# Convective Heat and Mass Transfer

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## Module No. # 01
## Lecture No. # 32
## Stefan Flow Model

We are now familiar with the 3 simplifications of the boundary layer flow model. The first one was the Stefan flow model; second one was the Couette flow model and the third is the Reynolds flow model.

In today's lecture, I shall develop the Stefan flow model further for variety of mass transfer problems that we encounter in engineering practice.

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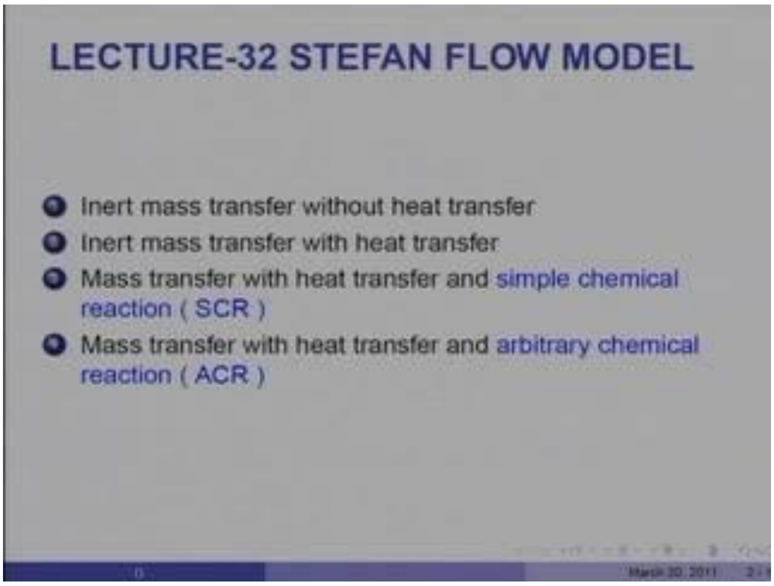

LECTURE-32 STEFAN FLOW MODEL

- Inert mass transfer without heat transfer
- Inert mass transfer with heat transfer
- Mass transfer with heat transfer and simple chemical reaction ( SCR )
- Mass transfer with heat transfer and arbitrary chemical reaction ( ACR )

The types of problems that we encounter are as follows: first is the inert mass transfer without heat transfer or chemical reaction. This means, something like simple evaporation of water where the water and the environment are all at the same temperatures. So, there is no temperature gradient and as a result, there is no heat transfer. Nor is there any chemical reaction because, water simply evaporates without any chemical reaction and the mass transfer however takes place because of the concentration gradients.

The next would be where let us say, the water droplet or water itself in a lake is at a different temperature from that at the infinity state or the environment. In which case, there would be heat transfer either through the water or from the water to the environment depending on which temperature is larger. Then, we will move to the situation in which there would be chemical reaction or combustion in which, let us say, liquid droplet or a would be burning along with heat transfer from the environment to the droplet and finally, but there we would use what is called as simple chemical reaction and I will explain what that means.

But many a times, particularly when dealing with burning of solids, we need the reaction. Mechanism is so complex that we have to deal with that situation somewhat differently. So, I will develop the forms of mass transfer relationships that evolve for the 4 types of mass transfer problems.

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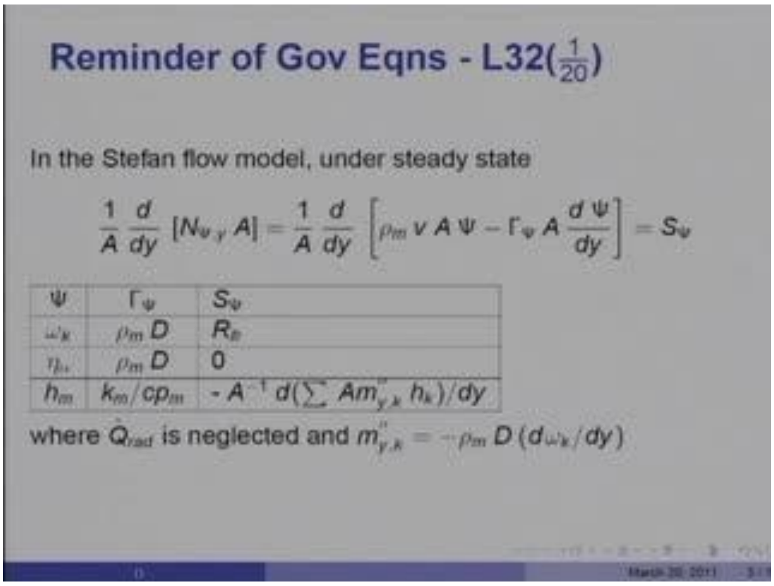

## Reminder of Gov Eqns - L32($\frac{1}{20}$)

In the Stefan flow model, under steady state

$$\frac{1}{A}\frac{d}{dy}\left[N_{\Psi,y}\,A\right] = \frac{1}{A}\frac{d}{dy}\left[\rho_m\,v\,A\,\Psi - \Gamma_\Psi\,A\,\frac{d\Psi}{dy}\right] = S_\Psi$$

| $\Psi$ | $\Gamma_\Psi$ | $S_\Psi$ |
|---|---|---|
| $\omega_k$ | $\rho_m D$ | $R_k$ |
| $\eta_\alpha$ | $\rho_m D$ | 0 |
| $h_m$ | $k_m/cp_m$ | $-A^{-1}\,d(\sum A m''_{y,k}\,h_k)/dy$ |

where $\dot{Q}_{rad}$ is neglected and $m''_{y,k} = -\rho_m D\,(d\omega_k/dy)$

As you recall, under steady state, the mass transfer equation would be something like this: N psi y A equal to all that and psi can stand for mass fraction or element mass fraction or enthalpy; the radiation term is of course a neglected in this energy equation and m dot double prime y k as you will recall, is the diffusion mass flux as per the Fick’s law of diffusion.

So, these are the source terms in each of these equations; R k would be finite when there is a chemical reaction; if not, it will be 0.

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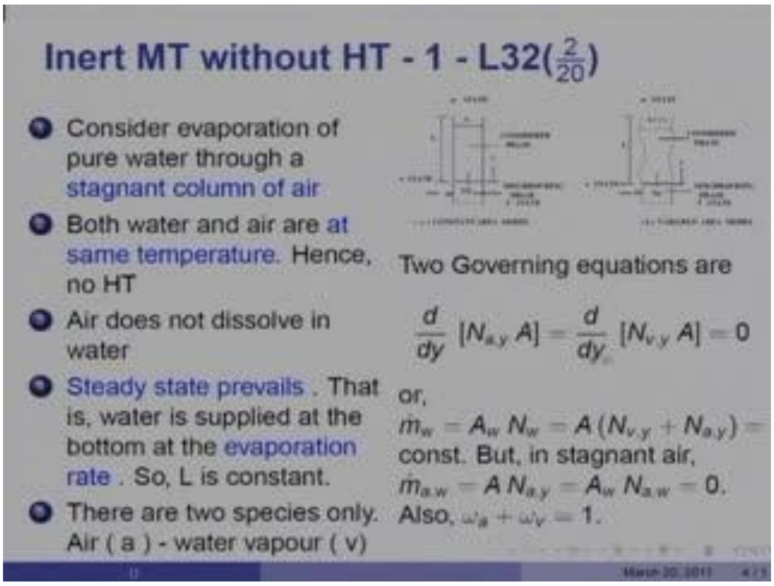

## Inert MT without HT - 1 - L32($\frac{2}{20}$)

- Consider evaporation of pure water through a stagnant column of air
- Both water and air are at same temperature. Hence, no HT
- Air does not dissolve in water
- Steady state prevails . That is, water is supplied at the bottom at the evaporation rate . So, L is constant.
- There are two species only. Air ( a ) - water vapour ( v)

Two Governing equations are

$$\frac{d}{dy}\left[N_{a,y}\,A\right] = \frac{d}{dy}\left[N_{v,y}\,A\right] = 0$$

or,
$\dot{m}_w = A_w\,N_w = A\,(N_{v,y} + N_{a,y}) =$ const. But, in stagnant air, $\dot{m}_{a,w} = A\,N_{a,y} = A_w\,N_{a,w} = 0$. Also, $\omega_a + \omega_v = 1$.

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So, let us consider the first type: inert mass transfer without heat transfer. Let us say I have a tank here with water in it and the water evaporates because in the infinity state the air is either dry or has relativity less than 100 percent. Therefore, there would be mass transfer from water to the infinity state. The column height that I have considered above the water is L and the air in the column is stagnant. Stefan flow model as you know, is primarily applied to diffusion mass transfer; both water and air are at the same temperature and therefore there is no heat transfer.

Air also does not dissolve in water; as such there will be no transfer of air from the considered phase into the water phase steady state prevails. That is, water is supplied at the bottom at the evaporation rate. What this means is that, somehow we have constructed an apparatus in which water is supplied at the same rate at which it is evaporating and therefore the column height L would remain constant or the water level will remain constant.

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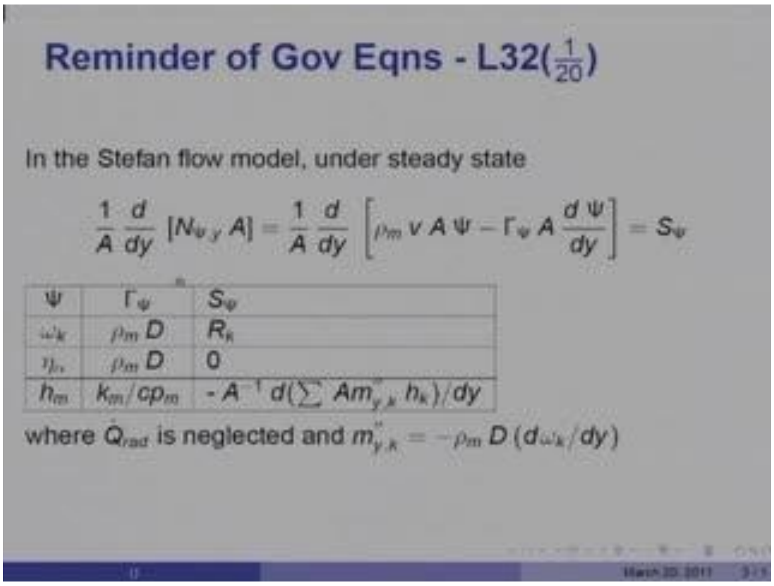

## Reminder of Gov Eqns - L32($\frac{1}{20}$)

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| $\Psi$ | $\Gamma_\Psi$ | $S_\Psi$ |
|---|---|---|
| $\omega_k$ | $\rho_m D$ | $R_k$ |
| $\eta_{fr}$ | $\rho_m D$ | 0 |
| $h_m$ | $k_m/cp_m$ | $-A^{-1}\,d(\sum A m''_{y,k}\,h_k)/dy$ |

where $\dot{Q}_{rad}$ is neglected and $m''_{y,k} = -\rho_m\,D\,(d\omega_k/dy)$

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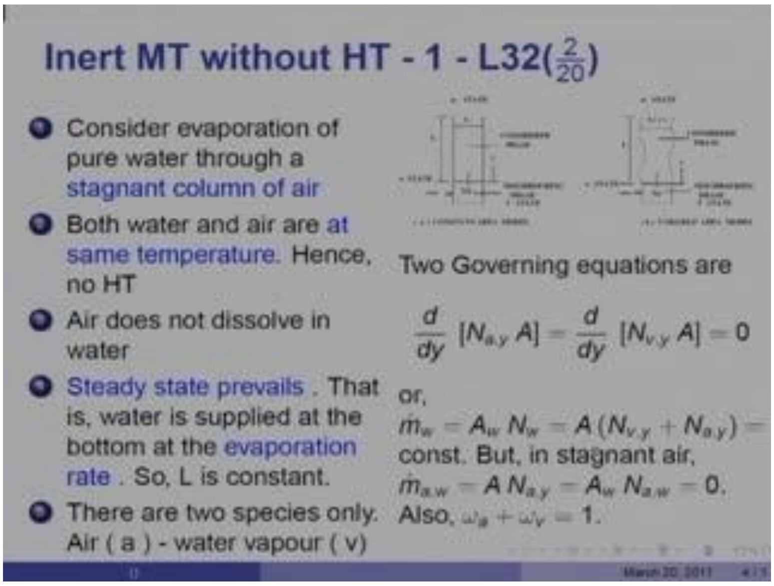

## Inert MT without HT - 1 - L32($\frac{2}{20}$)

- Consider evaporation of pure water through a stagnant column of air
- Both water and air are at same temperature. Hence, no HT
- Air does not dissolve in water
- Steady state prevails . That is, water is supplied at the bottom at the evaporation rate . So, L is constant.
- There are two species only. Air ( a ) - water vapour ( v)

Two Governing equations are

$$\frac{d}{dy}\,[N_{a,y}\,A] = \frac{d}{dy}\,[N_{v,y}\,A] = 0$$

or,
$\dot{m}_w = A_w\,N_w = A\,(N_{v,y} + N_{a,y}) =$ const. But, in stagnant air,
$\dot{m}_{a,w} = A\,N_{a,y} = A_w\,N_{a,w} = 0.$
Also, $\omega_a + \omega_v = 1$.

Now, in this problem we have two species: one is the air a and the water vapour v and therefore the governing equation for this, because there is no mass there is no chemical reaction as you will see in the previous equation, N v y A would be equal to 0 for both air and water. So, that is what I have written here; this means that the mass transfer rate m dot w in kg per second would be area A w into N w equal to A times N v y plus N a y equal to the constant because this is gradient, is 0.

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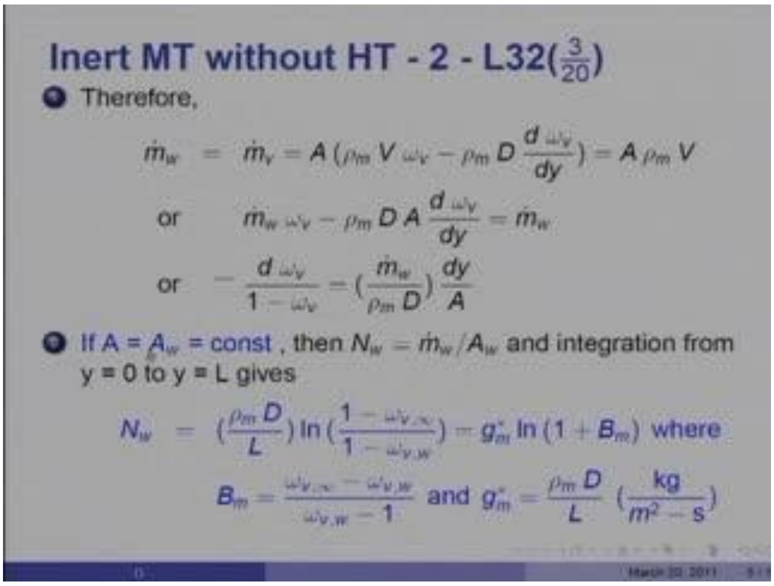

## Inert MT without HT - 2 - L32($\frac{3}{20}$)

- Therefore,

$$\dot{m}_w = \dot{m}_v = A\,(\rho_m\, V\, \omega_v - \rho_m\, D\, \frac{d\,\omega_v}{dy}) = A\,\rho_m\, V$$

$$\text{or} \quad \dot{m}_w\, \omega_v - \rho_m\, D\, A\, \frac{d\,\omega_v}{dy} = \dot{m}_w$$

$$\text{or} \quad -\frac{d\,\omega_v}{1 - \omega_v} = (\frac{\dot{m}_w}{\rho_m\, D})\, \frac{dy}{A}$$

- If A = $A_w$ = const , then $N_w = \dot{m}_w / A_w$ and integration from y = 0 to y = L gives

$$N_w = (\frac{\rho_m\, D}{L})\, \ln\,(\frac{1 - \omega_{v,\infty}}{1 - \omega_{v,w}}) = g_m^*\, \ln\,(1 + B_m) \text{ where}$$

$$B_m = \frac{\omega_{v,\infty} - \omega_{v,w}}{\omega_{v,w} - 1} \text{ and } g_m^* = \frac{\rho_m\, D}{L}\ (\frac{\text{kg}}{m^2 - \text{s}})$$

But, in stagnant air, m a w is equal to 0 because there is no mass transfer of air in this. It is stagnant and therefore, omega a plus omega v is equal to 1. We note this and therefore in this equation you will see that N a y will be set to 0 and m dot w would be simply A times N v y and that is what I have written here (Refer Slide Time: 06:15).

So, m dot w will be m dot of vapour only equal to A times N v y and N v y as you know is the convective mass flux plus diffusion mass flux and that would be equal to A rho m V. Or if I rearrange this, notice that A rho m V is simply m dot w. Therefore, m dot w into omega v minus rho m D A into d omega v by dy would be equal to m dot w or if I transfer this on the right hand side then, you will see it will be 1 minus omega v equal to m dot w by rho m D dy by A (Refer Slide Time: 06:50).

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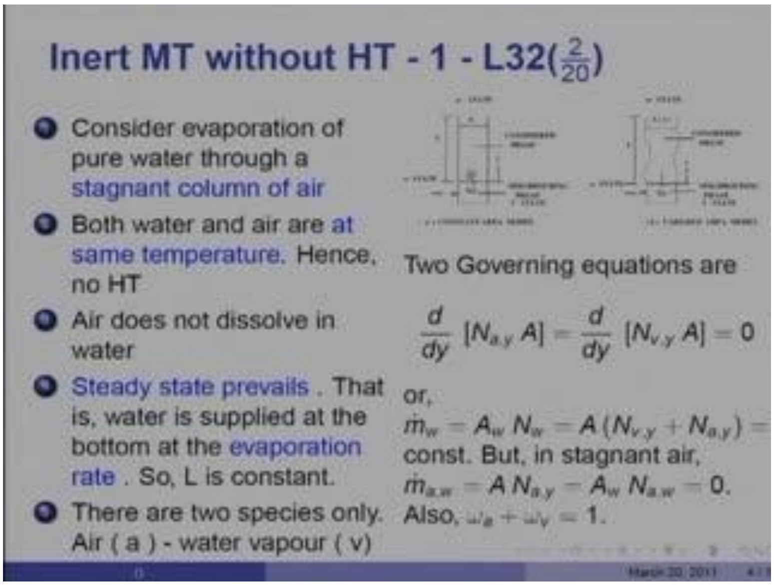

## Inert MT without HT - 1 - L32($\frac{2}{20}$)

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Also, $\omega_a + \omega_v = 1$.

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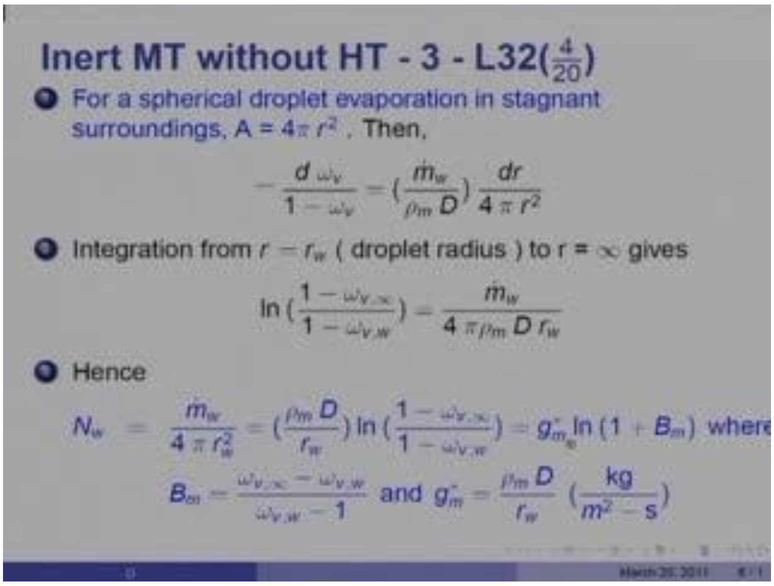

## Inert MT without HT - 3 - L32($\frac{4}{20}$)

- For a spherical droplet evaporation in stagnant surroundings, A = $4\pi\,r^2$ . Then,

$$-\frac{d\,\omega_v}{1-\omega_v} = (\frac{\dot{m}_w}{\rho_m\,D})\,\frac{dr}{4\,\pi\,r^2}$$

- Integration from $r = r_w$ ( droplet radius ) to r = $\infty$ gives

$$\ln\,(\frac{1-\omega_{v,\infty}}{1-\omega_{v,w}}) = \frac{\dot{m}_w}{4\,\pi\,\rho_m\,D\,r_w}$$

- Hence

$$N_w = \frac{\dot{m}_w}{4\,\pi\,r_w^2} = (\frac{\rho_m\,D}{r_w})\,\ln\,(\frac{1-\omega_{v,\infty}}{1-\omega_{v,w}}) = g_m^*\,\ln\,(1+B_m)\ \text{wher}$$

$$B_m = \frac{\omega_{v,\infty}-\omega_{v,w}}{\omega_{v,w}-1}\ \text{and}\ g_m^* = \frac{\rho_m\,D}{r_w}\,(\frac{\text{kg}}{m^2-\text{s}})$$

If A is equal to A w is equal to constant, that is, if I take the constant area model then simply, N w which is equal to m dot w by A w and integration would give me rho m D by L ln 1 minus omega v infinity over omega 1 minus omega v w equal to g star m into ln 1 plus B m. This bracket 1 minus omega v infinity into 1 minus omega v w, can also be written as omega v infinity minus omega v w over omega v w minus 1 and g m star would be rho m D by L. It has the same units as the mass flux N w and this is a constant and written in this form. B m is called the driving force from mass transfer to occur and it is given in this fashion.

So, you get a very simple logarithmic formula for mass flux or the evaporation rate, instantaneous evaporation rate of water in the spherical system. Supposing, I have a droplet then, the surface area A would go on changing with the radius and A would be equal to 4 phi r square as we go along and A would go on changing. So, A would be function of y if you like. In that case, you will see d omega v 1 minus omega v would equal m dot w rho m D into dr by 4 pi r square. Integration from r equal to r w which is the droplet radius to r equal to infinity, gives ln 1 minus infinity over 1 minus omega v w equal to this quantity with r w in the denominator. Remember, 1 over r infinity would be 0 and therefore only r w survives.

If I rearrange this equation it would be written in this fashion N w will be equal to m dot w divided by area of the spherical droplet 4 pi r w square, would be equal to rho m D r w into this quantity (Refer Slide Time: 09:30) which again can be written as 1 ln 1 plus B m, where B m is again as before omega B infinity minus omega v w into omega v w minus 1 and g m star which is the coefficient. If you like a is rho m D divided by r w instead of l in the previous problem when the area is constant.

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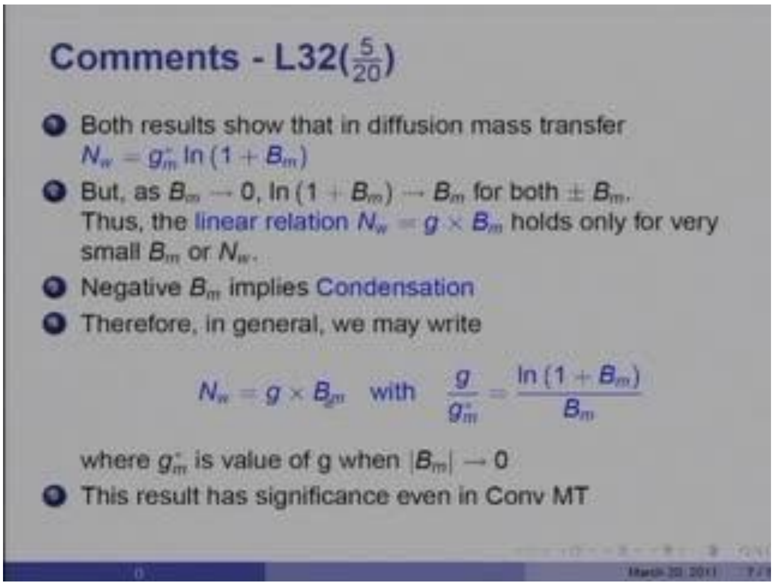

## Comments - L32($\frac{5}{20}$)

- Both results show that in diffusion mass transfer $N_w = g_m^* \ln(1 + B_m)$
- But, as $B_m \rightarrow 0$, $\ln(1 + B_m) \rightarrow B_m$ for both $\pm B_m$. Thus, the linear relation $N_w = g \times B_m$ holds only for very small $B_m$ or $N_w$.
- Negative $B_m$ implies Condensation
- Therefore, in general, we may write

$$N_w = g \times B_m \quad \text{with} \quad \frac{g}{g_m^*} = \frac{\ln(1 + B_m)}{B_m}$$

where $g_m^*$ is value of g when $|B_m| \rightarrow 0$

- This result has significance even in Conv MT

So, we get mass flux in this. Again, logarithmic form, even area changes; so both the results show that in diffusion mass transfer, you get N w equal to g star m ln 1 plus B m.

But, as B m tends to 0 you can check out on your pocket calculator that ln 1 plus B m tends to B m for both positive or negative B m. Thus, the linear variation N w g into B m holds only for very small mass transfer rate B m or N w whichever way you want to look at it. Typically, when B m is of the order of 0.02 or less, this relationship holds very well and when B m is negative then of course, it will imply condensation. When B m is positive it would imply evaporation therefore, in general we may write N w equal to g B m where g over g star m is equal to ln 1 plus B m by B m and where g star is the value of g when B m tends to 0.

Now, although this result has been found from for diffusion mass transfer, we shall later on show that the result has significance even in convective mass transfer. Let us now consider the case of inert mass transfer with heat transfer in which case let us say, the water and the environment are at different temperatures.

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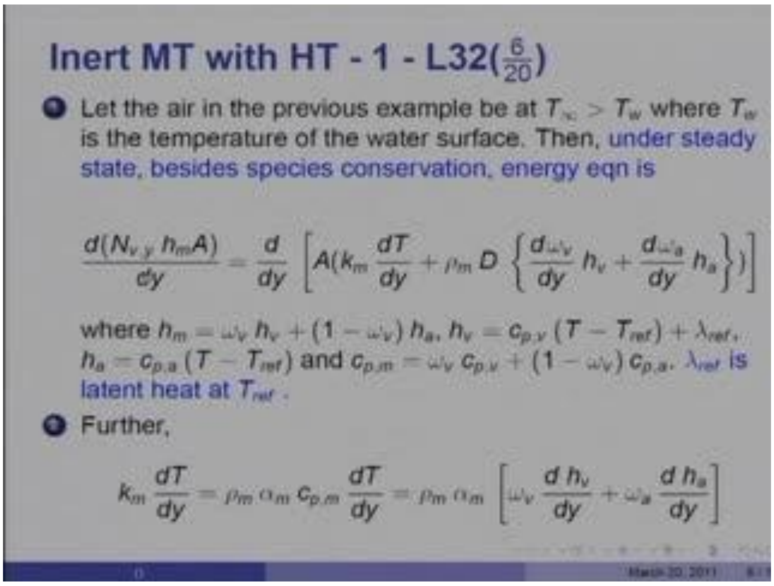

### Inert MT with HT - 1 - L32($\frac{6}{20}$)

- Let the air in the previous example be at $T_\infty > T_w$ where $T_w$ is the temperature of the water surface. Then, under steady state, besides species conservation, energy eqn is

$$\frac{d(N_{v,y}\, h_m A)}{dy} = \frac{d}{dy}\left[A(k_m \frac{dT}{dy} + \rho_m D \left\{\frac{d\omega_v}{dy} h_v + \frac{d\omega_a}{dy} h_a\right\})\right]$$

where $h_m = \omega_v\, h_v + (1 - \omega_v)\, h_a$, $h_v = c_{p,v}\,(T - T_{ref}) + \lambda_{ref}$, $h_a = c_{p,a}\,(T - T_{ref})$ and $c_{p,m} = \omega_v\, c_{p,v} + (1 - \omega_v)\, c_{p,a}$. $\lambda_{ref}$ is latent heat at $T_{ref}$.

- Further,

$$k_m \frac{dT}{dy} = \rho_m\, \alpha_m\, c_{p,m} \frac{dT}{dy} = \rho_m\, \alpha_m \left[\omega_v \frac{d\, h_v}{dy} + \omega_a \frac{d\, h_a}{dy}\right]$$

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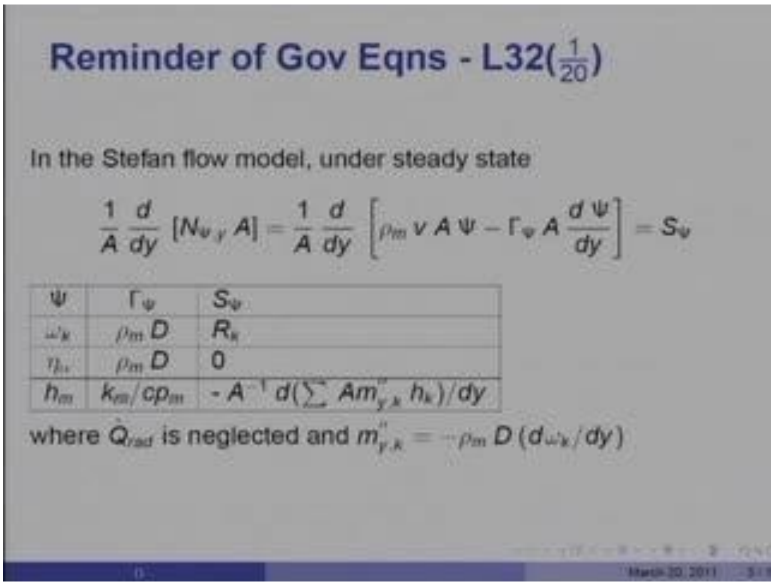

## Reminder of Gov Eqns - L32($\frac{1}{20}$)

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| $\Psi$ | $\Gamma_\Psi$ | $S_\Psi$ |
|---|---|---|
| $\omega_k$ | $\rho_m D$ | $R_k$ |
| $\eta_k$ | $\rho_m D$ | 0 |
| $h_m$ | $k_m/cp_m$ | $-A^{-1}\,d(\sum A m''_{y,k}\,h_k)/dy$ |

where $\dot{Q}_{rad}$ is neglected and $m''_{y,k} = -\rho_m D\,(d\omega_k/dy)$

So, let us say in temperature the environment is greater than that of the surface where T w is the temperature of the water surface. Then, under steady state besides species conservation equation, we must now invoke the energy equation and that for as you will see from our first line here we are invoking this equation.

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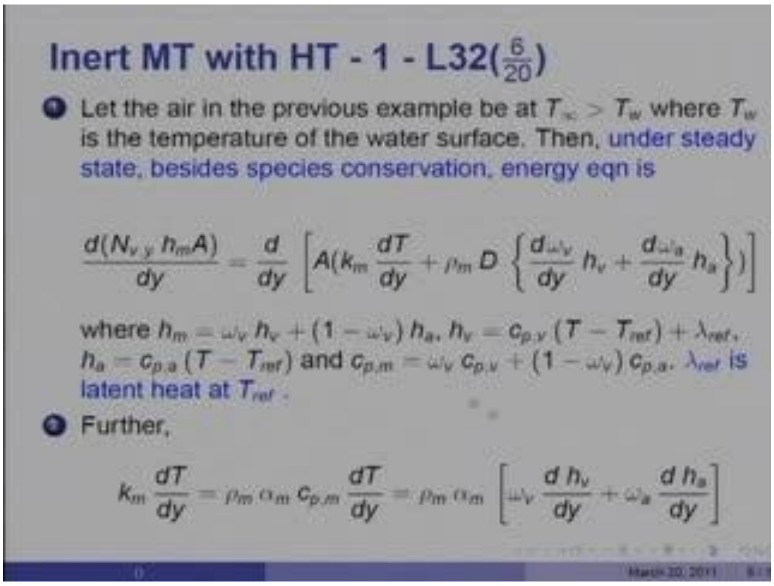

## Inert MT with HT - 1 - L32($\frac{6}{20}$)

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$$\frac{d(N_{v,y}\,h_m A)}{dy} = \frac{d}{dy}\left[A(k_m\frac{dT}{dy} + \rho_m D\left\{\frac{d\omega_v}{dy}h_v + \frac{d\omega_a}{dy}h_a\right\})\right]$$

where $h_m = \omega_v\,h_v + (1-\omega_v)\,h_a$, $h_v = c_{p,v}\,(T - T_{ref}) + \lambda_{ref}$, $h_a = c_{p,a}\,(T - T_{ref})$ and $c_{p,m} = \omega_v\,c_{p,v} + (1-\omega_v)\,c_{p,a}$. $\lambda_{ref}$ is latent heat at $T_{ref}$ .

- Further,

$$k_m\frac{dT}{dy} = \rho_m\,\alpha_m\,c_{p,m}\frac{dT}{dy} = \rho_m\,\alpha_m\left[\omega_v\frac{d\,h_v}{dy} + \omega_a\frac{d\,h_a}{dy}\right]$$

So d N v y h m A divided by dy would be d by dy of A k m dT dy which is the conduction heat transfer plus this is the Fick’s law of diffusion, multiplied by h k which is in this case would be omega v by dy h of vapour plus d omega a by dy h of air and the mixture enthalpy in this case would be the mass fraction of water vapour into enthalpy water vapor plus 1 minus omega v which is the mass fraction of air into enthalpy of air. h v would be given by specific heat of vapor into T minus T ref plus lambda ref which is the latent heat at temperature T ref and h a would have only the sensible part C p a into T minus T ref. The mixture specific heat could be simply omega v into C p v plus omega air into C p a; so that is what this formula end.

Now, let us look at this term k m dT dy; now k m can be written as rho m alpha m C p m dT by dy and that is equal to rho m alpha m. If I absorb C p m dT dy and put C p m equal to all this, you will see that you get omega v into d h v by dy plus omega a into d h a by dy.

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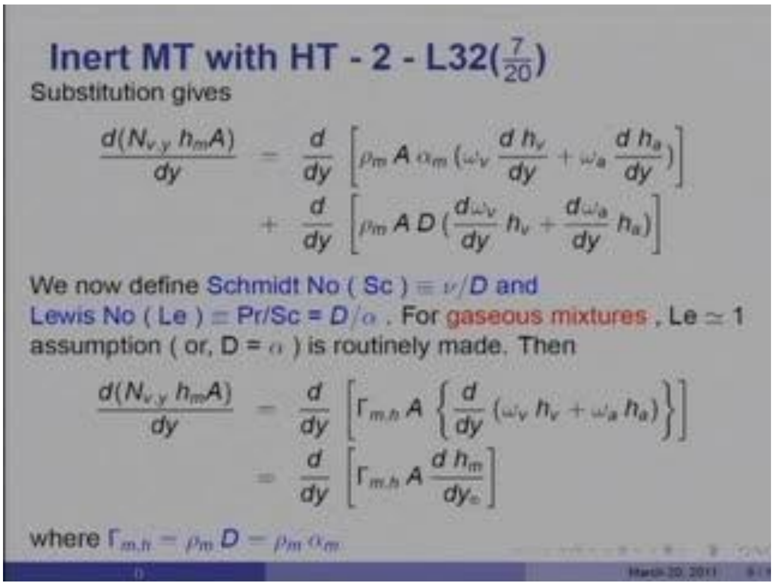

## Inert MT with HT - 2 - L32($\frac{7}{20}$)

Substitution gives

$$\frac{d(N_{v,y}\, h_m A)}{dy} = \frac{d}{dy}\left[\rho_m A\, \alpha_m \left(\omega_v \frac{d\,h_v}{dy} + \omega_a \frac{d\,h_a}{dy}\right)\right] + \frac{d}{dy}\left[\rho_m A\, D \left(\frac{d\omega_v}{dy} h_v + \frac{d\omega_a}{dy} h_a\right)\right]$$

We now define Schmidt No ( Sc ) $\equiv \nu/D$ and Lewis No ( Le ) $\equiv$ Pr/Sc = $D/\alpha$ . For gaseous mixtures , Le $\simeq$ 1 assumption ( or, D = $\alpha$ ) is routinely made. Then

$$\frac{d(N_{v,y}\, h_m A)}{dy} = \frac{d}{dy}\left[\Gamma_{m,h} A \left\{\frac{d}{dy}(\omega_v h_v + \omega_a h_a)\right\}\right] = \frac{d}{dy}\left[\Gamma_{m,h} A \frac{d\,h_m}{dy}\right]$$

where $\Gamma_{m,h} = \rho_m D = \rho_m \alpha_m$

This result… this is an important one which we are now going to substitute here. So, you will get d N v y h m A by dy equal to d by dy of rho m A alpha m omega v d h v by dy plus omega d h a by dy plus the second term which is the Fick’s law of diffusion term which carries with it enthalpy of the species.

Now, in mass transfer problem it is common to define Schmidt number as nu by diffusivity and we define Lewis number as Prandtl divided by Schmidt number which is equal to diffusivity of mass divided by diffusivity of heat.

Now, for gaseous mixtures, Lewis number is very close to 1 for example, as you know Prandtl number is about 0.7, Schmidt number would be about 0.672.68 in this kind of system. So, in effect Lewis number Le can be taken as very nearly 1 which implies that D is equal to alpha. If I make that assumption, then you will see that and replace these rho m into D into rho m into alpha as gamma. As I have defined here (Refer Slide Time:15:00), gamma m h equal to rho m D equal to rho m alpha m. Then you will see this becomes gamma m h into A plus d by dy of simply product of omega v h v plus omega h a, which is nothing but the mixture enthalpy d h m by dy so we essentially get d by dy N v by h m A equal to d by dy of this.

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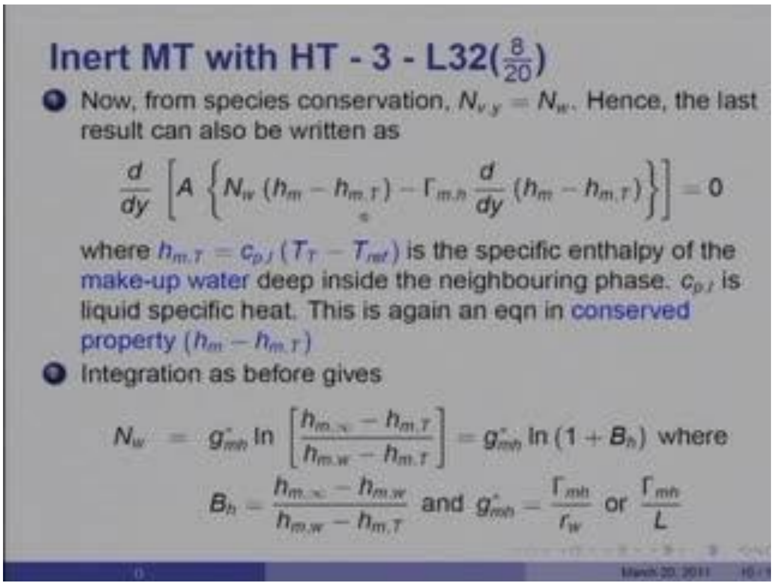

## Inert MT with HT - 3 - L32($\frac{8}{20}$)

- Now, from species conservation, $N_{v,y} = N_w$. Hence, the last result can also be written as

$$\frac{d}{dy}\left[A\left\{N_w\,(h_m - h_{m,T}) - \Gamma_{m,h}\frac{d}{dy}(h_m - h_{m,T})\right\}\right] = 0$$

where $h_{m,T} = c_{p,l}\,(T_T - T_{ref})$ is the specific enthalpy of the make-up water deep inside the neighbouring phase. $c_{p,l}$ is liquid specific heat. This is again an eqn in conserved property $(h_m - h_{m,T})$

- Integration as before gives

$$N_w = g^*_{mh}\ln\left[\frac{h_{m,\infty} - h_{m,T}}{h_{m,w} - h_{m,T}}\right] = g^*_{mh}\ln(1 + B_h)\ \text{where}$$

$$B_h = \frac{h_{m,\infty} - h_{m,w}}{h_{m,w} - h_{m,T}}\ \text{and}\ g^*_{mh} = \frac{\Gamma_{mh}}{r_w}\ \text{or}\ \frac{\Gamma_{mh}}{L}$$

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Now from species conservation we have learnt that N v y remains constant which is equal to N w. This the mass transfer as the surface itself remains constant throughout in a constant area problem. Then hence, the last result can be written as d by dy of N N w h m minus h m T minus gamma m h into d by dy h m minus h m T where h m T is simply the enthalpy of the transferred substance and it is a constant.

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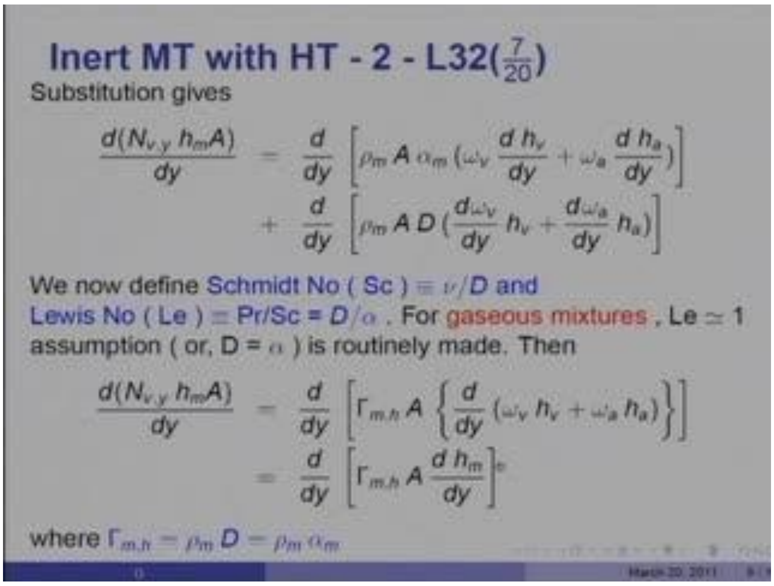

## Inert MT with HT - 2 - L32($\frac{7}{20}$)

Substitution gives

$$\frac{d(N_{v,y}\,h_m A)}{dy} = \frac{d}{dy}\left[\rho_m A\, \alpha_m \left(\omega_v \frac{d\,h_v}{dy} + \omega_a \frac{d\,h_a}{dy}\right)\right] + \frac{d}{dy}\left[\rho_m A\, D \left(\frac{d\omega_v}{dy} h_v + \frac{d\omega_a}{dy} h_a\right)\right]$$

We now define Schmidt No ( Sc ) $\equiv \nu/D$ and Lewis No ( Le ) $\equiv$ Pr/Sc = $D/\alpha$ . For gaseous mixtures , Le $\simeq$ 1 assumption ( or, D = $\alpha$ ) is routinely made. Then

$$\frac{d(N_{v,y}\,h_m A)}{dy} = \frac{d}{dy}\left[\Gamma_{m,h} A \left\{\frac{d}{dy}(\omega_v h_v + \omega_a h_a)\right\}\right] = \frac{d}{dy}\left[\Gamma_{m,h} A \frac{d\,h_m}{dy}\right]$$

where $\Gamma_{m,h} = \rho_m D = \rho_m \alpha_m$

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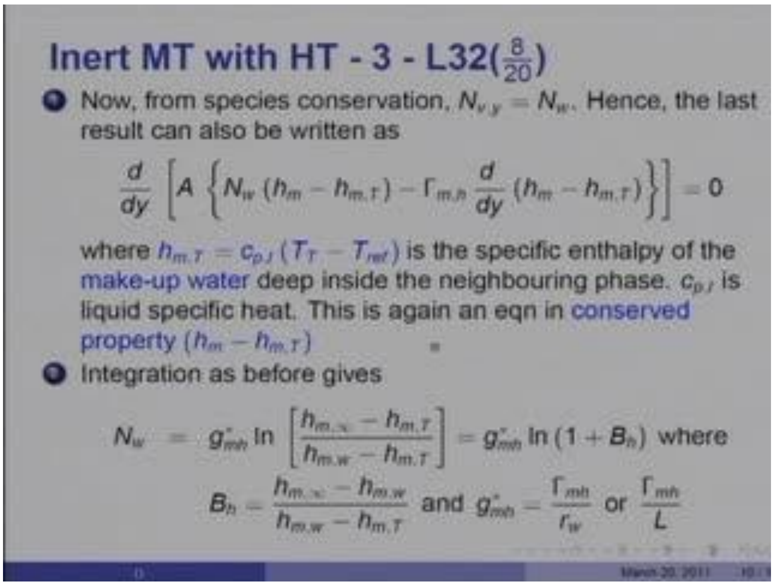

## Inert MT with HT - 3 - L32($\frac{8}{20}$)

- Now, from species conservation, $N_{v,y} = N_w$. Hence, the last result can also be written as

$$\frac{d}{dy}\left[A\left\{N_w (h_m - h_{m,T}) - \Gamma_{m,h} \frac{d}{dy}(h_m - h_{m,T})\right\}\right] = 0$$

  where $h_{m,T} = c_{p,l}(T_T - T_{ref})$ is the specific enthalpy of the make-up water deep inside the neighbouring phase. $c_{p,l}$ is liquid specific heat. This is again an eqn in conserved property $(h_m - h_{m,T})$
- Integration as before gives

$$N_w = g^*_{mh} \ln\left[\frac{h_{m,\infty} - h_{m,T}}{h_{m,w} - h_{m,T}}\right] = g^*_{mh} \ln(1 + B_h) \text{ where}$$

$$B_h = \frac{h_{m,\infty} - h_{m,w}}{h_{m,w} - h_{m,T}} \text{ and } g^*_{mh} = \frac{\Gamma_{mh}}{r_w} \text{ or } \frac{\Gamma_{mh}}{L}$$

So, all I have done is really added or subtracted h m minus h T and here h m minus h T and therefore, I have made really no change. So, h m T would be the C p of the liquid into temperature of the transfer substance which is water in this particular example minus T ref is the specific enthalpy of the makeup water deep inside the neighboring phase. C p l is the specific heat of the liquid. Now, this is again a conserved property equation in h m minus h m T. So, it is exactly same as the equation we had for omega v and therefore its solution 2 would be identical m w equal to g star m h ln h m infinity minus h m T over h m w minus h m T which I would write as g star m h into ln 1 plus B h. In this case, where B h is now formed from h m minus infinity minus h m w over h m w minus h m T and g star m h would be gamma m h over r w in case of a spherical system and this would be the case in case of a linear system (Refer Slide Time: 16:50).

So, we find that the logarithmic form is again retrieved from the solution of the energy equation because just as we had recovered the logarithmic form in case of mass transfer without heat transfer

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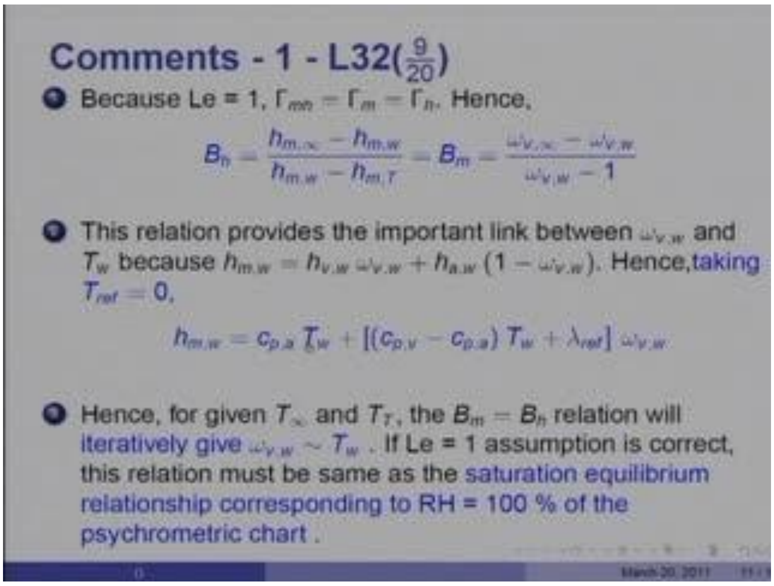

## Comments - 1 - L32($\frac{9}{20}$)

- Because Le = 1, $\Gamma_{mh} = \Gamma_m = \Gamma_h$. Hence,

$$B_h = \frac{h_{m,\infty} - h_{m,w}}{h_{m,w} - h_{m,T}} = B_m = \frac{\omega_{v,\infty} - \omega_{v,w}}{\omega_{v,w} - 1}$$

- This relation provides the important link between $\omega_{v,w}$ and $T_w$ because $h_{m,w} = h_{v,w}\,\omega_{v,w} + h_{a,w}\,(1 - \omega_{v,w})$. Hence, taking $T_{ref} = 0$,

$$h_{m,w} = c_{p,a}\,T_w + [(c_{p,v} - c_{p,a})\,T_w + \lambda_{ref}]\,\omega_{v,w}$$

- Hence, for given $T_\infty$ and $T_T$, the $B_m = B_h$ relation will iteratively give $\omega_{v,w} \sim T_w$ . If Le = 1 assumption is correct, this relation must be same as the saturation equilibrium relationship corresponding to RH = 100 % of the psychrometric chart .

Now, since Lewis number is equal to 1, gamma m h would be equal to gamma m and gamma h because thermal diffusivity is equal to mass diffusivity and hence you will see B h which is h m infinity minus h m w or h m w minus h m T would also be equal to omega v infinity minus omega v w omega v w minus 1. Now, this relation provides the important link with between the mass fraction of vapor at the wall and the temperature of the wall. Remember, when the temperature of the surroundings is at T infinity and the liquid which is T; T is at some other temperature we still do not know what the surface temperature T w will be and we need to determine that in such problems.

So, how do we determine that? We use this relationship that is, B m is equal to B h and h m w would be then h v w omega v w h a w into 1 minus omega v w and if I take T ref equal to 0 then h m w will be simply C p a into T w into C p v minus C p a T w plus lambda at 0 degree lambda at T ref equal to 0 into omega v w.

Hence, for a given T infinity and T T the B m equal to B h relationship will iteratively give omega v n omega and T w. So, what one does is, one simply assumes a value of T w h m infinity of course, can be obtained because you know already omega v infinity. You know T infinity and therefore that can be obtained; these two can be obtained as we saw on the last slide. This can be obtained knowing T w. Only thing is, you do not know omega v w. So, how do we get that omega v w? We can get that by saying that at the surface of the water saturation conditions would prevail and corresponding to RH equal to 100 percent. Therefore, that value of omega v w can be noted either from the psychometric chart or you can also use steam tables in which case, you will get a partial pressure so that from which you can recover the mass fraction of the wall.

If, after assuming T w you have determined omega v w in this way, you substitute in the B m expression; if you find that B h is equal to B m then obviously, your choice of T w was correct and therefore you already have T w omega v w relationship. If however, the 2 equations do not balance then, you must change the value of T w till you get balance between B m and B h. So, iterations are involved in discovering temperature T w.

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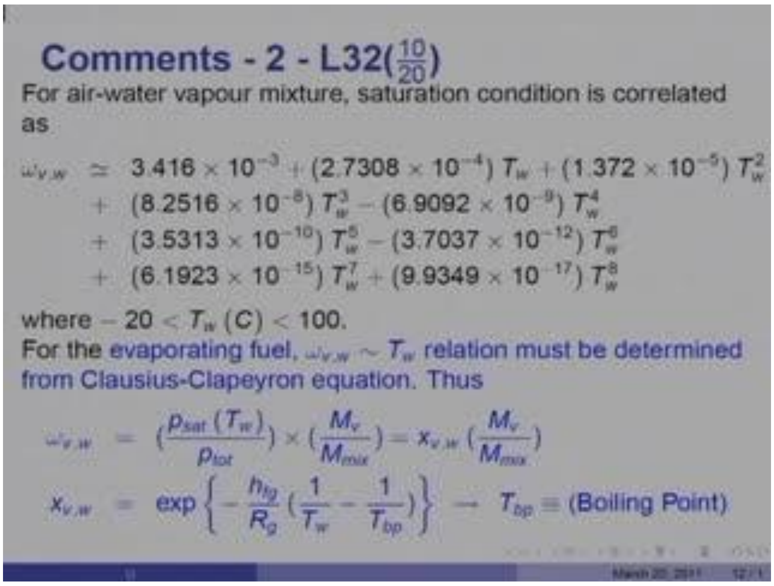

## Comments - 2 - L32($\frac{10}{20}$)

For air-water vapour mixture, saturation condition is correlated as

$$\begin{aligned}\omega_{v,w} &\simeq 3.416 \times 10^{-3} + (2.7308 \times 10^{-4})\, T_w + (1.372 \times 10^{-5})\, T_w^2 \\ &+ (8.2516 \times 10^{-8})\, T_w^3 - (6.9092 \times 10^{-9})\, T_w^4 \\ &+ (3.5313 \times 10^{-10})\, T_w^5 - (3.7037 \times 10^{-12})\, T_w^6 \\ &+ (6.1923 \times 10^{-15})\, T_w^7 + (9.9349 \times 10^{-17})\, T_w^8\end{aligned}$$

where $-20 < T_w\,(C) < 100$.

For the evaporating fuel, $\omega_{v,w} \sim T_w$ relation must be determined from Clausius-Clapeyron equation. Thus

$$\omega_{v,w} = \left(\frac{p_{sat}(T_w)}{p_{tot}}\right) \times \left(\frac{M_v}{M_{mix}}\right) = x_{v,w}\left(\frac{M_v}{M_{mix}}\right)$$

$$x_{v,w} = \exp\left\{-\frac{h_{fg}}{R_g}\left(\frac{1}{T_w} - \frac{1}{T_{bp}}\right)\right\} \rightarrow T_{bp} \equiv \text{(Boiling Point)}$$

Now, in order to help, if you are doing something on the computer then for air water vapor mixture saturation condition, that is the saturation curve on the psychometric chart is given like this; where this is the T dribble and this is the specific in humidity and you have the 100 percent RH line (Refer Slide Time: 20:15). The values corresponding to this have been correlated here in omega v w as a function of T w for the range of T w minus 20 to 100. For computer applications, in order to help iterations you one can use this relationship or simply try by hand; this is just for your information.

So, now of course, when we consider a fuel we do not know the omega v w T w relationship by way of a psychometric chart or anything like that and therefore, such a relationship must be determined from what is called that the Clausius-Clapeyron equation. In that case, you will see that omega v w is related to X v w into molecular weight of vapor divided by molecular weight of the mixture and X v w of course, is equal to P saturation at T w divided by P tot. X v w would be given by exponential of minus h fg divided by R g into 1 over T w minus T bp for all liquid fuels. Typically, you will have a boiling point known and h fg known and therefore X v w can be recovered for a given T w.

So, one try is in such cases assume at T w recover or calculate X v w; from that, you calculate omega v w and check whether B m is equal to B h as on the previous slide or otherwise go on changing T w till you get convergence.

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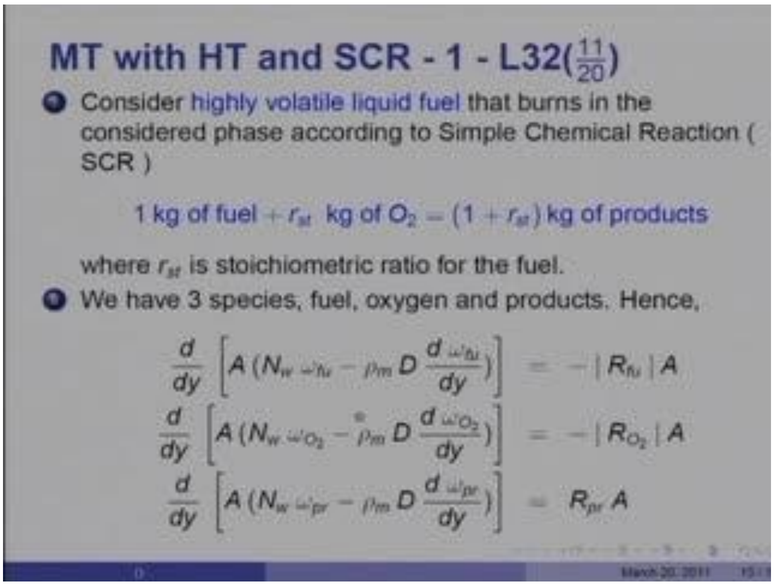


## MT with HT and SCR - 1 - L32($\frac{11}{20}$)

- Consider highly volatile liquid fuel that burns in the considered phase according to Simple Chemical Reaction ( SCR )

  1 kg of fuel + $r_{st}$ kg of $O_2$ = $(1 + r_{st})$ kg of products

  where $r_{st}$ is stoichiometric ratio for the fuel.
- We have 3 species, fuel, oxygen and products. Hence,

$$\frac{d}{dy}\left[A\,(N_w\,\omega_{fu} - \rho_m D\,\frac{d\,\omega_{fu}}{dy})\right] = -\,|\,R_{fu}\,|\,A$$

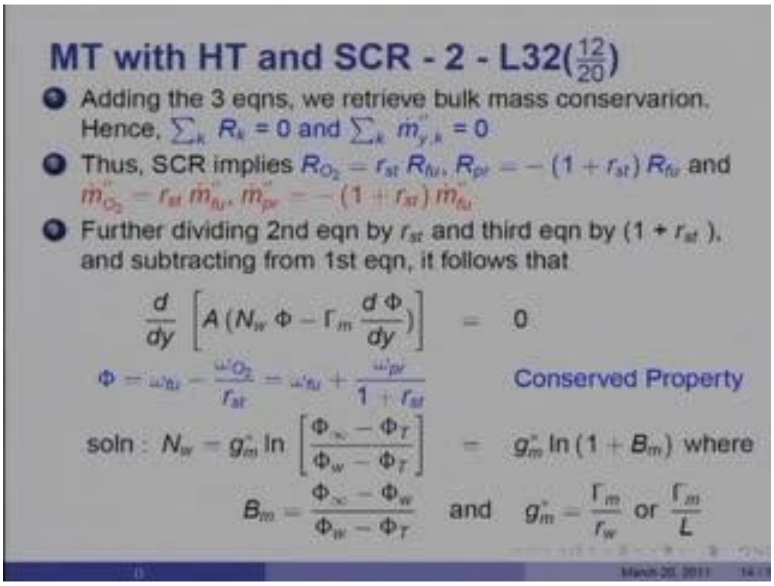
$$\frac{d}{dy}\left[A\,(N_w\,\omega_{O_2} - \rho_m D\,\frac{d\,\omega_{O_2}}{dy})\right] = -\,|\,R_{O_2}\,|\,A$$

$$\frac{d}{dy}\left[A\,(N_w\,\omega_{pr} - \rho_m D\,\frac{d\,\omega_{pr}}{dy})\right] = R_{pr}\,A$$

Now, I turn to the problem of mass transfer with heat transfer and simple chemical reaction. Now, what is simple chemical reaction? Let us consider highly volatile liquid fuel that burns in the considered phase according to the Simple Chemical Reaction SCR. Liquids usually burn by first evaporating in the gas phase without any change in composition. That is, the chemical formula of the fuel does not change when it comes out in the form of a vapor; but then, it burns in the vapor phase, in the gaseous phase as the homogeneous combustion and we say that simple chemical from reaction simply implies that 1 kilogram of fuel combines with r st kilograms of oxygen to give you 1 plus r st kilograms of products. Now, where r st is the stoichiometric ratio of the fuel and you must have run from your stoichiometry, how to evaluate say for example, hydrocarbon fuel? How to evaluate the value of r st?

In this problem therefore, we have 3 species. The fuel specie, the oxygen specie and the product which is itself a mixture but it is a product specie which we take it as a single specie. Then we will have 3 equations and because the chemical reaction is present, you will have convective flux and a diffusion flux d by dy of wood, equal rate of depletion of fuel. We therefore, I have said magnitude of R fu with a negative sine likewise, oxygen 2 would deplete minus R o. Whereas, the product would increase and therefore it has a plus sign in front of here so we have 3 equations because we have 3 species - 3 mass transfer equations.

(Refer Slide Time: 24:14)



## MT with HT and SCR - 2 - L32($\frac{12}{20}$)

- Adding the 3 eqns, we retrieve bulk mass conservarion. Hence, $\sum_k R_k = 0$ and $\sum_k \dot{m}''_{y,k} = 0$
- Thus, SCR implies $R_{O_2} = r_{st} R_{fu}$, $R_{pr} = -(1 + r_{st}) R_{fu}$ and $\dot{m}''_{O_2} = r_{st} \dot{m}''_{fu}$, $\dot{m}''_{pr} = -(1 + r_{st}) \dot{m}''_{fu}$
- Further dividing 2nd eqn by $r_{st}$ and third eqn by $(1 + r_{st})$, and subtracting from 1st eqn, it follows that

$$\frac{d}{dy}\left[A\left(N_w \Phi - \Gamma_m \frac{d\Phi}{dy}\right)\right] = 0$$

$$\Phi = \omega_{fu} - \frac{\omega_{O_2}}{r_{st}} = \omega_{fu} + \frac{\omega_{pr}}{1 + r_{st}} \qquad \text{Conserved Property}$$

$$\text{soln}: \; N_w = g_m^* \ln\left[\frac{\Phi_\infty - \Phi_T}{\Phi_w - \Phi_T}\right] = g_m^* \ln(1 + B_m) \text{ where}$$

$$B_m = \frac{\Phi_\infty - \Phi_w}{\Phi_w - \Phi_T} \quad \text{and} \quad g_m^* = \frac{\Gamma_m}{r_w} \text{ or } \frac{\Gamma_m}{L}$$

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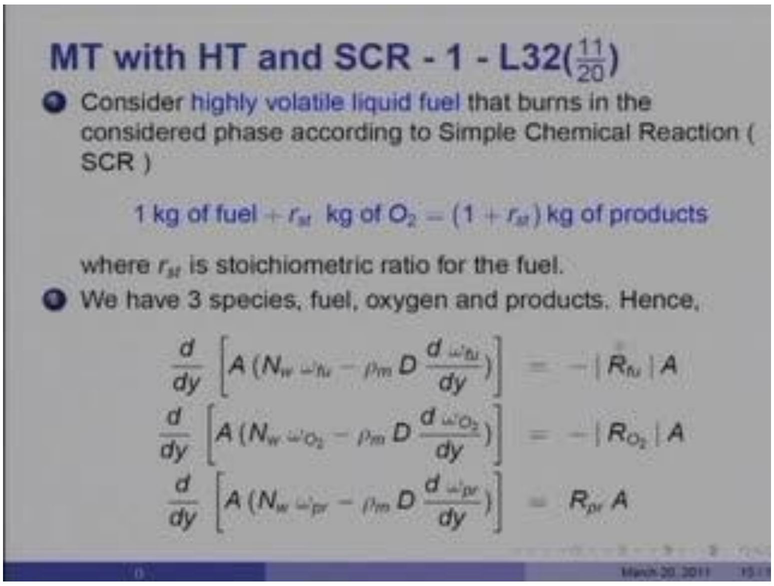

## MT with HT and SCR - 1 - L32($\frac{11}{20}$)

- Consider highly volatile liquid fuel that burns in the considered phase according to Simple Chemical Reaction ( SCR )

  1 kg of fuel + $r_{st}$ kg of $O_2$ = $(1 + r_{st})$ kg of products

  where $r_{st}$ is stoichiometric ratio for the fuel.
- We have 3 species, fuel, oxygen and products. Hence,

$$\frac{d}{dy}\left[A\,(N_w\,\omega_{fu} - \rho_m D\,\frac{d\,\omega_{fu}}{dy})\right] = -\,|\,R_{fu}\,|\,A$$

$$\frac{d}{dy}\left[A\,(N_w\,\omega_{O_2} - \rho_m D\,\frac{d\,\omega_{O_2}}{dy})\right] = -\,|\,R_{O_2}\,|\,A$$

$$\frac{d}{dy}\left[A\,(N_w\,\omega_{pr} - \rho_m D\,\frac{d\,\omega_{pr}}{dy})\right] = R_{pr}\,A$$

If we add the 3, we must retrieve the bulk mass conservation equation because the addition of the fuels, fuel oxygen and product; some of these will be equal to 1 and when all these are sum that will be equal to 1, so D 1 by dy would be 0.

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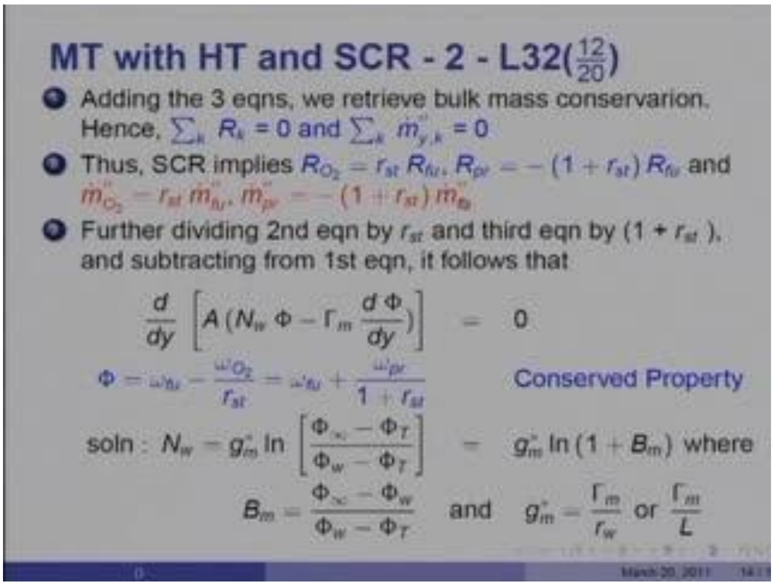

## MT with HT and SCR - 2 - L32($\frac{12}{20}$)

- Adding the 3 eqns, we retrieve bulk mass conservarion. Hence, $\sum_k R_k = 0$ and $\sum_k \dot{m}''_{y,k} = 0$
- Thus, SCR implies $R_{O_2} = r_{st}\,R_{fu}$, $R_{pr} = -\,(1 + r_{st})\,R_{fu}$ and $\dot{m}''_{O_2} = r_{st}\,\dot{m}''_{fu}$, $\dot{m}''_{pr} = -\,(1 + r_{st})\,\dot{m}''_{fu}$
- Further dividing 2nd eqn by $r_{st}$ and third eqn by $(1 + r_{st})$, and subtracting from 1st eqn, it follows that

$$\frac{d}{dy}\left[A\,(N_w\,\Phi - \Gamma_m\,\frac{d\,\Phi}{dy})\right] = 0$$

$$\Phi = \omega_{fu} - \frac{\omega_{O_2}}{r_{st}} = \omega_{fu} + \frac{\omega_{pr}}{1 + r_{st}} \qquad \text{Conserved Property}$$

$$\text{soln}: \; N_w = g_m^*\,\ln\left[\frac{\Phi_\infty - \Phi_T}{\Phi_w - \Phi_T}\right] = g_m^*\,\ln\,(1 + B_m) \text{ where}$$

$$B_m = \frac{\Phi_\infty - \Phi_w}{\Phi_w - \Phi_T} \quad \text{and} \quad g_m^* = \frac{\Gamma_m}{r_w} \text{ or } \frac{\Gamma_m}{L}$$

So, some of the diffusions, fluxes would add to 0 and therefore you will be having A N w d by dy of N w into sum of all these quantities and therefore, the sum of the R k must also be equal to 0 because the bulk mass conservation.

So, therefore SCR simply implies that R o 2 the oxygen depletion rate is equal to r st times fuel depletion rate. On the other hand, the product generation rate would be minus 1 plus r st times fuel depletion rate. Likewise, the diffusion of oxygen would be r st times m dot f double preview and product will be minus 1 plus r st m double dot fu.

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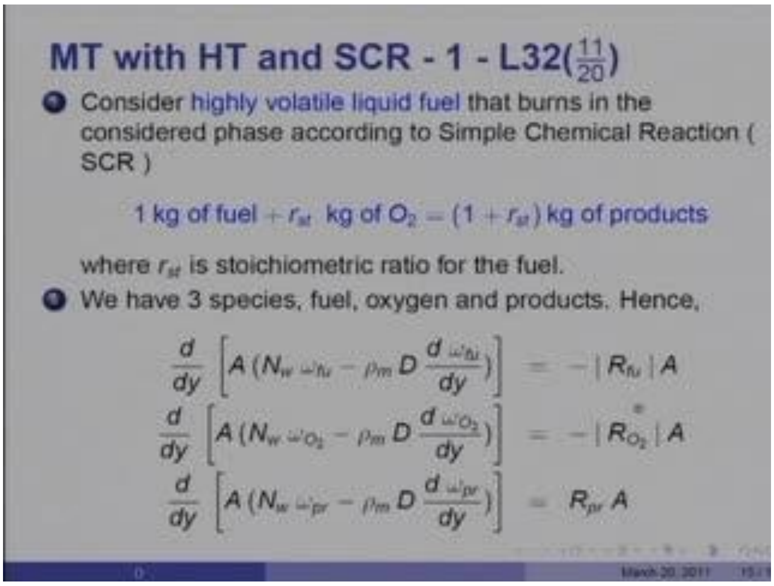

## MT with HT and SCR - 1 - L32($\frac{11}{20}$)

- Consider highly volatile liquid fuel that burns in the considered phase according to Simple Chemical Reaction ( SCR )

  1 kg of fuel $+ \; r_{st}$ kg of $O_2 = (1 + r_{st})$ kg of products

  where $r_{st}$ is stoichiometric ratio for the fuel.
- We have 3 species, fuel, oxygen and products. Hence,

$$\frac{d}{dy}\left[A\,(N_w\,\omega_{fu} - \rho_m D\,\frac{d\,\omega_{fu}}{dy})\right] = -\,|R_{fu}|\,A$$

$$\frac{d}{dy}\left[A\,(N_w\,\omega_{O_2} - \rho_m D\,\frac{d\,\omega_{O_2}}{dy})\right] = -\,|R_{O_2}|\,A$$

$$\frac{d}{dy}\left[A\,(N_w\,\omega_{pr} - \rho_m D\,\frac{d\,\omega_{pr}}{dy})\right] = R_{pr}\,A$$

If I now divide this equation by r st and then subtract the resulting equation, I divide by r st throughout and which is a constant and subtract that equation from this equation, then you will see that R fu minus R o 2 divide by r st would be 0.

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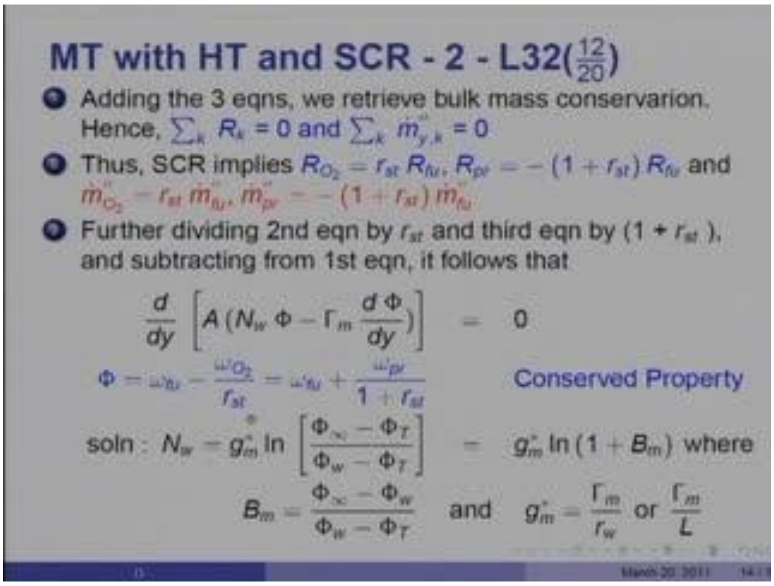

## MT with HT and SCR - 2 - L32($\frac{12}{20}$)

- Adding the 3 eqns, we retrieve bulk mass conservarion. Hence, $\sum_k R_k = 0$ and $\sum_k \dot{m}''_{y,k} = 0$
- Thus, SCR implies $R_{O_2} = r_{st}\,R_{fu}$, $R_{pr} = -\,(1 + r_{st})\,R_{fu}$ and $\dot{m}''_{O_2} = r_{st}\,\dot{m}''_{fu}$, $\dot{m}''_{pr} = -\,(1 + r_{st})\,\dot{m}''_{fu}$
- Further dividing 2nd eqn by $r_{st}$ and third eqn by $(1 + r_{st})$, and subtracting from 1st eqn, it follows that

$$\frac{d}{dy}\left[A\,(N_w\,\Phi - \Gamma_m\,\frac{d\,\Phi}{dy})\right] = 0$$

$$\Phi = \omega_{fu} - \frac{\omega_{O_2}}{r_{st}} = \omega_{fu} + \frac{\omega_{pr}}{1 + r_{st}} \qquad \text{Conserved Property}$$

$$\text{soln}: \; N_w = g_m^{*}\,\ln\left[\frac{\Phi_\infty - \Phi_T}{\Phi_w - \Phi_T}\right] = g_m^{*}\,\ln\,(1 + B_m) \;\text{where}$$

$$B_m = \frac{\Phi_\infty - \Phi_w}{\Phi_w - \Phi_T} \quad \text{and} \quad g_m^{*} = \frac{\Gamma_m}{r_w} \;\text{or}\; \frac{\Gamma_m}{L}$$

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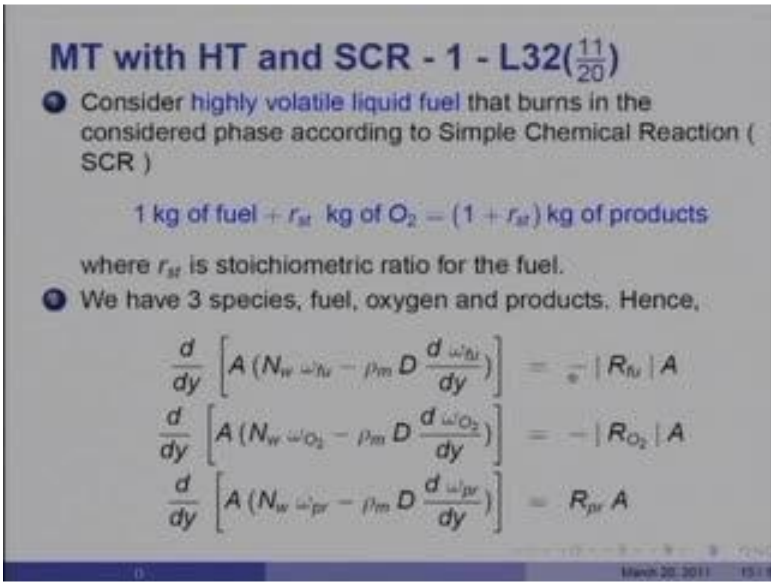

## MT with HT and SCR - 1 - L32($\frac{11}{20}$)

- Consider highly volatile liquid fuel that burns in the considered phase according to Simple Chemical Reaction ( SCR )

$$1 \text{ kg of fuel} + r_{st} \text{ kg of } O_2 = (1 + r_{st}) \text{ kg of products}$$

where $r_{st}$ is stoichiometric ratio for the fuel.

- We have 3 species, fuel, oxygen and products. Hence,

$$\frac{d}{dy}\left[A\left(N_w\,\omega_{fu} - \rho_m D\,\frac{d\,\omega_{fu}}{dy}\right)\right] = -\,|R_{fu}|\,A$$

$$\frac{d}{dy}\left[A\left(N_w\,\omega_{O_2} - \rho_m D\,\frac{d\,\omega_{O_2}}{dy}\right)\right] = -\,|R_{O_2}|\,A$$

$$\frac{d}{dy}\left[A\left(N_w\,\omega_{pr} - \rho_m D\,\frac{d\,\omega_{pr}}{dy}\right)\right] = R_{pr}\,A$$

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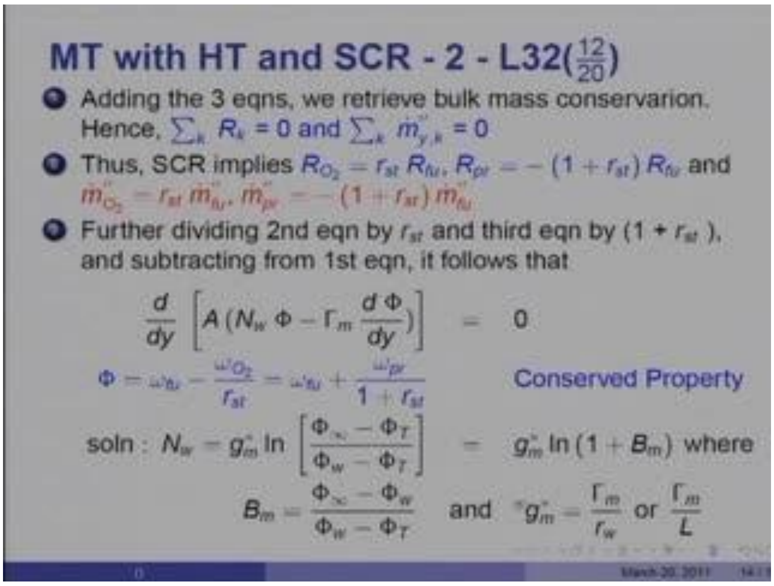

## MT with HT and SCR - 2 - L32($\frac{12}{20}$)

- Adding the 3 eqns, we retrieve bulk mass conservarion. Hence, $\sum_k R_k = 0$ and $\sum_k m''_{y,k} = 0$
- Thus, SCR implies $R_{O_2} = r_{st}\,R_{fu}$, $R_{pr} = -\,(1 + r_{st})\,R_{fu}$ and $m''_{O_2} = r_{st}\,m''_{fu}$, $m''_{pr} = -\,(1 + r_{st})\,m''_{fu}$
- Further dividing 2nd eqn by $r_{st}$ and third eqn by (1 + $r_{st}$ ), and subtracting from 1st eqn, it follows that

$$\frac{d}{dy}\left[A\left(N_w\,\Phi - \Gamma_m\,\frac{d\,\Phi}{dy}\right)\right] = 0$$

$$\Phi = \omega_{fu} - \frac{\omega_{O_2}}{r_{st}} = \omega_{fu} + \frac{\omega_{pr}}{1 + r_{st}} \qquad \text{Conserved Property}$$

$$\text{soln}: \; N_w = g^*_m \ln\left[\frac{\Phi_\infty - \Phi_T}{\Phi_w - \Phi_T}\right] = g^*_m \ln(1 + B_m) \text{ where}$$

$$B_m = \frac{\Phi_\infty - \Phi_w}{\Phi_w - \Phi_T} \quad \text{and} \quad g^*_m = \frac{\Gamma_m}{r_w} \text{ or } \frac{\Gamma_m}{L}$$

As a result, you will get an equation which is like this d by dy of A N w phi minus gamma m d phi by dy equal to 0 and phi will be omega fu minus omega o 2 by r st. Likewise, if I divide this third equation by 1 plus r st throughout n add then again the right hand side would be 0 and I will have a phi which you will stand for omega fu plus omega product over 1 plus r st. We said that any equation of this form with the 0 source term implies that phi is a conserved property.

So, we have again got an equation of conserved property like the inert mass transfer problem and therefore we would have N. Solution of that would be N w equal to g star m ln phi infinity minus phi T over phi w minus phi T equal to g star m ln 1 plus B m, where B m would be given. Now, by this quantity phi infinity minus phi w over phi w minus phi T and phi, can stand for this group or it can stand for this group, do not one use the appropriate group depending on the convenience of the problems at hand. g star m would be gamma m d or by r w in spherical system or gamma m divided by L in the plane system. The important thing to note is that, even in a problem involving combustion we are able to reduce the problem mathematically to a form just like that of evaporation of water in the absence of heat transfer.

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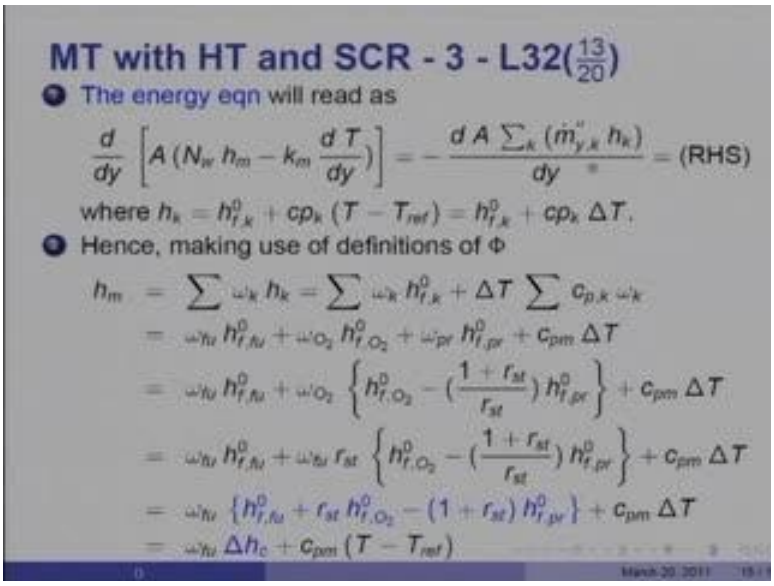

## MT with HT and SCR - 3 - L32($\frac{13}{20}$)

- The energy eqn will read as

$$\frac{d}{dy}\left[A\left(N_w\, h_m - k_m\,\frac{d\,T}{dy}\right)\right] = -\,\frac{d\,A\,\sum_k \left(\dot{m}''_{y,k}\, h_k\right)}{dy} = \text{(RHS)}$$

where $h_k = h^0_{f,k} + cp_k\,(T - T_{ref}) = h^0_{f,k} + cp_k\,\Delta T$.

- Hence, making use of definitions of $\Phi$

$$\begin{aligned} h_m &= \sum \omega_k\, h_k = \sum \omega_k\, h^0_{f,k} + \Delta T \sum c_{p,k}\,\omega_k \\ &= \omega_{fu}\, h^0_{f,fu} + \omega_{O_2}\, h^0_{f,O_2} + \omega_{pr}\, h^0_{f,pr} + c_{pm}\,\Delta T \\ &= \omega_{fu}\, h^0_{f,fu} + \omega_{O_2}\left\{h^0_{f,O_2} - \left(\frac{1+r_{st}}{r_{st}}\right) h^0_{f,pr}\right\} + c_{pm}\,\Delta T \\ &= \omega_{fu}\, h^0_{f,fu} + \omega_{fu}\, r_{st}\left\{h^0_{f,O_2} - \left(\frac{1+r_{st}}{r_{st}}\right) h^0_{f,pr}\right\} + c_{pm}\,\Delta T \\ &= \omega_{fu}\left\{h^0_{f,fu} + r_{st}\, h^0_{f,O_2} - (1 + r_{st})\, h^0_{f,pr}\right\} + c_{pm}\,\Delta T \\ &= \omega_{fu}\,\Delta h_c + c_{pm}\,(T - T_{ref}) \end{aligned}$$

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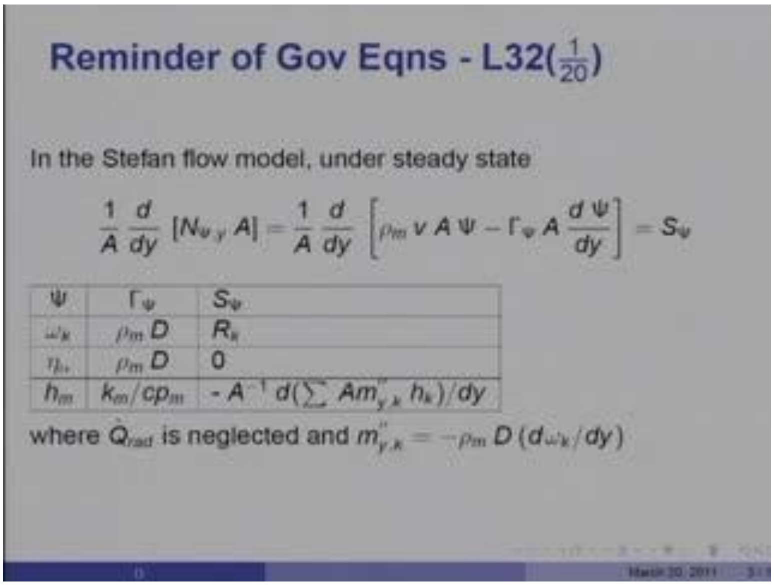

## Reminder of Gov Eqns - L32($\frac{1}{20}$)

In the Stefan flow model, under steady state

$$\frac{1}{A}\frac{d}{dy}\left[N_{\psi,y}\,A\right] = \frac{1}{A}\frac{d}{dy}\left[\rho_m\, v\, A\,\Psi - \Gamma_\psi\, A\,\frac{d\,\Psi}{dy}\right] = S_\psi$$

| $\Psi$ | $\Gamma_\psi$ | $S_\psi$ |
|---|---|---|
| $\omega_k$ | $\rho_m D$ | $R_k$ |
| $\eta_{k*}$ | $\rho_m D$ | 0 |
| $h_m$ | $k_m/cp_m$ | $-A^{-1}\, d(\sum A m''_{y,k}\, h_k)/dy$ |

where $\dot{Q}_{rad}$ is neglected and $m''_{y,k} = -\rho_m D\,(d\omega_k/dy)$

So let me now turn to energy part of the… because, whenever combustion takes place there would be heat transfer. Now, we would have d by dy of A N w h m minus k m d T dy would equal d of the diffusion mass transfer of dy. You remember from slide 1, this is the right hand side and this is the equation that I am writing now.

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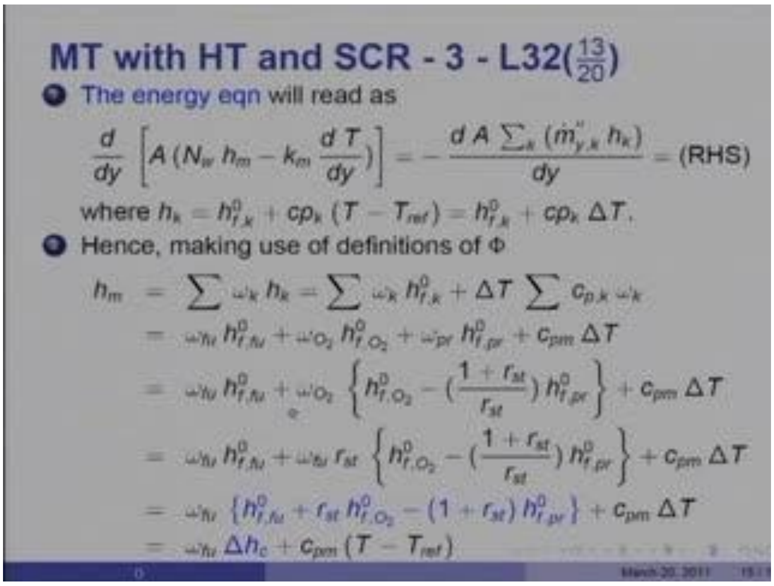

## MT with HT and SCR - 3 - L32($\frac{13}{20}$)

- The energy eqn will read as

$$\frac{d}{dy}\left[A\,(N_w\, h_m - k_m\,\frac{d\,T}{dy})\right] = -\frac{d\,A\sum_k (\dot{m}''_{y,k}\, h_k)}{dy} = (\text{RHS})$$

where $h_k = h^0_{f,k} + cp_k\,(T - T_{ref}) = h^0_{f,k} + cp_k\,\Delta T$.

- Hence, making use of definitions of $\Phi$

$$\begin{aligned}
h_m &= \sum \omega_k\, h_k = \sum \omega_k\, h^0_{f,k} + \Delta T \sum c_{p,k}\,\omega_k \\
&= \omega_{fu}\, h^0_{f,fu} + \omega_{O_2}\, h^0_{f,O_2} + \omega_{pr}\, h^0_{f,pr} + c_{pm}\,\Delta T \\
&= \omega_{fu}\, h^0_{f,fu} + \omega_{O_2}\left\{h^0_{f,O_2} - (\frac{1+r_{st}}{r_{st}})\, h^0_{f,pr}\right\} + c_{pm}\,\Delta T \\
&= \omega_{fu}\, h^0_{f,fu} + \omega_{fu}\, r_{st}\left\{h^0_{f,O_2} - (\frac{1+r_{st}}{r_{st}})\, h^0_{f,pr}\right\} + c_{pm}\,\Delta T \\
&= \omega_{fu}\,\{h^0_{f,fu} + r_{st}\, h^0_{f,O_2} - (1+r_{st})\, h^0_{f,pr}\} + c_{pm}\,\Delta T \\
&= \omega_{fu}\,\Delta h_c + c_{pm}\,(T - T_{ref})
\end{aligned}$$

This is the energy equation, then equal to the right hand side; this is the right hand side where h sub k is the species enthalpy and as you will know that whenever you have reacting fuel we write it as h naught f k as the enthalpy of formation plus C p k into T minus T ref or for short I will write as h naught f k C p k delta T. Hence, making use of definitions of phi, let us say h m would be simply omega k h k and therefore, it will be omega k h naught f k plus delta T into sigma C p k omega k. Then the first part of the term would simply result in omega fu h naught fu plus omega o 2 h naught f o 2 plus omega product into h naught f product and C p k omega k would be simply C p m the mixture specific heat into delta T.

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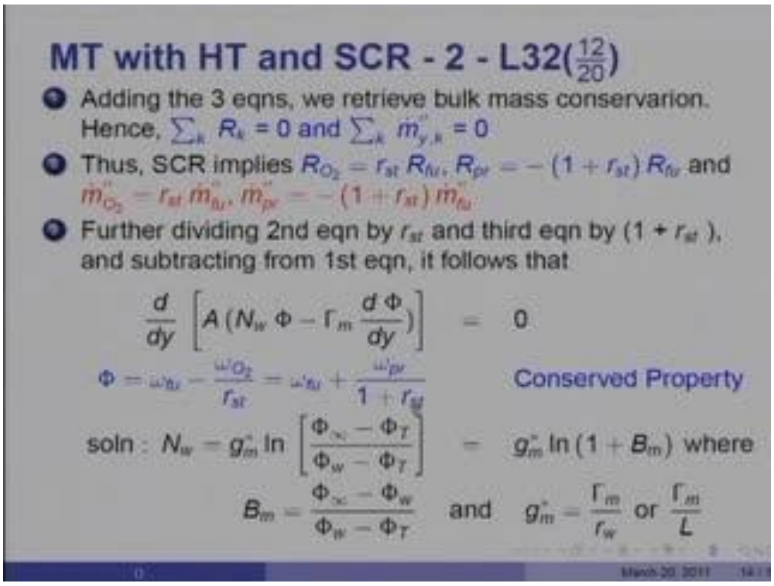

## MT with HT and SCR - 2 - L32($\frac{12}{20}$)

- Adding the 3 eqns, we retrieve bulk mass conservarion. Hence, $\sum_k R_k = 0$ and $\sum_k \dot{m}''_{y,k} = 0$
- Thus, SCR implies $R_{O_2} = r_{st}\, R_{fu}$, $R_{pr} = -(1 + r_{st})\, R_{fu}$ and $\dot{m}''_{O_2} = r_{st}\, \dot{m}''_{fu}$, $\dot{m}''_{pr} = -(1 + r_{st})\, \dot{m}''_{fu}$
- Further dividing 2nd eqn by $r_{st}$ and third eqn by $(1 + r_{st})$, and subtracting from 1st eqn, it follows that

$$\frac{d}{dy}\left[A\,(N_w\,\Phi - \Gamma_m\,\frac{d\,\Phi}{dy})\right] = 0$$

$$\Phi = \omega_{fu} - \frac{\omega_{O_2}}{r_{st}} = \omega_{fu} + \frac{\omega_{pr}}{1 + r_{st}} \qquad \text{Conserved Property}$$

$$\text{soln}:\ N_w = g^*_m \ln\left[\frac{\Phi_\infty - \Phi_T}{\Phi_w - \Phi_T}\right] = g^*_m \ln(1 + B_m)\ \text{where}$$

$$B_m = \frac{\Phi_\infty - \Phi_w}{\Phi_w - \Phi_T} \quad \text{and} \quad g^*_m = \frac{\Gamma_m}{r_w}\ \text{or}\ \frac{\Gamma_m}{L}$$

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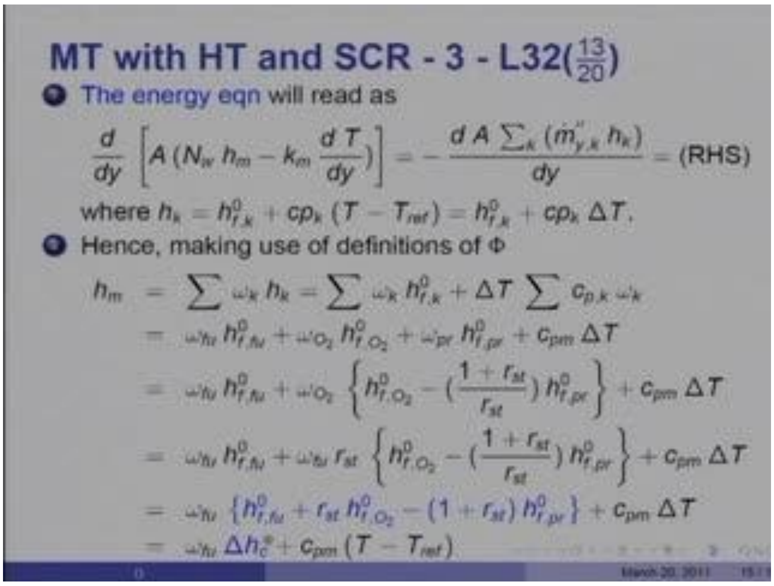

## MT with HT and SCR - 3 - L32($\frac{13}{20}$)

- The energy eqn will read as

$$\frac{d}{dy}\left[A\,(N_w\,h_m - k_m\,\frac{d\,T}{dy})\right] = -\frac{d\,A\sum_k(\dot{m}''_{y,k}\,h_k)}{dy} = (\text{RHS})$$

where $h_k = h^0_{f,k} + cp_k\,(T - T_{ref}) = h^0_{f,k} + cp_k\,\Delta T$.

- Hence, making use of definitions of $\Phi$

$$\begin{aligned} h_m &= \sum \omega_k\,h_k = \sum \omega_k\,h^0_{f,k} + \Delta T \sum c_{p,k}\,\omega_k \\ &= \omega_{fu}\,h^0_{f,fu} + \omega_{O_2}\,h^0_{f,O_2} + \omega_{pr}\,h^0_{f,pr} + c_{pm}\,\Delta T \\ &= \omega_{fu}\,h^0_{f,fu} + \omega_{O_2}\left\{h^0_{f,O_2} - (\frac{1 + r_{st}}{r_{st}})\,h^0_{f,pr}\right\} + c_{pm}\,\Delta T \\ &= \omega_{fu}\,h^0_{f,fu} + \omega_{fu}\,r_{st}\left\{h^0_{f,O_2} - (\frac{1 + r_{st}}{r_{st}})\,h^0_{f,pr}\right\} + c_{pm}\,\Delta T \\ &= \omega_{fu}\,\{h^0_{f,fu} + r_{st}\,h^0_{f,O_2} - (1 + r_{st})\,h^0_{f,pr}\} + c_{pm}\,\Delta T \\ &= \omega_{fu}\,\Delta h^*_c + c_{pm}\,(T - T_{ref}) \end{aligned}$$

Now, from the previous slide we have defined phi. So, I can say that omega o 2 by r st will be minus omega product over r st. So, that is what I am going to use here. I am replacing omega product by 1 minus r st over r st omega o 2 equal plus C p m delta T.

Omega o 2 itself would be equal to omega fu r st into all these and as a result, you will see I get omega fu into h naught fu plus r st h naught o 2 h naught f o 2 minus 1 plus r st h naught f product plus C p m delta T.

This essentially means that this is the enthalpy, total enthalpy of the reactants. 1 mole of fuel combines with r st moles of oxygen therefore, this is the enthalpy of the reactants and this is the enthalpy of the products. So, the reactant enthalpy minus product enthalpy as you know, is the heat of the combustion and therefore, you will get omega fu del h c plus C p m into T minus T ref, where delta T is written as T minus T ref; again, as I have said there.

So, we get a, enthalpy equal to heat of combustion into omega fuel now, I have here replaced omega o 2 and omega product in terms of omega fu but I could do it the other way. I can replace omega fu and omega o 2 in terms of omega product or I can replace omega fu and omega product in terms of omega 2 and I will get different expressions involving del h c which I will show you shortly.

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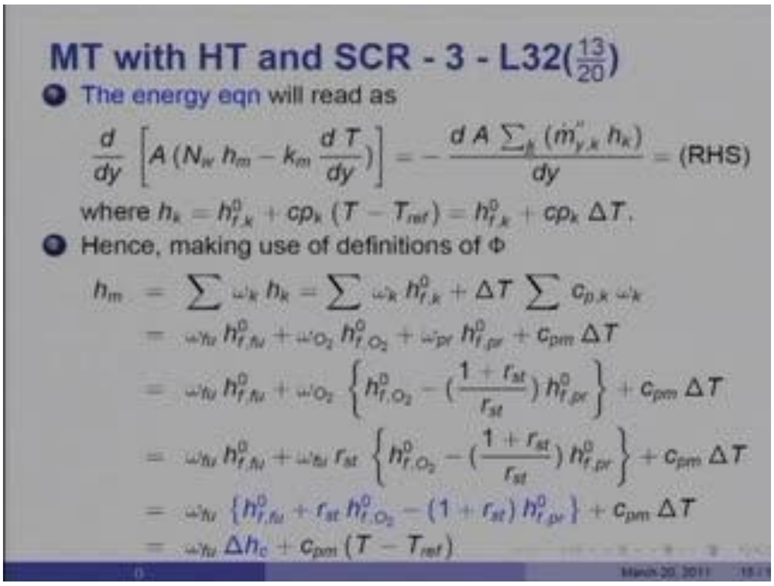

## MT with HT and SCR - 3 - L32($\frac{13}{20}$)

- The energy eqn will read as

$$\frac{d}{dy}\left[A\left(N_w\, h_m - k_m \frac{d\,T}{dy}\right)\right] = -\frac{d\,A \sum_k (\dot{m}''_{y,k}\, h_k)}{dy} = \text{(RHS)}$$

where $h_k = h^0_{f,k} + cp_k\,(T - T_{ref}) = h^0_{f,k} + cp_k\,\Delta T$.

- Hence, making use of definitions of $\Phi$

$$\begin{aligned} h_m &= \sum \omega_k\, h_k = \sum \omega_k\, h^0_{f,k} + \Delta T \sum c_{p,k}\, \omega_k \\ &= \omega_{fu}\, h^0_{f,fu} + \omega_{O_2}\, h^0_{f,O_2} + \omega_{pr}\, h^0_{f,pr} + c_{pm}\,\Delta T \\ &= \omega_{fu}\, h^0_{f,fu} + \omega_{O_2}\left\{h^0_{f,O_2} - \left(\frac{1+r_{st}}{r_{st}}\right) h^0_{f,pr}\right\} + c_{pm}\,\Delta T \\ &= \omega_{fu}\, h^0_{f,fu} + \omega_{fu}\, r_{st}\left\{h^0_{f,O_2} - \left(\frac{1+r_{st}}{r_{st}}\right) h^0_{f,pr}\right\} + c_{pm}\,\Delta T \\ &= \omega_{fu}\left\{h^0_{f,fu} + r_{st}\, h^0_{f,O_2} - (1+r_{st})\, h^0_{f,pr}\right\} + c_{pm}\,\Delta T \\ &= \omega_{fu}\,\Delta h_c + c_{pm}\,(T - T_{ref}) \end{aligned}$$

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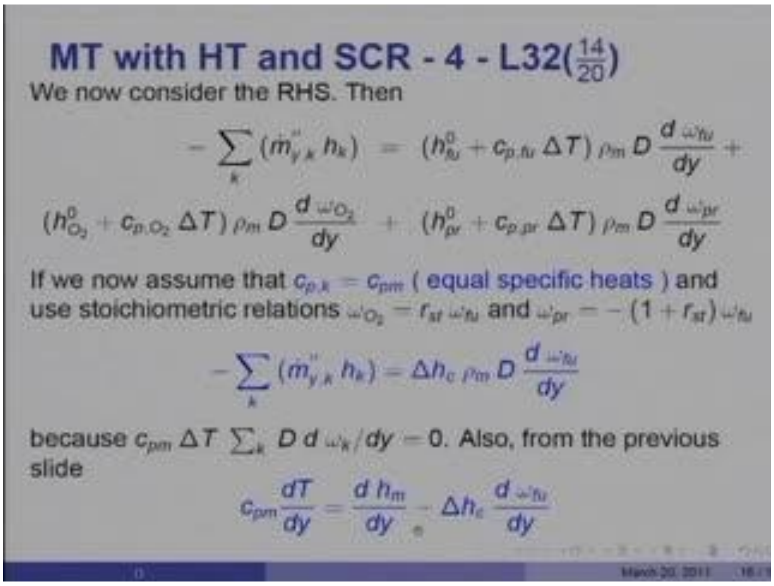

## MT with HT and SCR - 4 - L32($\frac{14}{20}$)

We now consider the RHS. Then

$$-\sum_k (\dot{m}''_{y,k} h_k) = (h^0_{fu} + c_{p,fu} \Delta T) \rho_m D \frac{d \omega_{fu}}{dy} + (h^0_{O_2} + c_{p,O_2} \Delta T) \rho_m D \frac{d \omega_{O_2}}{dy} + (h^0_{pr} + c_{p,pr} \Delta T) \rho_m D \frac{d \omega_{pr}}{dy}$$

If we now assume that $c_{p,k} = c_{pm}$ ( equal specific heats ) and use stoichiometric relations $\omega_{O_2} = r_{st}\, \omega_{fu}$ and $\omega_{pr} = -(1 + r_{st})\, \omega_{fu}$

$$-\sum_k (\dot{m}''_{y,k} h_k) = \Delta h_c\, \rho_m D \frac{d \omega_{fu}}{dy}$$

because $c_{pm} \Delta T \sum_k D\, d\, \omega_k / dy = 0$. Also, from the previous slide

$$c_{pm} \frac{dT}{dy} = \frac{d\, h_m}{dy} - \Delta h_c \frac{d \omega_{fu}}{dy}$$

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But, now let us consider the right hand side which is minus d by dy of A sigma k m y k h k. So, here summation of k m double prime y k h k would be simply h naught fu C p fu delta T into this, is the expression for m y fu plus the same quantity for oxygen and the same quantity for product.

Now, we make an important assumption which is always made for gaseous mixtures. Specific heats are functions of temperature and for different species they are somewhat different; but they are not so different that we cannot make an assumption of equal specific heats. If you assume that the each specie has the same specific heat then it would simply equal in the mixture specific heats.

So, that is the assumption I am going to make and use the stoichiometric relation omega o 2 equal to r st fu omega fu and omega product equal to minus 1 minus r st omega fu. If I do that then this entire relationship can be written as minus del h c rho m D omega fu by dy, because C p m delta T into sigma k D d omega k by dy would because sigma omega k would simply add up to 1 and that is equal to 0. Also, from the previous slide we have shown that C p m dT by dy is equal to d h m by dy minus, remember?

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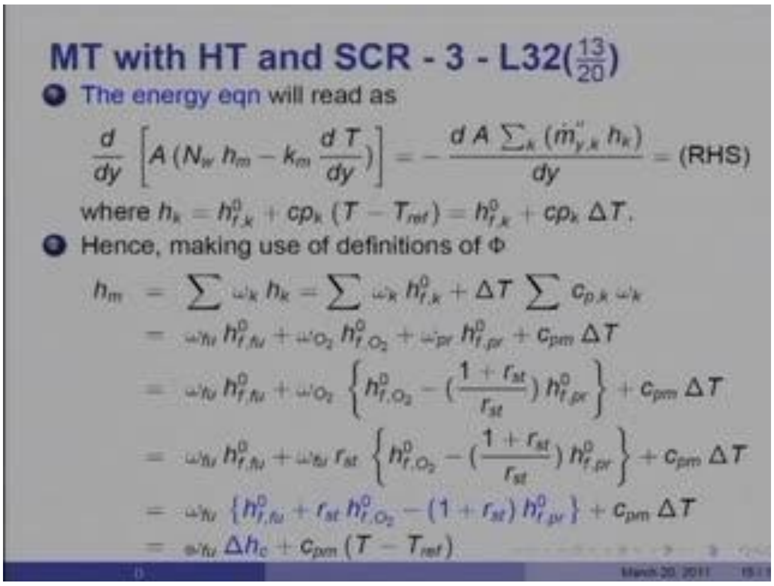

## MT with HT and SCR - 3 - L32($\frac{13}{20}$)

- The energy eqn will read as

$$\frac{d}{dy}\left[A\,(N_w\,h_m - k_m\,\frac{d\,T}{dy})\right] = -\,\frac{d\,A\,\sum_k(\dot{m}''_{y,k}\,h_k)}{dy} = \text{(RHS)}$$

where $h_k = h^0_{f,k} + cp_k\,(T - T_{ref}) = h^0_{f,k} + cp_k\,\Delta T$.

- Hence, making use of definitions of $\Phi$

$$\begin{aligned} h_m &= \sum \omega_k\,h_k = \sum \omega_k\,h^0_{f,k} + \Delta T \sum c_{p,k}\,\omega_k \\ &= \omega_{fu}\,h^0_{f,fu} + \omega_{O_2}\,h^0_{f,O_2} + \omega_{pr}\,h^0_{f,pr} + c_{pm}\,\Delta T \\ &= \omega_{fu}\,h^0_{f,fu} + \omega_{O_2}\left\{h^0_{f,O_2} - (\frac{1+r_{st}}{r_{st}})\,h^0_{f,pr}\right\} + c_{pm}\,\Delta T \\ &= \omega_{fu}\,h^0_{f,fu} + \omega_{fu}\,r_{st}\left\{h^0_{f,O_2} - (\frac{1+r_{st}}{r_{st}})\,h^0_{f,pr}\right\} + c_{pm}\,\Delta T \\ &= \omega_{fu}\,\{h^0_{f,fu} + r_{st}\,h^0_{f,O_2} - (1+r_{st})\,h^0_{f,pr}\} + c_{pm}\,\Delta T \\ &= \omega_{fu}\,\Delta h_c + c_{pm}\,(T - T_{ref}) \end{aligned}$$

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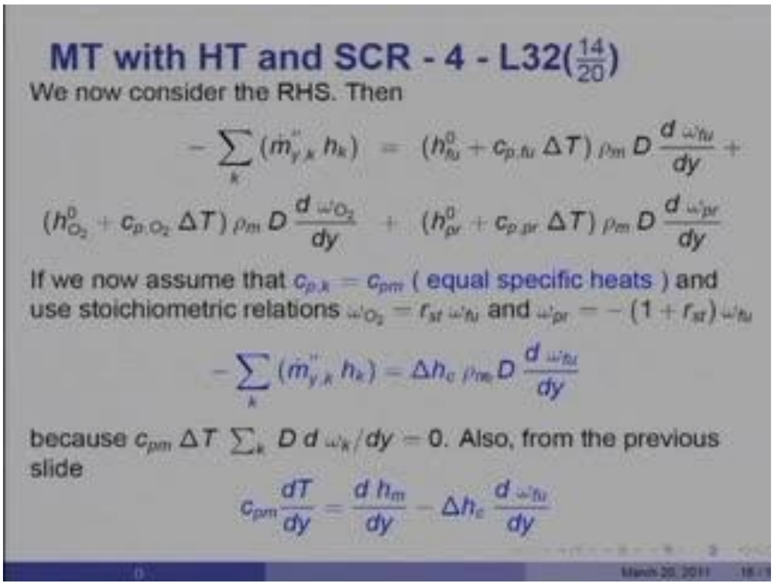

## MT with HT and SCR - 4 - L32($\frac{14}{20}$)

We now consider the RHS. Then

$$-\sum_k(\dot{m}''_{y,k}\,h_k) = (h^0_{fu} + c_{p,fu}\,\Delta T)\,\rho_m\,D\,\frac{d\,\omega_{fu}}{dy} + (h^0_{O_2} + c_{p,O_2}\,\Delta T)\,\rho_m\,D\,\frac{d\,\omega_{O_2}}{dy} + (h^0_{pr} + c_{p,pr}\,\Delta T)\,\rho_m\,D\,\frac{d\,\omega_{pr}}{dy}$$

If we now assume that $c_{p,k} = c_{pm}$ ( equal specific heats ) and use stoichiometric relations $\omega_{O_2} = r_{st}\,\omega_{fu}$ and $\omega_{pr} = -\,(1 + r_{st})\,\omega_{fu}$

$$-\sum_k(\dot{m}''_{y,k}\,h_k) = \Delta h_c\,\rho_m\,D\,\frac{d\,\omega_{fu}}{dy}$$

because $c_{pm}\,\Delta T\,\sum_k D\,d\,\omega_k/dy = 0$. Also, from the previous slide

$$c_{pm}\frac{dT}{dy} = \frac{d\,h_m}{dy} - \Delta h_c\,\frac{d\,\omega_{fu}}{dy}$$

So, if I take differential of this equation with respect to y then C p m into dT by dy would be d h m by dy into minus del h c into d omega fu by dy and that is what I have written here. Now, I have this expression and I have this expression which I am going to make use of in deriving the right hand side.

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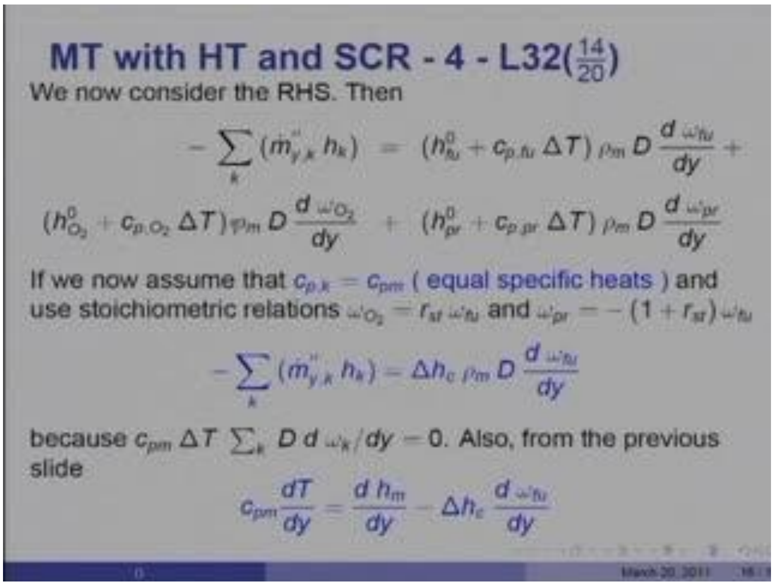

### MT with HT and SCR - 4 - L32($\frac{14}{20}$)

We now consider the RHS. Then

$$-\sum_k (\dot{m}''_{y,k}\, h_k) = (h^0_{fu} + c_{p,fu}\,\Delta T)\,\rho_m\, D\,\frac{d\,\omega_{fu}}{dy} + (h^0_{O_2} + c_{p,O_2}\,\Delta T)\,\rho_m\, D\,\frac{d\,\omega_{O_2}}{dy} + (h^0_{pr} + c_{p,pr}\,\Delta T)\,\rho_m\, D\,\frac{d\,\omega_{pr}}{dy}$$

If we now assume that $c_{p,k} = c_{pm}$ ( equal specific heats ) and use stoichiometric relations $\omega_{O_2} = r_{st}\,\omega_{fu}$ and $\omega_{pr} = -(1 + r_{st})\,\omega_{fu}$

$$-\sum_k (\dot{m}''_{y,k}\, h_k) = \Delta h_c\,\rho_m\, D\,\frac{d\,\omega_{fu}}{dy}$$

because $c_{pm}\,\Delta T\,\sum_k D\, d\,\omega_k/dy = 0$. Also, from the previous slide

$$c_{pm}\frac{dT}{dy} = \frac{d\,h_m}{dy} - \Delta h_c\,\frac{d\,\omega_{fu}}{dy}$$

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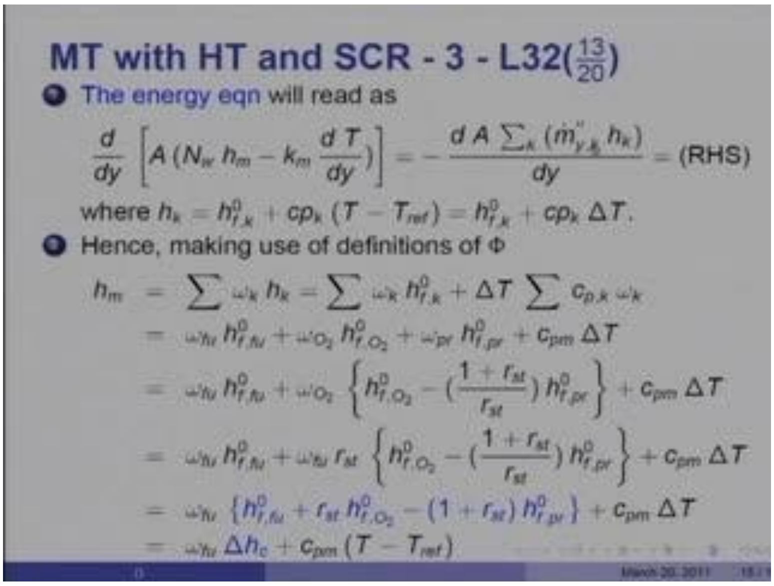

### MT with HT and SCR - 3 - L32($\frac{13}{20}$)

- The energy eqn will read as

$$\frac{d}{dy}\left[A\,(N_w\, h_m - k_m\,\frac{d\,T}{dy})\right] = -\frac{d\,A\,\sum_k (\dot{m}''_{y,k}\, h_k)}{dy} = (\text{RHS})$$

where $h_k = h^0_{f,k} + cp_k\,(T - T_{ref}) = h^0_{f,k} + cp_k\,\Delta T$.

- Hence, making use of definitions of $\Phi$

$$\begin{aligned} h_m &= \sum \omega_k\, h_k = \sum \omega_k\, h^0_{f,k} + \Delta T \sum c_{p,k}\,\omega_k \\ &= \omega_{fu}\, h^0_{f,fu} + \omega_{O_2}\, h^0_{f,O_2} + \omega_{pr}\, h^0_{f,pr} + c_{pm}\,\Delta T \\ &= \omega_{fu}\, h^0_{f,fu} + \omega_{O_2}\left\{h^0_{f,O_2} - (\frac{1 + r_{st}}{r_{st}})\, h^0_{f,pr}\right\} + c_{pm}\,\Delta T \\ &= \omega_{fu}\, h^0_{f,fu} + \omega_{fu}\, r_{st}\left\{h^0_{f,O_2} - (\frac{1 + r_{st}}{r_{st}})\, h^0_{f,pr}\right\} + c_{pm}\,\Delta T \\ &= \omega_{fu}\,\{h^0_{f,fu} + r_{st}\, h^0_{f,O_2} - (1 + r_{st})\, h^0_{f,pr}\} + c_{pm}\,\Delta T \\ &= \omega_{fu}\,\Delta h_c + c_{pm}\,(T - T_{ref}) \end{aligned}$$

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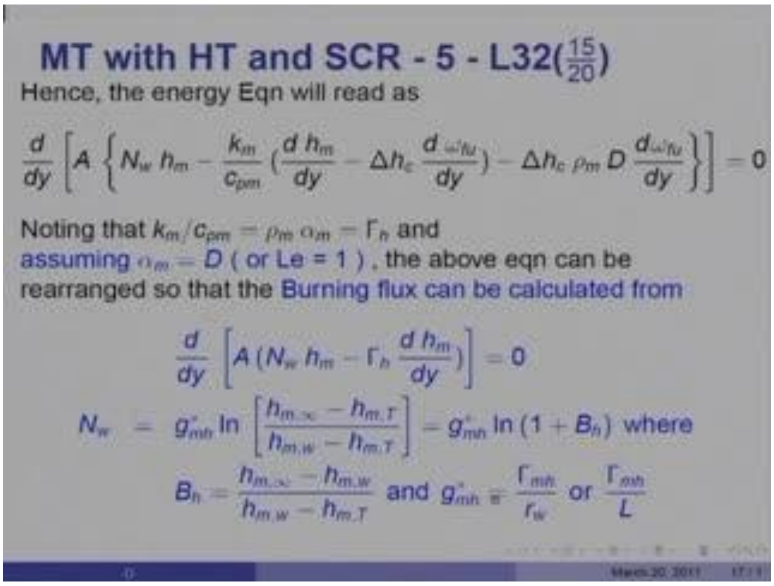

## MT with HT and SCR - 5 - L32($\frac{15}{20}$)

Hence, the energy Eqn will read as

$$\frac{d}{dy}\left[A\left\{N_w\,h_m - \frac{k_m}{c_{pm}}\left(\frac{d\,h_m}{dy} - \Delta h_c\,\frac{d\,\omega_{fu}}{dy}\right) - \Delta h_c\,\rho_m\,D\,\frac{d\omega_{fu}}{dy}\right\}\right] = 0$$

Noting that $k_m/c_{pm} = \rho_m\,\alpha_m = \Gamma_h$ and assuming $\alpha_m = D$ ( or Le = 1 ) , the above eqn can be rearranged so that the Burning flux can be calculated from

$$\frac{d}{dy}\left[A\,(N_w\,h_m - \Gamma_h\,\frac{d\,h_m}{dy})\right] = 0$$

$$N_w = g^*_{mh}\ln\left[\frac{h_{m,\infty} - h_{m,T}}{h_{m,w} - h_{m,T}}\right] = g^*_{mh}\ln\,(1 + B_h)\ \text{where}$$

$$B_h = \frac{h_{m,\infty} - h_{m,w}}{h_{m,w} - h_{m,T}}\ \text{and}\ g^*_{mh} = \frac{\Gamma_{mh}}{r_w}\ \text{or}\ \frac{\Gamma_{mh}}{L}$$

So, the right hand side can now be formed which is as you know is d by dy. I can now take everything on this side and you can see that I can transform this equation into A times N w h m into k m by C p m d h m by dy minus del h c d omega fu by dy. This is essentially the k d T dy term. This is the del h c rho m D d omega fu by dy which is the right hand side term. Then, if I again make the Lewis number equal to 1 assumption that is alpha m equal to D, then you will see that gamma m this k m by C p m can also be written as rho m into D and equal to gamma h. let us say So, then you will see that this term would get cancelled with that term and I would get A times N w h m gamma h into d h m by dy equal to 0 and again a conserved property relationship has been obtained from the energy equation.

The solution would be again same as that for inert mass transfer with B h defined as h m infinity minus h m w h m w minus h m T and this is the g star m h. Again, you will see that in a simple chemical reaction we have got the same formula; both from mass transfer equation as well as from the energy equation.

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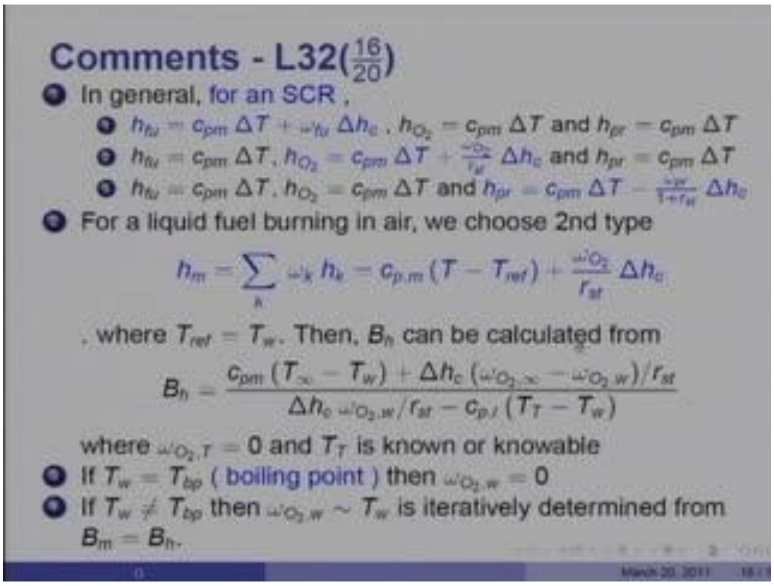

## Comments - L32$(\frac{16}{20})$

- In general, for an SCR ,
  - $h_{fu} = c_{pm}\,\Delta T + \omega_{fu}\,\Delta h_c$ , $h_{O_2} = c_{pm}\,\Delta T$ and $h_{pr} = c_{pm}\,\Delta T$
  - $h_{fu} = c_{pm}\,\Delta T$ , $h_{O_2} = c_{pm}\,\Delta T + \frac{\omega_{O_2}}{r_{st}}\,\Delta h_c$ and $h_{pr} = c_{pm}\,\Delta T$
  - $h_{fu} = c_{pm}\,\Delta T$ , $h_{O_2} = c_{pm}\,\Delta T$ and $h_{pr} = c_{pm}\,\Delta T - \frac{\omega_{pr}}{1+r_{st}}\,\Delta h_c$
- For a liquid fuel burning in air, we choose 2nd type

$$h_m = \sum_k \omega_k\,h_k = c_{p,m}\,(T - T_{ref}) + \frac{\omega_{O_2}}{r_{st}}\,\Delta h_c$$

, where $T_{ref} = T_w$. Then, $B_h$ can be calculated from

$$B_h = \frac{c_{pm}\,(T_\infty - T_w) + \Delta h_c\,(\omega_{O_2,\infty} - \omega_{O_2,w})/r_{st}}{\Delta h_c\,\omega_{O_2,w}/r_{st} - c_{p,l}\,(T_T - T_w)}$$

where $\omega_{O_2,T} = 0$ and $T_T$ is known or knowable

- If $T_w = T_{bp}$ ( boiling point ) then $\omega_{O_2,w} = 0$
- If $T_w \neq T_{bp}$ then $\omega_{O_2,w} \sim T_w$ is iteratively determined from $B_m = B_h$.

So, as I said earlier that we can define our mixture in variety of ways for a simple chemical reaction, one is to say h fu is equal to C p m delta T omega fu del h c which is what I did earlier. h o 2 will then be equal to C p m delta T and h product will be C p m delta T, because of the equal specific heat. But, I can also associate del h c with oxygen in which case h fu will be C p m delta T h o 2 will be C p m delta T into omega 2 o 2 r st del h c and h product. Then, again you have h fu h o 2 and I can associate now with omega del h c with the product mass fraction.

So, there are three ways in which you can do it for a liquid fuel burning in air. We often choose second type because we often know omega o 2 concentrations much better. Then h m will be omega k h k would be simply C p m into T minus T ref omega o 2 by r st del h c which is from these three relationships. If I now take for a moment that T ref is equal to T w which I do not know usually, then I will get for convenience, rather I am not knowing where T ref. If, I take T ref equal to T w then B h which was h m infinity minus h m w over h m w minus h m T would transform to C p m T infinity minus T w del h c omega o 2 infinity minus omega o 2 w divided by r st del h c into omega o 2 w by r st minus C p l T T minus T w, where of course omega o 2 T is 0 because, oxygen does not exist in the fuel and T T is known or knowable somehow.

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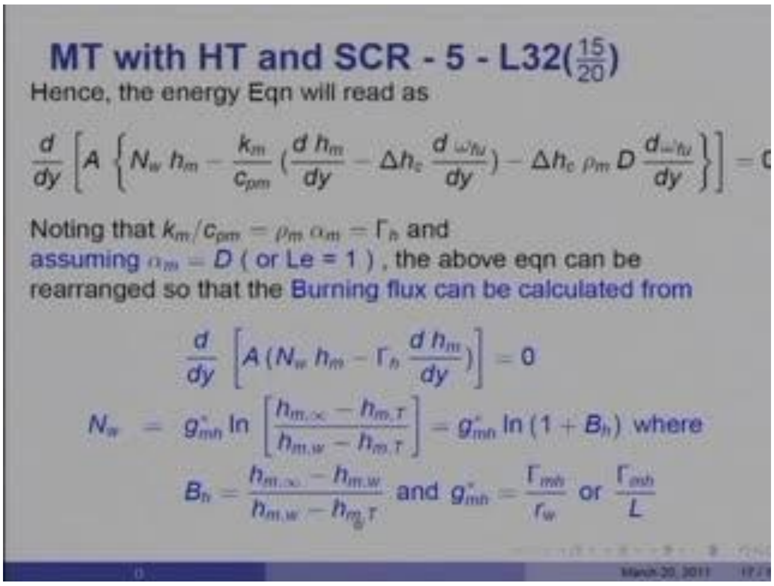

## MT with HT and SCR - 5 - L32($\frac{15}{20}$)

Hence, the energy Eqn will read as

$$\frac{d}{dy}\left[A\left\{N_w\, h_m - \frac{k_m}{c_{pm}}\left(\frac{d\,h_m}{dy} - \Delta h_c\frac{d\,\omega_{fu}}{dy}\right) - \Delta h_c\,\rho_m\, D\,\frac{d\omega_{fu}}{dy}\right\}\right] = 0$$

Noting that $k_m/c_{pm} = \rho_m\,\alpha_m = \Gamma_h$ and assuming $\alpha_m = D$ ( or Le = 1 ) , the above eqn can be rearranged so that the Burning flux can be calculated from

$$\frac{d}{dy}\left[A\left(N_w\, h_m - \Gamma_h\,\frac{d\,h_m}{dy}\right)\right] = 0$$

$$N_w = g^*_{mh}\ln\left[\frac{h_{m,\infty} - h_{m,T}}{h_{m,w} - h_{m,T}}\right] = g^*_{mh}\ln(1 + B_h) \text{ where}$$

$$B_h = \frac{h_{m,\infty} - h_{m,w}}{h_{m,w} - h_{m,T}} \text{ and } g^*_{mh} = \frac{\Gamma_{mh}}{r_w} \text{ or } \frac{\Gamma_{mh}}{L}$$

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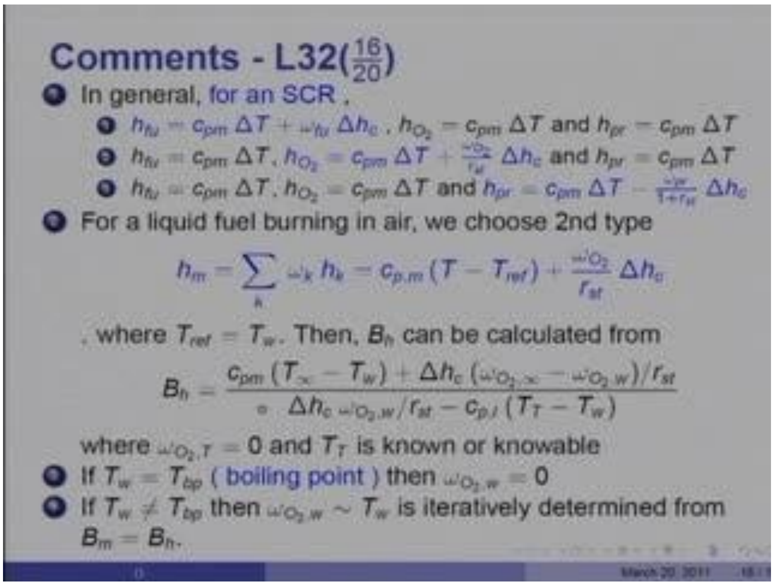

## Comments - L32($\frac{16}{20}$)

- In general, for an SCR ,
  - $h_{fu} = c_{pm}\,\Delta T + \omega_{fu}\,\Delta h_c$ , $h_{O_2} = c_{pm}\,\Delta T$ and $h_{pr} = c_{pm}\,\Delta T$
  - $h_{fu} = c_{pm}\,\Delta T$ , $h_{O_2} = c_{pm}\,\Delta T + \frac{\omega_{O_2}}{r_{st}}\,\Delta h_c$ and $h_{pr} = c_{pm}\,\Delta T$
  - $h_{fu} = c_{pm}\,\Delta T$ , $h_{O_2} = c_{pm}\,\Delta T$ and $h_{pr} = c_{pm}\,\Delta T - \frac{\omega_{pr}}{1+r_{st}}\,\Delta h_c$
- For a liquid fuel burning in air, we choose 2nd type

$$h_m = \sum_k \omega_k\, h_k = c_{p,m}\,(T - T_{ref}) + \frac{\omega_{O_2}}{r_{st}}\,\Delta h_c$$

, where $T_{ref} = T_w$. Then, $B_h$ can be calculated from

$$B_h = \frac{c_{pm}\,(T_\infty - T_w) + \Delta h_c\,(\omega_{O_2,\infty} - \omega_{O_2,w})/r_{st}}{\Delta h_c\,\omega_{O_2,w}/r_{st} - c_{p,l}\,(T_T - T_w)}$$

where $\omega_{O_2,T} = 0$ and $T_T$ is known or knowable

- If $T_w = T_{bp}$ ( boiling point ) then $\omega_{O_2,w} = 0$
- If $T_w \neq T_{bp}$ then $\omega_{O_2,w} \sim T_w$ is iteratively determined from $B_m = B_h$.

If T w was already, the fuel surface was already at the boiling point T w by T bp, then of course, no oxygen would survive at the surface and therefore that would be 0 because the fuel concentration there would be 1.

If T w is not equal to T bp then of course, omega o 2 and T w relations must be established iteratively by balancing B m and B h, where B h is given by this (Refer Slide Time:37:05). So, you assume at T w evaluate the B h value then evaluate the B m then evaluate the omega v w rather omega o 2 w and then again get the balance done in favor of till convergence is obtained.

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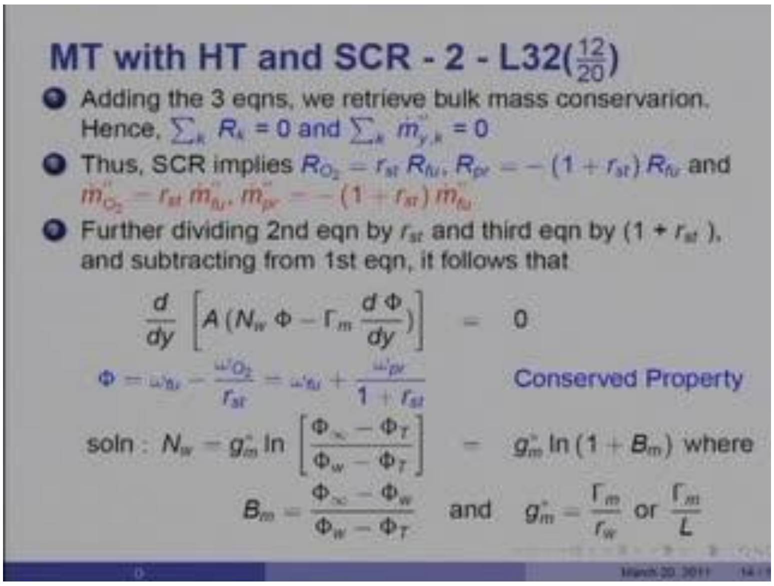

### MT with HT and SCR - 2 - L32($\frac{12}{20}$)

- Adding the 3 eqns, we retrieve bulk mass conservarion. Hence, $\sum_k R_k = 0$ and $\sum_k \dot{m}''_{y,k} = 0$
- Thus, SCR implies $R_{O_2} = r_{st} R_{fu}$, $R_{pr} = -(1 + r_{st}) R_{fu}$ and $\dot{m}''_{O_2} = r_{st} \dot{m}''_{fu}$, $\dot{m}''_{pr} = -(1 + r_{st}) \dot{m}''_{fu}$
- Further dividing 2nd eqn by $r_{st}$ and third eqn by $(1 + r_{st})$, and subtracting from 1st eqn, it follows that

$$\frac{d}{dy}\left[A\left(N_w \Phi - \Gamma_m \frac{d\Phi}{dy}\right)\right] = 0$$

$$\Phi = \omega_{fu} - \frac{\omega_{O_2}}{r_{st}} = \omega_{fu} + \frac{\omega_{pr}}{1 + r_{st}} \quad \text{Conserved Property}$$

$$\text{soln}: \ N_w = g_m^* \ln\left[\frac{\Phi_\infty - \Phi_T}{\Phi_w - \Phi_T}\right] = g_m^* \ln(1 + B_m) \ \text{where}$$

$$B_m = \frac{\Phi_\infty - \Phi_w}{\Phi_w - \Phi_T} \quad \text{and} \quad g_m^* = \frac{\Gamma_m}{r_w} \ \text{or} \ \frac{\Gamma_m}{L}$$

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Where B m is given by this relationship and you will see phi value now will be known in all states for example omega fu in the infinity state is 0. Whereas, omega o 2 is known so phi infinity is known in the T state omega fu will be 1 and omega o 2 will be 0 so phi T is known. omega w will depend on the temperature which will determine omega fu and the remaining will be omega o 2.

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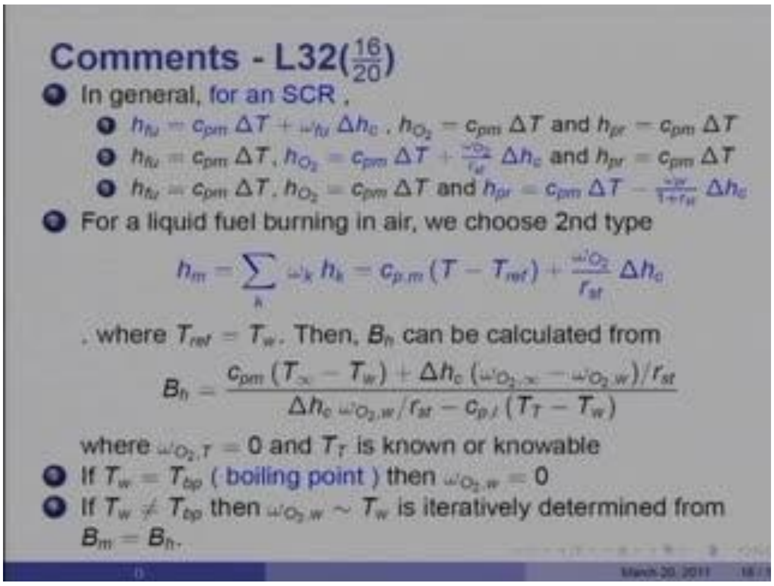

### Comments - L32($\frac{16}{20}$)

- In general, for an SCR ,
  - $h_{fu} = c_{pm}\,\Delta T + \omega_{fu}\,\Delta h_c$ , $h_{O_2} = c_{pm}\,\Delta T$ and $h_{pr} = c_{pm}\,\Delta T$
  - $h_{fu} = c_{pm}\,\Delta T$, $h_{O_2} = c_{pm}\,\Delta T + \frac{\omega_{O_2}}{r_{st}}\,\Delta h_c$ and $h_{pr} = c_{pm}\,\Delta T$
  - $h_{fu} = c_{pm}\,\Delta T$, $h_{O_2} = c_{pm}\,\Delta T$ and $h_{pr} = c_{pm}\,\Delta T - \frac{\omega_{pr}}{1+r_{st}}\,\Delta h_c$
- For a liquid fuel burning in air, we choose 2nd type

$$h_m = \sum_k \omega_k h_k = c_{p,m}(T - T_{ref}) + \frac{\omega_{O_2}}{r_{st}}\,\Delta h_c$$

, where $T_{ref} = T_w$. Then, $B_h$ can be calculated from

$$B_h = \frac{c_{pm}(T_\infty - T_w) + \Delta h_c(\omega_{O_2,\infty} - \omega_{O_2,w})/r_{st}}{\Delta h_c\,\omega_{O_2,w}/r_{st} - c_{p,l}(T_T - T_w)}$$

where $\omega_{O_2,T} = 0$ and $T_T$ is known or knowable

- If $T_w = T_{bp}$ ( boiling point ) then $\omega_{O_2,w} = 0$
- If $T_w \neq T_{bp}$ then $\omega_{O_2,w} \sim T_w$ is iteratively determined from $B_m = B_h$.

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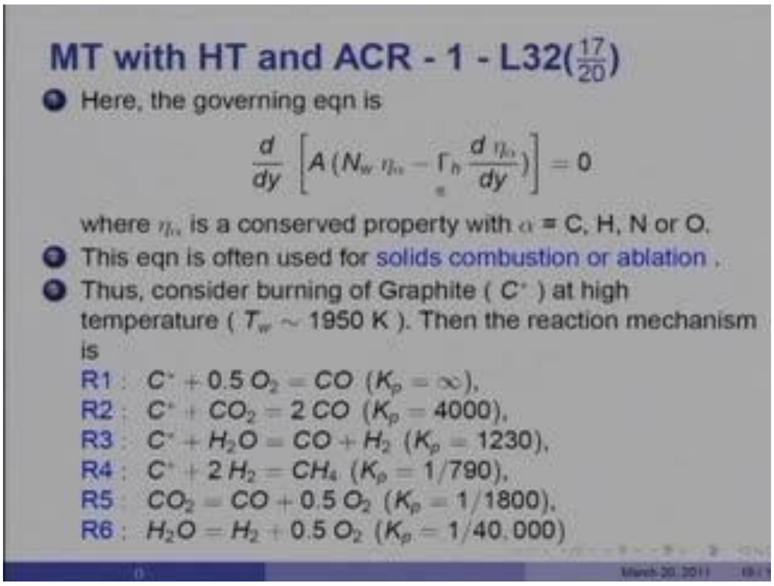

**MT with HT and ACR - 1 - L32($\frac{17}{20}$)**

- Here, the governing eqn is

$$\frac{d}{dy}\left[A\left(N_w\,\eta_{\alpha} - \Gamma_h\,\frac{d\,\eta_{\alpha}}{dy}\right)\right] = 0$$

where $\eta_{\alpha}$ is a conserved property with $\alpha$ = C, H, N or O.

- This eqn is often used for solids combustion or ablation .
- Thus, consider burning of Graphite ( $C^*$ ) at high temperature ( $T_w \sim 1950$ K ). Then the reaction mechanism is

R1 : $C^* + 0.5\,O_2 = CO$ ($K_p = \infty$),
R2 : $C^* + CO_2 = 2\,CO$ ($K_p = 4000$),
R3 : $C^* + H_2O = CO + H_2$ ($K_p = 1230$),
R4 : $C^* + 2\,H_2 = CH_4$ ($K_p = 1/790$),
R5 : $CO_2 = CO + 0.5\,O_2$ ($K_p = 1/1800$),
R6 : $H_2O = H_2 + 0.5\,O_2$ ($K_p = 1/40{,}000$)

We can determine the relationships in that manner and carry out iterations. Now, I come to mass transfer with heat transfer and arbitrary chemical reaction. What do I mean by arbitrary chemical reaction? By arbitrary chemical reaction, is typically a curve in solids combustion? For example, let us say consider burning of a graphite C star is given by C star at very high temperature T w of the order of 1950 or much higher then, there are several reactions taking place. The first 4 reactions take place at the surface of the graphite C star half o 2 equal to CO whose equilibrium constant infinity C star CO 2 would be 2 CO whose again K p is 4000 C star H 2 O equal to CO plus H 2 1230 and C star equal to 2 H 2 equal to CH 4 1 by 790.

CO does generate a hydrogen and C H 4 would then burn in the gas phase. Of course, here the K p so low at 1950, there hardly any C H 4 would be formed and therefore, we can say that very small amounts of C H 4 will be present. CO would then react with oxygen to produce CO 2 which would then dissociate in this fashion. The CO 2 will be CO plus half O 2, giving you CO 2 and H 2 O would dissociate to give H 2 plus O 2. Again, the K p is very low and therefore the reverse reactions would be dominant you producing CO 2 and H 2 O.

So, for such a complex mechanism it is best to conserve the elements we have. Elements, C H and O in this case; you can see elements are C H and O of course, nitrogen is not present in the fuel and therefore we write this and as you recall, the element mass fraction equation is always a conserved property equation.

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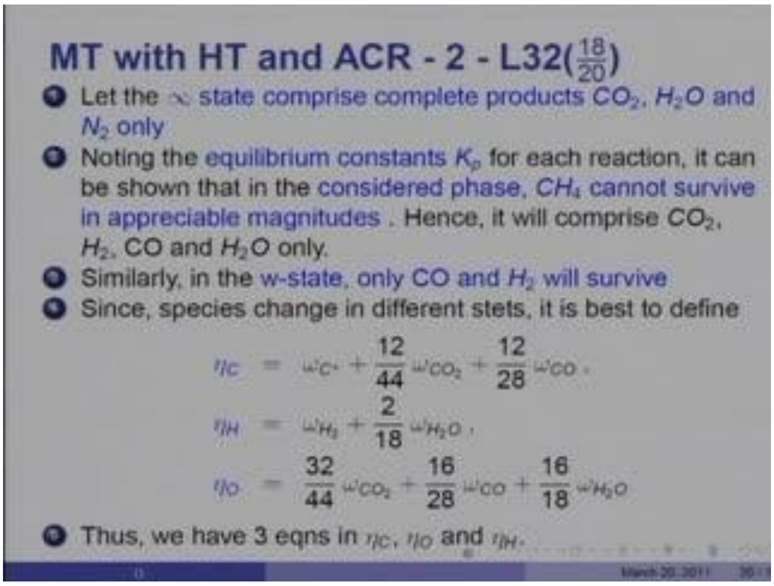

## MT with HT and ACR - 2 - L32($\frac{18}{20}$)

- Let the $\infty$ state comprise complete products $CO_2$, $H_2O$ and $N_2$ only
- Noting the equilibrium constants $K_p$ for each reaction, it can be shown that in the considered phase, $CH_4$ cannot survive in appreciable magnitudes . Hence, it will comprise $CO_2$, $H_2$, CO and $H_2O$ only.
- Similarly, in the w-state, only CO and $H_2$ will survive
- Since, species change in different stets, it is best to define

$$\eta_C = \omega_{C^*} + \frac{12}{44}\omega_{CO_2} + \frac{12}{28}\omega_{CO},$$

$$\eta_H = \omega_{H_2} + \frac{2}{18}\omega_{H_2O},$$

$$\eta_O = \frac{32}{44}\omega_{CO_2} + \frac{16}{28}\omega_{CO} + \frac{16}{18}\omega_{H_2O}$$

- Thus, we have 3 eqns in $\eta_C$, $\eta_O$ and $\eta_H$.

So let the infinity state comprise of CO 2 H 2 O and N 2 only that is the full products that so, the noting the equilibrium constant K p for each reaction it can be shown that in the considered phase, CH 4 cannot survive in appreciable magnitudes. Hence, it will comprise, the considered phase will essentially comprise of CO 2 H 2 CO and H 2 O only.

Similarly, in the w-state only CO and H 2 will survive and therefore since species change in different states it is best to define a eta C equal to omega C star over 12 by 44 omega CO 2 then eta H would be given by that and eta O it will be given by that composition.

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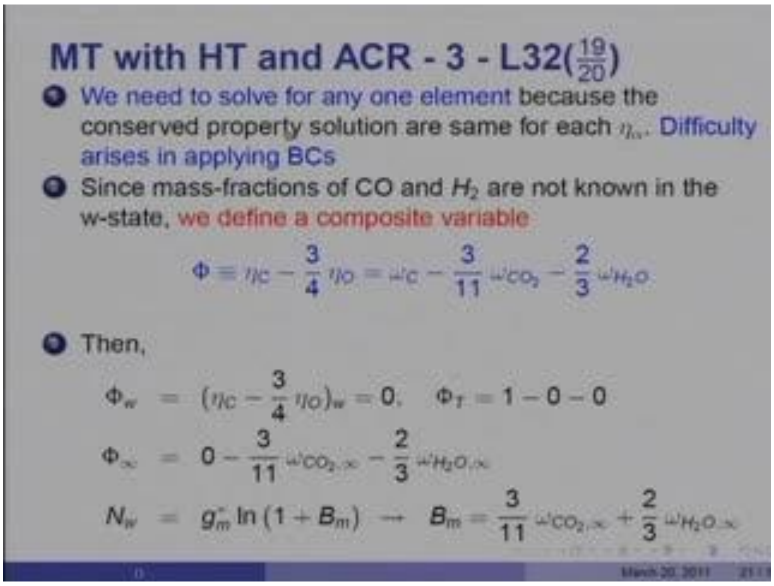

## MT with HT and ACR - 3 - L32($\frac{19}{20}$)

- We need to solve for any one element because the conserved property solution are same for each $\eta_k$. Difficulty arises in applying BCs
- Since mass-fractions of CO and $H_2$ are not known in the w-state, we define a composite variable

$$\Phi \equiv \eta_C - \frac{3}{4}\eta_O = \omega_C - \frac{3}{11}\omega_{CO_2} - \frac{2}{3}\omega_{H_2O}$$

- Then,

$$\Phi_w = (\eta_C - \frac{3}{4}\eta_O)_w = 0, \quad \Phi_T = 1 - 0 - 0$$

$$\Phi_\infty = 0 - \frac{3}{11}\omega_{CO_2,\infty} - \frac{2}{3}\omega_{H_2O,\infty}$$

$$N_w = g_m^* \ln(1 + B_m) \rightarrow B_m = \frac{3}{11}\omega_{CO_2,\infty} + \frac{2}{3}\omega_{H_2O,\infty}$$

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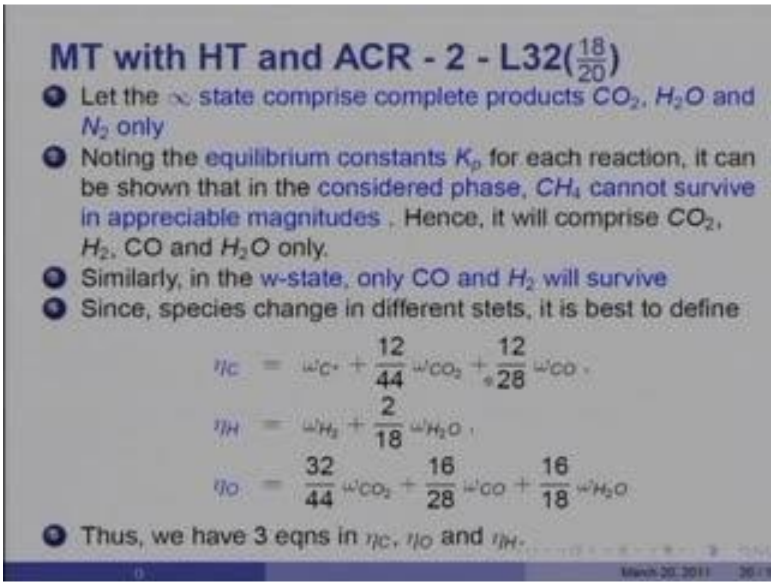

## MT with HT and ACR - 2 - L32($\frac{18}{20}$)

- Let the $\infty$ state comprise complete products $CO_2$, $H_2O$ and $N_2$ only
- Noting the equilibrium constants $K_p$ for each reaction, it can be shown that in the considered phase, $CH_4$ cannot survive in appreciable magnitudes . Hence, it will comprise $CO_2$, $H_2$, CO and $H_2O$ only.
- Similarly, in the w-state, only CO and $H_2$ will survive
- Since, species change in different stets, it is best to define

$$\eta_C = \omega_{C^*} + \frac{12}{44}\omega_{CO_2} + \frac{12}{28}\omega_{CO},$$

$$\eta_H = \omega_{H_2} + \frac{2}{18}\omega_{H_2O},$$

$$\eta_O = \frac{32}{44}\omega_{CO_2} + \frac{16}{28}\omega_{CO} + \frac{16}{18}\omega_{H_2O}$$

- Thus, we have 3 eqns in $\eta_C$, $\eta_O$ and $\eta_H$.

Thus we have 3 equations for eta C eta H and eta O. So, instead of solving 3 equations any 1 can be solved; but you will find that not all these quantities on the right hand side are very well known in the three states.

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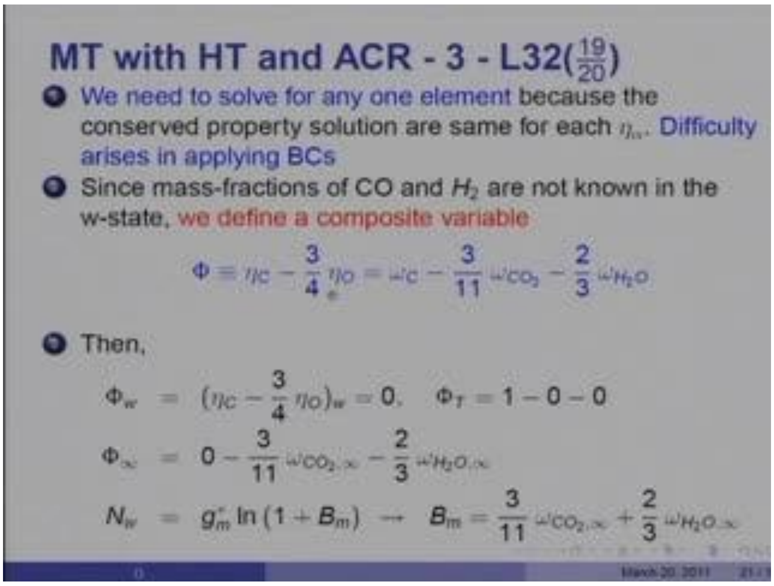

## MT with HT and ACR - 3 - L32($\frac{19}{20}$)

- We need to solve for any one element because the conserved property solution are same for each $\eta_{\alpha}$. Difficulty arises in applying BCs
- Since mass-fractions of CO and $H_2$ are not known in the w-state, we define a composite variable

$$\Phi \equiv \eta_C - \frac{3}{4}\eta_O = \omega_C - \frac{3}{11}\omega_{CO_2} - \frac{2}{3}\omega_{H_2O}$$

- Then,

$$\Phi_w = (\eta_C - \frac{3}{4}\eta_O)_w = 0, \quad \Phi_T = 1 - 0 - 0$$
$$\Phi_\infty = 0 - \frac{3}{11}\omega_{CO_2,\infty} - \frac{2}{3}\omega_{H_2O,\infty}$$
$$N_w = g_m^* \ln(1 + B_m) \rightarrow B_m = \frac{3}{11}\omega_{CO_2,\infty} + \frac{2}{3}\omega_{H_2O,\infty}$$

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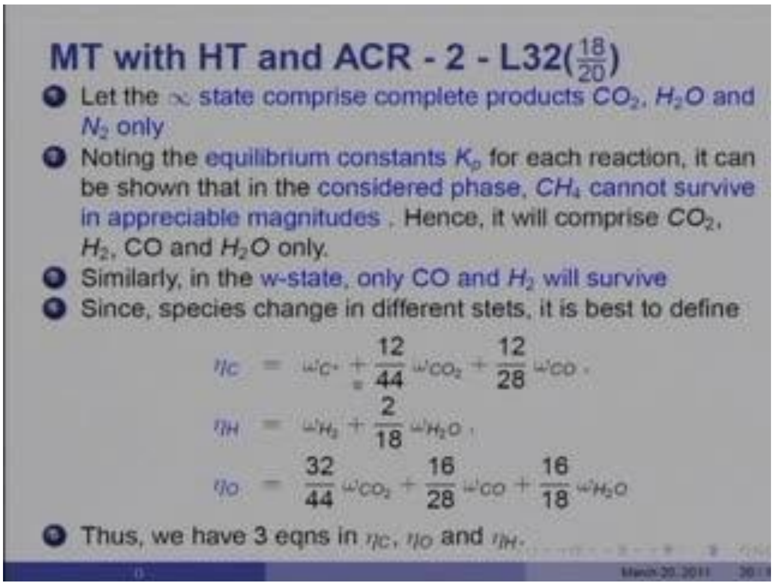

## MT with HT and ACR - 2 - L32($\frac{18}{20}$)

- Let the $\infty$ state comprise complete products $CO_2$, $H_2O$ and $N_2$ only
- Noting the equilibrium constants $K_p$ for each reaction, it can be shown that in the considered phase, $CH_4$ cannot survive in appreciable magnitudes . Hence, it will comprise $CO_2$, $H_2$, CO and $H_2O$ only.
- Similarly, in the w-state, only CO and $H_2$ will survive
- Since, species change in different stets, it is best to define

$$\eta_C = \omega_{C^*} + \frac{12}{44}\omega_{CO_2} + \frac{12}{28}\omega_{CO},$$
$$\eta_H = \omega_{H_2} + \frac{2}{18}\omega_{H_2O},$$
$$\eta_O = \frac{32}{44}\omega_{CO_2} + \frac{16}{28}\omega_{CO} + \frac{16}{18}\omega_{H_2O}$$

- Thus, we have 3 eqns in $\eta_C$, $\eta_O$ and $\eta_H$.

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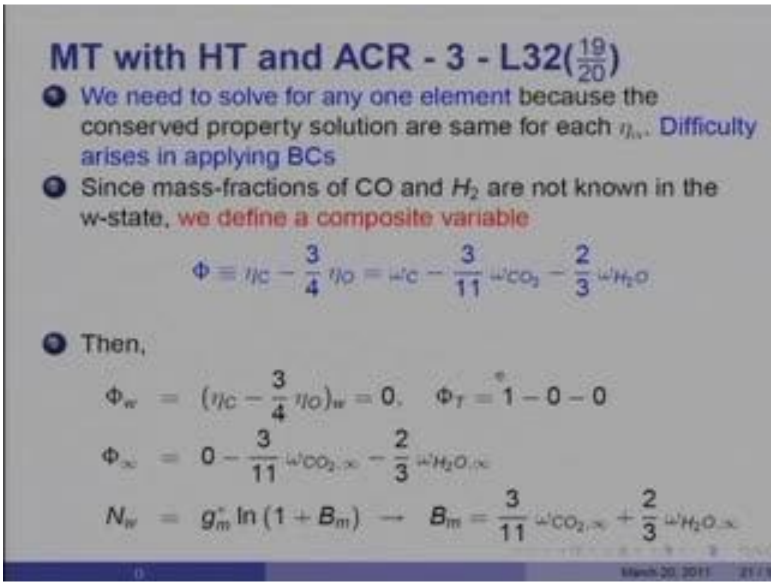

MT with HT and ACR - 3 - L32($\frac{19}{20}$)

- We need to solve for any one element because the conserved property solution are same for each $\eta_{k}$. Difficulty arises in applying BCs
- Since mass-fractions of CO and $H_2$ are not known in the w-state, we define a composite variable

$$\Phi \equiv \eta_C - \frac{3}{4}\eta_O = \omega_C - \frac{3}{11}\omega_{CO_2} - \frac{2}{3}\omega_{H_2O}$$

- Then,

$$\Phi_w = (\eta_C - \frac{3}{4}\eta_O)_w = 0, \quad \Phi_T = 1 - 0 - 0$$

$$\Phi_\infty = 0 - \frac{3}{11}\omega_{CO_2,\infty} - \frac{2}{3}\omega_{H_2O,\infty}$$

$$N_w = g_m^* \ln(1 + B_m) \rightarrow B_m = \frac{3}{11}\omega_{CO_2,\infty} + \frac{2}{3}\omega_{H_2O,\infty}$$

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So, the best thing is to derive a composite quantity which we shall take as eta C minus 3 by 4 n o and from this, these relationship you will see that I can form a nu variable omega C minus 3 by 11 omega CO 2 minus 2 by 3 omega H 2 O. Now, I can definitely form phi w equal to n c minus 3 by n o w equal to 0 in the w state in the T state only eta C will be 1 or omega C will be 1. All these are zeros, so they have put to 0. And in the infinity state, I do not have any carbon but there is omega CO 2 and omega H 2 O so they are retained and as a result I will get N w equal to ln 1 plus B m, where B m would be 3 by 11 CO 2 infinity plus 2 by 3 omega H 2 O infinity.

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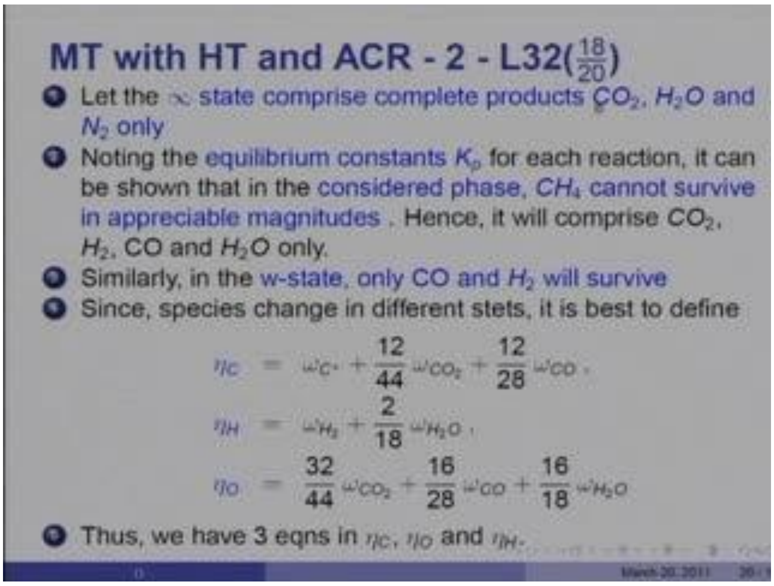

## MT with HT and ACR - 2 - L32($\frac{18}{20}$)

- Let the $\infty$ state comprise complete products $CO_2$, $H_2O$ and $N_2$ only
- Noting the equilibrium constants $K_p$ for each reaction, it can be shown that in the considered phase, $CH_4$ cannot survive in appreciable magnitudes . Hence, it will comprise $CO_2$, $H_2$, CO and $H_2O$ only.
- Similarly, in the w-state, only CO and $H_2$ will survive
- Since, species change in different stets, it is best to define

$$\begin{aligned}\eta_C &= \omega_{C^*} + \frac{12}{44}\omega_{CO_2} + \frac{12}{28}\omega_{CO},\\ \eta_H &= \omega_{H_2} + \frac{2}{18}\omega_{H_2O},\\ \eta_O &= \frac{32}{44}\omega_{CO_2} + \frac{16}{28}\omega_{CO} + \frac{16}{18}\omega_{H_2O}\end{aligned}$$

- Thus, we have 3 eqns in $\eta_C$, $\eta_O$ and $\eta_H$.

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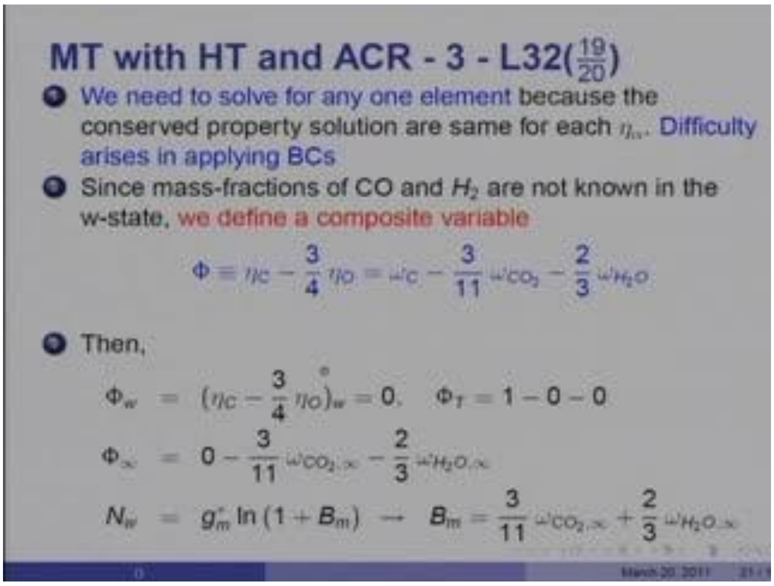

## MT with HT and ACR - 3 - L32($\frac{19}{20}$)

- We need to solve for any one element because the conserved property solution are same for each $\eta_{k}$. Difficulty arises in applying BCs
- Since mass-fractions of CO and $H_2$ are not known in the w-state, we define a composite variable

$$\Phi \equiv \eta_C - \frac{3}{4}\eta_O = \omega_C - \frac{3}{11}\omega_{CO_2} - \frac{2}{3}\omega_{H_2O}$$

- Then,

$$\begin{aligned}\Phi_w &= (\eta_C - \frac{3}{4}\eta_O)_w = 0, \quad \Phi_T = 1 - 0 - 0\\ \Phi_\infty &= 0 - \frac{3}{11}\omega_{CO_2,\infty} - \frac{2}{3}\omega_{H_2O,\infty}\\ N_w &= g_m^* \ln(1 + B_m) \rightarrow B_m = \frac{3}{11}\omega_{CO_2,\infty} + \frac{2}{3}\omega_{H_2O,\infty}\end{aligned}$$

Both these are known in the infinity state, because that is what we said that we know. only the product in the infinity state. So, and therefore, I can calculate the mass transfer rate of graphite burning simply by knowing CO 2 at in the infinity state and omega H 2 O in infinity state.

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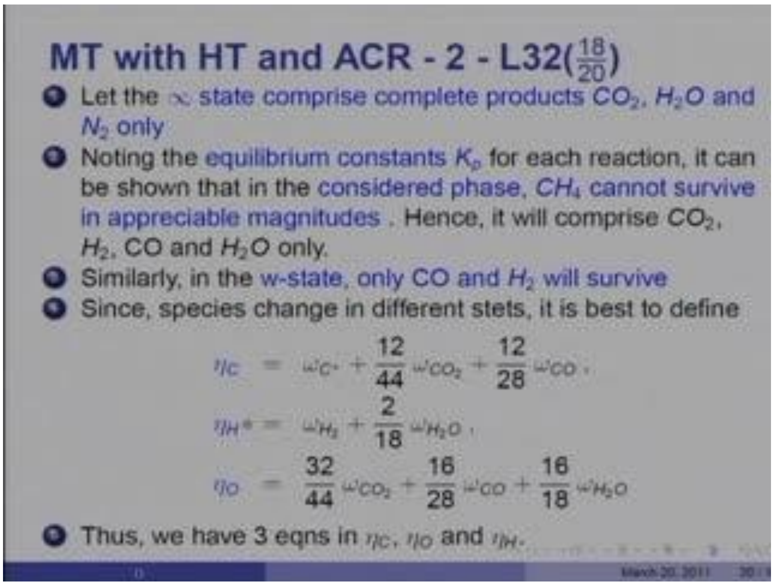

## MT with HT and ACR - 2 - L32($\frac{18}{20}$)

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$$\eta_{H^*} = \omega_{H_2} + \frac{2}{18}\omega_{H_2O},$$
$$\eta_O = \frac{32}{44}\omega_{CO_2} + \frac{16}{28}\omega_{CO} + \frac{16}{18}\omega_{H_2O}$$

- Thus, we have 3 eqns in $\eta_C$, $\eta_O$ and $\eta_H$.

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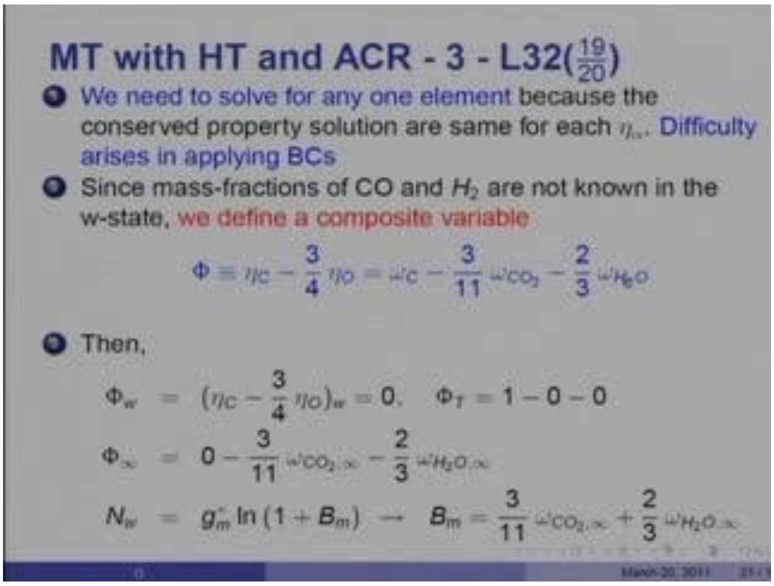

## MT with HT and ACR - 3 - L32($\frac{19}{20}$)

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$$\Phi \equiv \eta_C - \frac{3}{4}\eta_O = \omega_C - \frac{3}{11}\omega_{CO_2} - \frac{2}{3}\omega_{H_2O}$$

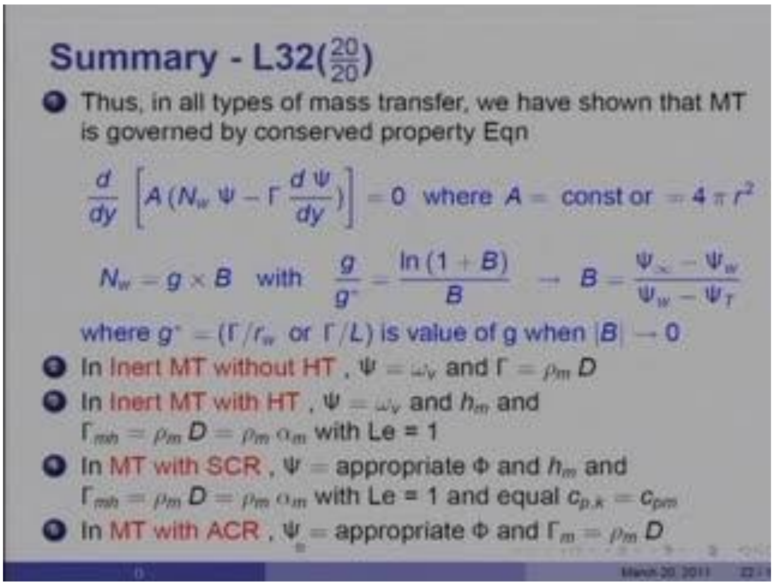
- Then,

$$\Phi_w = (\eta_C - \frac{3}{4}\eta_O)_w = 0, \quad \Phi_T = 1 - 0 - 0$$
$$\Phi_\infty = 0 - \frac{3}{11}\omega_{CO_2,\infty} - \frac{2}{3}\omega_{H_2O,\infty}$$
$$N_w = g_m^* \ln(1 + B_m) \rightarrow B_m = \frac{3}{11}\omega_{CO_2,\infty} + \frac{2}{3}\omega_{H_2O,\infty}$$

So in this case, because I know the relationship connecting element mass fractions with the species mass fractions, I am able to create a composite phi as eta C minus 3 by 4; simply by observation, this and this manipulation is simply by observation.

Such that I do not want any of the things, any specie on the right hand side whose concentration I would not know in the infinity w and T states and this is what I have been able to achieve; so that the calculation of mass transfer becomes easy.

(Refer Slide Time: 43:35)



## Summary - L32($\frac{20}{20}$)

- Thus, in all types of mass transfer, we have shown that MT is governed by conserved property Eqn

$$\frac{d}{dy}\left[A\left(N_w\,\Psi - \Gamma\,\frac{d\,\Psi}{dy}\right)\right] = 0 \quad \text{where } A = \text{const or } = 4\,\pi\,r^2$$

$$N_w = g \times B \quad \text{with} \quad \frac{g}{g^*} = \frac{\ln\,(1+B)}{B} \quad \rightarrow \quad B = \frac{\Psi_\infty - \Psi_w}{\Psi_w - \Psi_T}$$

where $g^* = (\Gamma/r_w$ or $\Gamma/L)$ is value of g when $|B| \rightarrow 0$

- In Inert MT without HT , $\Psi = \omega_v$ and $\Gamma = \rho_m\,D$
- In Inert MT with HT , $\Psi = \omega_v$ and $h_m$ and $\Gamma_{mh} = \rho_m\,D = \rho_m\,\alpha_m$ with Le = 1
- In MT with SCR , $\Psi$ = appropriate $\Phi$ and $h_m$ and $\Gamma_{mh} = \rho_m\,D = \rho_m\,\alpha_m$ with Le = 1 and equal $c_{p,k} = c_{pm}$
- In MT with ACR , $\Psi$ = appropriate $\Phi$ and $\Gamma_m = \rho_m\,D$

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So, in some way I would say that we have analyzed all types of mass transfer problems from the by converting every problem to a conserved property equation and psi has to be defined appropriately, so that N w equal to g B with g by g star equal to ln 1 plus B by B and B is equal to psi infinity minus psi w over psi w minus psi T and A is 4 pi r square or a constant.

For inert mass transfer without heat transfer, psi was equal to omega v, as you know, and gamma was simply equal to rho m D. For inert mass transfer with heat transfer, we had psi equal to omega v and h m and we made the assumption of Lewis number equal to 1. from mass transfer with heat transfer and chemical reaction we choose psi equal to appropriate phi and mixture enthalpy. We make the Lewis number equal to 1 - assumption and also we say that the specific heats of participating species would be equal to specific heat of mixture and in the mass transfer with arbitrary chemical reaction, we showed that psi can be simply appropriate phi and gamma m as rho m diffusivity.

So you can see that a variety of problems have been reduced to conserve property relationship through appropriate and justifiable assumption which makes calculation of the mass transfer rate simple and we are able to derive analytically derived relationship connecting N w, the mass transfer to the driving force B and the relationship we have found is a logarithmic one. In the next lecture, we will see how, what the Couette flow model has to say.