Convective Heat and Mass Transfer
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Module No. # 01
Lecture No. # 24
Near-Wall Turbulent Flows-01

In the previous two lectures, we considered the formal aspect of turbulence and asked a question how turbulence sustains itself. We showed this process of sustenance through breakdown of varies. We explained this process in three ways; one was scale analysis, second was spectral analysis and the third was the vorticity dynamics. Now, we must turn to more predictive aspects, after all we wish to - we were able to compute or calculate friction factor and Nusselt numbers in turbulent flows.

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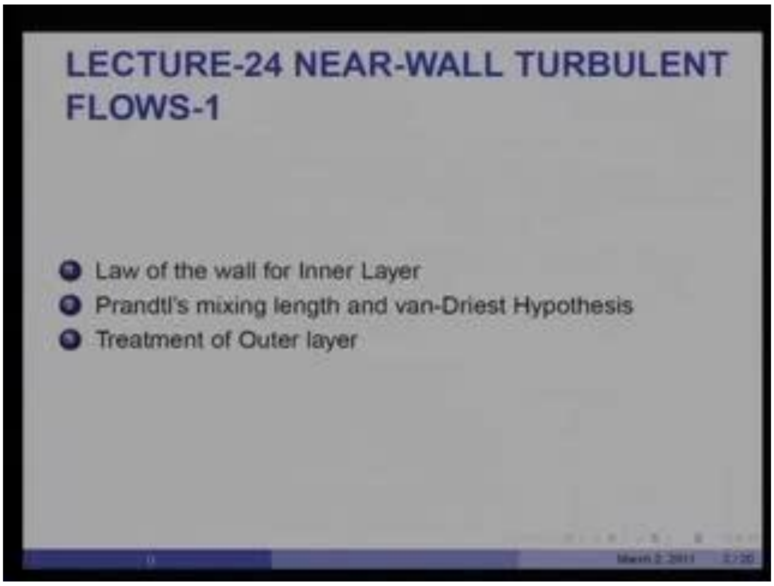

So, I am going to turn today to regions of flow close to the wall. If one analyses experimentally, then one would find an inner layer, as well as an outer layer. There are some special features of this inner layer, which make it possible for us to determine the friction factor and Nusselt number. I will also ensure how Prandtl's mixing length idea can be employed to predict velocity distribution in inner layer.

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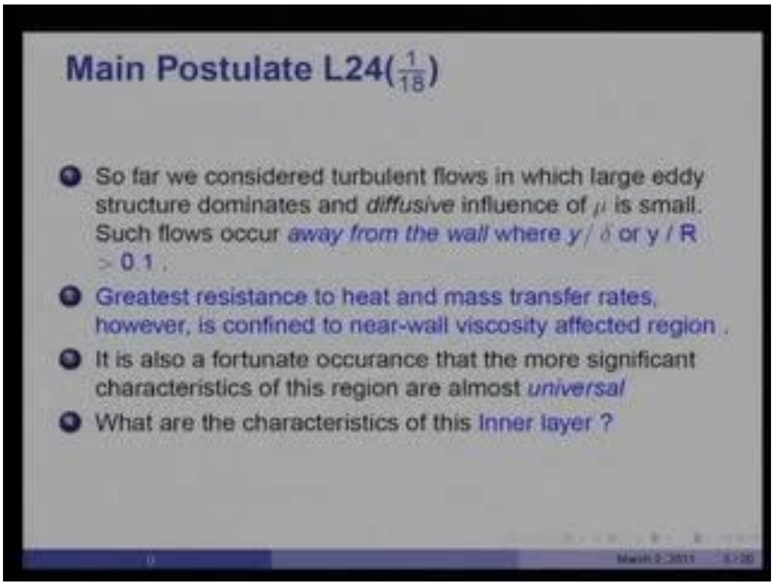

So, let us turn to the main postulates. So, in our formal aspect, we dealt with turbulent flows, whose structure is dominated by large eddies that is where the production takes place. The diffusive influence of viscosity was rather small, being confined only to carry out dissipation at the smallest scales. Near the wall, however, viscosity also plays its role in bringing about diffusion. What is the size of this inner layer? To begin with, let me say, approximately y divided by delta in a boundary layer would be of the order of 15 percent. Likewise, y divided by radius, where y distance from the wall in a pipe flow would again be of the order of 15 percent.

So, we are talking about a fairly narrow region close to the wall, about 15 percent; whereas, away from the wall, where diffusive influence of viscosity is very small, would be greater than 0.15. However, it is the inner region, which is of great importance to us; because, the greatest resistance to heat and mass transfer occurs close to the wall, where or in the region, in which, viscosity play this dominant role; that is where the fluid flow is sluggish. Therefore, likewise, the heat and mass transfer is also very sluggish. Therefore, we are very much interested in this inner region.

Now, it is of course a very fortunate occurrence - quite an accident of nature, really; that the most significant characteristics of this inner region are almost universal. We are going to exploit this universality of the inner layer to predict friction factor Nusselt number.

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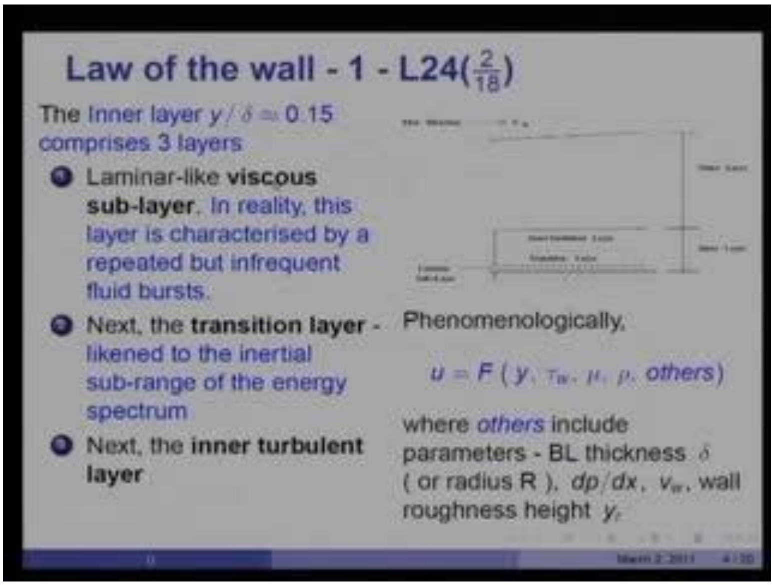

So, let us ask a question what are the characteristics of this inner layer? This is the flow against a wall, this is the free stream velocity U infinity and this is the total boundary layer thickness delta, let us say.

Then, the inner layer which as I said is about 15 percent of the total, itself comprises of three characteristic layers, as I shown here. The inner most layers is often called the viscous sub-layer, is almost laminar-like, because that is where the effect of viscosity is so great that all fluctuations are almost killed and you get essentially laminar flow.

In reality, however, this layer is characterized by repeated but infrequent fluid burst. What happens is, the laminar sub layer here grows a little, becomes unstable and weak. At this point, lumps of fluid from the outer layer hit the inner viscous sub-layer and uproot fluid out into the outer layer again or the outer parts of the inner layer. This kind of fluid being flung out from the sub layer is quite visible, if you did flow visualization of a typical turbulent boundary layer.

Then, this region is very intermittent. When I say it is intermittent, it is infrequent. So, for all practical purposes to begin with, we might say the region is almost like laminar layer.

The next is the transitional layer, which we may likened to the inertial sub range that we identified during our formula aspects of turbulence; that is called the transitional layer. Now, here, at both turbulent fluctuations, as well as fluid viscosity, both are equally dominant. Then, there is the fully turbulent part of the inner layer, where essentially the flow is very much like fully turbulent flow.

So, as I said, inner layer has three layers; laminar sub layer, transitional layer and the inner turbulent layer. The outer layer is definitely turbulent, so we will take up the outer layer towards the end of this lecture. But, presently we wish to concentrate on the inner layer, because in this part of the boundary layer that really offers significant rates resistances to heat transfer. It is also the region in which greater part of the temperature velocity and concentration gradients take place, whereas the outer part has more or less uniform profiles. So, it is the inner layer which is of great importance to us.

Now, phenomenologically I may postulate that the velocity parallel to the wall u would be function; first of all of the fluid property - two properties being used; rho, the density and viscosity. Of course, u must vary with distance from the wall, therefore y is included. tau wall would determine the shear stress at the wall that is the velocity gradient at the wall. Because, tau wall is mu d dy at y equal to 0, therefore shear stress is also included here. Then, there are many other factors that are likely to influence the velocity profile in the inner layer.

Now, what are those other factors? The other factors would be the boundary layer thickness; itself could well influence the nature of the velocity of the profile. The pressure gradient and it is variation in the x direction could also affect velocity distribution. If there is transpiration or mass transfer, then of course, v w will also influence the velocity distribution. Finally, in order to enhance the rate of heat transfer, particularly in gases, we often employ rough surfaces, so even the roughness height would influence the nature of the velocity profile in this.

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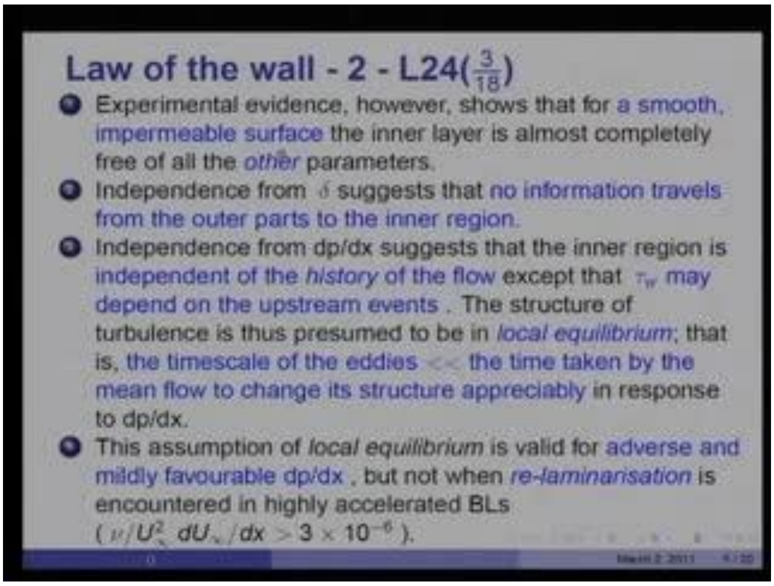

Experimental evidence, however, shows that for a smooth, impermeable surface; smooth meaning roughness height is 0; impermeable means v w is 0; the inner layer is almost completely free of all other parameters. Now, this is very interesting that for v w equal to 0, for a smooth surface, the other parameters play a very minor role. I will explain why it is so.

So, for example, independence from delta suggests that no information travels from the outer parts to the inner regions. So, inner region is sort of insular region that is not really effected by what happens very far outside into the outer layers. Independence from dp dx suggest that the inner region is also independent of the history of the flow. Except that the shear stress variation along the wall, may influence a little bit the velocity profile, but the influence would be expected to be not so great, as far as the velocity profile is concerned.

Structure of turbulence is thus presumed to be in local equilibrium; that is, the time scale of eddies in the inner layer are much smaller than the time taken by the mean flow to change its structure appreciably in response to dp dx.

You may like to think about this. I often give the example of a city and a village. A city quickly response to what happens in distant places; for example, Bombay would get influenced by what happens in New York or London, but a village, say 100 kilometers outside Mumbai, would be hardly influenced by what happens in the world around. I am using an analogy, so that you might remember what we mean by insularity of the inner region and the slowness of the inner region.

This assumption of local equilibrium is valid for adverse, as well as mildly favorable dp dx, but not when re-laminarisation is encountered at very high accelerated boundary layers. Of course, we are talking about here fairly moderate range of plus and minus pressure gradients. So, highly accelerated boundary layers, this pressure gradient parameter m nu by U infinity square dU infinity by dx would be greater than 3 into 10 raise to minus 6.

So, we are not talking about such highly accelerated boundary layer, we are talking about only those which are more frequently encountered, having moderate plus and minus pressure gradients.

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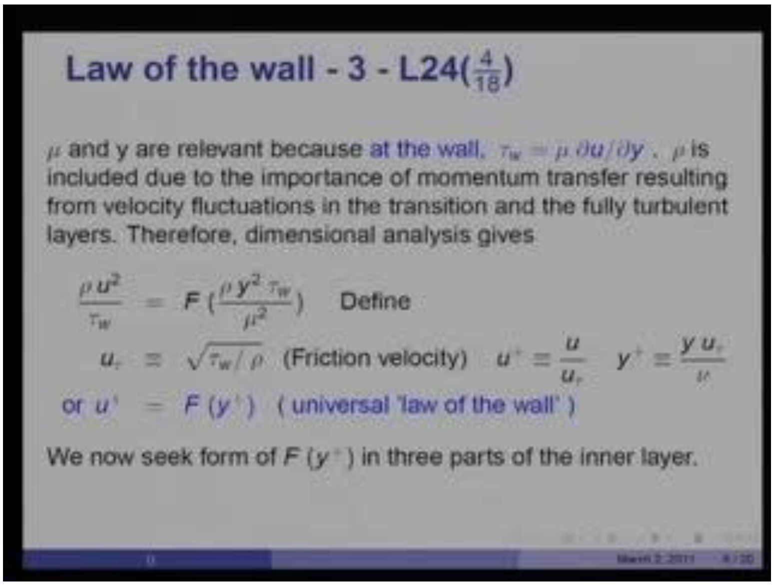

We included viscosity and distance from the y, simply because, as I said, tau wall is equal to mu du dy, therefore, we must include mu and y. The density rho is included due to the importance of momentum transfer resulting from velocity fluctuations, in the transition and fully turbulent layers.

As I said, there are bursts of fluid, which come in breakdown of the laminar sub layer and there is a burst of fluid coming out of the laminar sub layer. This requires momentum transfer from the outer layers to inner layer, of the inner layers. Therefore, density would definitely pay an important role. Therefore, we have included density.

So, if I were to go back to these slides (Refer Slide Time: 04:08). So, we are essentially excluded all others and only we are going to concentrate on these four parameters, because they define more or less the equilibrium of the inner layer. So, if I were to carry out the dimensional analysis, I would find that rho u square divided by tau wall would be function of rho y square tau wall by mu square.

Now, I define u tau equal to under root tau wall by rho; this is often called the friction velocity; square root of tau wall by row has the dimensions of velocity. I will define a dimensionless velocity u plus as u divided by u tau and y plus would be defined as y u tau divided by nu. You can see this is a kind of a Reynolds number, based on distance from the wall and the friction velocity - y plus. So, both u plus and y plus are dimensionless, so you will readily recognize that this is nothing but u plus square and this is nothing but y plus square or another way of saying is, u plus will be a function of y squares. At least, the phenomenology suggests that this relationship will be universal and it is often called the universal law of the wall.

The exact forms of F y plus, we must now find out, what the form it will take in the three layers; that is, the laminar sub layer, the transitional layer and the fully turbulent layer of the inner turbulent layer.

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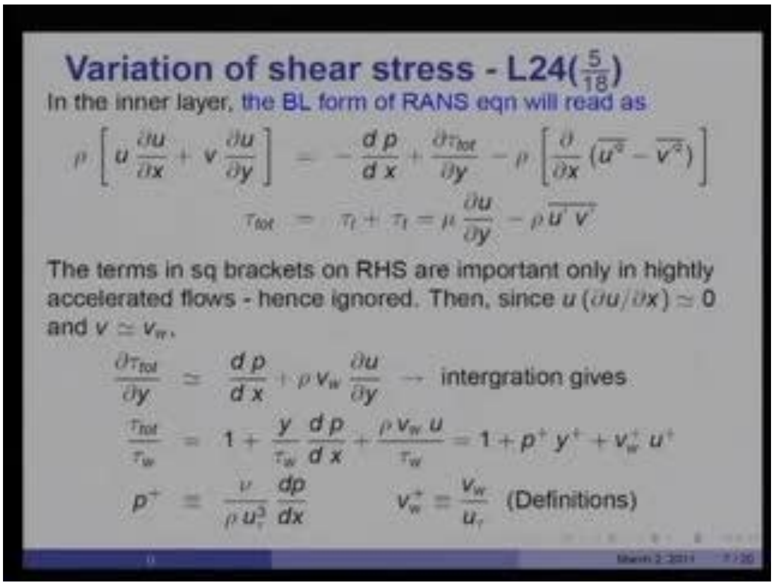

So, let us go layer by layer. To do that you will recall that the RANS equations for actual momentum would look like this; these are the convection terms; this is the pressure gradient term; this is the total shear stress. There would be these terms, which would arise only which are necessary to be included, only when highly accelerated flows are considered.

But, as I said, we are not going to consider, so those terms will be dropped. Also, very close to the wall, u is very very small in the inner layer, which is about 15 percent. u by du dx would be much much smaller than this quantity v du by dy. Therefore, this term can also be dropped; as a result what we will get is, d tau tot total divided by dy would be approximately equal to d p by d x plus rho v w du by dy.

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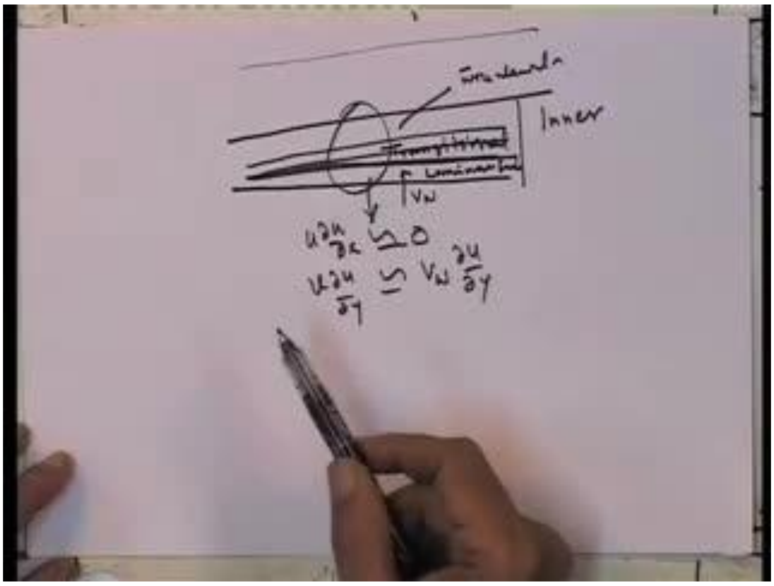

Now, of course, v equals v w; this is the inner layer; v w is present only here; this is the turbulent layer, sorry; this is the transitional layer; this is the laminar sub layer. So, we say that u du by dx will be approximately 0 in these region. v du by dy would be approximately equal to v w du by dy, these are very very approximate in the sense that v w extends it effect in the inner 15 percent of the total boundary layer thickness.

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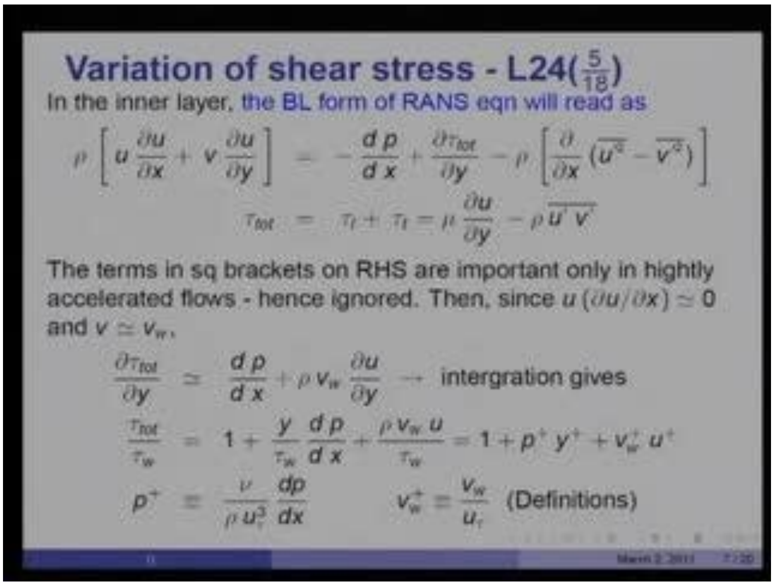

If I make these assumptions, then you will see that I can non dimensionalize this. First of all I must integrate this, so tau tot would be d p d x into y plus rho v w into u and tau tot would be equal to tau wall at y equal to 0.

So, in other words, this integration would result into tau tot divided by tau wall equal to 1 plus y divided by tau wall d p by d x plus rho v w u divided by tau wall. Now, you can see, this is nothing but, this is tau wall divided by rho. Here, in the denominator, which is u tau square; so I can form a v w plus, which is v w over u tau. u over u tau will give me u plus, so this v plus u plus.

I will define this term as p plus y plus, in which case, p plus would be defined in this fashion, nu divided by rho u tau cube dp by dx; v w plus would be v w by u tau; these are the definitions.

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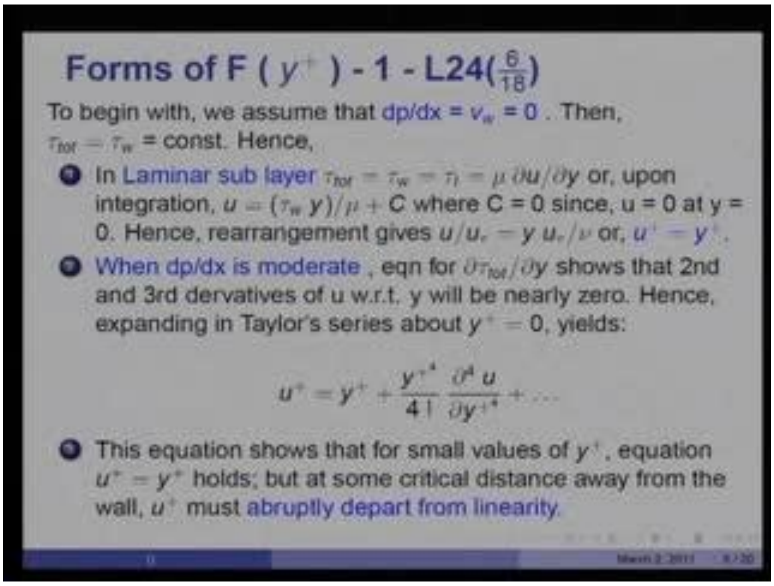

So, hold this in your mind that the ratio of shear stress - a total stress to the shear stress at the wall is 1 plus pressure gradient times y by tau wall and rho v w u by tau wall. So, in effect, this ratio is in fact influenced by the pressure gradient, as well as the effect of v w, as it should be.

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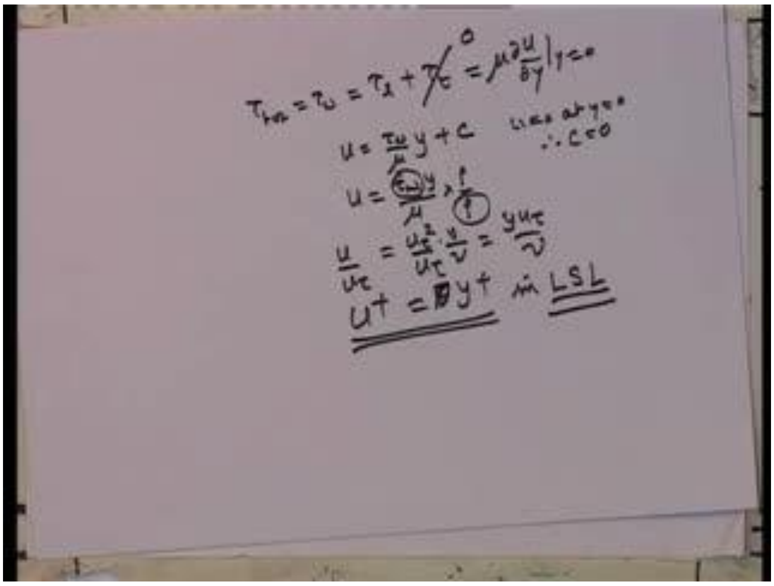

So, now, let us look at layer by layer. To begin with, we shall assume that dp by dx and v w are both 0; that means, we are considering the case in which this is 0 and this is 0, so tau tot would be equal to tau wall throughout the inner layer; therefore, shear stress is a constant. Now, in the laminar sub layer, tau tot would be equal to tau wall equal to tau l and tau t - the turbulence stress would be 0. That would be equal to mu times du by dy at y equal to 0.

So, integration of this would give me u into tau wall divided by mu times y plus constant, but u is equal to 0 at y equal to 0 and therefore, c is equal to 0; therefore, I get u equal to tau wall y by mu. If I multiply and divide this by rho, then you will see, I will get this tau wall divided by rho will become u tau square into y by nu. If I take 1 nu tau on this side, I will get u over u tau divided by u tau, which is y u tau by nu. In effect, this is u plus is equal to y plus in the laminar sub layer. In the laminar sub layer, u plus would be simply equal to y plus and that is what I shown here, so u plus is equal to y plus.

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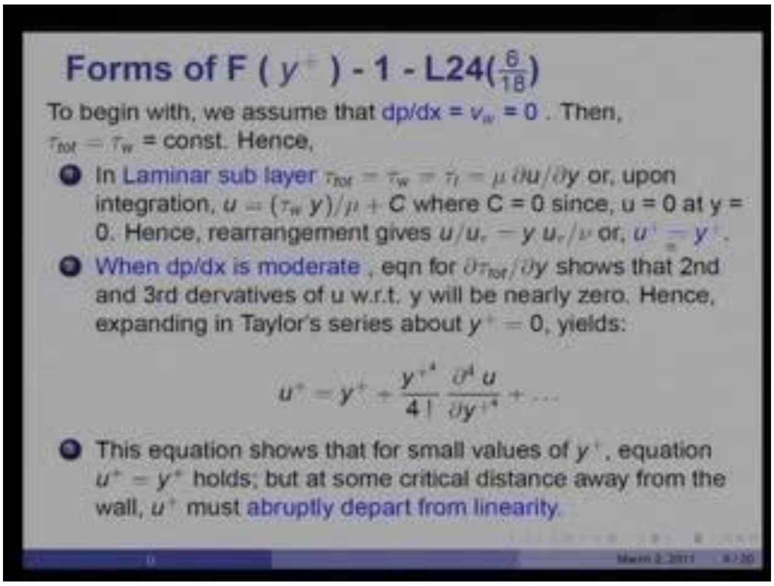

Now, when dp by dx is moderate, the equation for d tau tot by dy - now of course, v w is still 0 - shows there would be little bit of y dependence, there will be y dependence plus pressure gradient; that term is 0.

So, tau tot by tau wall would actually be influenced little bit by distance from the wall and therefore, the second and third derivatives of u with respect to y will be nearly 0. Hence, if we expand this u plus y plus relationship in Taylor series, then about y equal to 0 you will get u plus equal to y plus plus y plus 4 by 4 factorial d 2 u by dy plus 4 plus so on and so forth.

Now, this equation shows that for small values of y plus u plus equal to y plus holds, which is the laminar sub layer, but at some critical distance away from the wall, u plus must abruptly depart from linearity. All right, so there is a way useful little deduction that we will carry over to the next transitional layer.

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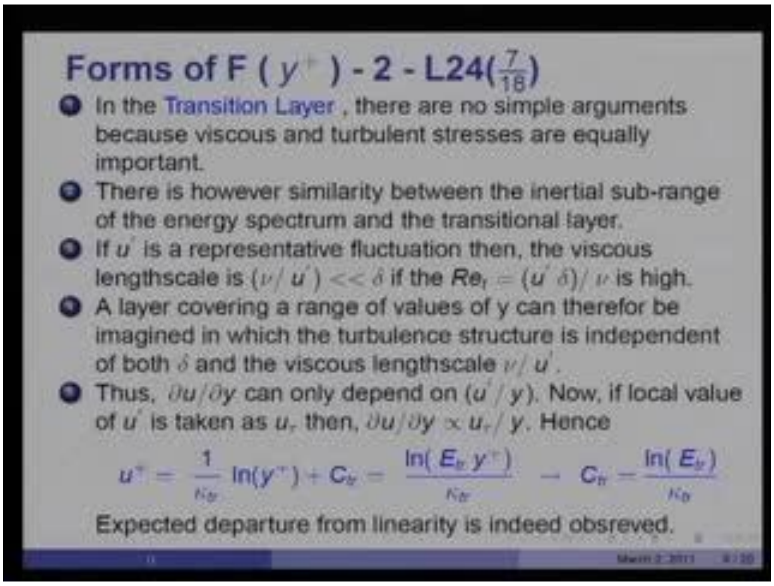

Now, in the transitional layer, there are no simple phenomenal logical arguments that one can give, because both viscous and turbulent stresses are equally important in the transitional layer. There is however a similarity between the inertial sub range of the energy spectrum and the transitional layer. In that if we said, if u dash is a representative velocity to fluctuation, then the viscous length scale would be nu by u dash, would be much much less than delta, as we already seen; if the turbulent Reynolds number, u dash delta by nu is high.

A layer covering a range of values of y can therefore be imagined, in which the turbulence structure is independent of both delta - the large scale, as well as the viscous - very very viscous length scale nu by u dash. This is how we characterize the inertial sub range, as being relatively uninfluenced by either the very large scale or very very viscous scales.

How should du dy vary then, in this region? The du dy can only depend on u dash divided by y. If we for a moment say that u dash would be proportional to u tau, then du by dy would be proportional to u tau divided by y.

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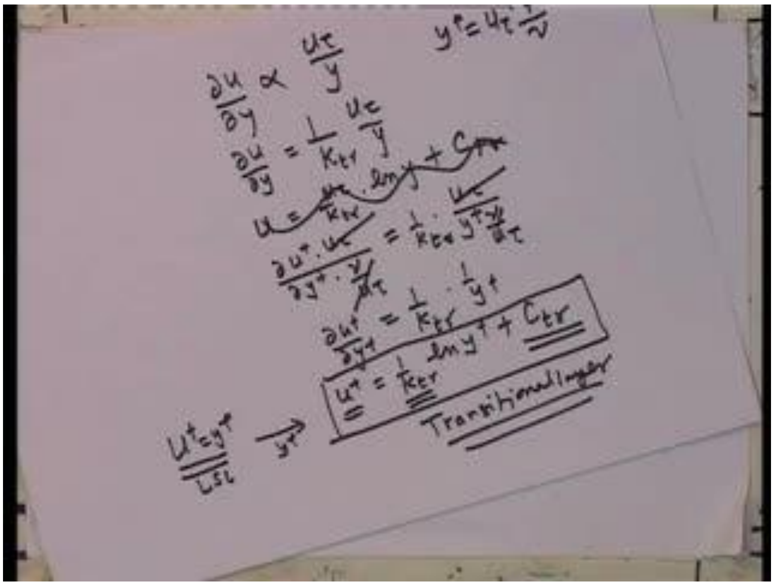

If I said that du by dy is proportional to u tau by y, then I will get, let us say du by dy equal to - I am going to now call kappa transition u tau by y and if I were to integrate this, I will get u equal to 1 over kappa transition u tau into ln of y plus a constant of integration, which is c transition. Before I do that in fact, I can say that if I make this du plus multiplied by u tau and make this dy plus multiplied by - because y plus is equal to u tau y by nu, so dy being changed to - that will become nu by u tau that will equal 1 over kappa transition u tau divided by y nu by u tau. Then, you will see that this u tau gets cancel with this, this gets cancelled with that. I will have essentially du plus by dy plus equal to kappa transition 1 over y plus or I would get, u plus equal to 1 over kappa transition ln y plus plus a constant of integration C tr.

If this is the law that applies to the transition layer, then it does indeed show that there is a clear departure from u plus equal to y plus, which was in the laminar sub layer; this is now in the transitional layer.

We had anticipated that there would be some distance y plus at which this transition would take place sudden departure in the slope du plus dy plus, which was equal to 1, now becomes 1 over kappa transition, 1 over y plus. So, there is a sudden change in the velocity gradient and therefore, the velocity itself.

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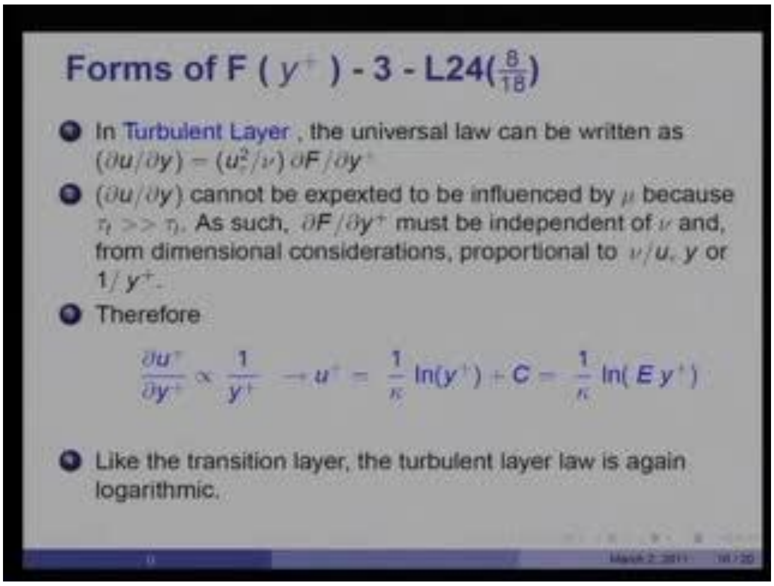

So, the expected departure from linearity in the u plus y plus law is already attained. Now, indecently this equation can also be recast as ln E transition y plus by K transition, where C transition would be ln E transition by K transition, which is simple.

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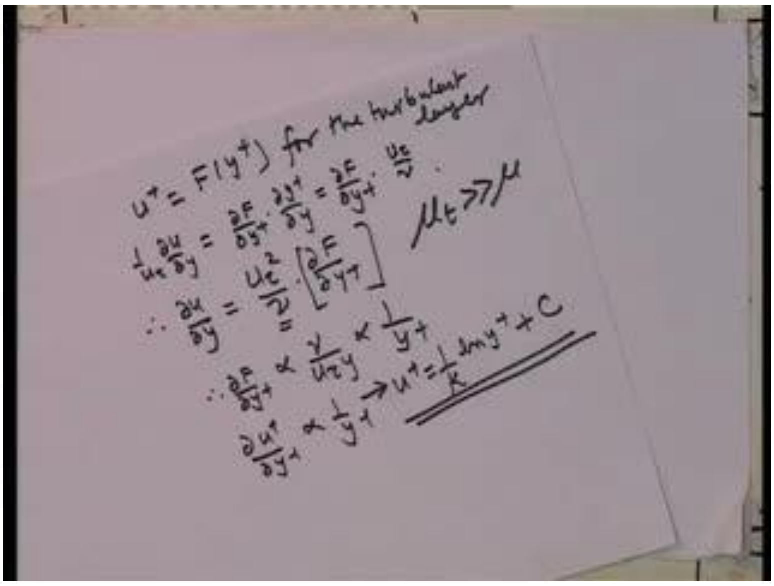

Let us turn to the fully turbulent layer. Now, what we are looking for is the u plus equal to F y plus for the turbulent layer. Therefore, this will be 1 over tau du by dy equal to d F by dy plus into dy plus by dy or that would be equal to d F by dy plus into u tau by nu.

Therefore, you see du by dy in the turbulent layer would be u tau square by nu d F by dy plus.

Now, of course, in the fully turbulent layer, mu t is much much greater then mu. So, we do not expect nu to play any significant role in the fully turbulent part of the inner layer. Therefore, this expression must be independent of nu. Therefore, d F by dy plus must be proportional to nu divided by u tau y, so that dimensionally the two sides are correct, or this is nothing but proportional to 1 over y plus.

Therefore, we get du plus by dy plus as being proportional to 1 over y plus, this again gives u plus equal to 1 over kappa ln y plus. Now, this will be plus for a constant of integration.

So, the fully turbulent layer also suggests a logarithmic law, but the values of kappa and C may be different from those of the transitional layer.

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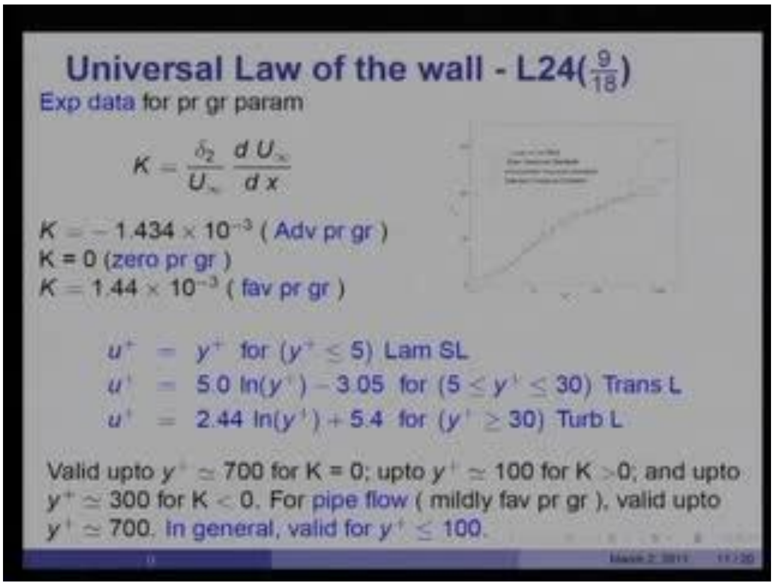

Now, let us look at the experimental data, because, so far, we have put up phenomenological argument. So, let us look at the experimental data, in which, we look at this parameter, which is the pressure gradient parameter; delta 2 divided by U infinity d U infinity by d X.

I am looking at three types of flows; one is the adverse pressure gradient boundary layer, K equal to minus 1.434 into 10 raise to minus 3; K equal to 0 is the 0 pressure gradient boundary layer, because U infinity would be constant; K equal to 1.44 into 10 raise to minus 3, which is the favorable pressure gradient; what does this show? The experimental data are circles, it show 0 pressure gradient boundary layer; squares, show favorable pressure gradient boundary layer; triangles, show adverse pressure boundary layer.

Now, you can see right up to 5, 10, 30 or almost, let us say, up to about a 100, there is a complete collapse of all experimental data on u plus versus y plus - u plus versus y plus. u plus is a linear scale and y plus is a logarithmic scale. up to bar 100 you will see that there is a complete universality irrespective of the pressure gradient. The favorable pressure gradient boundary layer data begin to depart from - let us say somewhere around about 300. The 0 pressure gradient data seem to do quite well, even up to Reynolds number of - I mean y plus of almost 700. But, the adverse pressure gradient data seem to begin to depart at about 300. The favorable pressure gradient data seem to depart say at about 150 or so, from the universality.

So, we can say, up to about 100, the velocity profile in these coordinates u plus versus y plus is almost universal. How does it fit? u plus equal to y plus, it seems is valid up to y plus less than or equal to 5, this is what we identify. This is y plus equal to 5 here, is identified as laminar sub layer. Then, there is a change in slope as you can see and that is the transitional layer up to about 30 that is given by 5 ln y plus minus 3.05. This implies that one over kappa transition is 5; therefore kappa transition must be 0.2. That region extends from y to about 30 that we identify as the transitional layer. u plus equal to 2.44 ln y plus plus 5.4 seems to apply for y plus greater than 30 and that we would say as the turbulent layer.

So, the main observations are for K equal to 0, 0 pressure gradient boundary layer. These laws apply up to y plus equal to 700. I have drawn these laws by the solid line that extend right till about 2000 - y plus up 2000. For favorable pressure gradient y plus of 100 seems to be the upper limit of applicability, whereas again for adverse pressure gradient, it seems to be about 300 y plus of 300. For pipe flow, which is a very mildly favorable pressure gradient, which I have not shown here, in fact then experimental data show that again, like K equal to 0 case, experimental data for pipe flow would also fall on the universal laws, till about y plus of 700.

But, it is safe, therefore to say that in general, irrespective of the pressure gradient that we would encounter, the region y plus less than 100 seems to be almost certainly universal - y plus less than 100 seems to be almost certainly universal.

So, we have discovered that the inner layer in the absence of v w, but very moderate pressure gradients does actually have a reasonably universal structure. But, the moment you exceed y plus of 100, the outside pressure gradient effects - that is the other switch we had ignored begin to play their role. Velocity profiles do depart from this universal law that we have identified.

The main changes occur only in the fully turbulent part of the inner layer. The laminar sub layer and the transitional layer are somehow completely insular; they are not affected by the pressure gradient at all. The inner smaller region of the inner layer is also up to about 100, is also universal, but beyond 100, the pressure gradient effect starts playing a constant of integrations role, all right.

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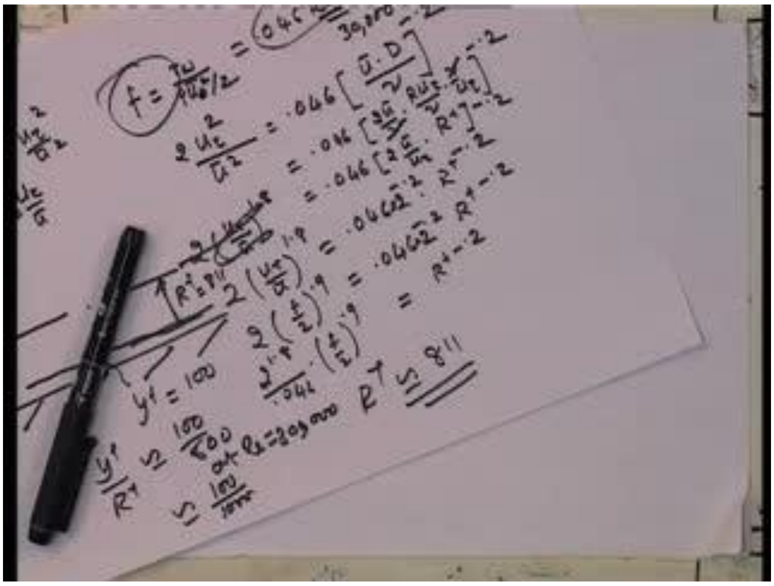

So, what is this special about y plus of 100? Well, it either - I will show shortly, it corresponds to say about 10 to 15 percent of the boundary layer thickness. We can explain this from for a pipe flow.

So, for example, in a pipe flow, f friction factor, which is tau wall divided by rho u bar square divided by 2 is 0.46 Reynolds raise to minus 0.2. So, if I take a Reynolds number of let us say 30000; then what am I saying? This relationship actually can be shown tau wall by rho is u tau squared divided by u bar square into 2 is equal to 0.046 into u bar into diameter divided by nu raise to minus 0.2.

If I were to define here, 0.046 into 2 u bar by nu into Ru tau by nu into nu by u tau raise to minus 0.2. Then, you will see this is equal 0.046 into this nu bar gets cancelled with this nu bar, I have 2 times u bar by u tau into R plus raise to minus 0.2 or you will see therefore, this becomes 2 times - if I take this term on this side, then you will see, I get 2 into u tau divided by u bar raise to 1.8 is equal to 0.046 into 2 raise to minus 0.2 into R plus raise to minus 0.2. All right, what is u bar by u tau? Well, this is the friction factor, the friction factor is actually equal to 2 times u tau square by u bar square and therefore, u tau by u bar is actually under root of f by 2.

So, in other words, I get here 2 times f by 2 raise to 0.9 equal to 0.046 2 raise to 0.2 minus 0.2 into R plus raise to minus 0.2. R plus raise to minus 0.2 would be equal to 2 raise to 1.8 divided by 0.046 into f by 2 raise to 0.9. Therefore, if I take now Reynolds number of 30000, then I can get the value of f from our usual relationship and therefore, I can show that R plus will be about 811; that is what I have shown here.

So, R plus would be about 811; that is at the axis of the pipe from the wall - R plus would be about 811, whereas in the inner layer where universality exists, y plus is about 100. So, y plus by R plus is approximately 100 by 800 at Reynolds number of 30000. If the Reynolds number was bigger, then this could go up to 100 by 1000 even or 1200 or something like that.

So, we are essentially talking about a region of the order of 12 percent, 15 percent region, which defines the inner layer, all right.

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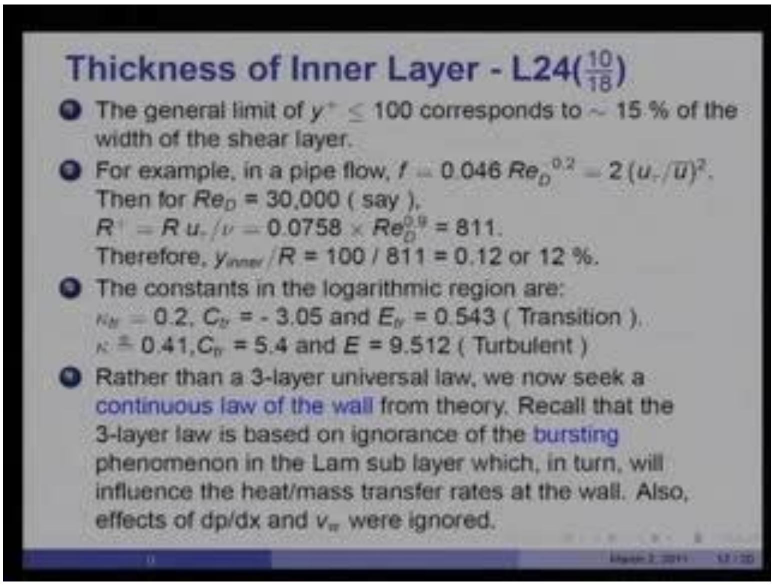

The constants, we have identified as I shown here. The constants of kappa transition will be 0.2, kappa for the turbulent layer from 2.44 would be 0.41 and C transition would be 5.4. Of course, likewise, each transition and E of the turbulent layer will be 0.543 and 9.512; this is just another way of writing these two.

The three layer law is very nice, but, as we said, it has very sharp discontinuities. What we would really like, is to have, is continuous law of the wall. How do we do it?

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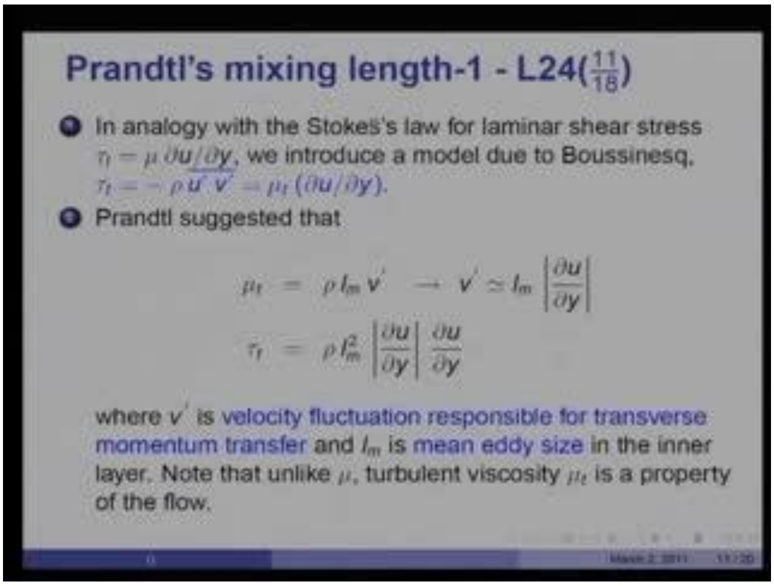

So, we seek now a continuous law of the wall, rather than this three layered description - mathematical description. To do that what I am going to do is, allow for this bursting phenomenon, as well as effects of dp dx and v w.

So, in analogy with stokes law, for laminar shear stress, tau l equal to mu du dy. We introduce a model due to Boussinesq, as tau t equal to minus rho u prime v prime equal to mu t du dy. Then, Prandtl suggested, in analogy with how laminar viscosity is defined, in rho times lm v dash equal to v dash was like lm into du by dy. Therefore, tau t becomes rho l m square du by dy du by dy, this as issues we considered in our formula aspects as well. Where, v dash is the fluctuation responsible for transverse momentum transfer and l m is the mean eddy size in the inner layer- sort of notional mean eddy size.

Note that unlike mu turbulent viscosity, mu t is a property of the flow, whereas mu is actually the property of the fluid. So, that is the difference between mu and mu t - the turbulent viscosity.

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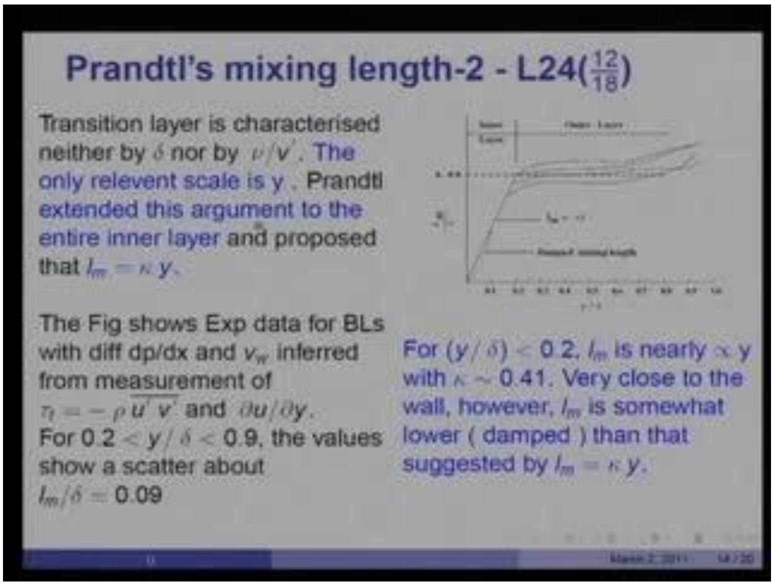

Now, the second question is how mixing length l m varies. So, the transitional layer is characterized neither by delta nor by nu by v dash, so the only relevant scale is y. Therefore, Prandtl extended this argument to the entire region of the inner layer and proposed that l m would be kappa times y; some constant that is directly proportional to y in the inner part of the layer.

Now, the experimental data for the measured, from velocity profiles of l m show this. This is y axis, l m divided by delta and this is y divided by delta, I am going from 0 to 1. Now, experimental data do show, except for this little damping, l m is in fact quite linear till about let us say 0.18 or 0.2, let us say this is what we call the inner layer, but, beyond that point, the l m begins to show lots of scatter.

Now, we are considering here flows in adverse pressure gradient, favorable pressure gradient, 0 pressure gradient, as well as pipe flows, many other ducted flows and so on so forth. So, this seems to be quite peculiar about mixing length that it is nearly constant in inner layer. It is nearly linear in the inner layer, say about up to about 20 percent or 15 percent and then it begins to show lots of scatter about a value of 0.09.

Mind you, it is somewhat difficult to accurately measure l m in the outer layer, because du dy in this region is also very small you see and therefore, it becomes difficult to measure this value of l m very accurately.

Now, indecently the boundary layers with different pressure gradients, as well as v w, as I said, I have been included for the 0.2 to 0.9 region. The values show a scatter about l m by delta equal to 0.09.

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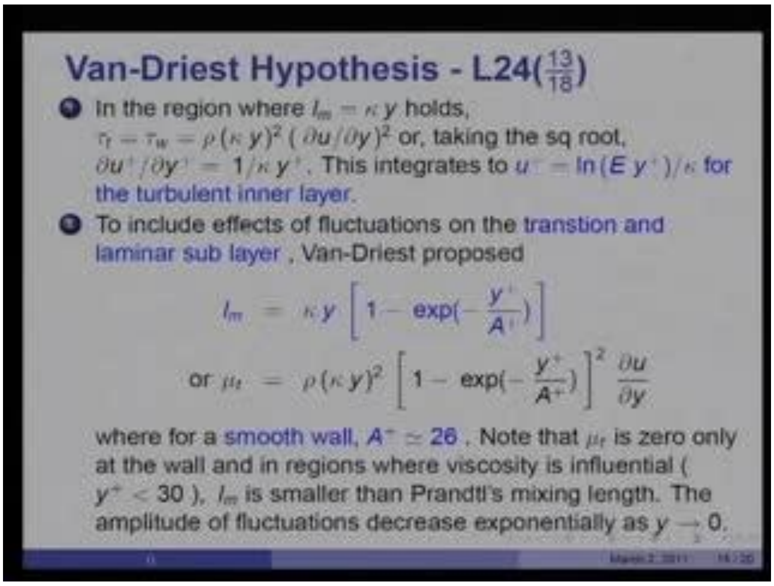

But, for y over delta less than 0.2, l m is nearly proportional to kappa - 0.41. Very close to the wall, l m is somewhat lower damped and then suggested by l m equal to kappa y.

Van Driest suggested that this damping actually occurs due to the effects of fluctuations in the transitional layer and therefore, he said the l m of Prandtl should be modified by introducing a damping function - 1 minus exponential of minus y plus by A plus. Therefore, from the previous slide, mu t would therefore be equal to rho into kappa y squared 1 minus kappa exponential of y plus by A plus whole squared du by dy.

Whereas, from experimental data, it is found that A plus equal to 26 for a smooth wall, will bring about the amount of damping required in line with what is observed in experiments. mu t of is 0 at the wall, because y is equal to 0 there and in regions where viscosity is influential that is y plus less than 30, l m is smaller than the Prandtl’s mixing length.

The amplitude of fluctuations decreases, thus exponentially as y tends to 0; that is what we expect, when viscosity begins to play its role in dissipation.

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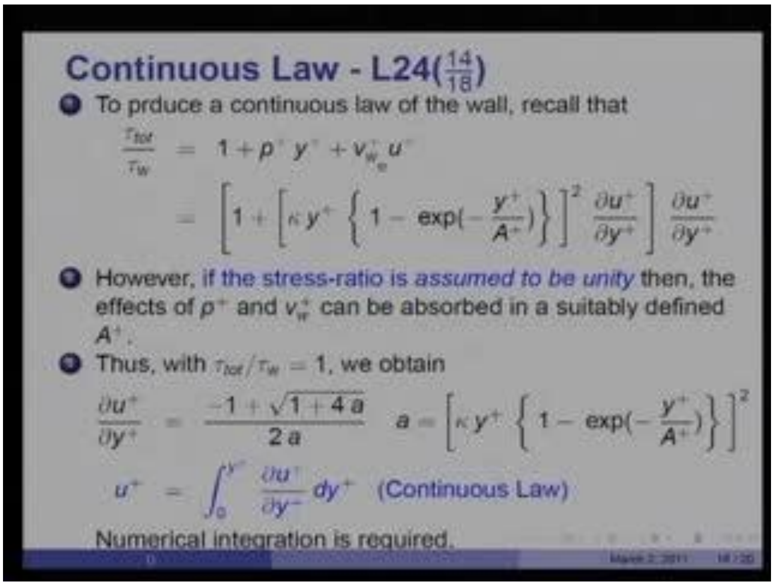

So, in order to develop continuous law of the wall - tau tot by tau wall, which was shown earlier to be 1 plus p plus y plus and equal to v w plus u plus. Now, it can appear in this fashion, where this mu t expression has been used to define tau tot divided by tau wall and this is a long expression.

However, if the stress ratio is assumed to be unity, then effects of p plus and v w plus can be absorbed in a suitably defined A plus.

So, what we are saying is we are going to cheat on this equation. We will say, let p plus equal to 0 and v w plus v equal to 0, so that tau tot by tau wall be exactly equal to 1 and would be equal to that relationship. But, to account for effect of p plus and v w plus we would simply tune the value of A plus, here that is the damping constant.

If you take tau tot by tau wall equal to 1, we obtain du plus by dy plus as a quadratic in du plus by dy plus. The solution is simply u plus equal to integral 0 to y plus du plus by dy plus into dy plus. The a here is given by that damping function.

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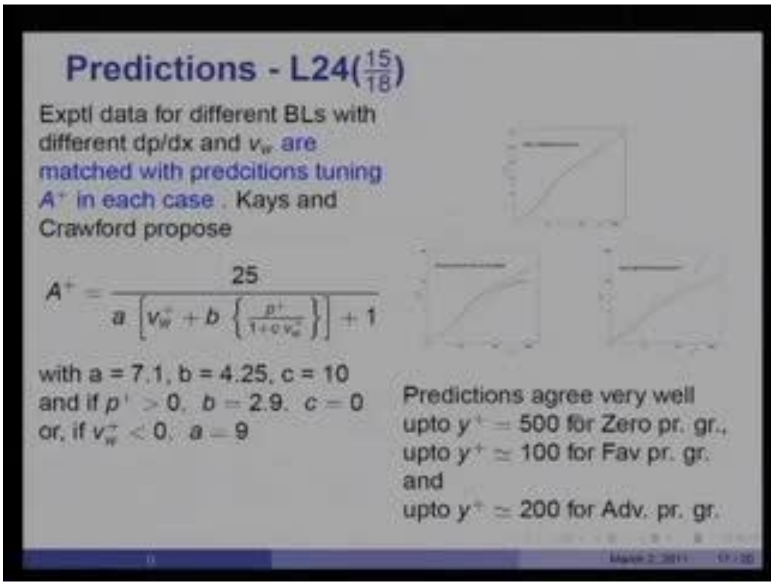

So, you need numerical integration to predict u plus as a function of y plus and that is what I have done here. Experimental data from different boundary layers with different dp dx and v w are matched with predictions by tuning A plus in each case. Kays and Crawford have proposed A plus to be 25 divided by a into v w plus plus b into all this function plus 1. We will make use of this a plus later on in actual computations of friction factors Nusselt number.

But, presently just see this; we do manage to predict the continuous law of the wall. Quite well predict the experimental data in favorable pressure gradient, adverse pressure gradient, as well as a in 0 pressure gradient boundary layers; up to 500 or 700 in 0 pressure gradient, up to 100 in favorable pressure gradient and up to - about 200 or 250 in adverse pressure gradient.

Therefore, we can say that we have now found a method for calculating universality of the inner layer in a continuous manner.

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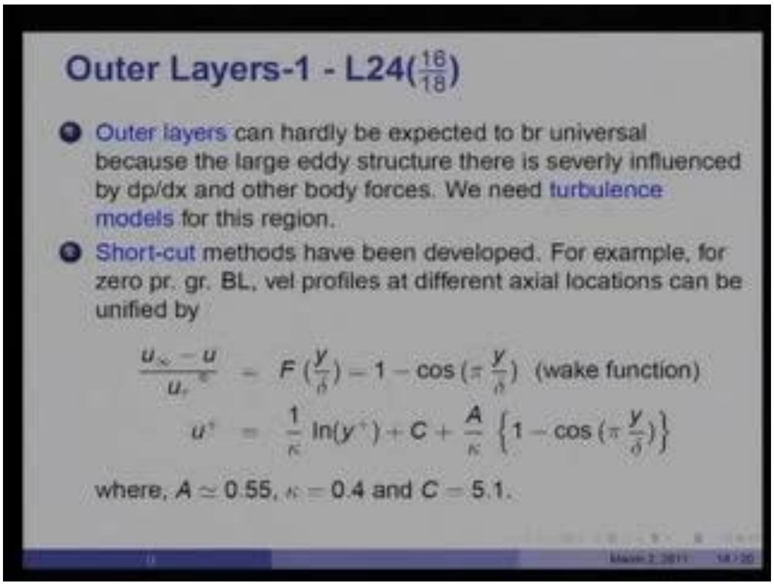

This is very useful when we do friction factor and number. Now, of course, the outer layers do not have any universality, they are big, they are significantly influenced by the pressure gradient and other effects. But, nonetheless efforts have been made - short cut methods have been made to universalized outer layers in this fan.

These are called the velocity defect law. u infinity minus u u tau equal to 1 minus cos y pi by delta. Therefore, the total velocity function is given by this A equal to 0.55 K equal to 0.4 and C equal to 0.51.

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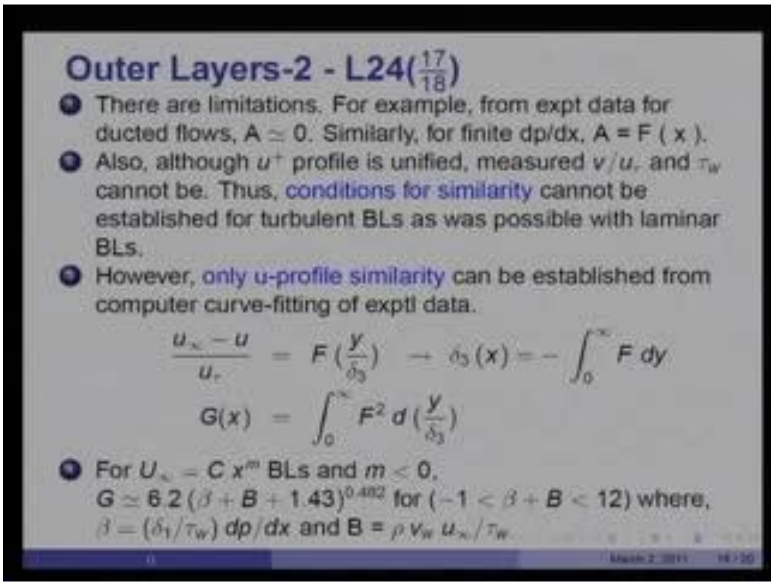

This of course applies only to 0 pressure gradient boundary layer, but not in general. There are other methods for outer layers, but I will not go into that at the moment.

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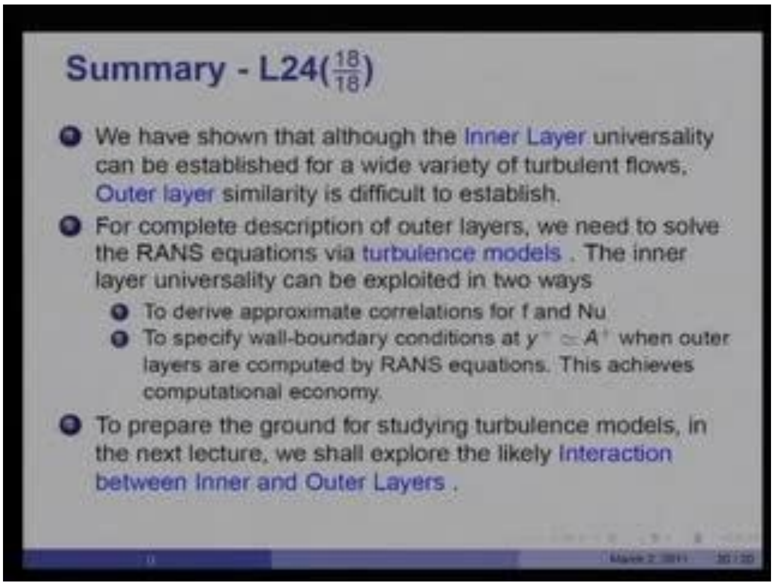

So, in summary, I would say that we have shown that although the inner layer universality can be established for a wide variety of turbulent flows, outer layer similarity is difficult to establish.

For complete description of the outer layers, we need to solve the RANS equations using turbulence models. The inner layer universality can be exploited in two ways. That is to derive approximate correlations for friction factor and Nusselt number, to specify wall boundary conditions at y plus equal to A plus when outer layers are computed by RANS equations. This achieves computational economy, but this is an aspect that the CFD analysis essentially worries about.

Now, to prepare the ground for studying turbulence models, in the next lecture, we shall explore the likely interaction between inner and outer layers; thank you.