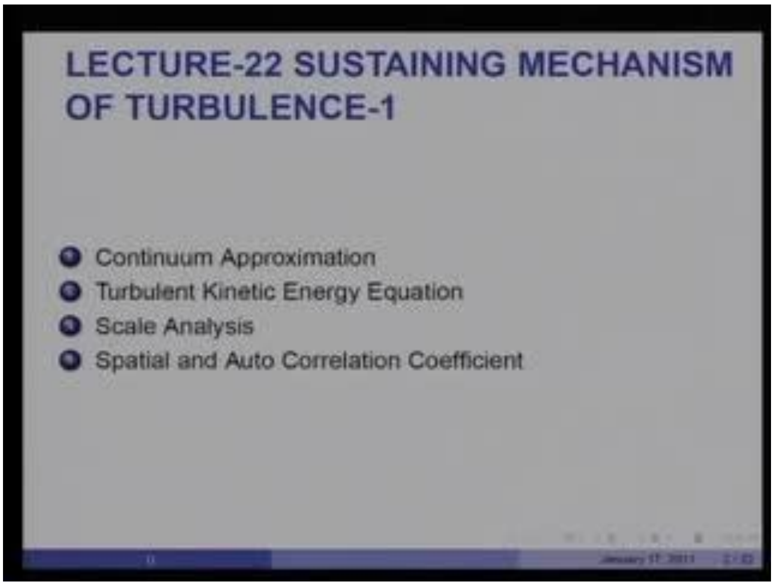
Convective Heat and Mass Transfer
Prof. A. W. Date
Department of Mechanical Engineering
Indian Institute of Technology, Bombay

Module No. # 01
Lecture No. # 22A
Sustaining Mechanism of Turbulence-I

In the previous lecture, we made two important observations: one was the characteristics of turbulent flow significantly different from those of laminar flow and secondly, turbulence once generated - say, inside a pipe - somehow sustained itself right through the length of the pipe; it does not die away which means that there must be some pumping action, which feeds turbulence continuously to sustain itself, while viscosity is trying to kill it.

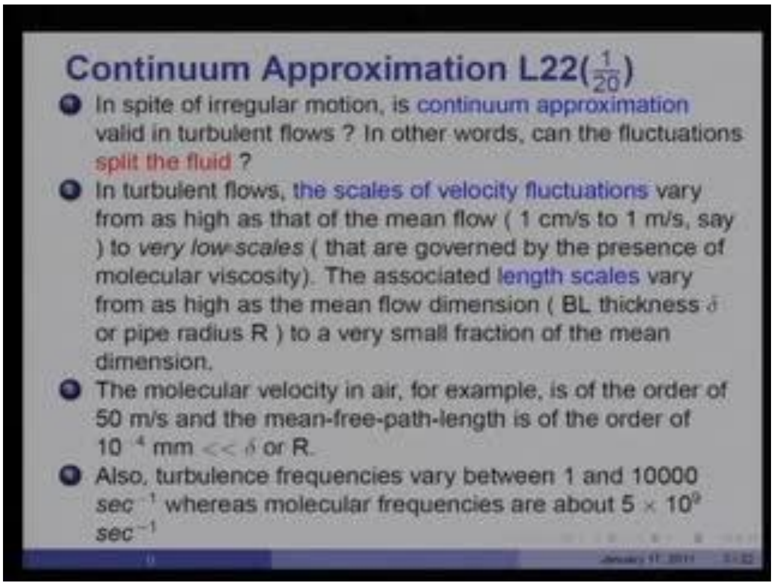
(Refer Slide Time: 01:24)



This lecture is the first of two lectures which will try and explain how turbulence sustains itself. We will do this firstly by some order - some magnitude analysis - by looking at and deriving the turbulent kinetic energy equation. Then, we will do scale analysis in which we will introduce the idea of length and time scales of turbulent eddies through special and auto correlation coefficients.

The first question that arises is that turbulent fluctuations are extremely random and sharp as I showed you in the previous slide. Could it be that these fluctuations actually split the fluid? For example, if I had a paper in my hand and if I stretch it, subjecting it to very random motion, it is likely that the paper will split. Will the fluid split? That is a question, because of the stretching and torturing by the vertices. Can the fluid actually split? The question can be answered by considering orders or magnitude.

(Refer Slide Time: 02:49)

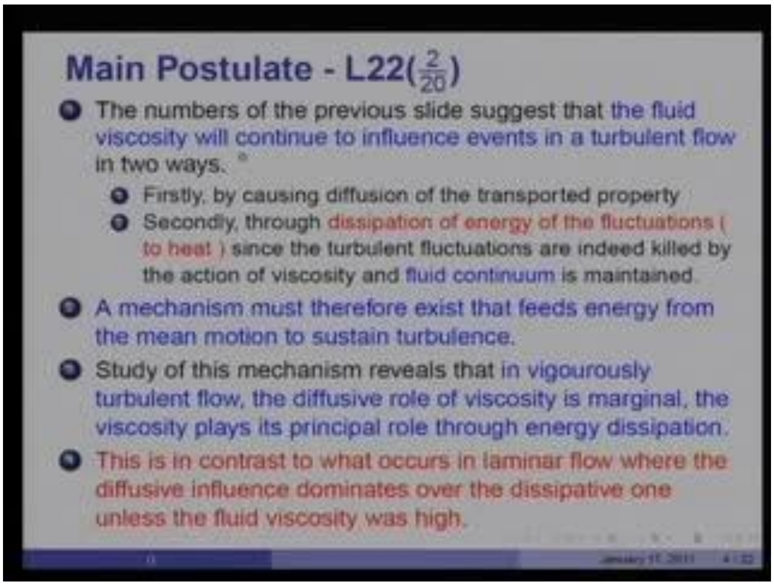


First of all, in turbulent flows, scales of velocity fluctuations vary from as high as that of the mean flow - say, in air, it could be anywhere up to 1 meter per second - to very low scales that are governed by the presence of molecular viscosity. The associated length scales would vary from as high as the mean flow dimension - say, boundary layer thickness or radius of a pipe - to a very small fraction of these quantities; the scales are associated with the turbulence fluid.

Let us ask ourselves what happens at the molecular level. For example, molecular velocity in air would be of the order of 50 meters per second, much greater than 1 meter per second. The mean-free-path-length would be of the order of 10 raised to minus 4 millimeters of delta and r; delta and r would be of the order of 1 millimeter to, let us say, 1 centimeter onwards. Velocity scales of a molecule are much greater than the mean flow velocity scales where the length scales are much smaller than the mean flow scales.

Similarly, in turbulence, frequencies of fluctuations are of the order of 10 raised to 4, whereas the molecular frequencies are of the order of 5 billion; there is a vast difference between what happens at the molecular level and what happens in practical turbulence. You can see that this vast difference implies that the two must be completely uncorrelated. Turbulence behaves in its own way, molecules continue to behave in their own way, and therefore, we could safely assume that turbulence does not, in any way, destroy the basic characteristic of a fluid; the fluid will always remain as a continuum because of the presence of viscosity and no splits at the molecular level would occur. This is a very important observation about turbulence, to make progress with the theory of turbulence.

(Refer Slide Time: 05:30)



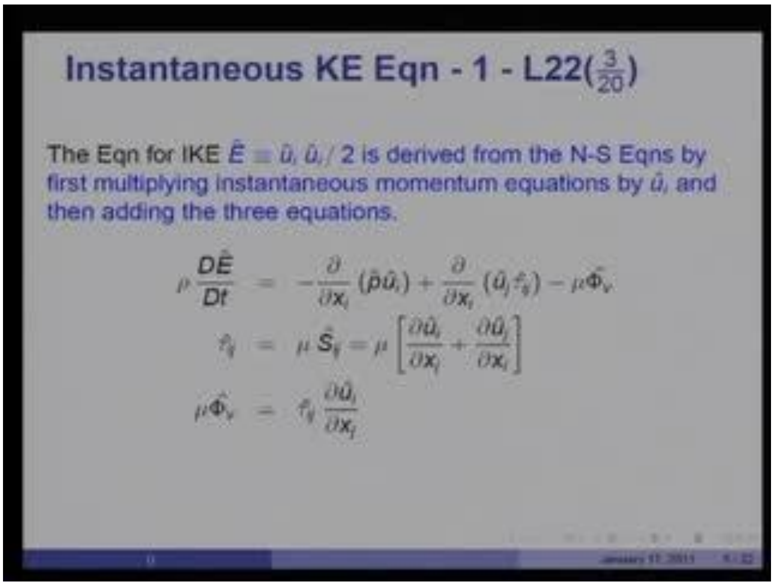
The numbers of the previous slides suggest that the fluid viscosity will continue to influence events in turbulent flow in two ways. Firstly, by causing diffusion of the transported property - it can be a momentum, it can be temperature, or mass structure or anything; so, turbulence will cause because of viscosity causing diffusion. Secondly, through dissipation of energy of the fluctuations to heat, since turbulent fluctuations are indeed killed by the action of viscosity; therefore, the fluid continuum is maintained.

In other words, fluctuations are no longer allowed by viscosity to sustain themselves and they are simply killed by viscosity. Therefore, the molecular behavior remains completely unaffected by turbulence.

Having made this observation that the continuum is maintained, we now turn to the main point that a mechanism must therefore exist that feeds energy from the mean motion to sustain turbulence, while viscosity kills turbulence.

Study of this mechanism reveals that in vigorously turbulent flows, the diffusive role of viscosity is marginal - that is the first one. But the viscosity plays its principal role through energy dissipation - that is, the motions are killed and therefore, the kinetic energy is dissipated.

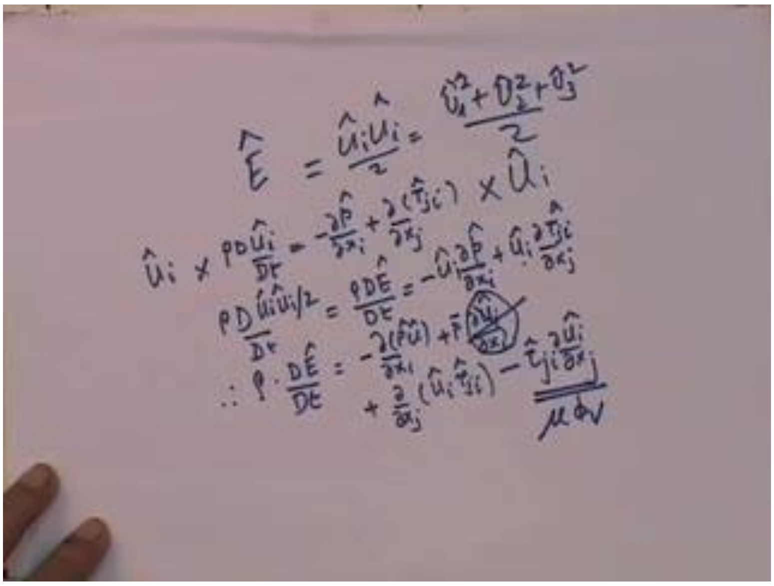
(Refer Slide Time: 07:30)



Now, this is in contrast to what occurs in laminar flow, where the diffusive influence dominates over the dissipative one unless the fluid viscosity was very high, as in oil flows. That is something we have already studied while considering laminar flows.

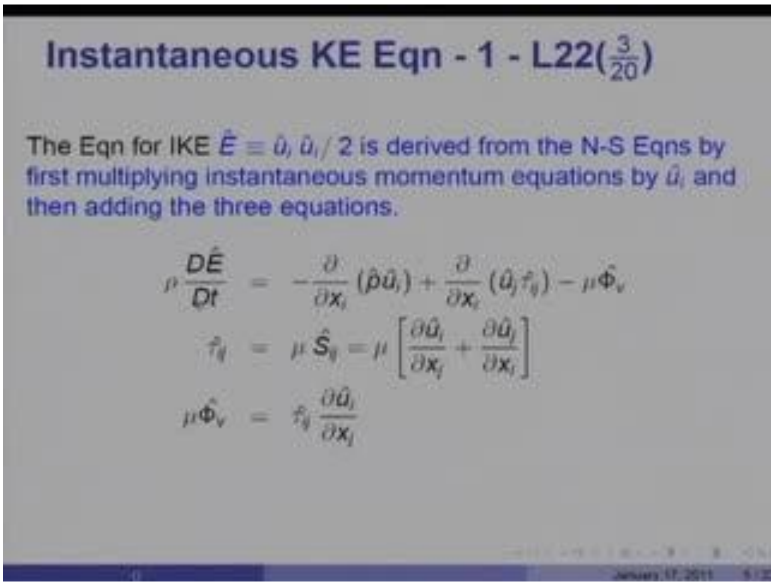
In order to explain whatever I have said through the Navier-Stokes equations, the first thing to appreciate is that the Navier-Stokes equation is written for an instantaneous velocity, u cap, which is a valid description of turbulent flow. Because the continuum prevails, all derivatives can be resolved for the instantaneous velocities, and therefore, the equations are valid. So, this is a fundamental assumption on which we will proceed.

(Refer Slide Time: 08:22)



For example, I can develop an equation for instantaneous kinetic energy. This is the equation for u cap u cap i by 2 which essentially means u1 square plus u2 square plus u3 square divided by 2 is the instantaneous kinetic energy. How do I derive then? Recall that the instantaneous momentum equation is D u cap i by D t equal to minus d p cap by d x i plus d tau j i cap d x j. If I multiply this equation by u cap i – that is, u cap i and u cap i – throughout, then you will notice that this will be rho times D u i u i cap divided by 2 by d t which is nothing but rho D E cap by D t. This will equal minus u cap d p cap by d x i plus u i cap d tau j i cap by d x j. Therefore, rho times D E cap by D t would be equal to minus p absorbing u inside by d x i plus p cap d u i d x i plus again observing this inside d by d x j of u i cap tau j i cap minus tau j i d u i d x j this is what it will be. But remember from continuity equation which also applies the instantaneous velocities this term will be 0 and that is why you get this equation, that is what I have written here D E by D t equal to d by d x i p by u i plus d by d x i of u j d u i d x j minus mu phi v, this is mu phi v the viscous dissipation term.

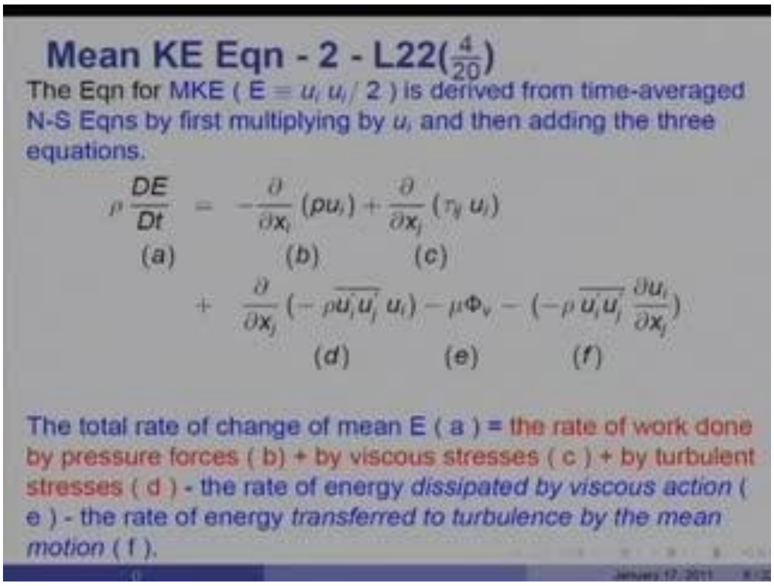
(Refer Slide Time: 11:39)



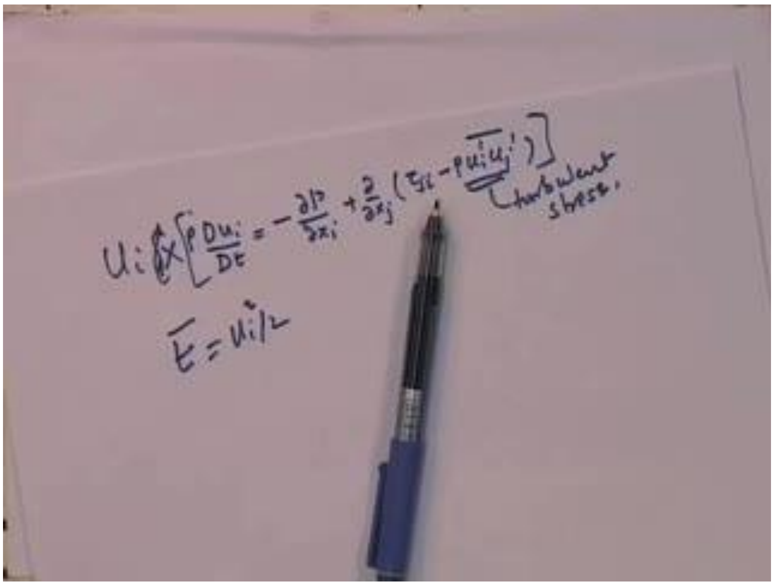
Essentially, what the equation says is the rate of change of instantaneous kinetic energy is the rate net rate of work done by pressure forces plus the net rate of work done by the stresses tau i j mu S i j; S i j is the strength rate d u i d x j plus d u j d x i minus viscous dissipation.

So, turbulent energy increases because of these terms, which can be positive or negative does not matter, but when they are positive it increases but mu phi v by definition would being positive would always decrease instantaneous kinetic energy. So, viscosity plays the role of destroying instantaneous kinetic energy.

(Refer Slide Time: 12:48)



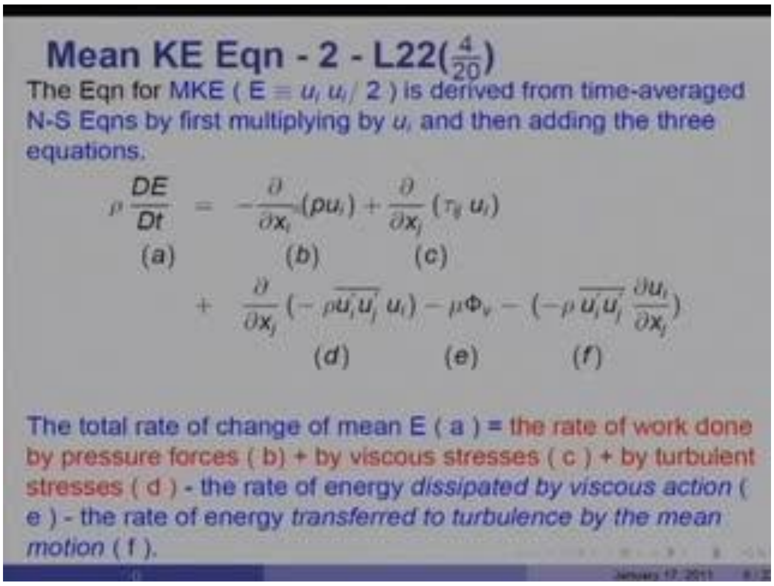
(Refer Slide Time: 12:52)



I do the same thing now to derive mean kinetic energy equation, but in this case what I will do is I will begin by writing the RANS equations where D u i by D t equal to minus d p by d x i plus d by d x j of tau j i minus rho u i prime u j prime d x j plus the body forces, which I am presently in ignoring and these where the turbulence stresses.

So, if I multiply this equation throughout by u i, I would get again an equation for u i square by two which is the mean kinetic energy equation; here is that equation.

(Refer Slide Time: 13:55)

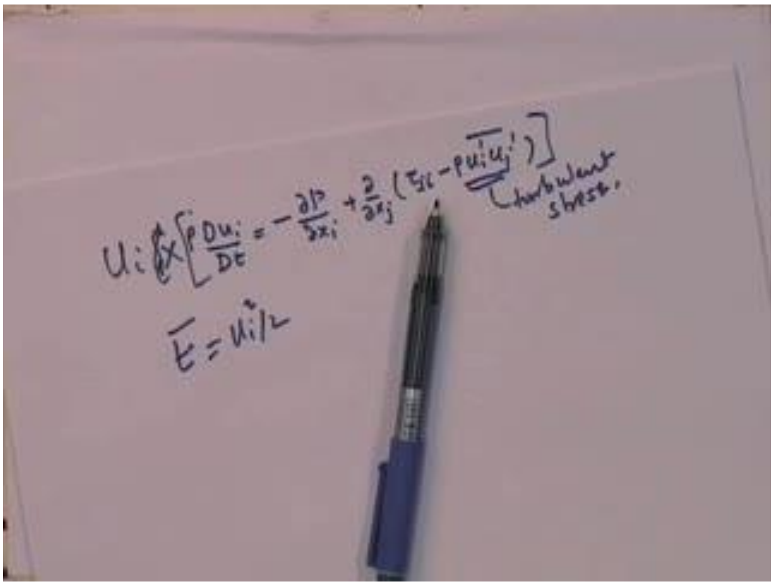


Its looks very similar in many terms - this is the pressure work term, this is the work done by laminar stress, this is the work done by turbulence stress, and this would be the viscous dissipation due to mean velocity gradients and minus minus turbulent stress multiplied by d u i by d x j the mean velocity gradient.

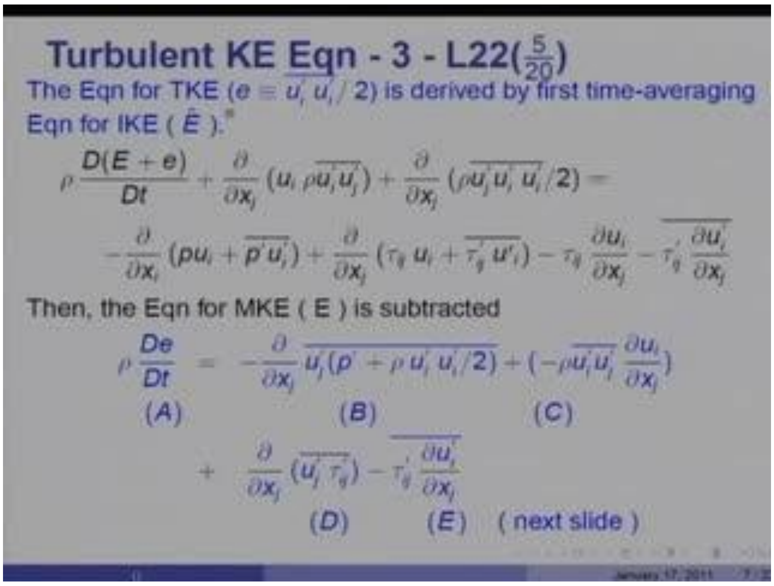
The equation essentially says - that the rate of change of mean kinetic energy E is equal to the rate of work done by pressure forces, rate of work done by viscous stresses, rate of work done by turbulence stresses; that is termed a minus the rate of energy dissipated by viscous action and minus this is the most important term f.

That the rate of energy transferred to turbulence by mean motion d u i d x j; now why do I say - rate of energy transferred to turbulence? That you will appreciate from the next slide.

(Refer Slide Time: 15:03)



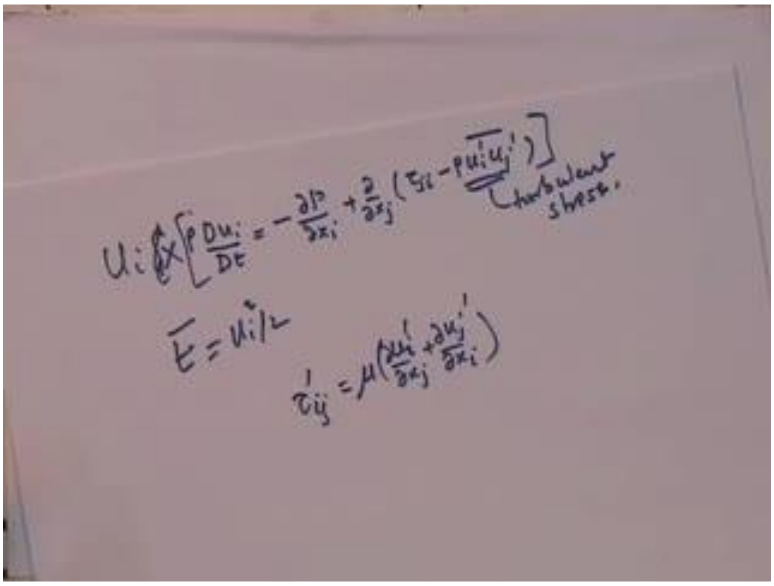
(Refer Slide Time: 15:06)



Now, I want to derive an equation for the turbulent kinetic energy, which is u i dash u i dash time average divided by 2; this is the turbulent kinetic energy; it is derived by first time averaging the instantaneous kinetic energy equation; in other words this equation i time average each term then time averaging of this term would give me mean E plus turbulent e and so on and so forth and the equation would look as I have shown here (Refer Slide Time: 00:00)

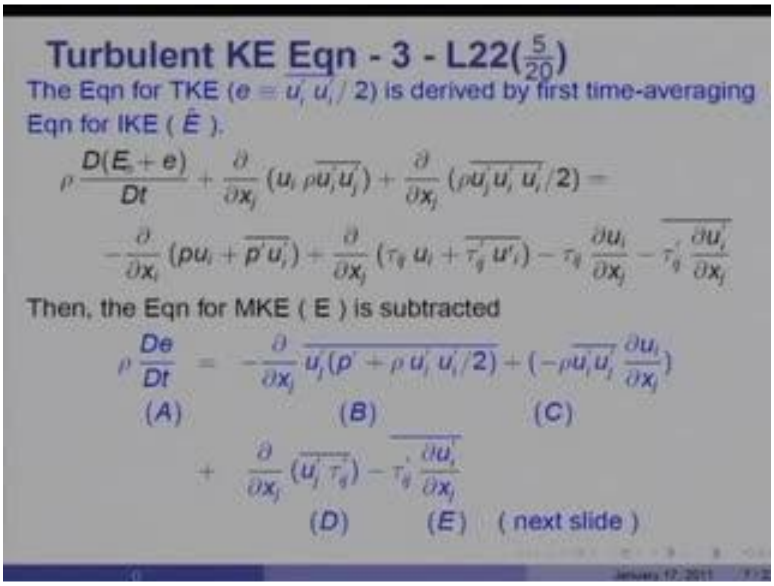
The equation would look like this - D rho D E e plus by D t plus d by d x j of u i rho u i prime u j prime plus d by d x j of rho u j u i prime u j prime u i prime with triple velocity correlation will appear equal to minus d p u i plus p dash u i dash plus d by d x j or tau i j i u i plus tau dash i u dash i minus tau i j d u i by d x j minus tau dash d u i dash by d x j, where turbulence stress tau dash i j.

(Refer Slide Time: 16:25)



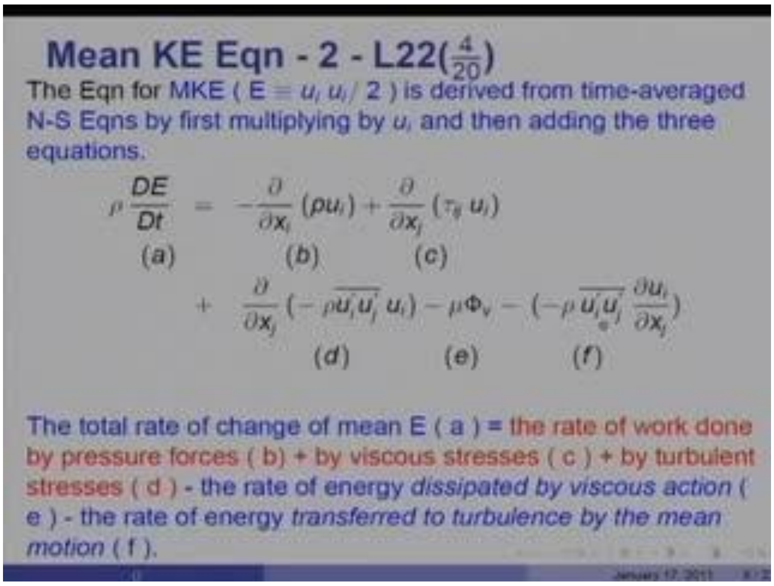
Tau dash i j is mu times d u dash by d x j plus d u j dash by d x i, this is the turbulent kind of prime or the turbulence stress based on fluctuating velocity strength rates.

(Refer Slide Time: 16:48)



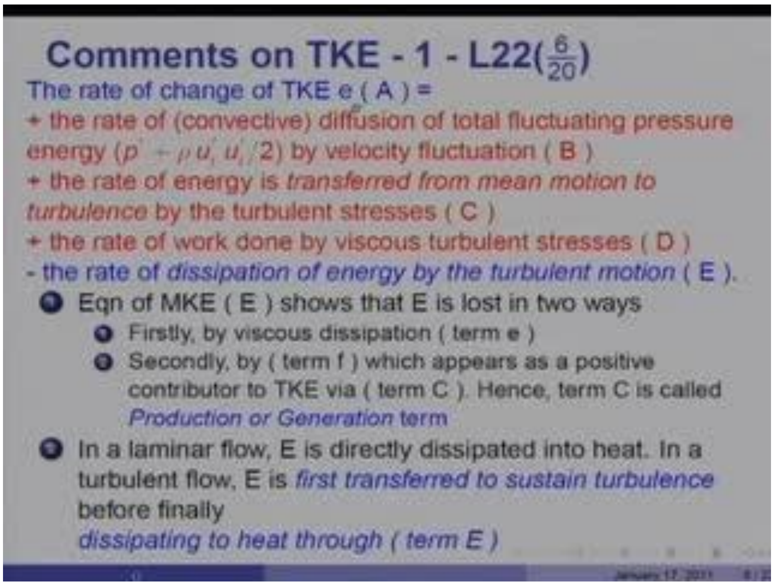
So, from this equation, which is time average form of the instantaneous kinetic energy equation, I now subtract the mean kinetic energy equation which I derived on the previous slide; I subtract this equation from this equation, then you will see that I would get an equation for row D e by d t equal to minus d by d x j u i j j dash p dash plus this plus minus rho u i j prime u j prime d u i d x j and then this term and this term.

(Refer Slide Time: 17:37)



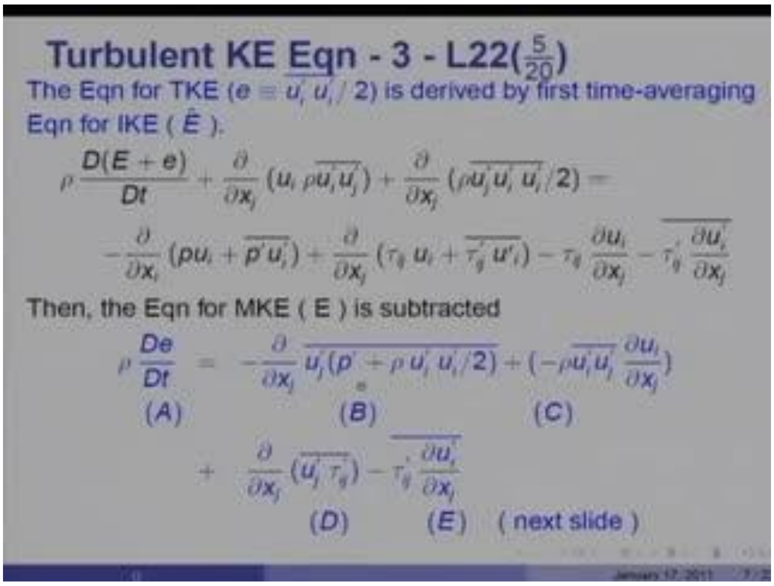
Notice that the C term here in the turbulent kinetic energy equation has exactly the opposite sign of the f term, they are both identical terms. But in one case you have a negative sign here and this is the positive sign here you have the positive sign

(Refer Slide Time: 18:06)



In otherwords, what is a law of kinetic energy? It turns out to be gain of turbulent kinetic energy and what do these terms represent? Well that is what is shown on the left on the next slide.

(Refer Slide Time: 18:21)

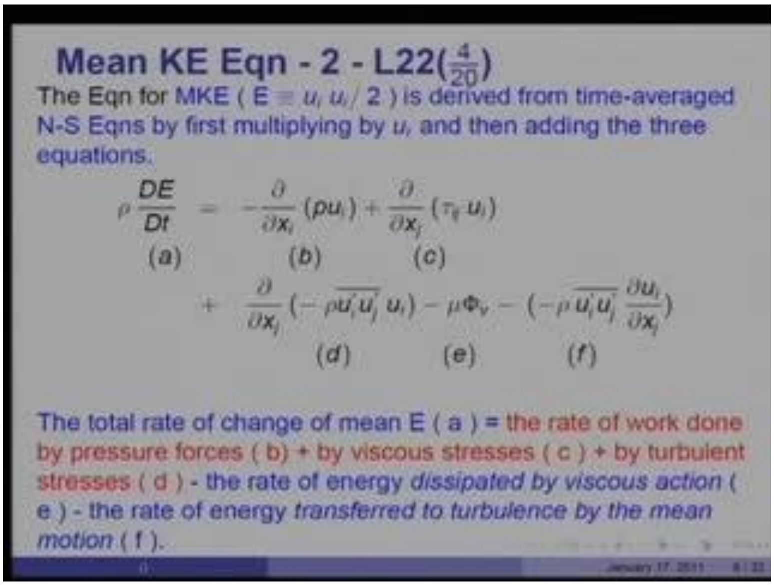


Rate of change of turbulent kinetic energy A which is the left hand side equals the rate of convective diffusion of total fluctuating pressure by velocity fluctuations to go back, this is the term p dash plus rho u i squared by 2 is the total fluctuating pressure, this is the static pressure, this is the dynamic pressure, and therefore the total term represents total fluctuating pressure and its diffusion due to velocity u dash j

plus the rate of energy transferred from mean motion to turbulence by turbulence stresses which is the term C plus by the turbulence stresses; it is the energy transferred to turbulence plus the rate of work done by viscous turbulent stresses this is the u j dash tau dash i j is and I explain what the definition of tau dash i j is, so that is the diffusion again of the stress tau or the stress work due to turbulent stress.

And finally minus the rate of dissipation of energy by turbulent motion and that is the term E here is a product of fluctuating stress multiplied by fluctuating velocity gradient and therefore this would be very much like the mu phi v term in which the phi v would now, we form from fluctuating velocity gradients.

(Refer Slide Time: 19:56)



Equation for mean kinetic energy shows that E is lost in two ways- firstly by viscous dissipation- term e, the mean energy is lost by viscous dissipation and secondly, by work done by stresses on mean velocity gradients and E is lost in two ways firstly by viscous dissipation term e and secondly by term f which appears as a positive contributor to turbulent kinetic energy C a term C.

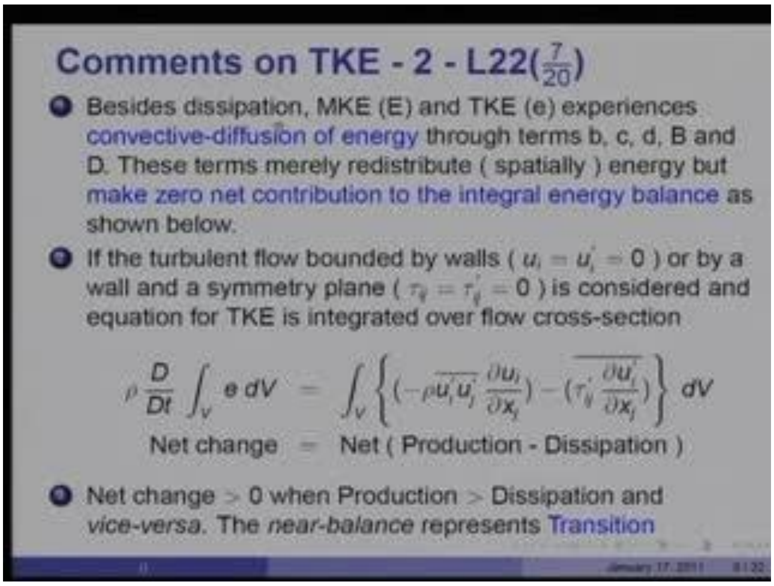
Hence term C is called the production or generation term, because it makes a positive contribution to rate of change of E. In laminar flow, the mean energy is directly dissipated into heat. In turbulent flow we can say that mean energy is first transferred to sustain turbulence before it is finally dissipated to heat through term E.

So, the first mean kinetic energy goes to turbulence through some C which increases that, but turbulent kinetic energy also decreases due to dissipation to heat before finally.

So, this first and before seems to be there ishave a time lag or a space lag in a fluid flow whichever way we is to look at it; so it is not an instance process, it happens perhaps in stages. Now, to explain that we will have to make some further explorations into turbulence which I will take up in the slides to flow as well as the next lecture.

So, remember mean energy is lost in two ways first by viscous dissipation and secondly by transferred to turbulence.

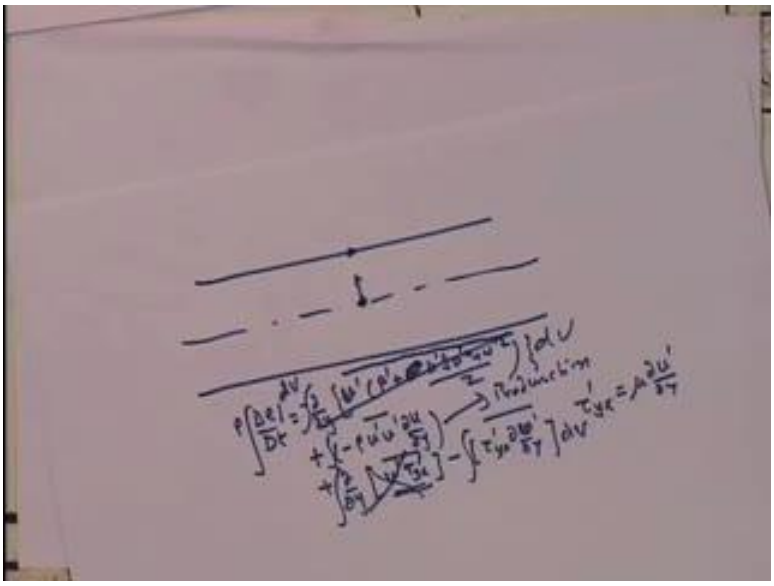
(Refer Slide Time: 22:13)



Turbulent kinetic energy on the other hand is sustained because of this transfer from mean energy from the mean energy and secondly it is destroyed by the fluctuating counter part of mu phi v which we called turbulent dissipation.

So, beside dissipation, transfer mean kinetic energy and turbulent kinetic energy experience convective-diffusion of energy through terms b, c, d, B and D. These terms simply redistribute energy specially but make no contribution or zero contribution to integral energy balance as we would see now.

(Refer Slide Time: 22:48)



So, for example, consider let us say flow between two parallel plates with an access of symmetry and I shall now integrate- I will now consider the kinetic energy equation, so it will look like rho D e by D t equal to and assuming that the gradients of all terms are much bigger in y direction then they are in any other direction.

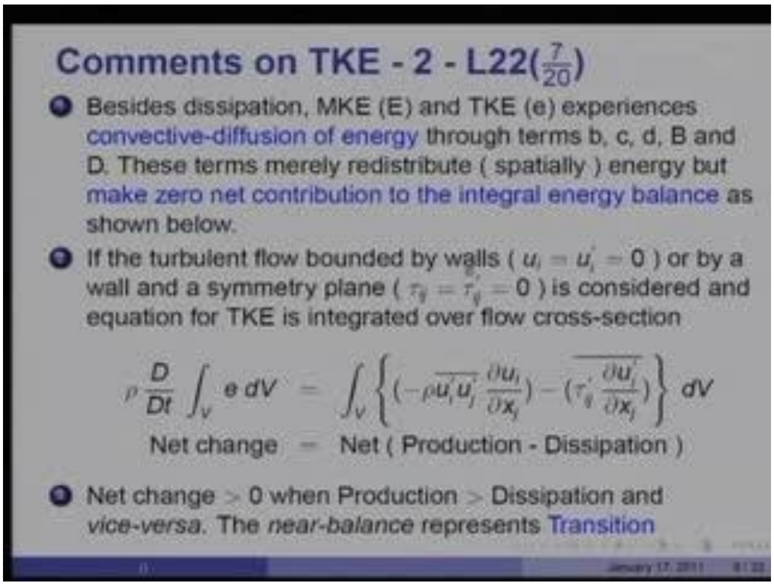
We would write d by d y minus d by d y of v dash into p dash plus u dash square plus v dash square plus w dash square by 2, which is nothing but the kinetic energy.

Minus or the plus minus rho u dash v dash into d u by d y plus d by d y of v dash into tau y x minus tau dash y x by d u dash d v dash by d y- let us say where tau dash y x is mu times d u dash by d y dash.

So, now if I integrate this from 0 to i that is over the volume of the channel; then you will notice that this term will have v dash at the wall and v dash at the at the access symmetry and therefore both of them would simply vanish.

So, integral of that d v is simply 0, integral of this term will survive, which we said was production term; this, like this term will also vanish integral whereas this term integral of d v will survive, because it is a product of velocity gradient and the fluctuating stress.

(Refer Slide Time: 25:38)



That is what I show here, so by wall and symmetry plane is considered, so if the turbulent energy is bounded by wall where the velocity fluctuations are 0 because of no slip or by wall and symmetry plane tau i j tau dash i j 0 is considered and equation for turbulent kinetic energy is integrated over the cross section.

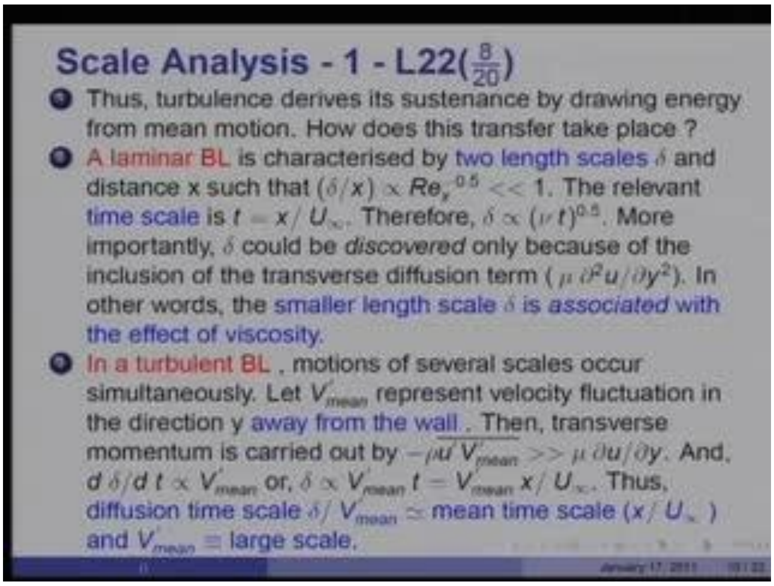
Then we would have D by D t e d v e equal to net production minus net dissipation, what this tells us this following? Is that the net change in kinetic energy over a cross section would be positive when net production exceeds net dissipation.

On the other hand, if the net dissipation exceeded net production, then e will simply die out the kinetic energy will be simply vanish; when there is near equilibrium that is production is very close to dissipation we would have transition.

The equation then sets the conditions for sustenance of turbulence, that the net production over a cross section must exceed the net dissipation then the turbulence would be sustained at every cross section downstream.

There are situations in fact, where even in a channel flow, a turbulence when generated can be made to relaminarize, one such possibility is to have a tube which is coiling around a steam with very high turbulent velocity fluctuations, so that the dissipation term begins to dominate over the production term and the turbulent could then relaminarize inside a pipe, but it is a coiled pipe very very special case not routine encountered in practical engineering.

(Refer Slide Time: 28:18)



When the production and dissipation are near balance, you would expect a kind of flow in which for a while the flow is laminar when dissipation over takes production and then the laminar flow will become unstable to produce in which production term it will take over from dissipation and little patches of turbulence and little patches of laminar fluid would appear in a flow and that is precisely what we called the transitional regime.

Thus, the turbulence derives this sustenance by drawing energy from the mean motions. Now, how does this transfer actually take place? That is what we want to ask, how does this transfer take place?

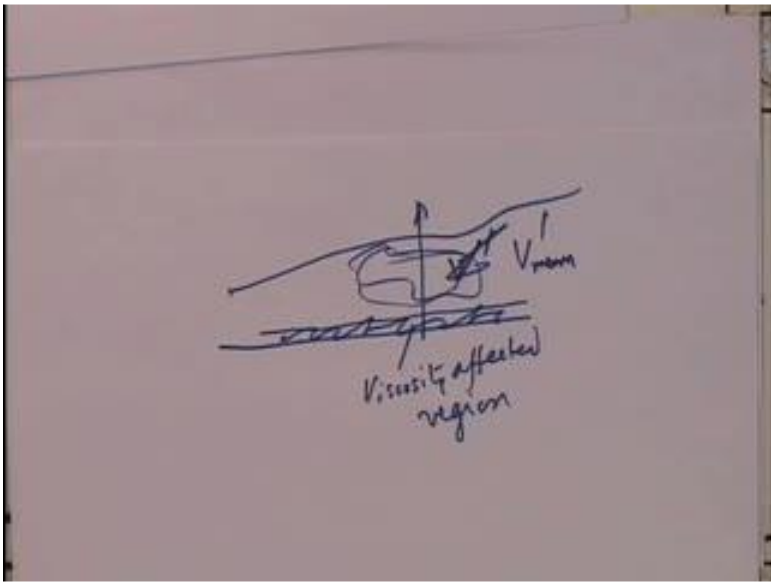
Now, to understand that, we must introduce the ideas of scale; so in a laminar boundary layer, for example, it is characterized by two length scales: one is delta, which is much much smaller, the transverse dimension is much much smaller than the stream wise distance x and that gives us delta by x has being proportional to Reynolds x to the minus 0.5 and this is usually much much smaller than one.

So, the relevant time scale however is t equal to x divided by U infinity and therefore, if I substitute that in here I would say delta is proportional to nu t raise to minus 0.5, where nu is a small number. Therefore, delta would be a very very small quality again as shown earlier.

More importantly, if you remember delta could be discovered only because of the inclusion of the transverse diffusion term mu d 2 u d y square in the laminar boundary layer equation.

One way to interpret this fact is to say the smaller length scale delta is associated with the effect of viscosity as shown here.

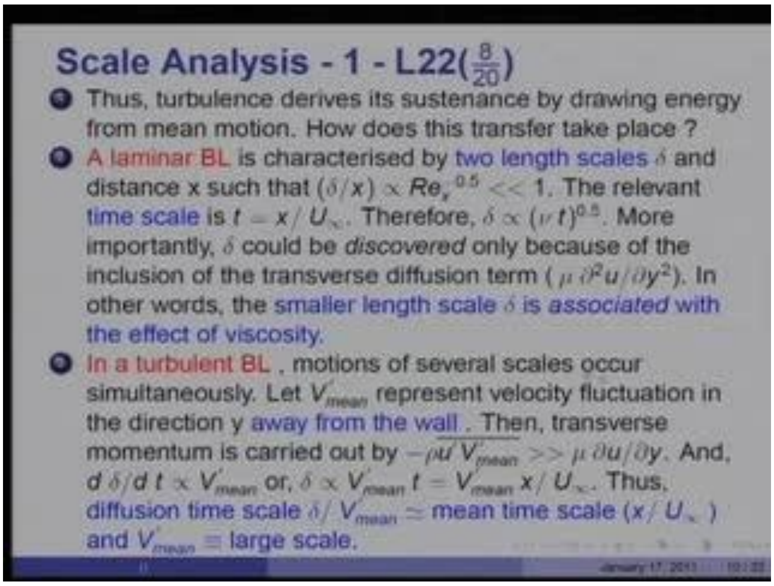
(Refer Slide Time: 30:11)



What about turbulent flow? Now, in a turbulent flow- turbulent boundary layer very close to the wall of course you have viscosity dominates- viscosity affected region, but the outer parts are certainly almost independent of the effects of viscosity, the laminar fluid viscosity.

So, in this region, if I were to say- what brings about transverse of momentum? Well, let us say- it is a representative velocity fluctuation V dash; mean, let us say- I call it V dash mean as the representative fluctuation which brings about transverse momentum.

(Refer Slide Time: 31:10)



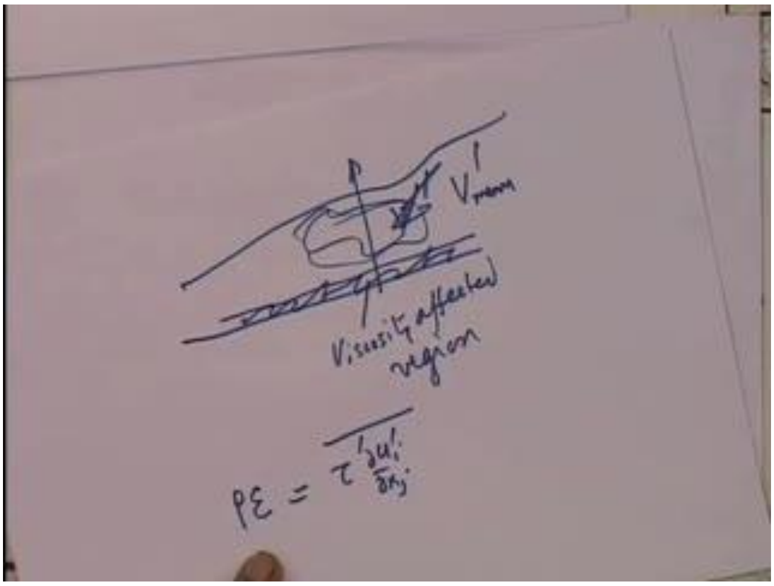
Then, you will see in turbulent boundary layer, motions of several scales occur simultaneously and we are going to choose V dash mean as the representative velocity fluctuation in the direction of y away from the wall.

Then, the transverse momentum is carried out by minus rho u dash V dash time average which would be much much greater than mu d u by d y and, d delta by d t, that is the rate of growth of turbulent boundary layer thickness would be proportional to V dash mean essentially and therefore delta would be proportional to V dash mean by t in multiplied by t but t as we observed would be x divided by U infinity even in a turbulent flow and therefore V dash x by U infinity, this would be V dash mean into x multiplied by (( ))

Thus, the diffusion time scale, delta by V dash mean would be approximately equal to mean time scale x by U infinity; this is very interesting, the delta is a small length scale; V dash mean is the representative fluctuating velocity in the turbulent core or the fully turbulent path of the boundary layer

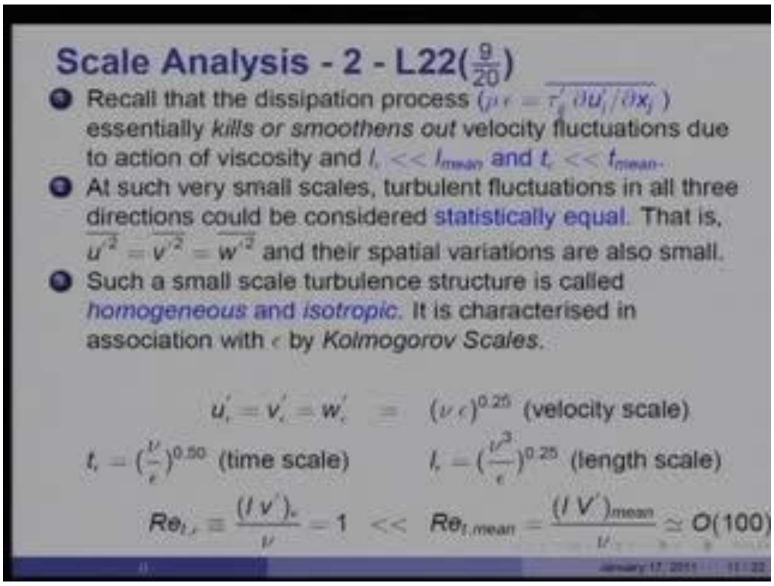
and the time scale associated with it is exactly same as the mean time scale and therefore, we shall regard V dash mean as a representative of the large scale motion; this is a very important idea that V dash mean would be taken as the representative of the large scale motion.

(Refer Slide Time: 33:12)



Recall that the dissipation process, mu tau dash d u dash i by d x j- this was the dissipation process; this I will represent as a rho times epsilon, where epsilon is called the dissipation rate of kinetic energy.

(Refer Slide Time: 33:37)



Now, this actually kills turbulence- rho into epsilon actually kills turbulence and smoothens out velocity fluctuations due to action of viscosity and therefore, the length scales associated with it would be much much smaller than the mean length scales and the time scales associated with it will also be much much smaller than the mean time scales.

At such very small scales of motion, turbulent fluctuations in all 3 dimensions- directions can be taken to be essentially statistically equal that is u prime square is equal to v prime square is equal to w prime square,

as well as the gradients will be 0. In another word, the spatial variations will also be very small; when special variations of fluctuating a time average quantities are 0 or this when the spatial gradients of the fluctuating quantities time average fluctuating quantities are 0, we say the structure is homogeneous and when the components of the velocity fluctuations are equal, we say it is isotropic and then therefore, we would essentially have where the viscosity plays its dominant role we would have essentially a homogeneous and isotropic turbulence structure.

It is characterized in association with epsilon by what are called Kolmogorov scales; so Kolmogorov use the idea, that is very small scale motions are essentially characterized by the effect of viscosity and by the effect of a turbulent dissipation and he brought in the quantity epsilon to represent the velocity scales of the associated with dissipation process as new epsilon raise to 0.25- this is dimensionally correct.

Similarly, the time scale t epsilon was taken as nu by epsilon raise to 0.5 and l by l sub epsilon was taken as nu cube by epsilon raise to 0.25 as the length scale.

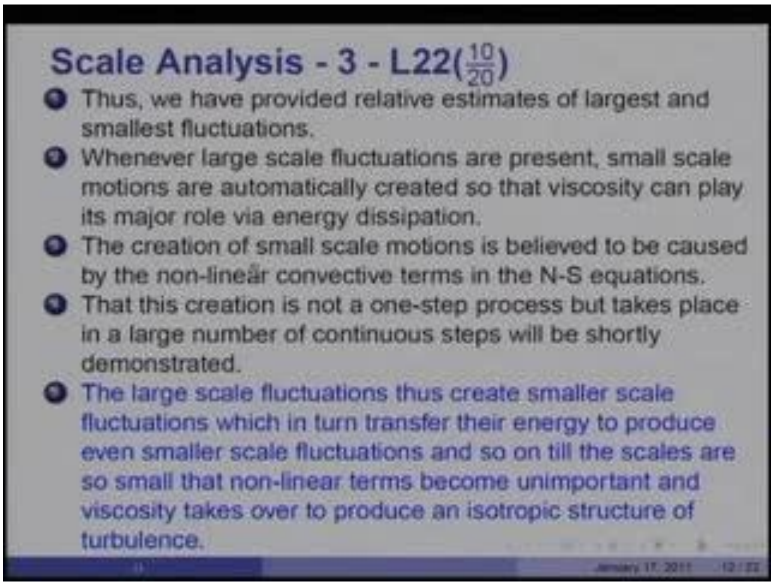
So, if I form Reynolds number based on length scale, velocity scale- I will get l v dash epsilon divided by nu and let us say- it is of the order of 1.

Then it follows that it would be much much smaller then l V dash mean by nu, which is the large scale motion and large scale length scale and that would be of the order of 100 or even more.

The Reynolds number associated with dissipative length scales and velocity fluctuation scales is much much smaller than the mean Reynolds number form from mean length scale and fluctuating velocity scales, which would be of the order of 100 or more.

Thus, we have provided relative estimates of the largest and the smallest scales; so largest scales belong to the mean dimensions, mean motion, where is the smallest one belonging to the dissipation scales.

(Refer Slide Time: 37:08)

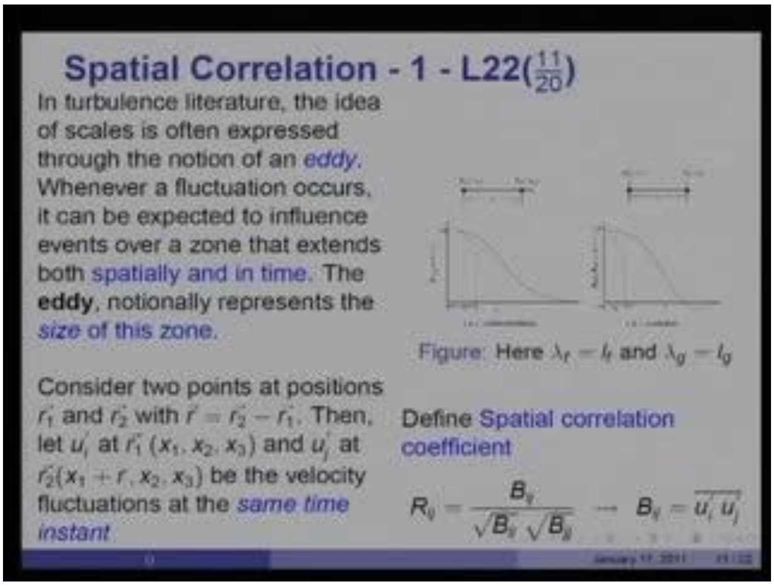


The most important to cut the story short, the most important aspect of it is that whenever large scale fluctuations are presents small scale motions are automatically created so that viscosity can play its major role via energy dissipation.

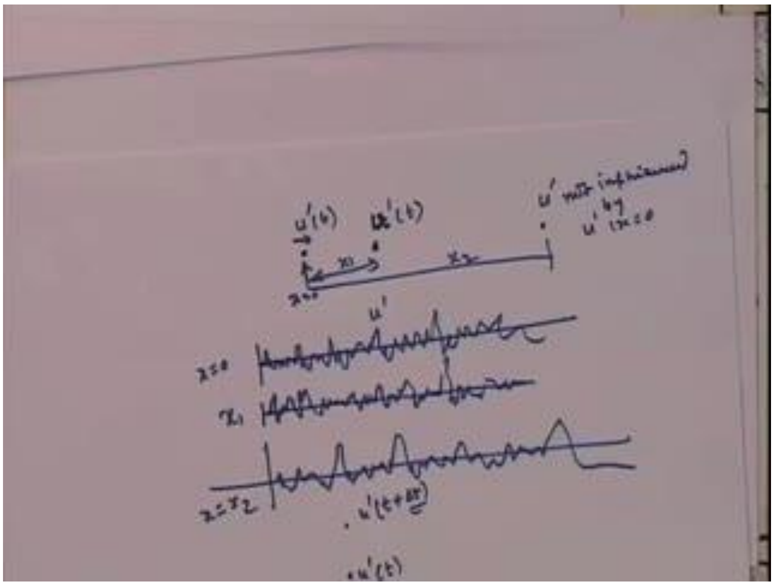
The creation of this small scale motions is believed to be cause by the non-linear convective terms in the Navier-Stokes equations. That this creation of smaller and smaller scales motion is not a one-step process but takes place in a large number of continuous steps will be demonstrated shortly- it can be done in more than one ways and I will try to do it in as simpler manner as possible.

The large scale fluctuations thus creates smaller scale fluctuations, which in turn transfer their energy to produce even smaller scale fluctuations and so on till the scales are so small that non-linear terms become unimportant and viscosity takes over to produce an isotropic structure of turbulence I mean that is the story that is the story of sustains of turbulence.

(Refer Slide Time: 38:16)



(Refer Slide Time: 38:36)



Now, in order to explain these ideas little further, it is customary to introduce the idea of a turbulence eddy; now you can imagine that- let us say I have 2 points in the flow and I consider- let us say, u dash here and v dash here at the same time t or let us say to begin with u dash and u dash itself, I consider the fluctuation in the x direction at the same time instant at two different points separated by a distance.

Now, we all know that if I a fluctuation at this point will influence this point if they were close to each other, so the fluctuation here would be influenced by fluctuation here.

However if this point was sufficiently for away, say here, then the u dash here will not be not influence by u dash at x equal to 0- let us say this is at x equal to 0 and this is the separation distance x 1 and this is definition distance x 2 let us say.

So, at this point, it is unlikely that u dash will sense what u dash at x equal to 0 is doing; in effect if I have to look at them in time at x equal to 0 then the fluctuations u dash would at x equal to 0 will look like that.

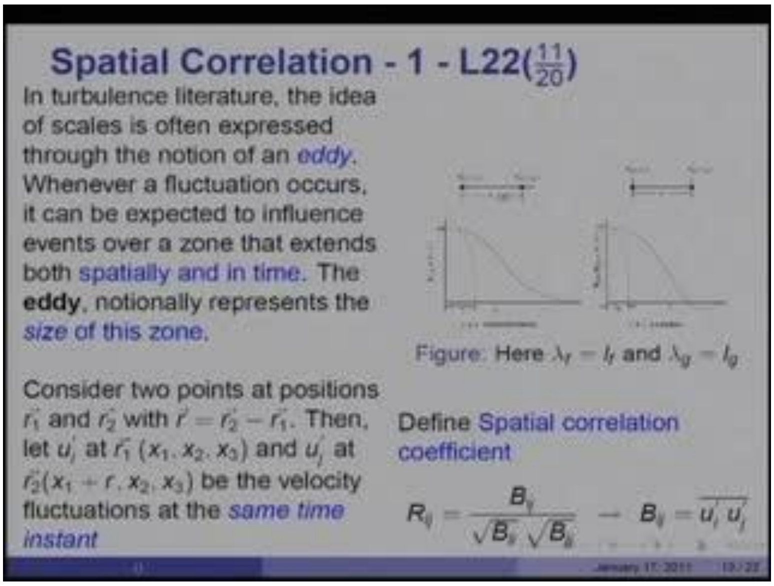
At x equal to x 2 let us say, they would look absolutely different and you can say that this form of u dash at x equal to 0 which is completely uncorrelated with what is happening at large distance.

What about intermediate distances? Here we can expect supposing x 1, which is very close to x equal to 0- I can explain something like that; marginally, good correlation at least it will look somewhat similar, but at very large distance it could be absolutely very very different,

I can say therefore and of course if I took this second point to merge with this, of course I will reproduce the same pattern, which means a complete correlation exist between the two points when they collapse on one another; a complete correlation exist when they are very very far, but a moderate correlation can exist for in between distances

and of course we do not know what that distance x 2 will be in real turbulent flow and that is what we wish to find out; this is spatial influence. but now if I take for example, u dash at time t and u dash at say another time t equal to t plus delta t separated by a distance delta t time distant delta t then a similar situation will arise.

(Refer Slide Time: 42:34)



If delta t was small- I would expect reasonably good correlation between the two; but if it was very very large, then of course they would be completely uncorrelated.

These ideas are expressed here in this figure, where I show two points u dash 1 and u dash 1 at two point separated in x 1 direction and is define a spatial correlation coefficient as B i j under root B i i under root B i B j j, where B i j is the u i prime u j prime at two different points but at the same time instant.

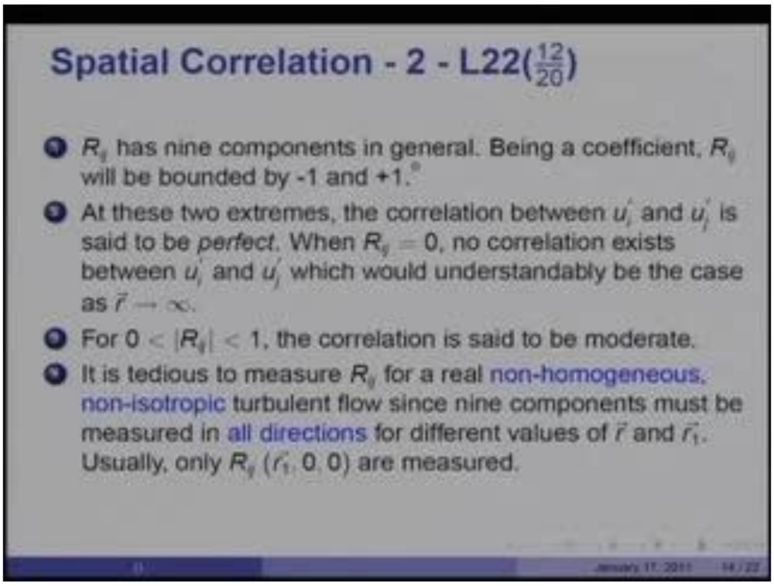
I do the same thing here for u 2 dash and u 2 dash separated in x 1 direction and then if I plot R i j from measurements of these quantities, then it would be it will look like perfect correlation of 1 at separation distance equal to 0 and the correlation would die out 0, that means there is no correlation beyond some long distance.

Similar thing would happen with respect to u 2 dash at the same time all though it will go through a negative before going to 0 at infinity, this is called the longitudinal time scale ah correlation;this is called the transverse correlation.

And for example, if I were to integrate this over a long time of infinity, then this would give me a length dimension which would be representative the average dimension over which a fluctuation at a point is going to influensive hence.

That I would call as the integral length scale of the fluctuation or the spatial size of the eddy in longitudinal direction- I can do the same thing for with respect to u 2 velocity and I would get a similar dimension in the transverse direction I can say; so I have estimated the size of the eddy physical size of or the zone over which turbulence is going to influensive hence.

(Refer Slide Time: 45:07)



Spatial correlation has nine components, R i j as defined has nine components in general and being a coefficient it would vary between minus 1 and plus 1. At these two extremes, we say the correlation is perfect, absolutely perfect because is magnitude is 1.

When R i j is equal to 0, of course no correlation exist between u i dash and u j dash which would understandably be the case when the separation distances r tends to infinity; between 0 and 1 we say the correlation is moderate.

It is steadiest to measure R i j in a real non homogenous isotropic turbulent flow, because nine components must be measured in all directions for different values of separation distance r and the direction r 1 r 2 and r 3 and therefore usually only in the direction r 1 or x 1 is the only direction which is taken to measure

Whenever measured by enlarge correlation coefficients are extremely difficult to measure the spatial correlations are extremely difficult.

I will stop here and continue with this lecture next class.