Convection Heat and Mass Transfer

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Module No. # 01
Lecture No. # 28
Turbulence Model – III

In the previous lecture, we looked at effects of low turbulence Reynolds numbers. Now, this low turbulence Reynolds number can occur close to the wall; that is one type of situation, but it can also occur along the length of a flat plate where the boundary layer is going and where the laminar flow turns turbulent through an intermediate region, which we normally define as the transitional region. In that transitional region, again the turbulence Reynolds number would tend to be low. Therefore, the models that we just discuss are very much applicable to such situations.

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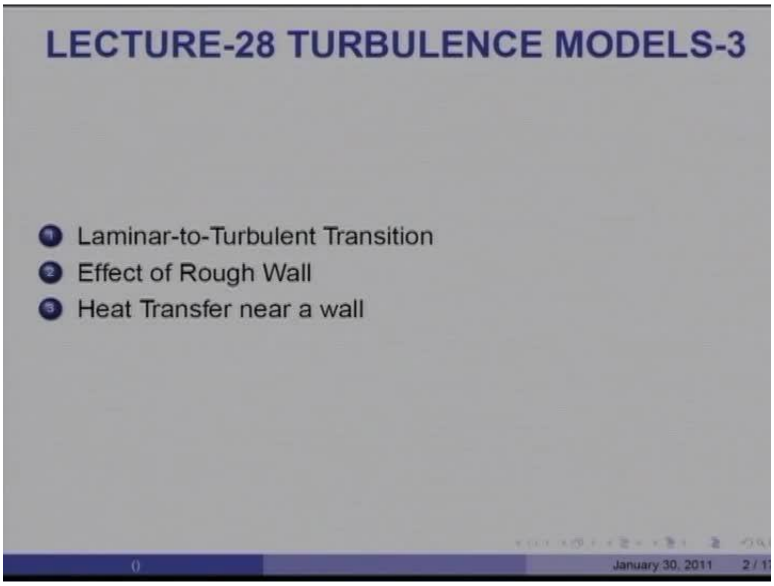

As I said, these turbulence models with lower turbulence Reynolds number require fairly

Heavy computation expense because of the very large number of grid nodes required closed to a wall. Therefore, at earlier times simpler approaches to modeling transitional regime where adopted and my purpose in this lecture is to really look at laminar to turbulent transition, in little more detail.

The second issue concerns rough walls. So, much of our discussion till now has been about smooth walls, but, in engineering, in order to enhance heat transfer, we often employ rough walls; deliberately structured or naturally rough walls. We want to see how to capture the effect of a rough wall either in a wall function or ultimately in generating the universal law of the wall for the inner layer of a turbulent boundary layer on a rough wall.

Finally, we will look at like we will derive the universal velocity law for the inner layer; is there a temperature law for the inner layer? What is that? These are the three topics we will take up in this lecture.

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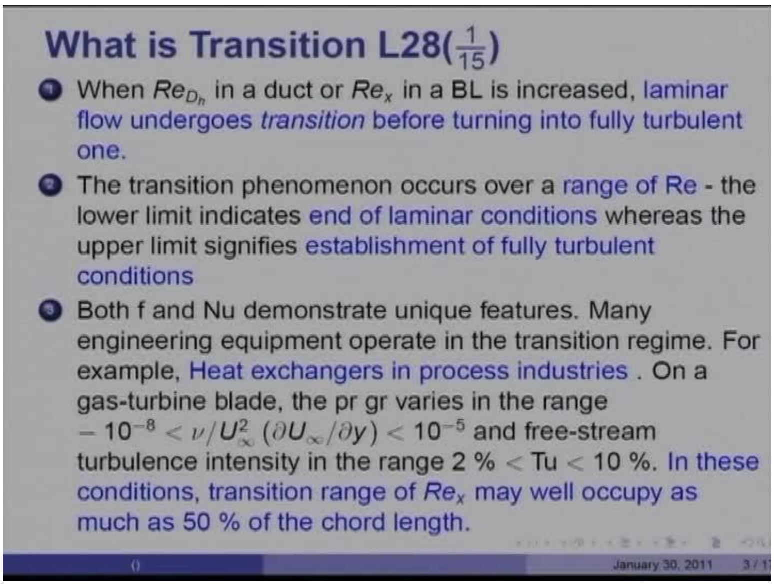

So, for example, in a duct when Reynolds number based on hydraulic diameter or in an external boundary layer when the Reynolds number based on actual distinct x is increase laminar flow undergoes transition before turning into fully turbulent wall.

This transitional Reynolds phenomenon actually occurs over a range of Reynolds number. Although in the under graduate work, we often believe that laminar to turbulent transition is very abrupt, but, actually the transition occurs over a range of Reynolds numbers. Although the range is very small compared to the normal values of fully turbulent Reynolds numbers that we normally encounter, but nonetheless, the range exits; we want to know how to account for that range.

The lower end of the range of Reynolds number indicates end of laminar conditions; whereas, the upper limit of the range signifies establishment of fully turbulent conditions and both f and Nu demonstrate unique features in the transitional layer.

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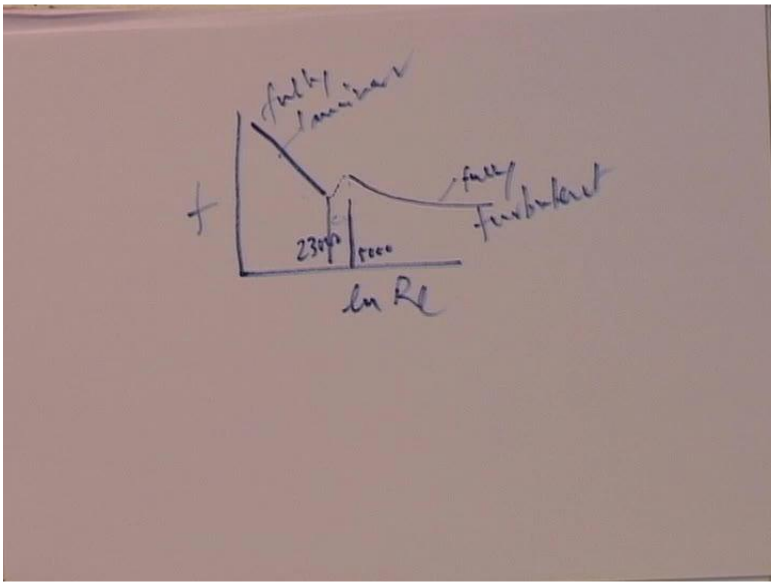

For example, you will recall that the friction factor in a pipe decreases linearly up to Reynolds number of let say about 2300. Then the friction factor actually increases a little, to say till about Reynolds number of 5000. Then, this is fully turbulent; this is fully laminar. In the transitional region here (Refer Slide Time: 04:40), the friction factor Reynolds number relation is quite reverse of what happens in the laminar and turbulent regions in that friction factor actually increases with Reynolds number; therefore, this curious phenomenon needs to be explained in somewhat (( )).

Now, many heat exchange equipments operate in the transitional region; particularly in process industries, this is often found that the Reynolds number inside a tube is often in the transitional region. Similarly, on a gas turbine blade, the pressure gradient varies in the range as low as minus 10 to the minus 8 of this parameter - rho by U infinity square du infinity by dy, to as high as 10 raise to minus 5; both of these are considering both pressure side and the suction side, and the free stream turbulence intensities vary from as low as 2 percent to 10 percent.

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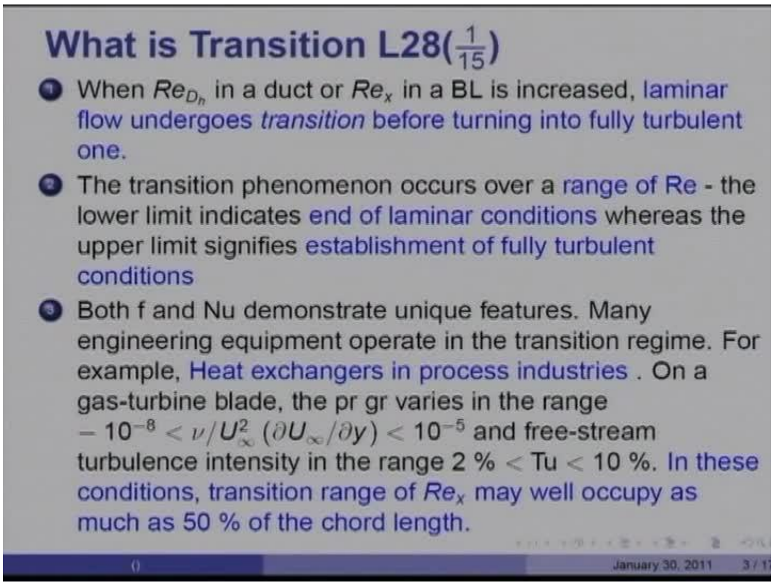

Now, in these conditions, transitional range of Re x may well occupy as much as 50 percent of the chord of the blade.

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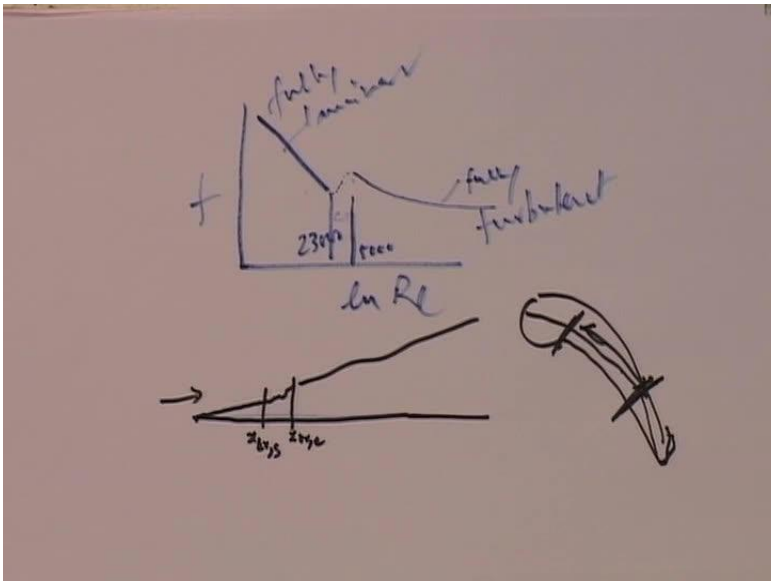

What is meant here is, for example, in a flat plate initially you will get laminar flow and then a small transitional region with x, then x transition n, and then you will get a turbulent boundary. So, this occupies a much smaller length of the total length of a boundary layer, but on a turbine blade, this is the chord, this is the chord length and you could get transitional Reynolds number as in transitional range occupying almost 50 percent of the chord lengths.

So, that is why we need to study turbulence in a transitional region more carefully.

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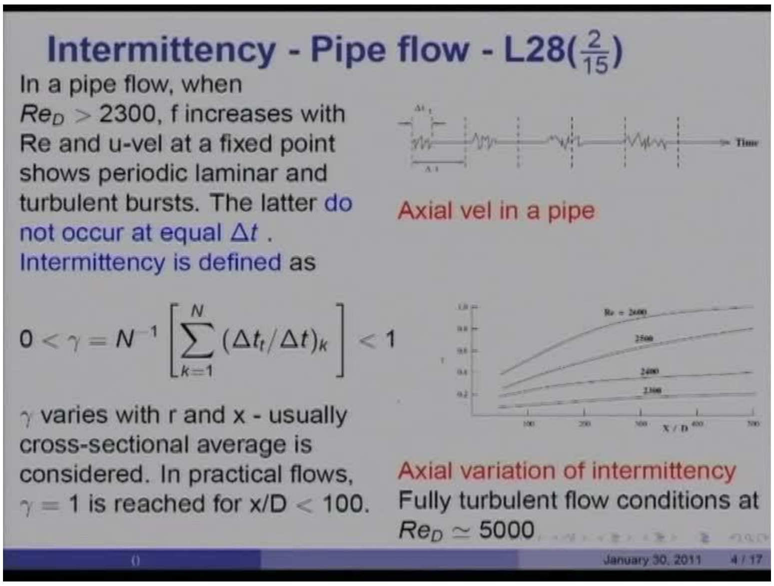

So, let us first of all get our ideas of transition in a pipe flow clearly. So, in a pipe flow for Reynolds number 2300, f increases with Reynolds number and if you were to measure u - velocity at a fixed point in the flow at any radius of the pipe, then you will see a picture, something like this; a flow will show turbulence for a while at a point; then for a certain period it will be quiet or laminar like; then it will be turbulent again; then it will be very quiet again; then turbulent, and then, so on and so forth.

If we measure this for a certain total time and divide this time period equally in steps of delta t, then in each delta t, a small fraction of it - delta t sub t, would be occupied by turbulent patch. Likewise, here this much would be the delta t whereas, this is the total is delta t (Refer Slide Time: 07: 57).

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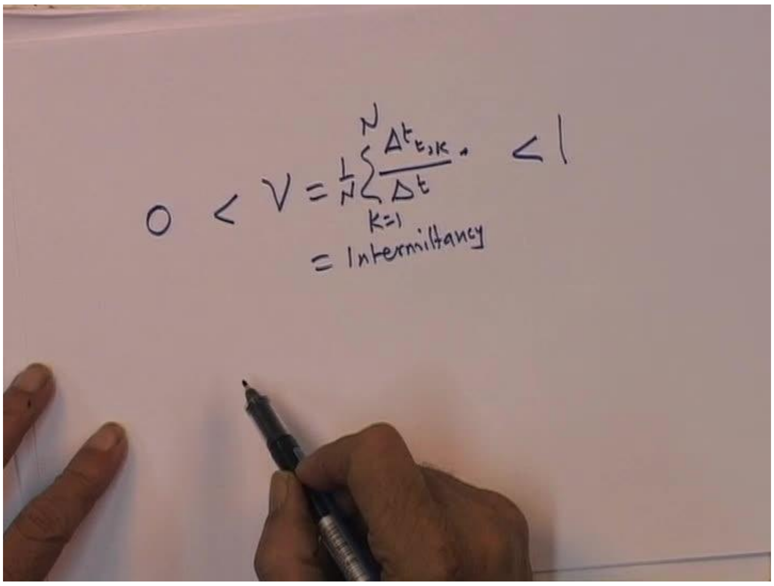

Now, we define intermittency as intermittency itself is defined as, gamma is defined as delta t of turbulent patch divided by the total time step delta t. This would be the kth patch, let us say. Then, if we sum up all this over k equal to 1 to N time steps and divide this by N, then that is called the intermittency; this is the intermittency definition. It tells you about the fraction of the time for which the turbulent patches occur. Now, obviously, it would only vary between 0 and 1, and that is what I have shown here (Refer Slide Time: 08:54) that, intermittency gamma would vary from 0 to 1.

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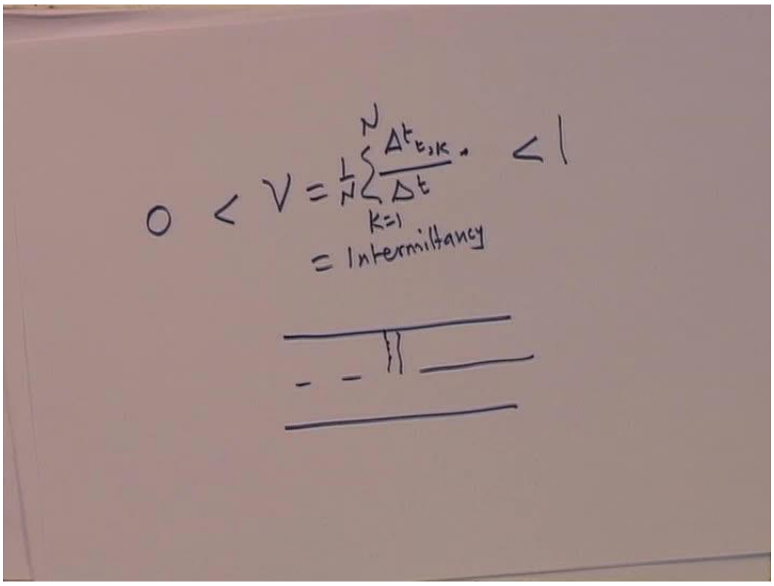

Now, of course, it would. The value of the gamma would vary in a pipe flow from axis to the wall at different points, but, usually, we speak of some kind of an average gamma. So, we do not worry about the radial variation too much.

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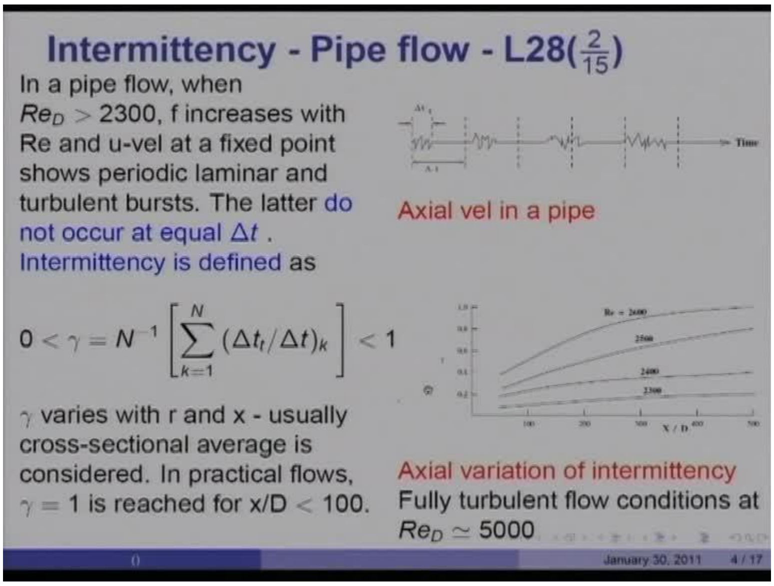

Here, I show the value of average gamma versus x by D of the pipe flow at different axial distances. This is the measurement made at Reynolds number of 2300 and you will see that the intermittency does not exceed 0.2, even after going well beyond 500.

So, in this case, the flow will never turn turbulent at all in practical links of pipes which would be usually 100 d or 200 d at best. But the flow will become fully turbulent at very long distance from the entrance. At 2400 at x by D of 500 it has reached about 0.4, but at 2500, you will see it has reached about 0.8, and at 2600 it has reached almost 1 at x by D of 500. Now, if you had a Reynolds number of 5000, then of course, it will reach the fully turbulent, gamma equal to 1 in a very short distance. It flows such that we say it is turbulent from the inlet itself.

So, remember, between 2300 and 5000, the transitional flow will continue for all practical lengths of the pipes. But for 5000 and above, the flow will be turbulent from the inlet itself in a pipe flow.

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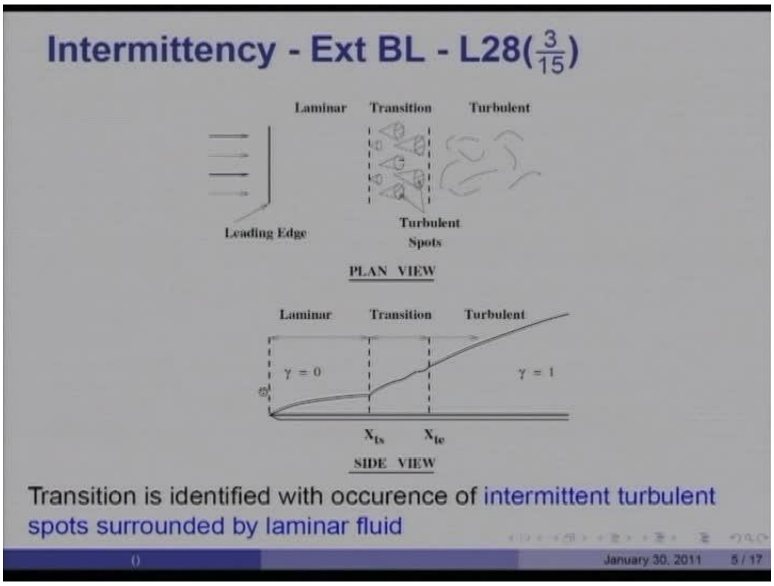

Similarly, if you were to look at the external boundary layer, you will see if there is a flow coming from the left. Then, there would be a laminar layer here with gamma equal to 0, but, then from here to here (Refer Slide Time: 11:12), which is the star of transition to end of transition, intermittency will go on increasing with distance. How will it increase? We will see shortly. Then fully turbulent conditions will be reached here at gamma equal to 1. Pictorially, if I have to look at the plan of this, it will look like this. If the flow is coming in from here, you will get very steady flow up to start of transition.

Then occasional spot sub turbulence will be seen and they tend to grow, engulfed by laminar fluid which kills the spots, but it is this energy transfer which gives rise to new spots of turbulence and then they would grow; but, they are again engulfed by laminar, surrounded by laminar fluid, and therefore, they get killed. But the process of energy transfer creates more spots and so on so forth. Ultimately, they grow not only in number, but also in strength; ultimately, at the end of transition, you will get a completely turbulent boundary layer or the turbulent fluid.

So, the transition is identified with the occurrence of intermittent turbulent spots surrounded by laminar fluid in an external; this is quite visible; you can do a flow visualization experiment to see this.

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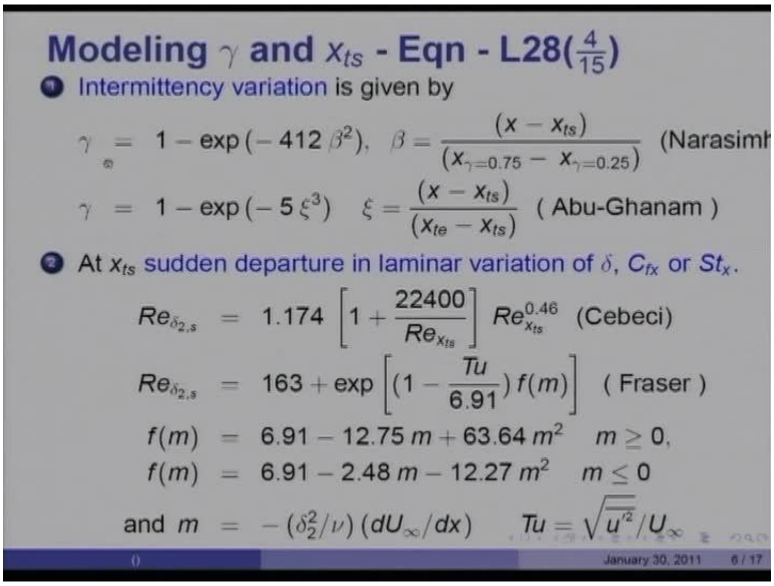

Now, the variation of intermittency with x in a boundary layer has been correlated. One of the first correlations was wrote by Narasimha, an Indian scientist working at Indian Institute of Science and it was 1 minus exponential of minus 400 and 12 beta square where, beta itself is a dimensionless quantity; X minus start of transition divided by X where the value of intermittency is 0.75 minus X where the value of intermittency is 0.25. Now, this is of course, very well to use in an experimental set up because you can see where the intermittency is 0.75 and 0.25 and therefore, determine the denominator quite exactly.

Computationally this is not very convenient and therefore, more recently in the 80s Abu-Ghannam and shah have come up with this expression gamma equal to 1 minus exponential of minus 5 times xi cubed where xi is X minus X ts divided by end up transition minus start up transition. So, all that remains is finding a waste to determine x ts and x te, and there are several suggestions.

So for example, for determining start of transition where there is a sudden departure in laminar variations of delta C fx and Stanton X, one can Cebeci is one author; Fraser is another author. So, Cebeci has suggested that at the start of transition the Reynolds number based on momentum thickness delta 2 is balanced by this expression where, the Reynolds number x is based on startup transition.

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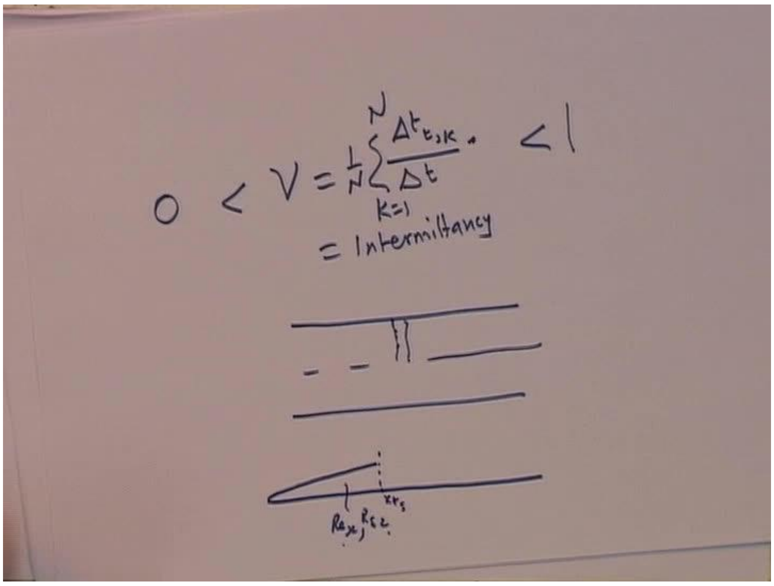

So, what one does is, if you were doing let us say applying integral method, then during the laminar flow, you will calculate Re x and Re delta 2 and see whether that expression that I showed you is actually satisfied.

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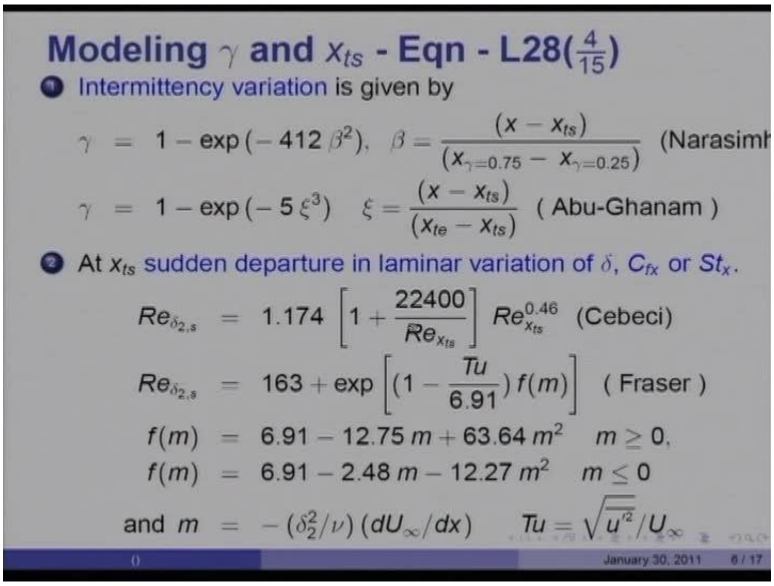

You will see at a certain point X ts, the Re x and Re delta 2 would be related by this expression and that would identify the start of transition. On the other hand, this expression, of course, does not take into account the effects of turbulence intensity which can vary quite a bit, particularly in gas turbine and compressive applications.

Therefore, Fraser has suggested that this formula Re delta 2 s is related to this f m and turbulence intensity Tu, and m is the pressure gradient parameter and includes du infinity by dx. So, when this equation you go on calculating Re delta 2 s and when it matches with this for m greater than 0 and m less than 0, then you say that is the start of transition m equal to 0. Of course, make it f m equal to 6.91; if turbulence intensity is not accounted and taken as 0 then this is simply exponential of 6.91 plus 163. That would be where the Re delta 2. You take the value of X ts where Re delta 2 is 163 into exponential of 6.91 for 0 pressure gradient boundary layer and turbulence intensity equal to 0. For all other cases, the X ts must respond to the pressure gradients.

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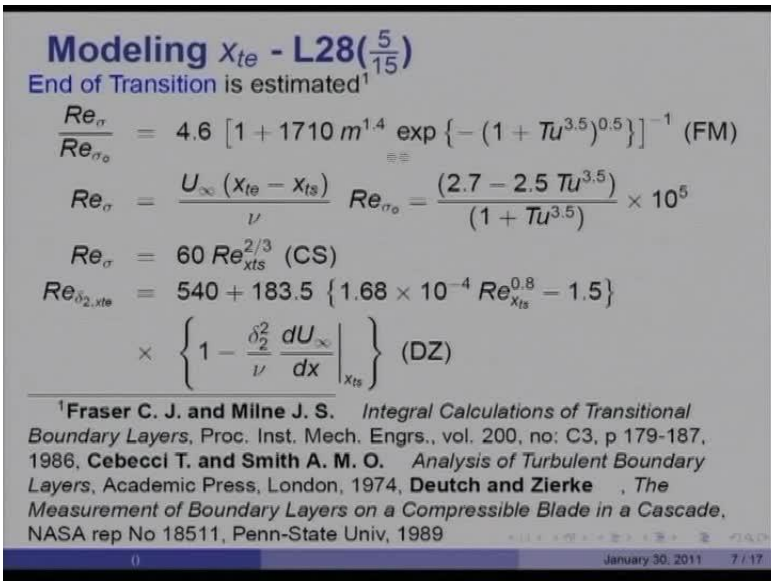

Now, how about end of transition for end of transition you would say you define Re sigma as U infinity X te minus X ts divided by nu.

So, it is based on the transitional length if you like Re sigma Re sigma naught is a function only of the turbulence intensity 4.6 into 1 plus 1710 m raise to point 1.4 into all this (Refer Slide Time: 17:20). Of course this term is valid only when m is greater than 0, which means that term a, is negative and that would occur in an adverse pressure gradient. So, we can locate the end of transition in the adverse pressure gradient as would be expected so this is the recommendation of Fraser and Milne and the paper was published in 1986 in Institution of Mechanical Engineers Proceedings.

There is another recommendation and that is by Cebecci who say that Re sigma which is based on transitional length should be when it equals 60 times Re X ts, which was identified on the previous slide (Refer Slide Time: 18:06) Re X ts raise to 2 by 3, then that would signify the end of transition. Then there is yet another one which is by Deutch and Zierke which again does not take into account turbulence intensity, but they recommend that Re delta 2 at end of transition should satisfy this expression where, now instead of m, like m which was based on delta 2 square dU infinity by dx, here also you see this really m 1 plus m at start of transition. So, that is the most important part this is at the value of start of transition.

So, there are three expressions particularly in adverse pressure gradients; all of these work very well.

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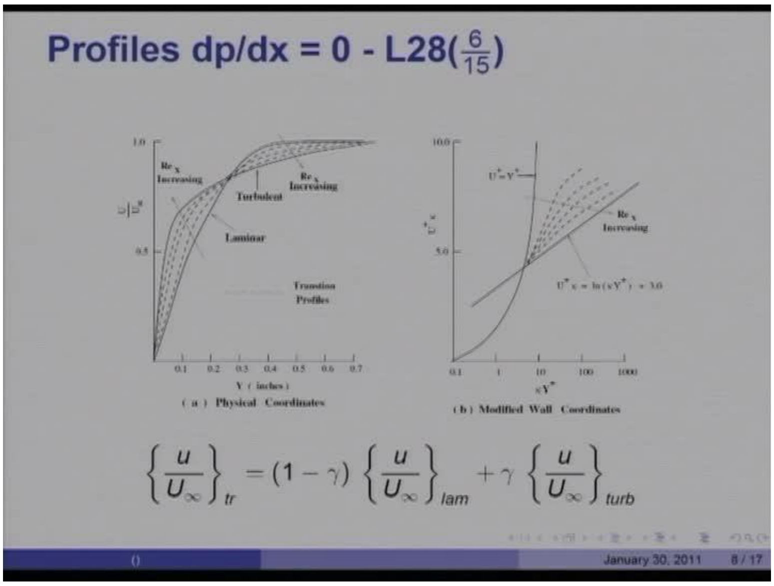

Now, in the integral method, if you wanted to use the integral method for solution of the integral equation through the transitional layer, you would need u over U infinity in the transitional layer.

Now, simply by observing how the experimental data looks like, this is u over U infinity divided by y given in inches here it would go from this would be the laminar profile, but in the transitional region, the profile would go something like that as the Re x increases, and here Re x increases in this passion (Refer Slide Time: 19:43). Finally, you would end up with a fully turbulent profile like that in u plus y plus coordinates. It would appear something like this and it is quite customary, therefore, to say that u over U infinity at transition would be 1 minus gamma times u over U infinity laminar plus gamma times u over U infinity turbulence.

So, having identified the values of X ts and X te, you can determine the gamma distribution with respect to X. Therefore, you can use this formula. This is pure pragmatism there is no big theory, but this kind of pragmatism seems to work and actually predicts a transitional boundary layer using integral method.

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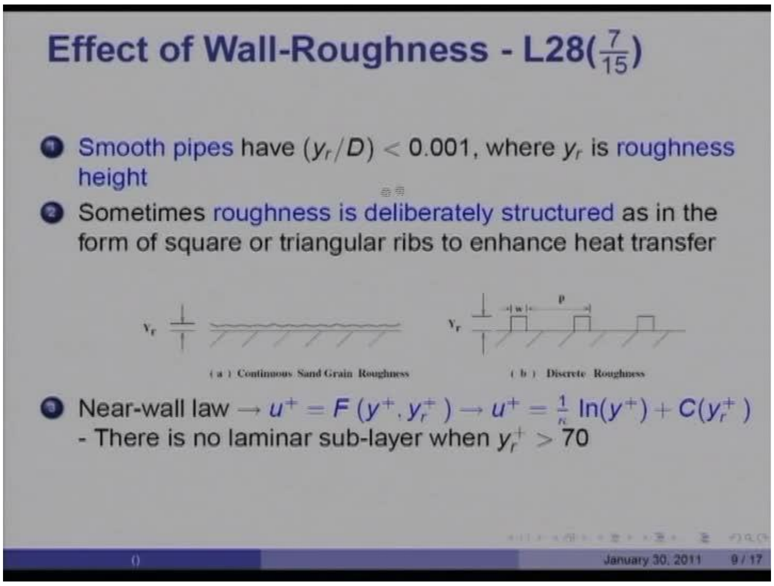

So, now, we turn to how to account for effect of wall roughness. Now, effect of wall roughness, as you will recall even from your under graduate days, you will recall that when the roughness height divided by the diameter is 0.001, then we say that the tube is smooth and you will get this kind of profile for smooth pipe turbulent region.

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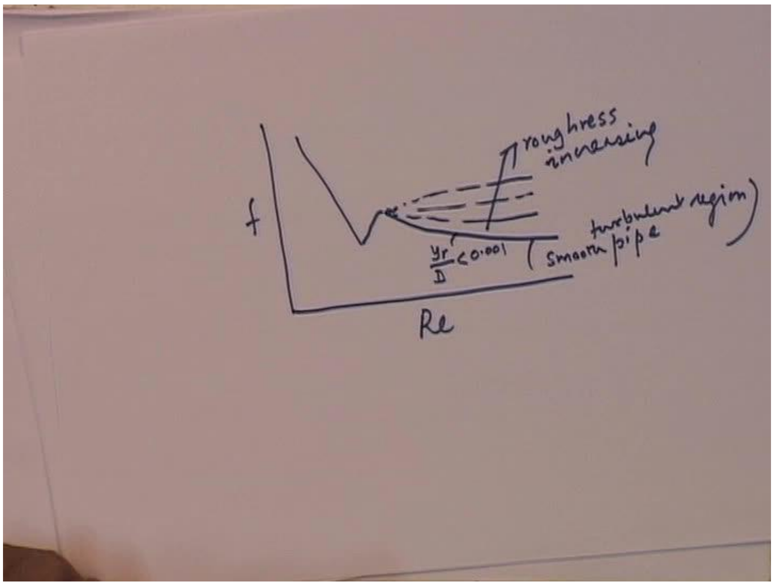

This would have the height roughness y r of D less than 0.001. But if you increase y r by D, then you will remember that you get friction factor which goes on like that so much, so that you may even get that, and this is where the roughness is increasing. So, the effect of roughness is to enhance the friction factor or really the pressure drop and this is what is observed in experimental data, that you will recall from your under graduate days.

We want to explain, what are the other features of rough surface. So, for example, a cement pipe is quite rough quite naturally, but many times as shown here. So, it will have a jagged surface, and experimentally such a surface is produced by pasting sand grains on the of various sizes on a surface of equivalent or average height y sub r. But many times, we actually have a rough surface which is deliberately structured by providing grids, for example like this, or studs - 3 dimensional studs can be provided of width W and pitch P, and it becomes quite difficult to a develop any universal law for a surfaces like this.

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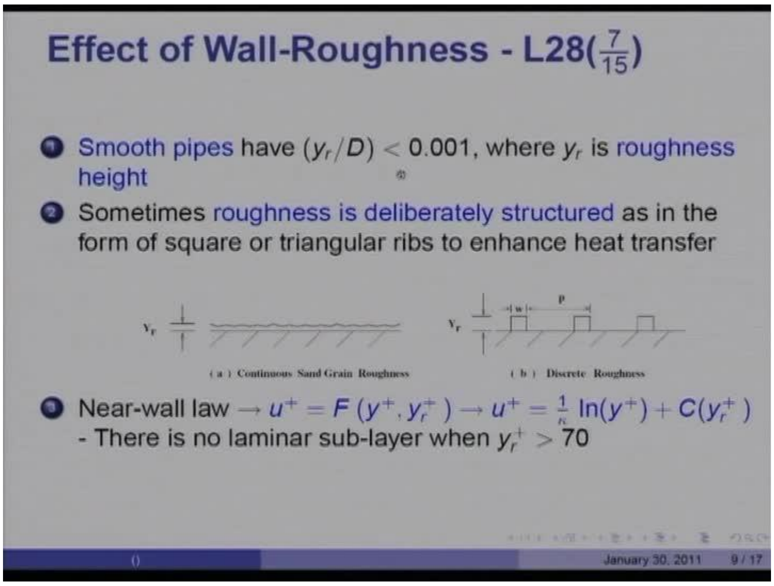

So, in order to circumvent this difficulty, what is done is, we always take sand grain roughness as a bench mark and determine the friction coefficient for that, the one for a structured surface. If the friction coefficient agrees with that of the sand grain roughness, then we say that the structured surface add an equivalent roughness height equal to the sand grain roughness height. So, sand grain roughness are done in a laboratory measurements and they provide the bench mark.

What you would expect near wall law to be u plus now to be not only function of distance from the wall y plus, but also the y r plus - the roughness side. Since we do not expect much of a laminar contribution, we will essentially have a turbulent layer developing. Therefore, we expect that u plus would be 1 over kappa ln y plus plus C, which would be function of the roughness height y r plus.

Usually, laminar sub-layer does not exists when y r plus is greater than 70 and we say such a surface is fully rough surface; but if y r plus is less than 70, then you can still get effects of a viscosity present and we have to account for those. So, that is what we have done in the next slide.

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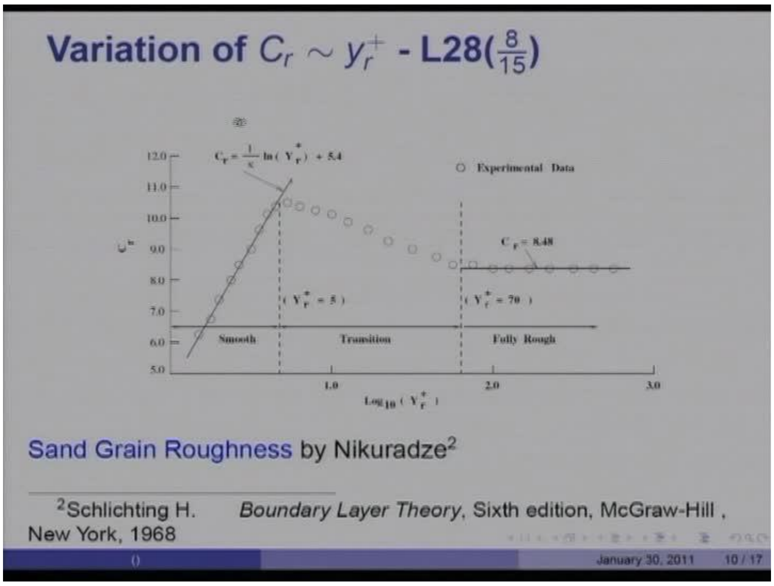

Here is a clock of C r which is a function of y r plus and the sand grain roughness data was generated by person called Nikuradze, a very well-known scientist from Soviet Union and this data is quoted in the book by Schlichting, called - The Boundary Theory, about which I mentioned even in during the laminar boundary layer discussion.

So, you will find C r versus log 10 or y r plus when it is perfectly linear and rising. It is a perfectly smooth surface that is y r plus itself is very low. Therefore, there is not really a rough surface. So, we say, up to here the surface is smooth. Then, the effect of roughness begins to come in, but the C r would then decline up to about say log y r plus to the base 10 of about 1.8 which is really y r plus about over 70.

Then, it is found that they experimental data remains really constant with increasing y r plus. So, this is taken as the indicator of the fully rough surface and C r value for that is 8.48.

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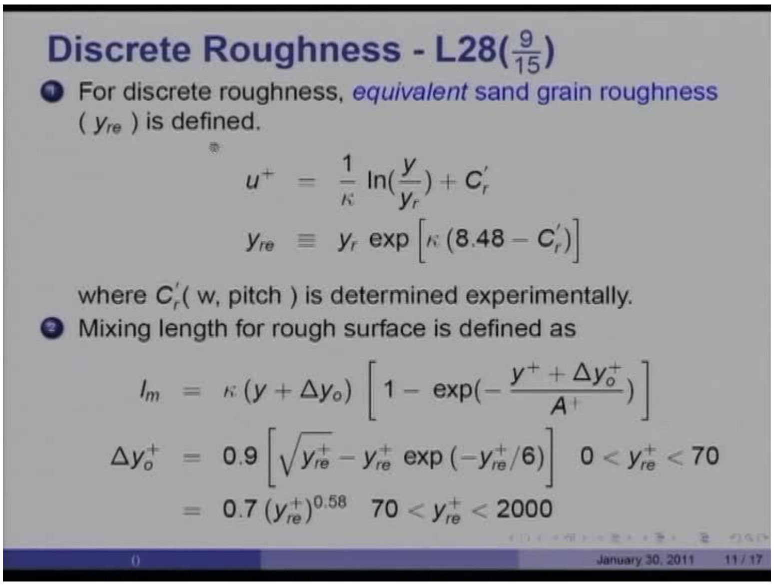

So, for a discrete roughness in equivalence sand grain roughness y re is defined. We say u plus will be equal to 1 over kappa ln y by y r plus C r dash and the equivalent roughness height then would be y r exponential kappa into 8.48 minus C r dash where, C r dash is the function of the width of the rib or the pitch of the rib, and is determined experimentally. So, that is how one would get the u plus - the universal law of the wall for a given structured surface.

One can also modify the mixing length. Here, instead of y plus, one would now take y plus plus delta y naught plus divided by A plus. Likewise, instead of y, we would take here y plus delta y naught; delta y naught is correlated in this way by Kays and Reynolds. This approach is no longer used now, but nonetheless, it provided excellent measurements, excellent prediction which compared with the experimental data very well.

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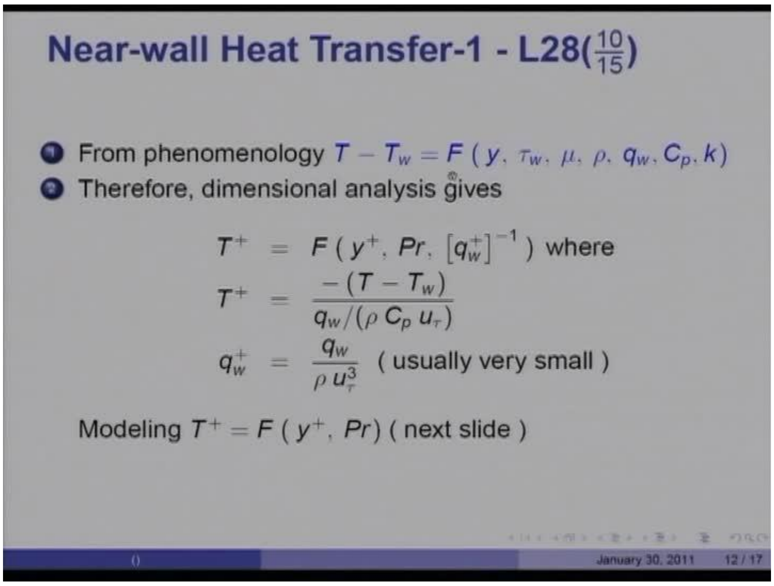

Finally, we turn to Near wall heat transfer. Like we develop the law of the wall for velocity from phenomenology where we postulated u as a function of T minus T w, the wall temperature is going to be a function of firstly y tau wall mu, and rho which really determine the four things which really determined the velocity profile. In addition, now you will have q wall because that would determine the temperature gradient at the wall.

C p is the specific heat of the fluid, and k - the conductivity of the fluid. Therefore, if one carries out the dimensional analysis, one would find that a quantity T plus; unusual looking, but what you have here is a T plus equal to minus T minus T w over q w into rho C p u tau; T plus would be a function of y plus, a Prandtl number, and q w plus q w plus is really q over rho u tau q. That is really the effect of friction at the wall we generate heat. But usually, compared to the actual heat transfer, this term is very small and therefore, it is not of great relevance. Effect of that can be taken up through property variation and relates times, but therefore, this term is really dropped and we say that T plus will be a function of y plus and Prandtl only.

So, let us see the forms that F y plus Prandtl will take in different layers of the inner layer.

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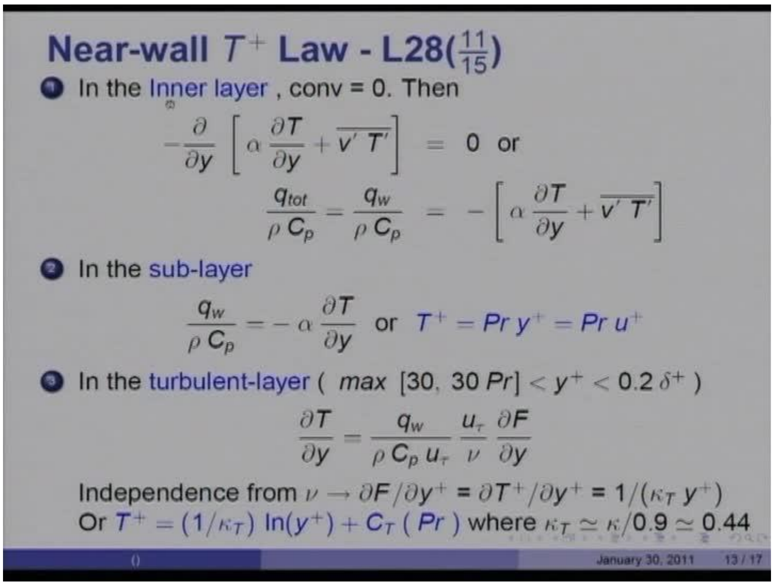

Now, to generate the T plus law, we first of all look at the differential equation where we say that for a smooth impermeable wall at any rate, the convection term will be very small and therefore, the total diffusion of heat flux would be equal to 0 or essentially q total divided by rho C p would equal q wall divided by rho C p equal to minus alpha dt by dy plus v prime T prime; all this would be equal to a constant.

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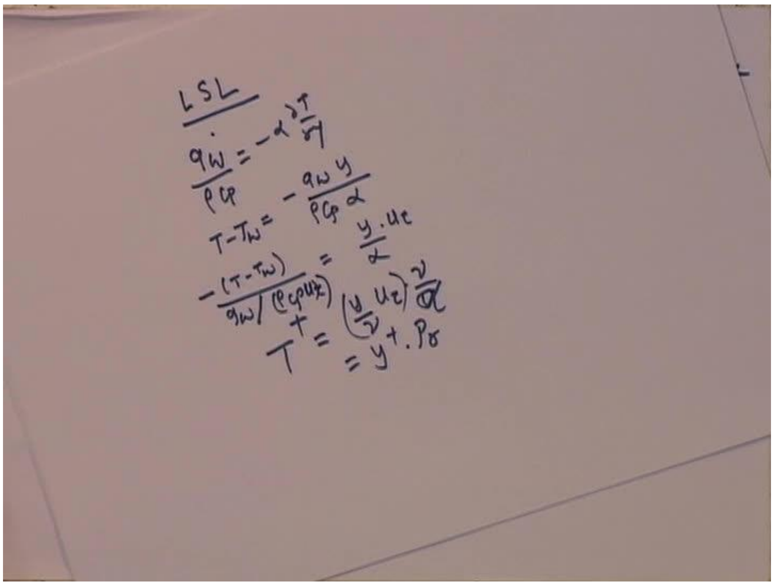

In the sub-layer, q wall over rho C p will be equal to minus alpha dt by dy. Hence, in the laminar sub-layer q wall over rho C p would be equal to minus alpha dt by dy. Therefore, q wall is constant. So, if I integrated that I will get T minus T wall equal to minus q wall y divided by rho C p into alpha. Now, if I said that the minus T minus T w divided by q wall divided by rho C p would be equal to y by alpha. If I divide this by u tau here, then this will be u tau and this is nothing but the definition of T plus and that will be equal to y by nu u tau into nu divided by alpha. Therefore, this is nothing but y plus and this is Prandtl number.

So, in the laminar sub-layer, it is straightforward to show that T plus would be equal to Prandtl times y plus and Prandtl times y plus is also equal to Prandtl times u plus.

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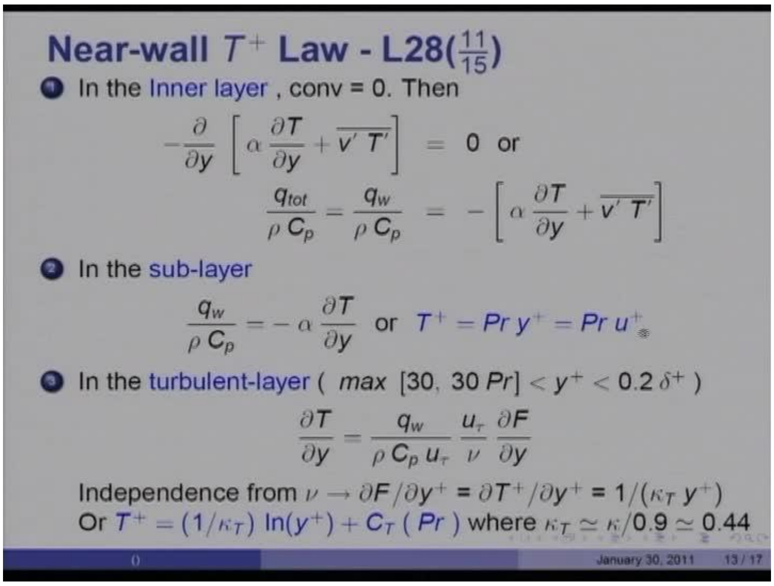

So, laminar sub-layer is fairly clear. Now, in the turbulent layer which is characterized by a maximum of 30 and 30 Prandtl number, less than y plus and less than 0.2 delta plus, because remember, the thickness of the thermal boundary layer may go into or the thermal thickness would be determined by the Prandtl number in the intermediate transition and sub-layer regions. Therefore, we take maximum of 30 to 30 Prandtl to 2 delta plus which is the end of the transitional layer.

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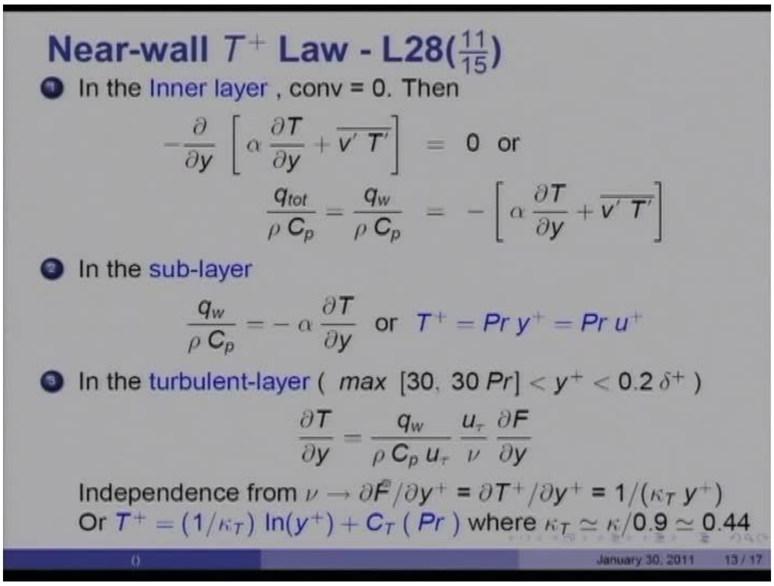

There the relationship dt by dy would read as dt by dy equal to q wall over rho C p u tau equal to dF by dy plus into dy plus by dy which is u tau by nu. This expression must be independent of nu which gives me that dF by dy plus must be equal to dt plus by dy plus equal to 1 over kappa T y plus, or T plus equal to 1 over kappa T ln y plus; all this for a fixed Prandtl number. Therefore, the constant of integration here C T would be a constant of integration would now be a function of Prandtl number and kappa T turns out to be kappa divided by 0.9 equal to 0.44.

So, the turbulent layer is in the fully turbulent layer, you will again get a logarithmic law for the temperature. In the laminar sub-layer, you will get this.

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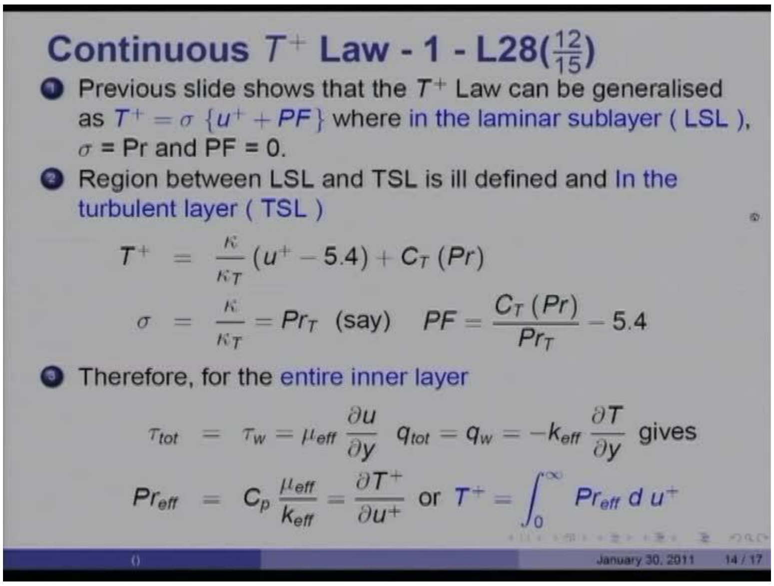

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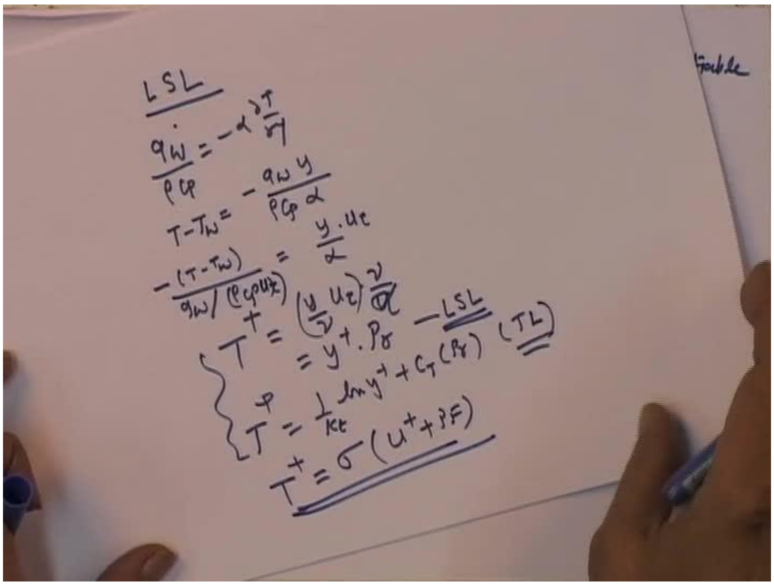

The transition region is somewhat complicated and therefore, not easy to define, but the two laws that we have got: T plus equal to y plus into Prandtl for laminar sub-layer and T plus equal to 1 over kappa T times ln y plus plus C T Prandtl for the turbulent layer; these two laws suggest that it is possible to generalize this as T plus equal to sigma times u plus plus PF.

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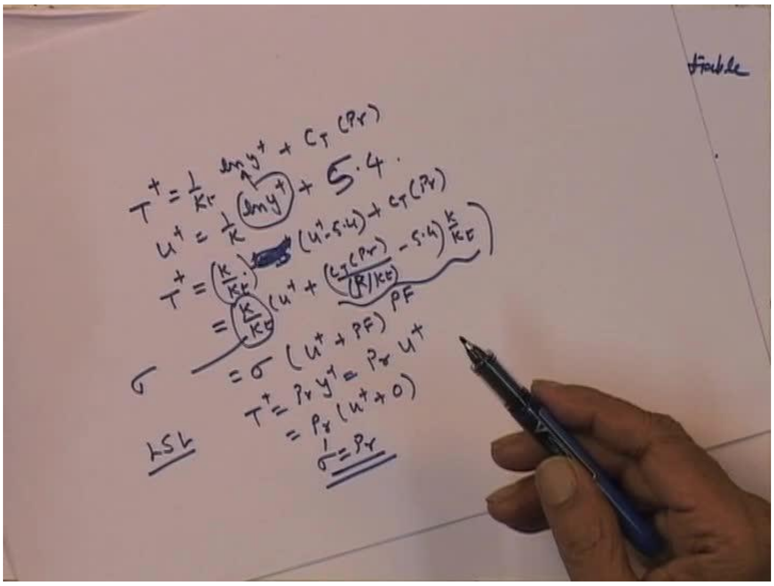

Now, to understand this, consider the turbulent law. For example, the turbulent law says T plus is equal to 1 over kappa T ln y plus plus C T Prandtl, but u plus is 1 over kappa ln y plus plus C which is 5.4, if you remember.

So I can substitute for ln y plus here. Then, I will say T plus equal to kappa over kappa T u plus minus 5.4 plus C T Prandtl; or another way of writing is kappa over kappa T into u plus plus another quantity called the C T Prandtl divided by Prandtl T kappa over kappa T minus 5.4 divided by kappa over kappa T. If I were to call this quantity PF for the time being, then you will see this. If I call this is to be sigma r, then it will look like sigma u plus plus PF.

Similarly in the laminar sub-layer you have T plus equal to Prandtl y plus is also equal to Prandtl u plus. So, I can write this as Prandtl times u plus plus 0 and say that this sigma is now Prandtl number, the laminar Prandtl number.

So, it is possible to generalize both these laws by an expression like, T plus u plus plus PF where, PF and sigma take different values in different layers.

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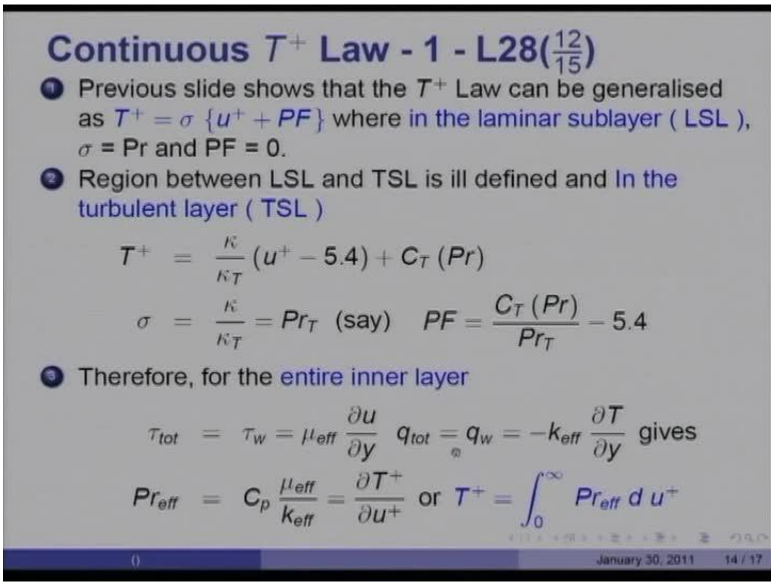

So, we are going to say now that this kind of representation T plus equal to sigma u plus plus PF applies across from laminar sub-layer onwards to fully turbulent layer, provided we interpret sigma correctly and PF correctly.

Now, sigma which is kappa over kappa T in a way represents, as you will see shortly, the slopes of the velocity and temperature profiles in the fully turbulent layer and therefore, it would amount to essentially turbulent Prandtl number. Therefore, I have replaced kappa by kappa T by Prandtl T. so PF for the fully turbulent layer would be that whereas, for the fully laminar sub layer, it would be 0 and sigma would be equal to laminar Prandtl number.

So, for the entire layer now, entire inner layer tau tot is equal to tau wall mu effective du dy where and q tot is equal to q wall minus k effective by dt dy and therefore, if I define Prandtl effective equal to C mu effective by k effective, then it will simply amount to dt plus by du plus by the definitions; that we have to understand.

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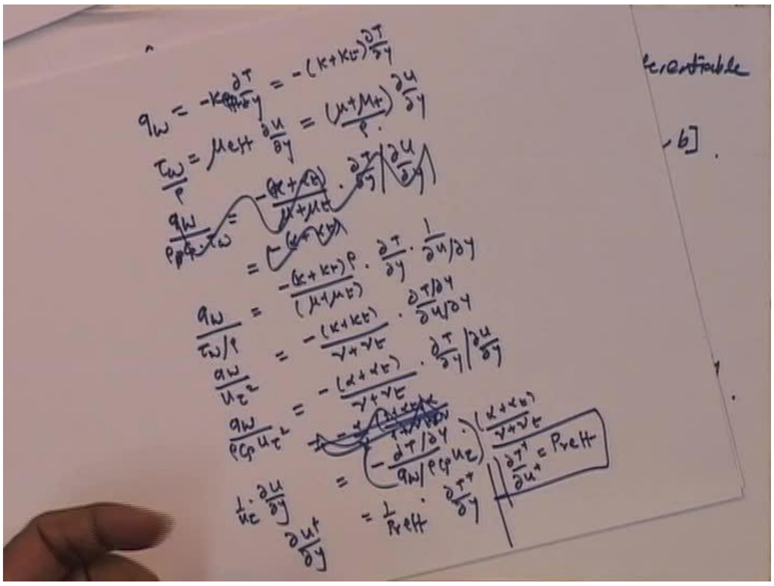

Let us say q wall is equal to minus k t dt dy or k effective rather, which is k plus k t and tau all is equal to mu effective du dy which is equal to minus k plus k t dt dy. This is equal to mu plus mu t du dy. Then, if I take a ratio of this you will see I get q wall divided by tau wall divided by rho; I say this.

Then this is will be rho here; so, tau wall divided by rho would be equal to minus k plus k t divided by mu plus mu t equal to dt by d y into 1 over du by dy. Therefore, this will become q wall divided by u tau squared equal to minus k plus k t divided by nu plus nu t dt by du and if I divide this now by rho C p, then I will get q wall divided by rho C p u tau squared will be equal to minus alpha plus alpha t over nu plus nu t dt by dy.

If I take nu and alpha common, then I will get minus alpha by nu into 1 plus alpha t by alpha over 1 plus nu t by nu, and this is nothing but, I do not even I have to do that at the moment, simply realize that I can write this now as minus dt divided by q wall divided by rho C p u tau is equal to dt by dy divided by du by dy dt by dy divided by du dy. So, dt by dy will be equal 1 over u tau into du by dy into alpha plus alpha t divided by nu plus nu t, which is nothing but, 1 over Prandtl effective. This (Refer Slide time: 40:07 to 40:13) can be written as dt plus by dy and this can be written as du plus by dy. Therefore, I get the expression that dt plus by du plus equal to Prandtl effect and that is what I have shown here.

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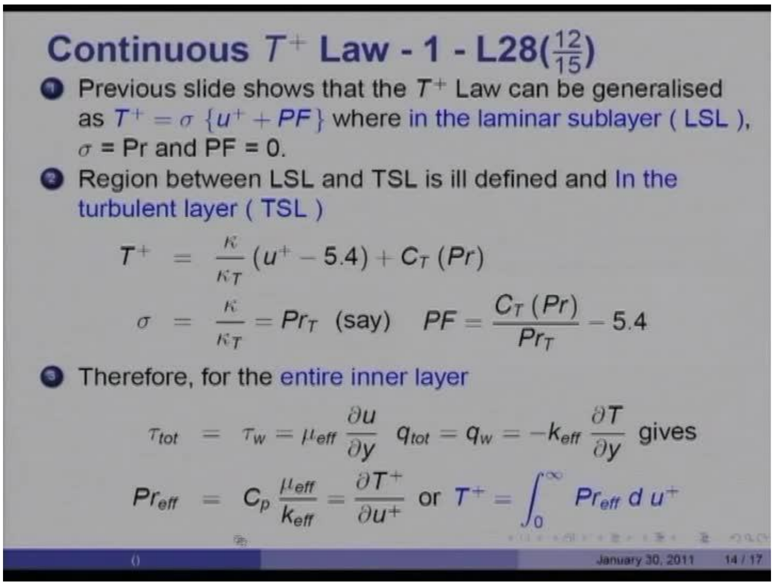

Therefore, you will see here Prandtl effective is equal to dT plus by du plus and therefore, T plus would be 0 to infinity Prandtl effective d u plus. This would give us a continuous temperature law, as the function of u plus or function of y plus depends on how one wants to read it.

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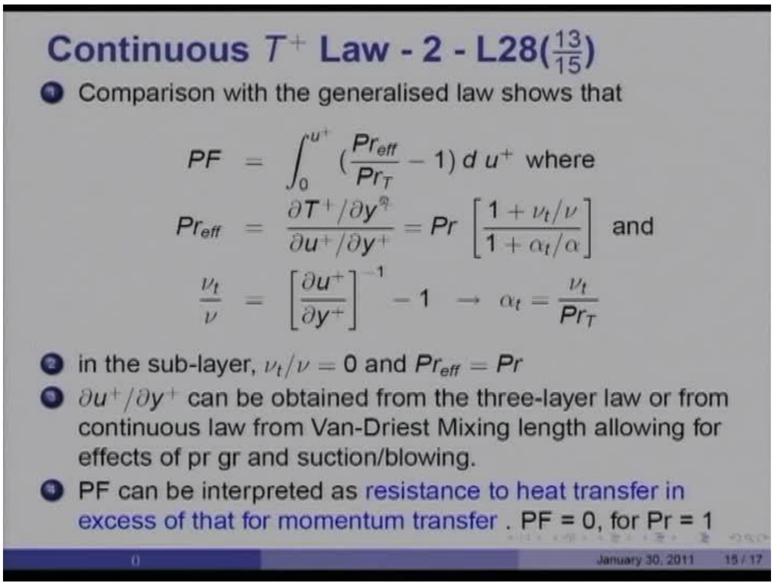

By comparison of this, if I compare this expression T plus equal to 0 to infinity Prandtl effective d u plus with sigma times u plus plus PF, then I can show that PF would be simply 0 to u plus Prandtl effective divided by Prandtl T minus 1 into d u plus where

Prandtl effective is Prandtl times 1 plus nu t by nu plus 1 plus alpha t by alpha and nu t by nu will be given by this.

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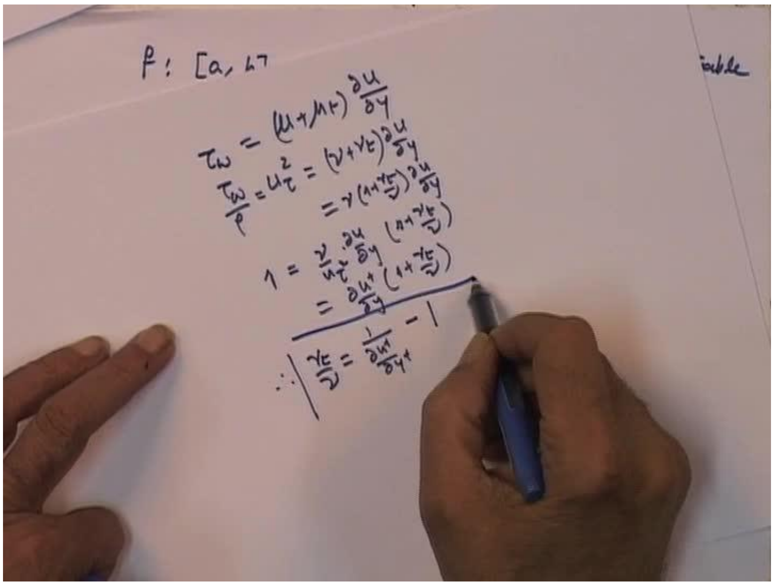

Why? Because remember, tau wall is mu plus mu t du dy. Therefore, tau wall over rho which is equal to u tau square will be nu plus nu t into du by dy. That will be equal to nu into 1 plus nu t by nu du by dy. Therefore, you will see that 1 will equal nu by u tau square into du by dy into 1 plus nu t by nu and this is nothing but, du plus by dy plus into 1 plus nu t by nu. Therefore, you will notice that nu t by nu will be simply 1 over du plus by dy plus minus1.

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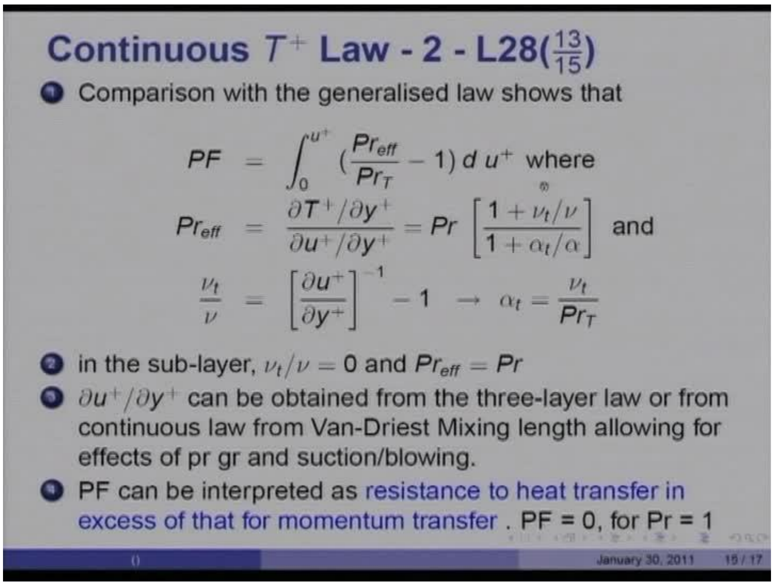

So, that is what I have shown here. So, we say nu t by nu is equal to du plus by dy plus raise to minus 1 minus 1 and alpha t would be simply nu t divided by Prandtl T. This expression shows that. Remember, du plus by dy plus is equal to 1 in the laminar sub-layer. Therefore, this will be 1 minus 1; so, in the laminar sub-layer, nu t by nu goes to 0 and Prandtl effective would simply go to Prandtl number because that dt plus by dy plus there, is equal to Prandtl number and du plus by dy plus is 1.

Therefore, now the task is simple. How do we determine PF as a function of u plus? Well, we need the values of du plus by dy plus. So, either we can take them from the three layer law which applies only to smooth surfaces, or we can use the Van-Driest Mixing Length model so that we can also allow for pressure gradient and suction and blowing for du plus by dy plus, and use it here to integrate this expression to obtain PF as a function of u plus.

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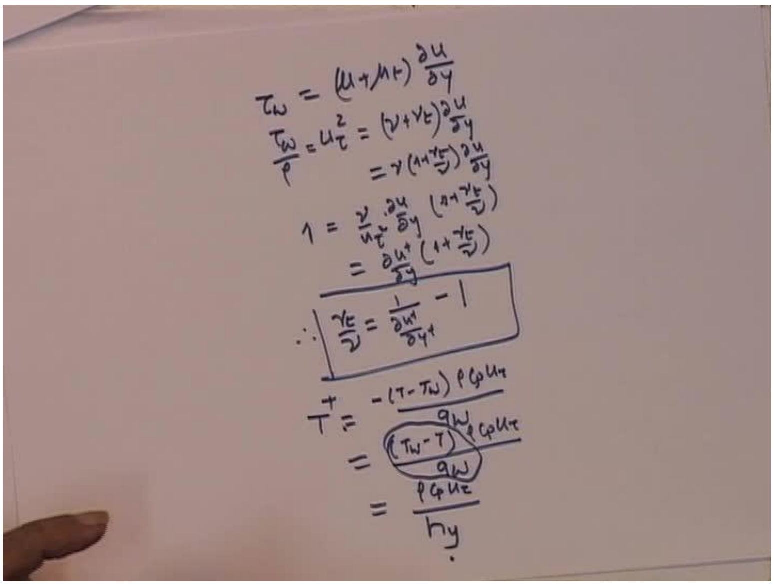

So, you will see that T plus in a way because of it is definition, T plus which is equal to minus T minus T w rho C p u tau divided by q w, which I can also write as T w minus T into rho C p u tau divided by q wall, or I can speak of this as 1 over rho C p u tau as h up to distance y q w over T w minus T can be taught of as rho C p u tau divided by heat transfer coefficient, up to distance y in a boundary layer. Therefore, T plus being inverse of the conductance can now be thought of as a resistance to heat transfer.

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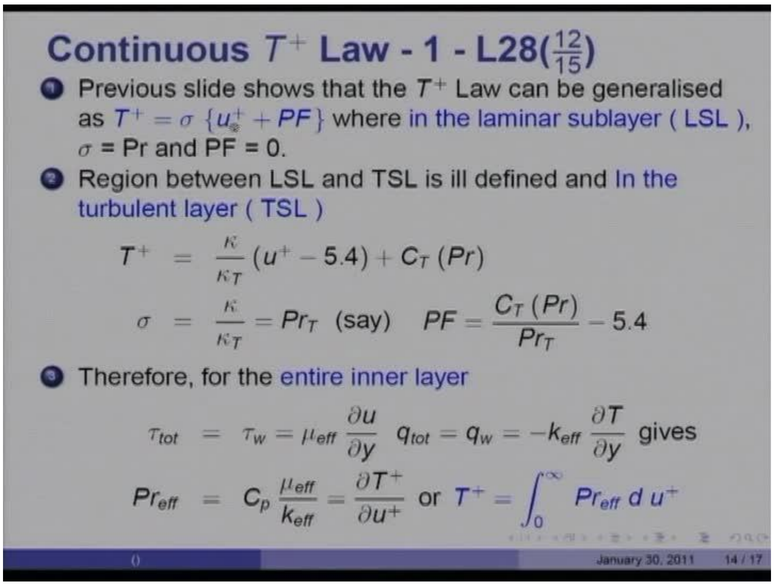

Therefore, the relationship that we have T plus equal to u plus plus PF multiplied by sigma is really, what it indicates is since u plus is related to resistance due to momentum transfer and T plus is resistance to heat transfer, we can say PF is really the excess resistance over a resistance to momentum transfer, and that is the interpretation one can now give to PF.

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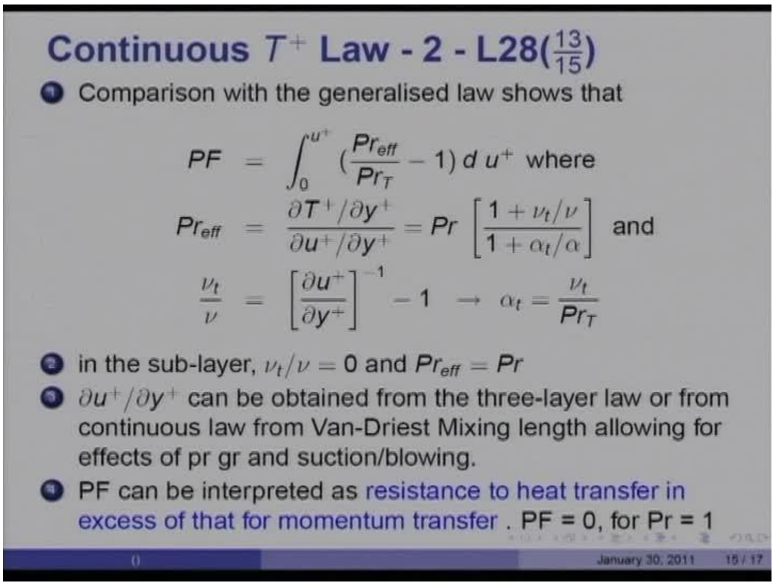

PF will be equal to 0 for Prandtl equal to 1 as a rho and then what I have done is I have integrated using du plus by dy plus of the from Van-Driest mixing length model and found the following.

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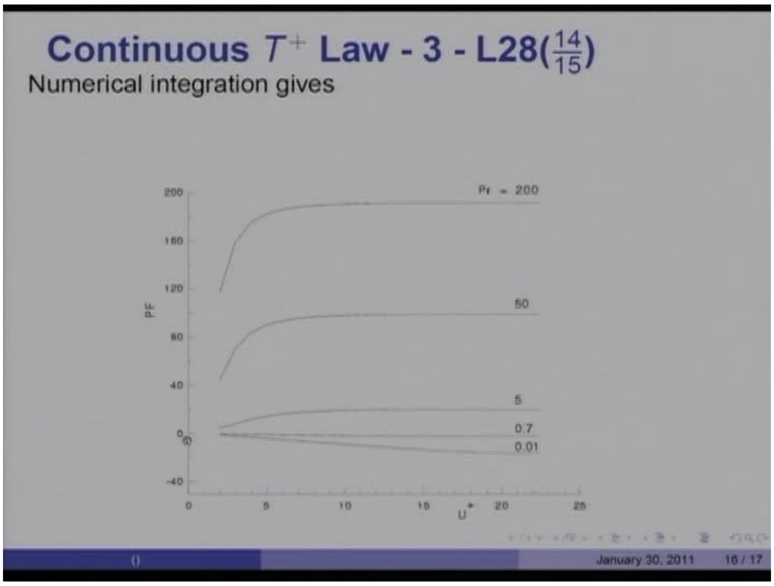

So, for example, here is the value of PF and here are the values of u plus; up to u plus equal to 5, you have laminar sub-layer and you do get the resistance to heat transfer, is an excess of momentum transfer, even in the laminar sub-layer. Then this is at Prandtl number of 50 and this is at Prandtl number of 5. At Prandtl number 1, it is all 0; there is no excess resistance and as we have observed earlier that, you would have perfect analogy between heat and momentum transfer

For 0.7, you have negative PF which means the resistance to heat transfer is less than the resistance to momentum transfer; in liquid metal, the resistance is even lesser then that to momentum transfer.

The transitional layer would occur somewhere around u plus of 13; end of transitional layer corresponding to y plus of 30 would occur somewhere here (Refer Slide Time: 46:30), and from here onwards, you have really fully turbulent layer and it so happens that, in the fully turbulent layer, the PF value reaches almost constancy. It remains only a function of Prandtl number. So, we can say PF in the limit is always a constant for a given Prandtl number and that these values of PF at large values of u plus have been correlated.

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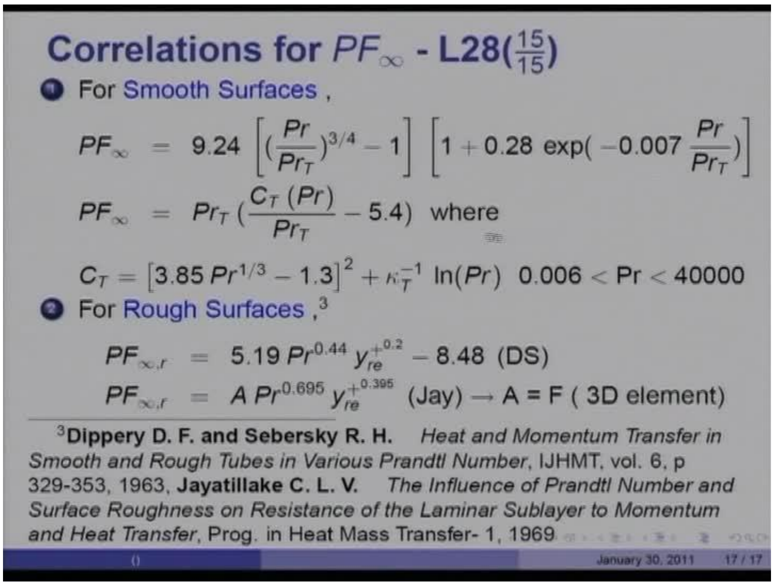

The most well-known among these is PF infinity is equal to 9.24 divided by Prandtl number Prandtl T raised to 3 by 4 and into another function of Prandtl divided by Prandtl T.

We can also interpret our C T Prandtl in this manner and PF infinity would be Prandtl T C T Prandtl by Prandtl T minus 5.4. These data were obtained by… This correlation for C T Prandtl was obtained by Russian scientist Khader and Imyaglom and it covered the range of Prandtl numbers from liquid metal range to very high viscous oils.

For rough surfaces, PF infinity is given both by Dippery and Sebersky, and also by Jayatillake who gave this formula. Therefore, I have given here, the references to both of them. For a rough surface, it is 5.19 Prandtl raised to 0.4 y r equivalent roughness height raised to 0.2 minus 8.48. Jayatillake's formula requires experimental information on a, which is element essentially for structured surfaces.

So, with this I end the lecture which covered the topics of transition; it covered the topic of a temperature law, and we found that a continuous temperature law is also possible to be derived for the inner layer.