Convective Heat and Mass Transfer

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Module No. # 01

Lecture No. # 22B

Sustaining Mechanism of Turbulence-I

To continue our discussion of spatial correlation coefficient, recall that I said these nine correlation coefficients are difficult to measure in a real non-homogeneous non-isotropic turbulence. It is usually measured in only one direction, say R 1.

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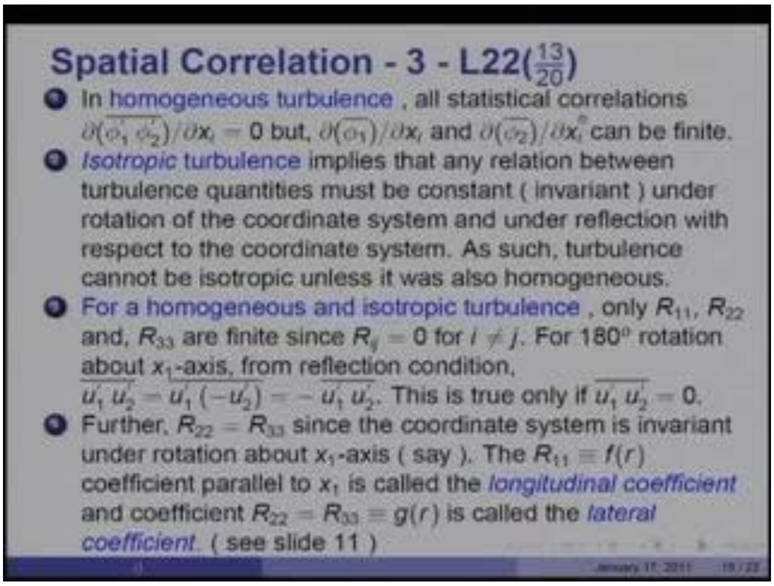

As we said earlier, in homogeneous turbulence, all statistical correlations of time average fluctuating components are 0. Their gradients are 0, but gradients of mean quantities can be finite and that is the definition of homogeneous. Isotropic turbulence implies that any relation between the turbulence quantities must be constant or invariant under rotation of the coordinate system and under the reflection with respect to the coordinate system.

As such, turbulence cannot be isotropic, unless it was also homogeneous. So, what this means? For a homogeneous isotropic turbulence, only R 11, R 22 and R 33 will be finite because all other quantities would involve spatial gradients of phi 1 dash phi 2 dash. Therefore, they would all be 0 for i not equal to j.

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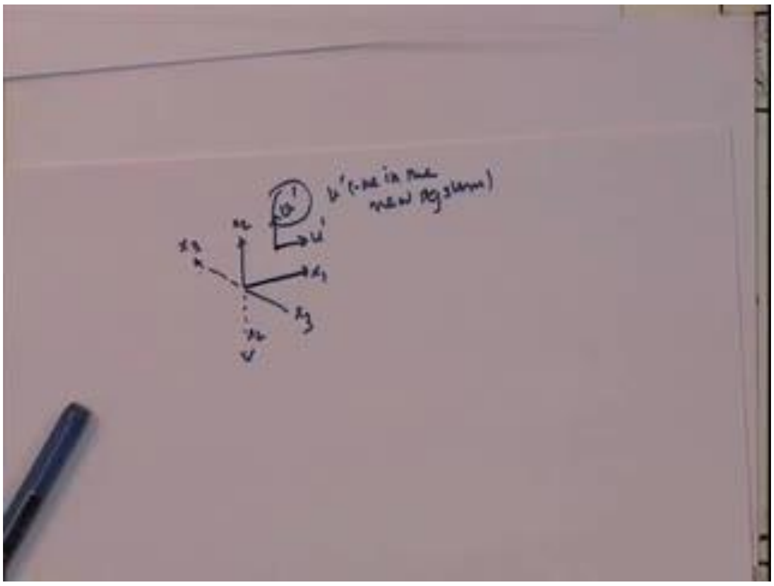

Secondly, for 180 degree rotation, if I had x 1, x 2 and x 3, I am considering at some point, the fluctuation as u dash and the fluctuation v dash. Let us say, if I turn this system through 180 degrees, so that x 2 takes this position. Therefore, x 3 would take that position. This is the negative x 2 and negative x 3 that is turning through 180 degrees. In this new coordinate system - x 1, x 2, x 3, you will see v dash will appear negative in the new system. Therefore, you will see the time averaging of the product.

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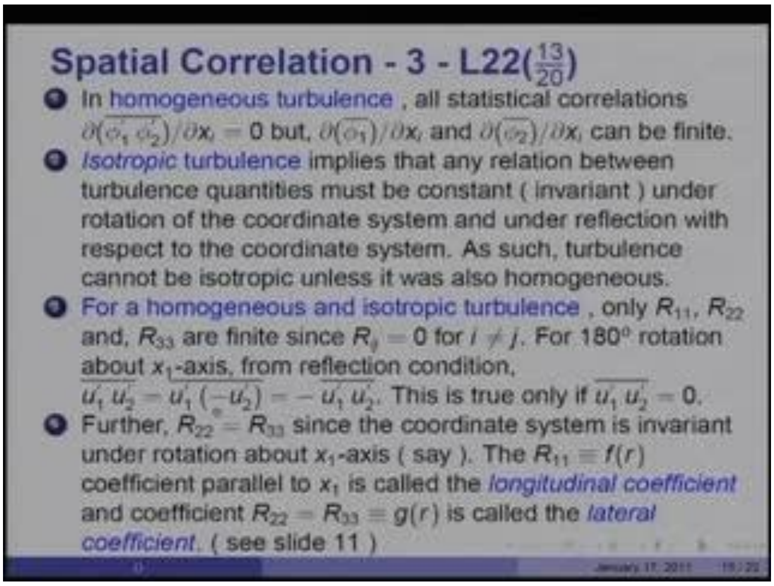

In the first system, it is u 1 dash u 2 equal to u 1 dash into minus u 2 dash in the second system and this would essentially be minus u 1 dash u 2 dash. Now, plus u 1 dash u 2 dash equal to minus u 1 dash u 2 dash can only be true, if u 1 dash u 2 dash were identically 0. That is the meaning of homogeneous isotropic turbulence and for that only R 11, R 22 and R 33 would be finite.

Further, R 22 and R 33 would also be equal, since the coordinate system is invariant under rotation about x 1 axis and that is what I showed. R 11 equal to f r coefficient parallel to x 1 axis is called the longitudinal coefficient; whereas, coefficient R 22 equal to R 33 equal to g r is called the lateral coefficient and that is what I showed in the slide. Here, this (Refer Slide Time: 04:19) is the lateral coefficient this is the longitudinal coefficient.

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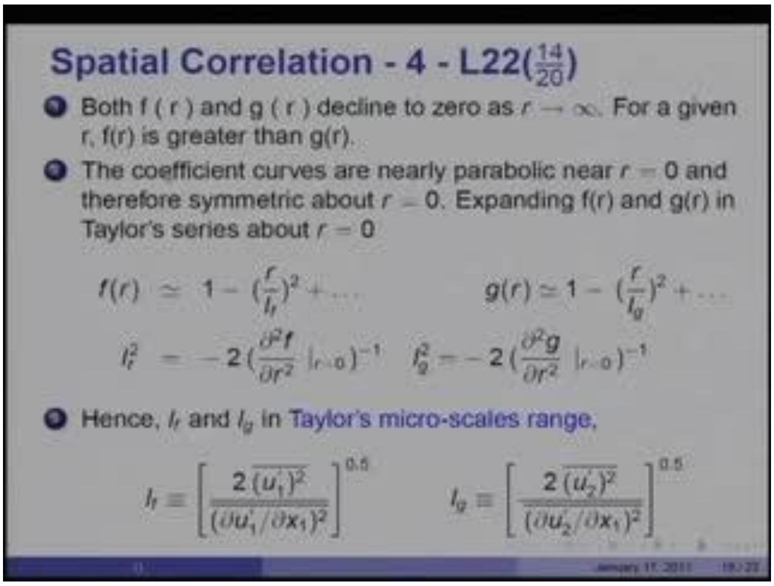

Both f r and g r are declined to 0 as r tends to infinity. For a given r, f r turns out to be a bigger magnitude than g r. The coefficient curves are nearly parabolic near r equal to 0. Therefore, it is symmetric about r equal to 0. Expanding f r and g r in Taylor's series about r equal to 0, you will see and retain only the first couple of terms. We will see f r would be equal to 1, when r is equal to 0. It is minus r by l f square plus additional term and I will define l f in a minute.

Likewise, g r would be approximately equal to 1 minus r by l g whole square plus several terms. Here, l f square would turn out to be minus 2 times d 2 f by dr square r times to 0 raise minus 1 and l g square will be minus 2 into d square g by dr square.

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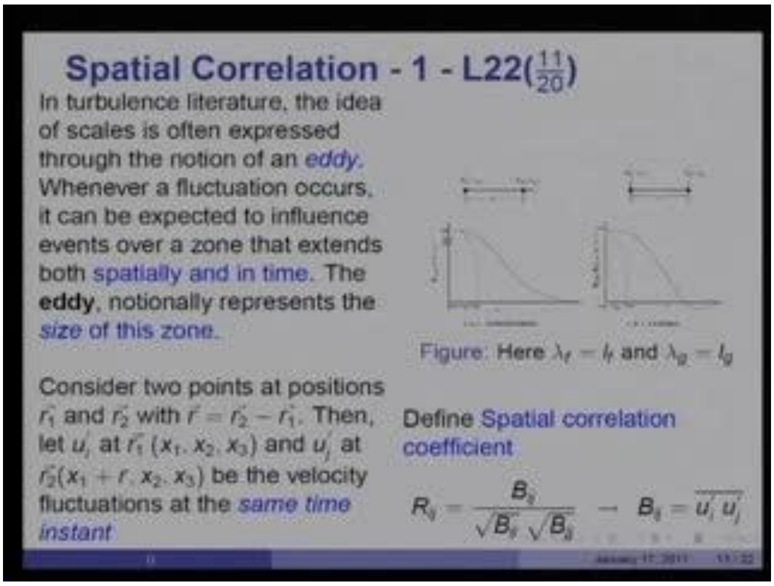

These are the projections of the second derivative of f. You will see for the transfer of longitudinal correlation. Now, l f would appear as that and l g would appear somewhere over there. That is the estimate of l g and this is the estimate of l f. In the Kolmogorov scale, a very small distance l epsilon would be somewhere here and the integral scales would be of that order. There is an integral of this curve between 0 and infinity and likewise here. So, l f and l g are somewhere between l epsilon and l integral. They are the two length scales, which we have we had identified earlier.

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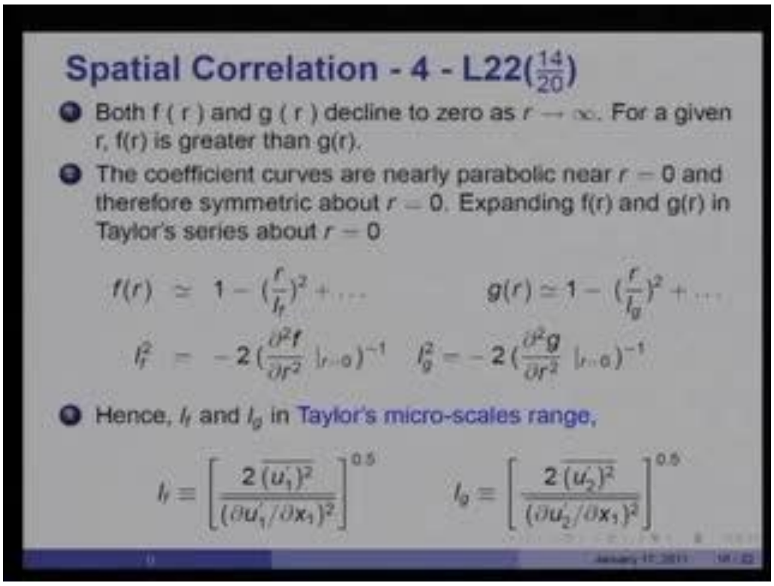

We have now identified a third length scale, which is in between these l f and l g. It is called as Taylor micro-scale. They are defined in this manner - 2 u 1 square divided by d u 1 dash by d x 1 whole square raised to 0.5. So, you can see this has a length dimension square is raised to 0.5 and therefore l f and l g would be based on u 2 prime square separated by distance x in the x direction. The special derivatives are very difficult to measure because simultaneously measuring fluctuating velocity at two adjoining points turns out to be quite a difficult task in a turbulent flow. Therefore, the gradients of fluctuations in x 1 direction would be quite difficult. We will see how to get over that difficulty. Before we do that we will make some important observations.

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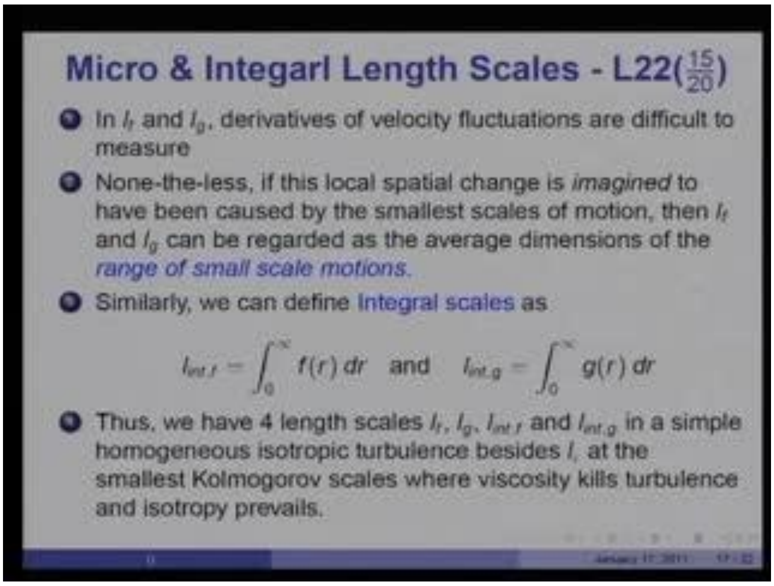

In l f and l g, the derivatives are difficult to measure. If this local spatial change is imagined to have been caused by the smallest scales of motion, then l f and l g can be regarded as the average dimensions of the range of small-scale motion. They are close to r equal to 0 and therefore, they can be considered to be in the range of small scale motions.

Similarly, if we integrate f r from 0 to infinity, we will get integral scales l int f and l int g. Thus, we have four length scales: the micro scale in longitudinal direction, micro scale in the transverse direction, integral scale in the longitudinal direction, integral scale in the transverse direction. In a simple homogeneous isotropic turbulence besides l, epsilon at the smallest Kolmogorov scales, where viscosity kills turbulence and isotropy prevails.

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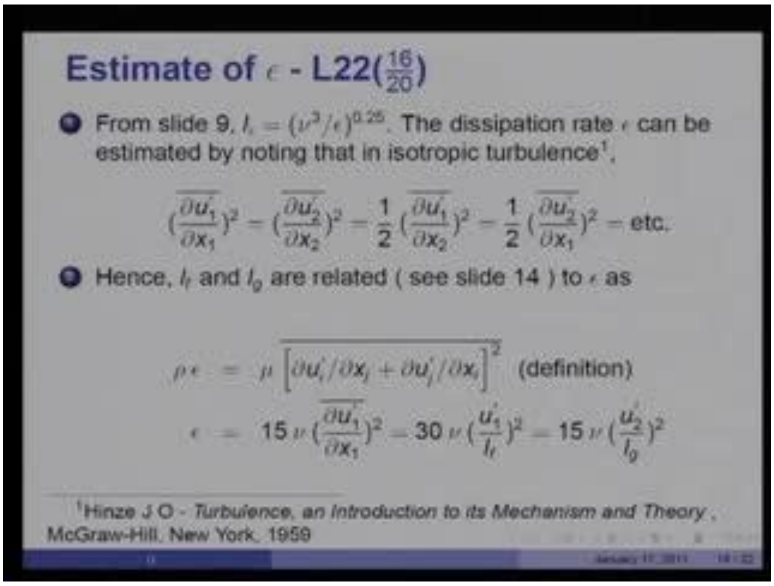

How do we estimate l epsilon? Recall that in slide 9, I showed that Kolmogorov related l epsilon to mu cube into kinematic viscosity cube divided by epsilon raised to 0.25. Therefore, l epsilon can only be estimated, if we can estimate the magnitude of the rate of kinetic energy dissipation. It can be estimated by noting that in isotropic turbulence.

There is a wonderful book by Hinze; it is called as Turbulence - an introduction to its mechanism and theory published by McGraw-Hill in 1959. Here, some properties of isotropic turbulence have been given. One of them is du 1 dash by dx 1whole square would equal du 2 dash by dx 2 whole square. It would only be equal to half of du 1 dash by dx 2 whole square equal to du 1 by 2 du 2 dash by dx 1 whole square and so on.

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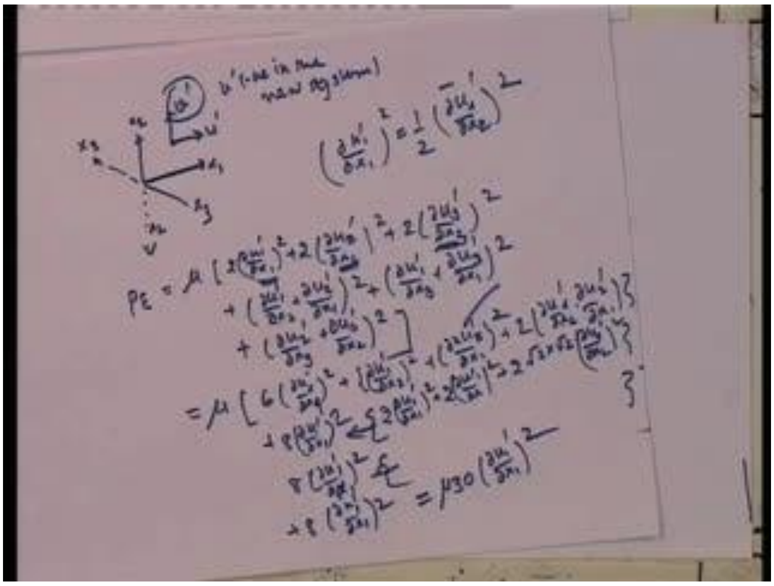

Remember, rho epsilon is actually equal to mu times 2 du 1 dash by dx 1 square plus 2 du 2 dash by dx 2 whole square plus 2 times du 3 dash by dx 3 whole square plus du 1 dash by dx 2 plus du 2 dash by dx 1 whole square plus du 1 dash by d x 3 plus du 3 dash by dx 1 whole square plus du 2 dash by dx 3 plus du 3 dash by dx 2 whole square.

Now, we showed that all these will be equal in isotropic turbulence. Therefore, I have 6 times du 1 dash by dx 1 whole square, but d u 1 dash by d x 1 is equal to half times d u 1 dash by d x 2 whole square. Therefore you will see du 1 dash by dx 2 square will become du 1 dash by dx 2 whole square plus du 2 dash by dx 1 whole square plus 2 times du 1 dash by dx 2 into du 2 dash by dx 1 as the first term.

Likewise, there will be second term and third term. If I make use of this relationship, then you will see this will become 2 times du 1 dash by dx 1 whole square. Then du 2 dash by dx 1 square would again become equal to 2 times du 1 dash by dx 1 whole square. This would equal 2 times du 1 by dash by f 2 will be 2 times square root into square root into du 1 dash by dx 2 square. Therefore, you will see this is nothing but 2 plus 2 into 4 and that is 2 plus 2 equals 4 plus 4 as 8. So, I will get 8 times du 1 dash by dx 1 whole square from this.

I will get 8 times du 1 dash by dx 1 square or from these also. This is du and so I get essentially 3 into… Therefore, all these will become 24 plus 6 equal to 30, so mu times

30 into du 1 dash by dx 1 whole square. That is what I have shown here as rho into epsilon.

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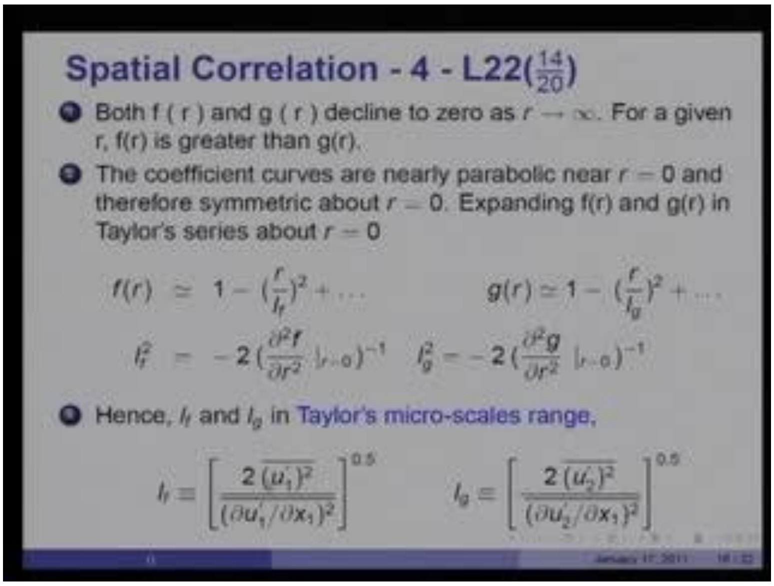

Remember, this definition l f is equal to 2 times u 1 dash square over du 1 dash by dx 1 square and that is what I have shown.

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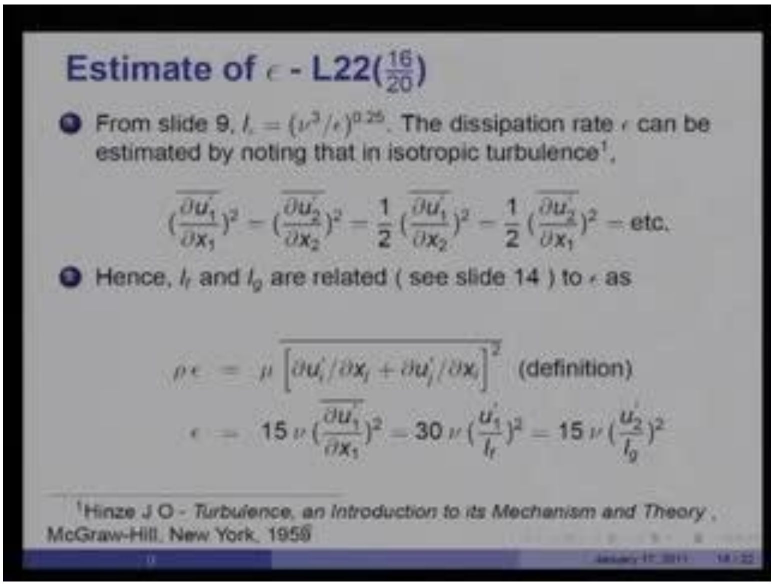

Epsilon would become essentially 15 times mu into du 1 dash by dx 1 whole square is equal to 30 times mu into u 1 dash whole square by l f and likewise 15 nu times u 2 dash by l g whole square. This is how one estimates epsilon, provided we know this quantity and this quantity.

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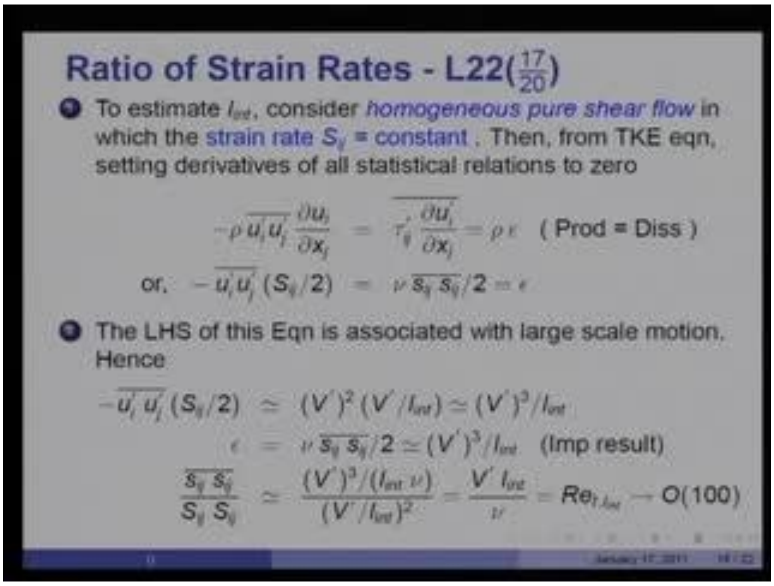

We can also estimate l f. To estimate integral length scale, consider homogeneous pure shear flow in which, the strain rate S i j of the mean velocity gradient is constant.

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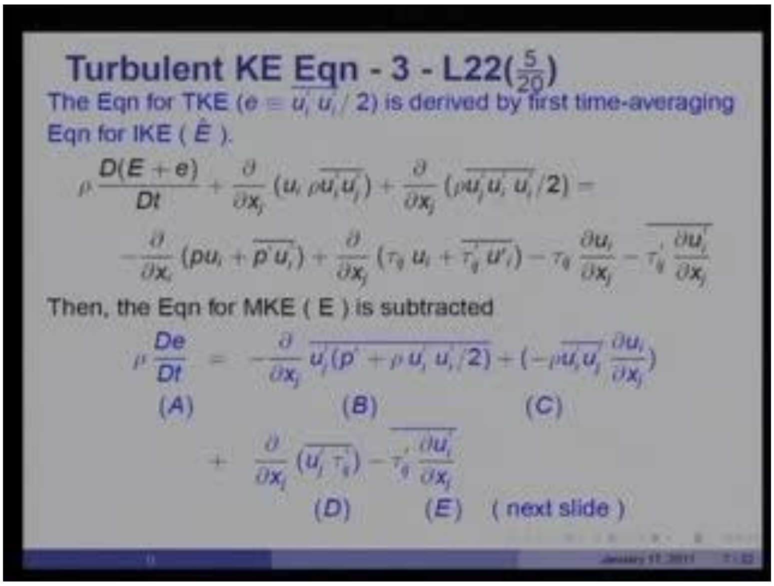

In the turbulent kinetic energy equation, all spatial gradients of product quantities would vanish, but the mean quantities would survive. So, this would survive but that would D term will go to 0, B term will go to 0 and production and dissipation would survive.

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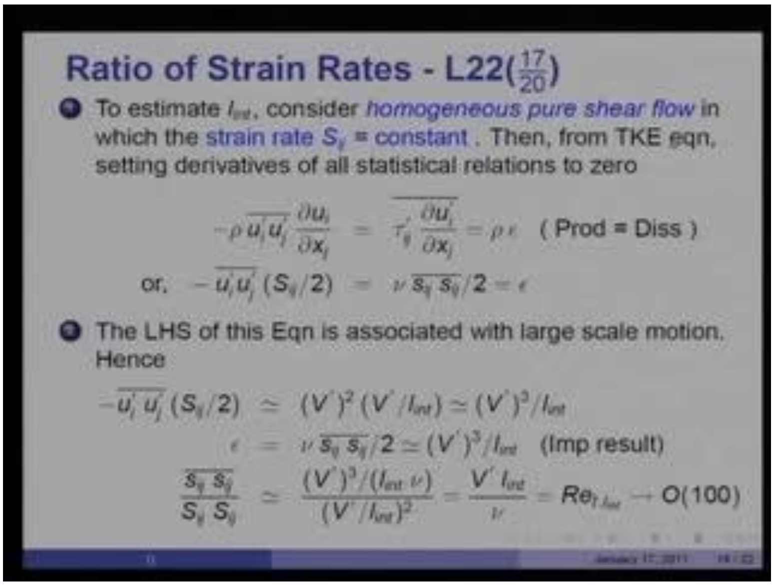

From turbulent kinetic equation and assuming steady state, we would have minus rho u prime u j prime mean velocity gradient equal to tau dash i j du dash by dx j. It is equal to rho into dissipation. In other words, production will exactly be equal to dissipation. This is called equilibrium state. Another way of writing this is u i j u prime j equal to S i j by 2 equal to mu times. Remember, this is tau dash as mu times strain rate s i j. Small s i j is d u i by d x j plus d u j dash by d x I, whereas S i j is from the mean quantities by d x j plus d u j by d x i that is the mean s i j. In other words, I get u i prime u j prime time average into S i j by 2 equal to mu times small s i j s i j divided by 2 and that is equal to epsilon.

I have divided throughout by density, so you get that as a very interesting result. Now, the left hand side of this equation is associated with large scale motion u i dash u i dash S i j by 2. It is really dimensional because u i dash u j dash is essentially V dash capital V dash square divided by l integral essentially large-scale motion because the mean velocity gradients are involved. That would equal V dash cube divided by l integral and that is equal to epsilon. Epsilon would also be equal V dash cube by l integral and this is a very important result.

Remember, epsilon is associated with very small-scale motion and action of viscosity and yet, it can be estimated from the representative scales of the large-scale velocity fluctuation and large scale integral length scale. Therefore, this is sometimes called as the first law of turbulence. It is the ability to estimate epsilon i j or epsilon from large-scale fluctuation velocities. Integral length scale is a very good result because it helps us later on in economic computation of turbulent flow. This result is routinely used in bimodulus of turbulent flow equations.

Another way of writing the same is that s i j divided s i j that is strain rates of smallest fluctuating motion divided by the strain rates of mean motion would be V dash cube l int nu divided by V dash l int square. That would be equal to V dash l int by mu or the turbulent Reynolds number is formed from fluctuations of the mean and integral lengths. So, this tend totally represents the large scale. Since Re t int l int is of the order of 100, it means that the strain rates of the fluctuating quantities at the smallest scales are much greater than the strain rates of the mean quantities like u. The strain rates formed from u dash are much greater than these. Another way of saying is we can expect it in terms of amount of straining. The small-scale motions are totally again uncorrelated with the large- scale motion.

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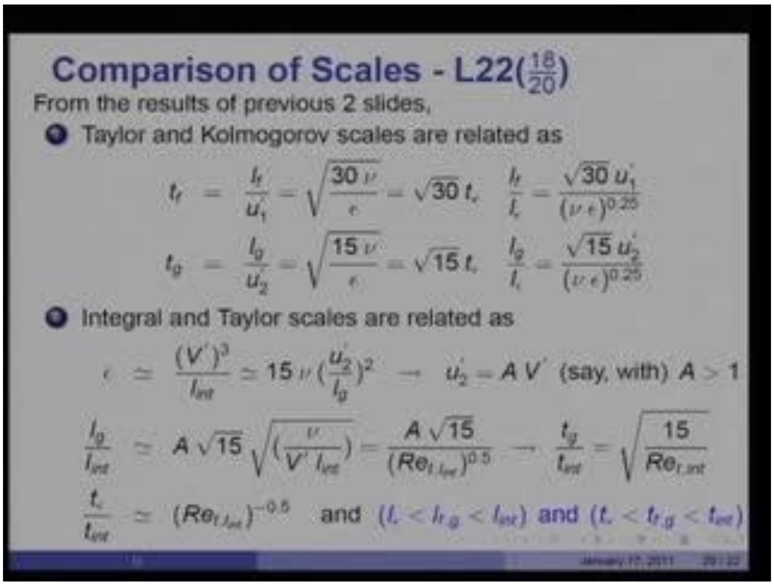

From the results of the previous two slides, we can now estimate and compare Taylor and Kolmogorov scales. So, time scale of t f would be l f by u 1 prime and that would be equal 30 mu divided by epsilon and under root 30 t epsilon because mu by epsilon is square root of…It is really the time scale of the Kolmogorov scales. So, this shows that the Taylor micro scale time scale is under root 30 times Kolmogorov time scale in the longitudinal direction. Similarly, l f divided by l epsilon would be again that quantity of time scale in the transverse direction; it would be root 15 times t epsilon. So, there is a considerable separation between time scales associated with Kolmogorov scales and the micro scales, but not as big as what we observed between integral scales and the Kolmogorov scales.

Integral and Taylor scales are related as follows. Since epsilon is V dash cube by l int cube equal to 15 mu times u dash square by l g. If I take u dash about A times V dash, then it follows that l g by l int would be A root into 15 under root mu by v dash l int equal to that Reynolds number of turbulence, which is of the order of 100. Therefore, t g by t integral would be under root 15 by that. In other words, the transverse micro scale would be much smaller than the integral scale. The same thing would also apply to the longitudinal time scale in comparison to integral time scale.

The only difference is 15 would be replaced by 30. Now, t epsilon by t integral is Reynolds t integral is raised to minus 0.5. So, we can say l epsilon is less than l f and g and is also less than l integral. Although the distance between this and this would be considerably smaller than the distance between this (Refer Slide Time: 22:13) and this and the separation distance overall separation distance would be determined be the Reynolds number.

The same story applies to the time scales. Kolmogorov time scales would be much smaller than integrals time scales, but just small compared to the Taylor micro scales. So, now we have discovered that there are three scales and the middle one is the representative.

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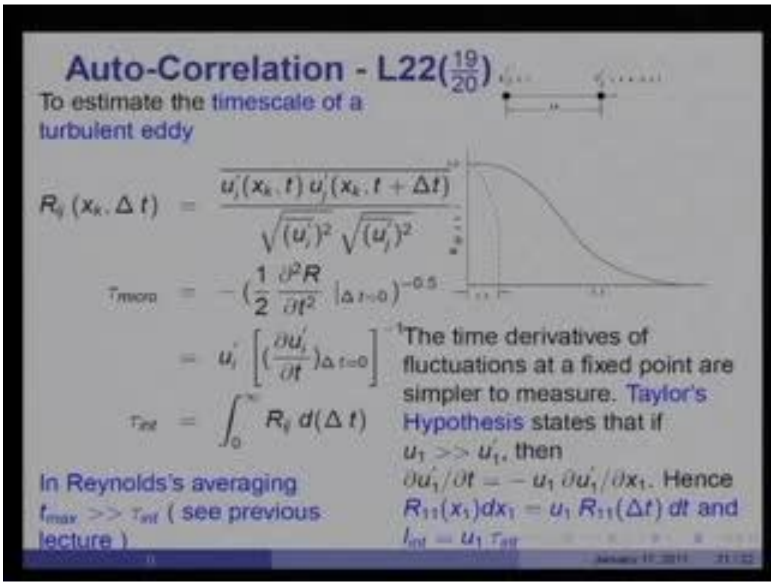

As I said, spatial correlations are very difficult to estimate. Therefore, it makes l f g very difficult to estimate. In order to do that we undertake measurement of what is called as a autocorrelation.

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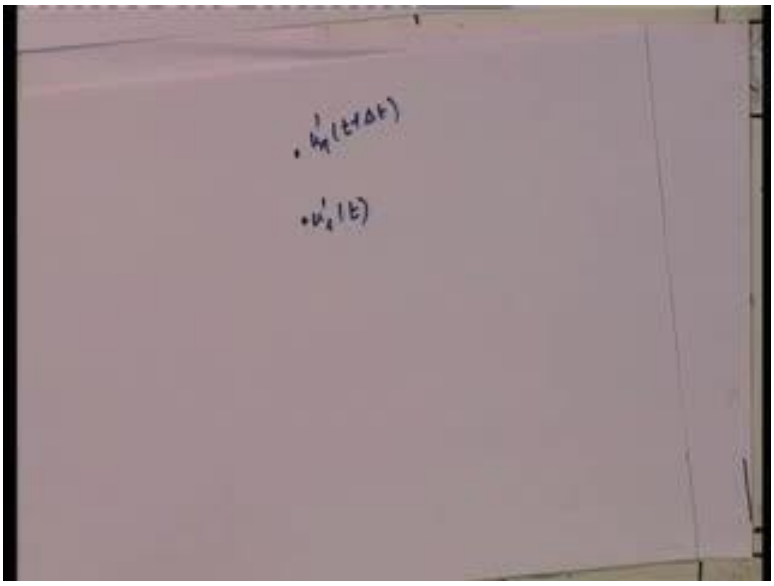

We will consider the same point, but separated by… Let this be u 1 dash at t and this will be u 1 dash at t plus delta t as separated by time. We would define exactly in the same fashion as we defined this spatial correlation coefficient.

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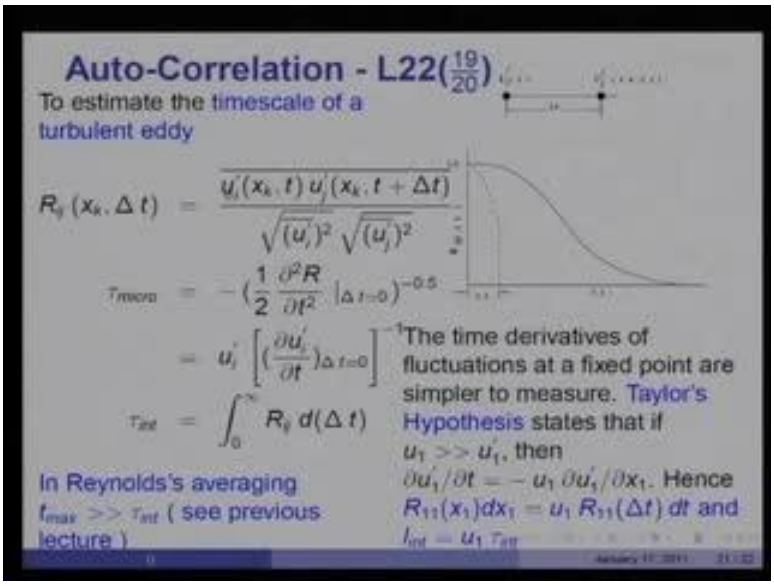

This is what I showed here. The value of u dash at t at the same point x k and the value of u dash j at t plus delta t is essentially b i j and u i dash square, which is b i i and u dash j is square and this will be at t plus delta t and this will be at t. Again, if I plot values of r i j for different values of delta t in the separation time, then I would get a perfect correlation, when delta t is 0. It will go on declining, as I go on increasing delta t and beyond a certain delta t, the fluctuations at the later time would be completely uncorrelated with the fluctuation at time t equal to 0.

So, like what we did earlier, I can estimate a micro time scale like tau micro associated with l f g equal to u dash i into du i dash by dt at delta t equal to 0 raised to minus 1 by taking advantage of the fact that the variation here is very nearly parabolic. Therefore, the projected delta t or tau micro would be that. Likewise, the integral time scale would be somewhere 0 to infinity R i j d delta t. Now, we can get an idea of what should be the smallest magnitude of t max required in Reynolds average.

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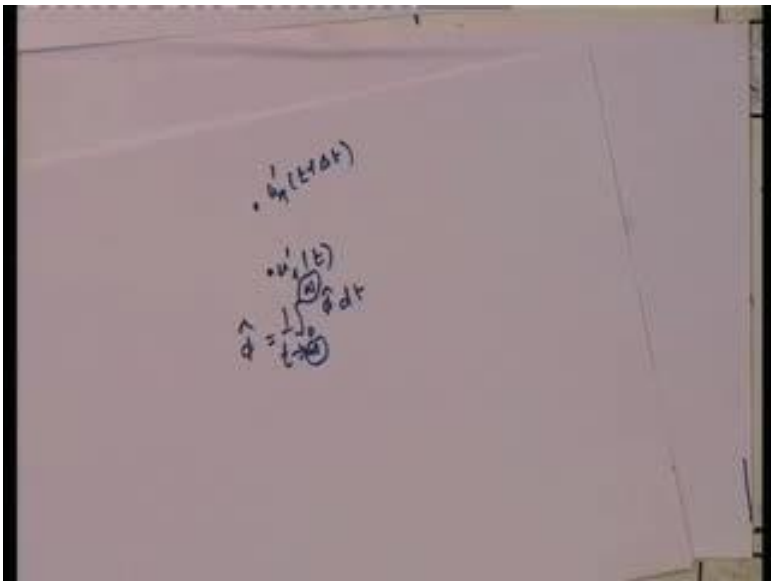

In Reynolds averaging, phi cap average equal to 1 over t, tending to infinity. 0 to infinity phi cap dt is what we said. For practical engineering measurements, we would of course need the estimate of epsilon. It has to be some finite time, as we cannot go on measuring for infinite time and that estimate key is now available.

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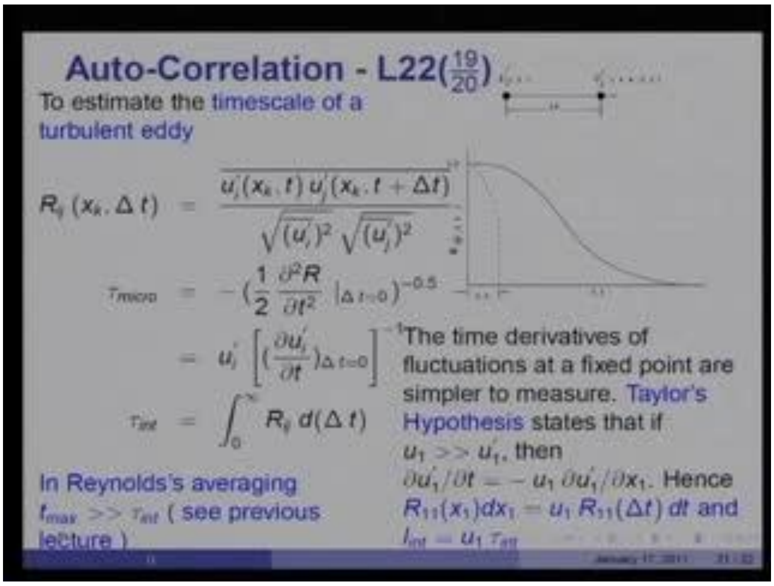

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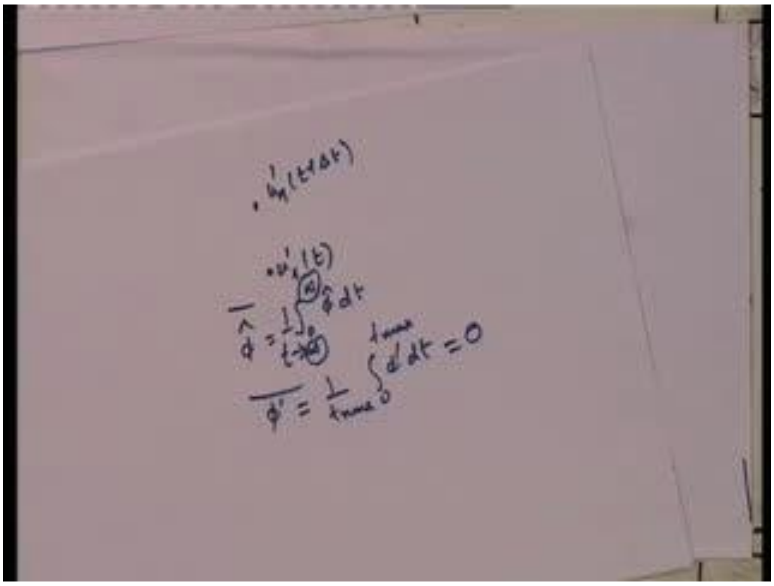

From this expression (Refer Slide Time: 26:00), we say for t max to be much greater than tau integral. This would ensure that phi dash bar would be equal to 1 over t max integral phi dash dt is 0 to t max and it would be equal to 0. That is the first importance of autocorrelation.

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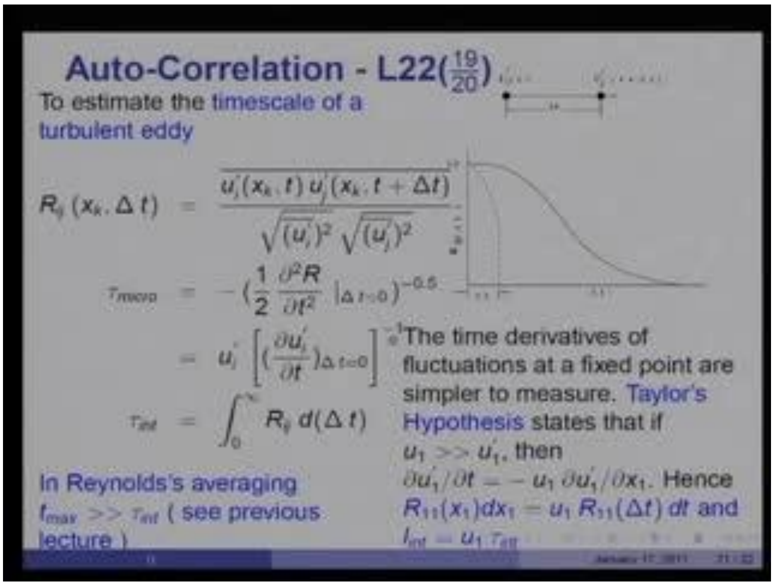

There is also another very important thing. As I said, it is not possible to measure special gradients of u i dash to estimate R i j, which is the special correlation coefficient. So, the time derivatives of fluctuations at a fixed point are easier to measure with a single instrument like a hot wire. It can enable us to measure that as a function of time. Taylor made a hypothesis, if mean u 1 is very much greater than u 1 dash, then du 1 dash by dt can be taken as minus u 1 into du 1 dash by dx 1. It gives us the estimate of du 1 dash by dx 1, which is required to estimate l f - the longitudinal Taylor micro scale. Therefore, we can say that R 11 of x 1 dx 1 would be equal to u 1 into R 11 of delta t dt. In other words, l integral would be u 1 times tau integral and that is given here.

Now, this is a very important deduction. There are two important deductions from autocorrelation. First of all, they are extreme and they are much easier to measure than the spatial correlation. The autocorrelation gives you the idea of what t max should be. Usually, four to five times, the integral tau in is taken in practical measurements. We are also able to estimate the spatial correlation coefficient and therefore, estimate the l integral from tau integral, which as I said is much easier to measure.

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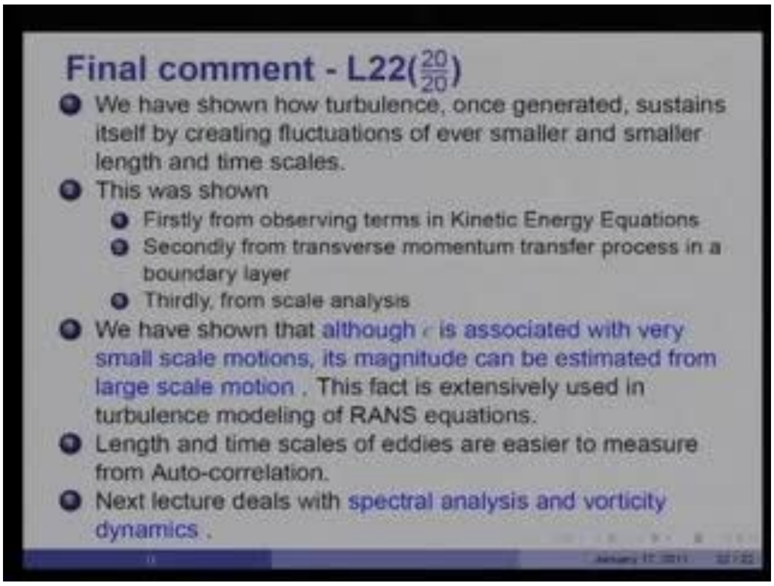

My final comment on these last two lectures is we have shown how turbulence, once generated, sustains itself by creating fluctuations of ever smaller and smaller length and time scales.

This was shown firstly, by observing terms in the kinetic energy equations. Secondly, from transverse momentum transfer processes in a boundary layer, where we got reasonably good idea of separation between the dissipation scale or viscosity affected scales and the large scale. Thirdly, we did the scale analysis and we have also shown that. Although epsilon is associated with very small-scale motions, magnitude can be estimated from large-scale characteristics of large-scale motion. This fact is extensively used in turbulence modeling of RANS equations. The length and time scales of eddies are easier to measure from auto correlation. Usually, most people measure autocorrelations and from that they derive the spatial correlation coefficients.

Energy from mean motion is somehow transferred down to very small scales, where viscosity takes over and kills turbulence. We have tried to understand in physical space with physical measurements on what can be done with physical measurements. By transforming equations in the wave number space, it is possible to illustrate this story even more convincingly and that is called spectral analysis.

The equations generated cannot be solved in physical space, unless they are brought back again in the physical space. The equations in the wave number space are difficult to solve, but it reveals a story of what really goes on sustaining turbulence. Which are the terms that actually carry out turbulent energy production? What is the role of the redistributive terms that vanish on the cross-section? What is the contribution of the dissipation motion? I will take up that story in the next lecture, where I will explain what spectral analysis is. There is also another possible explanation of this transfer process, which can actually be shown figuratively by imagining stretching and torturing of an element fluid element by vorticity dynamics equations. I will try to show you, how both these tell the same story that we have already revealed through equations in the physical space and measuring capabilities in the physical space.