Convective Heat and Mass Transfer
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Lecture No. # 25
Near-Wall Turbulent Flows-2

In the previous lecture, we saw that a turbulent flow past a wall can be differentiated into two main layers: the inner layer and the outer layer. And we said the inner layer occupies about 15-odd percent of the total layer thickness, say the boundary layer thickness or the radius of the pipe, and the outer layer about 80 to 85 percent. The inner layer itself has three further layers within it: the closest to the wall is almost laminar layer, then there is a transitional layer, and then, the fully turbulent layer. And we were able to develop a set of laws governing distribution of velocity with respect to the wall normal distance, while in terms of dimensionless variables u plus and y plus, and we found that these, this relationship between u plus and y plus, was quite universal, at least up to y plus of 100, which is really the 15 percent of the layer.

Today we wish to examine how the inner layer interacts with the outer layer and we shall use that understanding in a later lecture, to model, to carry out turbulence modeling.

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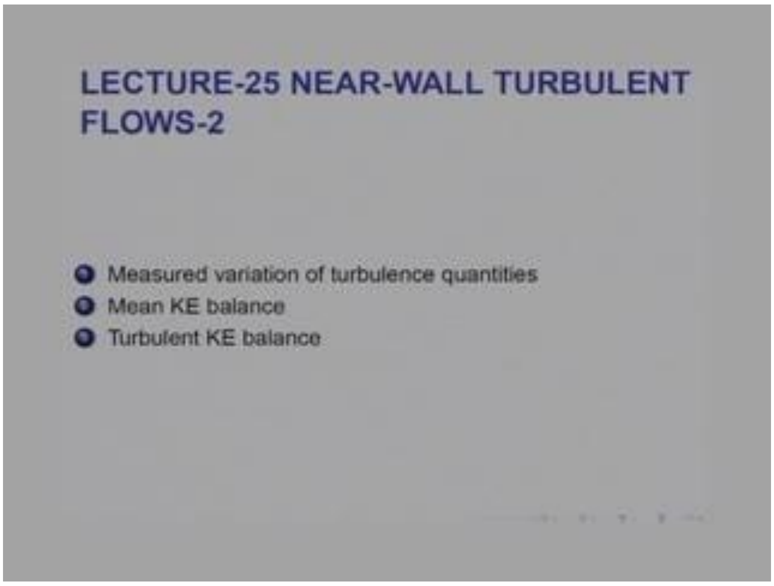

And we will develop this understanding through measured variations of turbulence quantities. We will look at the mean kinetic energy balance in the turbulent layer and also the turbulent kinetic energy balance in the layer, and that is how we will see what gives way, and what gains, in this interaction between outer and inner layers. So, let us have a first look at the measured variation of turbulence quantities.

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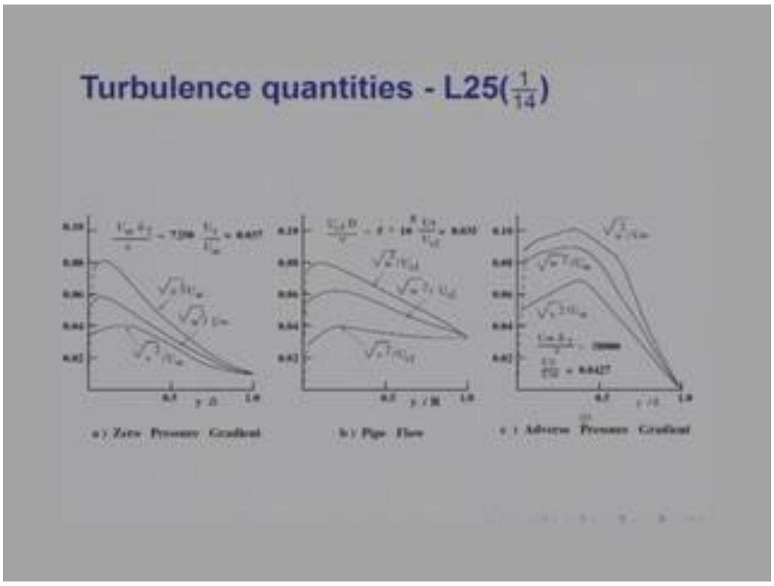

Here is a three plots, the one on the left (a) is called the zero pressure gradient boundary layer, (b) is for a mildly favorable pressure gradient as in a pipe flow, and the one on the right, (c), is quite fairly strong adverse pressure gradient boundary layer. What is plotted here is the u prime squared divided by u infinity v prime squared divided by u infinity and w prime squared divided by u infinity and versus y divided by delta going up to 1.

And u can see, the tendency is that u prime square divided by u infinity would be highest in all cases, followed by w prime square, and then, this is the wall normal velocity component - v prime square - which is the lowest. The intensity tends to be high near the wall. Say, at about in a zero pressure gradient boundary layer, the highest intensity would occur at about 15 percent of the boundary layer thickness; very similar even for a pipe flow, which is a mildly favorable pressure gradient. And, but, in a adverse pressure gradient, the peak somehow shifts to a much larger distance, say about 0.35 times delta.

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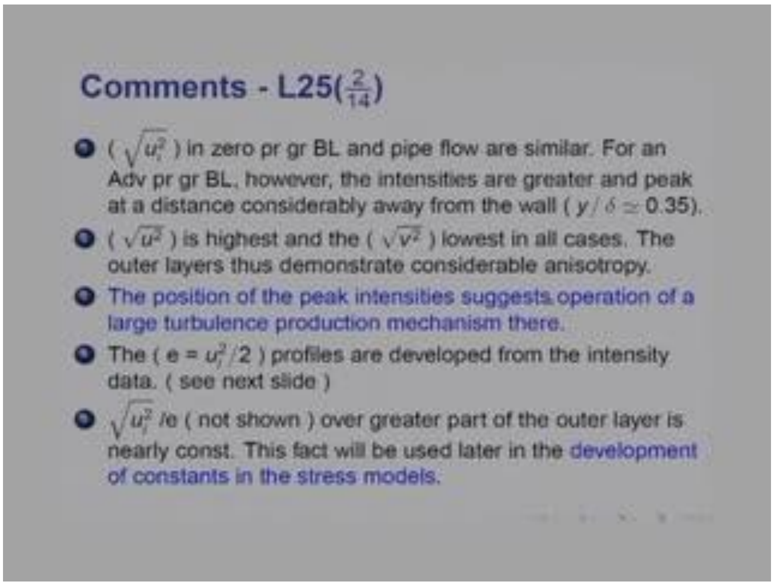

So, u prime squared in a zero pressure gradient boundary layer and pipe flow are very similar. For an adverse pressure gradient of boundary layer, the intensities peak, as I said, at y delta at about 0.35. U prime squared is highest and v prime squared is the lowest in all cases. The outer layer, thus demonstrates considerable anisotropy.

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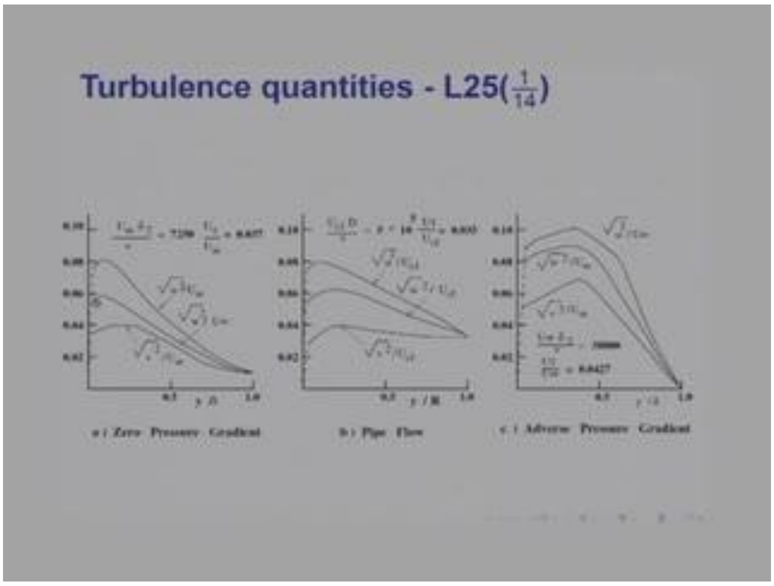

What is meant by anisotropy is that, there is a considerable difference between u prime squared v prime squared and w prime squared. And as I said, from here onwards you have the outer layer, the 85 percent of the layer. But when you come close to the wall, the anisotropy goes on reducing.

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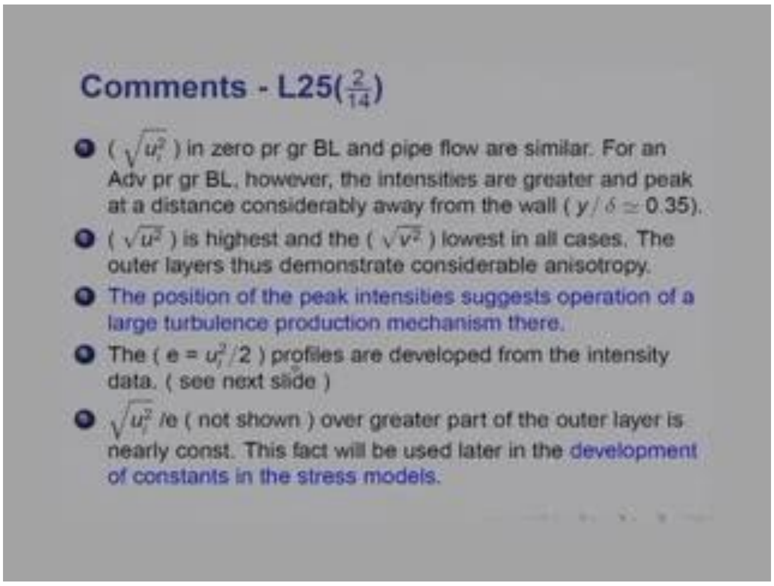

The position of the peak intensity suggests operation of a large turbulence production mechanism there. Close to the wall, apparently at the edge of the inner layer, there seems to be a large turbulence energy production mechanism.

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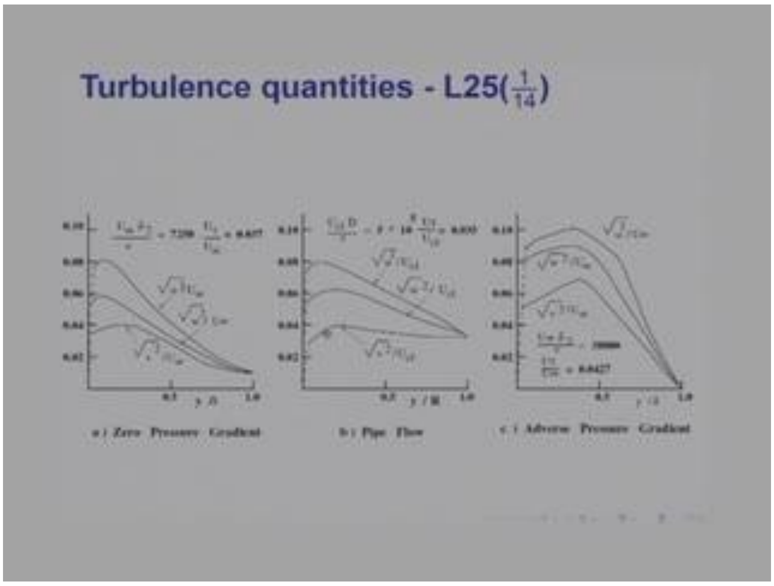

E, which is u prime square plus v prime square plus w prime square; profiles are developed from the intensity data; that is, you simply add up the three components and divide it by 2, will give you the kinetic energy data, which I will show on the next slide.

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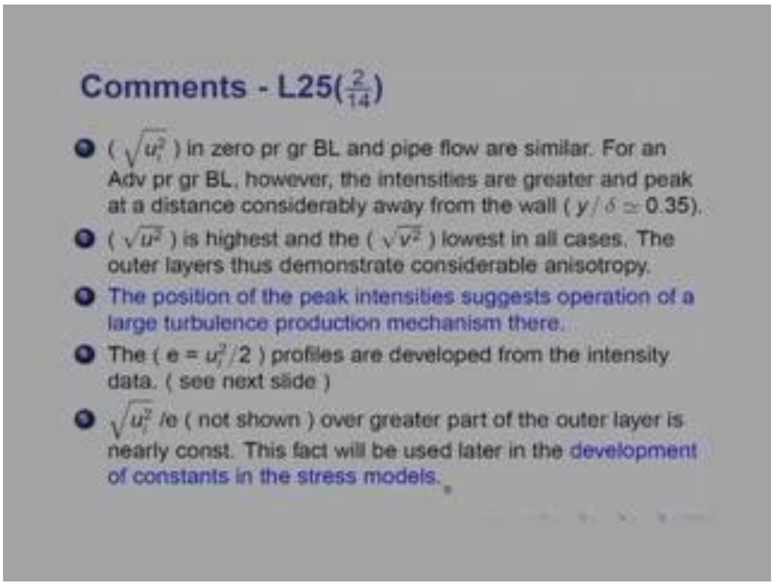

And u i square by e data, although we have normalized with respect to u infinity, you can also get data which is u i prime squared by e, over greater part of the outer layer is nearly constant. This fact will be used in the development of constants in the stress models that we shall discuss later.

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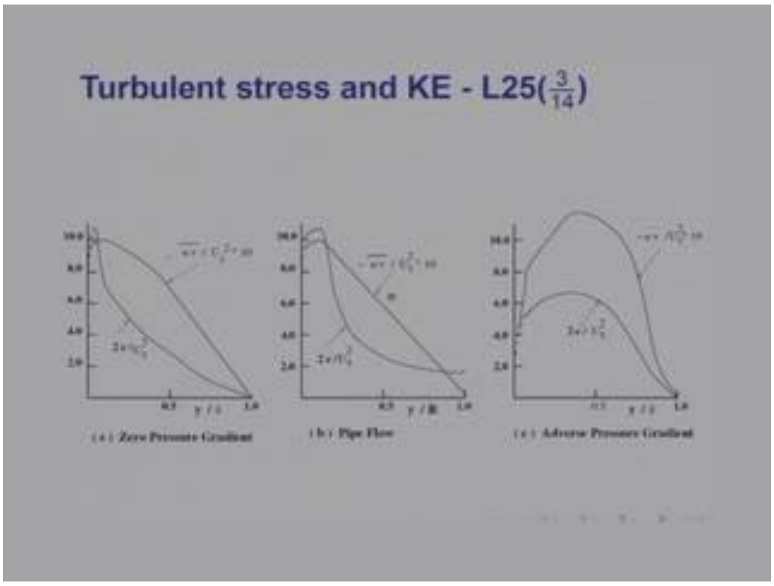

So, now let us look at two other things: called the turbulent stress and the turbulent kinetic energy. So, here is a plot of turbulent kinetic energy divided by friction velocity square, and you can see, that the turbulent kinetic energy indeed very high - of the order of 10 - for this quantity, and it declines through the layer for the zero pressure, and almost goes to 0 at the edge of the boundary layer.

But, in a pipe flow, where the order of magnitude seems to be very similar, the energy divided by friction velocity squared actually goes to it has a zero gradient, no doubt, but, goes to a finite value. This is because of the presence of the wall opposite if you like and this is very typical of channel flows. And in adverse pressure gradient, of course, the peak value shifts as I said to about 0.35, but again at the edge of the boundary layer, the kinetic energy does become 0.

Now, let us look at the turbulent stress u prime v prime bar divided by u tau squared and here it is multiplied by 10 to get the proper scaling. And you can see, that the turbulent stress seems to peak at about 10 to 15 percent of the layer, which is really the edge of the inner layer. And then, would drop down to 0 of course, when the viscosity begins to dominate; very similar tendency here also in a pipe flow, but see how it goes in case of adverse pressure gradient, again the peak occurs much later and there is a decline already of the shear stress. Now, the idea that the turbulent stress is actually linear turbulent; they are very, very linear in the outer parts, very nearly linear. Here as well in pipe flow with respect to distance from the wall, but in adverse pressure gradient, the variation is quite different; it is not at all linear till you went very, very far out into the boundary layer.

Remember, we developed the u plus y plus law, the universal law, by assuming a constant stress layer throughout the boundary layer, but it is remarkable that we were able to still predict a fairly good comparison with experiment for the u plus profile up to y plus of 700, which is almost to the edge of the outer layer. But frankly speaking, the shear stress does total stress, and here in this path, the laminar stress, of course, will be very, very negligible. But as you go close to the wall, the turbulent stress drops down, but the laminar stress would begin to dominate, and therefore, in the inner part of the layer, the constant stress layer assumption is not a bad one. The total stress remains constant in the inner part of the layer and that is what we found - that was the assumption we made in developing the u plus y plus relationships and they were found to be quite good up to 100, but not beyond, let us say, 200 in some cases, 300 or 700 in other cases.

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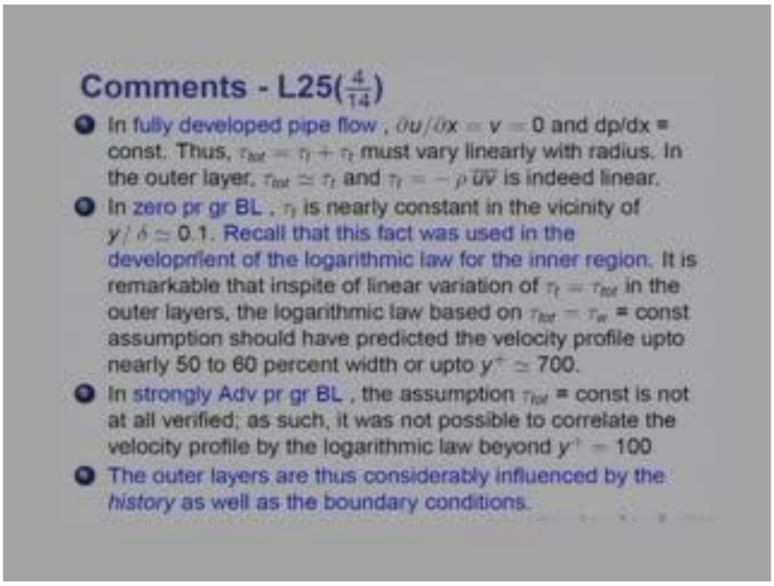

And this is what I discuss here, that in a zero pressure gradient boundary layer, the turbulent stress is nearly constant in the vicinity of the wall of y over delta 0.1.

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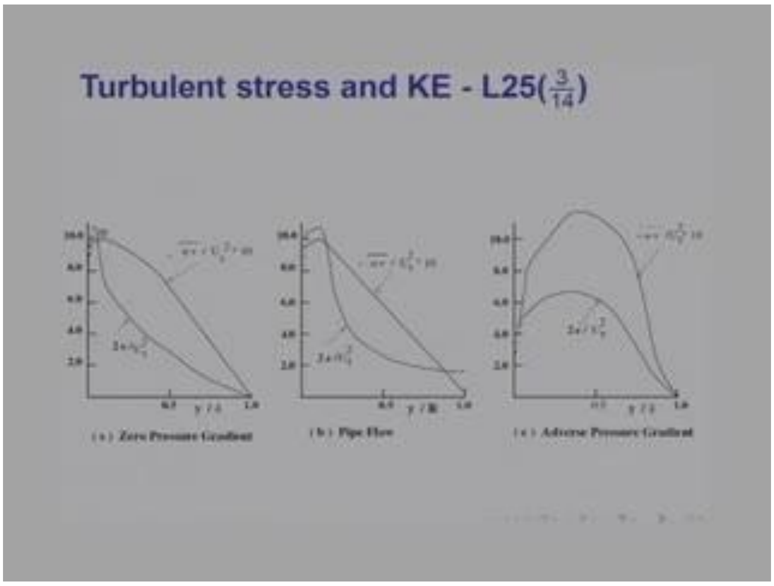

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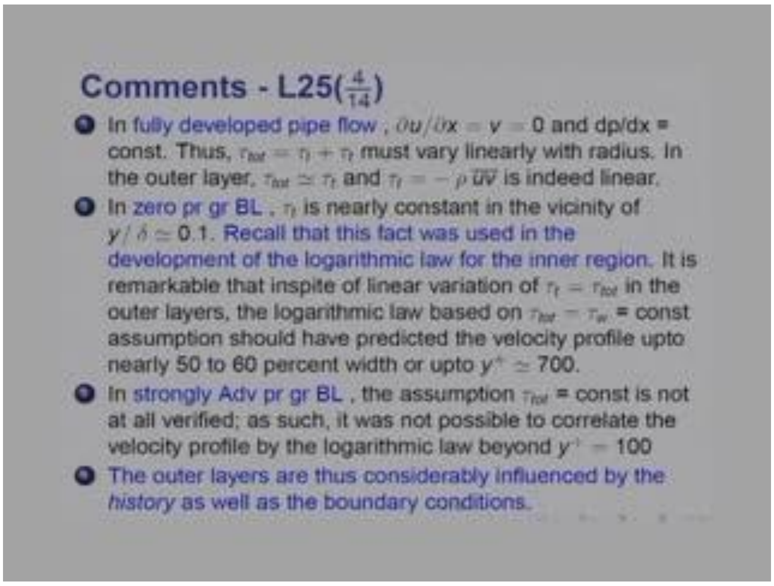

In the vicinity of the wall meaning somewhere about here, but then, of course, it drops down. The turbulent stress would drop down, the laminar stress would take over. So, we develop for the turbulent part of the inner layer using constant tau t equal to tau tot. And we were able to develop the logarithmic law for the inner region and we found that the we were able to get good agreement even up to y plus of 700, which is really surprising, but nonetheless quite good that the constant stress layer still is able to predict u plus y plus relationship.

In strongly adverse pressure gradient boundary layer, the assumption of tau tot equal to constant is not at all verified. As such it was not possible to correlate the velocity profile by the logarithmic law beyond y plus of 100. And the outer layers are thus considerably influenced by the history as well as the boundary conditions and that was our conclusion even earlier.

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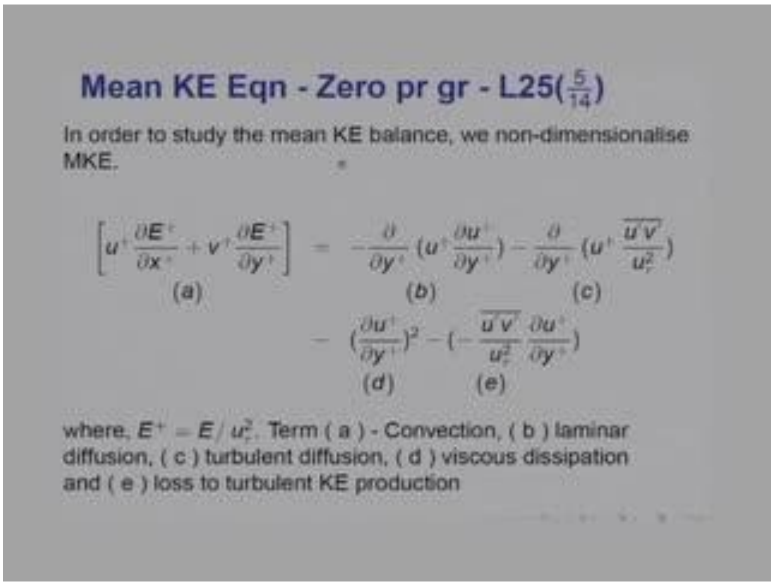

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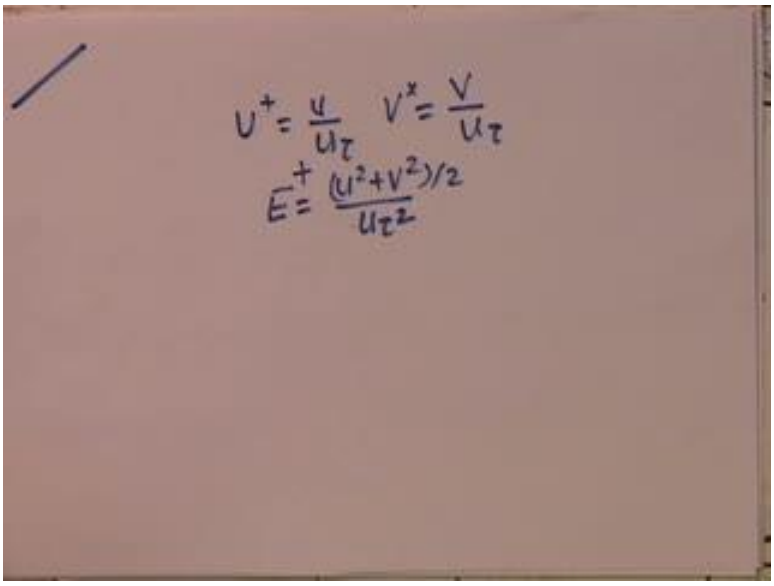

Now, let us look at the mean kinetic energy equation for a boundary layer and what I have done here is u and v have been normalized to u plus equal to u divided by u tau and likewise v plus equal to v divided by u tau. Energy is likewise u squared plus v squared divided by u tau, of course, divided by two u tau squared. So, that is the energy e plus. So, if u look at the equations here, term a is the convective term, term b is the laminar diffusion term, term c is the turbulent diffusion term, d is the viscous dissipation term and term e is the loss of mean energy due to turbulent kinetic energy production. So, let us see how each of these terms varies; by using our measured velocity profiles, we can construct the magnitudes of each of these terms and let us see what they look like.

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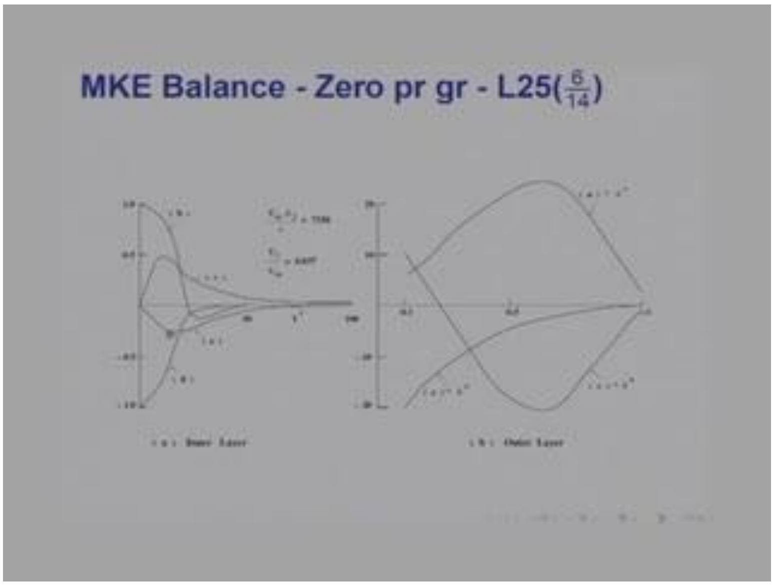

The figure on the left shows the inner layer going from 0 to y plus of 100 and this is y by delta now, from 0.1 onwards if you like; so, this figure ends here, and now in the physical coordinates y by delta, but here it is the dimensionless coordinates y plus have been used. So, you can see here the laminar diffusion is very, very strong close to the wall, we said viscosity does play its role a in diffusion and it is almost completely balanced by the viscous dissipation. That is the term d here, term c and term e, we will recall are… term c is the turbulent diffusion and term e is the loss of energy to production of turbulent kinetic energy. And that is what you see here that c, the turbulent diffusion term is positive in the 0 to y plus up to 100; whereas, the term e, the loss of energy to turbulent production turbulent kinetic energy production, is negative; that means, the energy is being given away to produce turbulence in this region of about y plus of 20 onwards; I mean it is maximum at about y plus of 20.

If you go to now outer layers, and here each of the term has been multiplied by delta plus, and delta plus as you know depending on where you are in the turbulent boundary layer, would be of the order of 800 to a 1000 or 1200.

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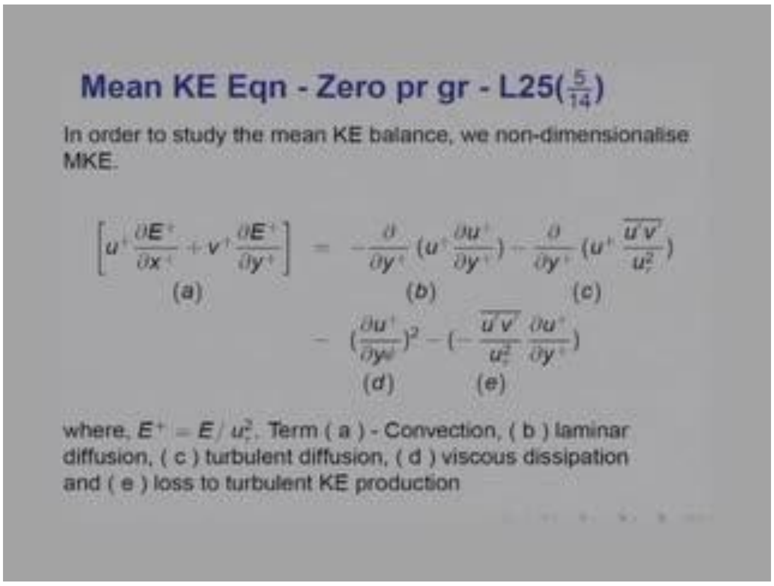

Now, you see the point, the convection of mean energy, which is this term is almost completely balanced by turbulent diffusion term - c term - and also by viscous dissipation term - which is the terms e here; sorry, this is the turbulent kinetic energy production term and this is the turbulent diffusion term. So, this term and this term dominate in the outer layers; whereas, this being laminar viscosity and laminar dissipation they are not that significant in that part.

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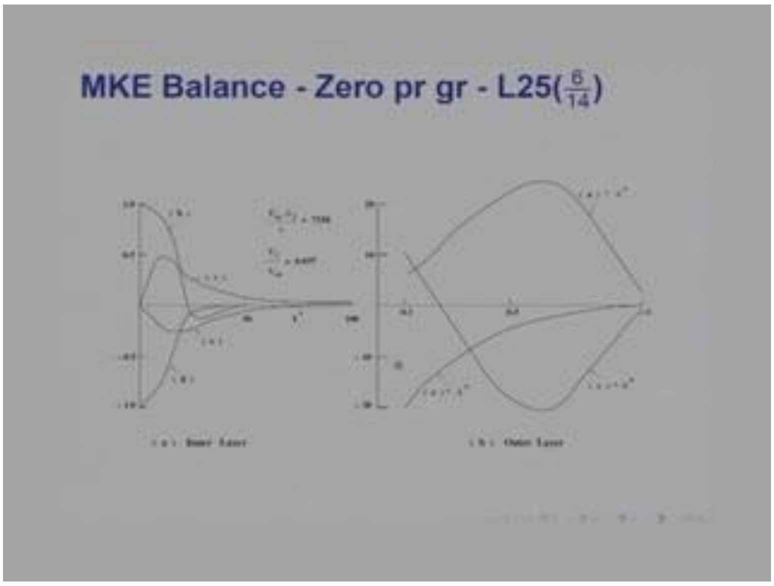

So, you will see that convection of mean energy is completely balanced almost by turbulent diffusion and by production of energy or turbulent kinetic energy, which is of course, it has a largest magnitude close to the wall about 0.1 and then it goes on reducing.

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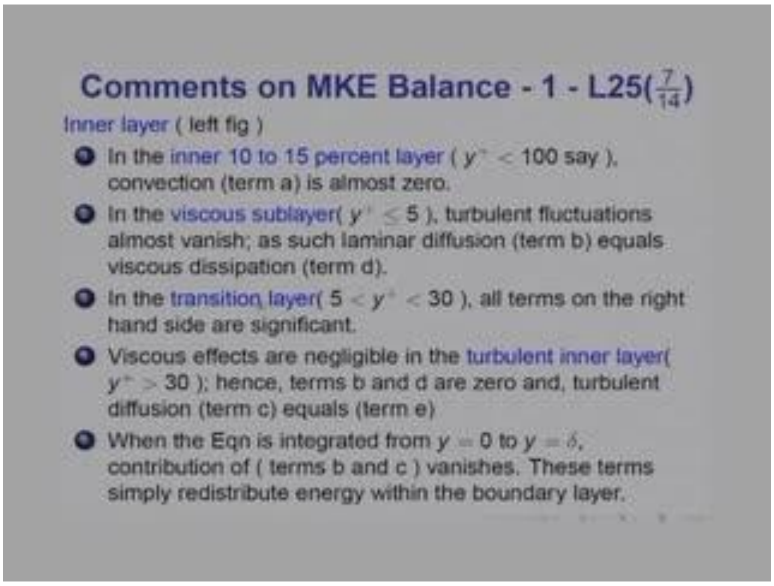

So, I made these comments in words now, that the in the inner 10 to 15 percent of the layer, y plus less than 100, convection term a is almost 0.

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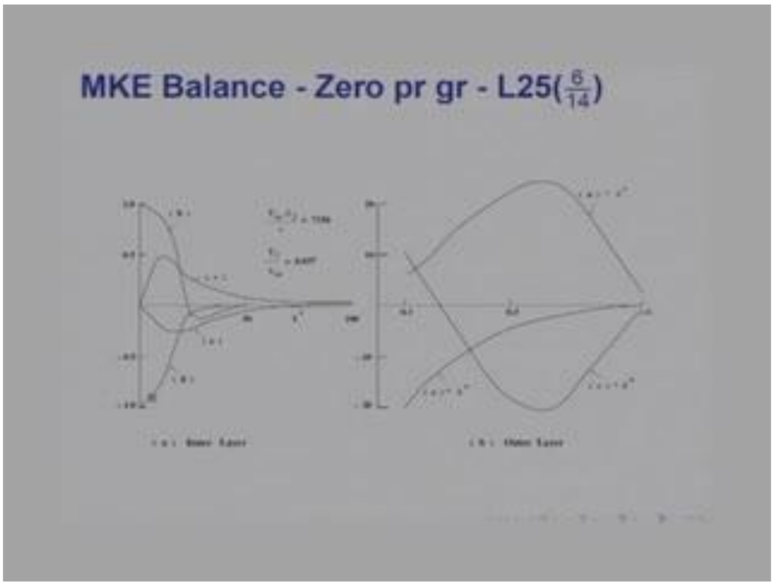

As we had assumed in our development of the log law, in the viscous sub layer, y plus less than 5, turbulent fluctuations almost vanish, as such the laminar diffusion term b equals the viscous dissipation term and that is what I had showed here - that the viscous y plus of less than 5 which is the laminar sub layer; you will see all other terms are almost 0 and viscous diffusion is balanced by viscous dissipation.

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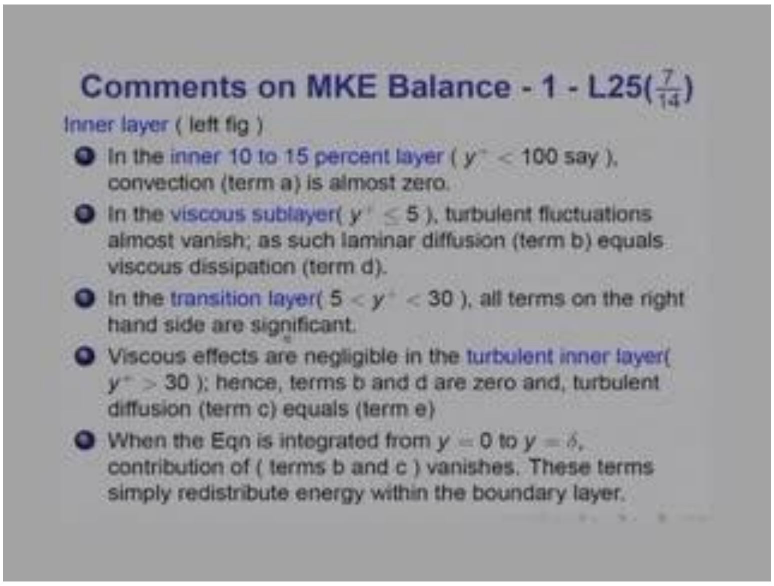

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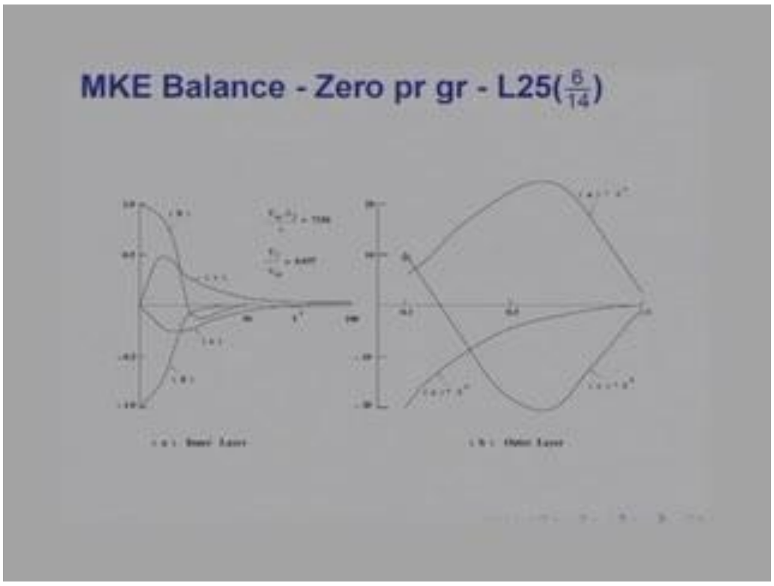

In the transitional layer, 5 to 30, all terms on the right hand side are significant and that is what we showed here. Now, you can see from around from 5 to about 30, all terms are significant - viscous diffusion, turbulent diffusion, turbulent viscous dissipation and transfer of energy to turbulent kinetic energy production. All are significant up to about, let us say 30 here, and then you find that the terms being the viscous diffusion and viscous dissipation terms, almost become negligible, and these terms assume a very small value, and that is why I have multiplied these terms by a factor of 10 to show how they respond in the outer layer. So, this point here corresponds to that point over there after multiplication by 10.

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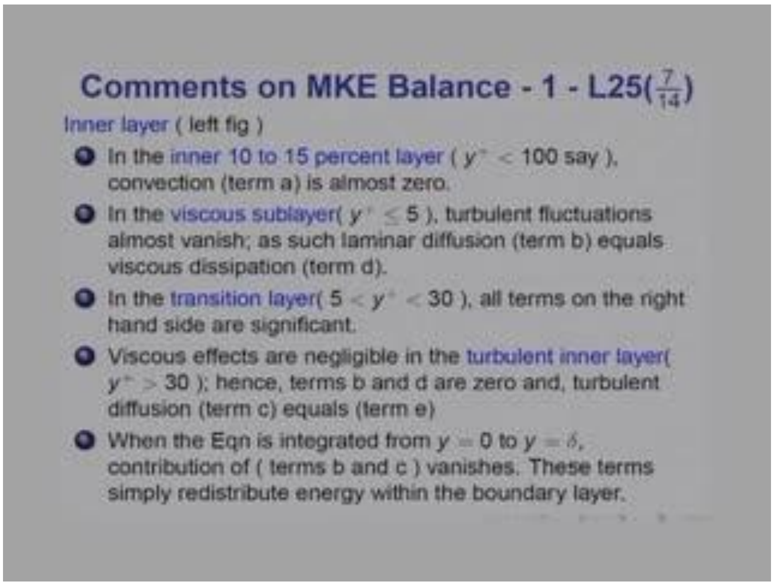

The viscous effects are negligible in the turbulent inner layer y plus greater than 30, hence the terms b and d are 0, and turbulent diffusion equals term e - the turbulent production of energy to turbulence.

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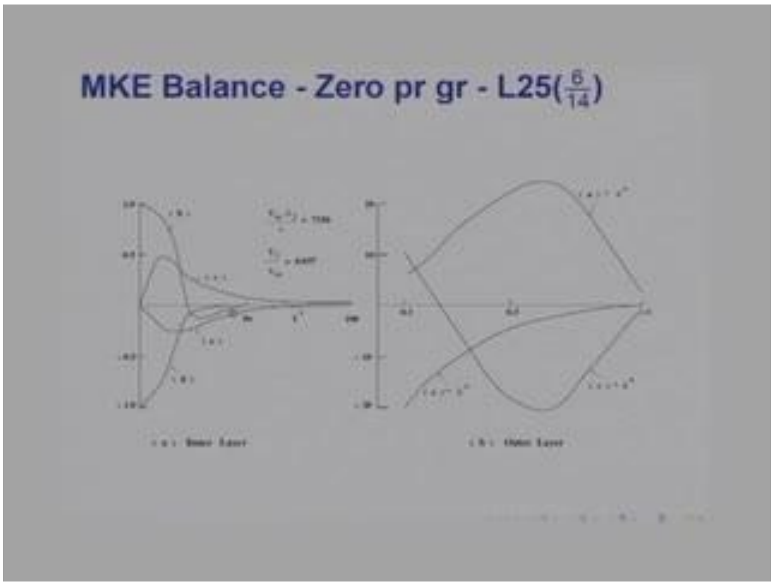

When the equation is integrated from y to delta, contributions of terms b and c vanishes; as you can see c was positive here, and then still positive here, becoming negative. So, the area under this part and under this part, just cancel each other; likewise the laminar diffusion which is negative here, and it turns slightly positive; the laminar diffusion is here it is positive, and then goes up to negative; and therefore, itself cancels up right here within the inner layer.

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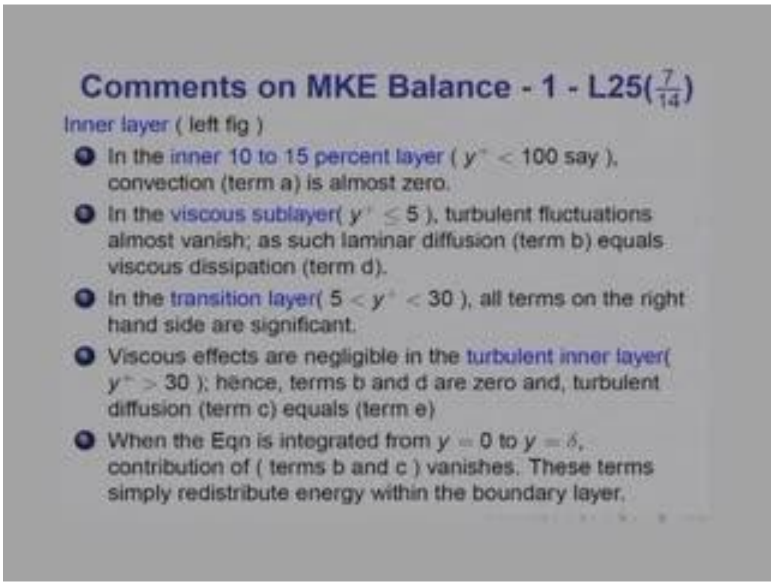

These terms simply redistribute energy within the boundary layer.

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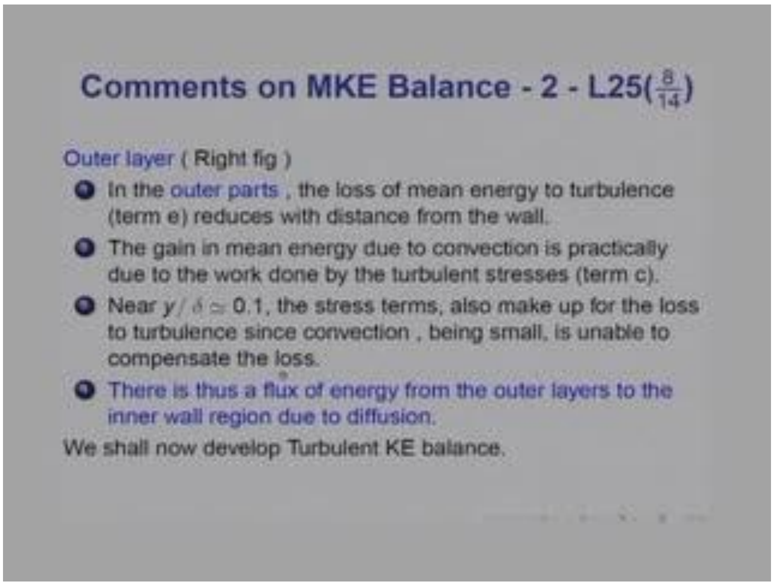

Now, if you look at the right figure on the outer parts, then the loss of mean energy to turbulence term e reduces with distance from the wall; the gain in the mean energy due to convection is practically due to the work done by turbulent stresses - this is the term c; near y by delta equal to 0.1, the stress terms also make up for the loss to turbulence, since convection being small is unable to compare this loss.

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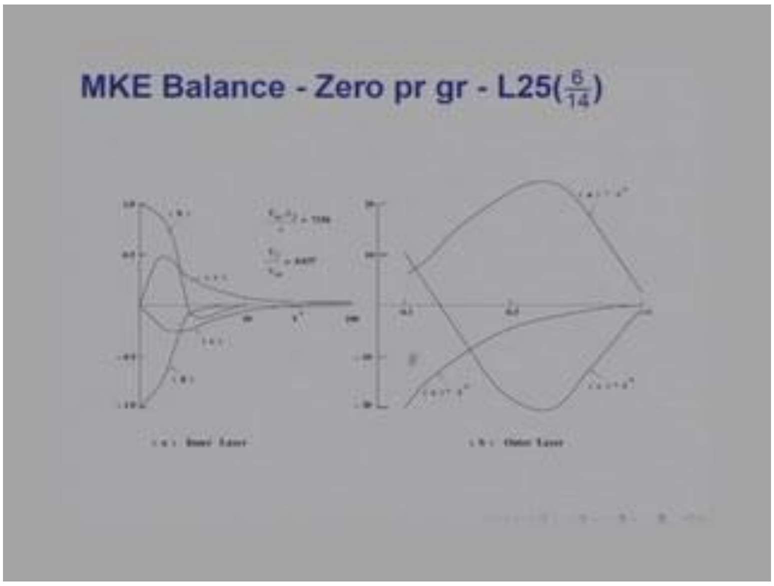

What is meant here is that, this little turbulent diffusion here makes up for the loss, compensates for the loss here. There is thus a flux of energy from the outer layers to the inner wall region, due to turbulent diffusion.

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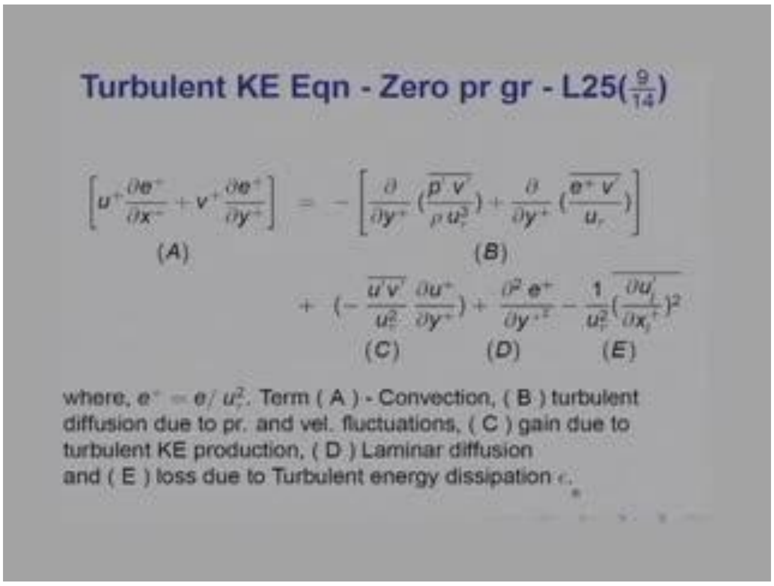

We shall now develop a similar balance for turbulent kinetic energy, and to do that, we shall derive, convert our E equation for a boundary layer, the turbulent kinetic energy equation and assuming zero pressure gradient. Then we have, this is the convection term of the turbulence kinetic energy; B is the turbulent diffusion due to pressure fluctuation and velocity fluctuation; C is the gain due to turbulent kinetic energy production, that is the term c; D is the laminar diffusion of turbulent kinetic energy; and finally, loss of due to turbulent energy dissipation, which we symbolize by epsilon.

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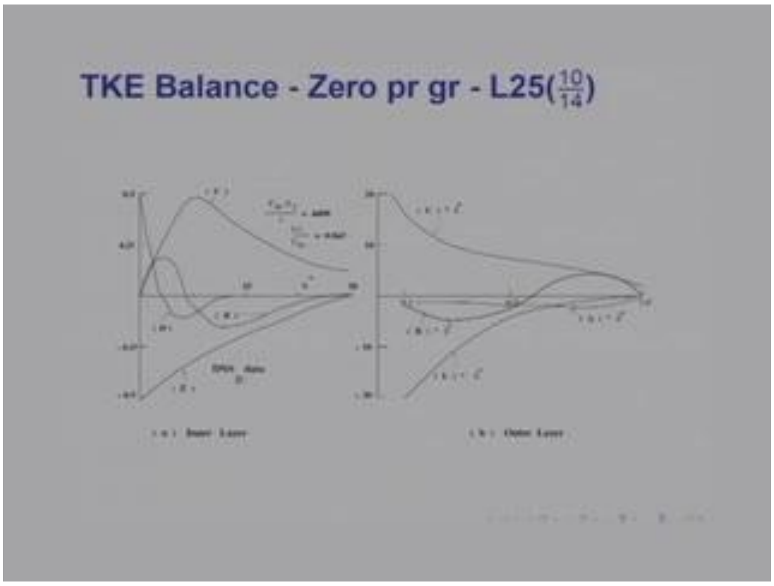

Let us look at how these terms vary. And again, I have a plot here; now what I am doing here is to show, not measure data, but data that were computed by direct numerical simulation by a person called Spalat. Again I am showing the inner region here, but this time going only up to y plus of 50 here, and then, the outer region here.

First look at term D. What was the term D? The term D is simply the laminar diffusion term that is making a positive contribution to energy, but it is also becoming negative after about let us say y plus of 5; and therefore, the its integrated value will be 0; term E is the turbulent energy dissipation, and again, that is quite large at the wall, but then goes on reducing to the beyond transitional layer itself it becomes almost negligible.

Now, let us look at term B which is the… as we will see term B is the diffusion due to pressure fluctuations and velocity fluctuations. So, there is a sudden increasing intensity of pressure fluctuations and velocity fluctuation near the wall. And you will see that therefore, you get B is positive, making positive contribution, but very quickly turns negative here, and in fact, you will see B will continue even in the outer parts and continue to make negative contribution from here. And then turn positive towards the end, but you will see that the net contribution of term B is almost 0, because positive areas negative areas balance, and therefore, very little contribution is made to the overall energy balance. But locally those terms are very important.

Finally, we look at term C, which you will recall, term C is the gain of turbulent kinetic energy due to… this is the loss term in the mean kinetic energy equation, which turns out to be the gain term in the turbulent kinetic energy and it is a gain due to turbulent kinetic energy production term. So, this C is the production term and that is what you see here, the production term peaks at about y plus of let us say, 20-odd, and then let us say about 15 or 18 odd, which is really the n middle of you like of the transitional layer and then begins to fall; then begins to fall; the term C begins to fall; it is transferred to this part here, and then, it will ultimately go down to 0. So, turbulent kinetic energy production is large here, but slowly it is becoming smaller and smaller and smaller towards the edge of the layer.

Finally, we will look at the term E, and just to capture here, term E is the turbulent energy dissipation, and that we said was large at the wall, decreasing here, and this is transferred here, and it goes on reducing, reducing and reducing till further, and therefore, very little dissipation - turbulent energy dissipation - takes place in the outer layers.

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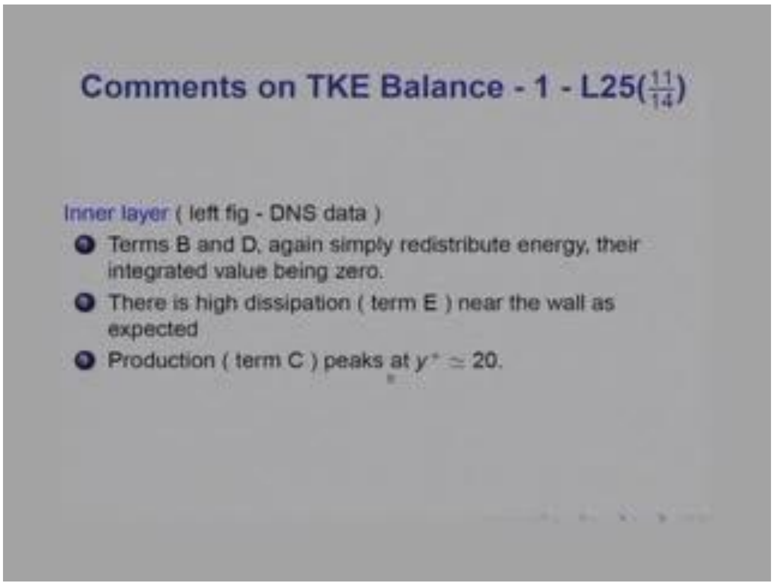

So, the comments then are terms B and D again simply redistribute energy, their integrated value being 0. There is a high dissipation term E near the wall, as expected, and the production term C peaks at around y plus of 20. So, now the outer layer data however, have been those corresponding to hot-wire measurements and what are the comments we can make on the outer layer.

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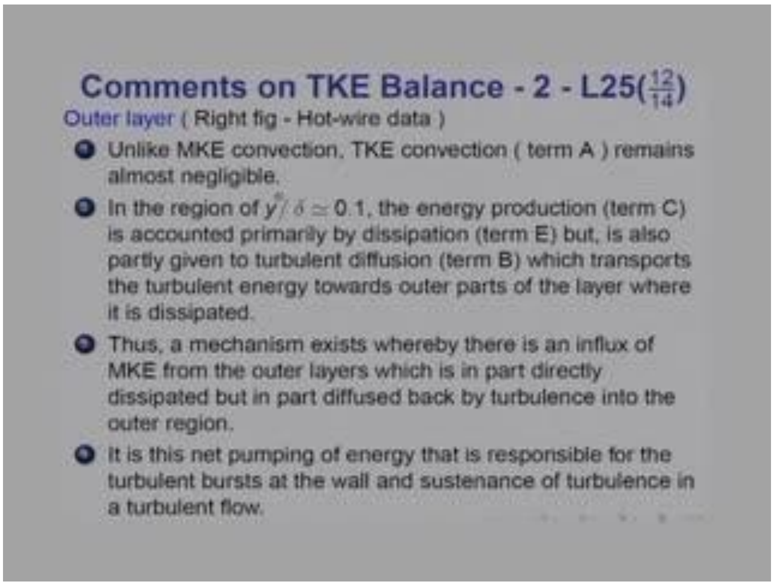

So, unlike the mean kinetic energy convection, the turbulent kinetic energy convection term A remains almost negligible.

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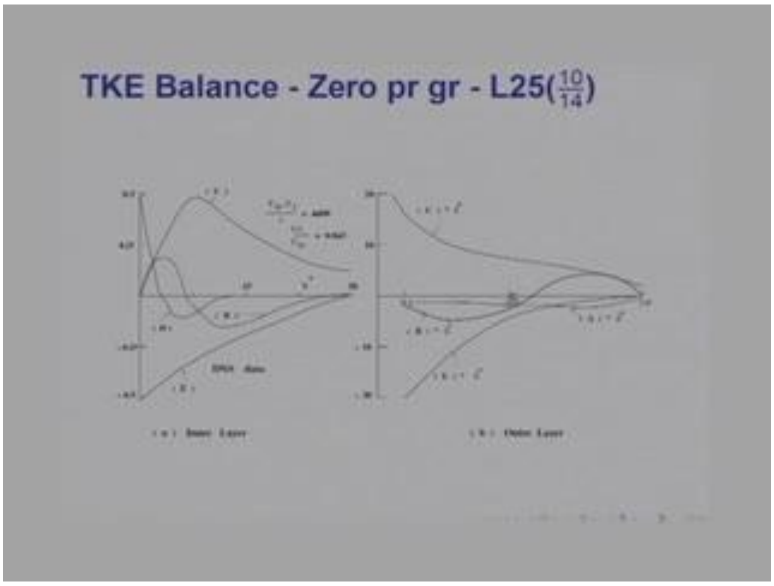

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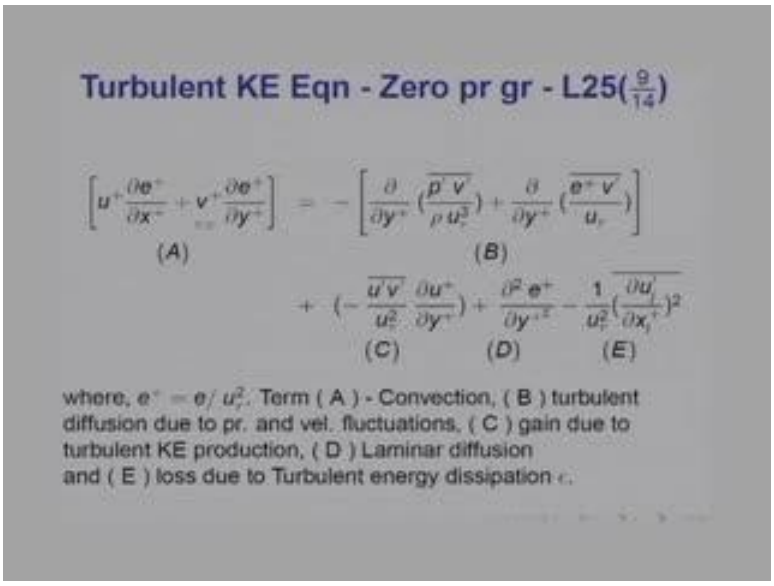

You will see the term A remains almost negligible right through the boundary layer; there is hardly any contribution made by term A, which is the convection term to the turbulent kinetic energy balance. Term A seems to make very little in comparison to other terms seems to make very little contribution to energy balance.

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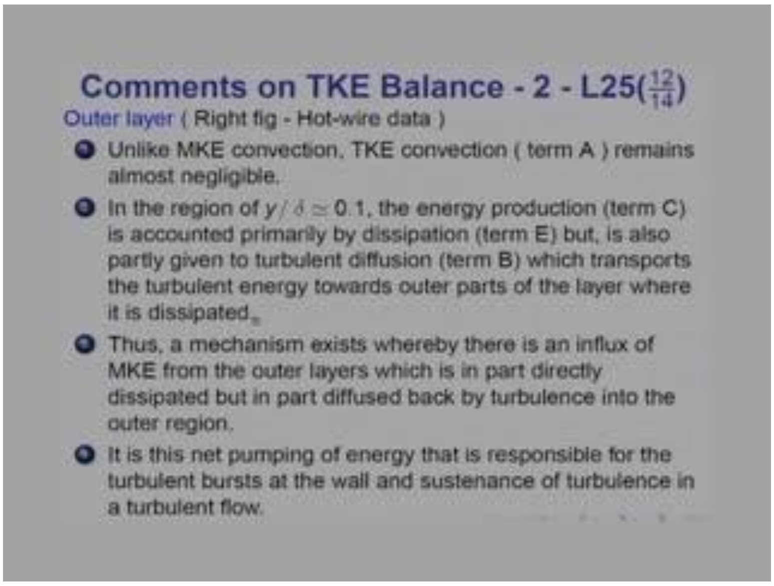

In the region of y by delta equal to 0.1, the energy production term C is accounted primarily by dissipation of energy. But is also partly given to turbulent diffusion term B, which transports the turbulent energy towards outer parts of the layer, where it is dissipated.

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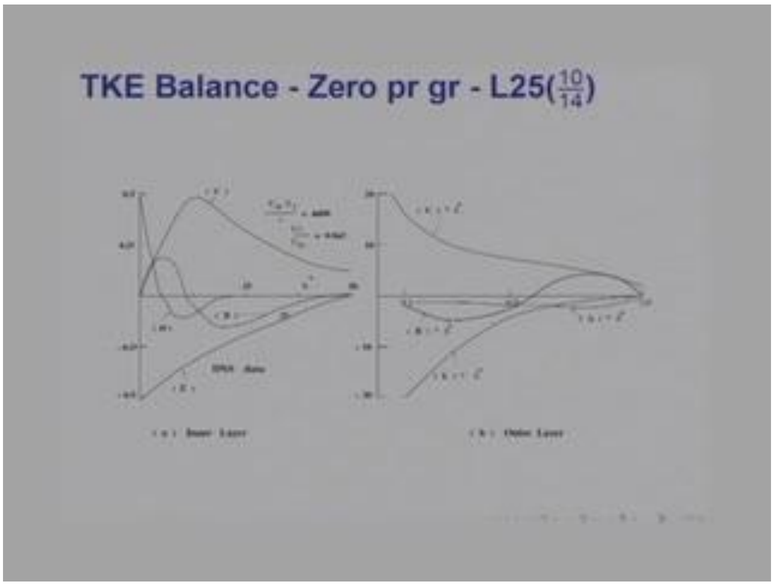

What is meant here is that, this is a positive term, no doubt, no doubt it is a positive term, that term simply transfers energy to the regions away from the wall, both through it term itself, as well as towards the outer parts of the layer, where it is finally, term E which where it is dissipated. Thus, a mechanism exists, whereby there is an influx of mean kinetic energy from the outer layers, which is in part directly dissipated within the outer layer, but in part diffused back by turbulence into the outer region.

So, let us read this again - a mechanism exists whereby there is an influx of mean kinetic energy from the outer layers towards the inner layer, which is in part directly dissipated, but in part diffused back by turbulence into the outer region. So, it is this pumping action from outer to inner and from inner to outer, that is responsible for turbulent bursts that I mentioned that laminar sub layer experiences lumps of fluid which come infrequently, but regularly towards the wall, and push a little amount lump of fluid out and push it back into the fluid layers above it. So, it is this pumping action which is explained by this mechanism. But it is this burst which comes out from time-to-time which really sustains turbulence ultimately. So, let us take the overall picture now.

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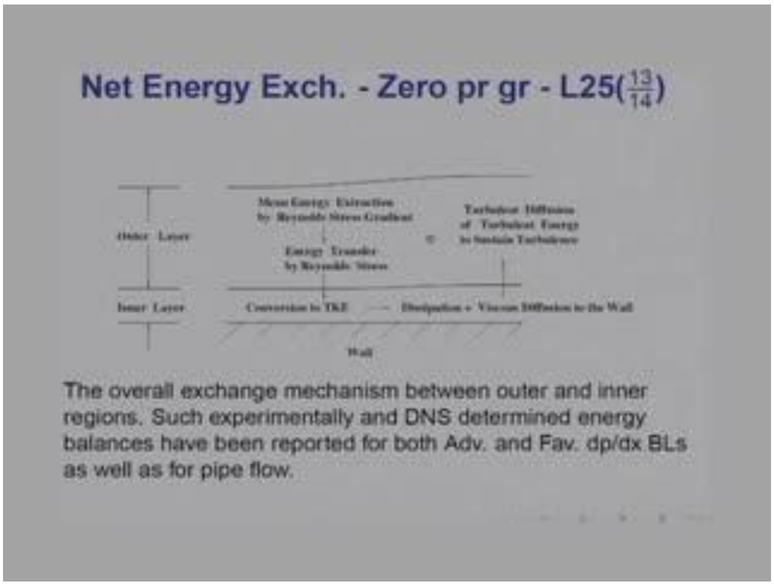

So, as I said this is the inner part of the layer let us say, and this is the outer part. So, what does the outer part do? The mean energy is extraction by Reynolds stress gradient that is what we observed. Energy transfer by Reynolds stresses to the wall, this energy gets converted to turbulent kinetic energy, because we saw the turbulent kinetic energy dominates at around y plus 15 or so.

So, in the inner layer turbulent kinetic, the energy coming from the outer layer gets converted to turbulent kinetic energy which is then, of course, dissipated within the inner region and also by viscous diffusion to the wall, because both these are influenced by effect of molecular viscosity, but then, we also observed that there is a mechanism, whereby turbulent diffusion of turbulent kinetic energy is from the inner layer to the outer layer, which really sustains turbulence. So, the overall exchange mechanism between outer and inner regions, such experimentally and DNS determined energy balances, have been reported for both adverse pressure gradient, favorable pressure gradient boundary layers as well as for pipe flows between two infinite parallel plates and so on, so forth. So, but overall story that this figure shows is very much observed in almost all flows past the wall and it is this story that is woven in turbulent modeling exercise.

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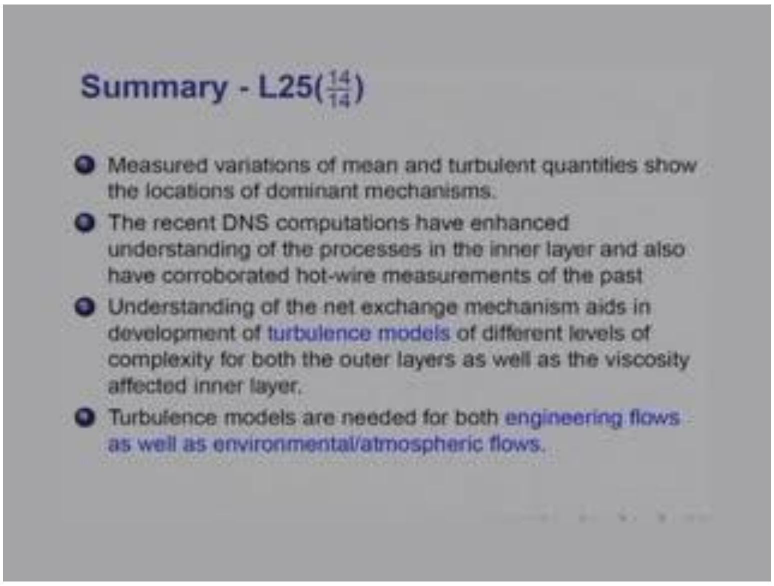

So, in conclusion, I would say that the measured variations of mean and turbulent quantities show the locations of the dominant mechanisms - that is the first thing. While we looked at the experimental data as well as the DNS data, the recent DNS computations have enhanced understanding of the processes in the inner layer and also have corroborated hot wire measurements of the past.

Remember, the inner region the laminar sub layer goes only up to y plus of 5, and the transitional layer goes only up to 30; and therefore, it was often doubted whether the earlier measurements made by hot wire anemometer, which has a diameter or of say, of the order of 5 microns, actually had made accurate measurements or not, but with the availability of very fast computers now, we can compute the instantaneous forms of the Navier Stokes equation - which is called the direct numerical simulation - and can place as many grid nodes as we want very close to the wall and can get very, very accurate computations done of the velocity fluctuations in that region. And therefore, DNS data are very valuable, it is fortunate that they corroborate the understanding that was developed earlier, by particularly, the measurements made by Townsend, those measurements have been verified by the DNS data.

Understanding of the net exchange mechanism aids in development of turbulence models of different levels of complexity for both the outer layers as well as for the viscosity affected inner layer. Turbulent models are needed for both engineering flows as well as for environmental and atmospheric flows. What I mean here, for example, in engineering flows the boundary layer thickness would be, let us say, of the order of a few centimeters or few millimeters, in fact.

Atmospheric boundary layers are of the order of meters. Is this description valid there? It turns out that although not in every detail, but the overall detail is very much reproduced even in atmospheric boundary layers. So, in that sense, these measurements done in the laboratory are very valuable; of course, nowadays measurements have been made even of atmospheric boundary layers with very sophisticated instrumentation to create thick boundary layers on the terraces of buildings and so on so forth; long terraces with very big wind tunnels to blow air, to create a large very thick boundary layer and measurements, so that the measurement accuracy increases, and we would therefore find the strain that we find that majority of the measurements made earlier in smaller diameter pipes and flaplets in small wind tunnels are actually getting reproduced even in thicker boundary layers.

So, we can trust our overall approach to understanding of the exchange mechanism on the basis of the data that was gathered in pipe flows and small thickness boundary layers.