Convection Heat and Mass Transfer

Prof. A.W. Date
Department of Mechanical Engineering
Indian Institute of Technology, Bombay

Module No. # 01

Lecture No. # 21

Nature of Turbulent Flows

In the next 8 to 9 lectures, I intend to cover the topic of turbulent flows.

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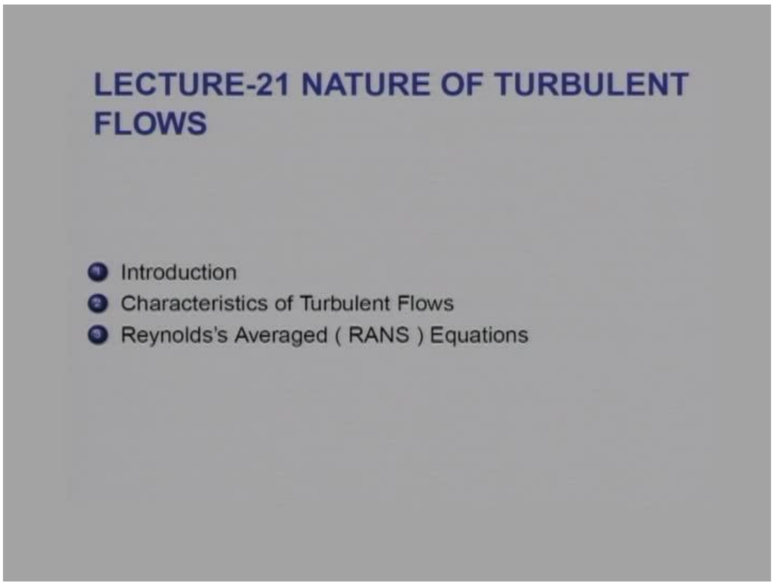

More often than not we encounter turbulent flows and heat transfer in turbulent flows is of great relevance to the heat transfer engineer. By way of an introduction, we must first understand what is so special about turbulent flows. Therefore, we would look at some important characteristics of turbulent flows and then understand why statistical averaging is absolutely essential to be able to track theoretically, the turbulent flow, which in turn would give us ability to predict the friction factor and the Nusselt number, just as we managed to do in laminar flows.

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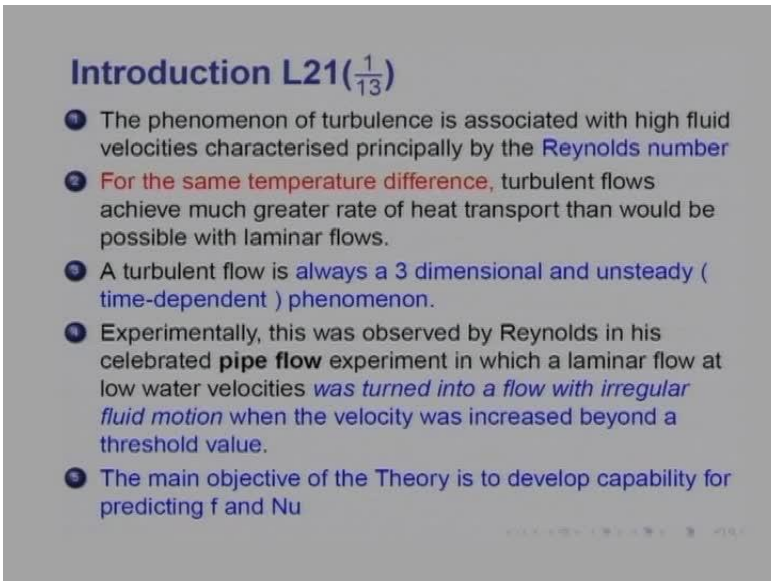

So, first of all let me start by saying that the phenomenon of turbulent is associated with high fluid velocities characterized principally by the Reynolds number. All of you will remember what the Reynolds number is; Reynolds number is equal to rho times velocity into some characteristic dimension l divided by the viscosity.

Therefore, it represents the ratio of inertial forces - the numerator, and the viscous forces - the denominator. For the same temperature difference, if you had a turbulent flow instead of a laminar flow, you would get much higher rate of heat transfer. In other words, the heat transfer coefficient would be much greater in turbulent flow than in laminar flow.

Now, why does that happen? We wish to find that out. The main thing to always keep in mind is the fact that the turbulent flow is always three dimensional and unsteady. Unsteady means time dependent. In fact, so time dependent and three dimensional that, turbulence is often described as random motion.

The word random often gives the impression that it can never be made tractable. We will see how this randomness can be combated. Experimentally, the formal study of turbulent flow began with Reynolds's celebrated pipe flow experiment, in which laminar flow at low velocities was turned into a flow with irregular fluid motion when the velocity was increased beyond a threshold value; all of you must have been introduced to this idea very well in your under graduate work.

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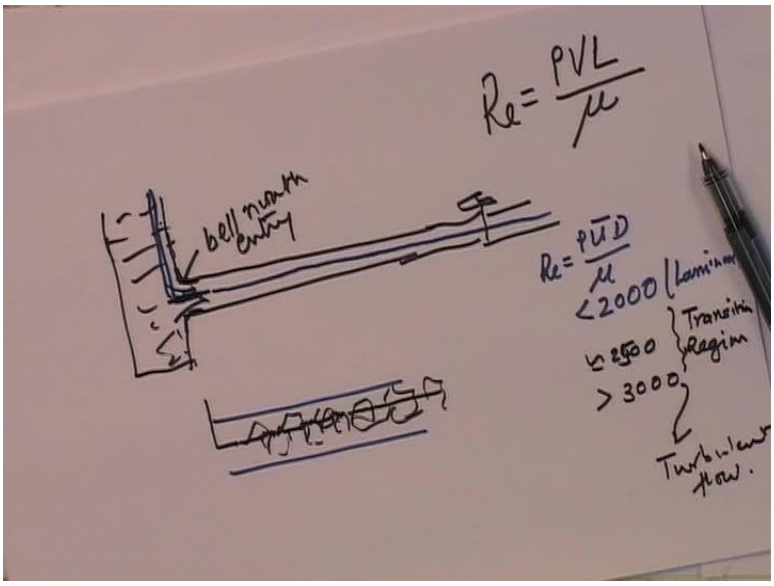

Let us look at what Reynolds actually did. Reynolds had a tank and a tube. The tank was filled with water and he had a bell mouth entry to the tube so that the fluid would enter very smoothly into the tube from this tank. At the exit of the pipe, there was a valve. By opening and closing the valve, he could therefore control the rate of flow through this tube. What he also had was a dye injector at the center line; let us say ink.

So, as long as the velocity was low, the dye would simply remain steady like a laminate and move straight out of the duct without any fluctuation or movement or anything like that - straight along the length. That is along the axis of the tube because the hypodermic needle through which he was injecting the dye was kept absolutely at the axis of the point.

This happened till Reynolds number equal to rho times u bar the mean velocity into diameter D by mu was less than 2000 and you have laminar flow. When Reynolds number increased beyond 2000, he observed a strange phenomenon. Let me draw a second diagram in which the dye would come out like so and it would be almost laminar like, but then turn a little turbulent. Then turbulent would dye out and then would again turn into turbulent, and then again dye out and then again dye out and so on and so forth.

So, in other words, he had a partly laminar partly turbulent or unsteady flow, locally over fixed times, he would get turbulent spots or turbulent patches if you like. This state of affairs would continue till say up to about 2400 Reynolds number or even 2500. But certainly when the Reynolds number was greater than 3000, he found that the entire tube would get engulfed by sinusoidal motion or what he calls sinusoidal motion, or really random turbulent vertical motion; the dye would completely mix out into the surrounding water and you had colored water coming out.

In this range - let us say between 2000 and 2500 or even up to 2000 to 3000 is called the transitional Reynolds number or transition regime, and certainly greater than 3000, we would call it as turbulent flow. So, this was the first formal experimental evidence of turbulent flow, although in nature people have always observed turbulent flow.

Then, the objective of the theory of turbulent flows is mainly to develop capability to predict friction factor and Nusselt number which is of relevance to engineering, but this randomness must somehow be combated. Only then can we make a sensible theory out of what Reynolds observed.

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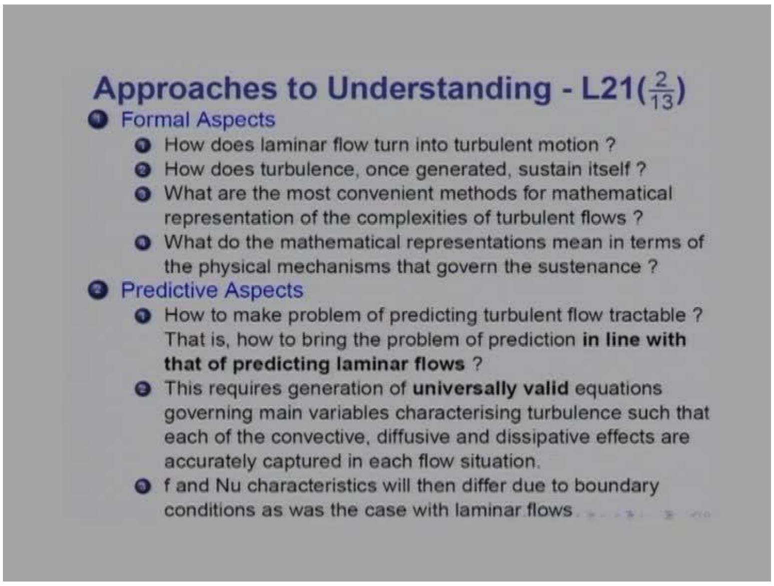

Now, if you look at turbulent literature, there is a great deal of literature which I will call as formal aspects of turbulence. It does not deal with prediction of friction factor and Nusselt number at all; instead it asks some very fundamental questions

Questions like - how does laminar flow turn into turbulent motion? That would be of great interest. How does turbulence, once generated, sustain itself? This is a most relevant question as far as practical turbulent flows are concerned, after all if the Reynolds number was greater than 3000; let us say 10000 or so or 1 lakh or 1 million whatever.

The flow becomes turbulent right from the entry to the tube and remains turbulent right to the exit from the tube. This has been evidenced in all practically encountered lengths of pipes of diameters. Whatever the diameter for all practical lengths of pipes, it has always been observed that a turbulent flow, once it is turbulent at entry, it always remains turbulent right through; that is along the length.

In other words, turbulence somehow finds a way to sustain itself in spite of the friction at the wall and so on and so forth; infact, the friction at the wall could be the main contributor to sustenance of turbulence; so, we have to find that out.

Third question would be - what are the most convenient methods for mathematical representations of the complexity of turbulent flows? How do you deal with randomness?

There are many ways. Do you deal with it in physical space which gives rise to statistical methods? Or do you deal with it in wave number space, which means you think of spectral representation of randomness?

Finally, once you have decided on either statistical or spectral, then you must understand two things: what do these mathematical representations, meaning each term in those equations really physically mean? What mechanisms do they represent? Can those mechanisms and the terms be independently measured in practical turbulent flows? What would be the difficulties of measuring such quantities? - These are all questions that are dealt with in the formal aspects of turbulence and this is a field in which right from physicist to mathematicians to engineers have contributed greatly.

It has been the most inviting and intriguing topic in physical sciences. Engineers are more interested in predictive aspects. In other words, how to make the problem of predicting turbulent flow tractable, first of all? By this we mean how to bring the problem of prediction of turbulent flows in line with that of predicting laminar flows. After all, we know already how to predict laminar flows through our earlier lectures.

Can turbulent flow be brought down to the same pattern as it were, so that friction factor and Nusselt number can be predicted? In order to do that, we would require generating universally valid equations governing the main variables characterizing turbulence, such that each of the convective diffusive and dissipative effects present in the turbulent flow are actually captured or accurately captured in each flow situation. That means we need equations governing turbulent motions that are as universal as they are in laminar flows; that is the first requirement.

Once we have such equations, then we would say that from one situation to another situation, simply we subject the equation to different boundary conditions to obtain f and Nu characteristic for each situation. This would be just like predicting laminar flow then. Only thing is - the equations are different or they might even be similar; that we will discover as we go along. The problem of predicting then would be brought down to the same footing as problem of predicting laminar flow.

Engineers are very eager to find equations from physicist and fundamental scientists who would give them universally valid equations of turbulent flows, which they can then readily apply, even if it was necessary to apply computer. They would do so and obtain solutions for friction factor and Nusselt number.

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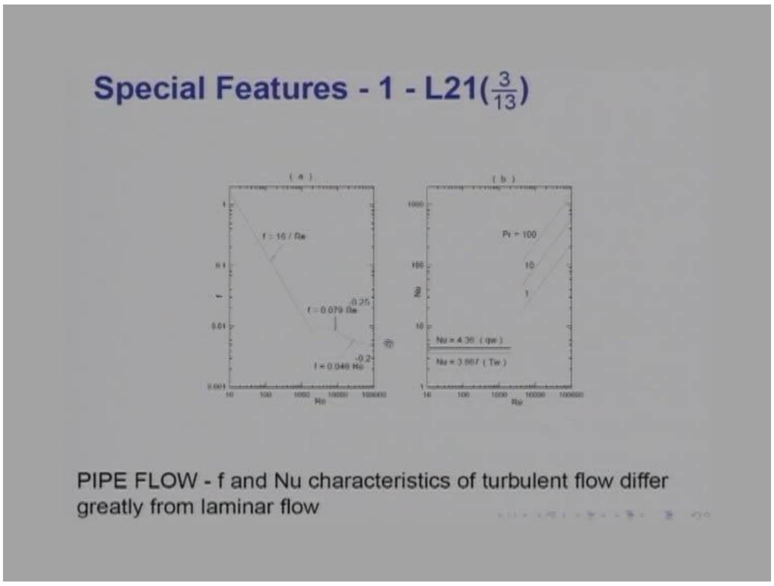

I will now take you through some features of turbulent flow which will help you to understand why we expect turbulent flow equations to have fundamentally different nature than the equations of laminar flow.

Look at this first slide with which you are all familiar from your under graduate work. The left hand side shows the variation of friction factor versus Reynolds number in a pipe flow and the right figure shows the variation of Nusselt number with Reynolds number.

Now, you will recall that the friction factor multiplied by Reynolds number is always a constant equal to 16 in a pipe flow. Therefore, on a log-log scale, it will look like a linear line till about Reynolds number of 2000. Now, the laminar flow equations do not know that the flow will turn turbulent or will change its character towards transition and then to turbulent, if Reynolds number was increased. There is no way in which the equations that we setup for laminar flow can sense that. In fact, if you continue to calculate the friction factor for Reynolds number greater than 2000, the line would be simply extended and we would predict a very low friction factor which does not accord with the experimental data.

The correlations that I have shown here (Refer Slide Time: 15:56) are the familiar correlations that you have always used. You will see that, in laminar flow, friction factor was inversely proportional to Reynolds number, but later it becomes inversely proportional to the Reynolds to the power of 0.2 and the slope of the line clearly changes - significant difference between laminar and turbulent flow.

Likewise, if I look at Nusselt number, then you will recall that in laminar flow under constant wall heat flux, Nusselt number would be 4.36 irrespective of the Prandtl number of the fluid, if the heat transfer was fully developed. Likewise, under constant wall temperature, it would be 3.66 irrespective of Prandtl number; in other words, Nusselt number is neither a function of Reynolds number nor Prandtl number, in fully developed heat transfer in a pipe.

On the other hand, the experimental correlations will show you that Nusselt number strangely becomes function of both Reynolds and Prandtl. Nusselt number increases with Reynolds number for a fixed Prandtl number, and for a fixed Reynolds number, it increases with Prandtl number. So, suddenly something happens and Nusselt number which was independent of Reynolds and Prandtl number in laminar flows, suddenly becomes function of both Reynolds and Prandtl number.

So, we can conclude that there is something fundamentally different in turbulent flows that laminar flow equations can hardly be expected to know.

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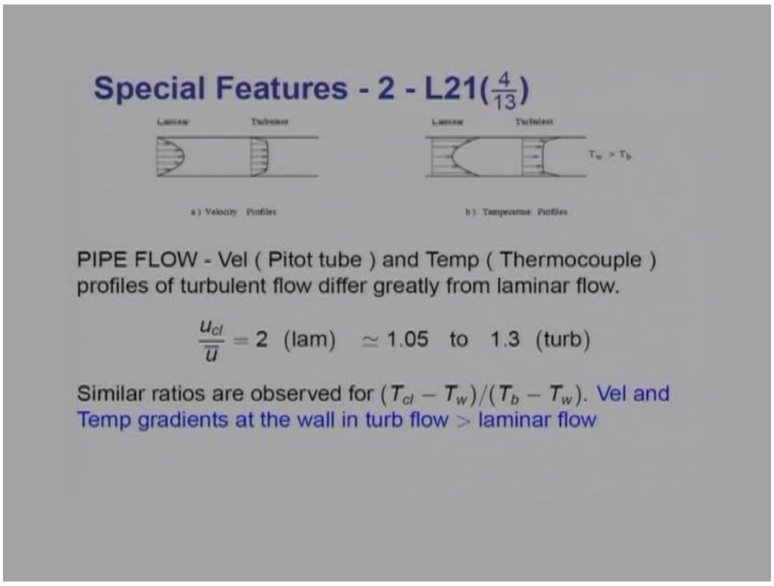

Let us look a little bit closer inside the tube and observe the major velocity and temperature profiles in turbulent flows; for example, in a pipe flow, you know that the fully developed profile would be parabolic in laminar flow. But in a turbulent flow, if you were to measure the velocity profile, it would show a very sharp gradient and very large flat core, and then back again to wall at 0. In fact, if you were to look at u center line divided by u bar, it is 2 in laminar flow; in turbulent flow, it will vary from as low as 1.05 to 1.3 depending on the Reynolds number. The higher the Reynolds number, the lower is the value.

So, the maximum to mean velocity at very high Reynolds numbers exceeding say 1 million would be about 1.05 - somewhere around 10000 or 8000. It would be of the order of 1.3. Similarly, if you were to measure the temperature in the flow, again you will recall the temperature profile T cl minus T w over T b minus T w; sorry, the temperature profile itself would be T wall here and T central line here and it would have a very parabolic nature. Whereas, the turbulent flow and the temperature profile will show characteristics which are like this - very sharp gradient there and flat core.

So, the ratio of T central line minus T wall over T bulk minus T wall again would be of this type in turbulent flow. Velocity and temperature gradients at the wall in turbulent flow are always much greater than laminar flow with a much more flat core than you would get in laminar flow; it is very special.

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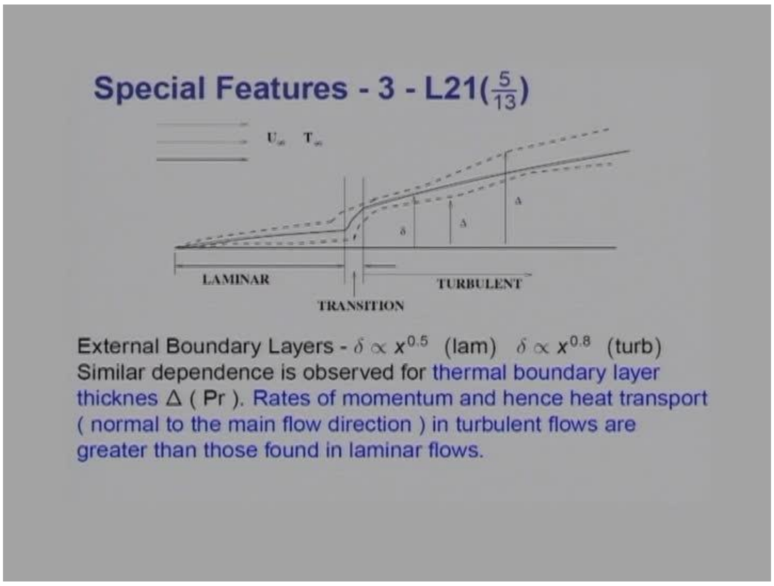

Let us look at external boundary layer - flat plate with a free stream velocity U infinity and temperature T infinity. Then, the solid line shows a laminar flow boundary layer will develop, but at some distance Re x based on u infinity x by nu. Suddenly, there will be the boundary layer thickness and it will grow over a short distance which we will call as transitional zone or regime. Then, the boundary layer will grow at a much greater rate with respect to x.

In fact delta, as we have seen already through our similarity solution, delta is proportional to x to the half in laminar boundary layer, but as we go along, we will see delta will be found to be proportional to x to the power of 0.8 in turbulent flows. So, the rate of growth of boundary layers is much faster in turbulent flows than it is in laminar flow. Similar would be the case as regards delta the thermal boundary layer thickness; although the manner in the ratio of delta by delta which was so strong, a function of Prandtl number here in laminar flow may not be so stronger function in turbulent boundary layers.

What is interesting is that even in external flow, not only inducted flows, but even in external flows, turbulent flow shows very different behavior than in laminar flow. I said turbulent flow is always unsteady. What do we mean by that?

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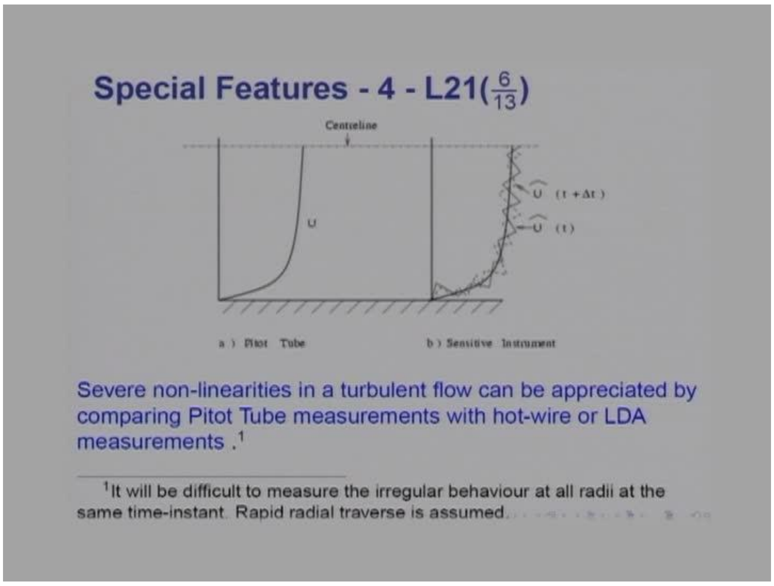

That is shown here in this figure. Suppose I have turbulent flow, let us say at Reynolds number of 30000; then, I use an instrument called the Pitot tube which you all have used in your under graduate experiments to measure velocity profile in a tube.

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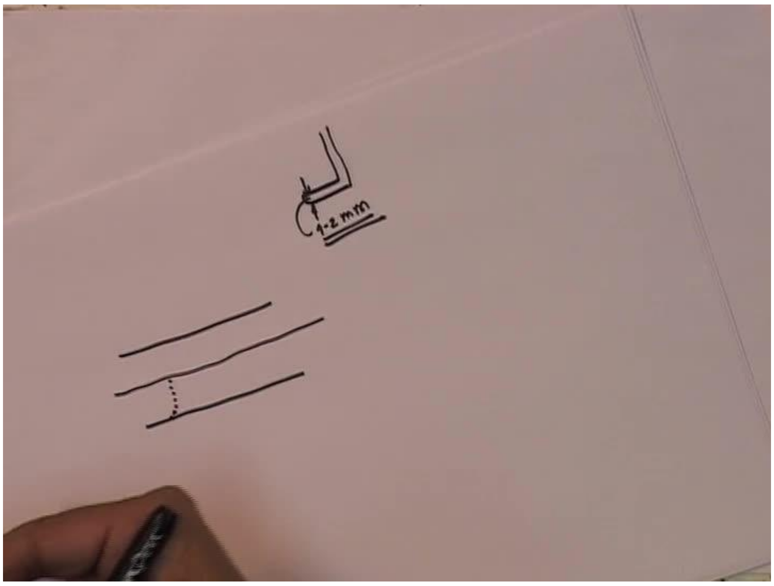

A Pitot tube is nothing but a hollow tube with a bend and the diameter of the mouth would be about 1 to 2mm; this stem is then connected to manometer to get…

So, let us say, I measure the velocity starting from here to the center of the pipe, then if I put the Pitot tube here here here here here (Refer Slide Time: 22:16) facing in actual direction, then the Pitot tube does not see any randomness, any unsteadiness at all because the flow is steady on the mean. That is your compressor or the blower is supplying the mean velocity at a constant rate. Therefore, on the mean, the time average value of the velocity, which is what? In fact, Pitot tube does not see something which happens at scales which are smaller than its diameter. It simply averages out what it sees and gives you a profile which would look like this would look like this (Refer Slide Time: 23:11) 0 velocity at the wall and so on.

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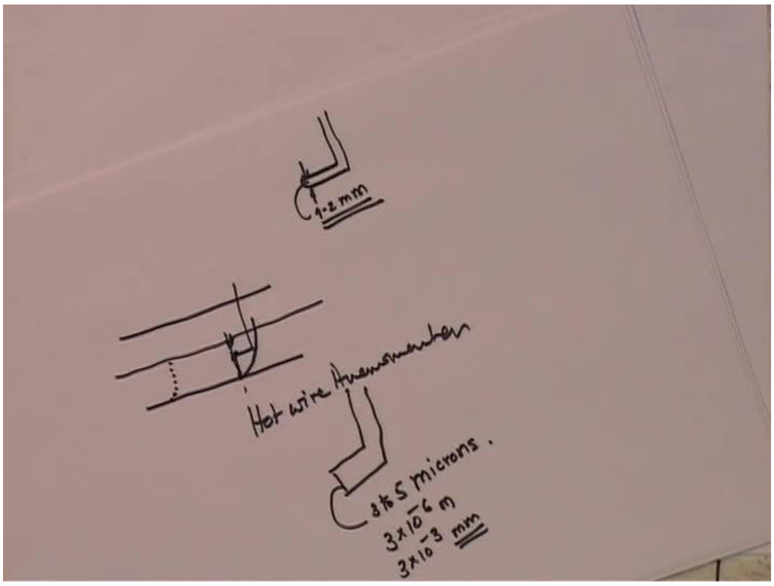

On the other hand, if I used an instrument known as hot wire anemometer, it has 2 stems and in this, a wire is connected. The diameter of the wire is 3 to 5 microns which means 3 into 10 raise to minus 6 meters or 3 into 10 raise to minus 3 millimeters; which means we have a measuring instrument which is about 300 times smaller than the Pitot tube dimension. The hot wire anemometer works on the principle - the hot wire is given electrical current such that it attains a certain temperature.

When the velocity flow, air or whatever flows over it, it cools down. But externally it has a compensating circuit which supplies additional energy in order to bring the temperature of the wire back to where it was set earlier. The amount of energy supplied to bring the wire back to its original temperature is a measure of the instantaneous velocity at that over the wire. Because the wire is so small, we can say, that is the instantaneous velocity at that point, where this instrument is kept - hot wire is kept. Now, a hot wire will only measure velocity as a function of time. If I put the hot wire here, then it will measure the velocity variation with time.

If I wanted a picture of entire profile at the same time, such an instrument does not exist, but we can do a thought experiment for a moment and say we move the hot wire from wall to the axis swiftly, very very fast such that it picks up all most instantaneous velocity at the same time instant at different position. Mind you, this is a thought experiment.

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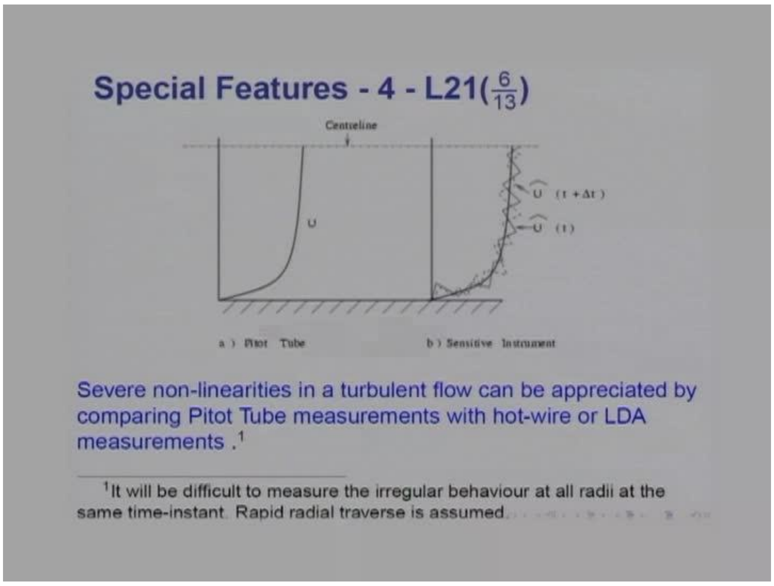

Then, what will it look like? Then, you will see it will measure something like this. It will measure velocities which are highly zigzag at time t and at time t plus delta t, it will measure something quite entirely different shown by dotted line. If you measure again at t plus 2 delta t, it will show something else and so on and so forth.

These days, instead of hot wire, people use laser Doppler anemometry for measurement of unsteadiness - unsteady motion. If you however to time average out or take average value of U at each point and draw a curve, then you will find that at different time, the curves will simply overlap each other. This is a very interesting idea that although the instantaneous velocities may differ greatly, the velocity on the mean is reproduced and the velocity on the mean would be exactly same as what was measured by the Pitot tube.

In other words, what was hidden and not discovered by Pitot tube is first of all discovered by the very sensitive instrument called hot wire anemometer, but what it measures if it is averaged out? It would be again same as what was measured by the Pitot tube.

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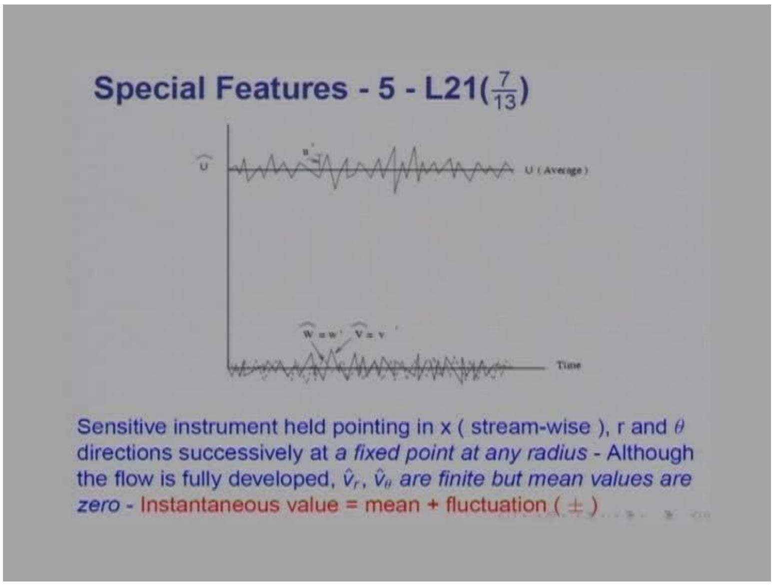

This idea of length scales is of great relevance in turbulent flow. Let us look at another picture: I said a sensitive instrument would measure velocity in a tube as a function of time.

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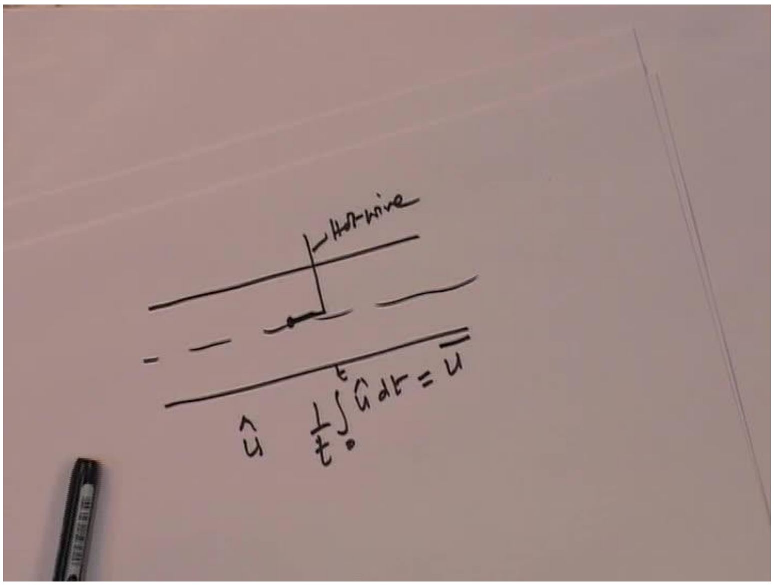

So, let us say I have kept a tube, a fully developed turbulent flow and I have kept a hot wire anemometer at the center of the pipe facing the actual direction. Then, you will see that you will get an instantaneous value of u which is given by u hat and plotted as a function of time. You can see the zigzagness, but if I were to time average this, which means, if I were to do integral u cap d t over time t 0 to t, then I would get a value which I call u bar.

Let us say the average value - that would remain almost constant; it will be constant at that point because the flow is fully developed. So, it is a flow steady on the mean, but instantaneously it is unsteady. Now, as you all know, you have used and you have experienced also, supposing I were to turn this Pitot tube to face the radial direction, then you know that because the flow is fully developed, the mean radial velocity will be 0, but in fact, it will be so at any radius of the pipe.

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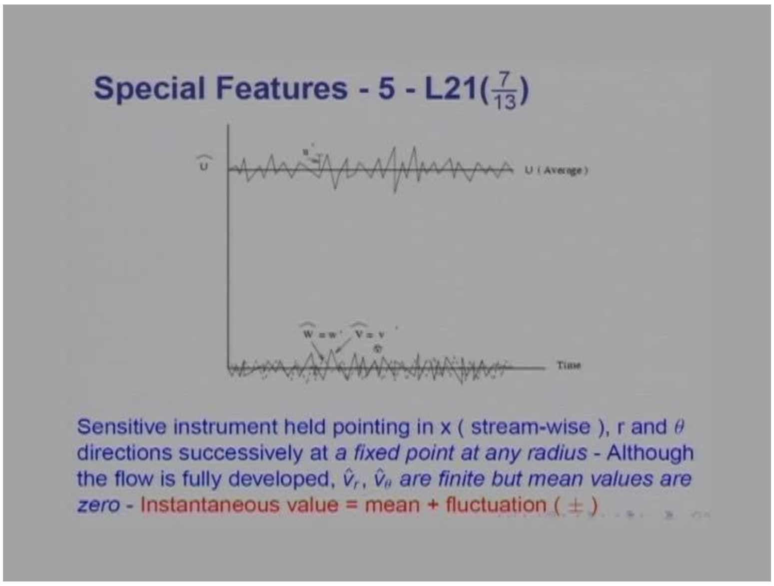

If I were to put a hot wire I would find that the radial velocity would show zigzagness - instantaneous value, but its average will be 0. That is why I have plotted it over here (Refer Slide Time: 30:26) and likewise the circumferential velocity would also show a finite value at different radii, instantaneous value will show, but its time average value will be 0 because the flow is fully developed; that is what is shown here.

This is very unusual that although the flow is fully developed, instantaneous values of U V r and V theta or W and V or whichever - are all finite. They vary randomly with time, but their averages of V and W are 0; whereas the average of u is a constant at a single point. What this figure shows? However is the following, we could say that the instantaneous value is always mean plus a fluctuation, and the fluctuation may be positive or negative.

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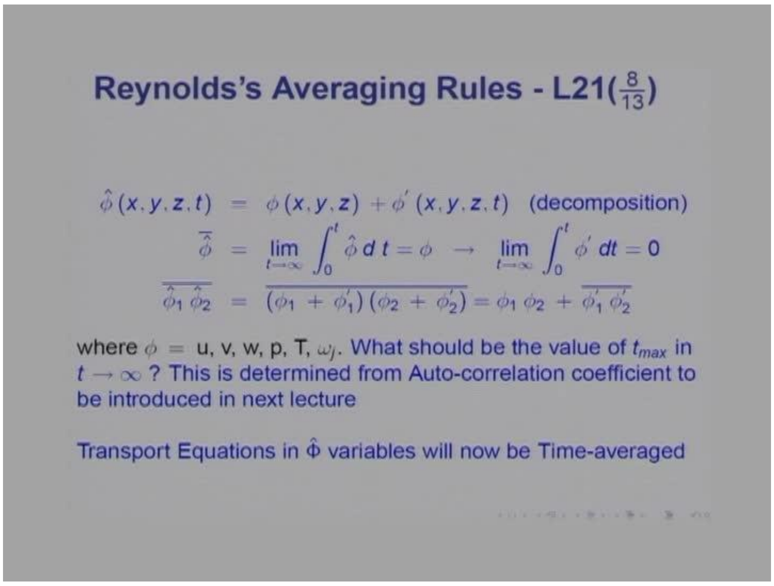

In fact, this is precisely what was done by Osborne Reynolds when he proposed his decomposition. It is shown here that the instantaneous value of phi where phi may be anything: velocity, u, v, w; it may be pressure; it may be temperature; it may be mass traction; does not matter what. The instantaneous value of each one of them would be function of x, y, z, t, but the mean value will only be a function x, y, z plus and a fluctuating value which is x, y, z, t and phi dash may be plus or minus.

Then, with this decomposition, Reynolds postulated that, time averaging of phi cap would be limit t tends to infinity 0 to t phi cap d t equal to phi - the mean value; in other words, the integrated value or integrated time average value of the fluctuations will always be 0; this is the postulate of Osborne Reynolds.

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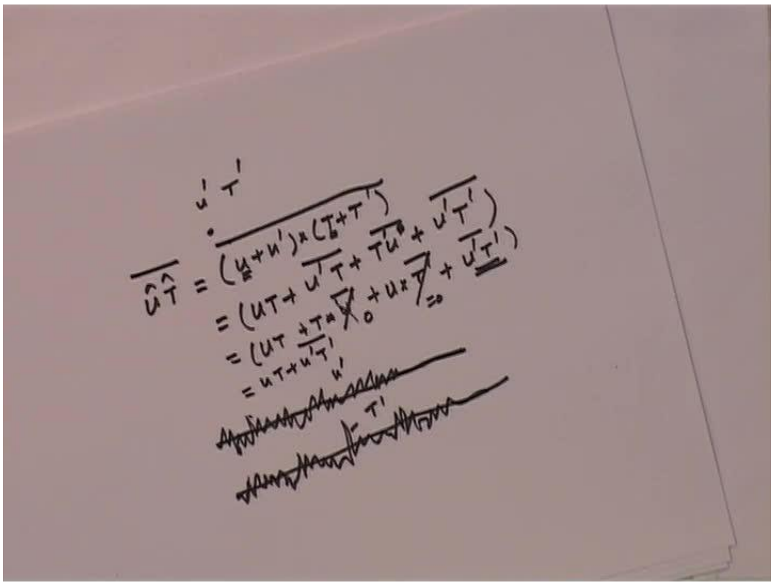

A very interesting issue then arises at a point in the flow. Let us say I have a velocity fluctuation and a temperature fluctuation also. I want to calculate what will be the time average of u T bar; that is what I want to calculate; let us say product. Then, it would be u plus u dash multiplied by T plus T dash time average which will be equal to, where u and T are the time average values and therefore they would remain the same; time average values do not change plus you will get u dash T time average plus T u dash time average plus u dash T dash time average; that is what you will get.

But notice that we have said that integrated value of any fluctuation will be always 0. Therefore, this would be u T plus T multiplied by u dash and that would be 0; sorry this should be T dash u plus u multiplied by T dash and that would too be 0, but u dash T dash will survive. Why? For example, let us look at this at the same point, let us say u varies like this (Refer Slide Time: 34:47) and this is u dash is plus or minus and I then brought T dash. So, these are T dashes.

So, you can see the product of u dash and T dash may be negative; it may be sometimes positive; it may be anything, but it will never be 0, even after the product has been time averaged. Therefore, u dash T dash will not be 0; time averaging of u dash T dash - it may be 0 sometimes, but very often it may even be positive or it may even be negative, and so on and so forth;

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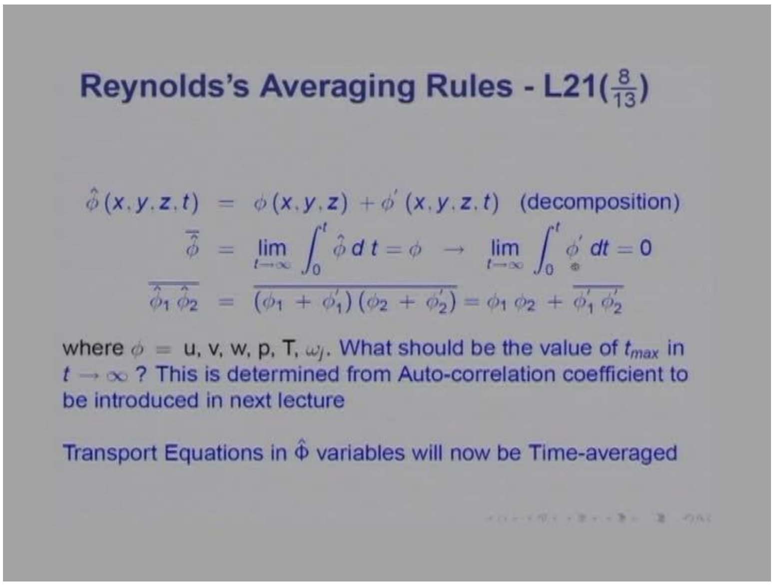

So, in other words, the time averaging of the product u T will be equal to u T plus u dash T dash. That is what I have shown here in this slide at phi 1 phi 2, time average is equal to phi 1 multiplied by phi 2 plus phi 1 dash phi 2 dash time average.

The next question is - what is this T equal to infinity? How long do I have to average? So, that phi dash or integration of phi dash will result into 0; that t max we would discover a little later in the next lecture from what is called as an Auto-correlation coefficient. So, we will defer that as to what this infinity should mean, to the next lecture. So, the transport equations of phi dash variables will be time average, now.

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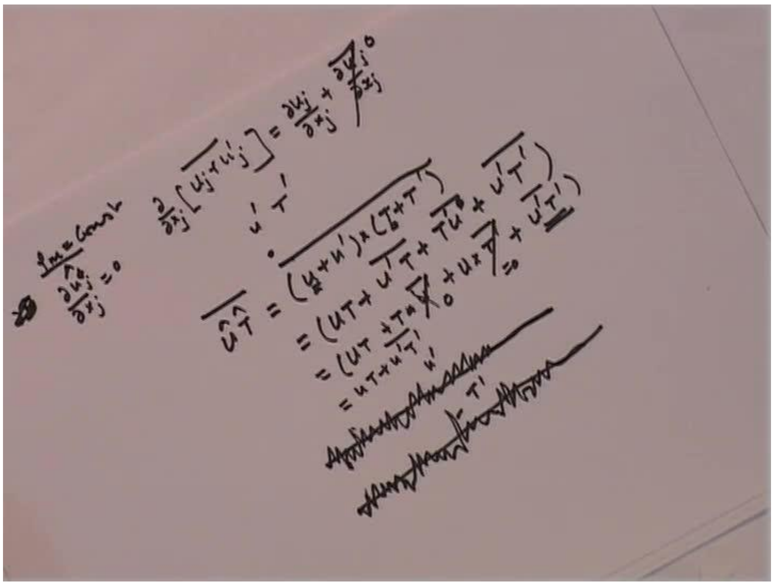

We already have these equations; for example, the continued equation of a turbulent flow would be written as: for constant density flow, it will be simply du cap by dx j equal to 0 and if I were to time average this, it will be simply d by dx j of u j plus u dash j of time average; that would be simply du j by dx j plus du dash by dx j, but then, we say that will be 0; that gives you du j by dx j equal to 0 the time average value.

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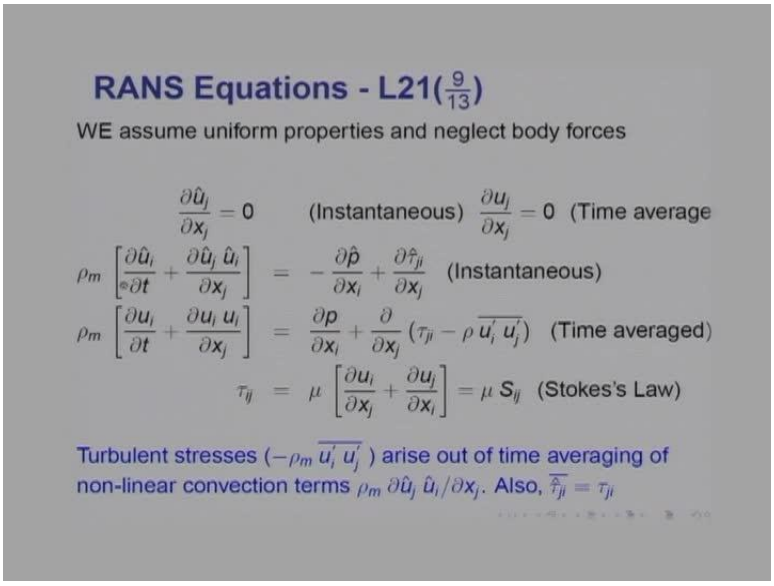

Similarly, the instantaneous momentum equation would look like this: rho m du i by dt and these are the convective term this is a pressure gradient term and these are the stresses, instantaneous. If I were to time average, then notice that this will simply transform to that because u i dash will go to 0. This also will have dp by d will become dp by dx, but remember this has a product in it. So, you will get d by dx j of u j u i plus d by dx j of u dash j u i j and that is what I have written; I have transferred that u dash term to the right hand side so that you get d by dx j tau ij which is the laminar stress, which is derived from this minus rho u i dash u j dash which is the time average.

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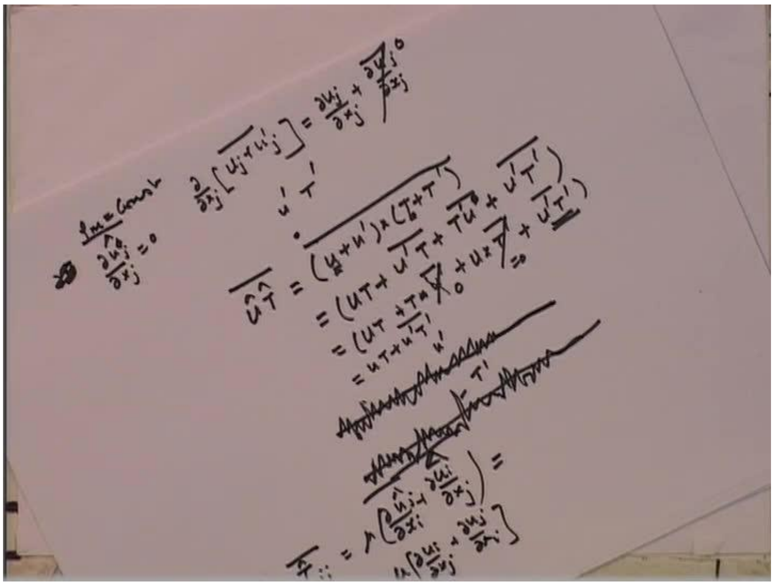

Remember, tau ij would be mu times du j by dx i plus du i by dx j and the time averaging of this would simply result into mu into du i dx j plus du j dx i because no product is involved in this definition. Therefore, tau ij would be that and a turbulent stress, instantaneous stress would be converted to a time mean stress represented by mean velocity gradients. Whenever I say mean, it means time averaged value and this is the Stokes’s law.

So, this newly appearing quantity, minus rho u i prime u j prime is what is called the turbulent stress in analogy with laminar stress tau ij. Always with a negative sign, turbulent stress is denoted as minus rho m u i prime u j prime and it arises out of time averaging of the convective term - rho m du cap j u i cap j dx j.

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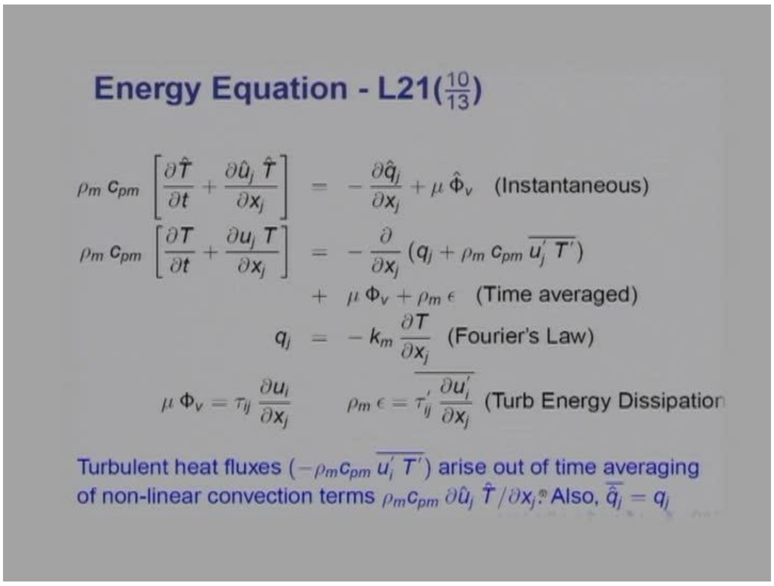

Likewise, if I look at the temperature equation, this would be the instantaneous form of the equation. Recall we had mu phi v, the viscous dissipation term, and this would be the conduction term and this is the convection term. Then the time averaging of that would simply result in dt by dt, but the time averaging of this which is a product, would result into du j by dx j here; the product term and of the fluctuation has been transferred to this side which will result in minus d by dx j of rho m C pm u j dash T dash. The time averaging of the conduction term would simply result into that q j mu phi cap v will result into mu phi v plus this (Refer Slide Time: 40:26 to 41:06).

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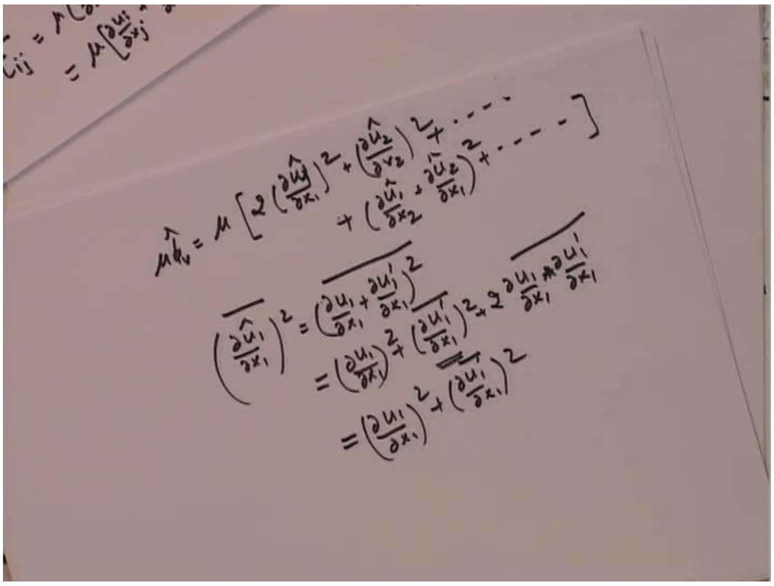

Now, this requires a little explanation: Remember, mu phi v is mu times 2 into du 1 by dx 1 whole square plus du 2 by dx 2 whole square - this is the instantaneous value I am writing; plus and likewise in the third direction plus du 1 by dx 2 plus du 2 by dx 1 square, and so on and so forth. If I time average these quantities, then you will see this quantity time averaged will be would be du 1 by dx 1 plus du dash 1 by dx 1 whole square time average and this will be equal to du 1by dx 1 whole square plus du 1 dash by dx 1 square plus 2 times du 1 by dx 1 multiplied by du 1 by dx 1 dash - all time average

You will see therefore, that this term will survive whereas this term will vanish because single u 1 dash appears here and this is the mean value. Therefore, you will simply get this as du 1 by dx 1 whole square plus du 1 dash by dx 1 whole square time average (Refer Slide Time: 42:58 to 43:20).

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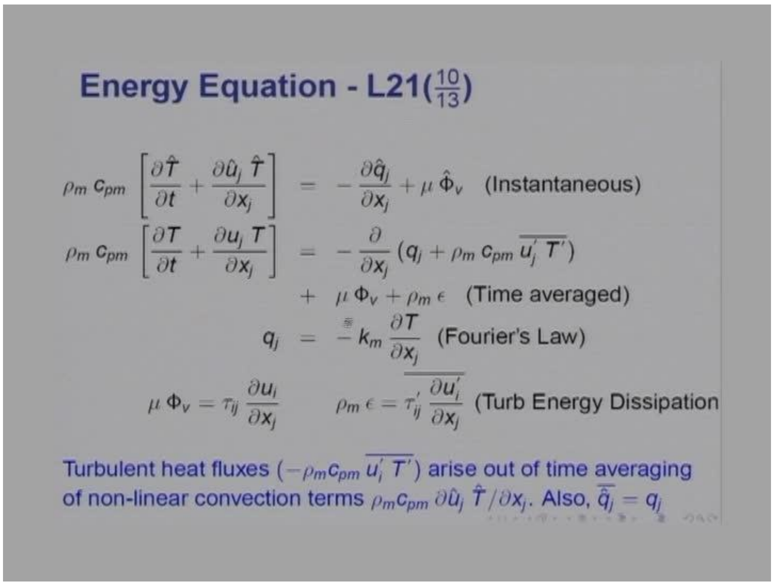

So, in other words, the mu phi v term will result into 2 terms: mu phi v which is formed from the product of tau ij mean multiplied by velocity gradients and tau prime ij multiplied by du i prime by dx j of the fluctuation part - the fluctuating stress and the fluctuating velocity gradient. This quantity is called the turbulent energy dissipation; usually it is very small compared to all other terms in the energy equation.

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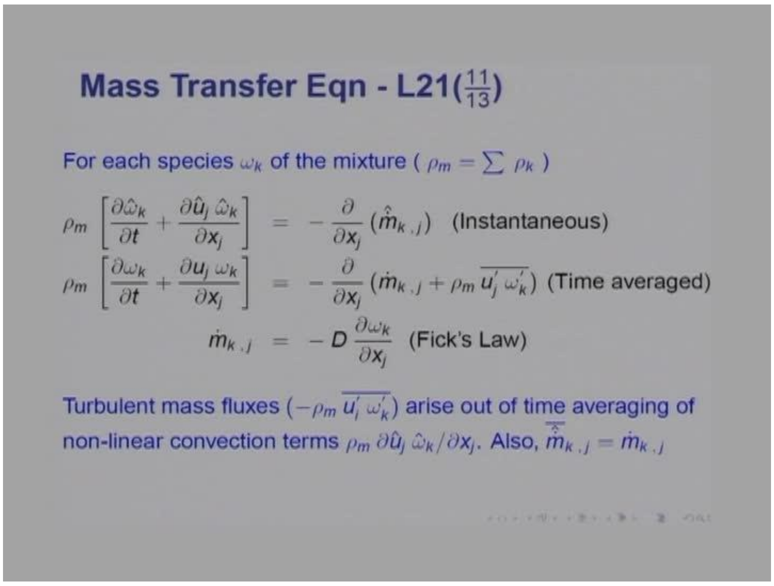

Later, we will find that that term plays a significant role in kinetic energy balance, but in thermal energy balance which usually very small, the same thing would hold even for

Mass Transfer Equation, and again you will have a turbulent mass flux with a negative sign, exactly same way as we did for the scalar temperature.

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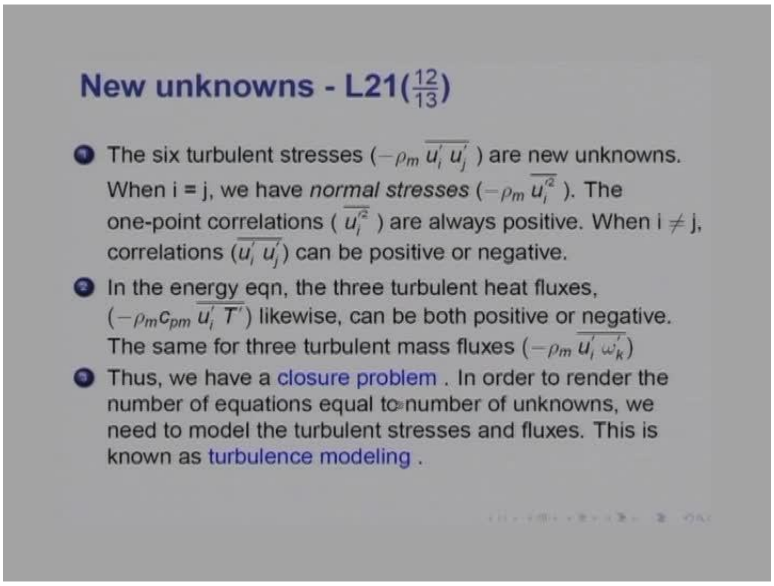

So, in summary, I would say we have now in the momentum equation 6 turbulent stresses. rho m u i prime u j prime; when i is equal to j, we call them normal stresses; rho m u i prime square with a negative sign when i is not equal to j, we will have shear stresses; u i prime square would always be positive because this is the square of the same quantity, but u i prime u j prime in general can be both positive or negative; depends on its location. In energy equation again u i dash prime T prime can likewise be positive or negative.

So, in effect, we have now a new closure problem. We simply have three momentum equations and 1 continuity equation, but we have created six more new unknowns called the turbulent stresses. So we have four equations and u, three velocities pressure and six stresses - ten unknowns.

So, in other words, unless we model these, we cannot make any progress with the solution of this Reynolds Averaged Navier Stokes equation which is often called - the RANS equations. So, in order to render the number of equations equal to the number of unknowns, we need to model the turbulent stresses and fluxes. This is known as turbulent modeling

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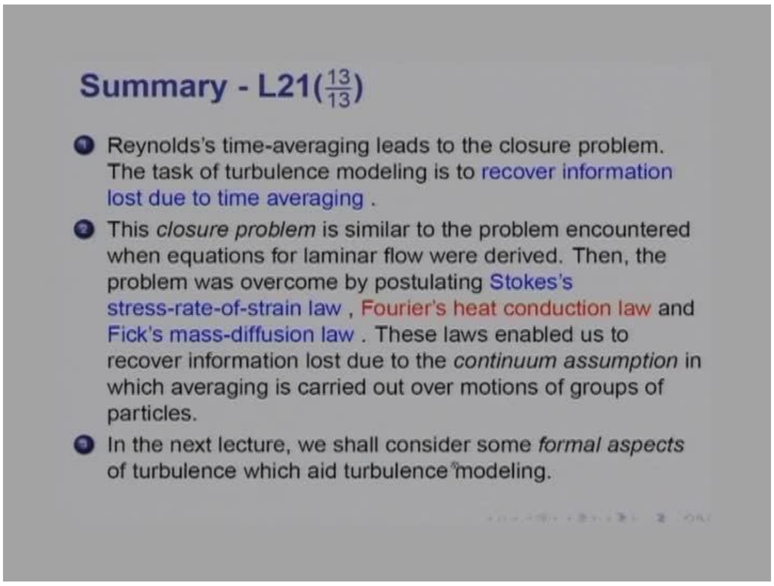

We will take that up essentially Reynolds time averaging has led to the closure problem. The task of turbulence modeling is to recover the information loss due to time averaging. This kind of a closure problem we have encountered earlier also, in laminar flow where we made a continuum approximation and we recovered the information lost in averaging over large number of particles, through viscosity and conductivity, and so on and so forth.

Likewise, we will have to do something to recover the lost information in time averaging and that is known as turbulence modeling. In the next lecture, we shall consider some of the formal aspect of turbulence which aid turbulence modeling.