Convective Heat and Mass Transfer

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Module No. # 01

Lecture No. # 27

Turbulence Models-II

In the previous lecture, we considered how we can treat the outer layers of a turbulent layer by differential equation or quantities called it characterizing turbulence. We employed the wall function approach so that the inner layer would not be computed at all; instead, we would use the consequences of the inner layer that is, the universality of the velocity profile there and apply the boundary conditions for kinetic energy and dissipation at the first node away from the wall. The node distance would be chosen that the y plus of that node would be somewhere between 30 and 100. This kind of modeling technique proves very useful for moderate pressure gradient flows.

There are situations where even the inner layers are influenced by situation or conditions far away from the wall. Such situations arise in strongly swirling flows or when a jet impinges on a flat surface and a very strong strain fields develop. In such situation one needs to go with this modeling technique even closer to the wall that is, right at the wall and the boundary conditions were then be given at the wall itself. So, we are going to examine how to capture low turbulence Reynolds number region in turbulence models, so that is the topic for today and let us see how we go.

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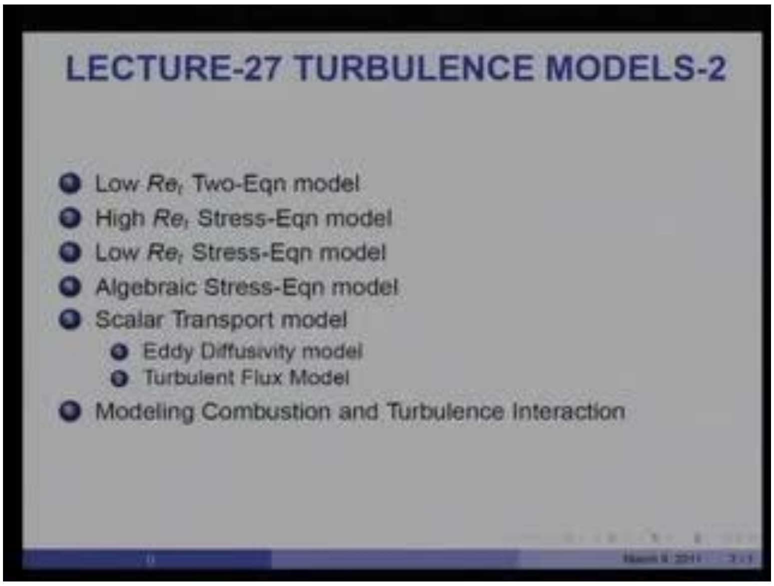

I would first describe the low Reynolds number turbulence; Reynolds number two-equation model and where the turbulent viscosity is calculated from kinetic energy and dissipation equations then, the other approach is not the eddy viscosity model but to actually solve the differential equation for Reynolds stresses. Again, there are high turbulent Reynolds number stress equation models and low turbulence Reynolds number stress equation models.

Computationally, these two turn out to be quite expensive and therefore a shortcut approach is often adopted which is called the algebraic stress equation model. This would complete our discussion on how to model a stress, but likewise we must also adopt a strategy for modeling turbulent scalar fluxes. Again, there are the eddy diffusivity models and the turbulent flux models analogues to the eddy viscosity model and the stress equation model. Finally, I will show couple of slides on how to model interaction between combustion and turbulence.

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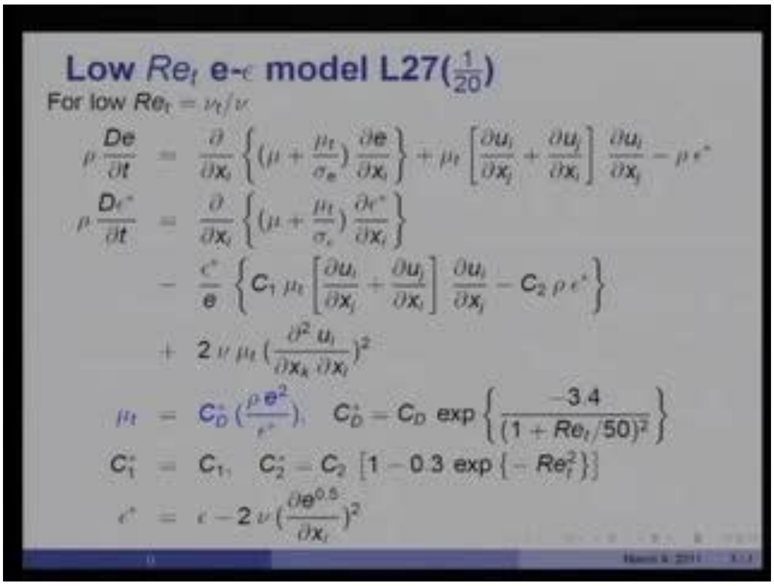

Let us consider the low turbulence Reynolds number forms of kinetic energy equation and the dissipation equation. We are going right up to the wall, the definition of epsilon is changed to epsilon star where epsilon star is defined here. Epsilon star is equal to the isotropic epsilon minus 2 nu de raise to half dx i square, I will explain why this change is necessary.

Then, there are the turbulent production terms and the dissipation term of the modified dissipation and likewise here also, but the constants are changed and are sensitized to distance to the lower turbulence Reynolds number regions. For example, one such term which is sensitized to distance from the wall and low turbulence Reynolds number region is 2 nu mu t d 2 U i dx k dx l, the reference to all this will appear shortly after few slides.

The turbulent viscosity is C D star raised to rho epsilon square divide by epsilon star and the C D star is now the C D, which was 0.09 in the high Reynolds number form exponential of minus 3.4 1 plus R e t by 50 whole square. C 1 star is taken as C 1 itself and C 2 star is C 2 into 1 minus 0.3 exponential of R e t square and epsilon star itself is this quantity. As you will see, when R e t becomes large that is, as we move away the wall all these constants will be rendered same as those that were used in the high turbulence Reynolds number model.

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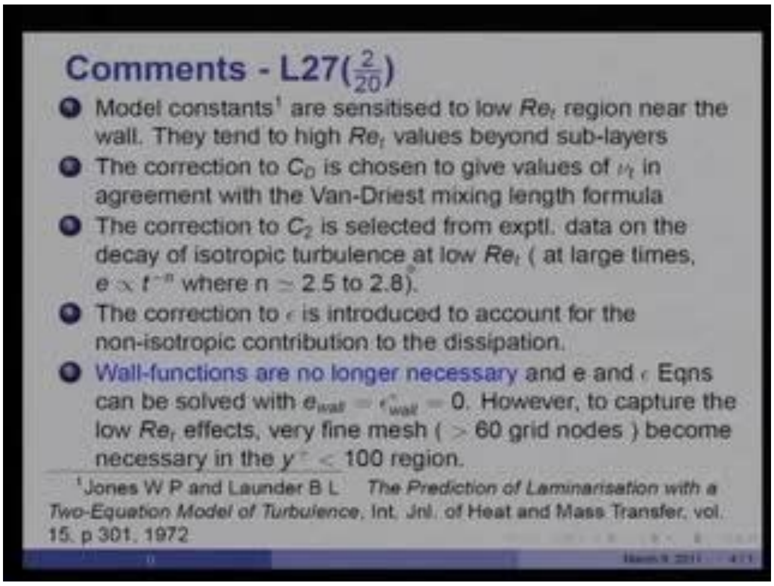

This is the reference Jones and Launder International Journal of heat and mass transfer in 1972. The model constants are sensitized to low turbulence Reynolds number region near the wall, they tend to hide R e t values beyond the sub layers the correction to C D is chosen to give a value of nu t in agreement with Van-Driest mixing length formula.

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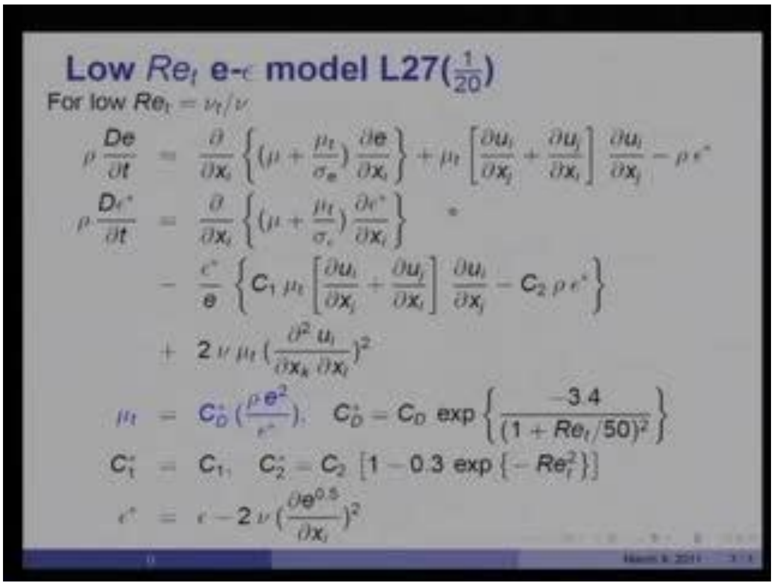

That is, this correction or the damping factor accounts for what was observed by Van-Driest, you will recall that we had determined a C 2 from decay of turbulence behind a grid.

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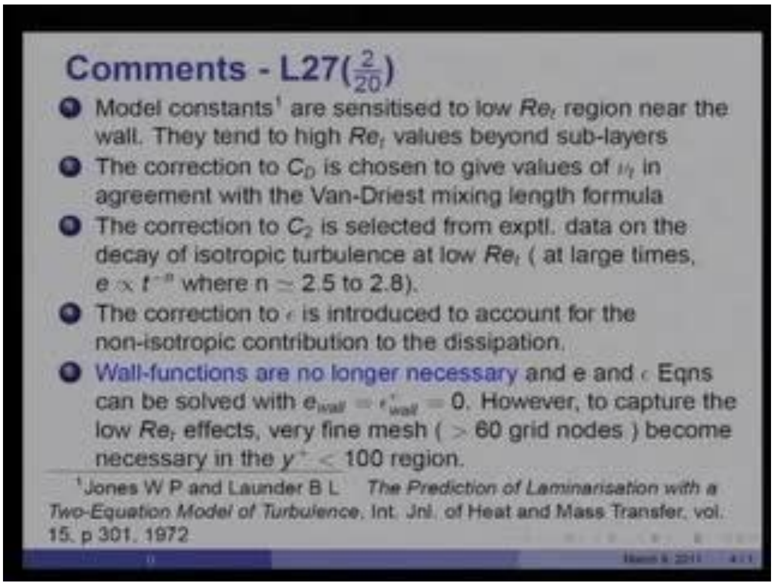

Now, in the early path at small times the kinetic energy decays again as t raise to minus n, but n would be then of the order of 1.1 to 1.2, but at large times it becomes 2.5 to 2.8. C 2 is again determined exactly from the same procedure as before, but the value of C 2 is corrected as shown here; it is sensitized to turbulence Reynolds number.

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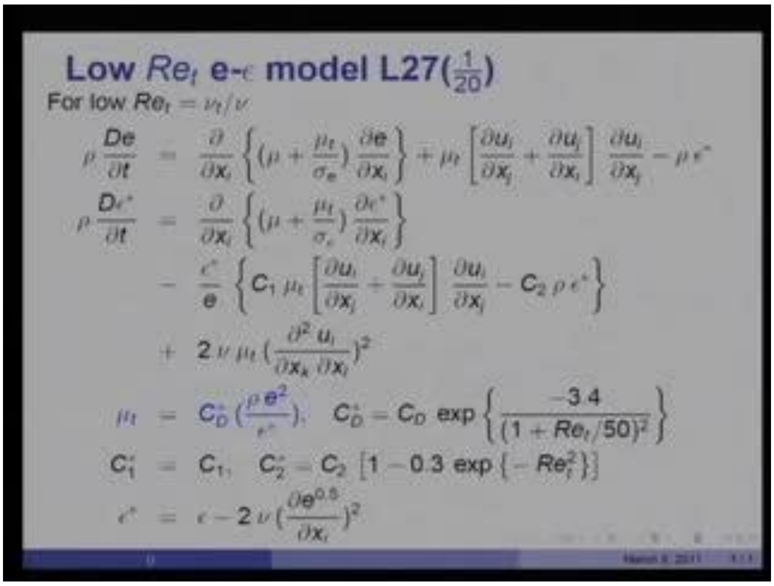

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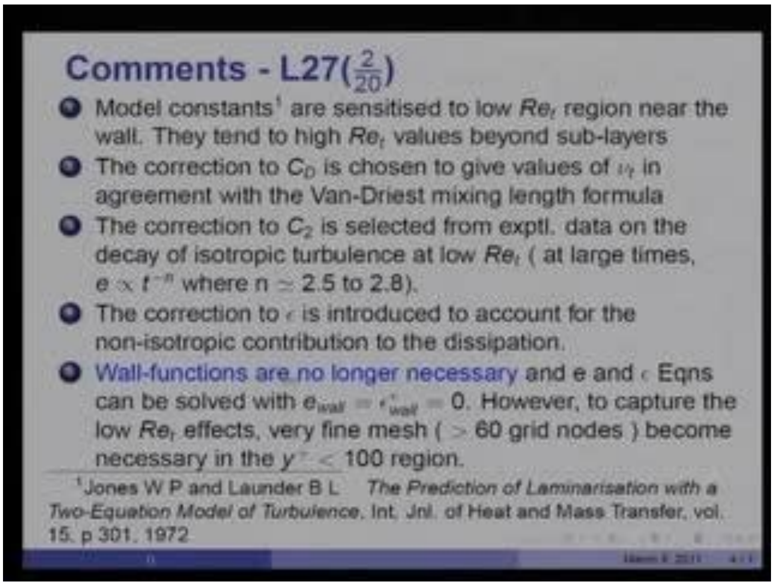

The correction to epsilon is introduced to account for non-isotropic contribution to dissipation - as you will recall - we said that as one moves close to the wall the amount of dissipation in different directions is actually different. This term epsilon star epsilon minus 2 nu e raise to half accounts for effect of non-isotropic.

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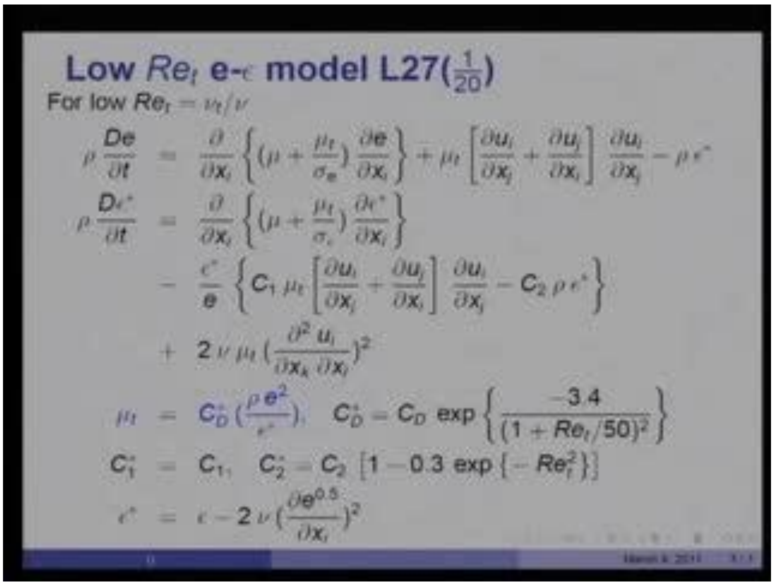

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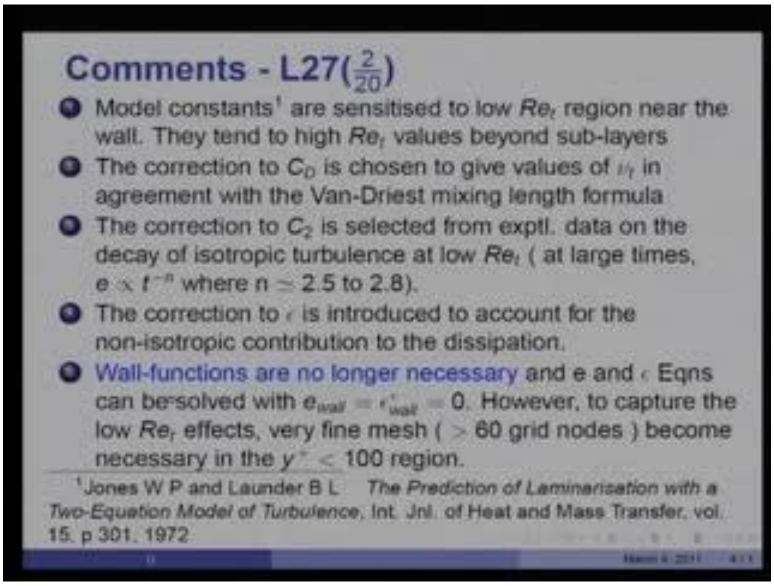

Since, we are able to now go right up to the wall; wall functions are no longer necessary; in other words, we are not going to use the universal law of the wall at all. The e and epsilon equations can now be solved with e wall equal to epsilon star wall equal to 0. To capture the effects of low turbulence Reynolds number in the region between - if you recall - this was the wall and in high Reynolds number form we had chosen P over here.

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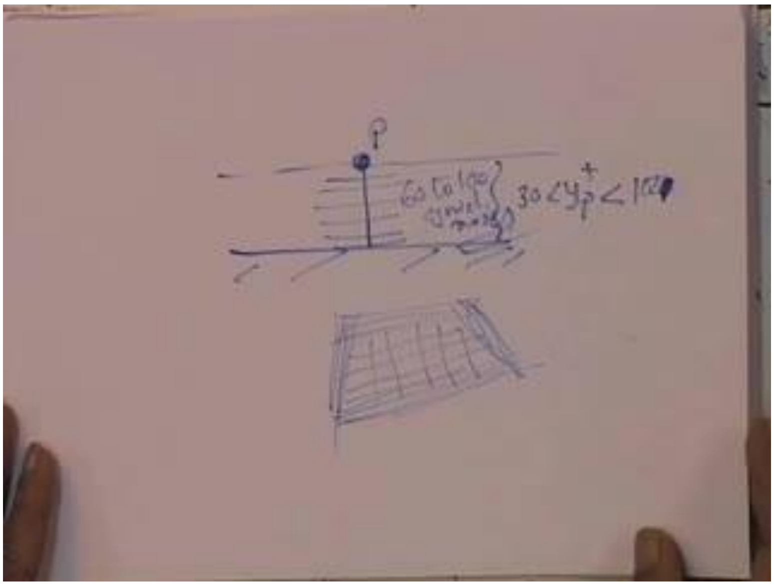

So, only one node to take care of entire 30 y p plus 100. Now, you require to capture very sharp variations of velocity and other thing as many as 60 to 100 grid nodes. This makes computation, if it was a simple two dimensional boundary layer then it is not into a with today's desktop computer is not a too much of an effort.

Supposing, we had an internal flow with two walls or three walls or whatever then, you will see that you require 60 nodes here, 60 nodes here, 60 nodes here - was a minimum 60, I mean. Then, there are the nodes required in the core of the domain and that makes the number of nodes required for computation very large and such low turbulence Reynolds numbers models is as much as their validity is far greater than that of the high turbulence Reynolds number model tend to be very expensive. Therefore, one needs to be little careful in using these models that means, we use them only when they are absolutely essential. So, extremely fine mesh usually greater than 60 grid nodes would be required in the y plus less than 100 region, so that completes the discussion on eddy viscosity models.

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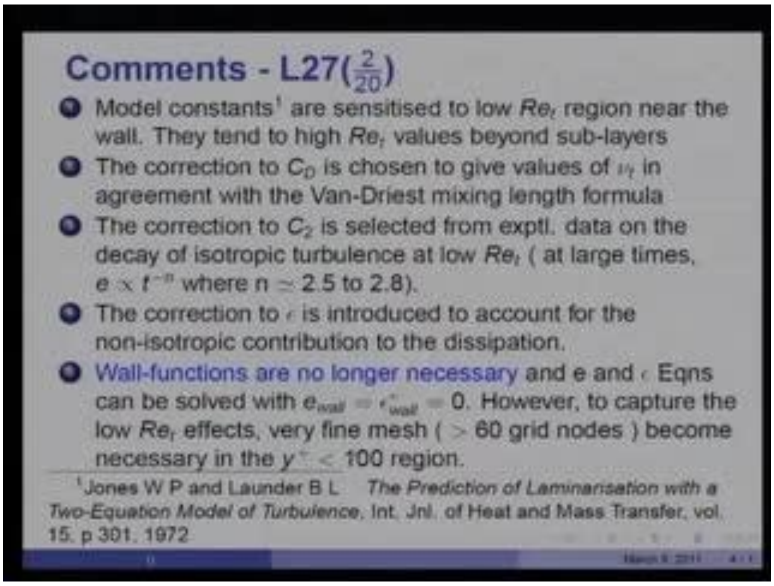

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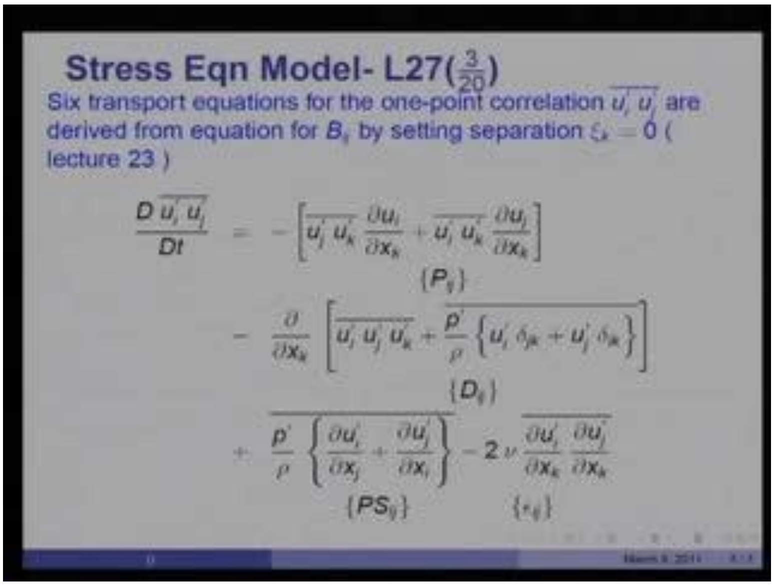

Now, turn to the stress equation models, so the six transport equations for one-point correlation u i prime u j prime are derived from the equation for B i j - which we had discussed in lecture number 23 - in which the gradients with respect to separation distance psi k is set to 0, because they are now looking for one-point correlation and only differentials with respect to x k would survive.

This is the convective transport of one-point correlation u i prime u j prime; this is the production rate of the stress; this is the diffusion rate of the stress and this is called the pressure strain term and I will explain the meaning of that in a minute. This is the dissipation or destruction of the stress itself is given by epsilon ij and it would be different in different directions.

If you notice this equation is a differential equation mainly because of the convection terms and diffusion term. The rest of the terms are simply the sources production term, pressure strain term and dissipation terms are simply sources. We will make use of this fact little later in deriving the algebraic stress models where we will point out that these terms which make these equation a differential equation can actually be simplified.

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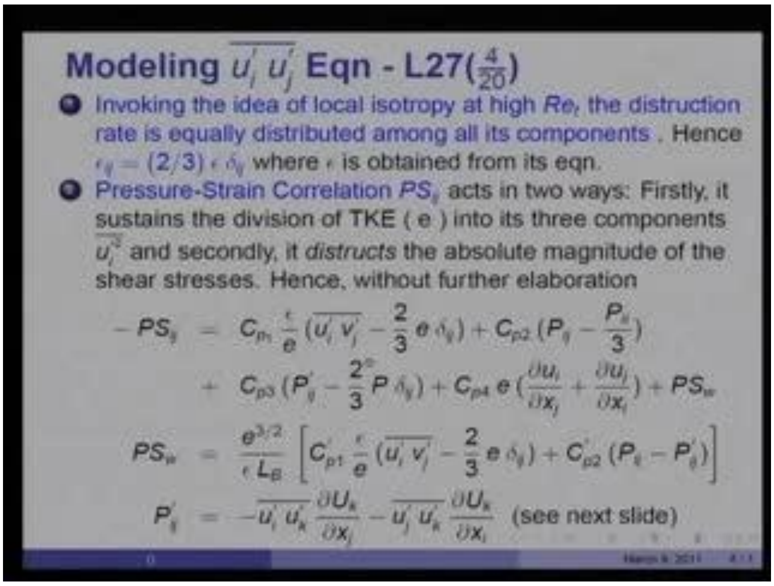

Invoking the idea of local isotropy at high turbulence Reynolds number, the destruction rate is equally distributed among all its components. Hence, epsilon ij is taken as two-thirds epsilon delta ij where epsilon is obtained from its equation - the epsilon equation. The pressure strain term acts in two ways; firstly, it sustains the division of turbulent kinetic energy into its three components u i square and secondly, it destructs the absolute magnitude of the shear stresses, it works in two ways, redistributes and destructs.

Complete discussion of the pressure strain term would be quite lengthy, so without further elaboration I will simply say, how the final form of the pressure strain model looks like. The minus PS ij is equated to difference between a stress minus two-thirds e delta ij C p2 likewise, is production term minus P ii by 3 where C p3 is P dash ij minus two-thirds P delta ij and C p4 is e times du i dx j plus du j dx i plus PS w.

Now, this is called the wall reflection term and it is modeled in this fashion. P dash ij which appears here is the turbulent stress production terms; let us see in next slide how these terms get further simplified.

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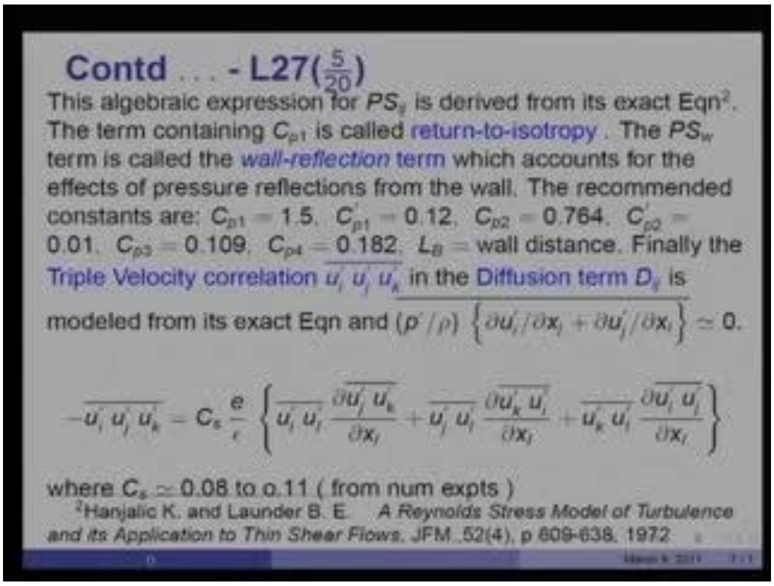

So, this algebraic expression for PS ij is derived from its exact equation by Hanjalic and Launder "A Reynolds Stress Model of Turbulence and Application to Thin Shear Flows" published in journal of fluid mechanics in 1972.

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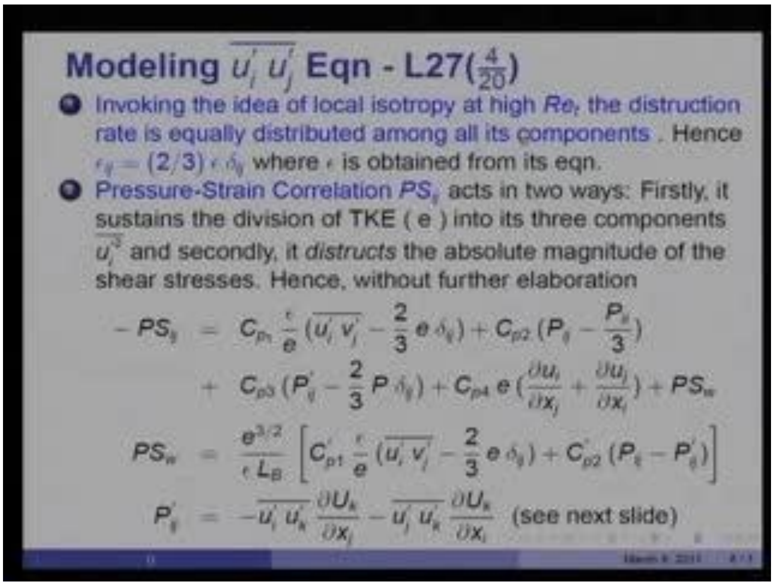

The term containing C p1 is called return-to-isotropy, there is always a tendency for a stress to return to its isotropy and the difference between this quantity minus two-thirds of kinetic energy; remember, when it is perfectly isotropic u i prime this should be u j prime; u i prime u i prime would be exactly equal to two-thirds of e.

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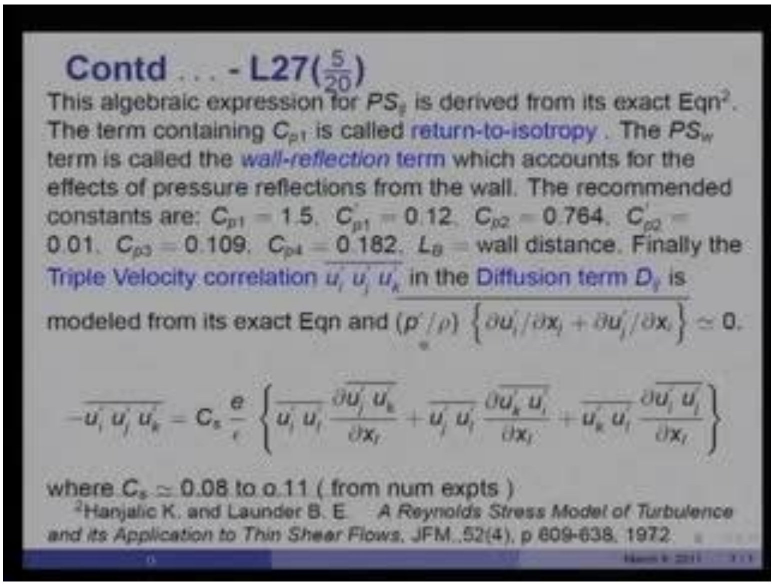

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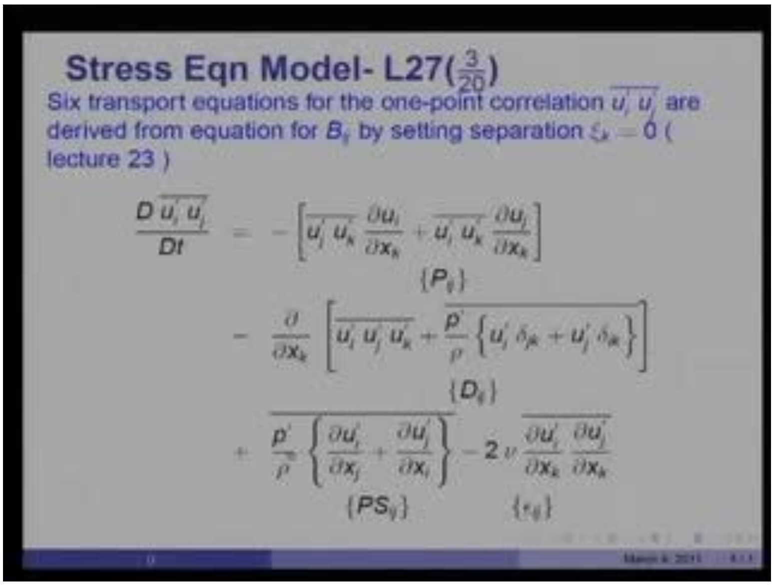

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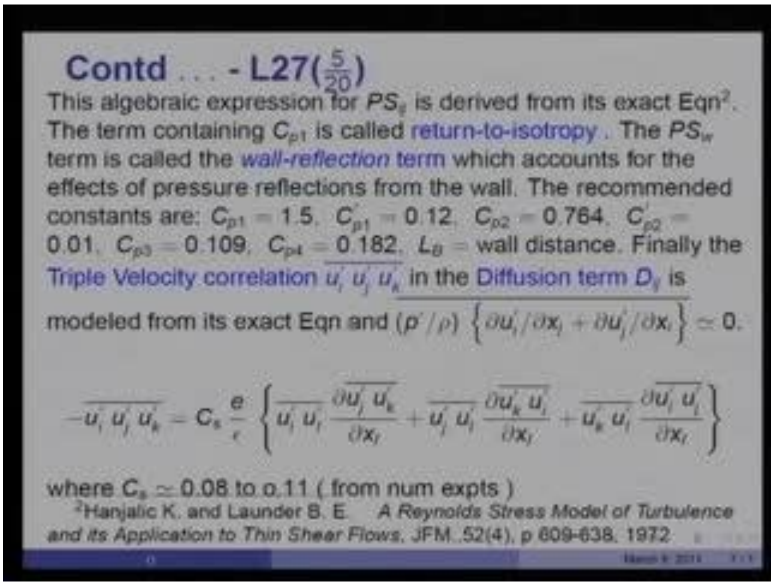

Therefore, simply in effect it is called the return to isotropy term. The PS w term is called the wall reflection term which accounts for the effect of pressure fluctuations or reflections from the wall, as you can see, in the pressure fluctuation term. Now, that becomes important close to the wall where the pressure fluctuations gives the effect which is primarily responsible for the presence of the wall. The recommended constants are C p1 equal to 1.5, C p1 dash equal to 0.12, all these were constants either derived from experimental data or tuned by numerical experiments.

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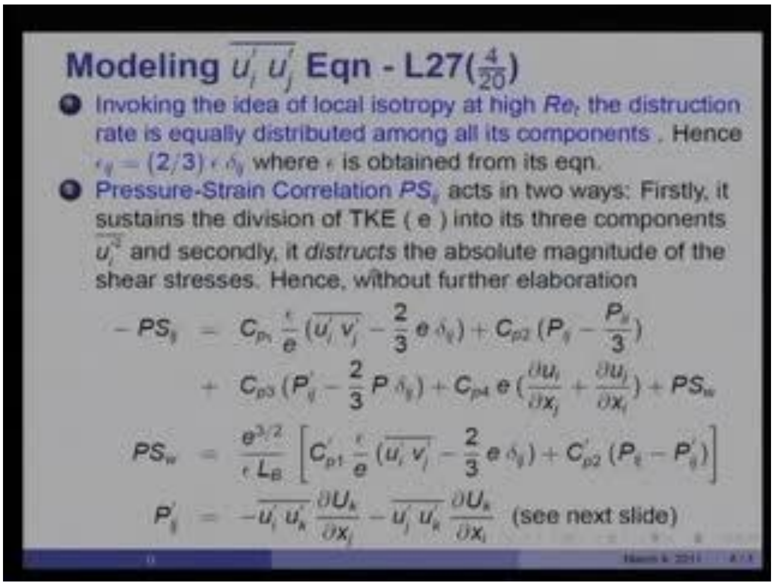

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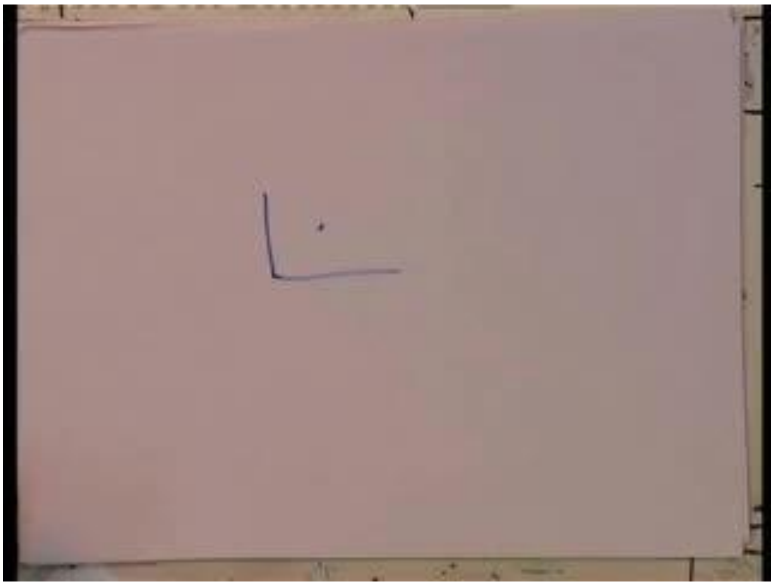

The appearance of L B here is really the distance from the wall and if you had the situation where there are more than two walls present then, one would take the minimum of the two walls to take care of L B. The triple velocity correlation u i prime u j prime u k prime in the diffusion term D ij is modeled from its exact equation again. It is possible to derive an exact equation for this quantity from instantaneous forms of Navier Stoke equations.

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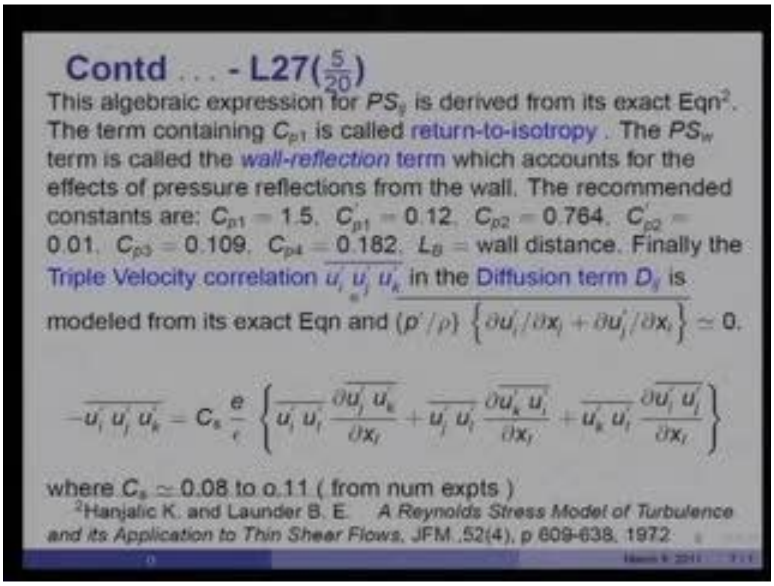

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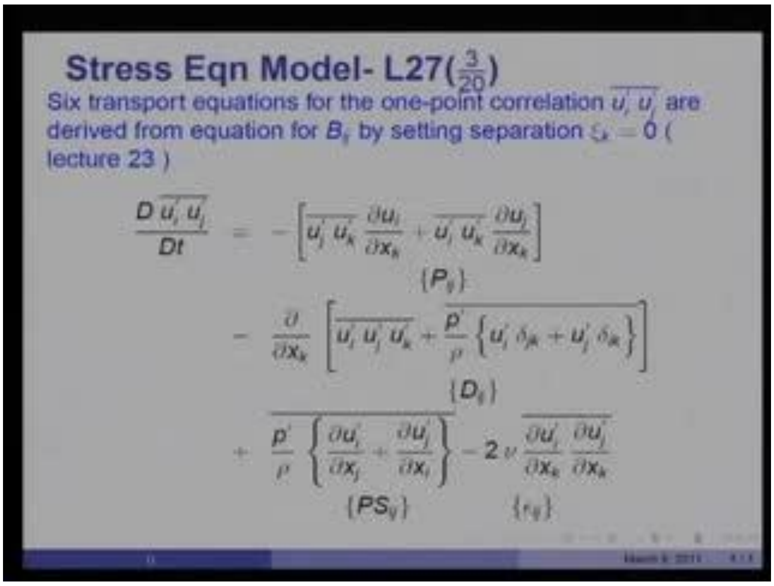

Again, it is a matter which I am not going to discuss what the equation looks like, but it is good to remember that the equation for u i prime u j prime gives the presence of u i prime u j prime u k prime there is a triple velocity correlation. Likewise, an equation for u i prime u j prime u k prime would give rise to 4 velocity fluctuation terms time average of u i square u j u k or u j square u i u k and so on and so forth.

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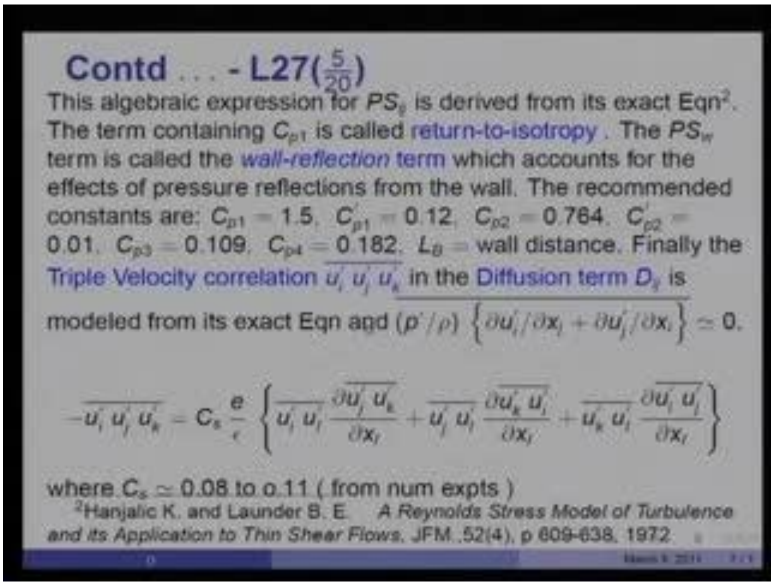

So, this is the problem with the statistical approach to deriving equations for turbulence, because there will always be terms of an order higher that need to be modeled and that is the closure problem. This is what presently the closure applied is the triple velocity correlation is made proportional to double velocity correlation and its gradients as you can see here, C S is 0.08 to 0.11 from numerical experiments.

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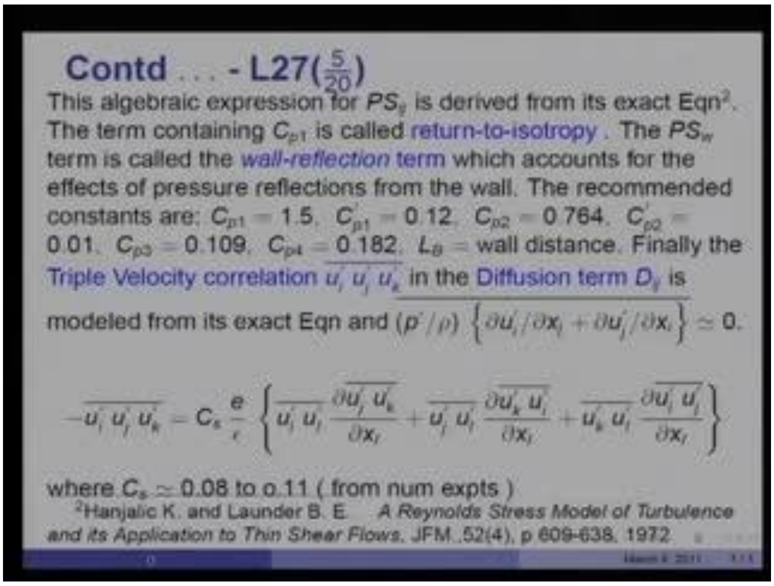

The whole process of modeling a stress equation is quite involved, but nonetheless at the end of the day it produces very useful differential equation which in fact can be solved in the manner of all other transport equations.

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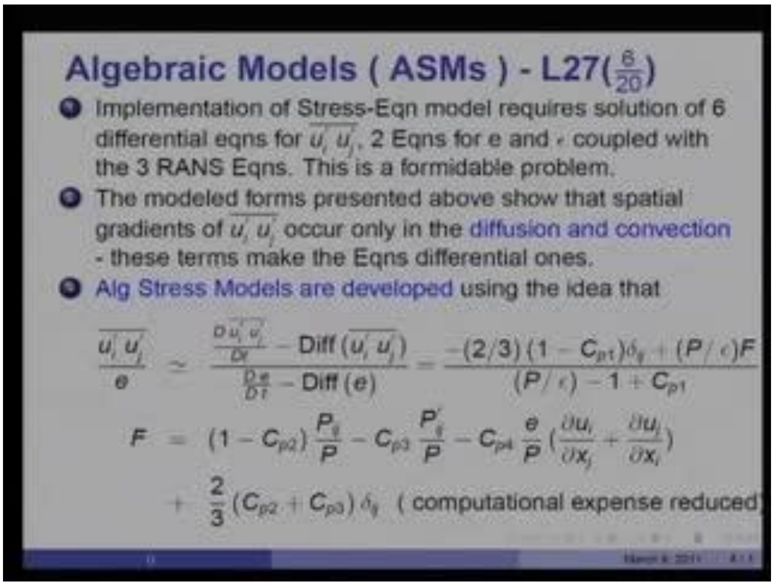

Now, the implementation of the stress equation models requires solution of 6 differential equations for u i prime u j prime; it requires 2 equations for e and epsilon that is a kinetic energy and dissipation coupled with 3 RANS equations that is, the original momentum equations. This makes it fairly a formidable problem, because 6 plus 2 for scalar equations and 3, that is 6 plus 2 plus 8 plus 3 is 11 equations and in a 3 dimensional flow this can be really a very formidable problem.

The model forms presented above show that the spatial gradients of u i prime u j prime occurs only in the diffusion and convection terms and these terms make the equation differential, this is the observation we made earlier.

So, the one way to get rid of this differential form is to say, we will develop an algebraic stress model and by using the idea that we can say that u i prime u j prime divided by e is approximately equal to convection minus diffusion of the stress divided by convection minus diffusion of the kinetic energy. That would reduce this right hand side to minus 2 by 3 1 minus C p1 delta ij plus P by epsilon divided by P by epsilon minus 1 plus C p1 multiplied by F here and F is equal to 1 minus C p2 P ij by P minus C p3 P dash ij by P minus C p4 e by P into the strain rate.

In other words, you can model the stress divided by kinetic energy by means of an algebraic expression though long, it nonetheless reduces the amount of effort involved. You need not solve the 6 stress differential equations for stresses instead, we use simply the algebraic expression for stresses and that reduces the computational time considerably.

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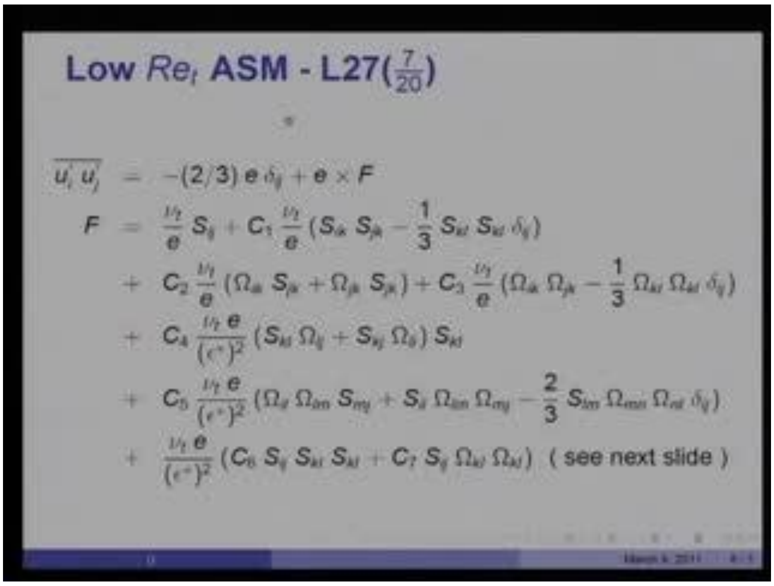

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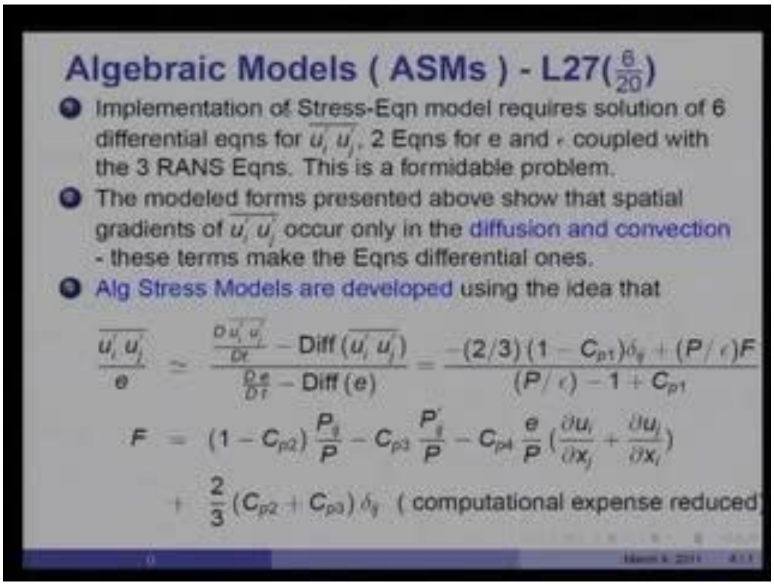

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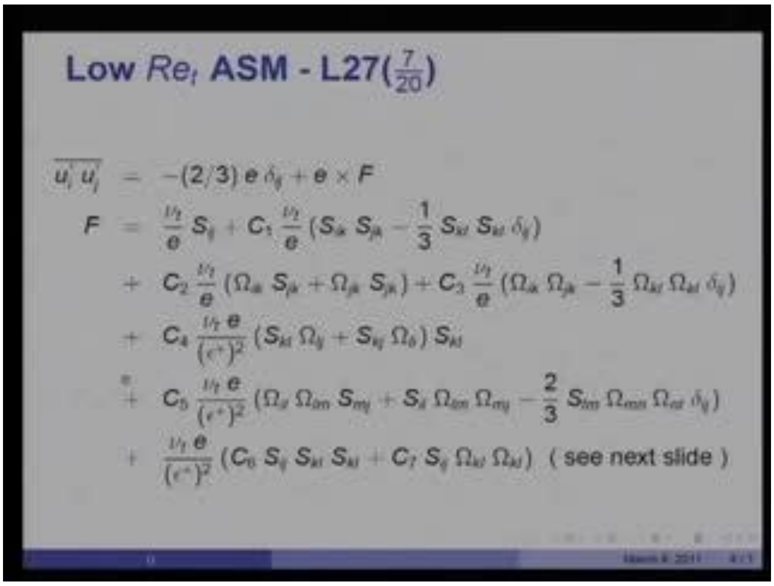

Like the high Reynolds number form of a stress - this is the high Reynolds number form of the algebraic stress model - there is also a low Reynolds number form of the algebraic stress model and that is given as u i prime u j prime equal to minus two-thirds e delta ij plus e multiplied by F and F here is a significantly longer term, it involves a strain rates S as well as vorticity rates omega.

So, simply accept for the moment that this modeled has a basis which I am not going to discuss here, but nonetheless it allows for the fact that the turbulence stress is not only related to its strain rate, but it is also related to its vorticity rate and that is what the next slide will show.

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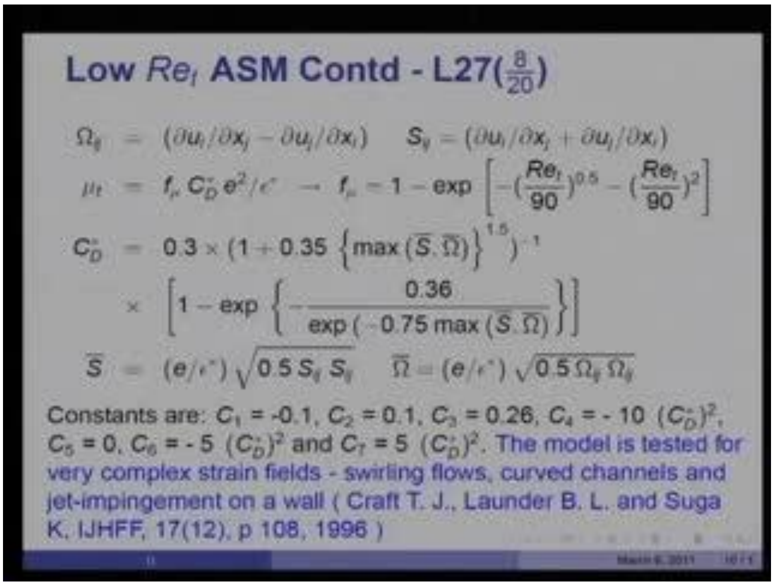

So, omega ij is du i dx j minus du j dx i and S ij is equal to du i dx j plus du j dx i, as we all know. Turbulent viscosity mu t is f mu C D star e square by epsilon star and f mu is sensitized in this way for the low turbulence Reynolds number region. C D star on the other hand is sensitized to the maximum of the mean strain or the mean vorticity multiplied by a damping function. Here, S bar the mean strain rate is taken as e by epsilon star square root of minus 0.5 S ij S ij and mean vorticity is taken as e by epsilon star square root of omega ij omega ij.

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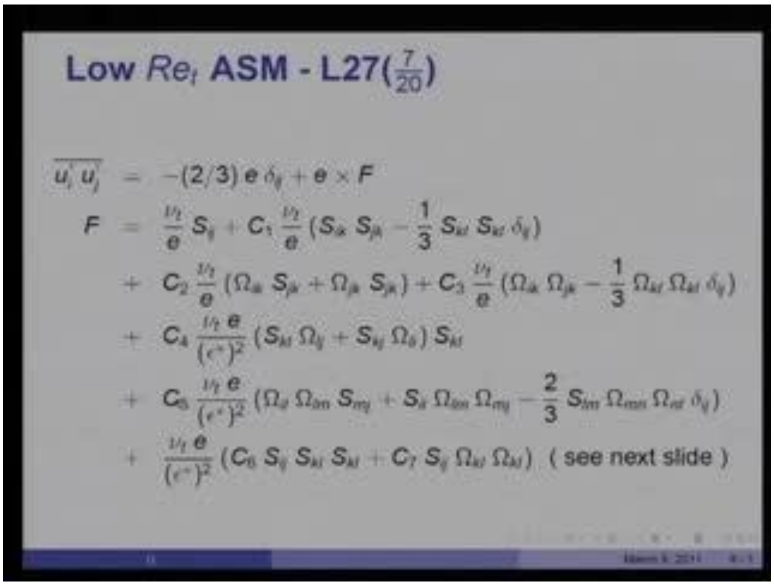

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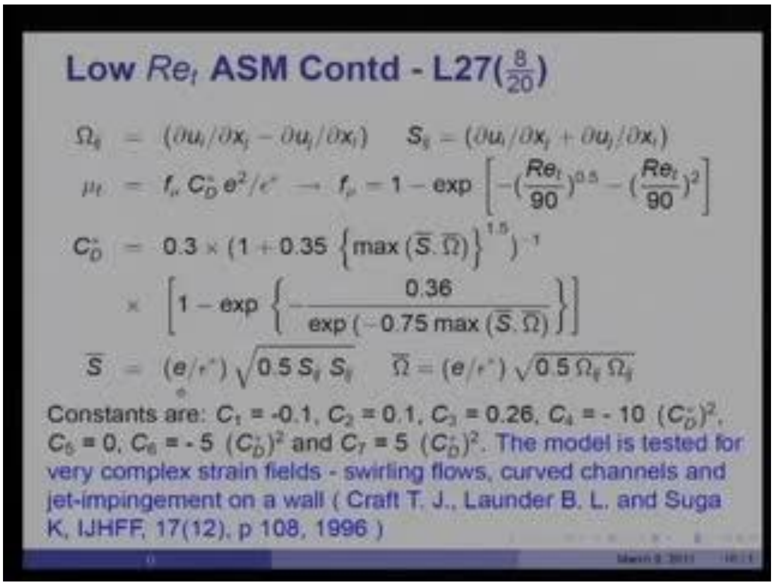

The constants that we are introduced earlier C 1, C 2, C 3, C 4 and C 5 are given here. The C 1 is minus 0.1, C 2 is 0.1, C 3 is 0.26 all these constants where derived from numerical experiments by Craft Launder and Suga at University of Manchester in 1996. The model is tested for very complex strain fields like swirling flows curved channels and jet-impingement on a wall.

The great advantage is, this model is algebraic and therefore, differential equations for stresses need not be solved, but at the same time it captures the effects of the low turbulence Reynolds number regions which are very important when you have very highly strain velocity fields and that is the great advantage of this model. This model has been used to predict quite accurately highly swirling flows in ducts in curved channels and in the situation of a jet impingement on a flat or a curved wall. So, that completes our discussion on high and low Reynolds number forms of turbulence stress.

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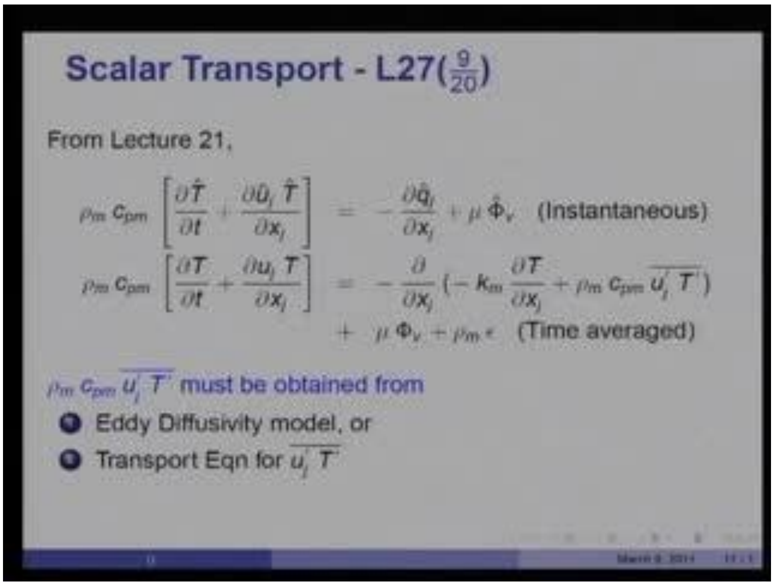

Now, we turn our attention to turbulent flux. You will recall in lecture 21, we have said that we begin by looking at the instantaneous form of the energy equation and then time average it and then, we get this laminar flux and this would be the turbulent flux. The task is to model this term rho m C pm u dash j T dash as the main turbulent flux term. This is the turbulent counterpart of viscous dissipation and this is the turbulent dissipation itself. Now, usually we know this quantity is much smaller than that quantity, how do we model this term? Again like the stress, we can adopt the eddy diffusivity model or we can look for a transport equation for u j dash T dash.

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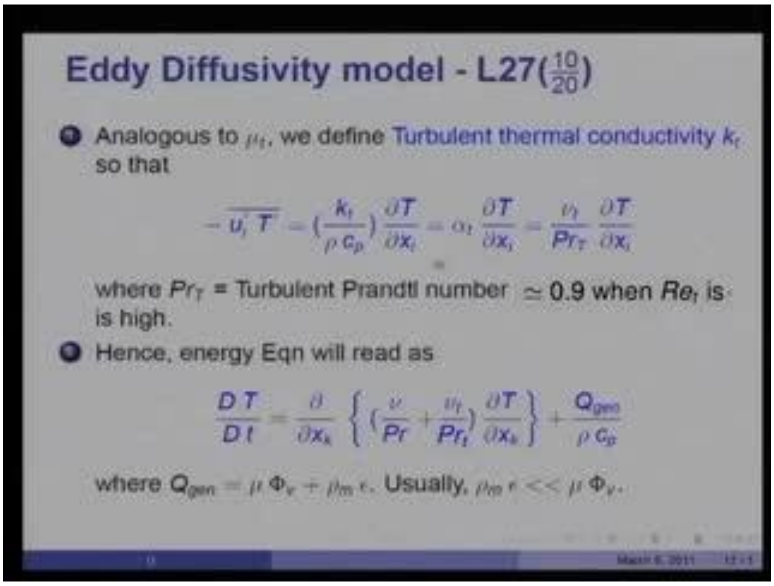

Eddy diffusivity model goes like this; analogous to mu t, we define turbulent thermal conductivity k t, so that minus u i prime T prime would be written as minus k t times rho C p dT by dx i equal to alpha t - the turbulent diffusivity - dT by dx i, which we can also write as nu t - the turbulent kinematic viscosity - divided by turbulent Prandtl number dT by dx i.

Unfortunately, the turbulent Prandtl number turns out to be an absolute constant about 0.9, when the turbulent Reynolds number is high. If you go close to the wall of course, you have to make turbulent Prandtl number a function of the turbulent Reynolds number itself. So, the energy equation would now look like DT by Dt equal to d by dx k nu by Prandtl plus nu t by Prandtl t into dT by dx k plus Q gen by rho C p, where this is the generation term and rho m epsilon usually is much smaller than the viscous dissipation term.

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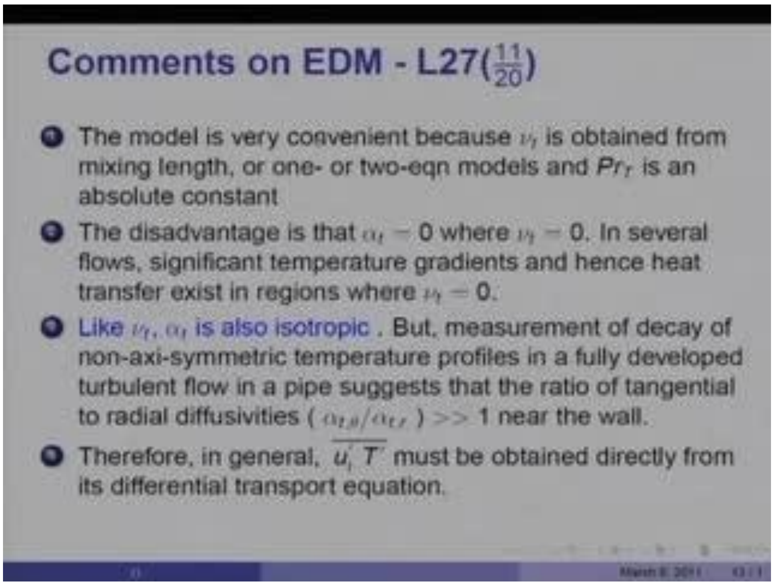

So, what are the comments on the eddy diffusivity model? The model is very convenient because nu t is obtained from mixing length, or one- or two-equation models of turbulence and Prandtl t is an absolute constant. So, everything is very well known and one can solve the turbulent energy equation in the manner of a laminar flow.

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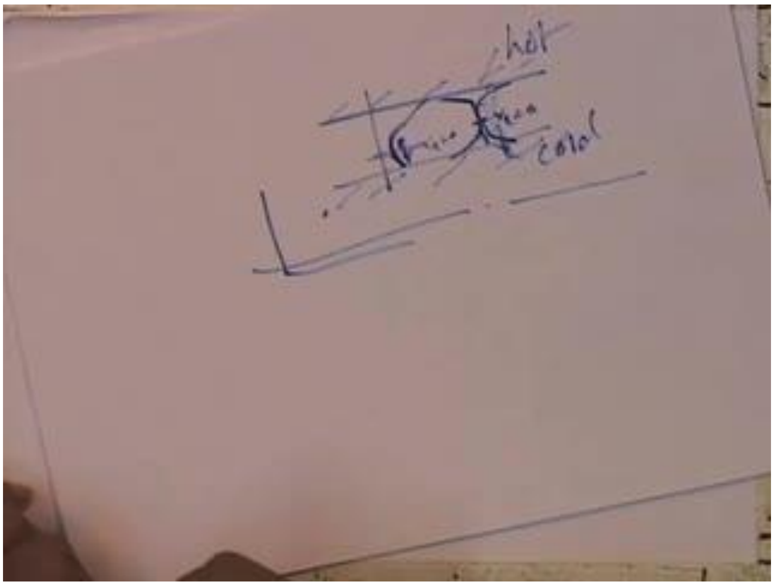

The disadvantage however is that, when nu t is equal to 0 alpha t will also be 0. Now, just imagine a situation, let say, we are talking about the flow in an annulus; this is the axis symmetry then, you will get a velocity profile which would be something like this.

Therefore, nu t will be equal to 0 over there, but now let us say this is hot and this is cold. So, we expect the temperature profile to be something of this type, not like this temperature profile to be something of this type and clearly, there would be temperature gradient here which would cross the heat transfer in that direction whereas, alpha t would be predicted 0 here and that is not acceptable to get practically correct predictions.

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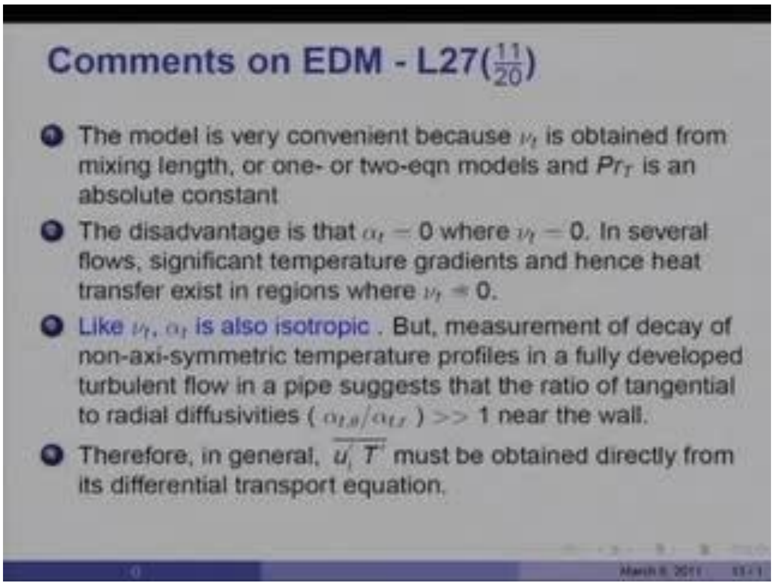

So, in such situations the model really fails but the advantage is like nu t alpha t is also isotropic. Again, measurements of decay of non-axi-symmetric temperature profiles in a fully developed turbulent flow in a pipe suggests that very close to the wall of the tube the tangential eddy diffusivity is actually greater than the radial diffusivity.

Actually, the turbulent eddy diffusivity itself is not isotropic as one move close to the wall, that fact cannot be captured by the eddy diffusivity model. Therefore, in general u i prime T prime must be obtained directly from its differential transport equations. So, we said after realizing the limitations of eddy viscosity model; we went to the stress equation model, we now having realized the limitations of the eddy diffusivity model you would go to turbulent flux model.

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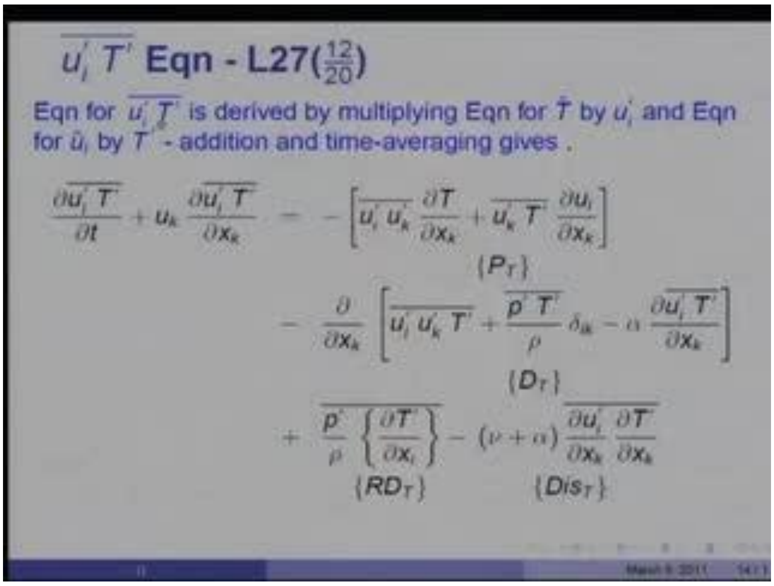

So, the equation for u i prime T prime is derived by multiplying equation for the instantaneous T by u i prime and equation for instantaneous u i by T prime the turbulent fluctuation adding and time averaging. Now this process gives rise to a convection of u i prime T prime, production of u i prime T prime, diffusion of u i prime T prime; firstly, by pressure fluctuations and secondly, by velocity fluctuations. This is the diffusion of the turbulent flux due to laminar diffusivity again like the pressure strain term, this is the redistribution term and then, there is the dissipation of u i prime T prime.

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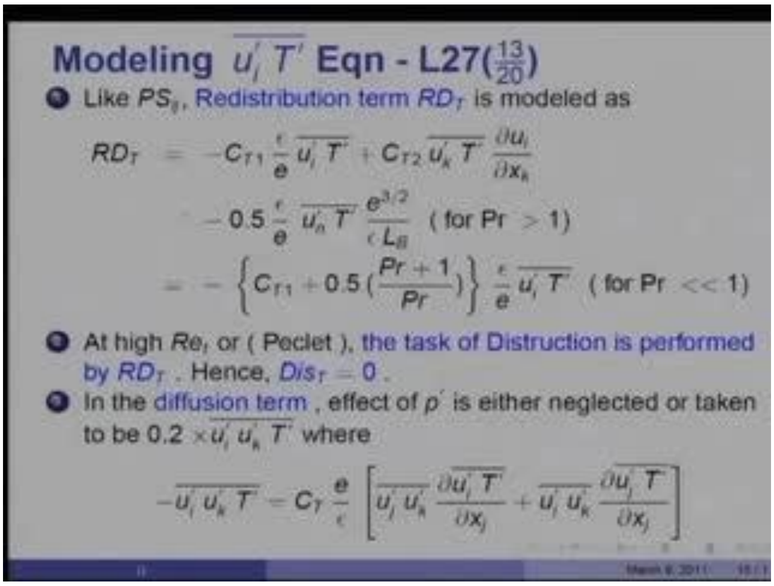

Again, you get very similar terms to those you had one at seen in the stress equation model. Like the pressure strain term the redistribution term is modeled has minus C t1 epsilon by e u i prime T prime plus C t2 u dash k T dash du i dx k this whole thing should be in the bracket equal to and this is further written in this manner for Prandtl greater than 1 and in this manner for Prandtl very small that is liquid metal heat transfer then, that is how the term is represented.

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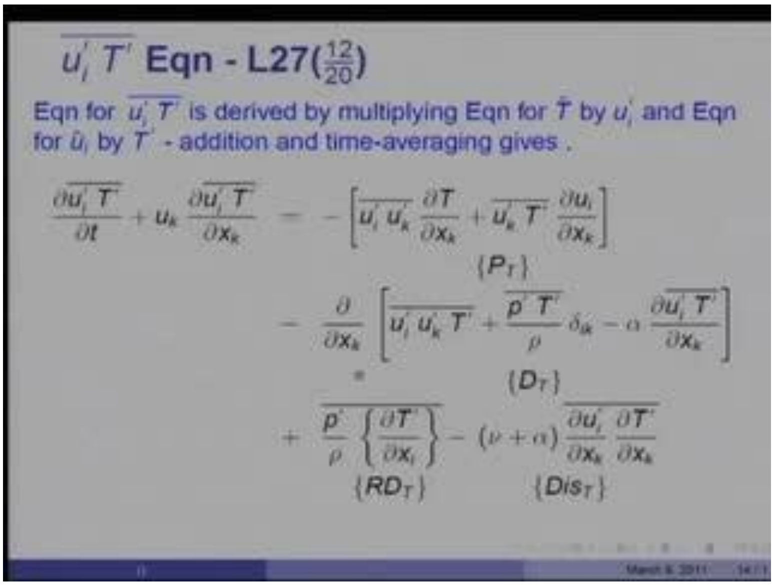

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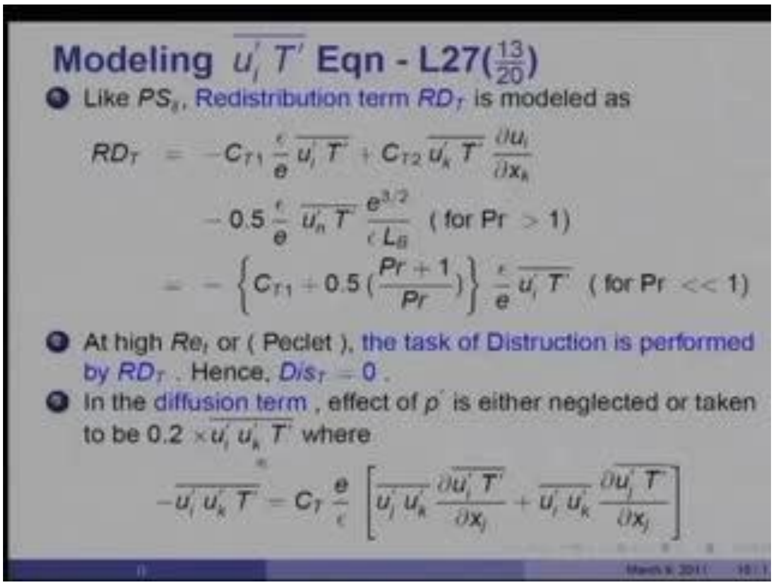

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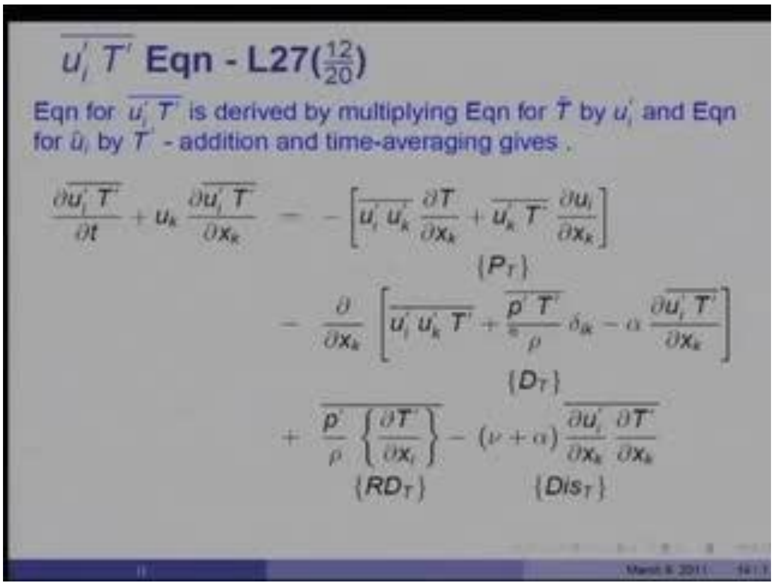

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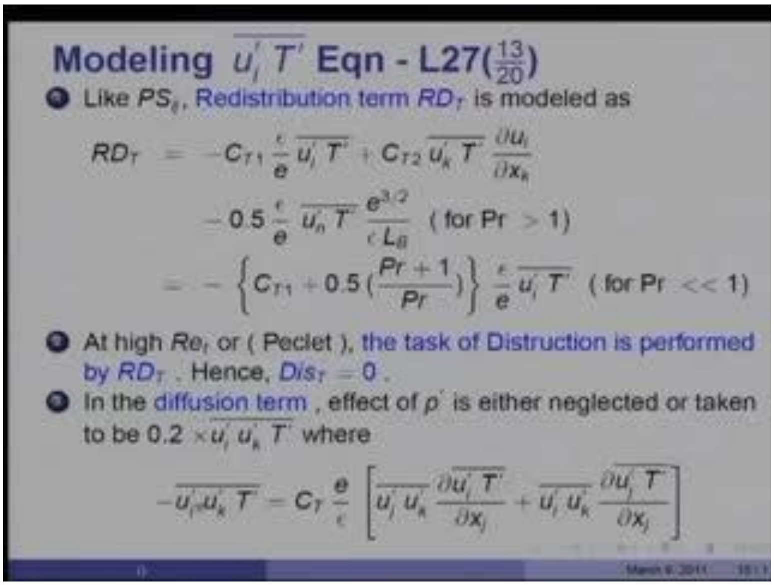

At high Re t or Peclet number the task of destruction term is performed by RD t and therefore, usually the destruction term that we mentioned here is taken to be 0. In diffusion term effect of p dash is either neglected; we either neglect this term all together or take it to be some fraction of that term and finally, the u i prime u k prime T prime is modeled as C T into a stress multiplied by the gradient of the flux in different directions.

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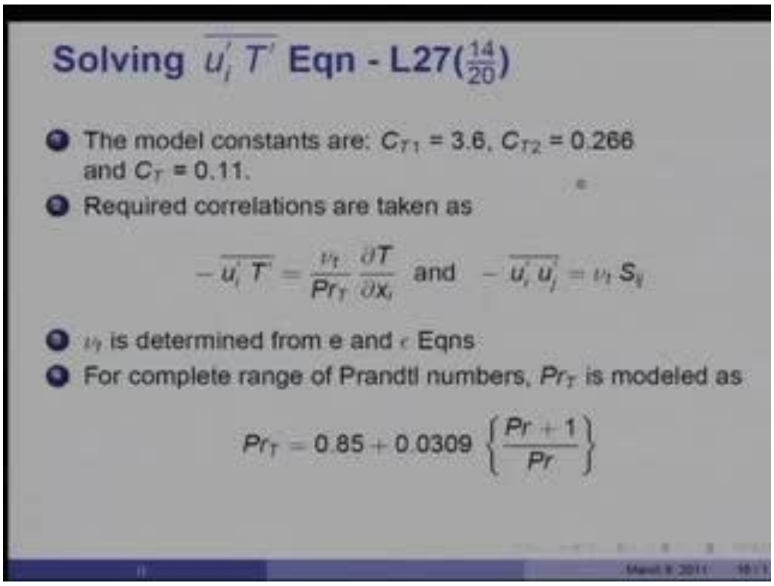

The model constants are C T1 3.6 C T2 0.266 and C T equal to 0.11 the required correlations are taken from the eddy viscosity model wherever required. For complete range of Prandtl numbers Prandtl T is modeled in this fashion. So, you will see that a Prandtl number was of the order of one then, this would be simply 2 and this term will be about 0.61 when added to that will give you about 0.92 or 0.91, but for very large Prandtl numbers the whole term would simply be about 0.88.

So, for Prandtl number greater than 1 Prandtl t equal to 0.9 can be taken to be fairly good representation. The term really makes a contribution for liquid metal heat transfer when the Prandtl number will be of the order of 0.002, 0.003 so on so forth then, the turbulent Prandtl number can exceed 1 itself.

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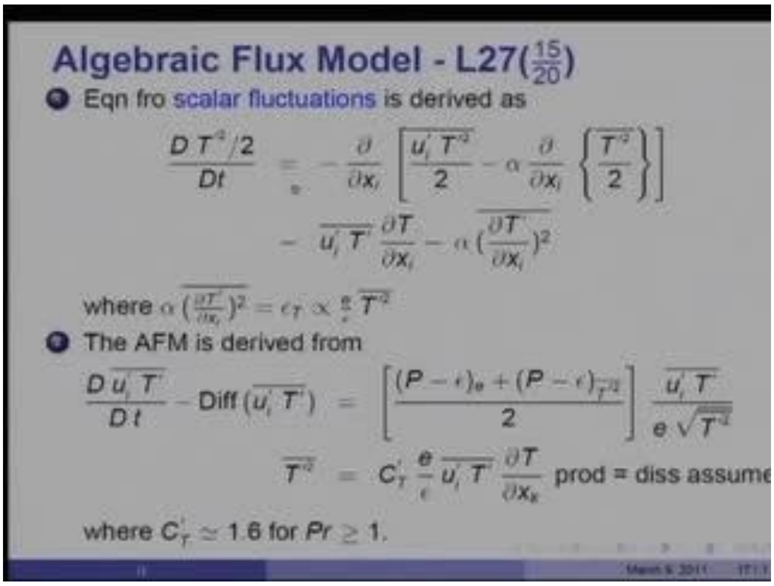

Like we derive the algebraic stress model, we can also derive algebraic flux model, but to do that we need to first of all derive an equation for turbulent temperature fluctuations and that is shown here. One can say T prime square by 2 is really - there should be a bar on top is - really there is some kind of a kinetic energy of the fluctuations if you like of temperature equals a diffusion of the same quantity into laminar diffusion and turbulent diffusion, the production of the same quantity and its destruction that they are the dissipation.

Now, it is this quantity which is usually modeled has being proportional to e by epsilon T prime squared that is, the dissipation rate of the turbulent temperature approximation is modeled as proportional to e by epsilon T prime square, so that term would be their e by epsilon T prime square.

Now, we can say algebraic flux model we would postulate that the convection minus diffusion of the flux would be taken as production minus dissipation of the kinetic energy plus production minus epsilon of T prime square divided by 2 some kind of averaging of the kinetic energy and turbulent fluctuation square into u i dash T dash divided by e into square root of T prime square.

This is pure pragmatism in order to obtain an algebraic equation for u i prime u T prime and T prime square would be C T dash e by epsilon u i prime T prime dT by dx k from production equal to dissipation that is, ignoring the convective and diffusive part we would get that.

So, one substitute for T prime square here minus dissipation gives you this quantity. Therefore, one can obtain u i prime T prime all these quantities are known very well from these two expressions. C T dash is taken as 1.6 for Prandtl greater than or equal to 1 for liquid metals things are somewhat little more difficult and therefore, not discussed here but algebraic flux model again says considerable computational time.

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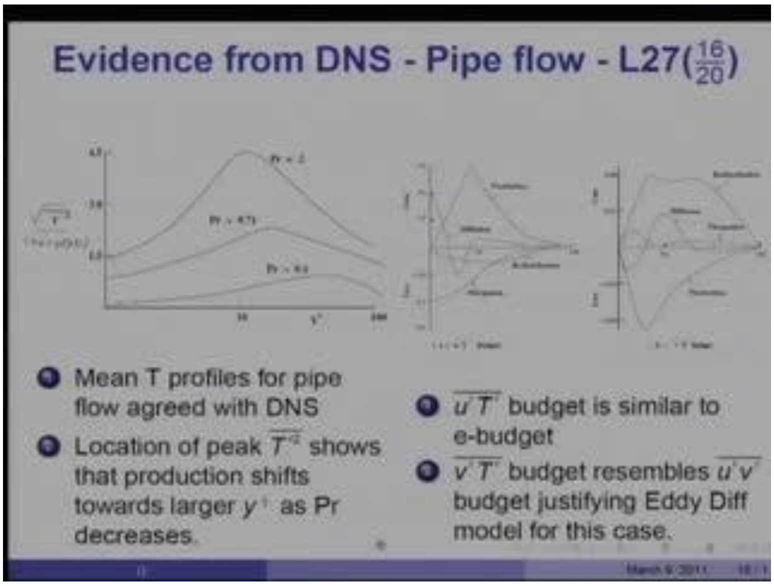

Now, there is evidence from direct numerical simulation of heat transfer or the turbulent fluctuation equation in pipe flow. So, the computations were made for Prandtl number 0.1, 0.7 and 2, I am showing here the inner region less than 100. So, first of all a comment is in order to say that the mean T profiles for pipe flow agreed with the DNS status. So therefore, we can say that at least the time average temperature profiles where in agreement with the measurements made at these three Prandtl numbers.

The left figure shows that the peak of T prime square shifts towards larger y plus as Prandtl number decreases as you can see and this is very much true that the production of the turbulent temperature fluctuations are closer to the wall at higher Prandtl numbers then, they are at low Prandtl numbers.

These two are the budgets for u prime T prime which looks very similar to the budget we had seen for turbulent kinetic energy and v dash T prime budget resembles the u dash v dash budget or the shear stress budget we had seen in the earlier and we justify the eddy diffusivity models.

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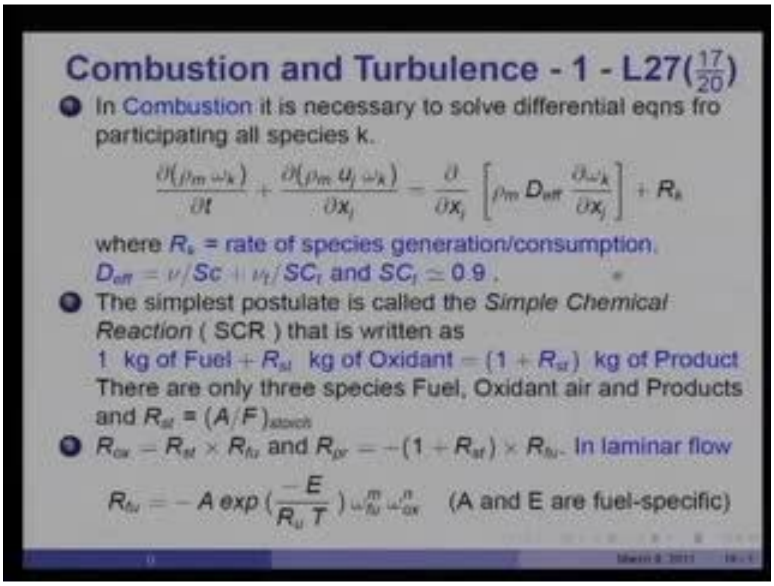

Lastly, I turn to the topic of interaction between combustion and turbulence. In combustion, it is necessary to solve for reacting species and product species that are postulated in a reaction mechanism. The equation reads like this, for each species k the mass fraction omega k convection of that into a equal diffusion of that plus the rate of production or destruction of that depending on the sign of R k.

So, D effective like nu alpha effective is the effective mass diffusivity and it is taken as nu divided by Schmidt number plus nu t divided by turbulent Schmidt number and at least in gaseous flows it is quite close to 0.9 like the turbulent Prandtl number.

Now, the simplest postulate of the reaction mechanism is the simple chemical reaction SC r which is written as 1 kilogram of fuel plus R st kilogram of oxidant equals 1 plus R st kilogram of product. So essentially, we have three species fuel, oxidant and product without defining what they are and R st will be the A by fuel stoichiometric air by fuel ratio.

Then, the rate of consumption or generation of oxidant air would be R st times their rate of consumption or destruction of fuel or product would be on the other hand will be minus 1 plus R st times R fu. Now, if it was the laminar flow R fu would be given by minus A exponential of E by R u T omega fu m omega ox n where A E m and n are fuel specific constants.

In a turbulent flow what we would like to do is to simply replace this omega fu by time average omega fu bar and omega ox bar, but they are not difficulties and that is what we want to appreciate on the next slide.

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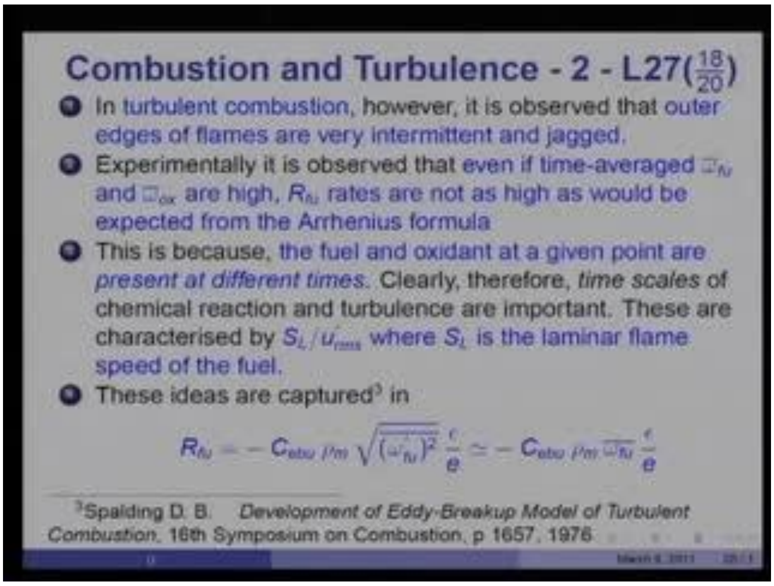

So, in turbulent and combustion it is observed that the outer edges of the flames are very intermittent and jagged, it is in this regions that really greater part of combustion takes place. Experimentally, it is observed that even if the time average omega fu bar and omega ox bar are high the actual R fu rates are not as high as would be expected from the Arrhenius formula that I showed on the previous slide.

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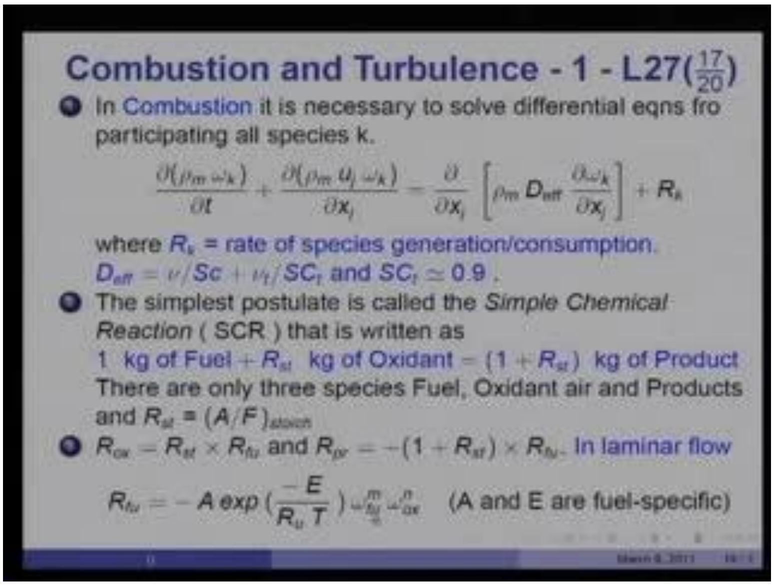

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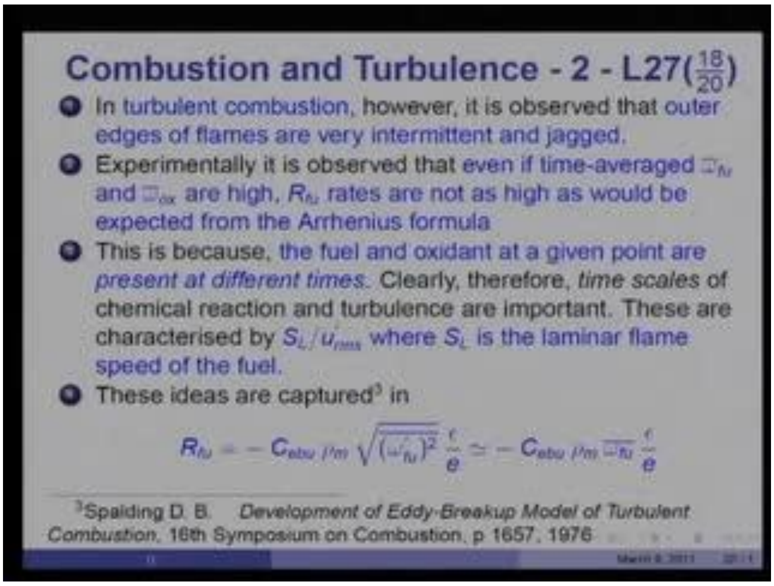

So, if you take bar values here the value of R fu predicted turns out to be much greater than actually what is observed in the extreme. Now, why does this happen? This happens because the fuel and oxidant at a given point are actually present at different times, because of the turbulence velocity fluctuations which actually transport the fuel. Although the time average values may be high, the actual reacting fractions are tend to be smaller.

Clearly therefore, the time scales of chemical reaction and turbulence must be considered simultaneously. Now of course, the subject matter governing these issues is quite large but here we would simply say that these time scales are S L by u dash in the rms sense where S L is the laminar flame speed of the fuel under question. This ratio determines where the reaction is dominated by combustion time scale or whether it is dominated by the turbulent time scales.

Now, these ideas are captured in a model suggested by Spalding, it is called the development of the eddy break up model of turbulent combustion presented at the 16th Symposium on Combustion in 1976.

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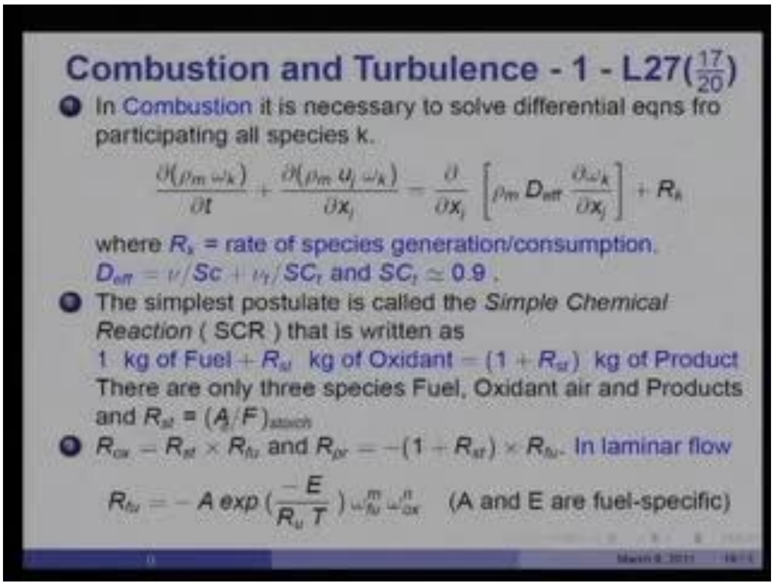

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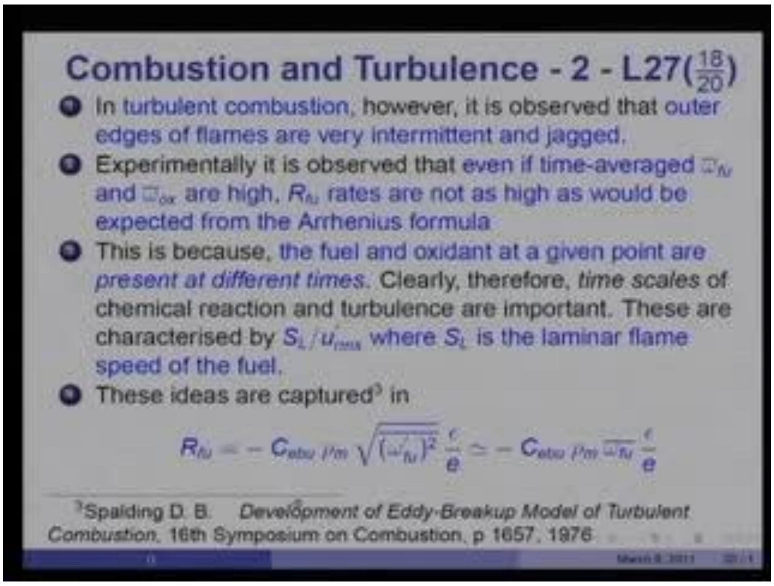

Spalding said that instead of the Arrhenius form that is used here what one have to use is R fu equal to minus C a constant into rho m into scalar fluctuation omega dash fu whole square which in practical computation is taken to be almost equal to omega fu bar and it makes very little difference only thing is, you must adjust C ebu appropriate plane.

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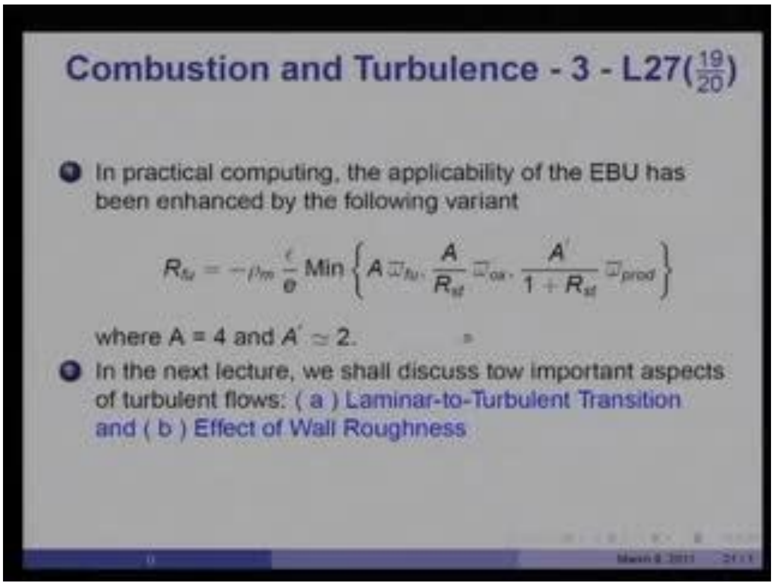

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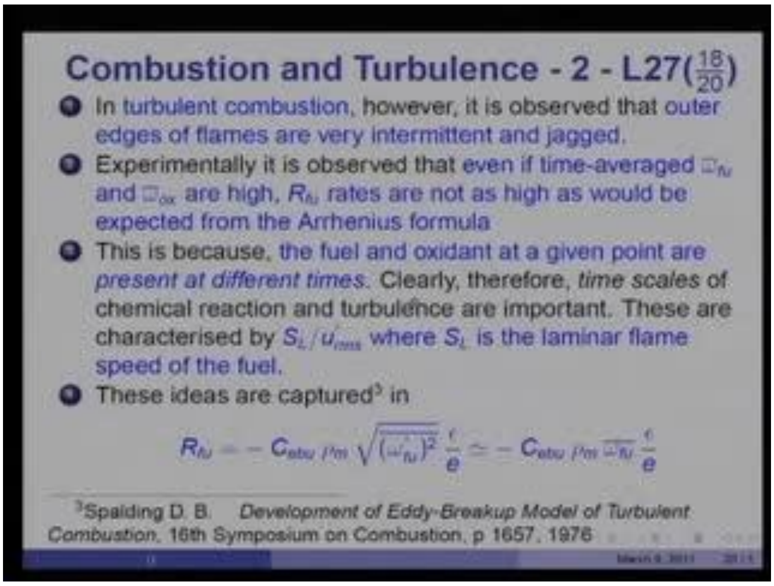

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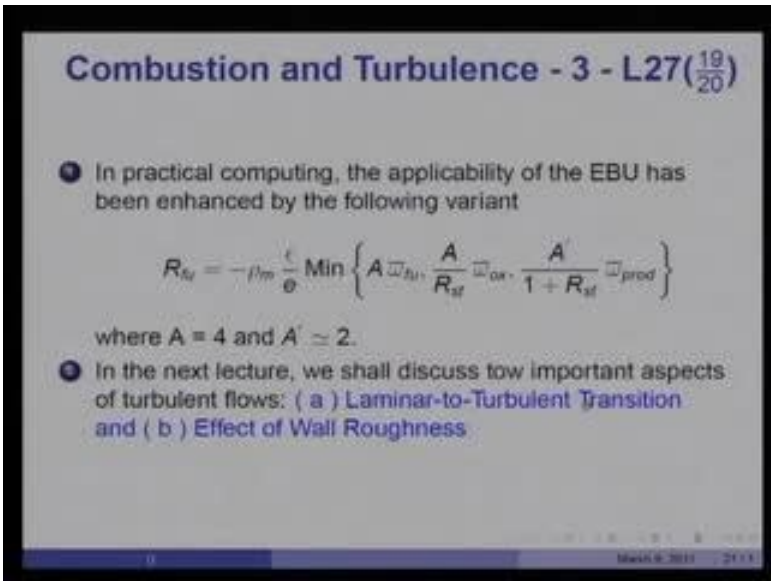

In as much as this model turned out to be quite good for turbulent combustion. Some more refinements have been made and it is said that instead of taking omega fu bar that that you saw here, what one would should take is really the minimum value of A fu bar omega ox bar by R st omega product bar by 1 plus R st with certain proportionality constants and recommended ones are A equal to 4 and A dash equal to 2. This is simply again pragmatism derived from experiments and has proved to be quite good at least in the certain types of combusting products.

In the next lecture, we shall discuss two important aspects of turbulent flows that is, the laminar to turbulent transition and the effect of wall roughness.