Convective Heat and Mass Transfer
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Module No. # 01
Lecture No. # 18
Fully–Developed Laminar Flow Heat Transfer–2

In the previous lecture, we considered the fully developed heat transfer in circular tube or flow between parallel plates under variety of boundary conditions. Today, our interest is to move on to Non-circular Ducts.

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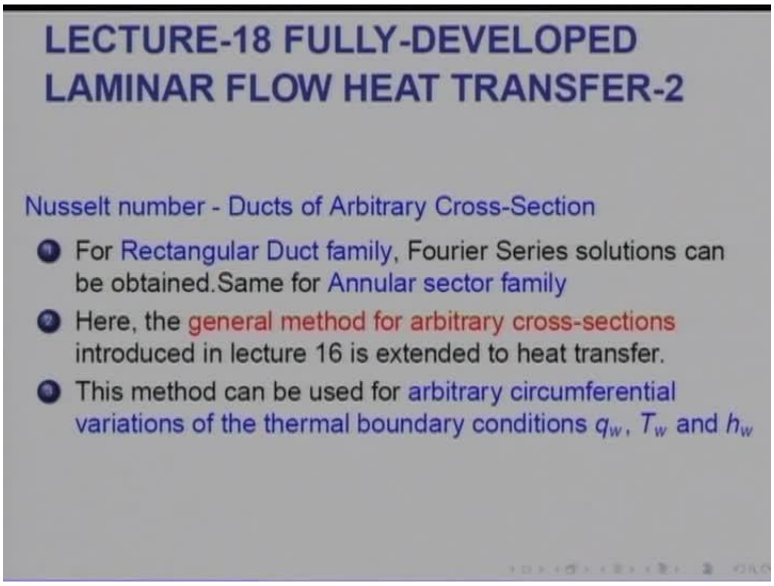

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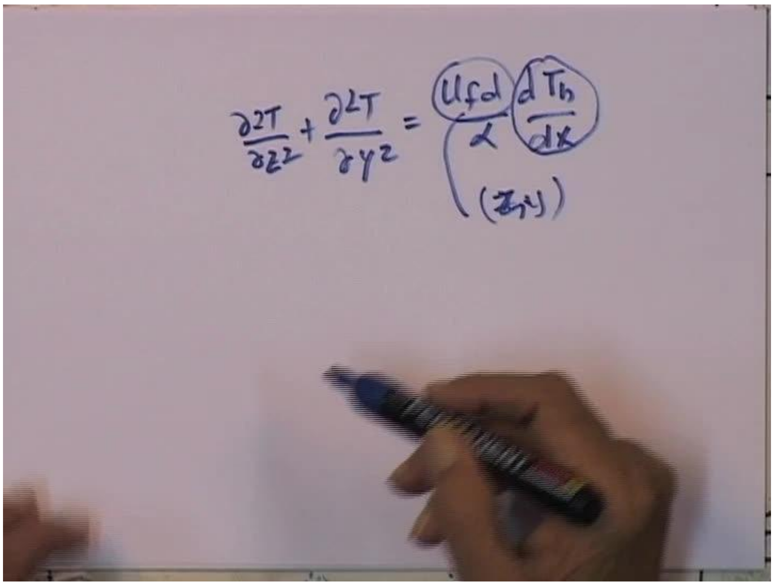

Non-circular ducts can be of regular shape such as rectangular duct or an annular sector duct, as we saw in the case of fully developed flow but, the ducts of this type can be solved exactly by the methods that we described there, because you get a Poisson equation of the type d 2 T d z square plus d 2 T d y square equal to u fully developed divided by alpha into d T bulk by d x.

The u fully developed for a non-circular duct, we have already obtained as a function of z and y; so, when you substitute for d T bulk by d x, you get a Poisson equation with a right hand side which is a function of x and y. You can use Fourier series to solve such problems.

The interest today is however very similar to what we saw in the lecture on fully developed flow in non-circular ducts. There we presented a method in which the method could be applied to ducts of arbitrary cross sections.

We are going to extend that method to include heat transfer. The whole purpose is to show the duct method can be applied also to heat transfer and with a very special provision and that is the method can be applied to any arbitrary variation of thermal boundary condition along the circumference.

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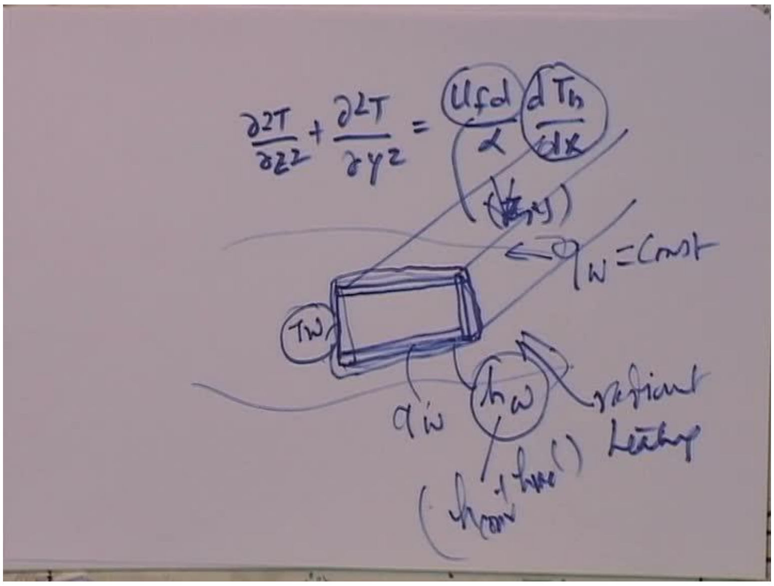

For example, if I have a rectangular duct I may have T w which is varying over the periphery. Although axially, the heat flux is constant - q w is constant - in the axial direction but circumferentially, T wall may vary or even q wall may vary or when we have a convective boundary condition even the heat transfer coefficient h w on the outside of the duct can vary.

So, we have three possibilities of T w varying, q w varying or h w varying. T w varying situation arises principally when you have a situation that a certain side of the duct is made of one material where the other side is made of another material that is one possibility. q w variation can arise because, you have subjected due to radiant heating which can come from one side. Likewise, heat transfer coefficient variation can also occur if there was a flow over this duct in which case, the heat transfer coefficient would vary along the periphery. including there can be This can be purely convective heat transfer or it can be h convection plus h radiation variation. So, both types are possible and effectively you have h w.

So, we want to track quite an extensive ground today in the sense that we should be able to have a method which can be applied to ducts of arbitrary cross section and ducts that have arbitrary circumferential variation of the thermal boundary condition.

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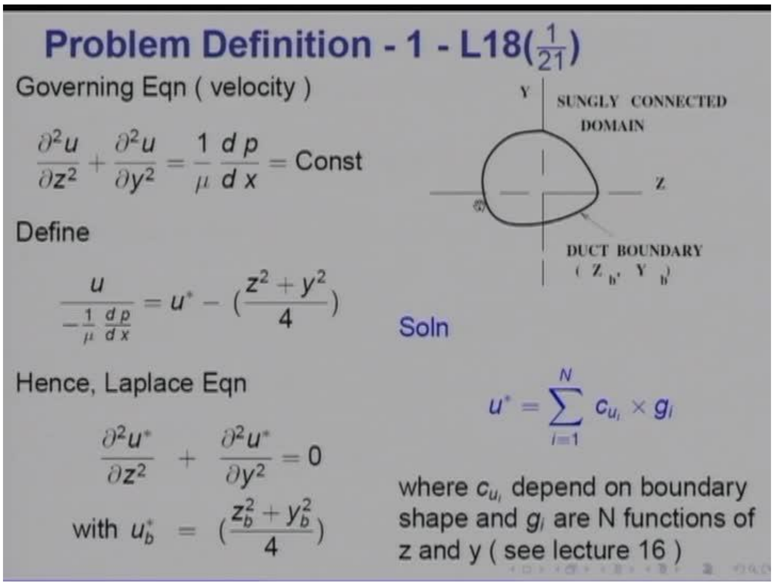

Just to recall, let us look at how we solve the velocity problem for a duct of non-circular cross section. This was done in lecture 16 - as you will recall - where we had the Poisson equation with the pressure gradient constant on the right hand side and we define a velocity u by that 1 over mu d p d x equal to U star minus z square plus y square by 4.

Therefore, the Laplace equation turned out to be d 2 u star d z square plus d 2 u star d y square equal to 0 and u star b the boundary value was equal to - where u is itself equal to 0 - z square z b square plus y b square divided by 4.

The solution that we had shown is given by this expression that u star is equal to i equal to 1 to N c i g i but, now I am calling it as c u i to indicate that these coefficients where for the velocity problem and g i functions were also given in lecture 16. N represents the number of boundary points you have chosen on the duct boundary.

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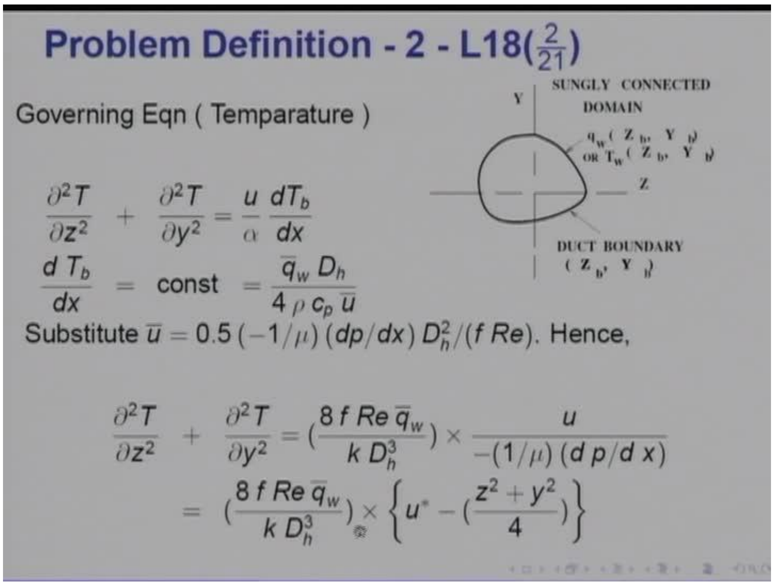

So, we will keep this in mind that this is how we had solve the velocity problem. We now turn to solving the heat transfer problem. Let us say, again I have shown the duct of arbitrary cross section with duct boundary coordinate z b y b known.

Again, it is also a singly connected domain as before, but the q wall or T wall or even heat transfer coefficient along the boundary may vary. The governing equation will be d 2 T d z square plus d 2 T d y square u fully developed divided by alpha - the thermal diffusivity - into d T bulk by d x. By overall heat balance, you will see d T bulk by d x is a constant given by q wall bar D h by 4 rho u bar c p and q wall bar is the average constant heat flux along the actual distance.

From the definition of friction factor multiplied by Reynolds number, we can readily represent u bar as 0.5 into 1 over mu d p d x d x square f Re and the negative here implies that the d p d x is negative so that u bar is positive. Therefore, if I substitute for d T bulk by d x and u bar then, I would get d 2 T by d z square plus d 2 T by d y square equal to 8 f Re q wall bar by k d h cube k being the thermal conductivity D h is the hydraulic diameter multiplied by u over minus 1 over mu d p d x.

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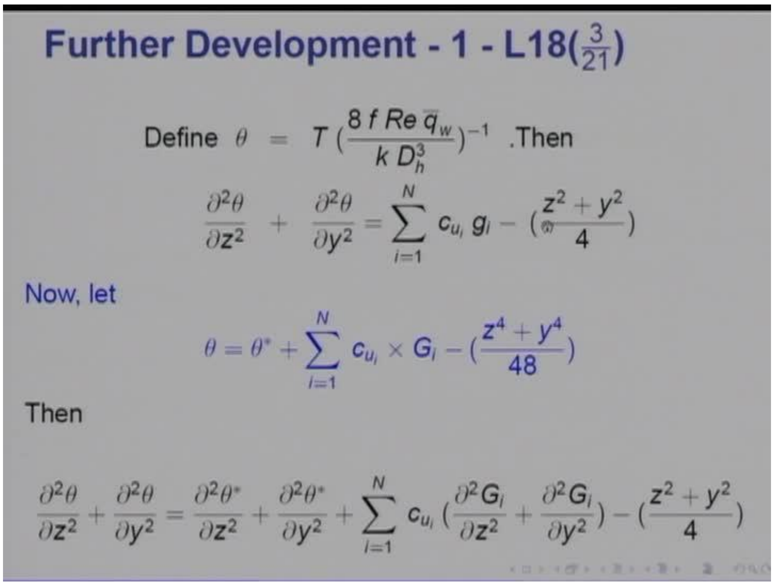

This quantity, we defined as u star minus z square plus y square by 4 with this as a multiply. I take this to the left hand side and then define theta equal to T into this quantity raise to minus 1. Then, the equation would simply read as d 2 3 d z square plus d 2 theta by d y square equal to c u i g i minus z square plus y square by 4.

Here onwards, the treatment would get somewhat complicated, so please be attentive in following the steps. I am now going to say, let theta - the temperature - be equal to some theta star plus sigma i equal to 1 to N c ui - the velocity coefficients - multiplied by another function called G i minus z 4 plus y 4 divided by 48, this is a postulate.

Then, you will see that substituting this into this equation would give me d 2 theta by d z square plus d 2 theta by d y square on the left hand side equal to the same thing in theta star plus c ui into second derivative of G i functions with respect to z plus second derivative of G i functions y square in minus z square plus y square by 4.

Now suppose, I say that this quantity the second derivative of G i with respect to z and y the sum of them is equal to small g i then, you will readily see that this quantity d 2 theta star by d z square plus d 2 theta star by d y star d y square would turn out to be 0, because this is exactly equal to c ui gi minus z square plus y square by 4.

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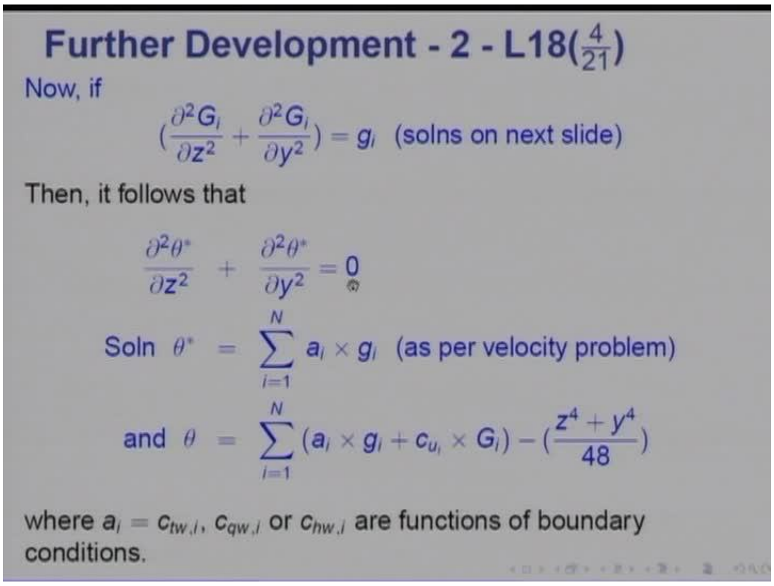

So, what I am going to do now is to substitute this equal to this or postulate that these functions G i are exactly equal to g i. So, if d 2 G i by d z square plus d 2 G i by d y square equal to g i then for each G i, I can generate a function g i and the solutions of that are given on the next slide.

The consequence is that as I said in the previous slide it would mean d 2 theta star by d z square plus d 2 theta star by d y star equal to 0. Again, I have a Laplace equation and for the Laplace equation as you recall, the solutions of x plus i y raise to n are all solutions of the Laplace equation for n varying from 0, 1, 2, 3, 4 up to anything. We have taken n equal to 8 in the velocity problem; we will stick to that and let us see what happens.

So, the solution would be As before as in the velocity problem, the solution now would be a i g i and theta would be a i g i plus c ui G i minus z square plus y square by 48. That is, if I substitute for theta star equal to a i small g i the solution to Laplace equation, we had shown is a i g i. So, I have substituted that for theta star, this term is theta star and these are the additional functions with the velocity coefficients c ui G i minus z to the 4 y to the 4 divided by 48.

Now, of course, the coefficient a i - we will show as you move go along - we will be different when the boundary condition for T w is given and therefore, those coefficients I have called c tw,i. If the q wall varies circumferentially, the coefficient will be called c w i and if heat transfer coefficient varies then, the functions would be called c hw i. So the a i here would could take any of these three depending on the boundary conditions that we have, so this is our solution to the temperature equation. Let us see how we can proceed further to develop Nusselt number, but before I do that here are the functions G i.

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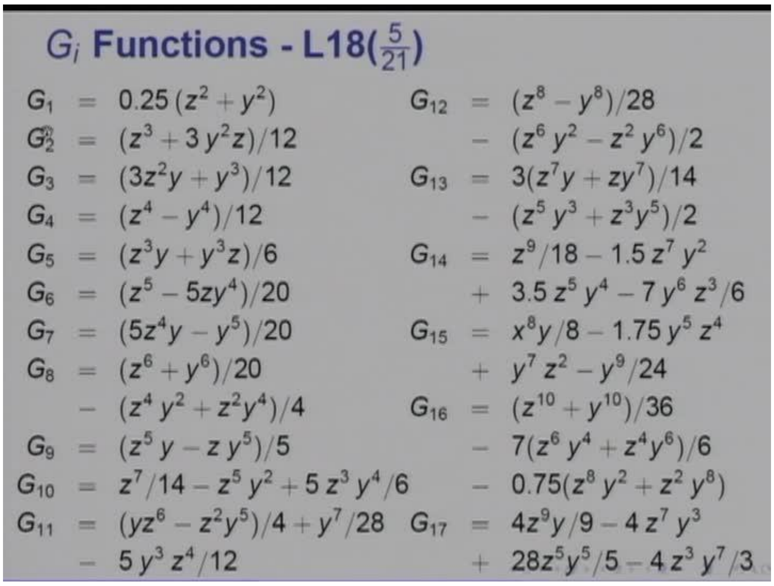

So, G 1 is for N equal to 0, G 2 and G 3 are for N equal to 1, G 4 and G 5 are for N equal to 2, G 6 and G 7 are for N equal to 3, G 8 and G 9 are for N equal to 5 so on and so forth. You could verify very well with the small g functions that I have.

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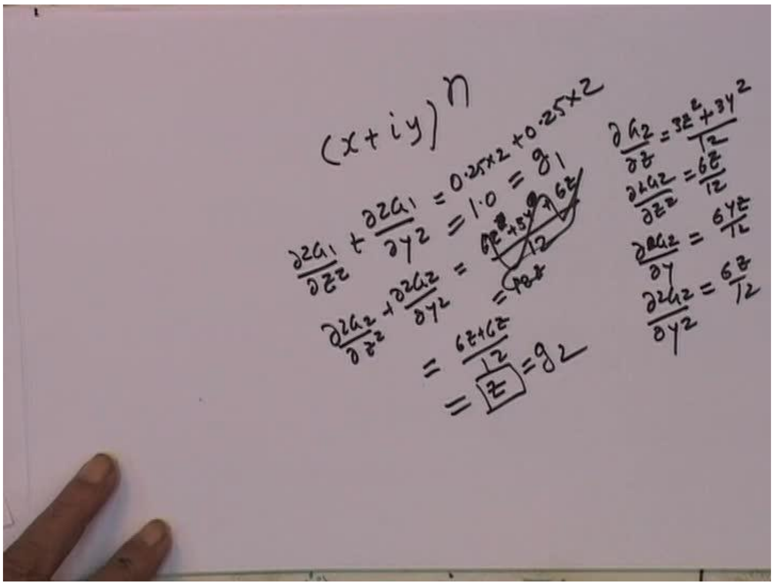

Basically in each case, if you wanted to see d 2 G 1 by d z square plus d 2 G 1 by d y square will give me essentially 0.25 into 2 plus 0.25 into 2 equal to 1 and that was precisely what g 1 was, if you recall.

Likewise, if I do d 2 G by g 2 by d z square plus d 2 G 2 by d y square then, you will see this will become 6 z 6 z square plus 3 y square plus 6 z plus divided by 12 and no sorry this would be this (Refer Slide Time:15:17). So, let us say d G 2 by d z will be 3 z square plus 3 y square by 12 and d 2 G 2 by d z square will be 6 z divided by 12. Similarly, d G 2 by d y would be 6 y z by 12 and d 2 G 2 by d y square will be 6 z by 12. Therefore, this quantity would turn out to be 6 z plus 6 z by 12 equal to z that is precisely was g 2, if you recall from our lecture number 16. Likewise, you can check out all of them and these are the last G 16 and G 17 for N equal to 8.

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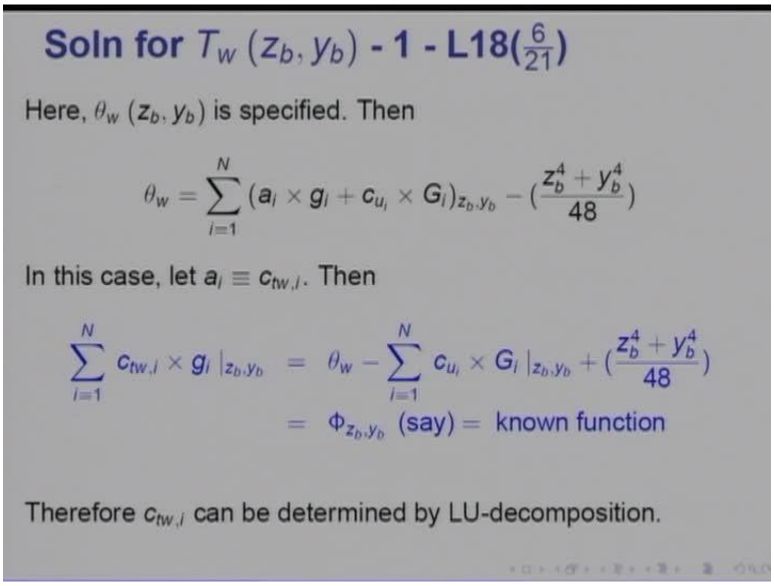

Now,, let us begin to develop solution for t w varies arbitrarily along this duct periphery. Here, theta w z b y b is specified then, you will see from the solution that we have got; we can get theta w equal to all these functions evaluated at z b y b - right hand side evaluated at z b y b - and that is what I have written here. So, you would get a i g i multiplied by c ui G i z b y b minus z b square plus y b square by 48.

As I said before, in this case a i will be taken as c tw,i then, you will see that c tw,i g i equal to 1 to N at z b y b would be equal to theta wall minus c ui into G i at z b y b and plus z b 4 plus y b 4 by 48. The entire right hand side is now known at z b y b, because theta wall has been specified. G functions can be evaluated for z b y b and c ui, the coefficients from the velocity problem are also already known and therefore, the entire right hand side can now be specified.

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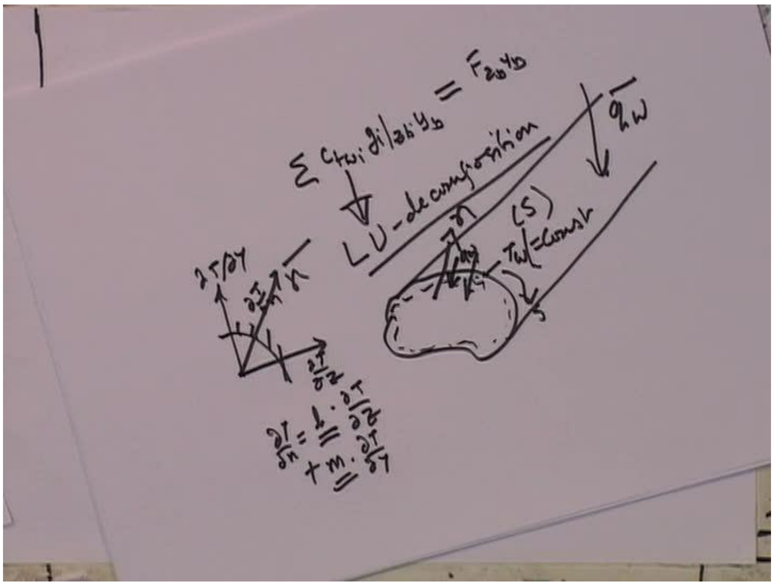

Now, essentially, you again have a problem very similar to that of the velocity problem c twi g i at z b y b is equal to some function of z b y b and the task is to determine c twi which we can do by L U decomposition, as before next. In this equation then we determine c tw,i by L U-decomposition. Once c tw,i are determined, I get the entire temperature solution as shown here.

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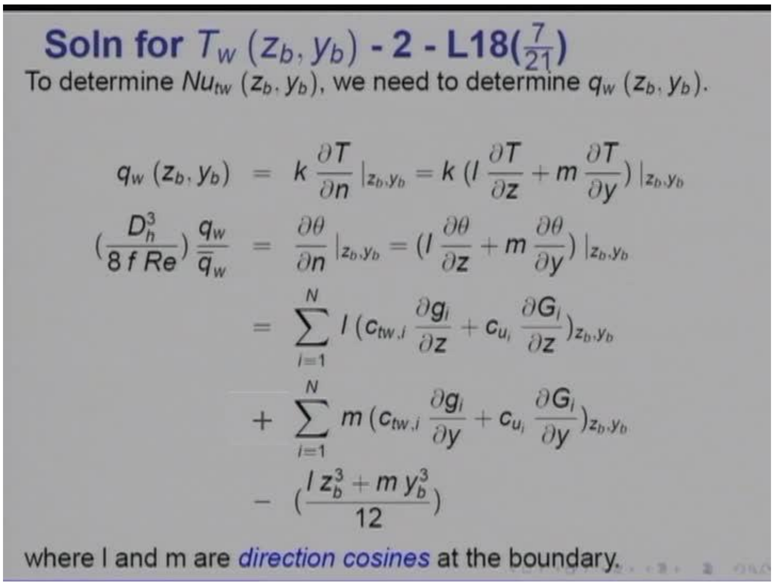

To determine the Nusselt number at the wall which will now vary along the periphery, because t w is varying along the periphery; the heat flux would also vary along the periphery because, the wall temperature is varying. In fact, the heat flux would vary in a non-circular duct and heat flux could vary along the periphery even when t w is constant, simply because the temperature profiles along say I have a duct like this even if t w was constant with respect to the periphery - the peripheral distance S - the velocity gradients and the temperature gradients can go on varying from point to point. Therefore, the local heat flux would be varying along the periphery, although its integral value would remain same as that specified which the axially constant heat flux q wall bar is.

Let us evaluate the temperature gradient at the wall it would be equal to k by d T d n at the wall where n is normal to the wall. So, q w which is coming into the duct would be plus k times d t by d n at the wall where n is normal to the boundary. You will recall from your first course in mathematics that a normal derivative can be split into derivative along direction z and direction y multiplied by direction cosines l and m, so i can write this as k l d T by d z plus m d T by d y at m.

Basically, if you have that as the boundary and this is the n direction then, d T d n along this will be simply d T d z the z direction d T d y along the y direction and d T d n would then be equal to l times d T d z plus m times d T d y, the l and m are direction cosines that is what we do here. So, you will see substituting those things here on the right hand side.

Therefore, if I now switch t to theta through this transformation q wall bar then, I will get D h cube divided by 8 f Re divided by q wall bar. Remember, I had defined theta equal to - that was the definition of theta - and that same thing I have substituted here to get D h cube 8 f Re q wall divided by q wall bar equal to d theta by d n equal to l d theta by d z plus m d theta by d y z b y b.

Now, each derivative here would be l times c tw,i d g i by d z plus c ui d G i by d z at z b y b plus m times c tw,i d g i by d y and c ui d G i by d y z b y b minus the l z cube plus m y b cube divided by 12, so that would be the right hand side which will enable me to evaluate q w. I already know c tw,i, I know d G i d y, I know G i function as well as small g i function. Therefore, the entire right hand side can be evaluated and the heat flux variation along the periphery can be evaluated.

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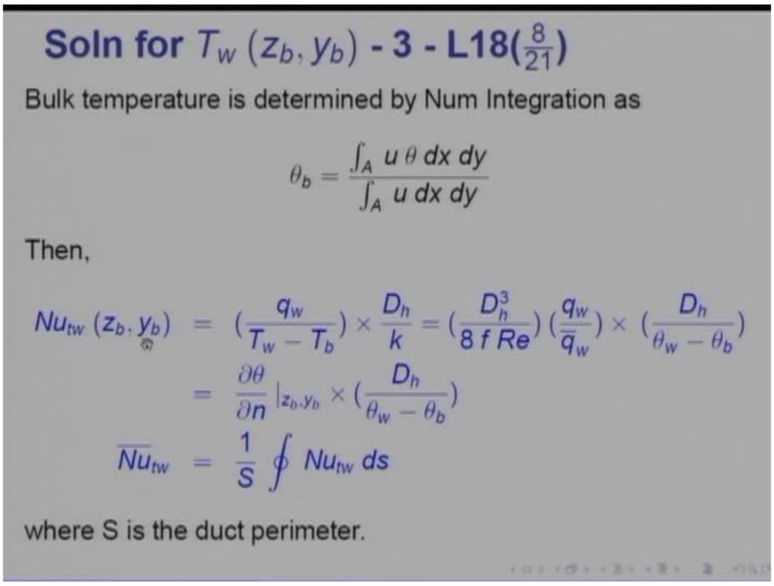

The next task of course, is to evaluate the bulk temperature u theta d x d y divided by u d x d y over the area of cross section of the duct, this is how we define the bulk temperature. Then, the Nusselt number which is q wall over T wall minus T bulk into D h by k would be simply equal to this quantity into D h by k which is equal to d theta by d n z b y b D h theta wall minus theta bulk.

Therefore, n u t w will be varying along the duct periphery and that quantity itself is very useful but, another useful quantity which is circumferentially average Nusselt number and that is what I have defined in this equation, S is a perimeter and the line integral of Nusselt number values along the periphery have been integrated with respect to perimeter.

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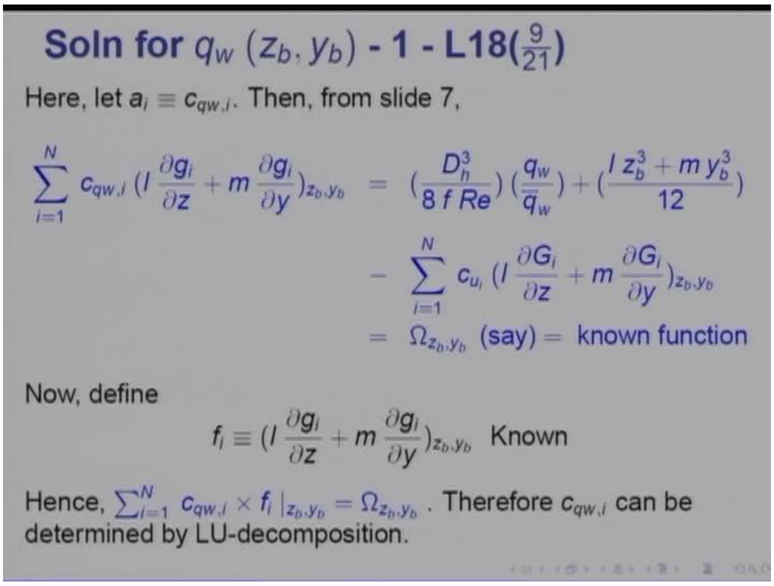

We now turn to the problem in which q w - the heat flux - is varying in the circumferential direction. The same temperature solution is to be used but the coefficients c qw,i are now to be evaluated. So, if I were to take q w specified then, the left hand side would become c qw,i l d g i by d z plus m d g i by d y at z b y b equal to D h cube over this into q wall bar by q wall bar plus this minus c ui l d G i by d z plus m d G i by d y z b y b and all these quantities are known because q wall z b y b has been specified.

Actually, uniform heat flux has also been specified and all these functions can be evaluated which I have called as gamma z b y b and l d g i by d z plus m d g i by d y at z b y b - if I were to call it as f i let us say - these are also known functions then, I would get again the velocity like problem c qw,i f i at z b y b equal to therefore, c qwi can be determined by L U-decomposition.

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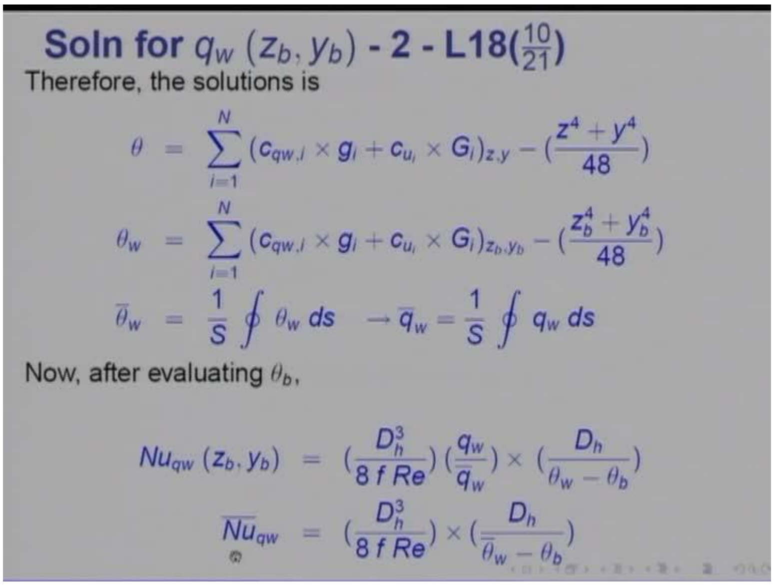

Therefore, the solution would be c theta equal to c qwi g i c ui from the velocity solution and this. So, I can readily evaluate theta w and knowing this, I can also evaluate theta bulk. Knowing theta wall, I can also evaluate theta wall bar as given and of course, q wall bar will be one over S line integral of q wall d s. Then, N u for the q wall varying would be given by that again as before.

The circumferentially averaged heat transfer coefficient would be given by the expression that I have shown at the bottom of the slide. So, N u cube bar q w is equal to D h cube at f Re D h by theta wall minus theta bulk where theta wall is given here and theta bulk is to be evaluated in the usual manner.

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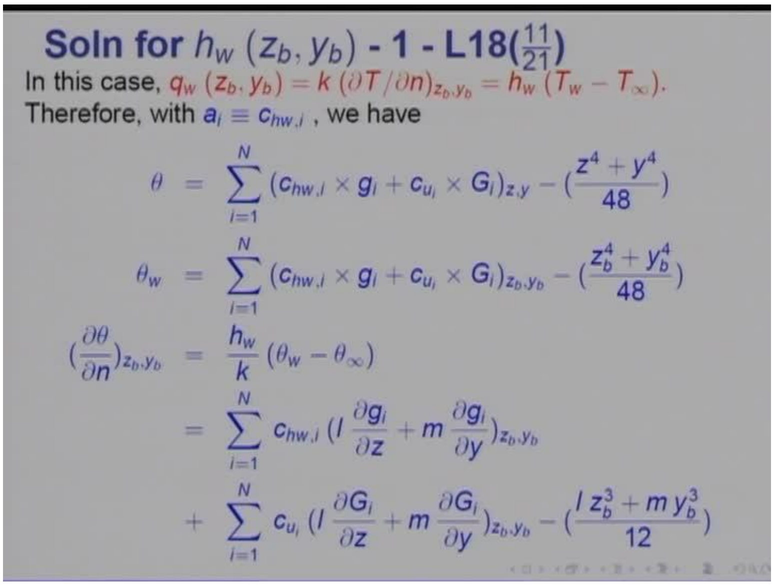

Now, I come to the very last boundary equation that the heat transfer coefficient can also be a function of circumference of the duct. So, in this case what would happen in which I have shown here? As you know, the definition of q wall is k d T d n at z b y b and that would be equal to h w into T wall minus T infinity where, h w has been specified and T infinity is some temperature outside the duct which is known.

Now, in this case a i would be hw i and therefore, the solution would read as theta times c hw,i g i plus c ui G i z y minus this quantity, theta wall would be simply that and d theta by d n z b y b h w over k theta wall minus theta infinity into c hw i l d g i by d z m d g i by d y z b y b plus c ui all these.

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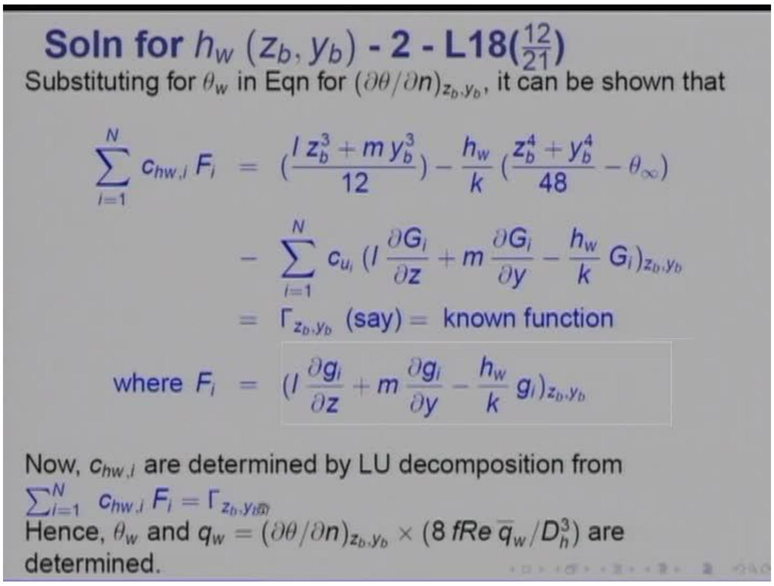

This equation will now enable us to evaluate c hw i as I show for example, this quantity would be equal to h w by k theta wall minus theta infinity minus the entire term on the bottom of the slide. So, I can now say that if I define F i equal to - it is simply l d g i by d z plus m d g i by d y h w g i z b y - what is given in the brackets. Then, the right hand side would be all this and the entire is known. So, again I get a equation which is i equal to 1 to N c hw i F i equal to gamma times z b y b.

Therefore, I can evaluate c hw i from which I can get theta wall as well as q wall because I can get d theta by d n. When the heat transfer coefficient has been specified, our main objective is to discover what will be the wall temperature and the local heat flux. The Nusselt number would of course, follow from it because h w is already known.

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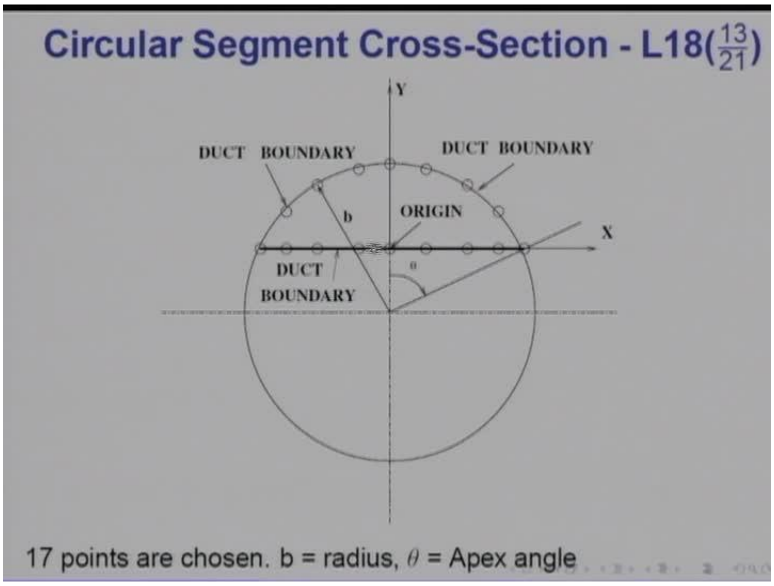

Now, let me take few examples, you will recall that for this circular segment cross section we had already developed the velocity solution. Therefore, the c ui coefficients are already known, theta naught is the apex angle and x is measured so and y is measured so. I am going to develop solutions in this case for T w variation and q w variation along this circular segment.

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Local Nu - T_w = const (θ = 90) - L18($\frac{14}{21}$)

i	z_b	y_b	l	m	q_w	Nu_{tw}
1	-1.0	0.0	0.0	-1.0	0.804E-04	0.023
2	-0.99	0.141	-0.99	0.141	0.380E-02	1.09
3	-0.75	0.661	-0.75	0.661	0.148E-01	4.24
4	-0.5	0.866	-0.5	0.866	0.181E-01	5.17
5	-0.25	0.968	-0.25	0.968	0.194E-01	5.55
14	0.0	0.0	0.0	-1.0	0.247E-01	7.06
15	-0.35	0.0	0.0	-1.0	0.207E-01	5.93
16	-0.75	0.0	0.0	-1.0	0.846E-02	2.42
17	-0.99	0.0	0.0	-1.0	0.359E-03	0.103

Exploiting symetry about z = 0, values for negative z are shown.
Low vaues of q_w correspond to hot-spot regions. $\overline{Nu}_{tw}$ = 4.02.

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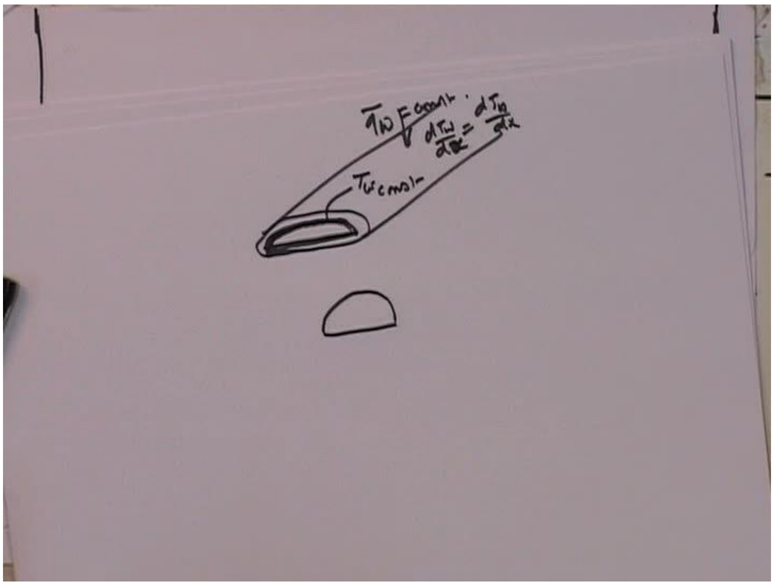

To begin with what I have done? I have simply said that over the entire periphery of this duct T w is constant. Of course, T w will vary with z d T w by d x will equal d T bulk by d x because q w is constant. Although, the circumferential which is a special case of arbitrary variation d T wall by d x equal to d T bulk by d x.

Axially, T wall increase with x but circumferentially, the temperature is constant. All it means that the metal of the duct has very high thermal conductivity so that any temperature variation is just even doubt and most of the solutions that are documented in literature are for this particular case of T w equal to constant.

I am going to take the case of theta equal to 90 degrees as a special case. Theta equal to 90 degree - as you recall - is nothing but, a very simple semicircle. So, have a look at this figure again, z b y b's are specified for all the boundaries 1, 2, 3, 4, 5 and what I have done now is only because of the symmetry on both sides I am giving you solutions on the left side of the boundary because the same solution would be reproduced. Circumferentially the wall temperature is constant, so we expect symmetry about this y axis.

So, I am giving you the solutions only on the left hand side to save space on this slide. So, you will see minus 1, minus 0.99, minus 0.75, minus 0.5, minus 0.5 all these are on the negative side of z and so these on the negative side of z and y b is equal to given.

This is on the curve top and this is where y b is 0. The direction cosines are of course, in each case turns out to be same and the q wall heat flux, you will see is like this. Right at minus 1 0 which means, this point the heat flux is very small it goes on increasing as you go towards the top, as you can see here 0 0 0 is the top most that is where it is 0.247.

On the flat side the heat transfer is 0.207 minus 1 again on the flat side again the heat flux is higher and goes on reducing. So, you have high heat flux coming here, high heat flux coming in here and it reduces along the periphery in this direction and in this in direction.

Likewise on this side as well, which I have not shown by exploiting symmetry. You will see how the Nusselt number varies 0.023, 1.09, 4.24 and so on and so forth. Now, if I take an circumferential average of the Nusselt number that is shown in this column, it turns out to be 4.02 for the semicircular duct. This value matches very well with what is published in the literature in which the solution has been obtained by Fourier series and that is possible for this elegant case of a collectively regular shape of theta equal to 90.

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Effect of θ - T_w = const - L18($\frac{15}{21}$)

	θ degrees				
c_i	90	60	45	30	10
c_3	-0.247e-1	-0.400e-2	0.893e-3	-0.928e-4	-0.136e-6
c_4	0.238e-7	0.239e-6	-0.450e-7	-0.345e-7	0.319e-8
c_7	-0.243e-1	-0.159e-1	-0.103e-1	-0.514e-2	-0.627e-3
c_8	0.833e-6	-0.613e-6	0.125e-5	0.563e-6	0.479e-7
c_{11}	0.305e-2	0.275e-2	0.295e-2	0.340e-2	0.405e-2
c_{12}	-0.245e-5	-0.154e-6	-0.599e-5	-0.314e-5	-0.334e-4
c_{15}	0.358e-3	0.35-e-3	0.587e-3	0.931e-3	0.147e-2
c_{16}	0.175-5	0.733e-6	0.744e-5	0.562e-5	0.624e-3
c_{17}	-0.105e-6	-0.111e-5	-0.490e-5	-0.141e-4	0.449e-3
Nu_{tw}	4.02	3.90	3.79	3.68	3.04

$\theta = 90$ corresponds to a duct of semi-circular cross section.

Even the finite difference solution for this problem gives you a very good argument with this value. Now, what I want to show is how were the coefficient c T w i coefficients - these are the c T w i coefficients - appear for T w equal to constant and this was the case of 90 degrees that we had earlier considered semicircular cross section.

This is the case of 60 degrees, this is the case of 45 degrees. Now, you will see c 1 and c 2 are 0 or c T w 1 and c T w 2 are 0, c 3 is finite, c 4 is finite, 5 and 6 are 0, 7 and 8 are finite, 9 and 10 are 0, 11 and 12 are finite, 13 and 14 are 0, 14 and 16 are finite and so is 17 although very small as you can see here.

In each case, the circumferentially averaged Nusselt number has been also been calculated. As you can see, as the angle becomes smaller the Nusselt number goes on decreasing of course, this Nusselt number is based on hydraulic diameter, so 4.02 is at 90 that we saw 3.90, 3.79, 3.66, 3.04 at 10 degrees.

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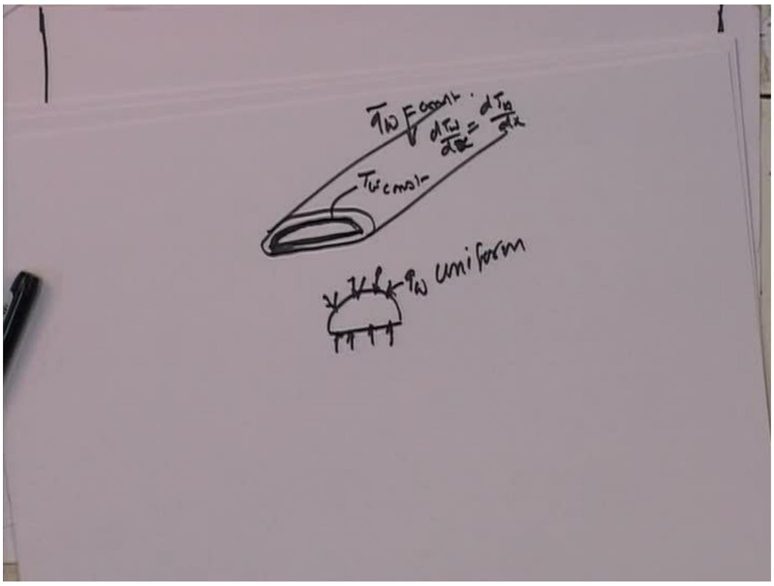

Now, let us turn our attention to the case in which circumferentially q wall is specified and again in this case, it is the q wall which is constant along the circumference, basically, q w is uniform around the circumference. Again, I have considered uniform simply because, it is a case in which some publish solutions are available.

Again exploiting symmetry, I am giving results in this case not 90 degrees but, 60 degrees just to make some change. I am considering the case of theta equal to 60 degrees and these are the points on the third boundary and these are the (()) points on the flat boundary, this is the l and m.

Now, you can see what the wall temperature looks like, the wall temperature is 0.237 minus 2 at 0.1 which is the point over here T w and it is the highest value as you can see from the table that 0.237 e to the minus 2 is the highest temperature.

Then, the temperature goes on reducing as you move towards the top of the duct and along the flat sides towards the center of the duct this represents hot spot. The bulk temperature in this case was 0.000122, so you can see how big this is, this is 1.22 raise to minus 4 whereas, T wall is this.

It is to discover such hot spots that solutions of this type are very important because circumferentially although the heat flux is uniform, wall temperature can vary and you can get hot spots at corner points. This is of great consequence both for example, if the corner point is also a highly stressed point and if it has very high temperature with very large temperature gradients around that point that could be cracking or thermal warping or anything of that kind can happen.

So, solutions of this type are essentially meant to discover hot spots along the wall. As you can see here at all the points that I have shown those points close to the corner are very high temperature. If you look at Nusselt number, it is 0.67 and it goes on increasing to 37.5 but look at this value of 0, 0, 0 right at the center on the flat surface. You will see y equal to 0, x equal to 0 means that temperature and that point, the Nusselt number is negative because T wall is 0 and it is less than T bulk.

So essentially, all it implies that it does not mean negative heat transfer but, simply the wall temperature is much lower than the bulk temperature resulting into a negative Nusselt number, then again T wall becomes greater than. So, such things can also happen that a wall temperature actually goes below the bulk temperature 4.84, 0.944 and 0.619 again.

So, in this particular case N u bar is 1.657 for 60 degrees it is 1.6 which is the circumferentially averaged heat transfer coefficient. Again, I have shown on this table what are the coefficients for q wall equal to constant condition. You can see again c 3 and c 4 are finite, c 7, c 8, c 11, c 12, c 15, c 16 and c 17 these are the coefficients same as before for constant wall temperature they are finite remainder are 0.

You can see for theta equal to 90 the circumferentially averaged heat transfer coefficient is 2.78, 1.61, 1.03, and 0.433 for a very narrow angle it is 0.049. You will see that the N u q wall bar is less than N u t wall for all angles.

Remember, we had 4.02 here, 3.90 something here so on and so forth. Now, these values have been verified by finite difference calculations also. So, this is a very convenient way of developing solutions for uniform.

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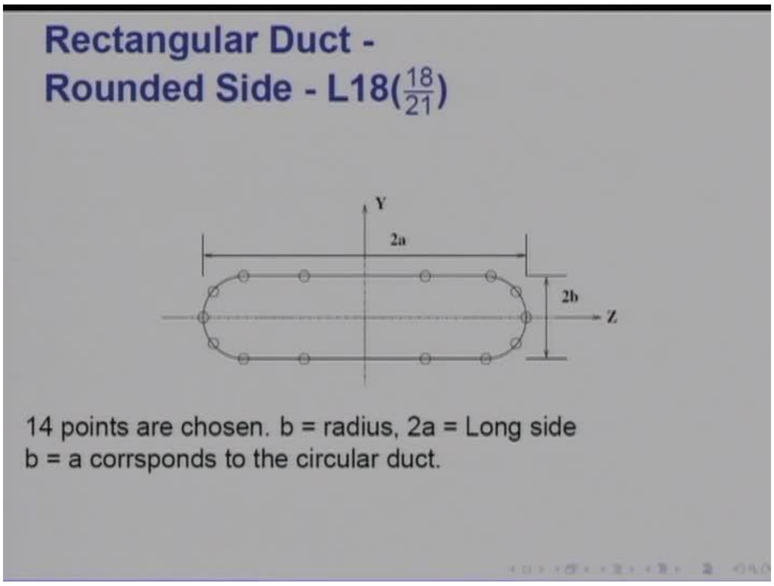

Now of course, just to remind you, I have taken circumferentially uniform wall temperature and circumferentially uniform heat flux just as special cases but, you could well have any arbitrary variation of t w and q w and still we would get the solutions exactly in the manner I have described.

I will now take a new kind of a duct for which we had earlier not obtained velocity solutions but, it is simply a duct in which the unrounded side is 2a along the x direction and 2b along the y direction. So, I have chosen 14 points b is radius of the rounded side and 2a is the long side.

Now, you will appreciate if b was equal to a, you will get a perfect circular duct. My interest is to show you that when b is equal to a in fact, you generate the circular tube solution that we are all very familiar with.

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Effect of b/a - L18($\frac{19}{21}$)

b/a	c_{u1}	c_{u4}	c_{u8}	c_{u12}	fRe
0.25	0.0292	0.262	-0.0372	-0.00358	19.78
0.50	0.110	0.182	-0.0597	0.0179	17.23
1.0	0.25	0.0	0.0	0.0	16.0
b/a	c_{t1}	c_{t4}	c_{t8}	c_{t12}	$\overline{Nu}_{tw}$
0.25	-0.693e-3	-0.783e-2	0.111e-2	0.113e-2	5.944
0.50	-0.940e-2	-0.176e-1	0.849e-2	-0.943e-3	4.73
1.0	-0.469e-1	0.0	0.521e-2	0.0	4.367
b/a	c_{q2}	c_{q4}	c_{q8}	c_{q12}	$\overline{Nu}_{qw}$
0.25	-0.348e-3	-0.432e-2	-0.874e-3	0.158e-2	-15.46
0.50	-0.253e-2	-0.929-2	0.276e-2	0.125e-2	5.056
1.0	0.0	0.0	0.521e-2	0.0	4.367

As $b/a \rightarrow 0$, $fRe \rightarrow 24$. For $b/a = 1.0$, $\overline{Nu}_{qw} = \overline{Nu}_{tw}$.
Negative $\overline{Nu}_{qw}$ at $b/a = 0.25$ indicates $\overline{T}_w < T_b$.

For different b by a I am showing you the values of c u because these were not shown earlier, because I had not obtained the velocity solution earlier but, the here is something that is shown. You will see that only 1, 4, 8 and 12 are finite. So for 0.25, I get 19.78 as the friction factor Reynolds number product; 0.5, I get 17.23 and 1, as I said is a circle or a circular duct for which only c u 1 is finite and it is f Re is equal to 16.

Although this is a limiting case of the geometry that I have shown, it does predict quite accurately the circular tube value. For the same cases I have considered the heat transfer under constant wall under uniform wall temperature around the duct. Again, you find c t 1, c t 4, c t 8 and c t 12 are finite for all of them and you can see this is 5.944, 4.73 and for b by a equal to 1 which is circle you will see only c t 1 and c t 8 are finite and you get a value of 4.367.

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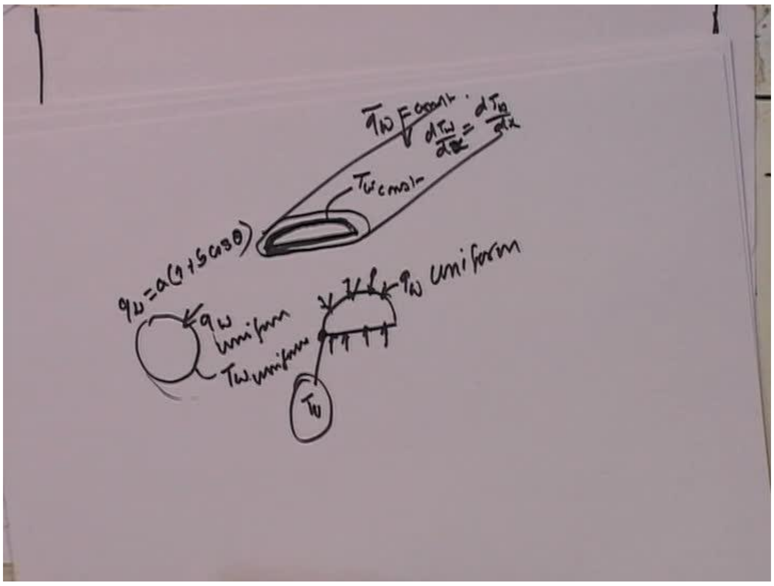

Remember, for a circular duct when q w is uniform, T w is also uniform and that is what is shown here. For circular duct that is b by a equal to 1 you get 4.367 as N u t wall bar and the same value is predicted also by specifying constant heat flux as that. The coefficients turn out to be the same and you will see that, sorry, the coefficients in constant T wall case is c t 1 is finite c t 8 is finite but, in constant heat flux case only c q 8 is finite.

Now, interesting case is that b by a equal to 0.25 and I have mean Nusselt number as minus 15.46. All it means is that, although the heat flux is finite on the wall when it is coming in the average T w bar is less than T bulk and therefore, you have negative Nusselt number. Such things can certainly happen in case of non-circular ducts.

You will recall that I had obtained finite different solution to a circumferentially varying boundary condition of the heat flux along the circular tube, q wall equal to 8 times 1 plus b cos theta was the circumferentially varying heat flux. I had obtained solutions by finite difference method, those solutions I have now obtained by the present method next.

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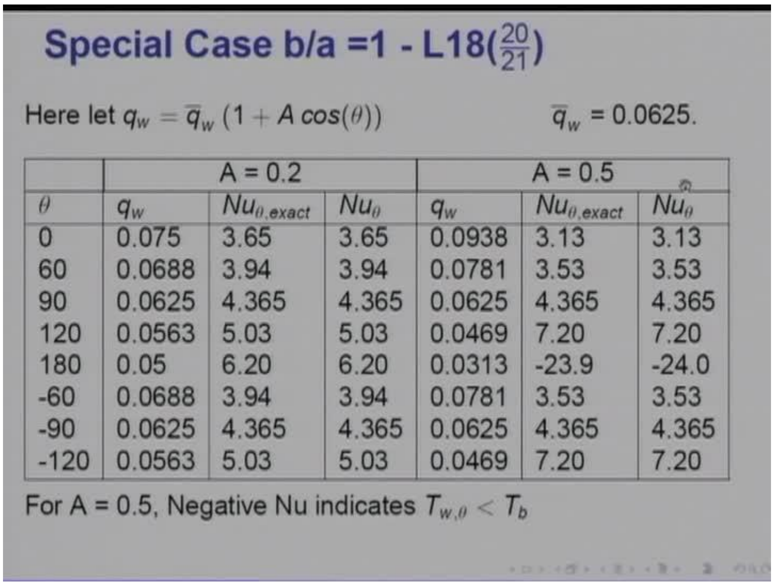

Special Case b/a =1 - L18($\frac{20}{21}$)

Here let $q_w = \overline{q}_w\,(1 + A\cos(\theta))$ $\overline{q}_w$ = 0.0625.

	A = 0.2			A = 0.5		
θ	q_w	$Nu_{\theta,exact}$	Nu_θ	q_w	$Nu_{\theta,exact}$	Nu_θ
0	0.075	3.65	3.65	0.0938	3.13	3.13
60	0.0688	3.94	3.94	0.0781	3.53	3.53
90	0.0625	4.365	4.365	0.0625	4.365	4.365
120	0.0563	5.03	5.03	0.0469	7.20	7.20
180	0.05	6.20	6.20	0.0313	-23.9	-24.0
-60	0.0688	3.94	3.94	0.0781	3.53	3.53
-90	0.0625	4.365	4.365	0.0625	4.365	4.365
-120	0.0563	5.03	5.03	0.0469	7.20	7.20

For A = 0.5, Negative Nu indicates $T_{w,\theta} < T_b$

You will see here, I have taken q wall equal to q wall bar plus 1 plus a cos theta, q wall bar has been specified here as 0.0625. You can see how the heat flux variation varies here and how the Nusselt number varies. Exact values as calculated from that formula that had given there and how the presently computed values absolute agreement for when A is 0.2.

When A is equal to 0.5 again there is a very good agreement except at this point where the value here predicted is somewhat because of rounding off it has been printed as minus 24 but actually it is minus 23.9.

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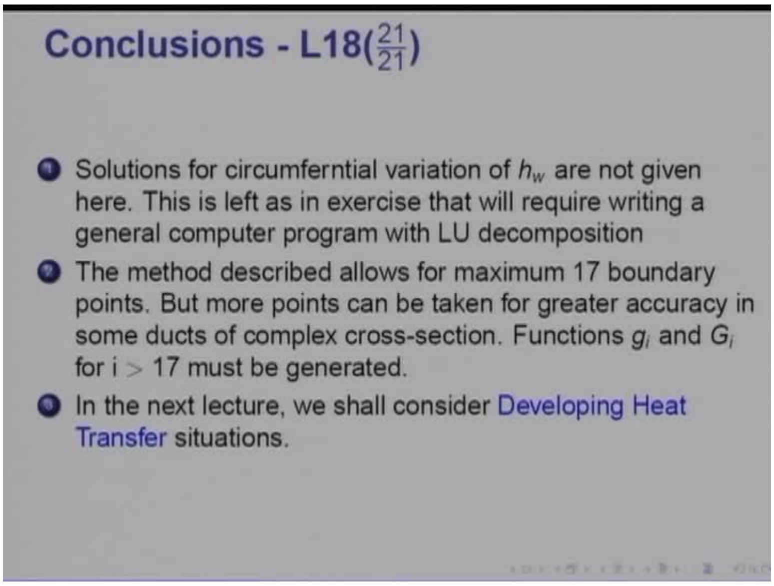

The solution has been very well reproduced by the new method. So, solutions for circumferential variation of h w are not given here, this is left as an exercise that will require writing a general computer program with L U decomposition.

Of course, as I said this method is extremely versatile, all you need to do is to write a general computer program with small g functions and big G functions and a routine that will evaluate u bar and theta bulk. All you do then is to have an input sub routine in which you give boundary coordinates and specify whether you want to solve for T w q w and h w.

So, a general computer program can be written and therefore, this method turns out to be very important. This is particularly of great importance because, we nowadays have micro tubes in which all kinds of complex cross section such as the moon shape duct that I had mentioned in my earlier lectures or sinusoidal duct these sort of ducts come about and therefore, this method is very valuable and in micro tubes you invariably get laminar flow. So in the next lecture, we shall consider developing heat transfer solutions.