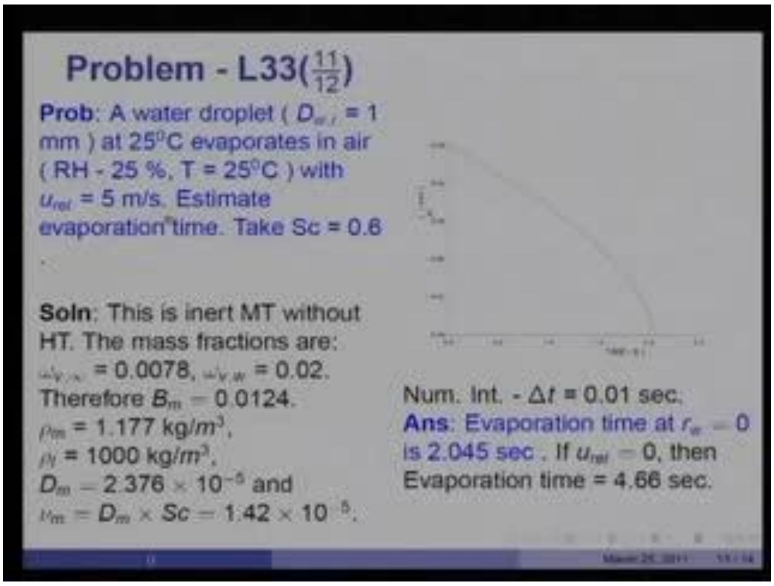


So, knowing nu m we can evaluate and u relative we can evaluate the Reynolds number that is required here. In the Reynolds number expression and D m which is required here to calculate gamma h because this is rho times D m. So, we can carry out the numerical integration with time step 0.01 second. Then evaporation time at r w equal to 0 would be 2.05 second and the radius would vary in this fashion 0.5 mm to start with and then it drops down very gradually to 0 value here at above 2.045 seconds.

(Refer Slide Time: 29:28)

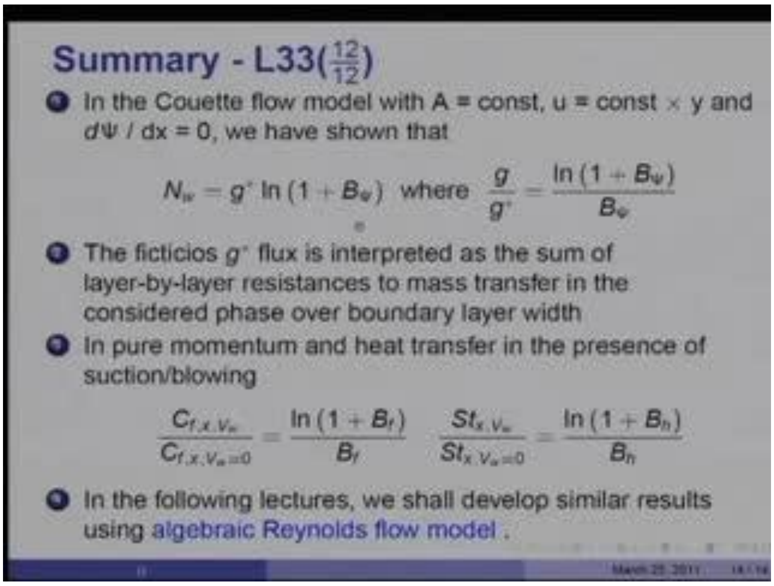


(Refer Slide Time: 29:34)



We can solve the same problem in stagnant surroundings by simply setting u relative equal to 0; which means that the Reynolds number here is 0 and therefore this will be 2 times gamma h. Then you will see that the evaporation time becomes 4.66 seconds. So, clearly having a relative velocity between the gas and the droplet has reduced the evaporation time and this is the principle that is used in cooling towers and in diesel engines for the purposes of reducing the size. In case of cooling towers and in diesel engines, this is of great value because it enhances the rate of burning of the fuel and which incidentally also reduces the cut off ratio of a diesel engine which in turn improves the efficiency of the engine.

(Refer Slide Time: 30:28)



So, finally in summary then we can say that a Couette flow model with area equal to constant and u equal to constant times y and d psi by dx equal to 0 gives us the formula N w equal to g star ln 1 plus B psi, where g over g star is equal to ln 1 plus to B psi by B psi.

We interpreted the g star flux as the sum of the layer by layer resistances to mass transfer in the considered phase over boundary layer width in pure momentum and heat transfer in the presence of suction and blowing. That is without gradients of species of any kind because the same species being sucked or blown. We have shown that C f x in the presence of suction and blowing divided by c f x in the absence of it is simply ln 1 plus B f by B f and the heat transfer likewise would be ln 1 plus B h by B h.

So, Couette flow model recovers essentially the results expected from a boundary layer flow model it also recovers the results expected from the Stefan flow model and in turn gives us an opportunity to evaluate the effect the property variations which we shall take up in subsequent lectures.

So, in the next lecture we will develop very similar results to what we have shown in Couette flow model via the algebraic Reynolds flow model and you will see what form N w and B relation has as per the Reynolds algebraic Reynolds flow model.