Convective Heat and Mass Transfer

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Module No. # 01

Lecture No. # 31

Convective Mass Transfer

In this 31st lecture, we will begin with Convective Mass Transfer. As we recall, we spent on laminar flows in external boundary layers and ducted flows in the first 20 lectures. Then, we spent 10 lectures on how to tackle turbulent flows and heat transfer.

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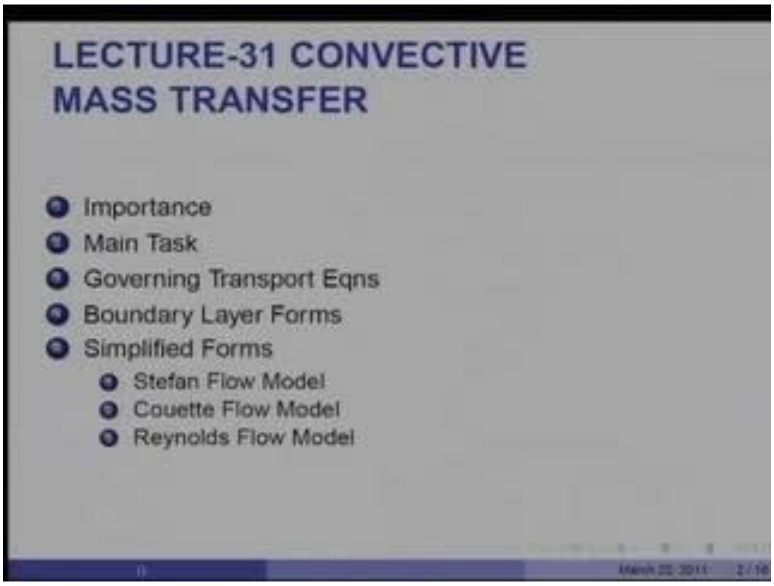

LECTURE-31 CONVECTIVE MASS TRANSFER

- Importance
- Main Task
- Governing Transport Eqns
- Boundary Layer Forms
- Simplified Forms
    - Stefan Flow Model
    - Couette Flow Model
    - Reynolds Flow Model

Now, we have turned to convective mass transfer. So, we have 10 lectures to do that. First of all, I will explain what the importance of the subject matter - convective mass transfer. What is our main task in convective mass transfer? What are the governing transport equations? Convert them into the boundary layer forms, but unlike what we did for a momentum and heat transfer, where we actually solved the boundary layer forms of equations in their differential form. What I am going to do here is develop very simplified models of boundary layer forms of equations, which are coupled and partial differential equations. You will see in the simplified forms that I develop, you would essentially have one-dimensional equations. They are good enough to represent what happens in a real boundary layer flow.

Finally, there are three models of this type. One is called the Stefan flow model; the other is the Couette flow model. Both these rely on one-dimensional ordinary differential equation. The Reynolds flow model goes even beyond and says that even an algebraic model is capable of predicting, what the full boundary layer flow model can predict. So, to that extent, our approach to the entire convective mass transfer is not based on solving the partial differential equations of a boundary layer, but it is actually based on solving this simplified model equations. I will be showing that it turns out that the solutions produced from these models, occurred very well with the boundary layer solutions that are obtained.

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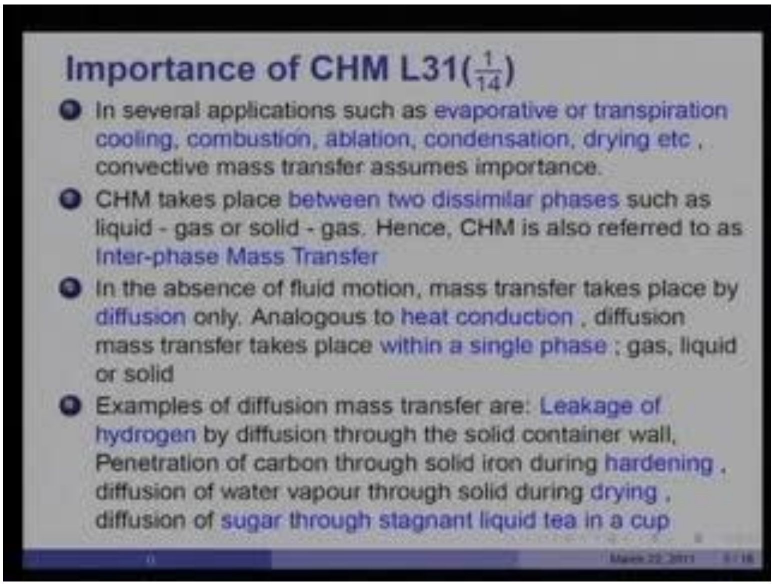

Importance of CHM L31($\frac{1}{14}$)

- In several applications such as evaporative or transpiration cooling, combustion, ablation, condensation, drying etc , convective mass transfer assumes importance.
- CHM takes place between two dissimilar phases such as liquid - gas or solid - gas. Hence, CHM is also referred to as Inter-phase Mass Transfer
- In the absence of fluid motion, mass transfer takes place by diffusion only. Analogous to heat conduction , diffusion mass transfer takes place within a single phase ; gas, liquid or solid
- Examples of diffusion mass transfer are: Leakage of hydrogen by diffusion through the solid container wall, Penetration of carbon through solid iron during hardening , diffusion of water vapour through solid during drying , diffusion of sugar through stagnant liquid tea in a cup

Several applications, such as evaporative or transpiration, cooling, combustion, ablation, condensation or drying etc, convective mass transfer assumes importance. Mechanical engineers need to deal with all of these in engineering practice.

The convective mass transfer always takes place between two dissimilar phases, such as liquid to gas or a solid to gas. So, many times, convective mass transfer is referred as inter phase mass transfer. For example, evaporation is liquid to gas; mass transfer ablation on the other hand is solid to gas; combustion can be of a wood particle or a coal particle, it is solid to gas mass transfer problem.

Now, in the absence of fluid motion, we can regard convective mass transfer as diffusion mass transfer. You will see that it is very analogous, but not exactly same as heat conduction diffusion mass transfer. It takes place within a single-phase gas solid or liquid, you will see as we go along. The examples of diffusion mass transfer are several. For example, leakage of hydrogen through solid container wall by diffusion. If you store hydrogen in a solid container, then hydrogen is very leaky. As I say, even through the solid wall, hydrogen would leak; diffused through the wall and come out. It could be dangerous from fire point of view. Penetration of carbon through iron would be an example. We use this method for carburizing iron to harden the surface of the iron. Penetration of carbon through solid iron would be an example of diffusion mass transfer.

Similarly, diffusion of water vapor through solid during drying would also be an example of diffusion mass transfer. Imagine that you have a tea cup and I have tea here, I will drink that but, you have a tea cup in which, you have put sugar and the liquid is stagnant. After a while, we will see that tea turns sweet that is because the sugar dissolves and diffuses through the tea and you taste the tea sweet. In order to hasten this rate of diffusion, what you do? You stir it, which essentially means you are bringing about convective mixing rather than diffused mixing. So that is the difference between convection and diffusion. The convective rates are much higher than the diffusion rates.

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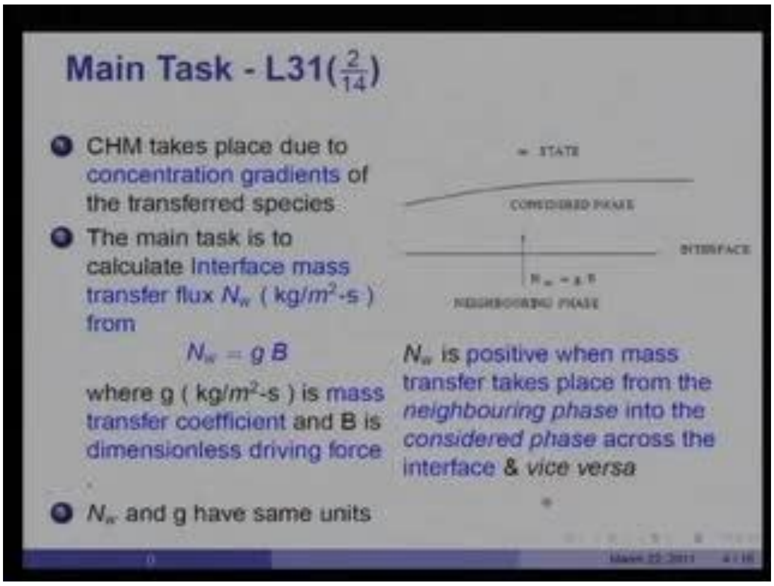

Main Task - L31($\frac{2}{14}$)

- CHM takes place due to concentration gradients of the transferred species
- The main task is to calculate Interface mass transfer flux N_w (kg/m^2-s) from

$$N_w = g\,B$$

where g (kg/m^2-s) is mass transfer coefficient and B is dimensionless driving force

- N_w and g have same units

N_w is positive when mass transfer takes place from the *neighbouring phase* into the *considered phase* across the interface & *vice versa*

Let us turn to convective mass transfer. As I said, it takes place due to concentration gradients of a transferred species. Let us say this as a flat surface. For example, It can be water and let us say, wind is blowing from left to right. The air is relatively dry compared to water; there would be an evaporation of water from liquid state into the vapor state. The air would simply take away the vapor.

In mass transfer problem, we always draw a figure like this to understand what we are doing. The air into which, mass transfer is taking place is called the considered phase. The water from which, the mass transfer is taking place is called the neighbouring phase. So, mass transfer always takes place from neighbouring phase to considered phase or from considered phase to neighbouring phase. In the latter case, the mass transfer flux N suffix w kilogram per meter square second would be negative and that is the interface, which is like the wall state. So, we always think of three states: infinity state, the wall interface or wall state and the transfer substance phase or the neighbouring phase, which I shall define very shortly.

Our task is to show that N w is actually directly proportional to the driving force of mass transfer B. B is a dimensionless driving force and the constant of proportionality The mass transfer coefficient, g is again equal to kilograms per meter square seconds. Unlike in heat transfer, where heat flux is related to a temperature difference and heat transfer coefficient has units different from the heat flux. In mass transfer, the mass transfer flux and the mass transfer coefficient have the same units because the driving force is dimensionless. N w is positive, when mass transfer takes place from the neighbouring phase into the considered phase across the interface and vice versa. So, what I mean here is that if evaporation takes place, then N w would be positive. On the other hand, if condensation would take place, then N w would be negative. Therefore, it is important to show the direction of N w, which is positive, when it is from neighbouring phase to the considered phase.

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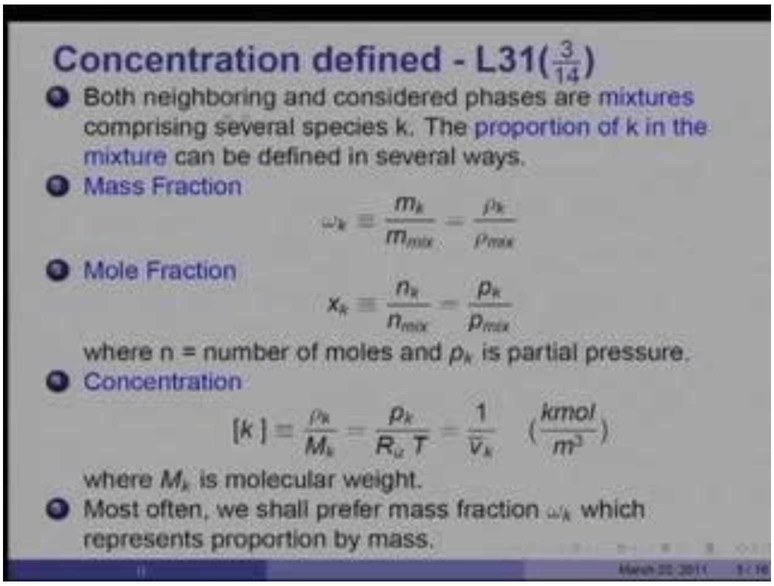

Concentration defined - L31($\frac{3}{14}$)

- Both neighboring and considered phases are mixtures comprising several species k. The proportion of k in the mixture can be defined in several ways.
- Mass Fraction

$$\omega_k \equiv \frac{m_k}{m_{mix}} = \frac{\rho_k}{\rho_{mix}}$$

- Mole Fraction

$$x_k \equiv \frac{n_k}{n_{mix}} = \frac{p_k}{p_{mix}}$$

where n = number of moles and p_k is partial pressure.

- Concentration

$$[k] = \frac{\rho_k}{M_k} = \frac{p_k}{R_u\, T} = \frac{1}{\bar{v}_k} \quad (\frac{kmol}{m^3})$$

where M_k is molecular weight.

- Most often, we shall prefer mass fraction ω_k which represents proportion by mass.

In mass transfer, we are always dealing with mixtures. For example, in an evaporation problem in the considered phase, you have a mixture of air and water. Only two species are involved. On the other hand, in a combustion problem, there can be several species like methane, oxygen, carbon dioxide, carbon monoxide, nitrous oxide and so on. I mean they depend on the temperature levels attained and so on. So, the considered phase could have several species. On the other hand, the neighbouring phase itself could be a fuel, if it is a pure liquid. Let us say, fuel like diesel or something like that and of course, there would only be single species called the fuel species.

For example, the diesel itself is mixed with water and the fuel itself is not a single specie, but a mixture of species. So, both the neighbouring phase and the considered phase are actually mixtures of several species. It depends on the problem at hand that how many species are involved in these two phases. Anything from one to several is possible. So, several species are involved. The proportion of species, k in the mixture must be defined. There are several ways to do it. One is called the mass fraction.

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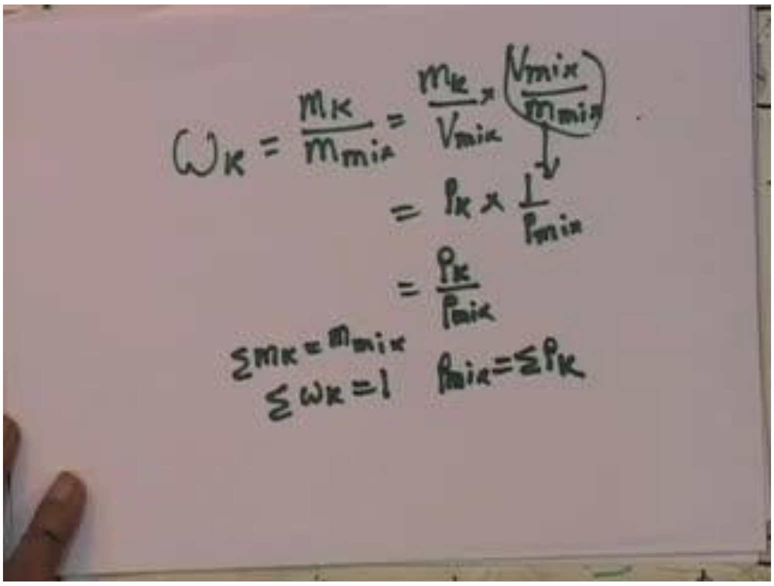

$$\omega_K = \frac{m_K}{m_{mix}} = \frac{m_K}{V_{mix}} \times \left(\frac{V_{mix}}{m_{mix}}\right)$$

$$= \rho_K \times \frac{1}{\rho_{mix}}$$

$$= \frac{\rho_K}{\rho_{mix}}$$

$$\Sigma m_K = m_{mix}$$

$$\Sigma \omega_K = 1 \quad \rho_{mix} = \Sigma \rho_K$$

For example, omega k is used as a symbol for mass of species k is divided by mass of the mixture. If I divide this by volume of the mixture and multiply by the volume of the mixture divided by mass of the mixture, then you will see that this quantity would readily transform to 1 over rho of the mixture. What about this quantity? Well, this quantity can be written as rho k. Why? Because, Dalton’s law says that in a mixture, each species behave as it had occupied the total volume of the mixture. Therefore, this would be rho k and therefore, the mass fraction omega k would be rho k divided by rho mixture. Since, sigma m k must be equal to m mix and therefore, sigma omega k must be equal to 1 and rho mix will be simply sigma rho k in a mixture. Now, this is very important relationships that we will be using to define the proportion of a species k in the mixture.

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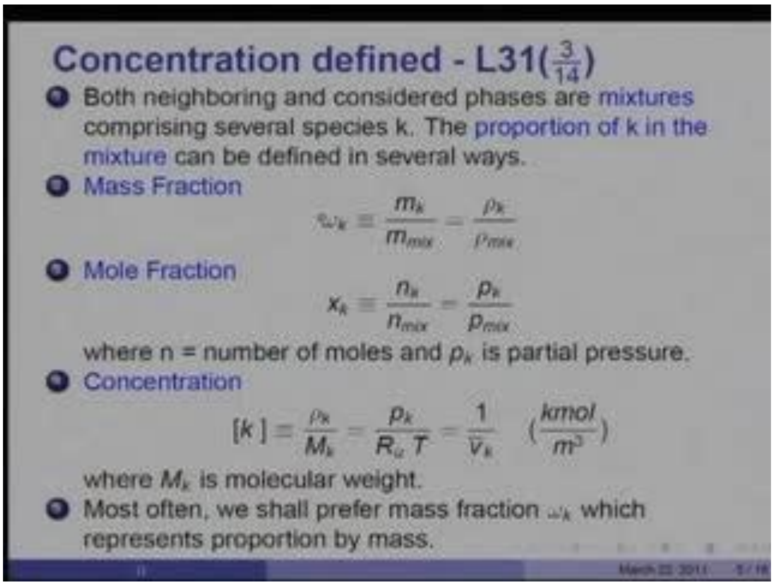

Concentration defined - L31($\frac{3}{14}$)

- Both neighboring and considered phases are mixtures comprising several species k. The proportion of k in the mixture can be defined in several ways.
- Mass Fraction

$$\omega_k \equiv \frac{m_k}{m_{mix}} = \frac{\rho_k}{\rho_{mix}}$$

- Mole Fraction

$$x_k \equiv \frac{n_k}{n_{mix}} = \frac{p_k}{p_{mix}}$$

where n = number of moles and p_k is partial pressure.

- Concentration

$$[k] = \frac{\rho_k}{M_k} = \frac{p_k}{R_u\,T} = \frac{1}{\bar{v}_k} \quad \left(\frac{kmol}{m^3}\right)$$

where M_k is molecular weight.

- Most often, we shall prefer mass fraction ω_k which represents proportion by mass.

Mass weighted proportions is defined by symbol omega suffix k rho k by rho mix and it uses Dalton’s law. Now, chemical engineers often use mole fractions and combustion engineers also use mole fractions to define species composition. It is defined as x k as number of moles of k divided by number of moles of the mixture. In gases, if you assume a validity of the perfect gas law, then this can also be shown equal to the partial pressure of species k divided by the partial pressure of the mixture. So, x k is simply the ratio of partial pressure of k divided by the total pressure of the mixture and n is the number of moles.

Chemists on the other hand, preferred to use a concentration. The symbol is square brackets and k is inside it. It is simply rho k divided by M k. They essentially innovate molar density. If we use perfect gas law, then you can see that can be written as p k by R u T, which is nothing but molar volume v bar k. Now, any quantity defined based on in molar units is given one over bar. So, this is kilo moles per meter cube and M k is the molecular weight of species k. Now, in all our calculation, we shall prefer to use omega k, which represents the proportion by mass.

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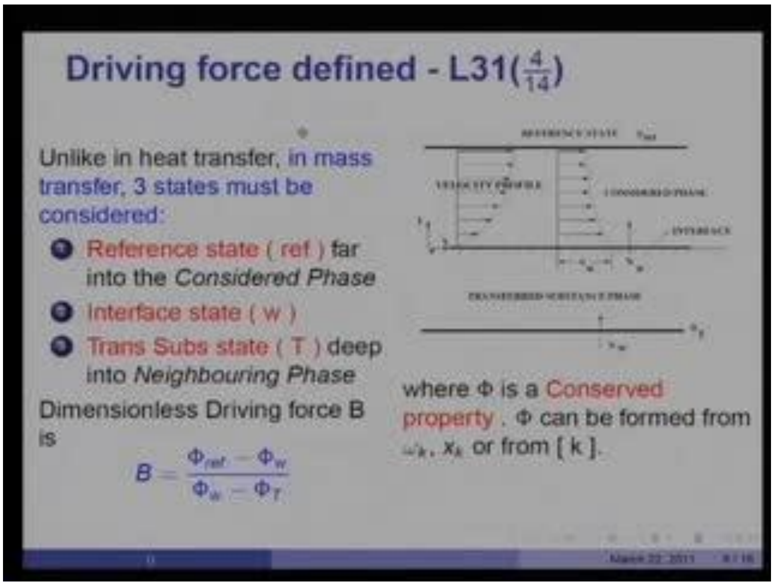

Driving force defined - L31($\frac{4}{14}$)

Unlike in heat transfer, in mass transfer, 3 states must be considered:

- Reference state (ref) far into the *Considered Phase*
- Interface state (w)
- Trans Subs state (T) deep into *Neighbouring Phase*

Dimensionless Driving force B is

$$B = \frac{\Phi_{ref} - \Phi_w}{\Phi_w - \Phi_T}$$

where Φ is a Conserved property . Φ can be formed from ω_k, x_k or from [k].

Let us define driving force. Now, unlike in heat transfer, we always refer to three states in mass transfer. The first, we will call the reference state into the considered phase, which is like the infinity state that we have already defined. We know, where the gradients of any quantity in the direction y are 0. Likewise, velocity gradients are 0 and any quantity temperature or omega k are all 0 in this infinity or reference state.

There is the interface state, which is defined with the symbol w. We must always consider the transfer substance state, which is deep inside the neighbouring phase. One of the principle assumptions of the transfer substance says very similar to the infinity state that there are no gradients of far of omega k or velocity or temperature or anything in this transfer substance state. So, all quantities are uniform in the transfer substance state and the mass transfer would take place. It can be shown to take place from here and it can be shown across the inter phase and into the considered phase.

This is the velocity profile. This is the profile of temperature or any other conserved property called phi, which I shall define now. The driving force is always defined based on this conserved property. It is the ratio of difference between phi in the reference state or the infinity state minus phi at the interface w divided by phi w minus phi T. We shall repeatedly encounter this dimensionless driving force.

Phi is called conserved property. Now, I will explain what conserved property is, in a short while. Can phi be formed from mass fraction of k? It can be formed from mole fraction of k. It can be formed from concentration of j, but as I said, we would always preferred to use mass fraction of k in all our development. So, note this definition of B, which we shall repeatedly encounter. This is a dimensionless driving force of the quantity of a property in the infinity state, in the w state and in the transfer substance phase.

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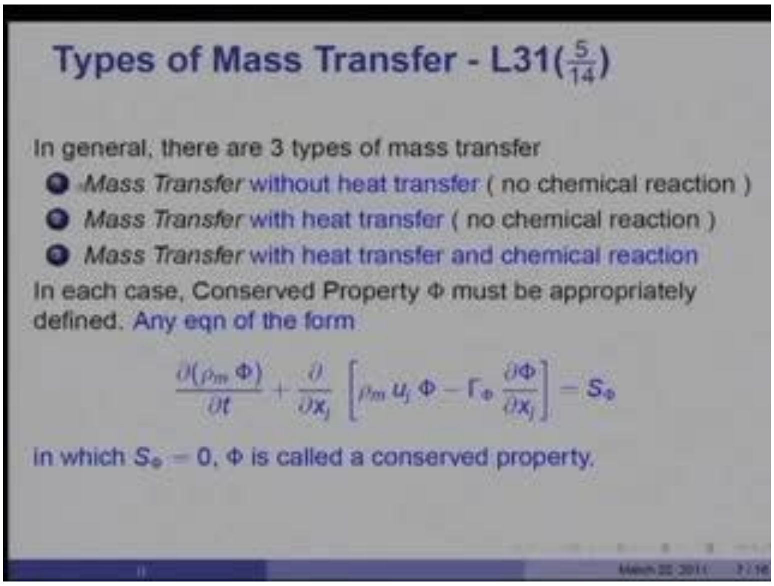

Types of Mass Transfer - L31($\frac{5}{14}$)

In general, there are 3 types of mass transfer

- *Mass Transfer* without heat transfer (no chemical reaction)
- *Mass Transfer* with heat transfer (no chemical reaction)
- *Mass Transfer* with heat transfer and chemical reaction

In each case, Conserved Property Φ must be appropriately defined. Any eqn of the form

$$\frac{\partial(\rho_m \Phi)}{\partial t} + \frac{\partial}{\partial x_j}\left[\rho_m u_j \Phi - \Gamma_\Phi \frac{\partial \Phi}{\partial x_j}\right] = S_\Phi$$

in which $S_\Phi = 0$, Φ is called a conserved property.

In general, there are three types of mass transfer problems. The first type is what I shall call inert mass transfer, without heat transfer and without chemical reaction. What I mean by that is that supposing, I have water at 30 degree centigrade placed in a beaker, which is also at 30 degree centigrade. Let us say the air around it is also at 30 degree centigrade. In other words, there is no temperature variation anywhere and the dry air is however relatively dry.

Let us say, it is 10 percent or 20 percent or 30 percent relative humid, whereas the humidity or the conditions at the surface of the water would saturated corresponding to these temperatures. So, there would be a concentration gradient in air, but there would be no temperature gradient from the transfer substance phase to the infinity phase to the considered phase and of course, there is no chemical reaction. Will the mass transfer takes place in the absence of any temperature variation? Of course, experience tells you that water will evaporate, although there are no temperature gradients. The principal reason is that there are concentration gradients, just like heat conduction. It is a spontaneous process, which arises as a result of temperature gradients. Mass transfer arises as a result of concentration gradient. In this case, the concentration of water vapor is in air. So, it is a spontaneous process and that is a situation, which we shall call as inert mass transfer without heat transfer and without chemical reaction.

Now, imagine that I have this water, which is at 25 degree centigrade. In the infinity state, I have the gas, which is at 50 degree centigrade. So, it is a hot gas and cold water and I do not know what the surface temperature of the water will be. Presumably, it will be between these two extremes. As you know, evaporation will takes place, but in this particular case, there will also be heat transfer from the gas considered phase to the transfer substance phase, which will bring about evaporation of water. In other words, the spontaneous evaporation in the absence of temperature gradient would now enhance its rate. It will be enhanced by the heat transfer, which is accompanying here.

Now, think of the other way. For example, I have hot water and let us say water is at 50 degree centigrade and the air is at 27 degree centigrade. Again, I do not know what the surface temperature will be. It will probably in between the two and we will find out the ways of determining that temperature in a minute. The main thing is - even if the water is hotter or the air is cooler, again evaporation would take place. So, positive mass transfer takes place, even when heat transfer is to the transfer substance or from the sub transfer substance to the considered phase. In other words, heat and mass transfer are simultaneously occurring, but they can be opposed or aiding each other. We consider this to be the situation of inert mass transfer with heat transfer, but no chemical reaction.

Third type of cause is the mass transfer, in which the liquid fuel is actually burning away. The manner in which liquid fuel burns is that it first evaporates, turns into a gaseous phase, where it undergoes chemical reaction and burning of the liquid fuel continuous. This is a situation of mass transfer with heat transfer and chemical reaction. So, I classified all mass transfer problems in three types. In each case, conserved property phi must be appropriately defined.

Now, what is the conserved property? Any transport equation of the form d rho m by phi by d t plus d by d x j that is a convective flux and a diffusive flux is equal to S phi. This is the well known form that we have seen so many times. Suppose, for a chosen phi, if it turns out that S phi is 0, then S phi is called the conserved property. In short, a conserved property is one, whose transport equation has no source term. It simply has a rate of change term; it has a convective term and a diffusion term. So, these are the only three terms that could be present in a conserved property equation. The task is to define appropriate conserved properties for each of the three types.

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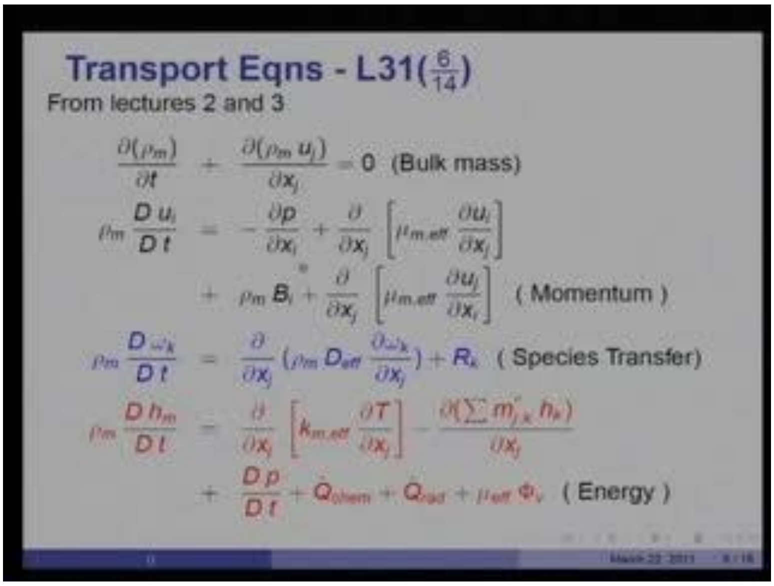

Transport Eqns - L31($\frac{6}{14}$)

From lectures 2 and 3

$$\frac{\partial(\rho_m)}{\partial t} + \frac{\partial(\rho_m u_j)}{\partial x_j} = 0 \quad \text{(Bulk mass)}$$

$$\rho_m \frac{D u_i}{D t} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu_{m,eff}\frac{\partial u_i}{\partial x_j}\right] + \rho_m B_i + \frac{\partial}{\partial x_j}\left[\mu_{m,eff}\frac{\partial u_j}{\partial x_i}\right] \quad \text{(Momentum)}$$

$$\rho_m \frac{D \omega_k}{D t} = \frac{\partial}{\partial x_j}\left(\rho_m D_{eff}\frac{\partial \omega_k}{\partial x_j}\right) + R_k \quad \text{(Species Transfer)}$$

$$\rho_m \frac{D h_m}{D t} = \frac{\partial}{\partial x_j}\left[k_{m,eff}\frac{\partial T}{\partial x_j}\right] - \frac{\partial(\sum m''_{j,k} h_k)}{\partial x_j} + \frac{D p}{D t} + \dot{Q}_{chem} + Q_{rad} + \mu_{eff}\Phi_v \quad \text{(Energy)}$$

We will simply recall this bulk mass equation. I have put suffix m in all this. The equations are written in the considered phase because in the transfer substance phase, there are no variations. So, all these equations are written only for the considered phase. This is the bulk mass with rho suffix m and suffix m implies mixture. This is the momentum equation; this is the species equation, so called mass transfer equation, which I had derived in the first few lectures.

This is the energy equation in which, this is the convective term, this is the conduction term and this is the convective mass transfer due to convective heat transfer due to diffusion mass transfer. This is the total derivative pressure; there would be chemical heat generation due to chemical reaction or radiation heat transfer or viscous dissipation. So, all these terms may vary and it depends on what type of problem that we are dealing with. We will be dealing the situation, in which (Refer Slide Time: 24:20) that is 0; that is 0 and that is 0. Sometimes, we may consider how to account for radiation heat transfer, but that we will see as we go along.

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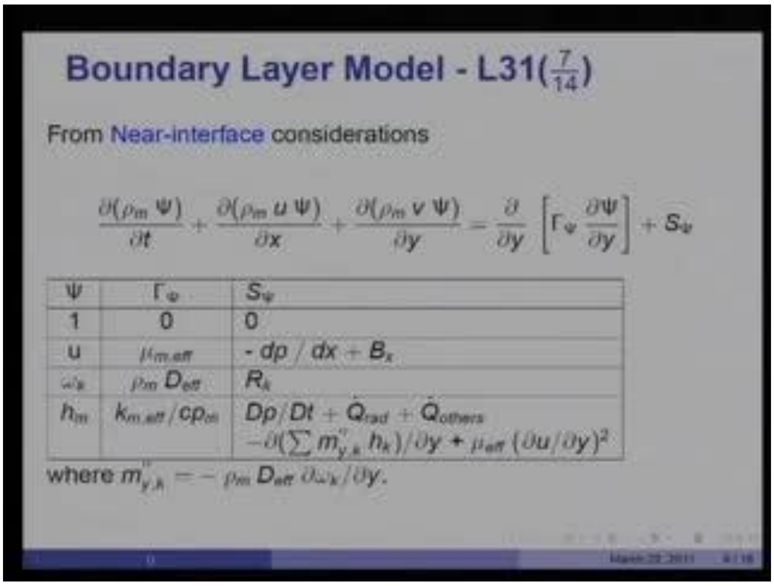

Boundary Layer Model - L31($\frac{7}{14}$)

From Near-interface considerations

$$\frac{\partial(\rho_m \Psi)}{\partial t} + \frac{\partial(\rho_m u \Psi)}{\partial x} + \frac{\partial(\rho_m v \Psi)}{\partial y} = \frac{\partial}{\partial y}\left[\Gamma_\Psi \frac{\partial \Psi}{\partial y}\right] + S_\Psi$$

Ψ	Γ_Ψ	S_Ψ
1	0	0
u	$\mu_{m,eff}$	$-dp/dx + B_x$
ω_k	$\rho_m D_{eff}$	R_k
h_m	$k_{m,eff}/cp_m$	$Dp/Dt + \dot{Q}_{rad} + \dot{Q}_{others}$ $-\partial(\sum m''_{y,k} h_k)/\partial y + \mu_{eff}(\partial u/\partial y)^2$

where $m''_{y,k} = -\rho_m D_{eff}\, \partial\omega_k/\partial y$.

The boundary layer flow model of forms of these equations (Refer Slide Time: 24:34) can be generalized in this way. You have d rho m psi by dt plus rho d rho m u by. So, these are the convective terms. This is the rate of change term; this is the diffusion term only in y direction because of the boundary layer model and S w, which is the source term. If I put psi equal to 1, it will simply mean that I am talking about bulk continuing bulk mass conservation. Here, psi is equal to u; it would mean momentum equation omega k would mean species equation with a rate of generation of species R k. Then, h m is the enthalpy of the mixture, which with a diffusion coefficient k m effective divided by cp m. All these are source terms that would arise for a boundary layer flow model.

Now, I have said here as effective and so on. This (Refer Slide Time: 25:36) is simply effective viscosity of the mixture. This is effective diffusivity of the mixture and this is the effective conductivity of the mixture. The diffusion rate m double dot y k essentially means diffusion in y direction of species k It is simply from Fick’s law and it is simply rho m D effective d omega k by d y. This is just by the way of remainder.

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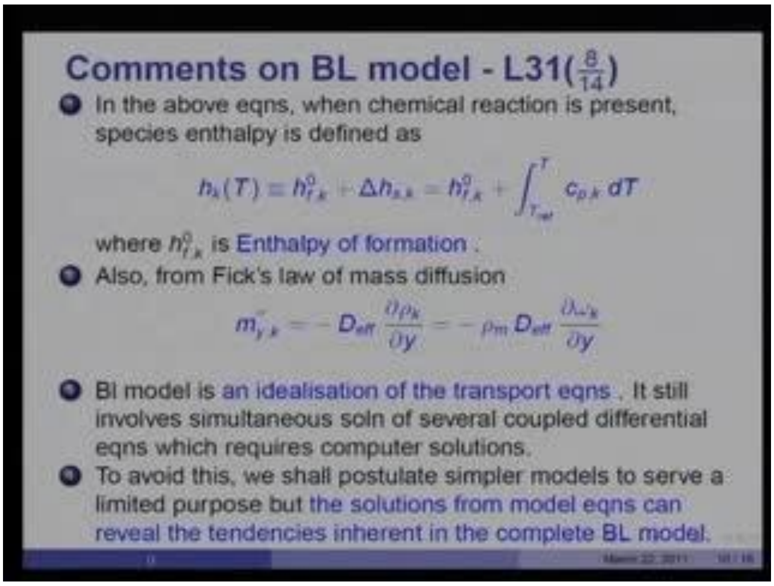

Comments on BL model - L31($\frac{8}{14}$)

- In the above eqns, when chemical reaction is present, species enthalpy is defined as

$$h_k(T) = h^0_{f,k} + \Delta h_{s,k} = h^0_{f,k} + \int_{T_{ref}}^{T} c_{p,k}\, dT$$

where $h^0_{f,k}$ is Enthalpy of formation .

- Also, from Fick's law of mass diffusion

$$m''_{y,k} = - D_{eff} \frac{\partial \rho_k}{\partial y} = - \rho_m D_{eff} \frac{\partial \omega_k}{\partial y}$$

- Bl model is an idealisation of the transport eqns . It still involves simultaneous soln of several coupled differential eqns which requires computer solutions.
- To avoid this, we shall postulate simpler models to serve a limited purpose but the solutions from model eqns can reveal the tendencies inherent in the complete BL model.

The general comments on the boundary layer model: the only thing that requires a little further elaboration is that the enthalpy h of the species k. You can see, this is where the species k is involved (Refer Slide Time: 26:25).So, this is the mixture enthalpy and this is the enthalpy of the species k. The enthalpy of the species k in a chemically reacting mixture is written as h naught f k plus delta h s k, where h naught f k is called the enthalpy of formation plus the sensible enthalpy to go from reference state to T state. Here, h naught f k is defined at the reference temperature, T ref to T. Sensible enthalpy is defined as integral T ref to T c p k dT.

Gaseous species often have c p k. It is a function of temperature, often fairly weak function, but nonetheless a function of temperature. Now, this h naught f k assumes different values for different species, when you have chemically reacting mixture. If it was an inert mixture, I could assign any value to h naught f k as a datum value. In a chemically reacting mixture, h naught f k is taken as 0 for naturally occurring substances like carbon, oxygen, nitrogen and so on. For all other compounds like CH 4 or CO 2 or CO, all of them would have a certain value for h naught f k from which, these are well tabulated in literature. So, there is no problem in getting the values of h naught f k. From Fick’s law of mass diffusion, as I said m double prime y k is simply D effective rho k by dy. Rho k can be written as rho m into omega k and that is what I have shown here.

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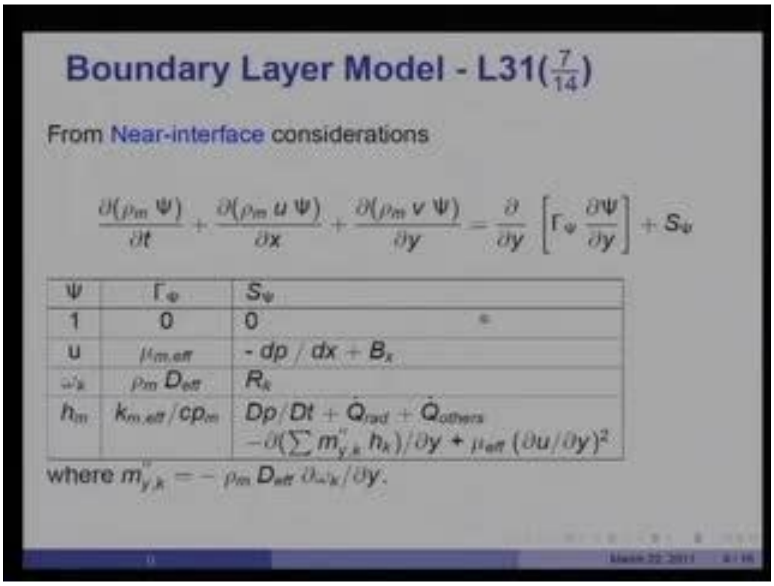

Boundary Layer Model - L31($\frac{7}{14}$)

From Near-interface considerations

$$\frac{\partial(\rho_m \Psi)}{\partial t} + \frac{\partial(\rho_m u \Psi)}{\partial x} + \frac{\partial(\rho_m v \Psi)}{\partial y} = \frac{\partial}{\partial y}\left[\Gamma_\Psi \frac{\partial \Psi}{\partial y}\right] + S_\Psi$$

Ψ	Γ_Ψ	S_Ψ
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u	$\mu_{m,eff}$	$-dp/dx + B_x$
ω_k	$\rho_m D_{eff}$	R_k
h_m	$k_{m,eff}/cp_m$	$Dp/Dt + \dot{Q}_{rad} + \dot{Q}_{others}$ $-\partial(\sum m''_{y,k} h_k)/\partial y + \mu_{eff}(\partial u/\partial y)^2$

where $m''_{y,k} = -\rho_m D_{eff}\, \partial\omega_k/\partial y$.

Now, boundary layer flow model itself is an idealization of the transport equation, but it still involves solution of simultaneous equations. Why? Remember, in heat transfer problems, you could often dissociate or decouple the momentum mass transfer problem from the heat transfer problem. We could do that by assuming constant properties. Even if I assume constant diffusivity, it is not possible to decouple u, omega k and energy equation. The main reason is as follows.

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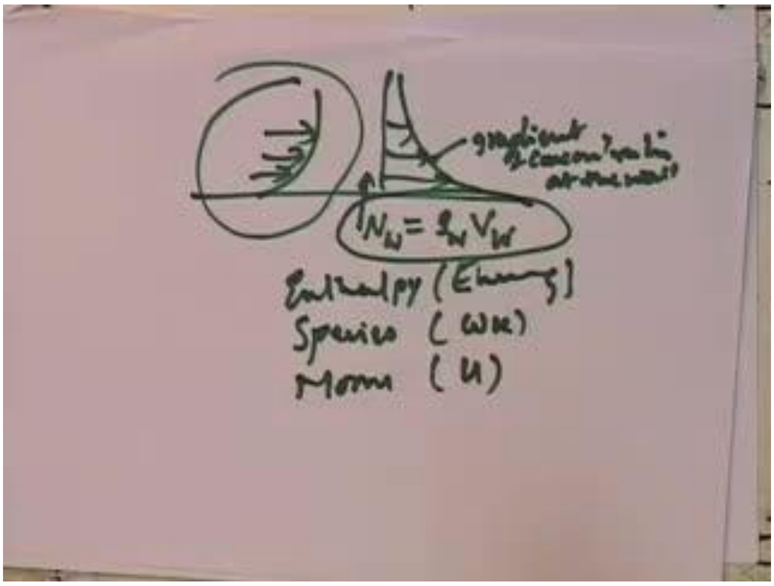

In a mass transfer problem, let us say N w influences the velocity profile, which is rho w into V w. Here, N w is governed by the gradients of concentration at the wall and that is what brings about mass transfer. Therefore, the velocity profile and the concentration profile are actually coupled to each other. So, you could not solve the momentum equation independently in the concentration problem.

If an energy equation was present and if heat transfer was also present, then you would have to solve the temperature equation or the enthalpy equation along with it. All the three equations: the enthalpy equation or the energy equation, the species equation that is for omega k and the momentum equation, u are always coupled through this boundary condition. In a way, a mass transfer problem is very similar to a natural convection problem, in which the momentum equation is coupled with the energy equation or the temperature equation.

The coupling arises due to the buoyancy term in the momentum equation, which you have done in your undergraduate work. The coupling arises through a source term in the momentum equation and not through the boundary conditions. In a mass transfer problem, the coupling is between momentum and the mass transfer equation. The energy equation arises through the boundary condition, which specifies N w. It is the mass flux as rho w times V w and V w is the boundary condition for the momentum equations.

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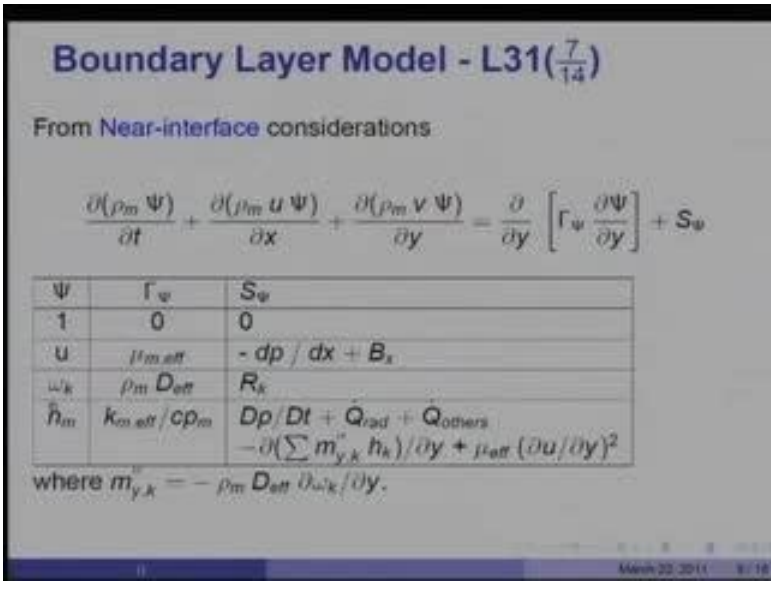

Boundary Layer Model - L31($\frac{7}{14}$)

From Near-interface considerations

$$\frac{\partial(\rho_m \Psi)}{\partial t} + \frac{\partial(\rho_m u \Psi)}{\partial x} + \frac{\partial(\rho_m v \Psi)}{\partial y} = \frac{\partial}{\partial y}\left[\Gamma_\Psi \frac{\partial \Psi}{\partial y}\right] + S_\Psi$$

Ψ	Γ_Ψ	S_Ψ
1	0	0
u	$\mu_{m\,eff}$	$- dp / dx + B_x$
ω_k	$\rho_m D_{eff}$	R_k
$\tilde{h}_m$	$k_{m\,eff} / cp_m$	$Dp/Dt + \dot{Q}_{rad} + \dot{Q}_{others}$ $-\partial(\sum m''_{y,k} h_k)/\partial y + \mu_{eff} (\partial u/\partial y)^2$

where $m''_{y,k} = - \rho_m D_{eff}\, \partial\omega_k/\partial y$.

Remember, we always have a mass transfer problem, which is a non-linear coupled problem. It requires simultaneous solution of all these and remember, there can be several species in a given problem. Evaporation of water is the simplest because it has only two species in a combustion problem, where a chemical reaction mechanism can involve as many as 40 to 50 species. What it means is that you need to solve so many species equation along with the momentum and the energy equation. So, the mass transfer problem can be very big. It depends on what kind of reaction mechanism that we have specified. In the absence of chemical reaction, life is simple and you would simply need to solve momentum and energy equations.

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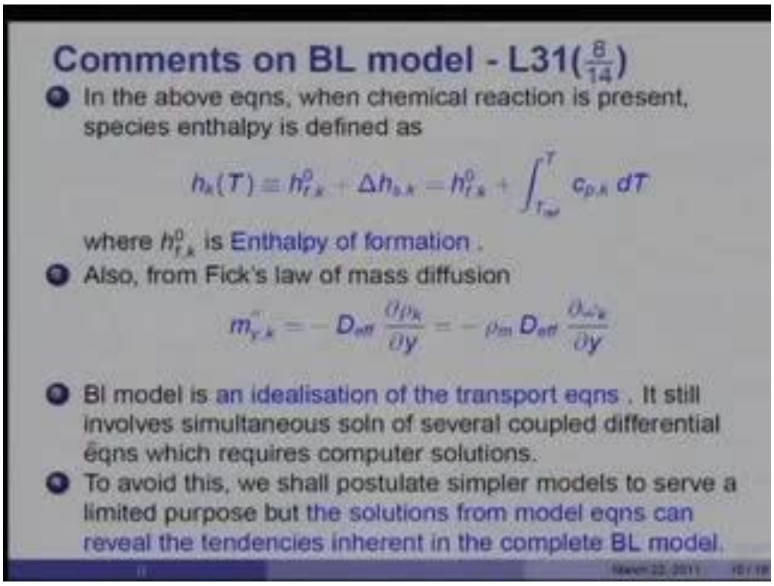

Comments on BL model - L31($\frac{8}{14}$)

- In the above eqns, when chemical reaction is present, species enthalpy is defined as

$$h_k(T) \equiv h^0_{f,k} + \Delta h_{s,k} = h^0_{f,k} + \int_{T_{ref}}^{T} c_{p,k}\, dT$$

where $h^0_{f,k}$ is Enthalpy of formation .

- Also, from Fick's law of mass diffusion

$$m''_{y,k} = -D_{eff}\frac{\partial \rho_k}{\partial y} = -\rho_m D_{eff}\frac{\partial \omega_k}{\partial y}$$

- Bl model is an idealisation of the transport eqns . It still involves simultaneous soln of several coupled differential eqns which requires computer solutions.
- To avoid this, we shall postulate simpler models to serve a limited purpose but the solutions from model eqns can reveal the tendencies inherent in the complete BL model.

The boundary layer flow model itself is an idealization of the full transport equation. It still involves simultaneous solution of several coupled differential equations. As I said, the coupling arises mainly through the boundary conditions. Therefore, you cannot separate the momentum fluid flow problem from the species equations, as we could in a pure heat transfer.

Essentially, I said we are going to describe three simple models. The main reason is that these models have essentially been developed to circumvent the need for solving the full mass transfer problem. The full mass transfer problem involves several differential transport equations. So, we will postulate simpler models to serve a limited purpose of essentially getting N w. The solution from the model equations can reveal tendencies inherent in the complete boundary layer model. So, without claim, everything that are consumable can be captured by the simpler model, but the major tendencies would certainly be captured by the simpler models. So that is our task, before I make foray into this simple model. Many times, the reactions takes place in such complex manner that it is simply not possible to deal with species k of the mixture.

You do not know what the concentrations of these species in different states of the neighbouring phase, the wall phase, the interface and the independent infinity state. So, it becomes very difficult to know the exact concentration or the exact mass fractions of all the species involved in these three states. In such a case, what one does? Invoke the element conservation. In a chemical reaction, all though the number of moles may not be conserved or may be conserved. Elements, which form these compounds must be conserved and you cannot destroy or generate elements.

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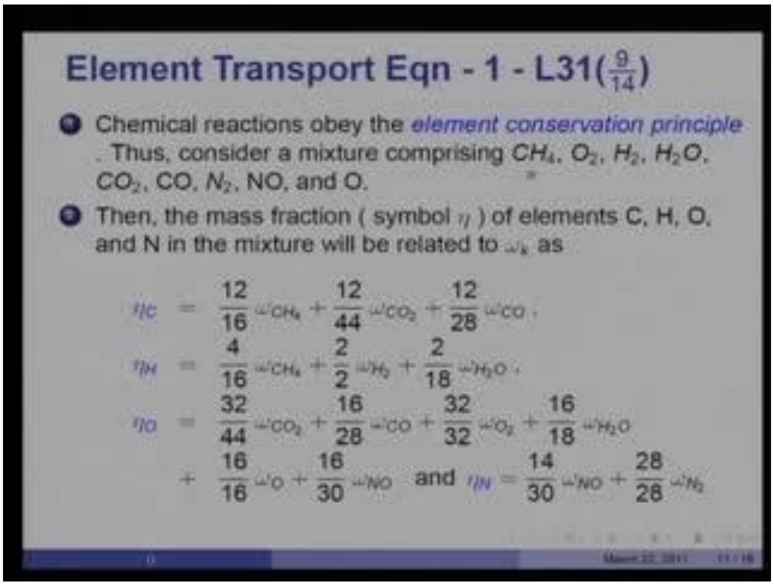

Element Transport Eqn - 1 - L31($\frac{9}{14}$)

- Chemical reactions obey the *element conservation principle* . Thus, consider a mixture comprising CH_4, O_2, H_2, H_2O, CO_2, CO, N_2, NO, and O.
- Then, the mass fraction (symbol η) of elements C, H, O, and N in the mixture will be related to ω_k as

$$\begin{aligned}\eta_C &= \frac{12}{16}\omega_{CH_4} + \frac{12}{44}\omega_{CO_2} + \frac{12}{28}\omega_{CO}\,,\\ \eta_H &= \frac{4}{16}\omega_{CH_4} + \frac{2}{2}\omega_{H_2} + \frac{2}{18}\omega_{H_2O}\,,\\ \eta_O &= \frac{32}{44}\omega_{CO_2} + \frac{16}{28}\omega_{CO} + \frac{32}{32}\omega_{O_2} + \frac{16}{18}\omega_{H_2O}\\ &+ \frac{16}{16}\omega_O + \frac{16}{30}\omega_{NO} \quad \text{and} \quad \eta_N = \frac{14}{30}\omega_{NO} + \frac{28}{28}\omega_{N_2}\end{aligned}$$

Consider a mixture comprising methane, oxygen, hydrogen, water vapor, carbon dioxide, carbon monoxide, nitrogen, nitric oxide and atomic O. Now, we define a element mass fraction symbol –eta. Unlike for the species, we use omega and for the element mass fraction, it is eta. What are the elements involved? C, H, O and N. So, we have four elements. These are the typical elements in any fuel. C, H, O, N systems are well known, element fraction is defined as eta. Sub C would be molecular weight of carbon divided by molecular weight of methane multiplied by mass fraction of methane plus molecular weight of carbon divided by molecular weight of CO2 omega CO2.

Basically, what it says is the element fraction in methane would be this much, element fraction of C in CO2 will be this much and element fraction in CO will be this much. Of course, other species do not have C and therefore, we need not worry about them. For example, hydrogen is involved in H2, H2O and in methane. Therefore, hydrogen element concentration or element fraction would be 4 by 16 into omega CH4 plus 2 by 2 into omega H2 and 2 by 18 into omega H20. O is found in oxygen, water vapor, carbon dioxide, carbon monoxide, nitric oxide and elemental O. This would be the case for O2, it is 32 by 44 into omega CO2.

In other words, element fractions can be formed from the species fraction. Finally, the eta N is the nitrogen element fraction. It would be 14 by 30 in nitric oxide and 28 by 28 in nitrogen itself. These element concentrations can be convected and diffused. They cannot be generated or destroyed, which means elements are always a conserved property. It is a great advantage because you do not need to know what is the rate at which they are generated or destroyed.

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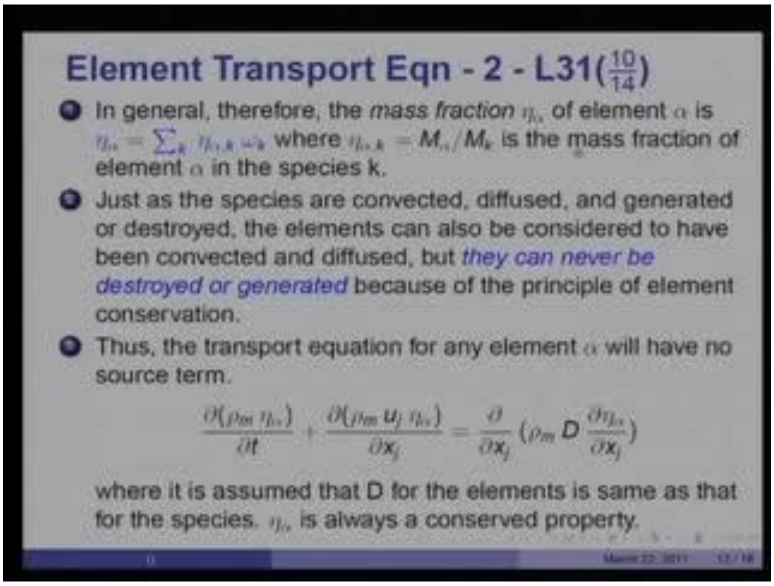

Element Transport Eqn - 2 - L31($\frac{10}{14}$)

- In general, therefore, the *mass fraction* η_{α} of element α is $\eta_{\alpha} = \sum_k \eta_{\alpha,k}\,\omega_k$ where $\eta_{\alpha,k} = M_{\alpha}/M_k$ is the mass fraction of element α in the species k.
- Just as the species are convected, diffused, and generated or destroyed, the elements can also be considered to have been convected and diffused, but *they can never be destroyed or generated* because of the principle of element conservation.
- Thus, the transport equation for any element α will have no source term.

$$\frac{\partial(\rho_m \eta_{\alpha})}{\partial t} + \frac{\partial(\rho_m u_j \eta_{\alpha})}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\rho_m D \frac{\partial \eta_{\alpha}}{\partial x_j}\right)$$

where it is assumed that D for the elements is same as that for the species. η_{α} is always a conserved property.

In general, the mass fraction is simply generalized and what I said on the previous slide is eta. Alpha is the mass fraction of the element and alpha is in species k. So, n alpha k omega k and n alpha k is simply the molecular weight of alpha. The element is divided by molecular weight of the species. Now, just as species are convected, diffused generated and destroyed, elements can also be considered to have been convected and diffused, but they can never be generated or destroyed As I said earlier, it is due to the principle of element of conservation. Therefore, the transport equation for element alpha will have no source term. It will simply have a rate of change term, a convective term and a diffusion term. It is assumed that diffusivity for the elements is same for the species n alpha, for the species k. Here, n alpha is always a conserved property.

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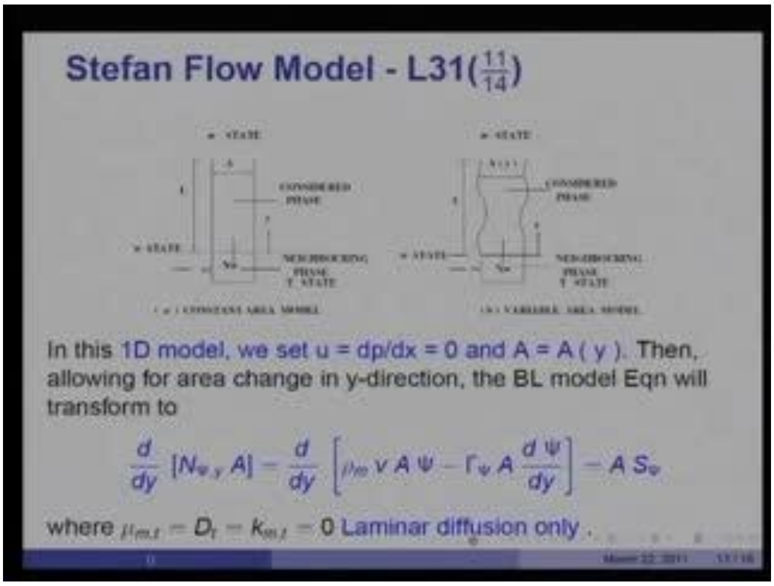

Stefan Flow Model - L31($\frac{11}{14}$)

In this 1D model, we set u = dp/dx = 0 and A = A (y). Then, allowing for area change in y-direction, the BL model Eqn will transform to

$$\frac{d}{dy}\left[N_{\Psi,y}\,A\right] = \frac{d}{dy}\left[\rho_m\, v\, A\, \Psi - \Gamma_\Psi\, A\, \frac{d\Psi}{dy}\right] = A\, S_\Psi$$

where $\mu_{m,t} = D_t = k_{m,t} = 0$ Laminar diffusion only .

Now, let me turn to these three simple models. The first one among these is the Stefan flow model. It is essentially a boundary layer flow model, in which we say u is 0 and we also say dp dx is 0. Now, we have included the area change and essentially, it would be a one-dimensional model.

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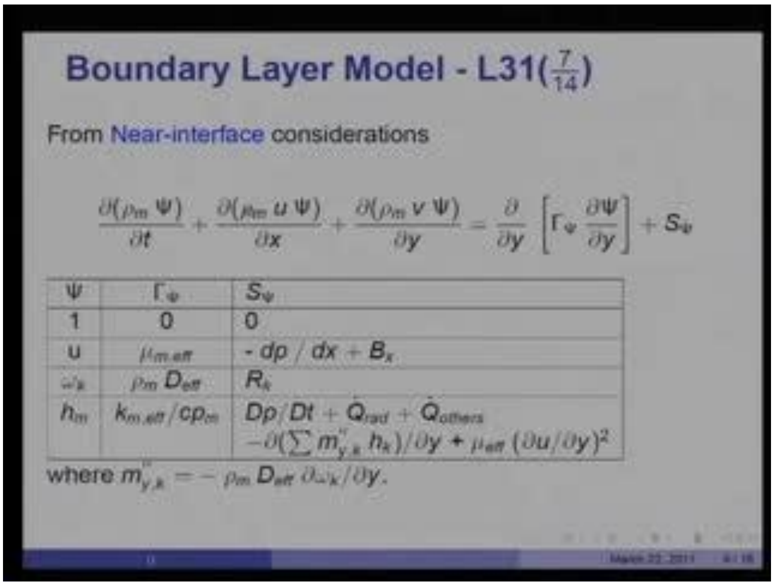

Boundary Layer Model - L31($\frac{7}{14}$)

From Near-interface considerations

$$\frac{\partial(\rho_m \Psi)}{\partial t} + \frac{\partial(\rho_m u \Psi)}{\partial x} + \frac{\partial(\rho_m v \Psi)}{\partial y} = \frac{\partial}{\partial y}\left[\Gamma_\Psi \frac{\partial \Psi}{\partial y}\right] + S_\Psi$$

Ψ	Γ_Ψ	S_Ψ
1	0	0
u	$\mu_{m,eff}$	$- dp / dx + B_x$
ω_k	$\rho_m D_{eff}$	R_k
h_m	$k_{m,eff} / cp_m$	$Dp/Dt + \dot{Q}_{rad} + \dot{Q}_{others}$ $-\partial(\sum m''_{y,k} h_k)/\partial y + \mu_{eff} (\partial u/\partial y)^2$

where $m''_{y,k} = - \rho_m D_{eff} \partial\omega_k/\partial y$.

As you can see from this equation, if I say d by dx is zero and u is 0 and therefore, no question of that sum being (.). I also have said that the pressure gradient is 0. There is no body force either. So, all these are simply 0 and rho m into v would be the mass transfer flux and that would be essentially be one-dimensional model.

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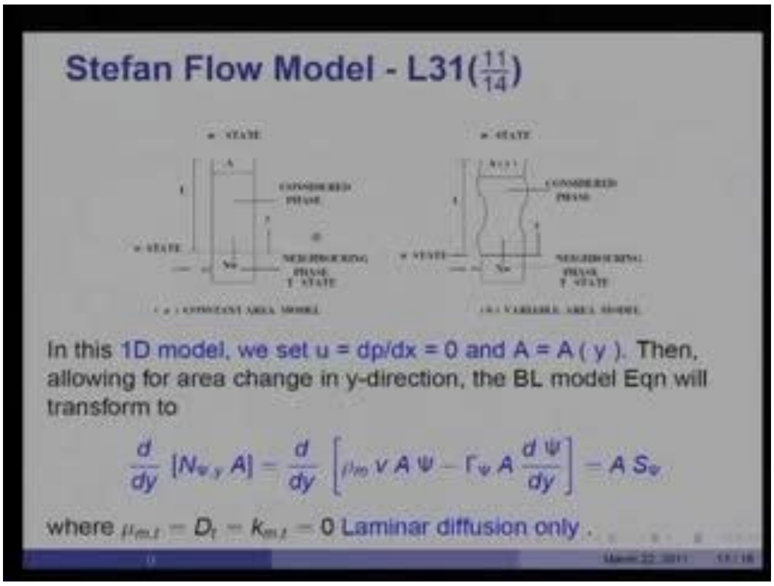

Stefan Flow Model - L31($\frac{11}{14}$)

In this 1D model, we set u = dp/dx = 0 and A = A (y). Then, allowing for area change in y-direction, the BL model Eqn will transform to

$$\frac{d}{dy}\left[N_{\Psi,y} A\right] = \frac{d}{dy}\left[\rho_m v A \Psi - \Gamma_\Psi A \frac{d\Psi}{dy}\right] = A S_\Psi$$

where $\mu_{m,t} = D_t = k_{m,t} = 0$ Laminar diffusion only .

It will simply say d by dy of N psi y into A. N psi y is the mass flux at any position y multiplied by area A. I am allowing for area change and here, it would be equal to d by dy of rho m v A psi minus the diffusion term. It would be equal to A times the source term.

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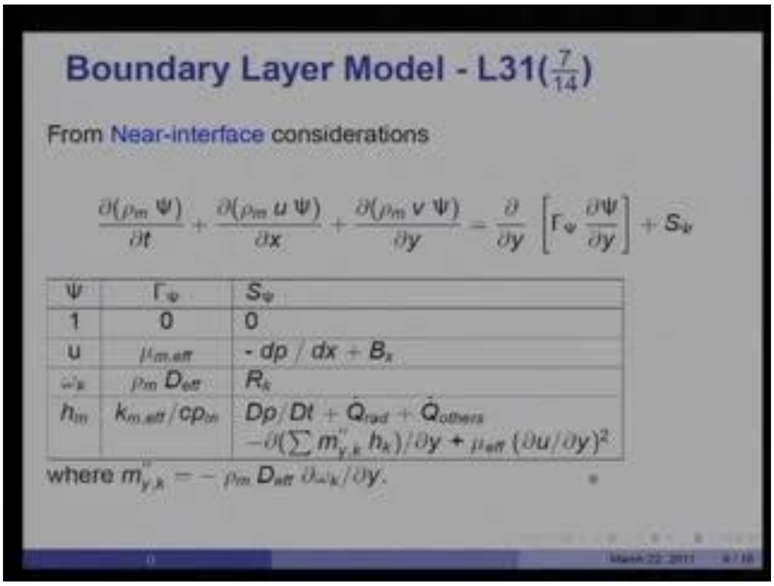

Boundary Layer Model - L31($\frac{7}{14}$)

From Near-interface considerations

$$\frac{\partial(\rho_m \Psi)}{\partial t} + \frac{\partial(\rho_m u \Psi)}{\partial x} + \frac{\partial(\rho_m v \Psi)}{\partial y} = \frac{\partial}{\partial y}\left[\Gamma_\Psi \frac{\partial \Psi}{\partial y}\right] + S_\Psi$$

Ψ	Γ_Ψ	S_Ψ
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u	$\mu_{m,eff}$	$-dp/dx + B_x$
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h_m	$k_{m,eff}/cp_m$	$Dp/Dt + \dot{Q}_{rad} + \dot{Q}_{others}$ $-\partial(\sum m''_{y,k} h_k)/\partial y + \mu_{eff} (\partial u/\partial y)^2$

where $m''_{y,k} = -\rho_m D_{eff} \partial \omega_k / \partial y$.

If psi is a conserved property, S psi would be 0, otherwise it would take the meaning that is associated here with psi. At the moment, I am using psi as a variable property, then that could be 0.

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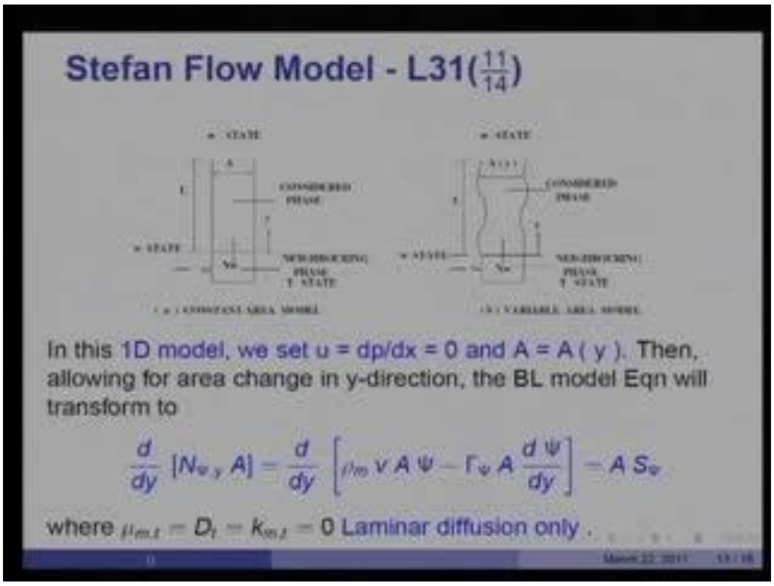

Stefan Flow Model - L31($\frac{11}{14}$)

In this 1D model, we set u = dp/dx = 0 and A = A (y). Then, allowing for area change in y-direction, the BL model Eqn will transform to

$$\frac{d}{dy}\left[N_{\Psi,y} A\right] = \frac{d}{dy}\left[\rho_m v A \Psi - \Gamma_\Psi A \frac{d\Psi}{dy}\right] = A S_\Psi$$

where $\mu_{m,t} = D_t = k_{m,t} = 0$ Laminar diffusion only .

Now, in the Stefan flow model, you do not need to solve the bulk conservation equation or the momentum equation because u itself is 0. Therefore, we simply saw that I have to solve psi. It can only be omega k or element alpha or it can be the enthalpy h m. It would have an appropriate source term.

Of course, u is 0 and it is a stagnant medium, where there can be no turbulent contribution to effective exchange coefficients. Essentially, it would always be a laminar diffusion through stagnant surroundings. Now, notice the nature of the model. I have said that this is the interface and let us say this is water; this is the stagnant air over it. In the infinity state, I may have air of any relative humidity less than 100 percent, so that the mass transfer will takes place.

As the mass transfer takes place, the level of the water would go on decreasing or the length L would go on changing. What is implicit in this model? It is somehow water is injected in the transfer substance state - T state, at the rate equal to the evaporation rate. So that L remains constant and therefore, you see no transient term written in this equation. So, Stefan flow model is for a steady state mass transfer, in which implicit is the assumption in the transfer substance state. The mass transferred amount is made up by supplying the transfer substance.

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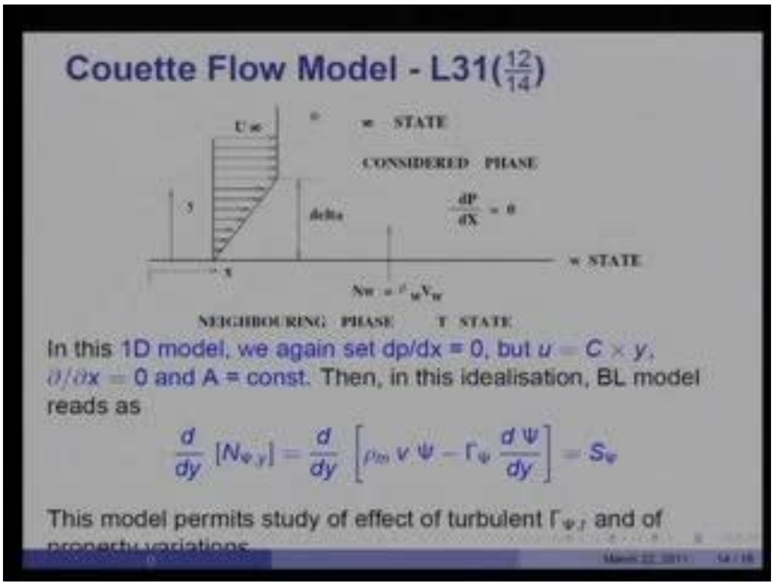

Couette Flow Model - L31($\frac{12}{14}$)

In this 1D model, we again set dp/dx = 0, but $u = C \times y$, $\partial/\partial x = 0$ and A = const. Then, in this idealisation, BL model reads as

$$\frac{d}{dy}\left[N_{\Psi,y}\right] = \frac{d}{dy}\left[\rho_m\, v\, \Psi - \Gamma_{\Psi}\frac{d\Psi}{dy}\right] = S_{\Psi}$$

This model permits study of effect of turbulent $\Gamma_{\Psi,t}$ and of property variations

Couette flow model goes closer to the boundary layer flow model.

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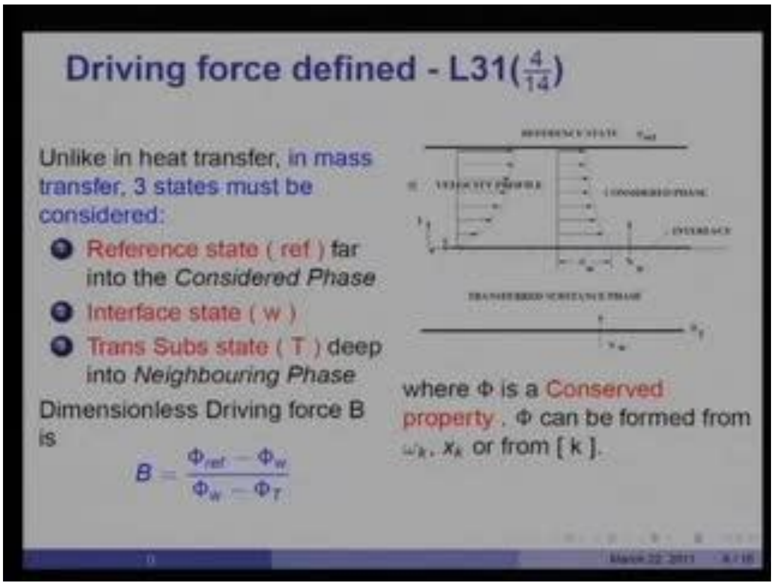

Driving force defined - L31($\frac{4}{14}$)

Unlike in heat transfer, in mass transfer, 3 states must be considered:

- Reference state (ref) far into the *Considered Phase*
- Interface state (w)
- Trans Subs state (T) deep into *Neighbouring Phase*

Dimensionless Driving force B is

$$B = \frac{\Phi_{ref} - \Phi_w}{\Phi_w - \Phi_T}$$

where Φ is a Conserved property . Φ can be formed from ω_k, x_k or from [k].

Instead of elaborate velocity profile, which is like this (Refer Slide Time: 42:52). It is actually influenced by the mass transfer rate and this shape is taken to be the simplest linear.

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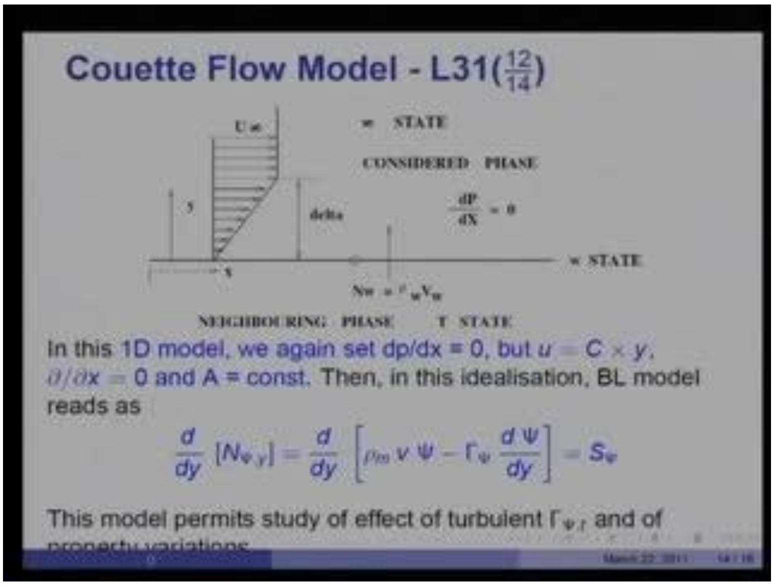

Couette Flow Model - L31($\frac{12}{14}$)

In this 1D model, we again set dp/dx = 0, but $u = C \times y$, $\partial/\partial x = 0$ and A = const. Then, in this idealisation, BL model reads as

$$\frac{d}{dy}\left[N_{\Psi,y}\right] = \frac{d}{dy}\left[\rho_m\, v\, \Psi - \Gamma_\Psi \frac{d\Psi}{dy}\right] = S_\Psi$$

This model permits study of effect of turbulent $\Gamma_{\Psi,t}$ and of property variations

In the Couette flow model, as a rule we say u is going to be a constant multiplied by y at all positions x. The pressure gradient will be 0 and in fact, all other gradients in x direction will be 0. Therefore, the equation would look remarkably similar to the previous equation, but psi can be equal to u because the flow has been allowed. Now, u is equal to C times y. Unlike, u equal to 0 in the Stefan flow model, you have u equal to C y. We said dP dX shall be 0, so there is no body force term or anything like that in the considered phase.

Now, merit of this model is -it deals with the flow. I can allow the variation of omega psi with respect to y. It can occur because variation of gamma psi, which is a diffusion coefficient or an exchange coefficient. It can be a function of temperature, it can be function of mass fraction in a turbulent flow and it can be a function of the position as indicated by the Prandtl's mixing length. For example, the exchange coefficient, turbulent viscosity goes on varying with y. So, the merit of the Couette flow model is that assuming linear profile, you can actually study the effects of property variations.

In fact, that is the limited purpose of this couette flow model. It is not to predict the mass transfer coefficient g or to predict mass transfer rate. It has to really examine the question of what would happen, if the properties were to vary from the wall state to infinity state. It turns out that the model shows the nature of variations that are captured by this model. They are actually found in the full boundary layer flow model or very close to it. That is very simple algebra. You can show what the effects of property variations are and that is where the merit of the Couette flow model lies.

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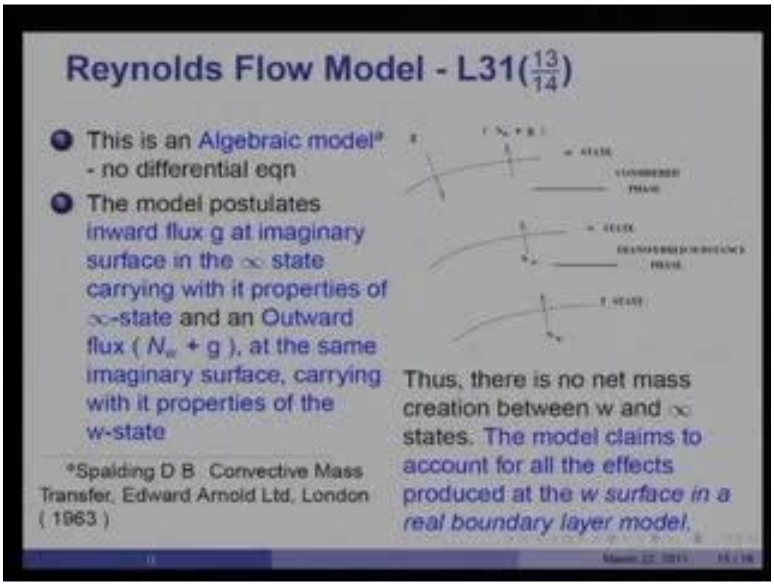

Reynolds Flow Model - L31($\frac{13}{14}$)

- This is an Algebraic model* - no differential eqn
- The model postulates inward flux g at imaginary surface in the ∞ state carrying with it properties of ∞-state and an Outward flux (N_w + g), at the same imaginary surface, carrying with it properties of the w-state

*Spalding D B Convective Mass Transfer, Edward Arnold Ltd, London (1963)

Thus, there is no net mass creation between w and ∞ states. The model claims to account for all the effects produced at the *w surface in a real boundary layer model.*

Finally, I come to the algebraic Reynolds flow model. Spalding D B postulated it in a book called Convective Mass Transfer published by Edward Arnold in London in 1963. This is an algebraic model and it is very simple. Therefore, there are no differential equations to be solved. What one says is that there is a considered, there is the interface there is the considered phase, there is the infinity state and there is the T state. So, T state has the same meaning as before. This is the interface and what the model postulates is that there is a fictitious inward g flux in the infinity state, which will bring the properties of the infinity state.

The model simply postulates a fictitious flux with a symbol g, which flows towards the interface from the infinity state and carries the properties of the infinity state. On the other hand, from the infinity state, there is N w plus g flux. It flows away from the w state, but it carries the values of the w state. N w plus g flows away from the interface and g flows towards the interface. Therefore, there is no net generation or depletion of mass, as a result of postulation of these two fluxes. Now, at first sight, this might appear to be highly arbitrary. The model claims to account for all effects produced at the w surface in a real boundary layer flow and that is the claim in the lectures to follow. We shall see how that claim is justified.

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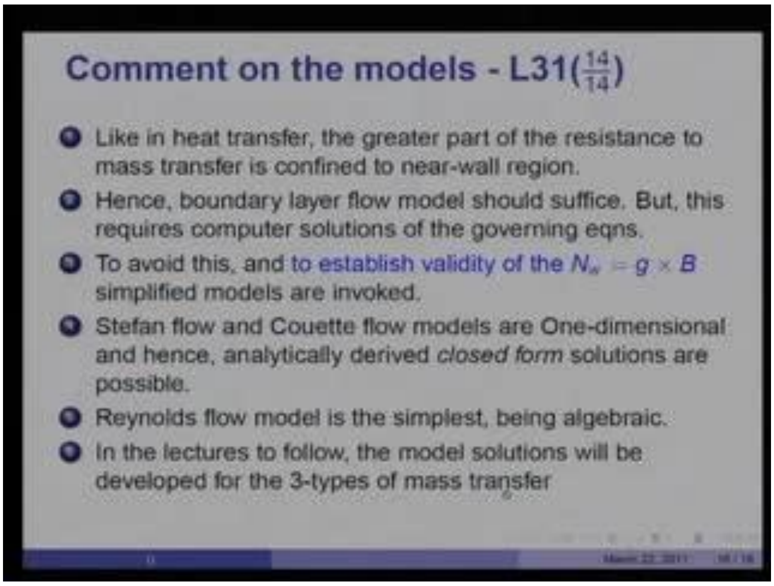

Comment on the models - L31($\frac{14}{14}$)

- Like in heat transfer, the greater part of the resistance to mass transfer is confined to near-wall region.
- Hence, boundary layer flow model should suffice. But, this requires computer solutions of the governing eqns.
- To avoid this, and to establish validity of the $N_w = g \times B$ simplified models are invoked.
- Stefan flow and Couette flow models are One-dimensional and hence, analytically derived *closed form* solutions are possible.
- Reynolds flow model is the simplest, being algebraic.
- In the lectures to follow, the model solutions will be developed for the 3-types of mass transfer

In summary, in heat transfer, the greater part of the resistance to mass transfer is confined to near-wall region. Therefore, boundary layer flow model should suffice, but this requires computer solutions to governing equations. In fact, a very large number of governing equations are coupled. To avoid this and to establish the validity of N w equal to g times B, simplified models are invoked. The Stefan and Couette flow models are one- dimensional. Hence, analytically, deriving closed form solutions becomes possible.

On the other hand, Reynolds flow model is the simplest, being algebraic. I will develop the mathematics of this model. A very simple mathematic that will convince you why all these models captured the real flow effect. It is N w equal to g that is the mass transfer coefficient multiplied by the dimensional driving force B.