# Convective Heat and Mass Transfer

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**Module No. # 01**
**Lecture No. # 20**
**Superposition Technique**

Today, I will take up the last lecture on laminar flow heat transfer in ducts. What I am going to consider is, the effect of actually varying thermal boundary condition. This is particularly of interest in oil flows, where the Prandtl number is very large and the thermal development length is considerably bigger than the velocity development length.

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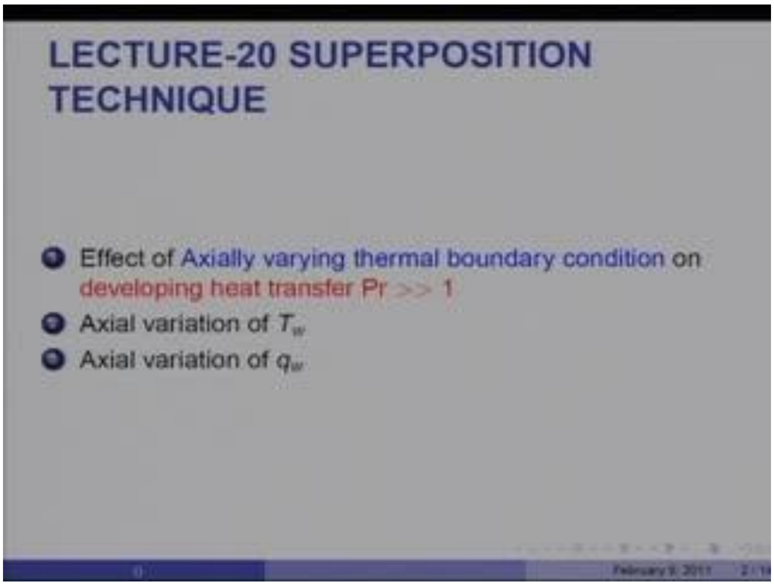


In this sort of situations, actually varying wall temperature as well as, actually varying heat flux has profound effect on the manner in which the heat transfer coefficient changes or varies with the actual distance.

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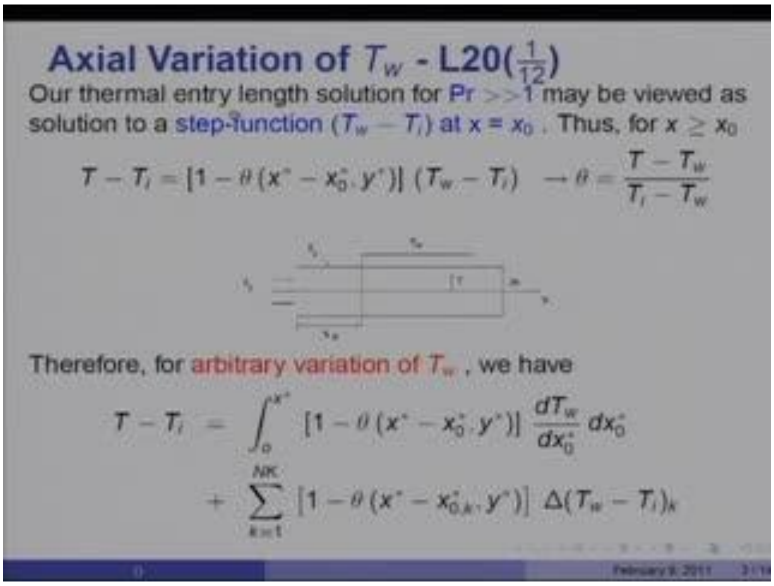


For example, let us consider the solution that we had obtained for Prandtl number very greater than 1. Now that solution was obtained for T equal to T w from x equal to 0. But, today I am going to consider the case of heat transfer begin from x equal to x naught, before that the wall temperature is same as the inlet fluid temperature.

In fact, we can view our earlier solution of lecture 18 as a step function in T w minus T i; the T i is inlet temperature at x equal to x naught. Therefore, for x greater than x naught, I may write T minus T i equal to 1 minus theta x star minus x naught star y star T w minus T i; theta, you will recall was defined as T minus T w over T i minus T w. So, just as we considered the external boundary layer situation; for the internal boundary layer situation, it would again look like this (Refer Slide Time: 02:25).

For arbitrary variation of T w, we will have T minus T i equal to 0 to x star 1 minus theta x star minus x naught star y star dT w by dx naught star, dx naught star; this would be for the continuous part of the variation. This would be 1 minus theta x star minus x naught k y star delta T w minus T i sub k; this would be for the discontinuous part of the solution. This is exactly following what we did in the case of external boundary layers (Refer Slide Time: 02:45).

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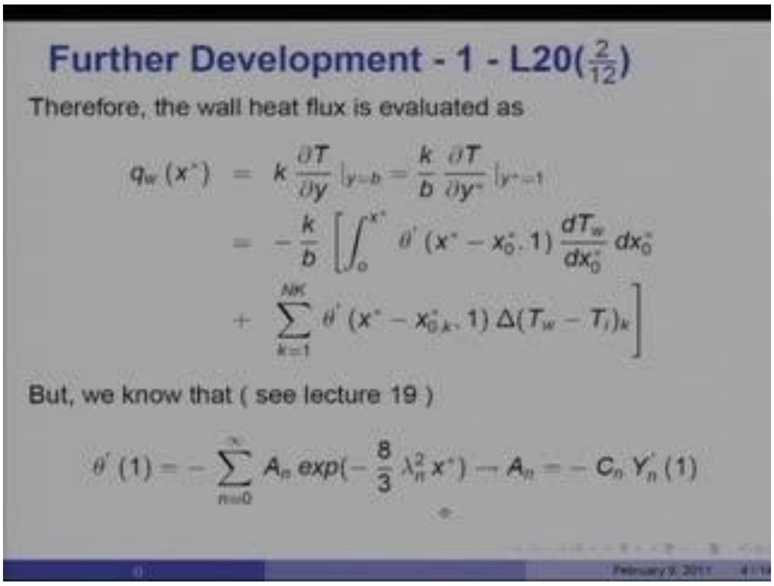


The heat flux response would be q w star, now I am considering the case of flow between parallel plates separated by distance to b. Then, that would be equal to q wall x star, would be equal to k dT dy at the wall y equal to b equal to k by b dT by dy star, y star equal to 1. That would read as theta dash, here as d theta by dy, x star x naught star 1- where y star is equal to 1. This will be for the continuous part; this would be for the discontinuous part (Refer Slide Time: 03:40).

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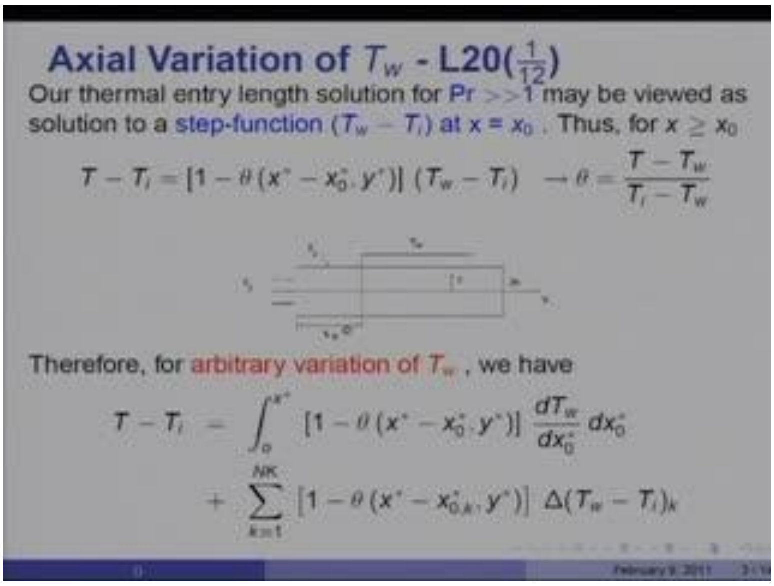

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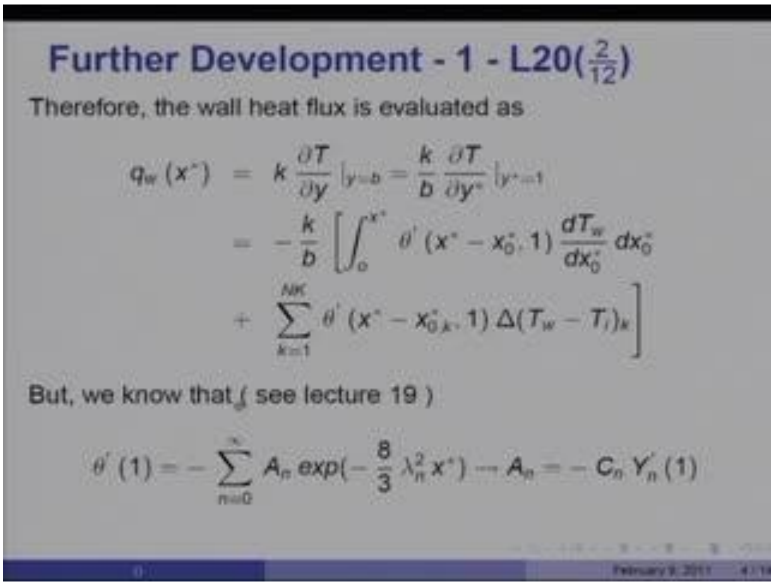


This is simply a differentiation of this equation with respect to y. But, you will recall that from lecture 19, we had found that theta dash 1 is actually equal to minus sum n equal to 0 to infinity A n exponential of minus 8 by 3 lambda n square x star equal to A n equal to minus C n Y n dash 1, where C n and Y n dash 1 were tabulated in lecture 19. So, we are going to use the same coefficients A n in this series; lambda values 2 were tabulated in lecture 19. If I substitute that here, for that I would get the heat flux response to arbitrary variation of T w along the wall.

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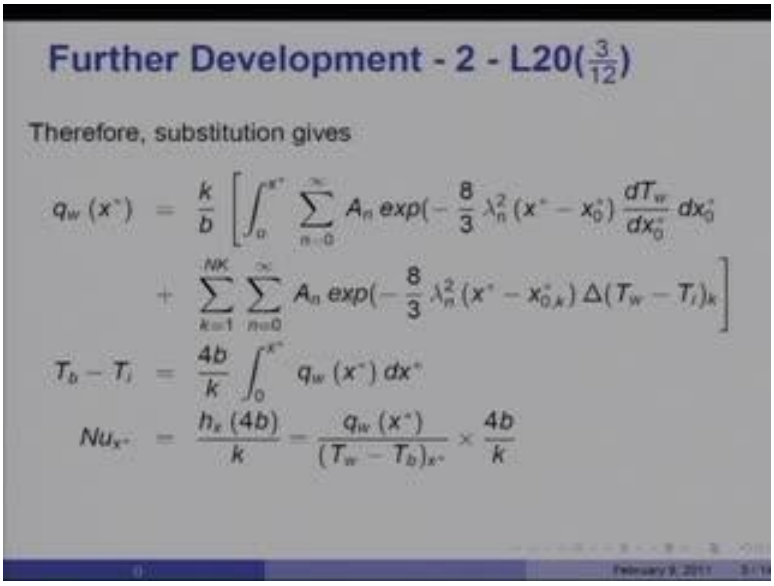

So, q w x star would become k by b 0 to x star n equal to 0 to infinity, this function which is written for x star minus x naught star dT w by dx naught star (Refer Slide Time: 04:45). This would be for the discontinuous part; the summation goes from n equal to 0 to infinity over n for the number of steps, a k equal to 1 to NK. Now that T bulk minus T, i would obviously be equal to 4b by k integrals 0 to x star q wall x star dx star and this is simply from the heat balance.

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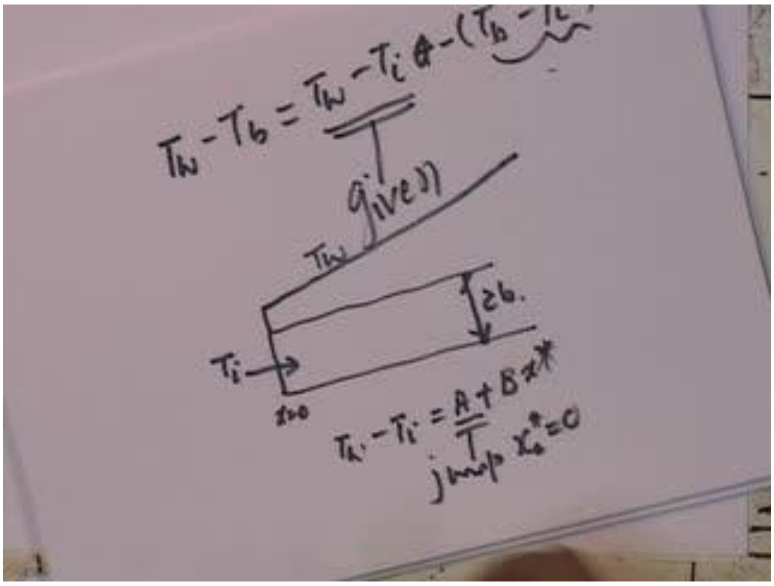


Therefore, the Nusselt number would be given by q wall x star divided by T wall minus T bulk x star 4b by k, where T wall minus T bulk is written as T wall minus T i minus T bulk minus T i. This is given and this is what we have evaluated just now (Refer Slide Time: 05:52). T wall minus T bulk is written in the fashion I have shown here, so that is how one gets T wall minus T bulk at any x star.

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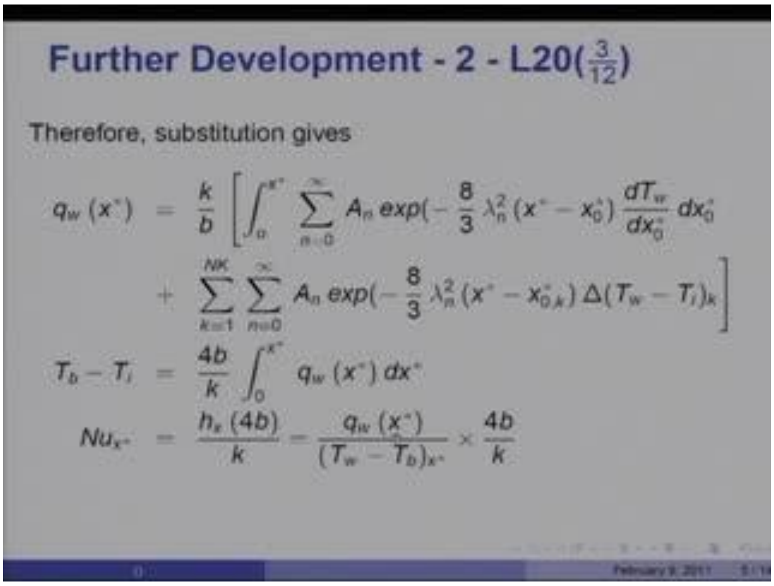


Let us move on to a problem in which I am going to consider, let us say there are two parallel plates separated by distance 2b and fluid enters at T i. At x equal to 0 itself there is a jump and then the temperature is linear. So, T w minus T i, this is T w equal to A plus Bx star. A is the jump at x naught star equal to 0 in this case.

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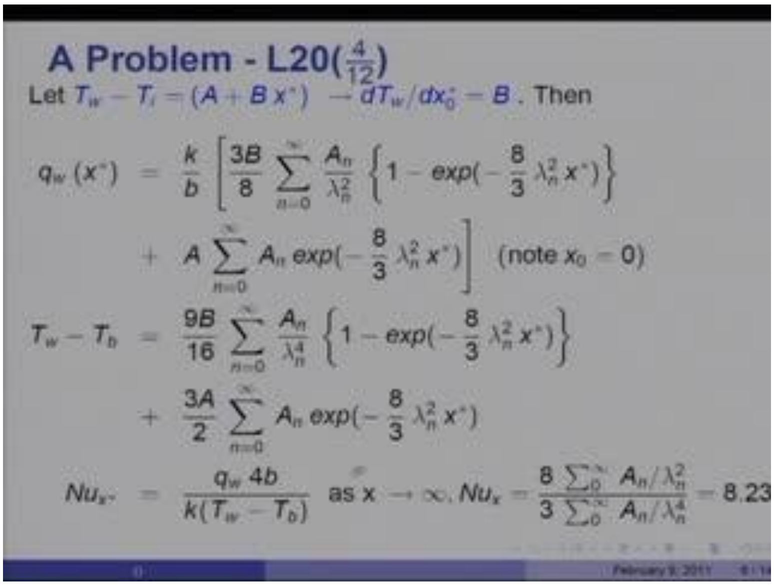

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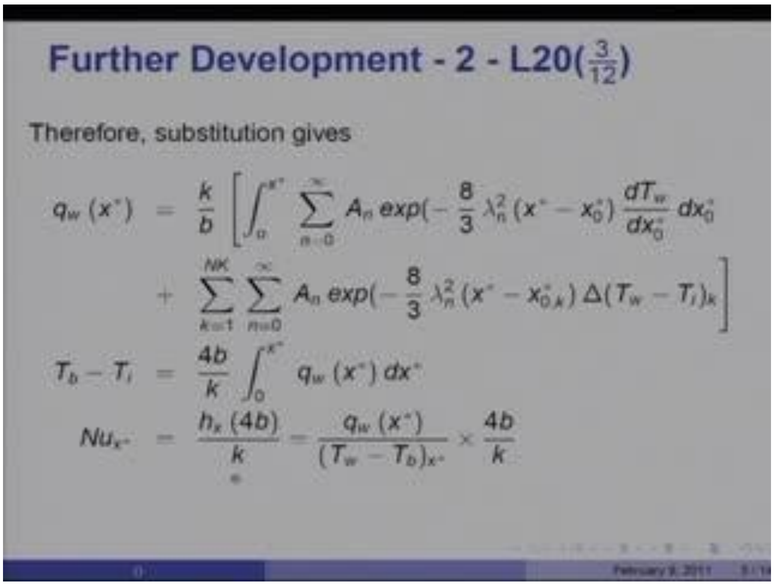


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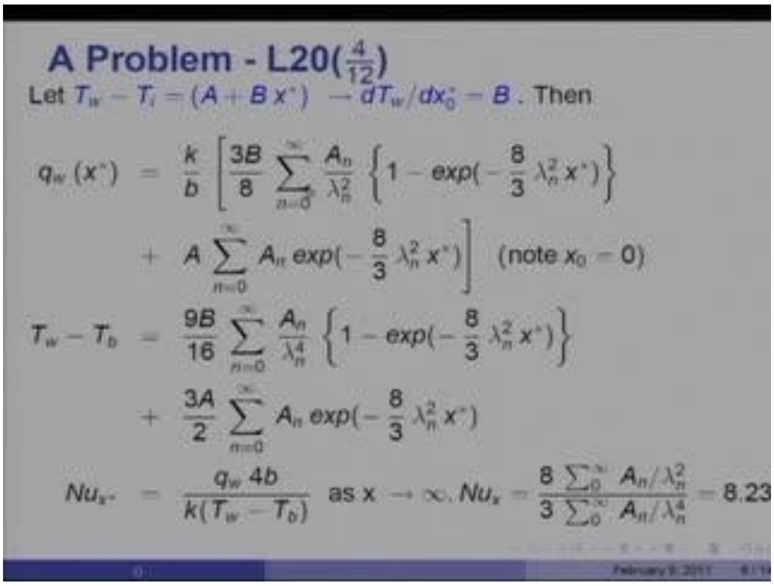

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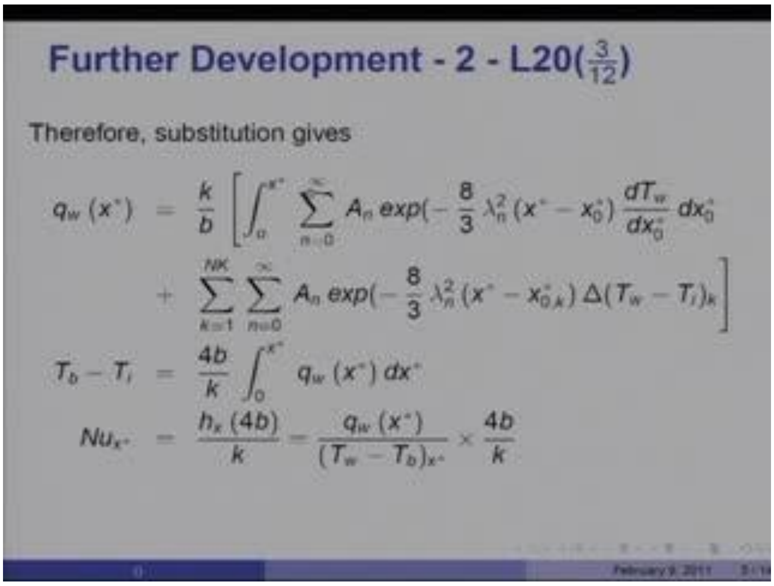


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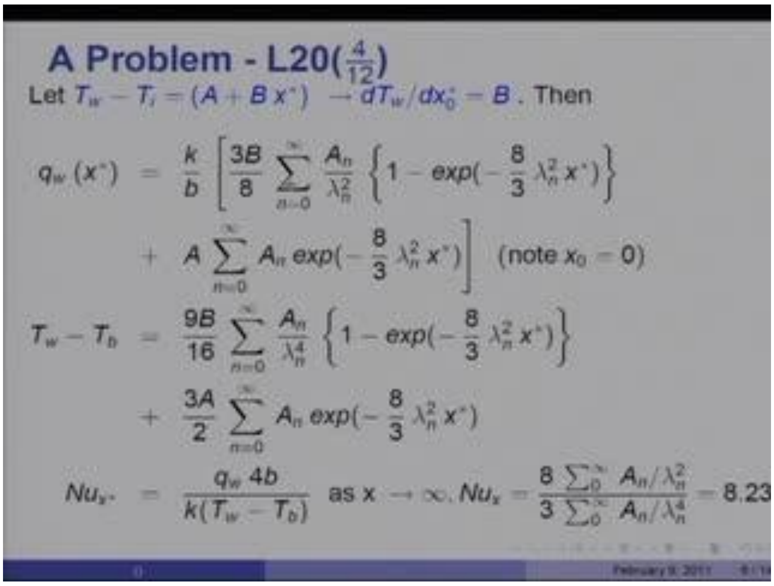


Let us see how the equations develop. If I have to substitute this variation of T w minus T i in the previous expression here, so dT w by dx naught star would be simply capital B; that is what this would be. The integration of that with b here, so b is simply is a constant and it would come out. The integration of this equation with x naught equal to 0 would be given by A n by lambda n square 1 minus exponential of minus 8 by 3 lambda n square x square, plus we must add the step jump right at x naught equal to 0, which is the entrance that will be plus A times n minus 0 equal to infinity A n exponential of minus n lambda n square x star (Refer Slide Time: 07:20). There are no other step jumps in this. Therefore, this one is a continuously right along and there are no further step jumps, so simple case.

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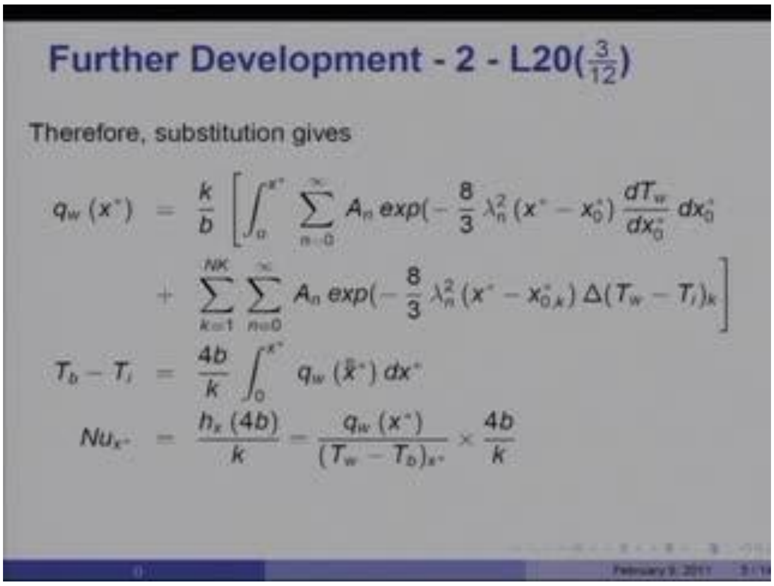


Once you obtain q wall in this manner, T wall minus T bulk would simply be given by 9B by 16 A n by lambda n 4, you can see that here T b minus T i which is an integral of q wall x star.

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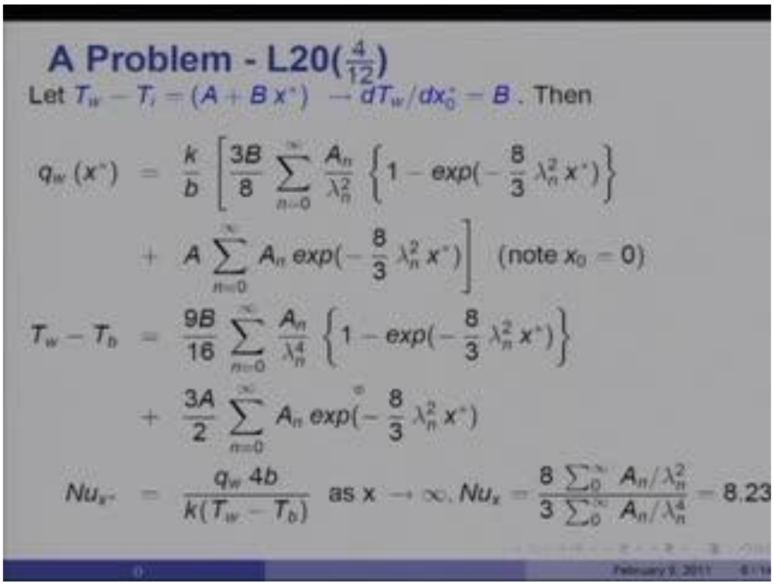

So, I have used that and added T wall minus T i, then they will get 9B by 16, n equal to 0 to infinity A n by lambda n 4 and this term plus the step jump part, which is done. As a result, you will get Nu x is equal to q wall 4b divided by k into T wall minus T bulk, but notice one thing that as x tends to infinity you will see that term will go to 0, that term will go to 0 (Refer Slide Time: 08:45). Therefore, T wall minus T bulk would be simply 9B by 16 n equal to 0 to infinity A n by lambda n 4. Likewise, q wall x star would be simply the A n by lambda n square, this term going to 0 and this term equal to 0.

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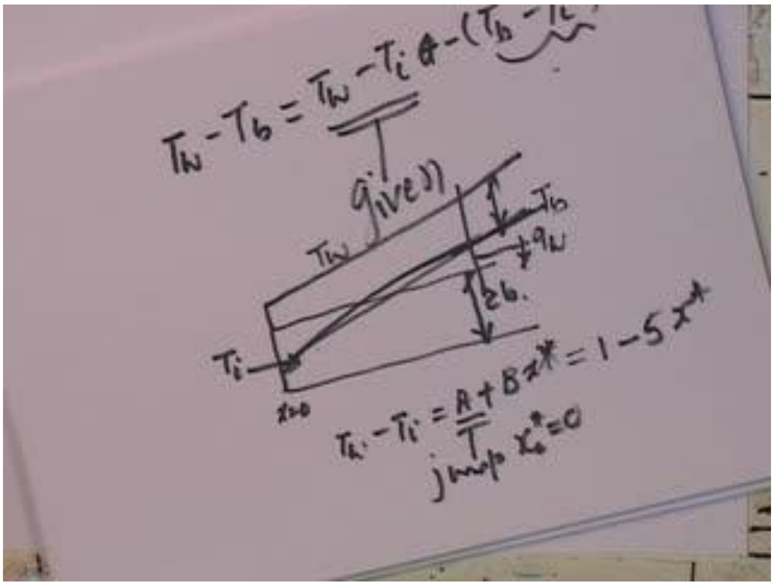


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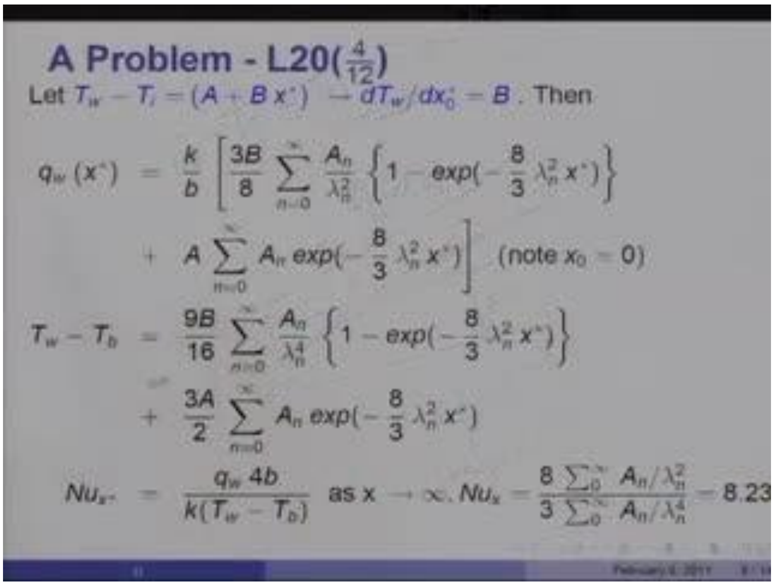

Now, what it shows is that the bulk temperature is also going to vary linearly after a long time although initially it may vary in an arbitrary manner, but after a long time T wall minus T bulk maintains a constant difference. Therefore, at this point onwards it would be a case of constant heat flux at the wall and that is what we find 8 by 3 n equal to 0 to infinity and that would be equal to 8.235.

This would be the general case, if we were to go to infinity we have verified that the solution obtain appears at least correct in the infinity state. Now, I am going to assign some values to A and B and see what happens.

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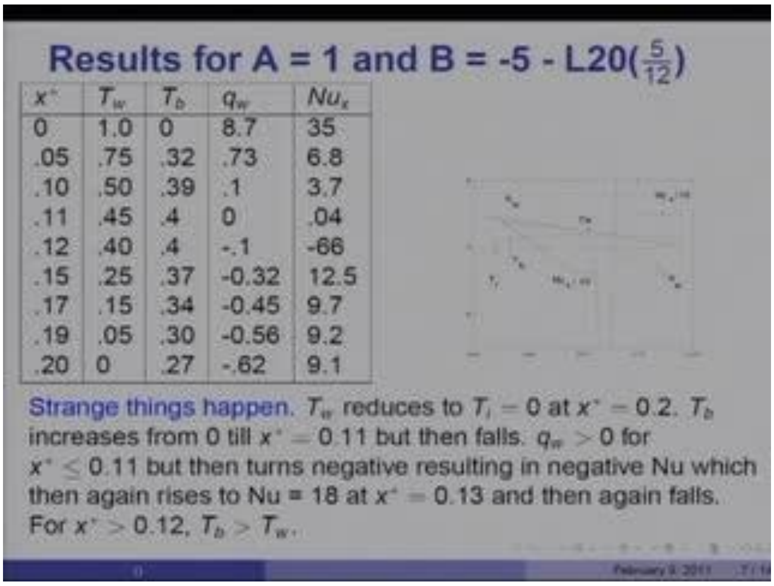


On this slide, I consider a case of T wall minus T bulk equal to 1 minus 5x star. The wall temperature declines as x increases, you can see that here, this is the variation of wall temperature on the graph that is shown (Refer Slide Time: 10:25). The values assigned are these, I have gone up to when T wall minus T i that is the wall temperature deduces back to the inlet temperature. The initial jump is A equal to 1 and it reduces to minus, so at x star equal to 0.2 the temperature of the wall is same as that of the inlet fluid you can see T wall decreases.

What happens to T bulk? T bulk as you can see on this graph first T bulk increases as you can see here and then its rate of increase is somewhat slowed down to this is the one that is shown by the dotted line (Refer Slide Time: 11:10). So, T bulk increases to 0.32

0.39, 0.4 at about 0.12 it is same as 0.4 and then begins to decrease when the wall temperature becomes equal to inlet fluid temperature, the bulk temperature that point is 0.27.

Notice, what happens to q wall; q wall is because T wall is greater than inlet fluid temperature, initially q wall is positive 8.7 reduces to 0.73 at x star equal to 0.05 reduces to 0.1. In fact, there is one point at 0.11 where there is no heat transfer that means this situation becomes adiabatic. Then, q wall changes sign that means the bulk temperature now is greater than the wall temperature. Therefore, you get negative heat fluxes from x star equal to 0.12 onwards.

What happens to Nusselt number? As you can see here, the Nusselt number is plotted along there initially, the Nusselt number falls as it should for any heating case, but then it suddenly drops to a very low value almost minus 66 at x star equal to minus 12 and then suddenly jumps back at 0.15 to plus 12.5 and then again reduces down to 9.1. Since, I have not computed long enough, I do not see 8.325 but, if I were to continue in that fashion but with a negative heat transfer, so even that is possible.

So, indeed strange things happen the wall temperature although is monotonically linearly decreasing, the bulk temperature has a hump at about 0.4 x star equal to 0.11 to 12.12 and then there is a decline and turning sign and likewise, Nusselt number is decline, but it goes negative and then becomes positive. This is a very interesting case of, how T bulk overtakes T wall and although q wall is negative, you still get Nusselt number positive.

This positive Nusselt number is for negative heat transfer, this positive Nusselt number is for positive heat transfer into the fluid and at this point you get negative heat transfer with negative Nusselt number; so indeed very strange things can happen (Refer Slide Time: 13:50).

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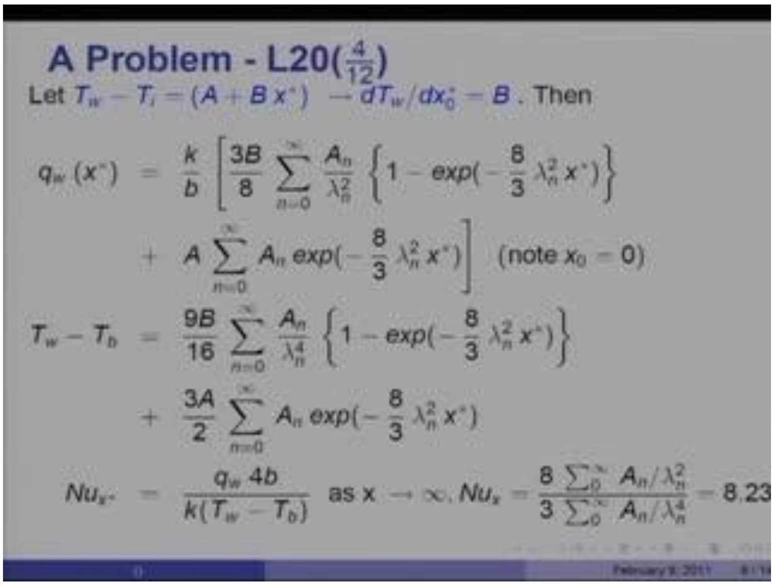


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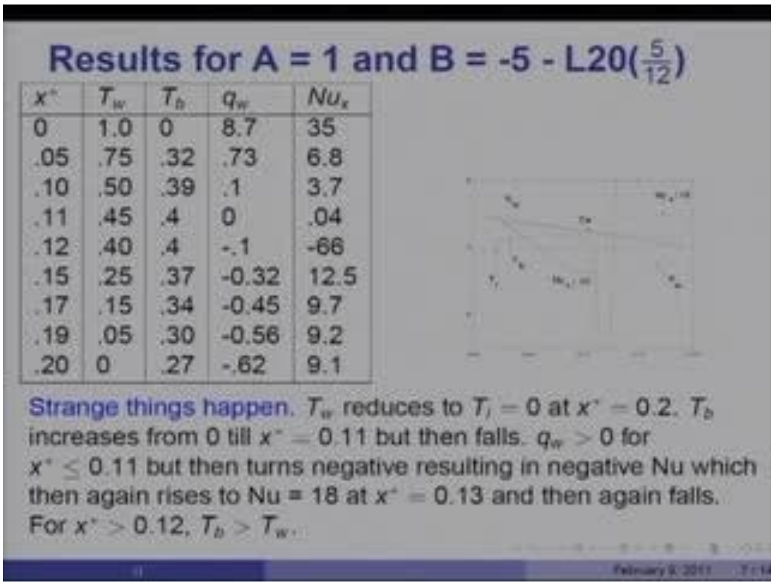


One can work out solutions for variety of A and B values to see what happens. As I said, one could generate these solutions also for a circular tube because as you will recall the A n and lambda n values remain the same for a circular tube also. Therefore, for such a case one can again carry out the integrations that I have shown here and evaluate Nusselt number for circular tube, which I shall not shown here. But it can be done and you do indeed get very strange variations of Nusselt number when the wall temperature varies actually.

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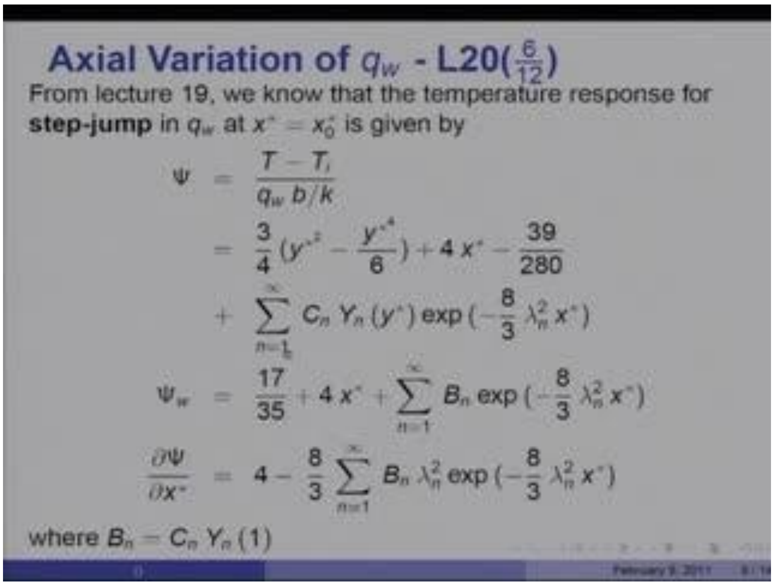


Let us consider the case of actual variation of heat flux; this is of a considerable interest in nuclear reactors as I will show a little later. From lecture 19 we know that the temperature response for step jump in q wall at x star equal to x naught star is given by T minus T i q wall b by k this is the fully developed solution, plus this one is the developing part of the solution (Refer Slide Time: 15:00). Remember, recall that we had said psi equal to psi f d plus psi developing and this part is the fully developed part and this part is the developing part.

So, writing this equation at y star equal to 1 would give me psi wall equal to 17 by 35 plus 4 by 4 x square plus B n exponential of 8 minus 8 by 3 lambda n x star, where B n is C n multiplied by Y n 1 which I have taken from y star equal to 1. If I were to take d psi by dx star it will be 4 minus 8 by 3 into all this into lambda n square and so on so forth.

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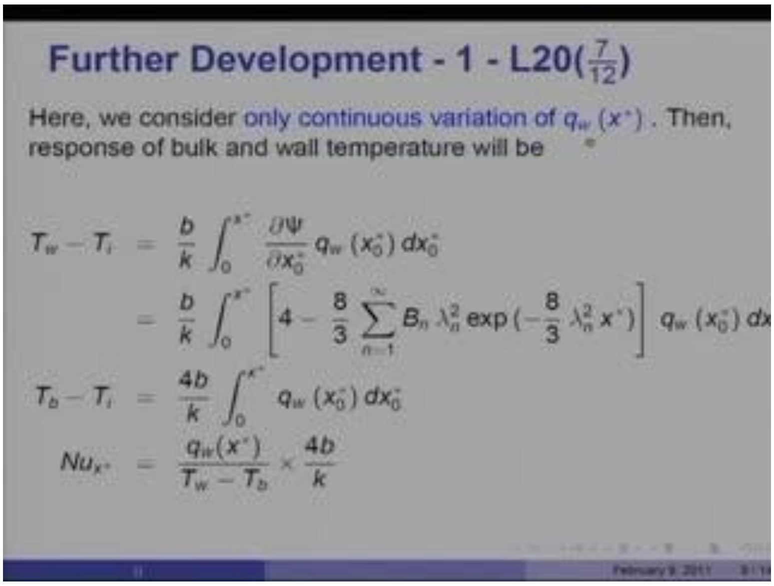


Here, we consider only continuous variation of q w x star; I am not going to consider situations in which step jumps in a q wall occur were there here. Then the response of the bulk and the wall temperature will be T w minus T i b by k d psi by dx naught star q wall x naught star dx naught star and that would be essentially equal to b by k equal to all that we derived on the previous slide into q wall x naught star dx star. T bulk minus T i would again be q 4b by k 0 to x star q wall x naught star dx naught star. Therefore, the Nusselt number would be q wall x star divided by T wall minus T bulk, simply take a difference of these two quantities and multiplied by 4b by k for flow between parallel plates.

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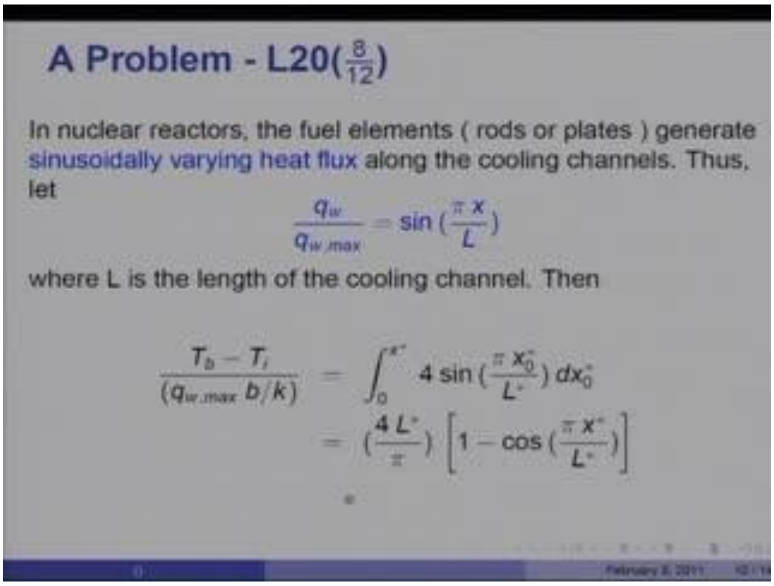


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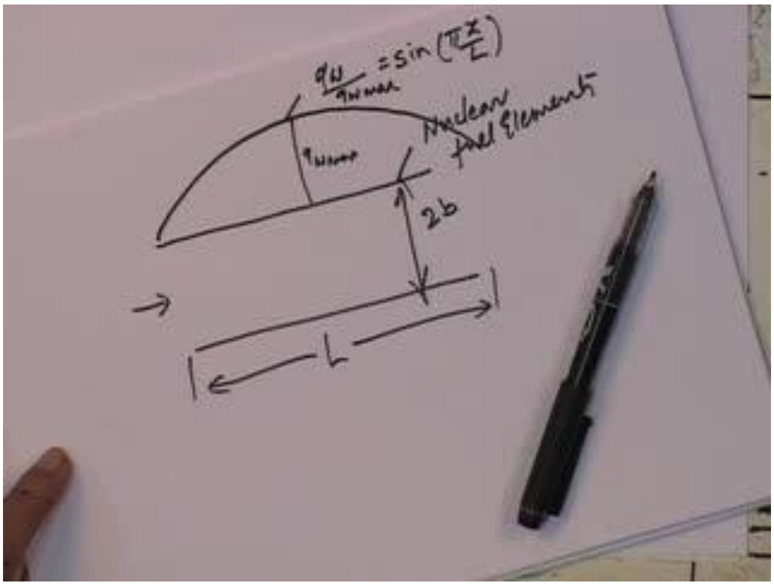


In nuclear reactors it is quite common. Consider flow between parallel plates and these are the nuclear fuel elements and let us say this is 2b. In fact, the earliest nuclear fuel elements were in fact flat sheets and not the circular rods that we find today and the coolant would be CO 2 at in the earliest nuclear reactor built.

Let us say, we have this 2b nuclear fuel elements generate generally heat in this fashion sinusoidal, so q wall would be q wall max. Let us say this is q wall max equal to sin pi x by L, where L is the length of the channel through which the coolant is flowing. I am going to consider this particular case, because of its relevance to nuclear reactors (Refer Slide Time: 17:40).

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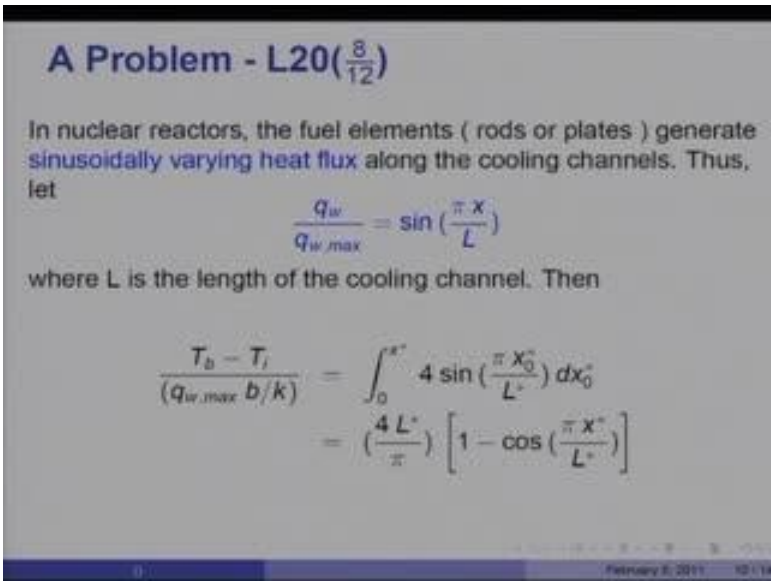


As I said, q wall over q wall max is let us say sine pi x by L, where L is the channel length. Then you will see T b minus T i q wall b by k would be simply 0 to x star 4 sin pi x naught star by L star dx naught star, so that gives us the value of T bulk at any x star would be simply given by that.

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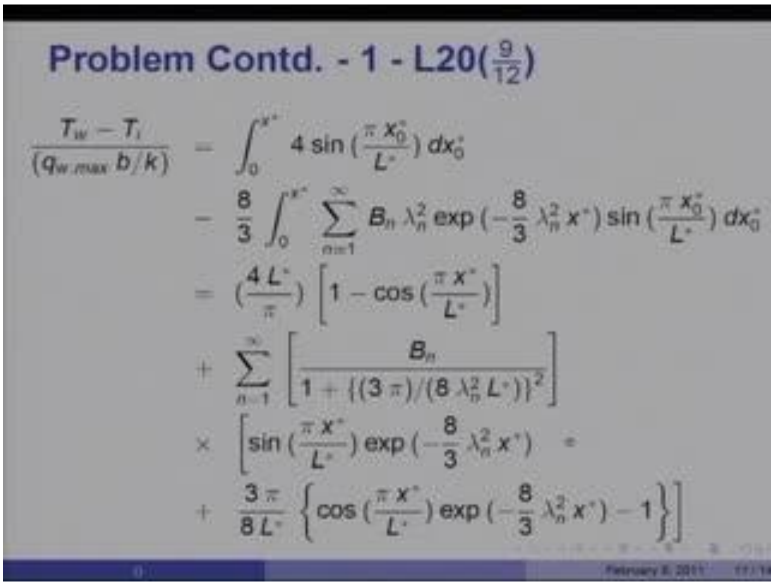

T wall minus T i would be given in this fashion, this is the 4x sin x pi x naught star dx naught star minus 8 by 3 all this and that would equal 4L star by pi cos pi x star by L star. This integration requires little effort because it is a product of exponential term and a sin term. So, the result is you get a very big bracket here with B n 1 plus 3 pi by 8 lambda n square L star whole square into sin pi x star by L star exponential of that term plus this term cos pi x star by l star exponential of minus 8 by 3 lambda n square x star minus 1.

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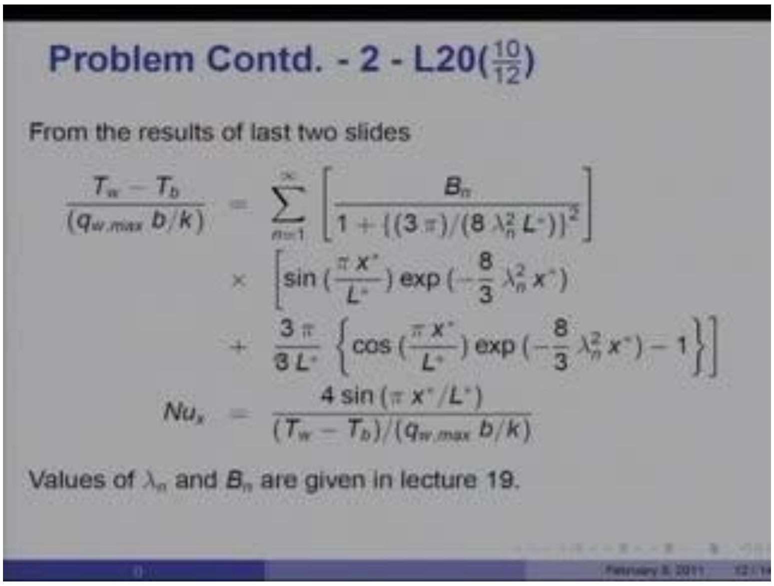


Now, we have obtained the variation of bulk temperature along the duct and we have obtained a variation of wall temperature along the duct. Therefore, we can calculate the Nusselt number as it is shown here. So, T wall minus T bulk would simply be different from the previous 2 slides and that would read in this fashion. We then apply Nu x is equal to q wall x, which is been specified sin pi x by L 4 times because hydraulic diameter is 4b and T wall minus T bulk into all this q wall max b by k. The values of lambda n and B n for constant heat flux cases, where given in lecture number 19.

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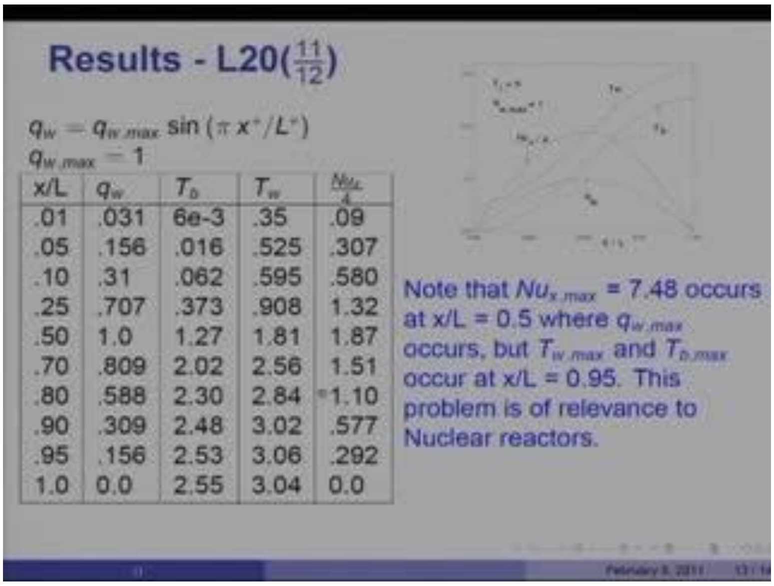


See now what happens? You can see the first column here and I have taken q wall max equal to 1 for convenience and x by L values I have taken are these from 0 to 1. The heat flux according to the sin function would vary in this fashion it reaches maximum at 1 at x by L equal to 0.5 and then again drops down to 0 value. So that is what has been shown here that the q wall is given in this fashion.

Notice how a T bulk varies? T bulk starts off with 0 at the inlet because that is when where the inlet fluid temperature it enters in inlet fluid temperature. Then it begins to rise it is not a linear rise although in the central part, it is linear where q wall is nearly constant. Therefore, T bulk varies more or less linearly in the linear path but then earlier it is not non-linear, so towards the end it is non-linear.

Most importantly see, how the wall temperature varies? The wall temperature increases, from there it changes the slope a little bit and then rises again to that value; this is the wall temperature variation. Therefore, Nusselt number divided by 4 for convenience to get a proper scaling here (Refer Slide Time: 21:45). You will see Nusselt number peaks at 1 at x by L equal to 0.5 at 1.87 as you can see, but then decreases down to 0.

So, the maximum Nusselt number is 7.48 and it occurs at 0.5 where q wall max occurs but T wall max and T bulk max continue to increase till x by L equal to 0.95. This problem is of relevance to nuclear reactors. Again, we can use the constants given in lecture 19 for flow in a circular tube.

So, you can see that if we were to assume simply that wall temperature was constant, we would get a very wrong picture of what happens and because we have assumed sin function we get T bulk and T wall variation which is different from what it would be if we had a linear constant distribution of constant wall heat flux.

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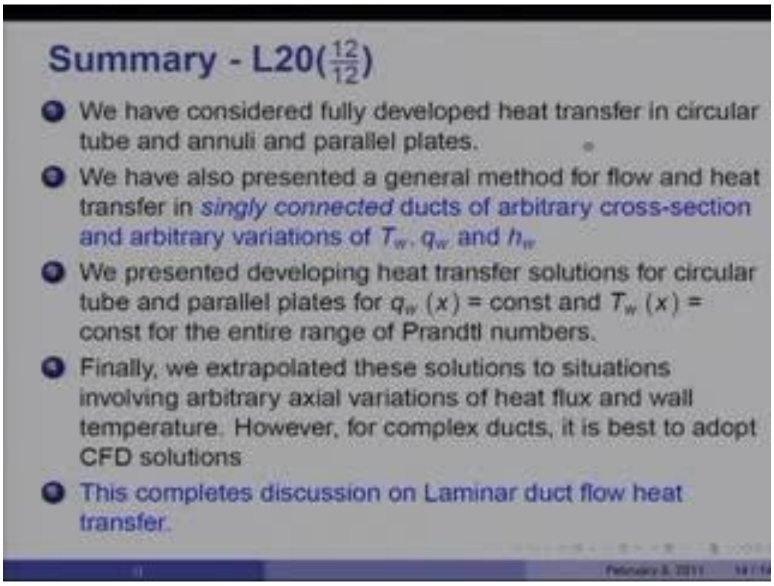


In summary I would say, we have considered fully developed heat transfer in circular tube and annuli and parallel plates. We have also presented general method for flow and heat transfer in singly connected ducts of arbitrary cross section and arbitrary variation of T w q w and h w. We have also presented developing flow heat transfer solutions for circular tube and parallel plates for q wall equal to constant and T w equal to constant for the entire range of Prandtl numbers.

Then finally, we extrapolated these solutions to situations involving arbitrary axial variations of heat flux and wall temperature, I will buy it for flow between parallel plates and or circular tubes. However, for complex ducts it is best to adopt the computational fluid dynamic technique and obtain the solutions.

So, with this I complete my discussion on laminar flow heat transfer, we have considered a typical case of what happens in the developing flow region and considered the fully developed flow situations and learn to obtain friction factor Reynolds number product for fully developed flows in ducts of arbitrary cross section. Then we extended that to the case situation of fully developed heat transfer in ducts of arbitrary cross section with arbitrary variations of thermal boundary conditions.

We found that for fully developed heat transfer as well as fully developed flow situation you could use Fourier series methods or cantor which variational methods, but we found that the general method based on conversion of the Poisson equations to Laplace equations turns out to be very general and can be applied to variety of ducts including regularly shaped ducts as well as arbitrary variation of boundary condition along the circumference.

Finally, in today's lecture, I indicated how solutions of constant wall heat flux and constant wall temperature that were obtained could be extended; the wall temperature and heat flux varies actually. So, with this I complete my discussion on laminar duct flow heat transfer. From next lecture onwards, we will be moving on to turbulent flow and heat transfer, thank you.