Convective Heat and Mass Transfer
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Module No. # 01
Lecture No. # 19
Laminar Developing Heat Transfer

In this lecture, we shall take up Laminar Developing Flow Heat Transfer. As we will recall, in the entrance region of a duct the velocity profile develops like so. This is the velocity profile or velocity boundary layer will develop like so, but the temperature boundary layer development would be governed by the value of Prandtl number.

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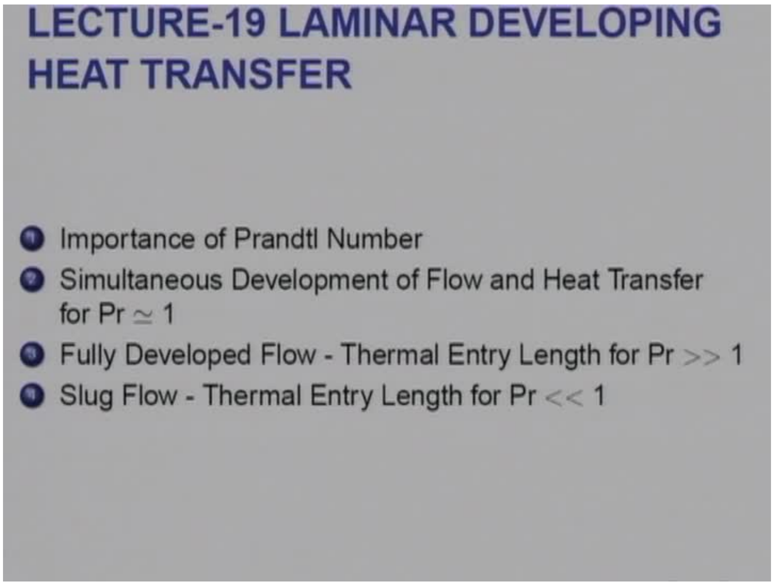

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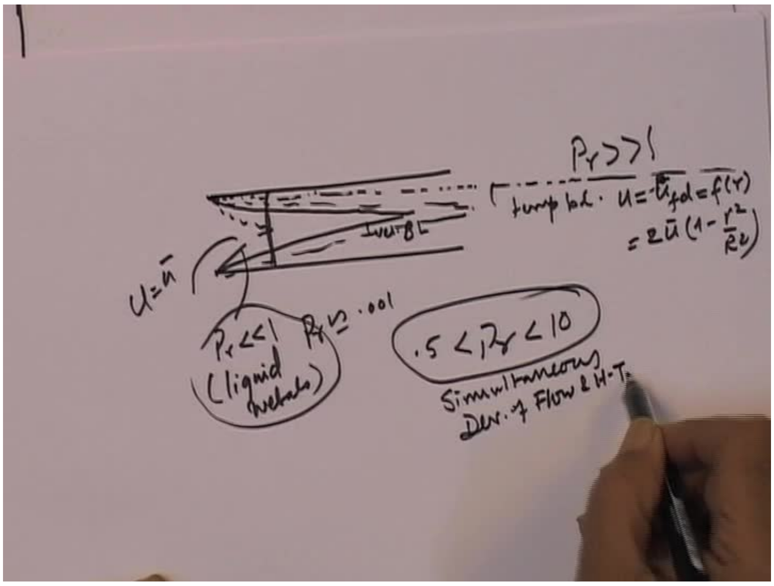

If Prandtl number was very small you will get very rapid development. This is for Prandtl number very much less than 1 because, the thermal boundary layers will develop much faster than the velocity boundary layer and this is the case of liquid metals. On the other hand, if Prandtl number was very large there is oils then, the thermal boundary layer development will be much slower and this would be the temperature boundary layer for Prandtl number much greater than 1 (Refer Slide Time: 01:36).

So much so, we can make very simple approximations for these two cases. For example, for liquid metals where Prandtl number is of the order of 0.001, over greater part of the length u will be simply equal to u bar whereas, Prandtl number very greater than 1 over greater part of the thermal development u will be simply equal to u fully develop and which will be function of r the array, which is for a circular tube it will be two times u bar into 1 minus r square by r square for example.

So, one can make very suitable approximations for these two extreme cases and obtain solutions but, when Prandtl number is between 0.5 and 10 then, both velocity and temperature profiles will develop at comparable rates and that is the case called as the Simultaneous Development of Flow and Heat Transfer.

That is the importance of Prandtl number in study of laminar developing heat transfer. So, for Prandtl number close to 1 in a small range from 0.5 to 10, one must consider both flow and heat transfer development simultaneously.

When Prandtl number is very greater than 1 then, flow will be fully developed and we will be calling it a thermal entry length problem because the velocity is specified likewise, when Prandtl number is very less than 1 then, u will be specified at the inlet value u and it is almost like a piston or a slug flow thermal entry length problem, so we will consider these cases separately.

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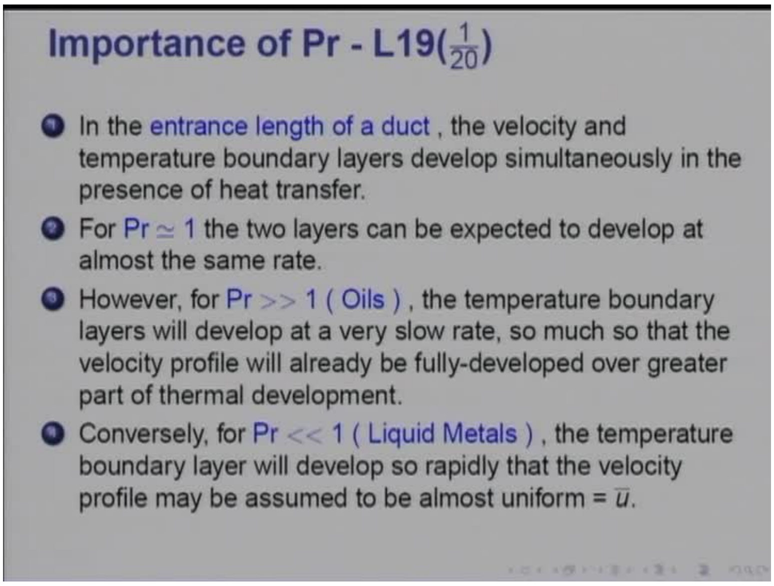

So, this is much of what I have already explain to you that for Prandtl numbers much greater than that is oils the flow will be fully develop over greater part of the thermal development length and in liquid metals it will be a taken as almost equal to u bar.

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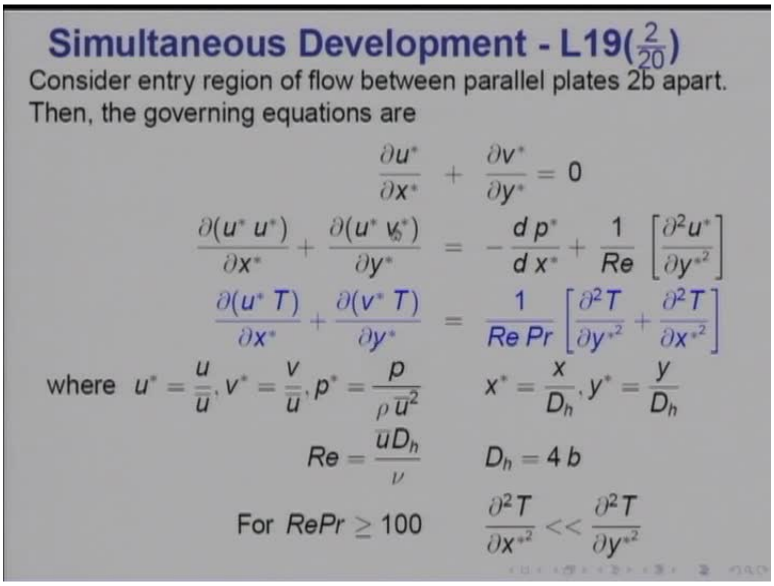

The real problem comes when there is a Prandtl number moderate and the two layers can be expected to develop at the same rate or comparable rates. Simultaneous development of both flow and heat transfer, we consider by way of an example the entry flow between two parallel plates 2b apart then, the governing equations will be this; this will be the continuity equation this you will recall is the momentum equation and now, we will have this as the temperature equation.

Of course, if we make boundary layer approximations and when Reynolds Prandtl - as you will recall from lecture number 17 - is greater than 100 then, temperature gradients in the y direction are much greater than the temperature gradients in the x direction, so this term would be dropped. We will be assuming that the Reynolds multiplied by Prandtl number is greater than 100.

The variables are shown here, so this is the convection term and there is one diffusion term that is this multiplied by 1 over Reynolds Prandtl.

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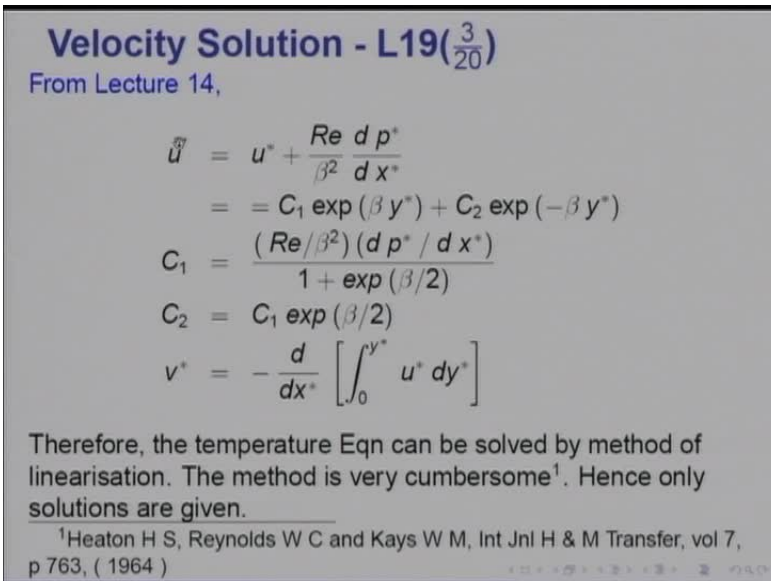

From lecture 14, you will recall that we had adopted Langhaar solution method and we have already obtained the velocity solution as given by C 1, C 2. For different values of beta, we printed out values of friction factor and others, but since u is known we can also get v from the continuity equation that is here.

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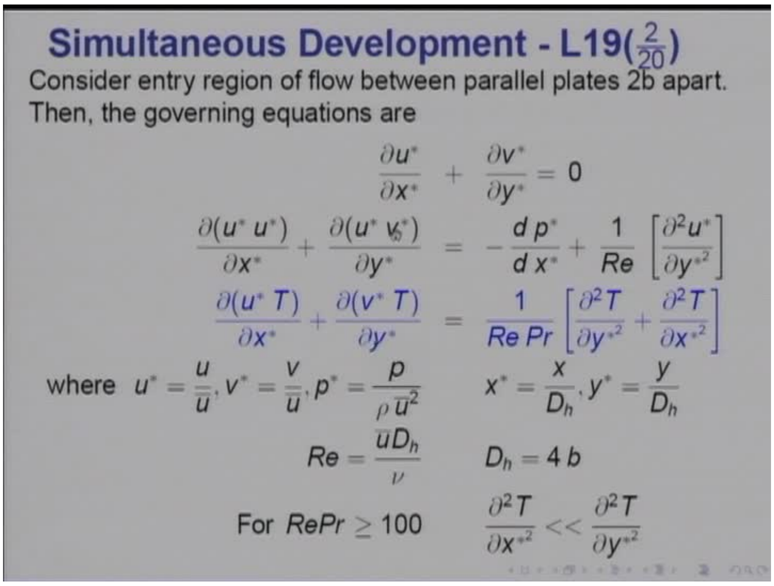

Knowing u star and v star, we have these two quantities unknown and therefore, we now intent to solve these equation only for temperature using the velocity solution from Langhaar’s case.

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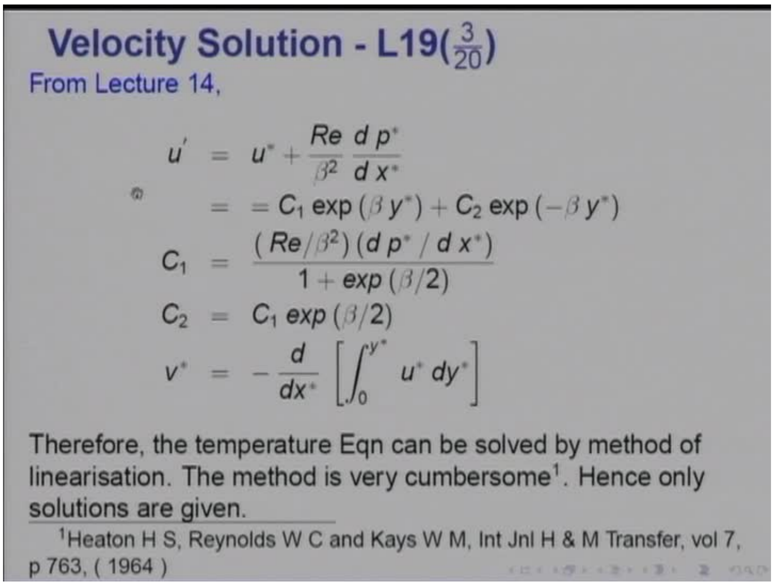

That resulting equation can again be solved by Langhaar's method of linearization. However, the algebra turns out to be very cumbersome as you will see from a paper by Heaton and Reynolds in case international general of heat and mass transfer volume 7 1964.

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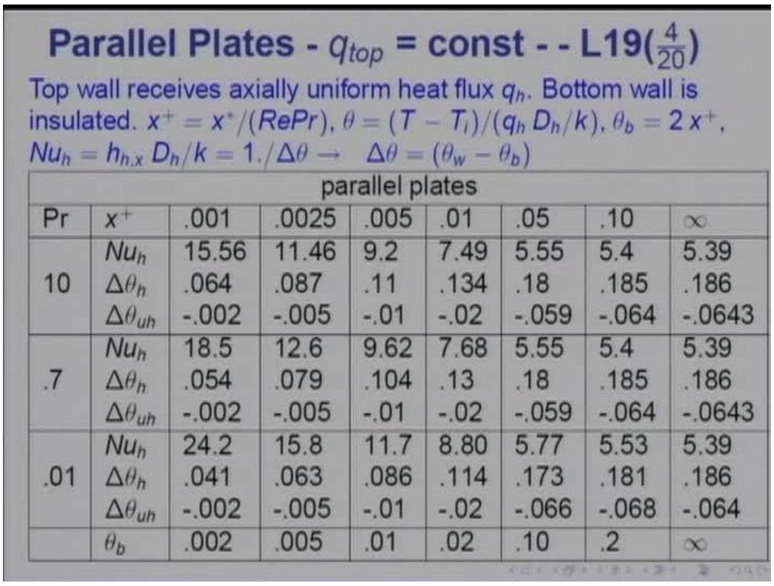

parallel plates								
Pr	x^+	.001	.0025	.005	.01	.05	.10	∞
10	Nu_h	15.56	11.46	9.2	7.49	5.55	5.4	5.39
	$\Delta\theta_h$	.064	.087	.11	.134	.18	.185	.186
	$\Delta\theta_{uh}$	-.002	-.005	-.01	-.02	-.059	-.064	-.0643
.7	Nu_h	18.5	12.6	9.62	7.68	5.55	5.4	5.39
	$\Delta\theta_h$	.054	.079	.104	.13	.18	.185	.186
	$\Delta\theta_{uh}$	-.002	-.005	-.01	-.02	-.059	-.064	-.0643
.01	Nu_h	24.2	15.8	11.7	8.80	5.77	5.53	5.39
	$\Delta\theta_h$	.041	.063	.086	.114	.173	.181	.186
	$\Delta\theta_{uh}$	-.002	-.005	-.01	-.02	-.066	-.068	-.064
	θ_b	.002	.005	.01	.02	.10	.2	∞

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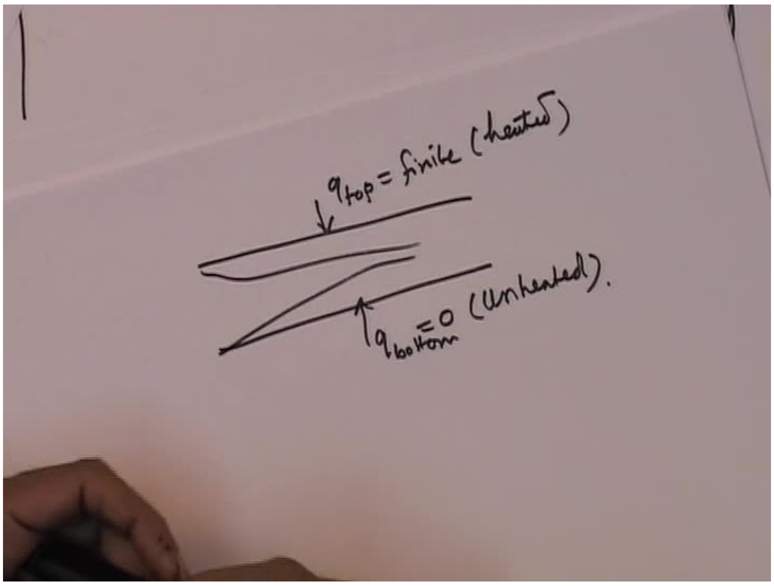

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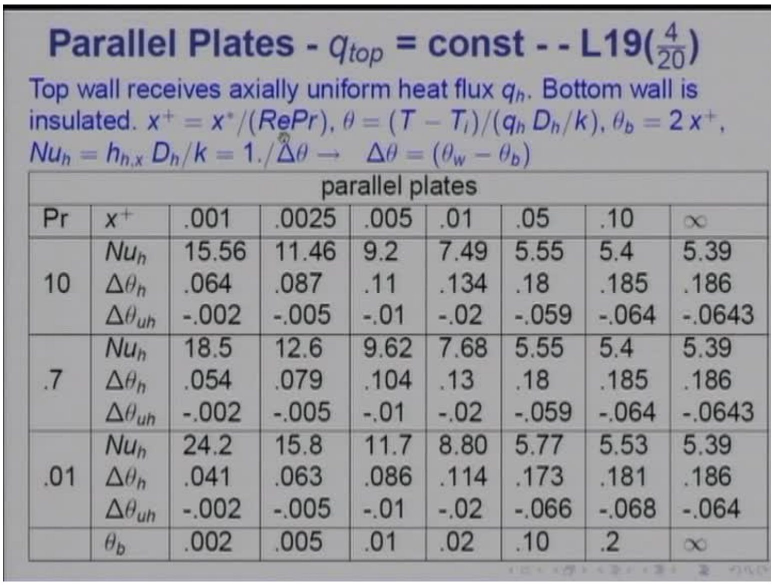

Parallel Plates - q_{top} = const - - L19($\frac{4}{20}$)

Top wall receives axially uniform heat flux q_h. Bottom wall is insulated. $x^+ = x^*/(RePr)$, $\theta = (T - T_i)/(q_h D_h/k)$, $\theta_b = 2x^+$, $Nu_h = h_{h,x} D_h/k = 1./\Delta\theta \rightarrow \quad \Delta\theta = (\theta_w - \theta_b)$

parallel plates								
Pr	x^+	.001	.0025	.005	.01	.05	.10	∞
10	Nu_h	15.56	11.46	9.2	7.49	5.55	5.4	5.39
	$\Delta\theta_h$	.064	.087	.11	.134	.18	.185	.186
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	$\Delta\theta_{uh}$	-.002	-.005	-.01	-.02	-.066	-.068	-.064
	θ_b	.002	.005	.01	.02	.10	.2	∞

So, the method is very cumbersome and therefore, what I am going to do is to present only the solution in order that you appreciate what the solutions under simultaneous development looks like. I am going to consider the case of flow between two parallel plates, q top is finite but q bottom is 0; this I will call heated side and this is unheated. The flow and temperature profiles are now simultaneously developing in this particular case. The x plus here is simply x divided by D h Reynolds Prandtl, theta is defined as temperature minus temperature in inlet divided by q on the hot side - I mean - q heated D h divided by k of course, the bulk temperature would vary linearly as - you will recall - 2 x plus. Therefore, the Nusselt number would be defined as h D h by k as 1 over delta theta where delta theta is T wall minus T bulk.

So, you will see that plotted for each Prandtl number there are three Prandtl numbers I have taken. You have Nu h on the heated side, theta wall on the heated side minus theta bulk and theta wall on the unheated side. I am showing here the solutions for x plus equal to 0.001, 0.0025, 0.005, 0.01, 0.05 and 0.1 and ultimately of course, at infinity the value is 5.39 for the Nusselt number and it is almost reached around let say 0.09 or you can say at about 0.1 at 0.7 Prandtl number again Nusselt number starts off from a very high value at close to the entrance and gradually goes on decreasing and again at about 0.1 you get very close to fully developed value but at 0.01 even at 0.1 the value is not very close to fully developed value, but it will be tend close to fully develop let say little longer at 0.12 at x plus equal to 0.1.

This can be deceptive, x plus equal to 0.1 at Prandtl number of 0.01 actually represents a shorter physical length when the Prandtl number is 10. So, as much as I say x plus equal to 0.1; it does not mean same physical length at low Prandtl number. At low Prandtl number the length is longer, the physical length is shorter then, it is for Prandtl number greater than 1 at 10 for example.

So, these are very instructive solutions, notice also that on the unheated side the wall temperature is less than the bulk value. As you can see in all cases that it is only on the heated side that the wall temperature exceeds the bulk temperature and the value of bulk temperature as evaluated from this is given right here (Refer Slide Time: 10:20).

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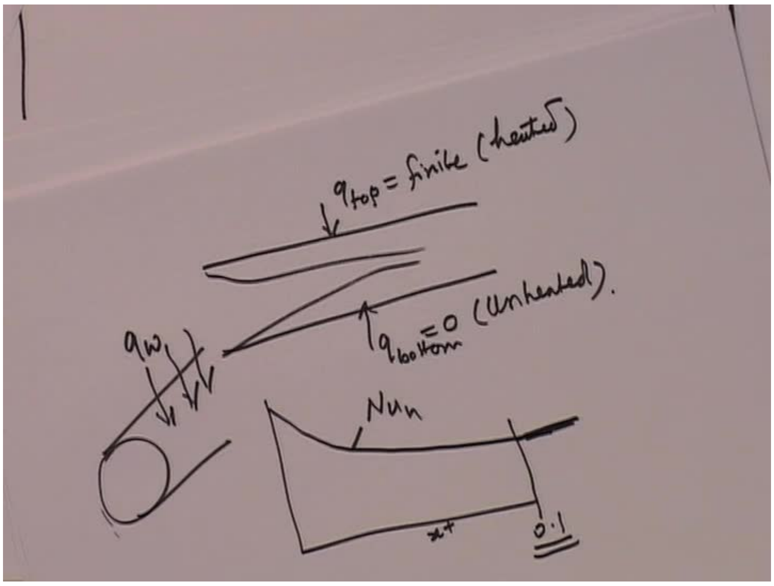

So, this is how the heat transfer coefficient would vary, on the heated side it will vary like that. This is Nu h versus x plus and it reaches 0.1 we said, when Nusselt number becomes constant with x that is, when we say fully develop heat transfer has been achieved.

So, similar thing is been done for circular tube - the cross section is a circular tube -and of course, it is uniform all around and also axially; q wall is constant axially as well as in circumferential direction. Let us see what happens when the flow and heat transfer develops simultaneous?

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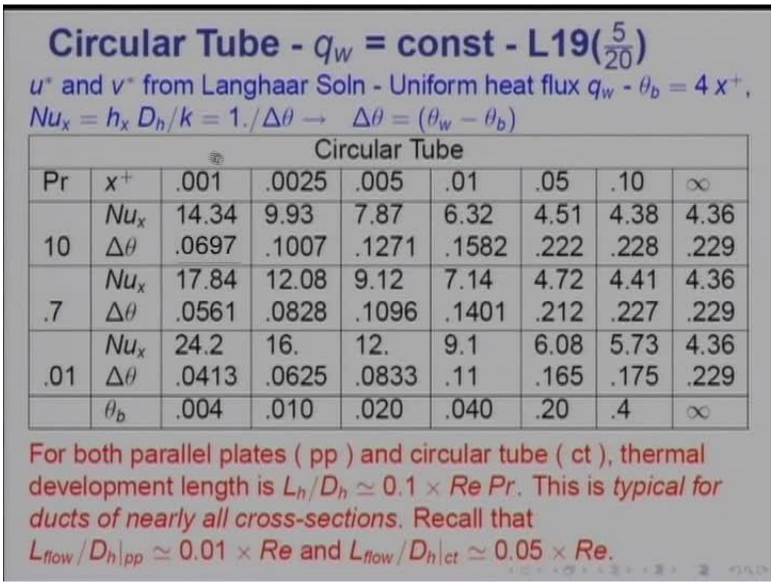

Again, as you will see in this case, theta bulk would vary as 4 x plus; u star and v star I have again taken from Langhaar solution to solve the temperature problem; Nusselt number is given by this and delta theta is theta wall minus theta bulk.

So, in this case for Prandtl equal to 10 and x plus equal to same values, you get very close to fully develop Nusselt number at x plus equal to 0.1. At 0.7, again very much so in fact it indicates that the development lengths are Prandtl number being very close to 1 it is almost the same, but at 0.01 again at x plus equal to 0.1 then, the Nusselt number is still higher and it would require larger x plus get to very close to fully developed value. These are the theta bulk and the wall temperatures are 0.697 to start with at Prandtl number of 10, 0.12, 0.15, 0.22 point a big apparent this should be 0.0697 I have made an error here and then, at the temperature difference goes on increasing but becomes progressively constant which results into constant Nusselt number in all Prandtl numbers. As you recall, in the fully developed state the Nusselt number is constant at 4.36.

So, for both parallel plates and circular tubes thermal development length is L h by D h approximately equal to 0.1 times Reynolds Prandtl. So obviously, when the Prandtl number is small L h by D h L h the physical length would be smaller then, when the Prandtl number is very large, let us say, about 10 this is the typical behavior for ducts of nearly all cross sections. You can easily take x plus equal to 0.1 as indicative of thermal development length, but for flow development length - you will recall - we had taken for parallel plates it was 0.1 times Reynolds and 0.05 times Reynolds for circular tube. So for thermal development and in the range of Prandtl number close to 1 as I said from 0.5 to 10 it is very good to approximate that as 0.1 times Reynolds Prandtl.

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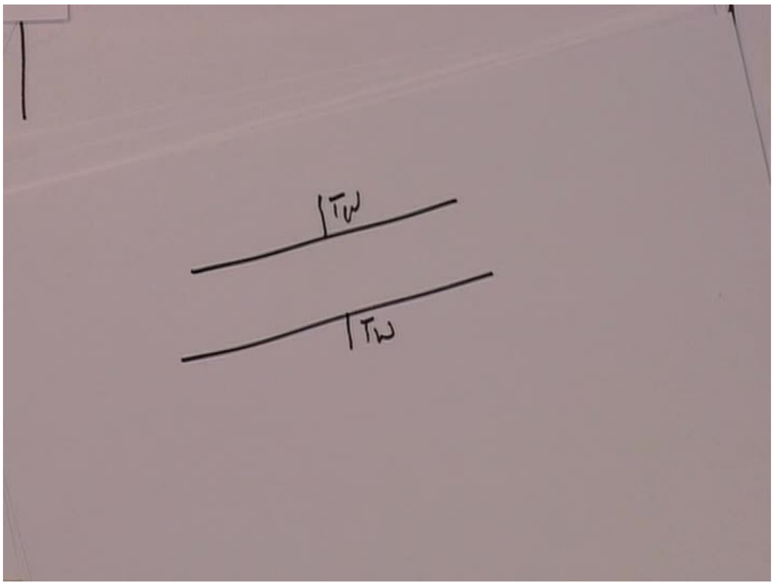

Now, I consider a case of both plates of parallel plates, here also T w and here also T w. So, we have case of uniform wall temperature and I am considering again Prandtl numbers is in the vicinity of 1, so we can see here 0.7 to 5.

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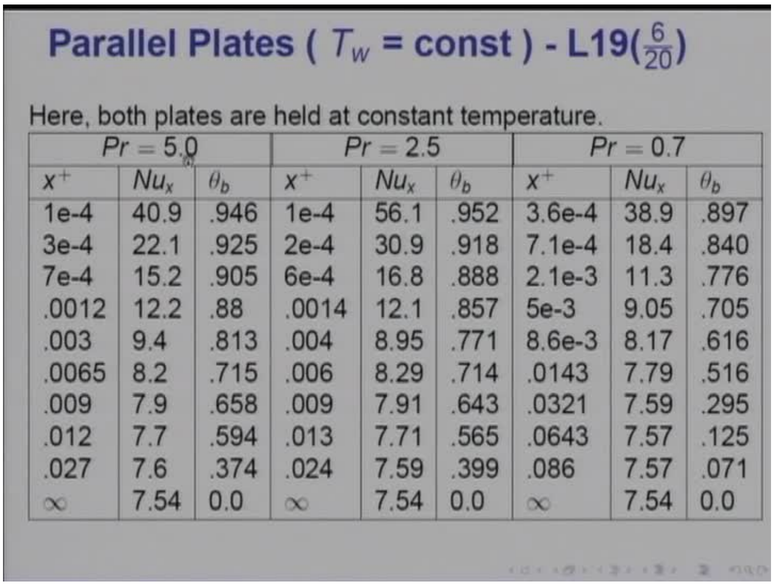

Parallel Plates (T_w = const) - L19($\frac{6}{20}$)

Here, both plates are held at constant temperature.

$Pr = 5.0$			$Pr = 2.5$			$Pr = 0.7$		
x^+	Nu_x	θ_b	x^+	Nu_x	θ_b	x^+	Nu_x	θ_b
1e-4	40.9	.946	1e-4	56.1	.952	3.6e-4	38.9	.897
3e-4	22.1	.925	2e-4	30.9	.918	7.1e-4	18.4	.840
7e-4	15.2	.905	6e-4	16.8	.888	2.1e-3	11.3	.776
.0012	12.2	.88	.0014	12.1	.857	5e-3	9.05	.705
.003	9.4	.813	.004	8.95	.771	8.6e-3	8.17	.616
.0065	8.2	.715	.006	8.29	.714	.0143	7.79	.516
.009	7.9	.658	.009	7.91	.643	.0321	7.59	.295
.012	7.7	.594	.013	7.71	.565	.0643	7.57	.125
.027	7.6	.374	.024	7.59	.399	.086	7.57	.071
∞	7.54	0.0	∞	7.54	0.0	∞	7.54	0.0

Again Langhaar’s method has been applied for constant wall temperature case and you will see, but both plates are heated now unlike the first problem in which only 1 plate was heated.

The plate temperatures are constant x plus equal to 1 raise minus 4 up to infinity Nusselt number goes from high of 40.9 at the close to inlet to 7.54. This is how the theta bulk varies theta bulk decreases to 0 x plus because theta bulk essentially is theta bulk minus t wall and therefore, that goes on reducing x plus is equal to this here for 2.5. You can see that nearly fully developed value is reached at about 0.027; you could even say 0.012, it will depends on how you define they are fully developed at point Prandtl equal to 0.7. Again, you get very good results from 0.012 onwards very close to fully developed value results at this value.

So, you can see that behavior of Nusselt number is very similar and thermal development length in such a case could be taken as let us say about 0.03 if you like or 0.02 is good enough to be close to 7.54.

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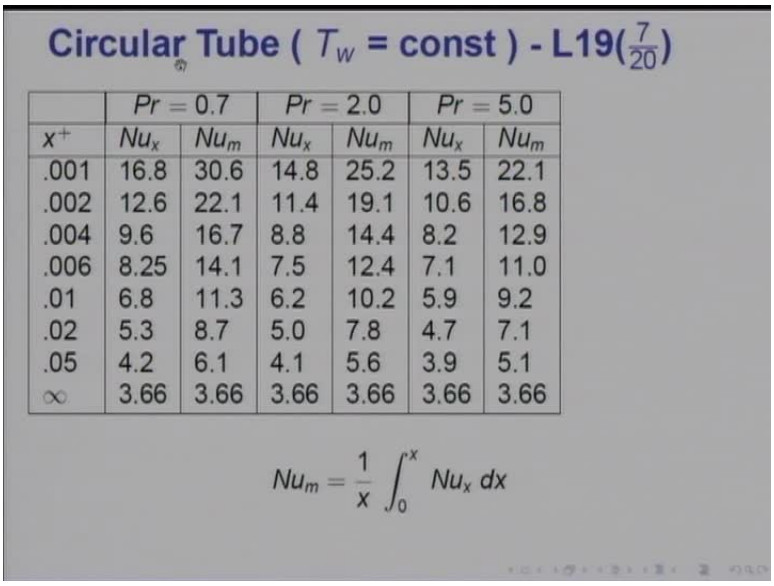

	$Pr = 0.7$		$Pr = 2.0$		$Pr = 5.0$	
x^+	Nu_x	Nu_m	Nu_x	Nu_m	Nu_x	Nu_m
.001	16.8	30.6	14.8	25.2	13.5	22.1
.002	12.6	22.1	11.4	19.1	10.6	16.8
.004	9.6	16.7	8.8	14.4	8.2	12.9
.006	8.25	14.1	7.5	12.4	7.1	11.0
.01	6.8	11.3	6.2	10.2	5.9	9.2
.02	5.3	8.7	5.0	7.8	4.7	7.1
.05	4.2	6.1	4.1	5.6	3.9	5.1
∞	3.66	3.66	3.66	3.66	3.66	3.66

$$Nu_m = \frac{1}{x}\int_0^x Nu_x\, dx$$

Similarly, we consider the case of T w equal to constant. Now, actually the circular tube as T w equal to constant and simultaneous development is being considered for three Prandtl numbers; Prandtl number for 0.7, 2 and 5. Again you will see that at 0.7 you get local Nusselt number as 16.8 and so on so forth to 3.66.

The mean Nusselt number goes on like that; the mean Nusselt number is defined as 1 over x 0 to x Nu x d x. Similarly at Prandtl 2, you get that at fully develop state. You get 3.66 which is in the unknown value, we had already computed that when we considered fully developed heat transfer Prandtl number 2 again is around at 0.05, you get 4.1 and 3.9.

So, in terms of local Nusselt numbers as the development length of 0.05 is a very good indicator of thermal development x plus equal to 0.05, it is very good for near unity Prandtl number fluids that for circular tube.

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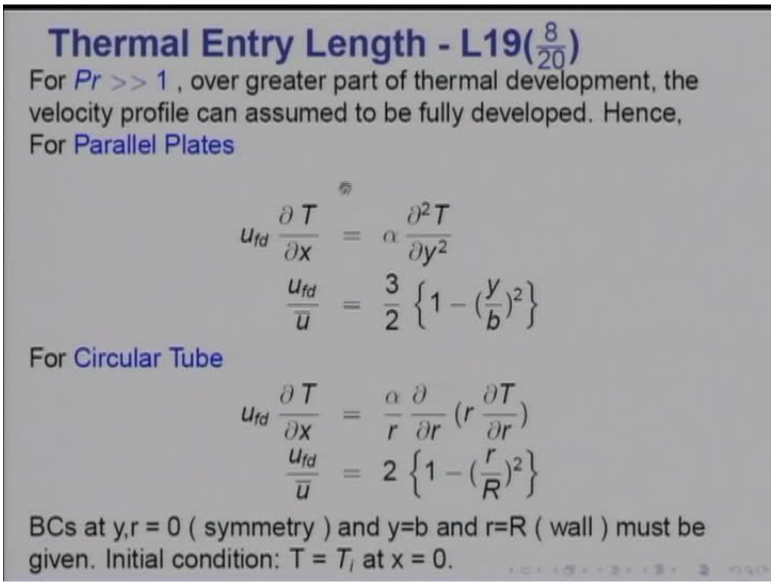

Now, we turn our attention to oils and Prandtl number very much greater than 1. As I said over greater part of the thermal development length the velocity profile can be assumed to be fully developed.

Hence, for parallel plates for example, it will be u f d dT by dx equal to alpha d 2 T by dy square and u f d by u bar square here will be 3 by 2 1 minus y by b square, where b is the half distance between the two plates. For a circular tube it will be u f d dT by dx will be equal to alpha divided by r d by bigger this is with d by dr into r dT by dr and u f d by u bar is equal to 2 into 1 minus r by r whole square.

The boundary conditions are at the symmetry plane that is y equal to 0 or r equal to 0 and y equal to b or y equal to R, you have the wall condition which must be specified with the inlet condition T equal T i at x equal to 0.

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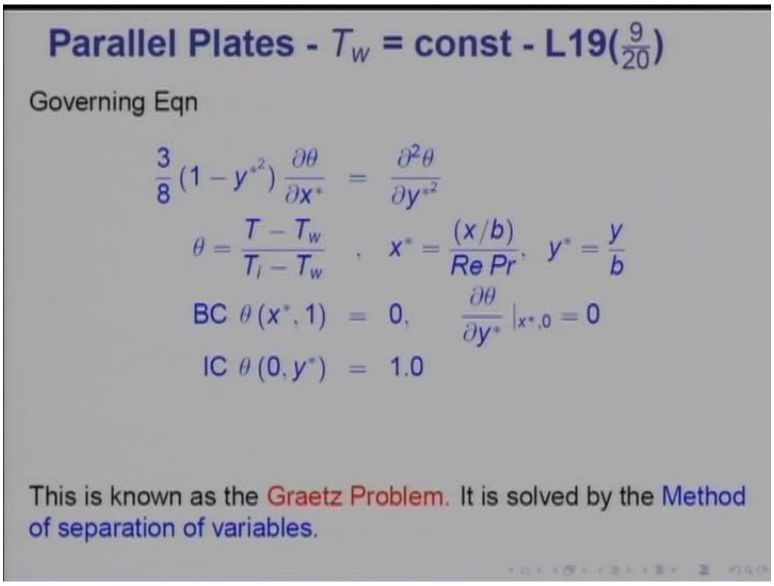

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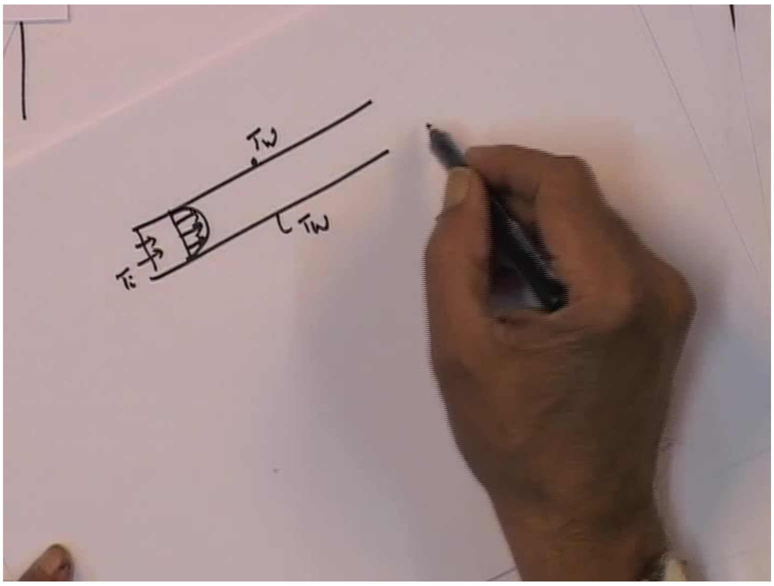

So, let us first of all take the case of flow between parallel plates and T wall is equal to constant flow between parallel plates and T wall is constant on both sides the fluid enters at T i but the velocity profile is already fully developed and it remains so throughout the length.

How do we obtain solution for this case? The governing equation would be if I substitute for u over u bar equal to 3 by 2 into 1 minus y by b square then, defining y star equal to y divided by b x star equal to x by b Reynolds Prandtl and T minus T w T i minus T w equal to theta then, the governing equation would simply be this and the boundary condition will be at the wall theta would be 0 because that is how we have defined theta d theta d y star would be 0 at the symmetric plane and on the inlet plane theta would be equal to 1

This problem with these boundary conditions is called the Graetz problem is very famous Graetz problem in heat transfer and it is solve by the method of separation of variables. Of course, now a days you can solve this problem by finite difference method quite easily but I am deliberately presenting here the method of separation of variables and indicating the solutions that are obtained mind you again all this is for Prandtl number very greater than 1 or oils.

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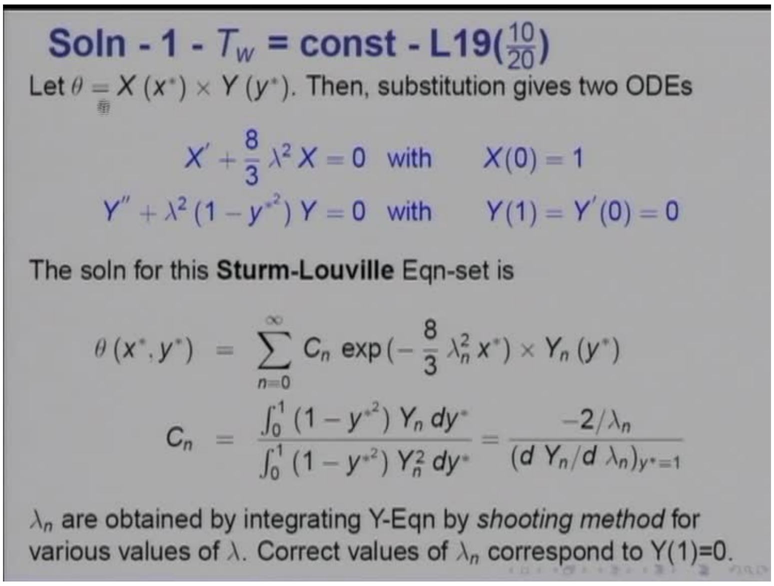

So, we say let theta be product of 2 functions X of x star and Y of y star then, if I substitute in this then, I would get two ODEs X dash plus 8 by 3 lambda square X equal to 0 with X 0 equal to 1 which is the inlet condition and Y double prime plus delta square 1 minus y star square Y equal to 0 with Y equal to 1 and Y dash equal to 0 is equal to 0 lambda square is the Eigen values.

Now, this solution is called the Sturm-Louville equation set and the solution to that is simply a product solution. For example, the first equation would have a straight forward solution x would be proportional to exponential of minus 8 by 3 lambda n square x star and multiplied by Y n star which is the solution to that equation.

For different values of n, you will have different values of constant of proportionality and the functions Y n. The C n values in this expression coefficient is evaluated from 0 to 1 1 minus y star square Y n dy star over that equal to minus 2 by lambda n dy n by d lambda n nu star equal to 1.

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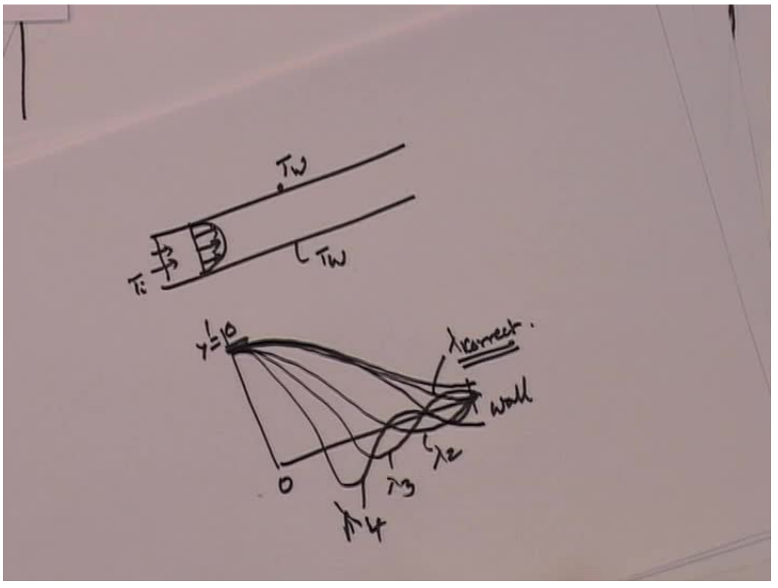

Now, how do we obtain lambda n's? That is simply by solving this equation by shooting method. So, you start at x is a symmetry and this is the wall, so you start with y dash equal to 0 as the known boundary condition, y itself can be anything and you choose different values of lambda. You may arrive at here and then, you may arrive at here and then, the correct value of lambda will be 1, this is lambda correct (Refer Slide Time: 21:15).

This will be the first value of lambda likewise; you go on changing the values of lambda. The next correct value will come out in that fashion, this is lambda 1, this is lambda 2 the third value will come out to be like so; the fourth correct value will come out to be so and so on and so forth. (Refer Slide Time: 21:39). So, this is lambda 3, this is lambda 4 and so on so forth.

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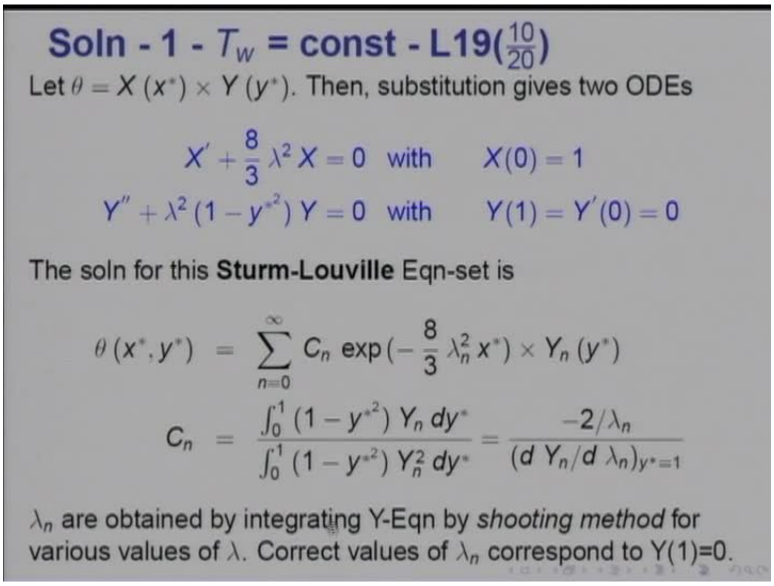

This is how we determine the lambda n. In all cases - I mean y star equal to 1 will turn out to be 0 which is the wall condition.

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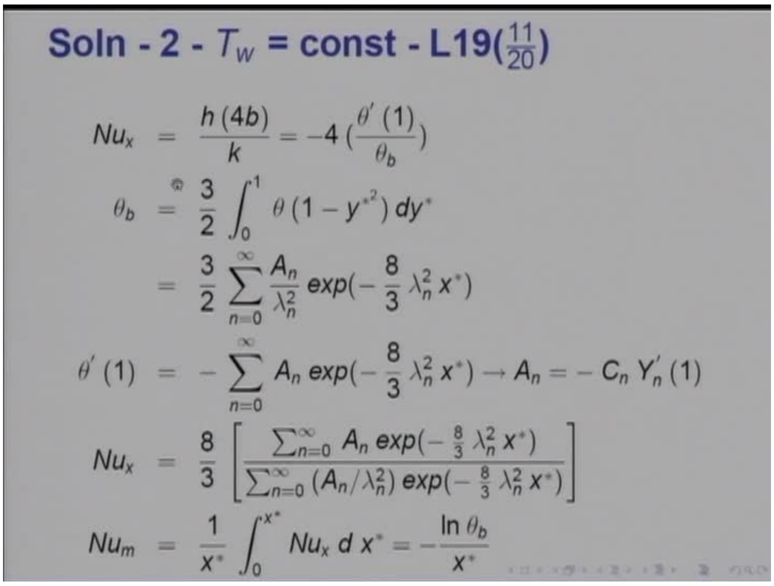

Once, you have determine lambda n's in this way, you can see Nusselt number itself will be h times hydraulic diameter because the plate widths is 2b; the distance between the plates is 2b, so hydraulic diameter is 4b and that would equal to minus 4 theta prime 1 divided by theta prime at the wall divided by theta bulk.

Theta bulk itself would be evaluated in this manner, which gives you 3 by 2 A n by lambda n square exponential of minus 8 by 3 lambda n square x plus. Theta prime will be given in this fashion, where A n is equal to minus C n Y n dash 1; the Y n dash 1 are noted down every time you get the solution at 1.

So, the slope of the y function for all correct solutions is noted down and that is stored into an array call Y n 1. So that is how you get the A n coefficient in this expression. Therefore, your Nusselt number expression looks like 8 by 3 into some of that divided by some of this (Refer Slide Time:23:21). The mean Nusselt number would be evaluated in this manner, which is simply minus ln theta bulk divided by x star.

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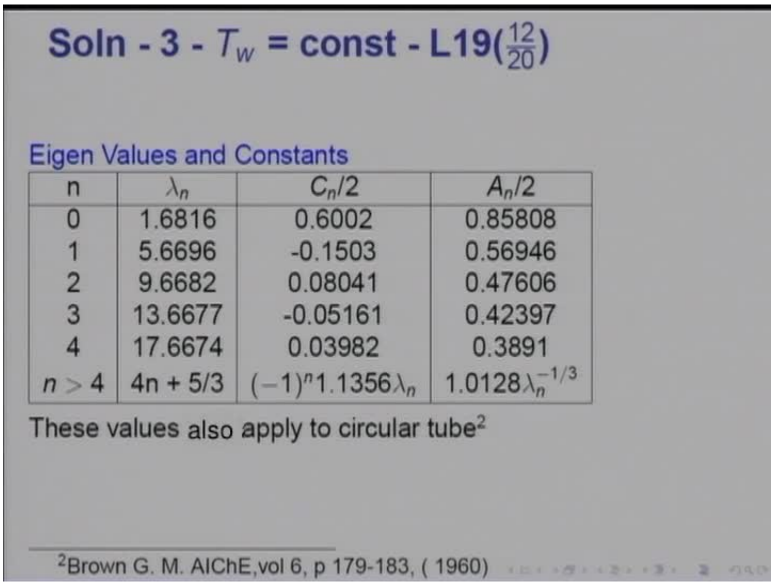

Here are the values of lambda n, C n and A n to be used in solving. So, these can be calculated once and for all for parallel plates 0, 1, 2, 3, 4 and for n greater than 4 they can be curve fitted in this manner. Incidentally, it so turns out that these coefficients also apply to the circular tube case; if I had circular tube with uniform wall temperature then the same coefficients would again apply to the circular tube case. This was shown by Brown in 1960.

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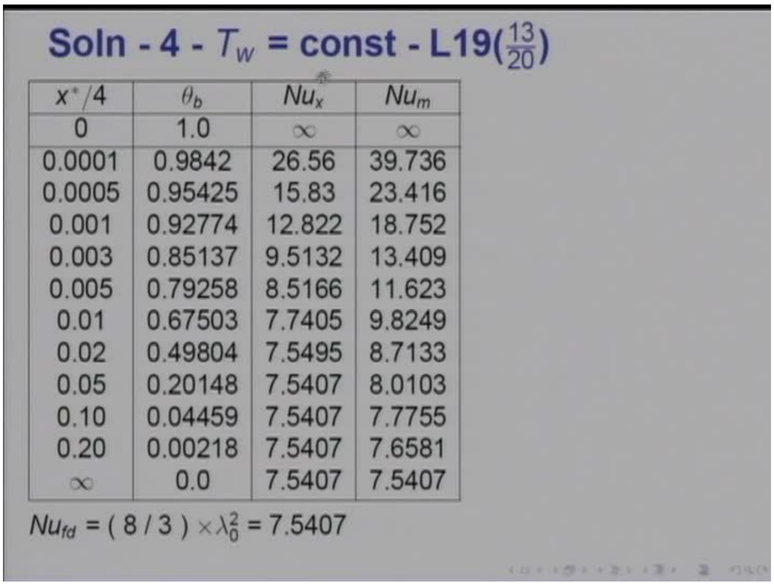

Soln - 4 - T_w = const - L19($\frac{13}{20}$)

$x^*/4$	θ_b	Nu_x	Nu_m
0	1.0	∞	∞
0.0001	0.9842	26.56	39.736
0.0005	0.95425	15.83	23.416
0.001	0.92774	12.822	18.752
0.003	0.85137	9.5132	13.409
0.005	0.79258	8.5166	11.623
0.01	0.67503	7.7405	9.8249
0.02	0.49804	7.5495	8.7133
0.05	0.20148	7.5407	8.0103
0.10	0.04459	7.5407	7.7755
0.20	0.00218	7.5407	7.6581
∞	0.0	7.5407	7.5407

$Nu_{fd} = (8/3) \times \lambda_0^2 = 7.5407$

Here are the computations of T wall equal to constant case. Here, I have got x star divided by 4 going from 0 to infinity. Theta bulk of course, at inlet would be 1 and it is reducing down to 0. Nusselt number would start with infinity of course, at x equal 0 but would begin to decline. You can see about by 0.02 the Nusselt number has almost become equal to the fully developed heat transfer and whereas, the mean Nusselt number takes much longer something like 0.3 or 0.4. Nu fully developed of course, can be simply evaluated from 8 by 3 into lambda naught square which is the lambda naught was given as 1.6816.

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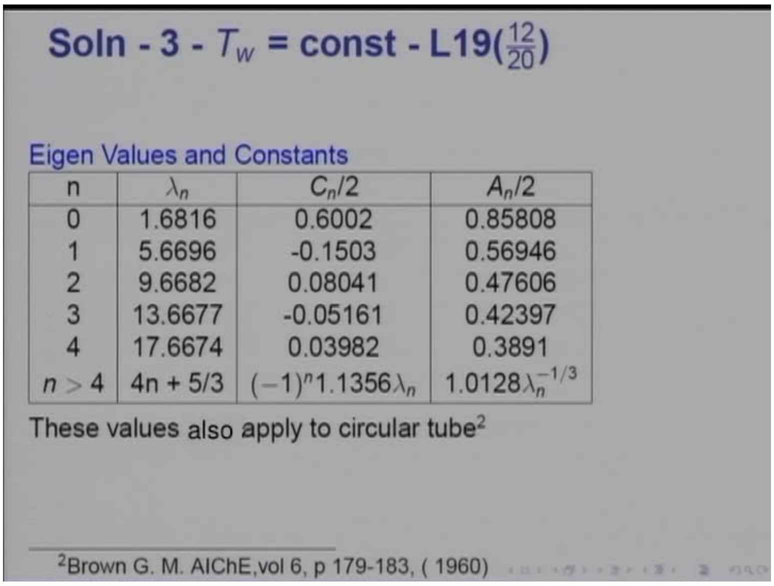

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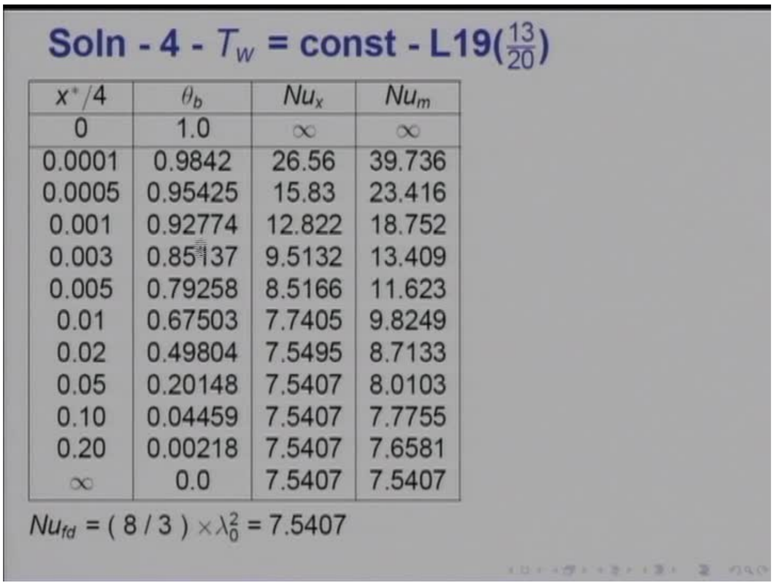

So, you can see that it will cannot to be 7.5407and that is what it is very well predicted here, because only first term is required for fully developed heat transfer.

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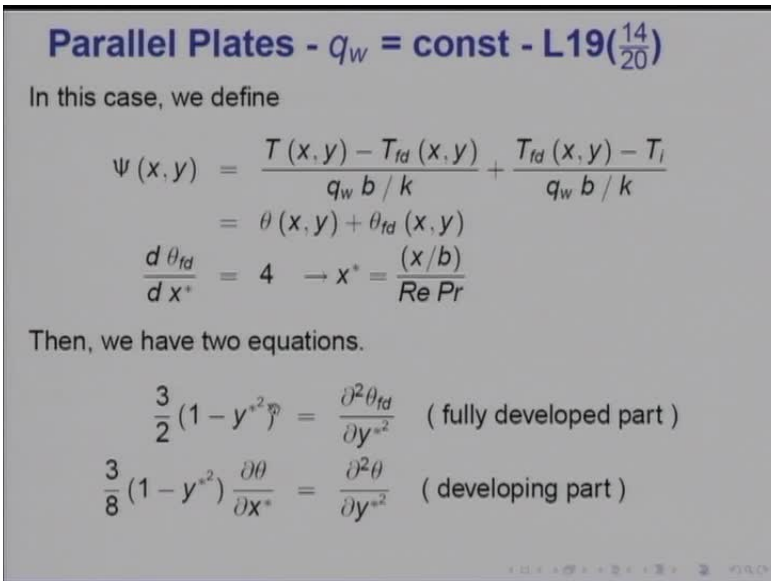

You can see Nusselt number does falls even in thermal entrance length problem of this type, where velocity has been assumed to be fully develop and the solution is applicable to oils, where Prandtl number is very large.

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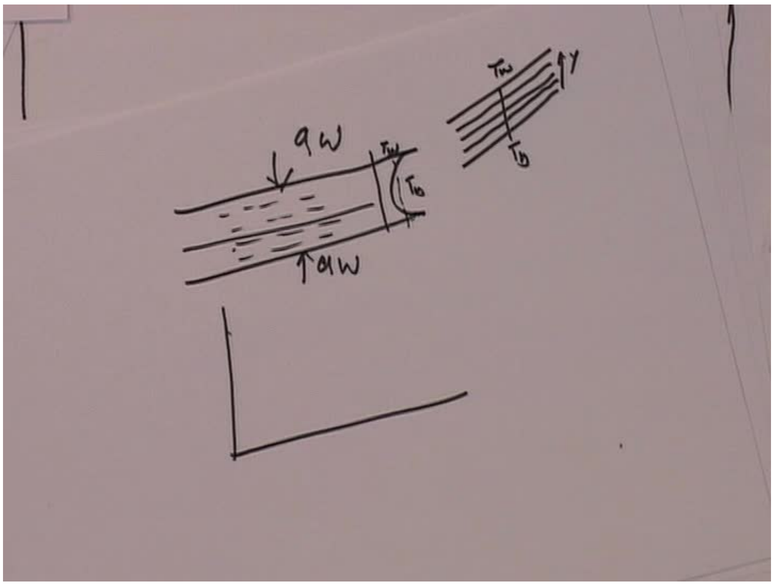

We now take the case of constant wall heat flux. When q w is constant on both the plates, ultimately the solution would become like this under the fully develop heat transfer the solution would be T wall there and there would be T bulk (Refer Slide Time: 25:44).

Although T wall and T bulk will increase with x, the difference between T walls minus T bulk would remain constant in the fully developed, whereas in the earlier cases, T wall and T bulk will increase at different rates and therefore at individual points also. Individual values of y T wall and T bulk will change actually at different rates, but once you reach the fully develop then, for all values of y the temperatures would increase linearly. Therefore, the solution can be split up in this manner as I shown.

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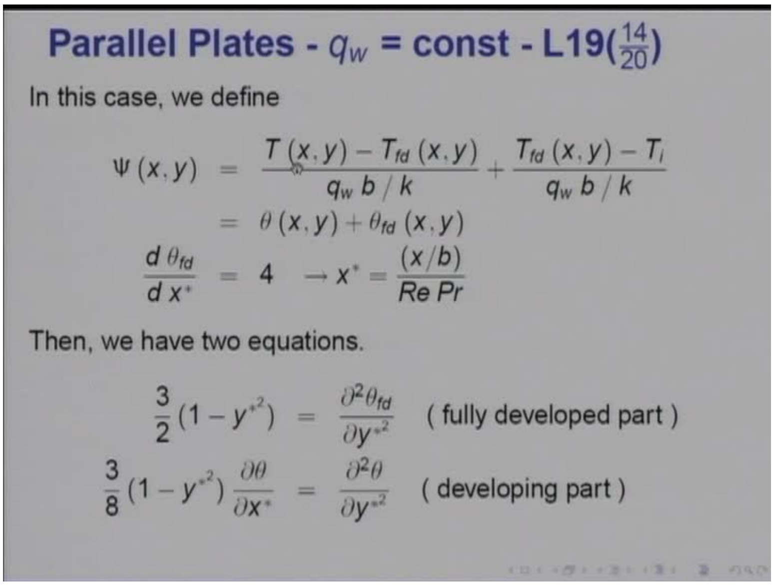

Let us say in this case, the dimensionless temperature is call psi instead of theta and that would be T x y minus T fully develop q wall minus q b by k. That is, in the developing part length and then onwards it is fully developed.

So, I can split it up as - but of course, as I said I do not know what T fd is but will discover that in a minute. So, this is taken as t theta x y plus theta fully develop x y, because, q wall equal to constant in given temperature has been normalized with respect to q wall b by k, as you can see here. In the fully develop path theta fd by theta x star would be 4, as I indicated here. All of them would be varying linearly and that would be exactly equal to dT bulk by dx and that is equal to x by b Reynolds Prandtl x star and theta fd by dx star would be 4.

We have 2 equations, this would be the full equation for the fully develop part 3 by 2 1 minus y star having substituted for theta fd by dx. This will be, I have substituted 4 here, so 3 by 8 into 4 is 3 by 2 d 2 theta fd by dy star square (Refer Slide Time: 27:52). This is the fully developed part of the solution and 3 by 8 1 minus y star square d theta by dx equal to d 2 theta by dy square is the developing part. This equation is very similar to what we had in the constant wall temperature case.

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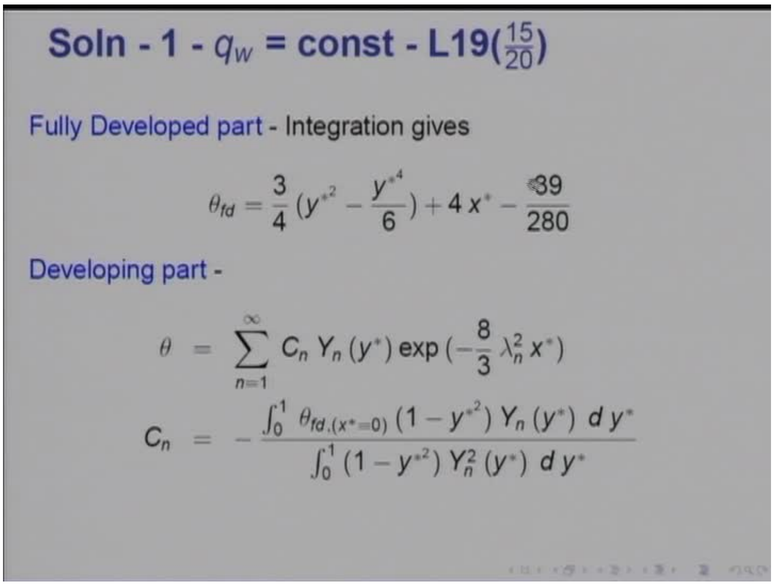

For the fully developed parts straight forward integration gives theta fd equal to 3 by 4 into a function of y plus 4 x star minus 39 by 280 - a very straightforward integration. For the developing part, it will be very similar to what we had. For the constant wall temperature, the function of coefficients C n Y n y star into a function of y and this would be a function of x. C n would be determine again in this fashion where theta fd at x star equal to 0 that is, putting 4 x star equal to 0 here; I will have a function of y into 1 minus y star y n y star dy star into all that. Y n distribution with y is already known, so I can determine C n.

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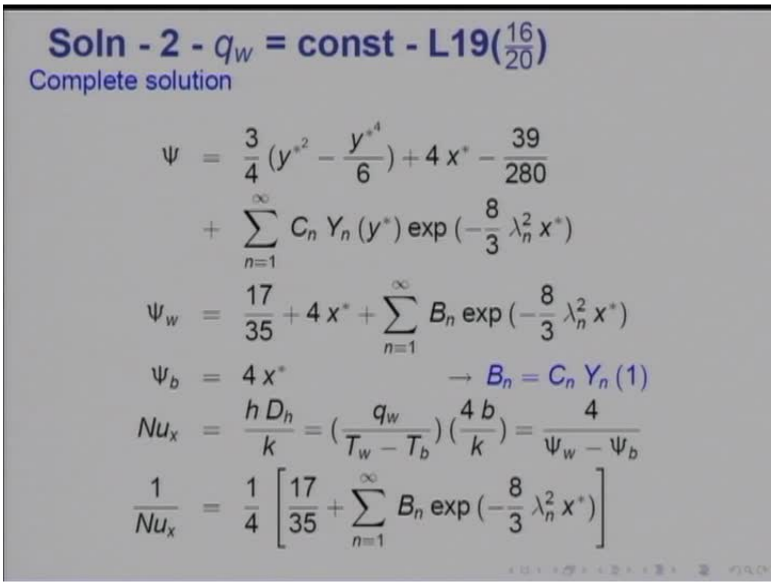

So the complete solution will look like this, 3 by 4 y star square minus y 4 by 6 plus 4 x star minus 39 by 280 plus C n Y n, which is the developing part. If I substitute y equal to 1, I will get psi wall which will vary in this fashion and if I carry out the integration to evaluate bulk temperature, I will get psi bulk but q wall is already constant and psi bulk will be 4 x star, but you can also verify that by integrating this with respect in the usual manner. B n here is C n Y n 1 then, the Nusselt number would become this 4 divide by psi wall minus psi bulk and 1 over Nusselt number would become 1 over 4 into 17 by 35 all this. So this is the complete solution to the constant wall heat flux problem.

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Soln - 3 - q_w = const - L19($\frac{17}{20}$)

Eigen values			Nu values		
n	λ_n	$-B_n$	$x^*/4$	Nu_x	Nu_m
1	4.2872	0.2222	0.0001	32.153	48.11
2	8.3037	0.07253	0.0005	19.113	28.33
3	12.3106	0.03737	0.001	15.427	22.65
4	16.3145	0.02328	0.005	9.9878	13.89
5	20.3171	0.01611	0.01	8.8031	11.58
6	24.319	0.01192	0.03	8.2458	9.446
7	28.3203	0.00923	0.05	8.2355	8.963
8	32.3214	0.0074	0.10	8.2353	8.599
9	36.3223	0.00609	0.20	8.2353	8.417
10	40.3231	0.00511	∞	8.2353	8.2353

For $n > 10$, $\lambda_n = 4n + 1/3$ and $-B_n = 2.401006\,\lambda_n^{-5/3}$

Here are this Eigen values and Eigen constants for the constant wall heat flux case. These are lambda n values; these are minus B n values they can be correlated for n greater than 10 by this expression. The Nusselt number value themselves looks like this x stars by 4 local Nusselt number varies from as I have 32 going down to 8.235.

You can see that the under constant wall heat flux case, the thermal development length for very high Prandtl number fluids is about 0.05; x star by 4 equal to 0.05 or x star equal to 0.2 mean Nusselt number varies in this fashions. This value we have already noted before that for parallel plates with constant wall heat flux on both sides it will be 8.235.

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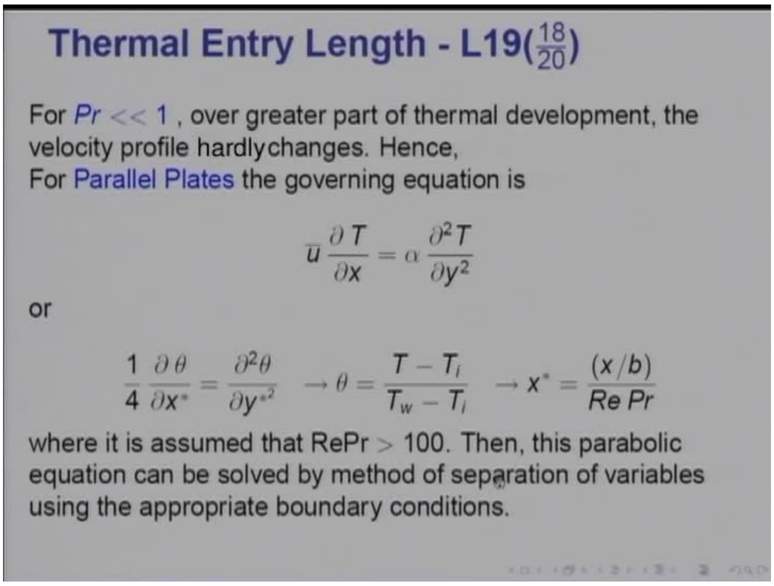

We now take up the final case of Prandtl number very much less than 1 that is liquid metal. Then, in this case the entrance region the velocity profile can be taken as equal to u bar and therefore, it will be u bar dT by dx equal to alpha d 2 T by dy square or 1 over 4 d theta by dx square d 2 theta by dy star square and these are the definition.

You will recall this is nothing but the heat conduction equation or unsteady heat conduction equation, if you replace x star by time. So, solutions to these can be obtained by method of separation of variables.

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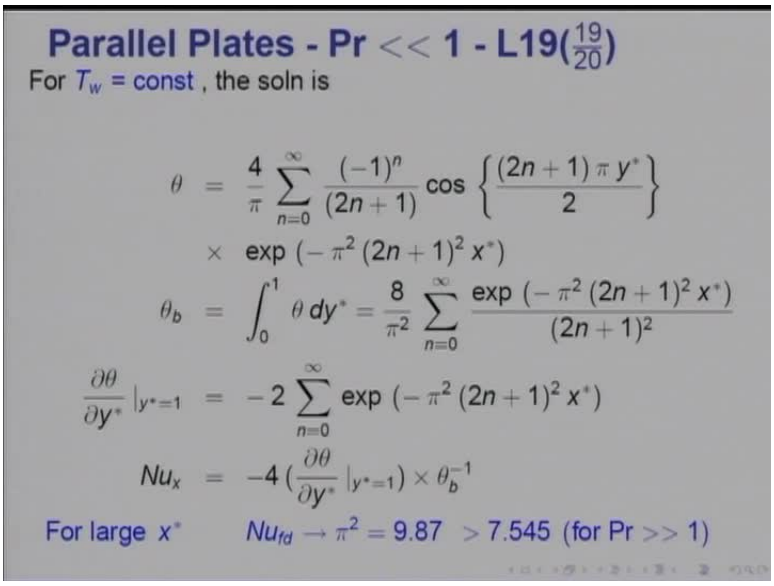

So, I will not develop that solution; it is a fairly well known solution and the solution is given for T wall equal to constant, the solution will turn out to be this. So, theta bulk is calculated in this fashion; d theta by dy star is the wall heat flux is calculated in this fashion and then, Nusselt number is calculated in this fashion. So, the evaluation of the series is what is required to obtain the Nusselt number.

For large x star Nu fully developed turns out to be equal to pi square and equal to 9.87 for liquid metals. For Prandtl number very much greater than 1 it was 7.545 and the fully develop Nusselt number here is 9.87.

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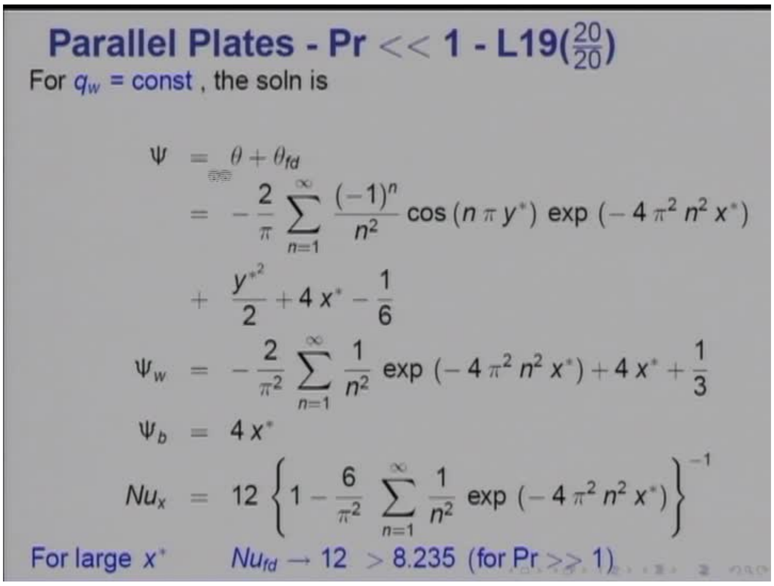

For constant wall heat flux case, again you can develop a solution which is psi equal to theta plus theta fd in this manner; this is the developing part of the solution and this is the fully develop part of the solution. You can evaluate psi wall, psi bulk is this and therefore, Nu x can be evaluated in this fashion.

Again, for larger x star Nu fd now turns to be 12, which is much greater than 8.235 for Prandtl number very much greater than 1. With this, I conclude everything on laminar developing heat transfer. As I said, for Prandtl numbers close to 1 - let us say - between Prandtl number ranges 0.5 to 10 one must solve the velocity and the temperature equation simultaneously. It is nowadays the best done by using CFD codes or you can write your own finite different course is very easy to write for this particular class of problems.

When Prandtl number is very much greater than 1 then, you can use fully develop velocity profiles which are available to you on our previous lectures. For Prandtl number very less than 1, you can easily take u equal to u bar and the problem becomes essentially that of heat conduction. So, it is a very straight forward to evaluate for Prandtl number very much less than 1.