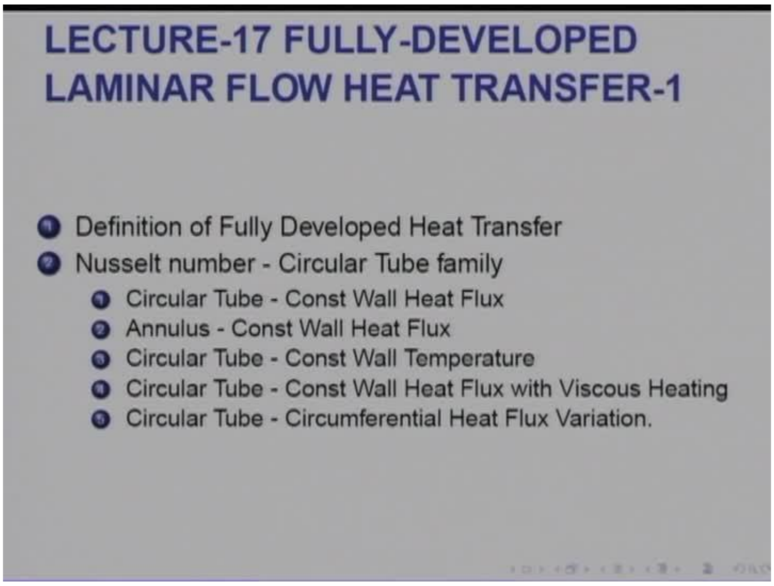
Convective Heat and Mass Transfer
Prof. A. W. Date
Department of Mechanical Engineering
Indian Institute of Technology, Bombay

Module No. # 01
Lecture No. # 17
Fully-Developed Laminar Flow Heat Transfer-1

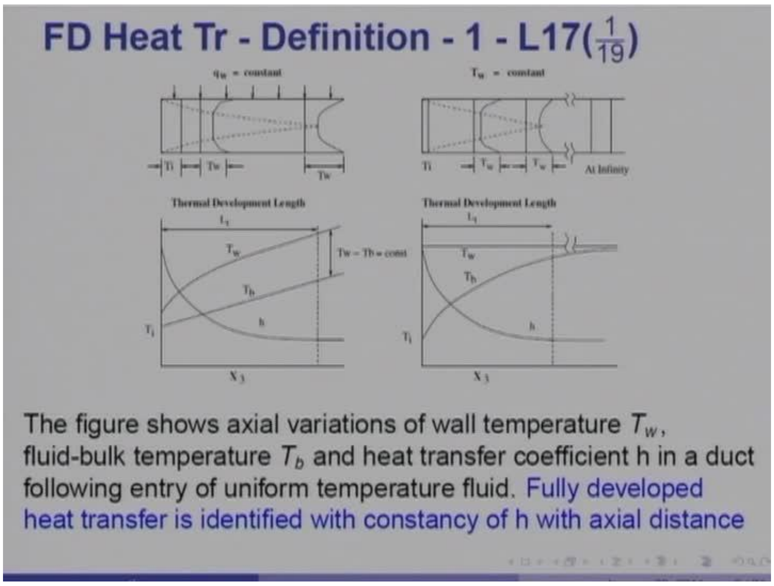
In the previous two lectures, we considered friction factors in fully developed laminar flow in regular section Ducts as well as Ducts of complex cross sections like any arbitrary cross section, as long as they were singly connected.

(Refer Slide Time: 00:56)



We now turn our attention to fully developed heat transfer and like we did in the case of friction factor we shall, first of all, look at very simple situations like the circular tube family annulus and so on so forth.

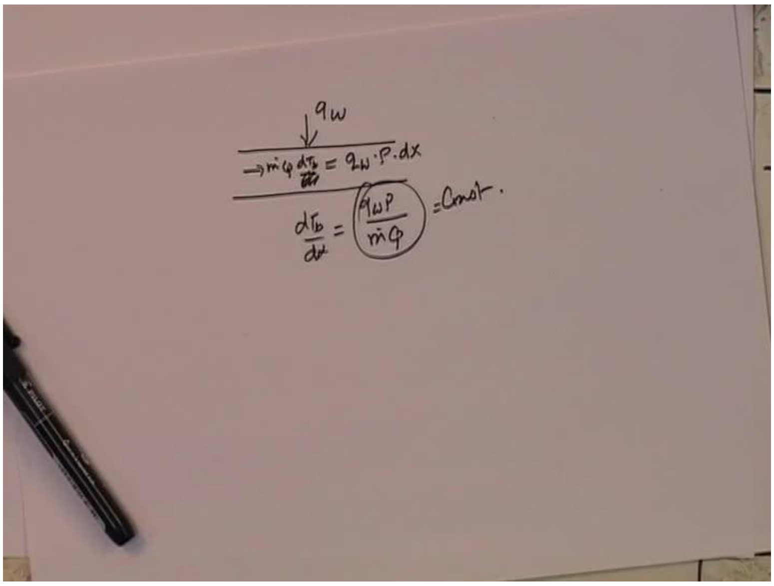
(Refer Slide Time: 01:15)



Under variety of boundary conditions, to do that we must define fully developed heat transfer. Consider these Ducts, in which fluid at uniform temperature enters and there is a constant wall heat flux supplied at the wall.

Then, you can see that the wall temperature will begin to rise and after some distance would rise at a linear rate. The bulk temperature however by first law of thermodynamic that is m dot c p into d T bulk by d x.

(Refer Slide Time: 02:15)

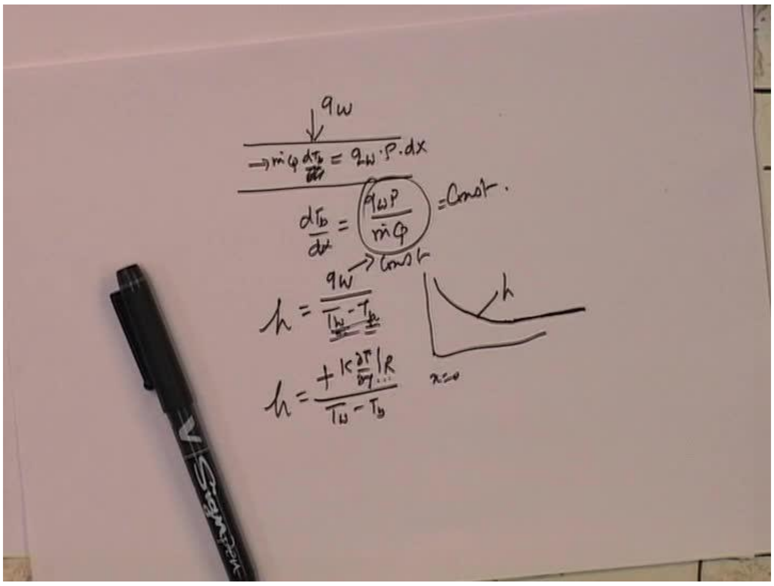


Heat flux is constant and m dot c p into d T bulk by d x into d T bulk will be simply equal to q wall into perimeter into d x. Therefore, d T bulk by d x will be simply q wall into perimeter divided by m dot c p and all these are constant.

You would see that the bulk temperature would rise linearly with x; right from the start. We assume that the velocity profile may or may not be fully developed.

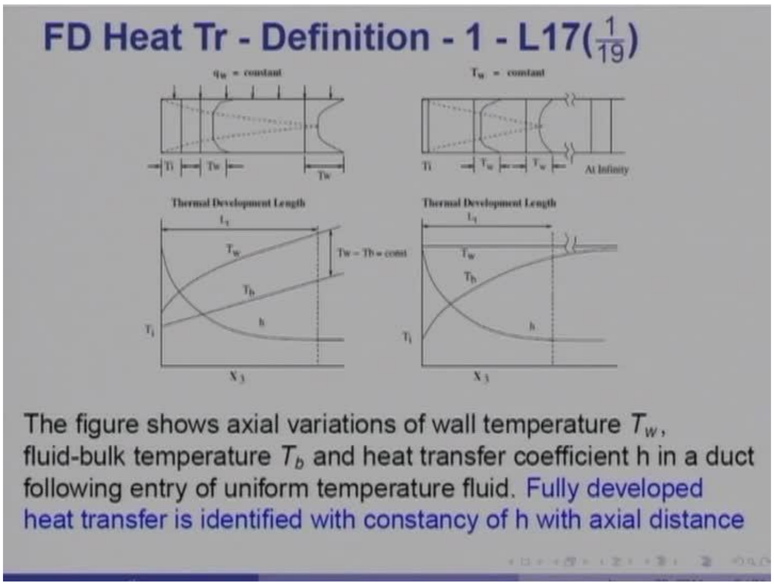
Therefore, this shows the thermal development of the temperature profile. We can see, it is started with a uniform temperature but then as the heat flux comes in, it assumes a curve shape. The gradient at each section would be constant because q wall is constant.

(Refer Slide Time: 03:07)



Ultimately, the profile would become like that. Since, heat transfer coefficient is defined as q wall over T wall minus t bulk and this is constant. The difference between T wall minus T bulk is small to begin with and heat transfer coefficient is high near x equal to 0. It progressively drops as difference between T wall minus T bulk increases and becomes constant because T wall minus T bulk itself becomes constant.

(Refer Slide Time: 03:50)



That is depicted on the left figure here. Now, let us consider another boundary condition, which is frequently met that of a constant wall temperature. This is like steam heating, in which the wall temperature is constant.

The fluid enters at a value lower than the wall temperature, which is shown here. The fluid bulk temperature would start rising. You can see here, unlike the constant wall heat flux case the bulk temperature does not raise linearly but it rises non-linearly with x.

To begin with, the gradient of temperature is very large and at the wall h, which is minus k d T by d y at R or plus here divided by T wall minus T bulk although, T wall minus T bulk is large.

So, k d T d y at R is very large. As a result, again you get a variation of heat transfer coefficient h, which is very similar to that in a constant heat flux case. That is what I have shown here (Refer Slide Time: 05:08); h goes on changing with x but decreasing with x. The temperature profile becomes curved because of the thermal boundary layer development.

At each section, the T wall will be same but only thing is the T central line will go on increasing. So, that the T bulk goes on increasing and this variation is non-linear as I have indicated here.

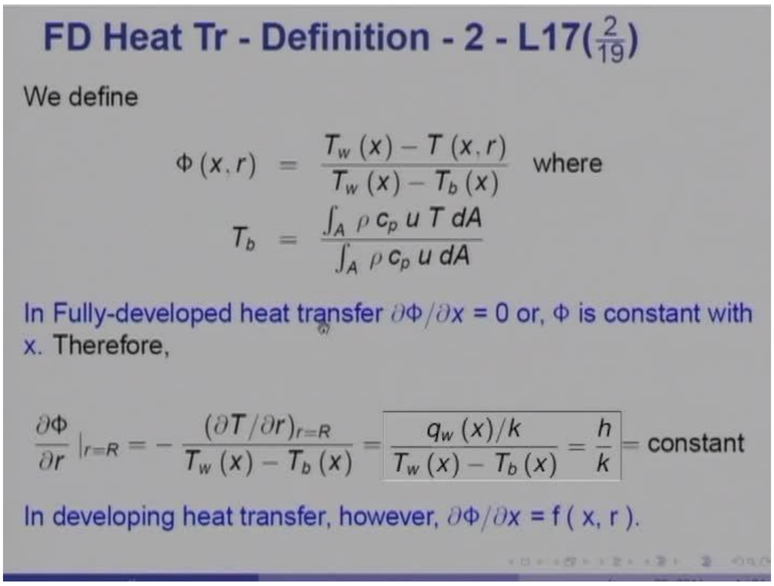
If the Duct was very long say, going up to infinity, the bulk temperature itself would become equal to the wall temperature. In fact, the temperature at all radii will become equal to the wall temperature.

But that would occur only at infinity and it is not a case of importance in practical Ducts, which have limited length to diameter ratio. Fortunately, it happens that as the temperature profile develops, the gradient at the wall goes on changing; as a result the heat transfer rate goes on changing with x.

A point is reached beyond which although T bulk is changing and heat transfer the gradient of the temperature is changing at the wall. A point is reached, where k d T d R divided by T wall minus T bulk becomes constant or h becomes constant.

In heat transfer, we say fully developed heat transfer is identified with constancy of h with axial distance. To show this, let us consider this - what does this imply for the temperature?

(Refer Slide Time: 07:09)



We define, for example phi as a function of x and r in case of a circular tube as T wall x minus T x r divided by T wall x divided by T bulk x, Now, when you T wall and T bulk can only be functions of x, where T bulk is rho c p u T dA and rho c p u dA, where dA is the is the area of cross section.

In fully developed heat transfer, we say that d phi by d x will be 0. The dimensionless temperature profile will go to 0 or dimensionless temperature profile will become constant with x.

Since, phi is a constant with x we would expect that d phi by d r at the wall will also be constant with x, which is was means that it is equal to minus d T by d r, r equal to R divided by T wall minus T bulk x equal to q wall over T wall x minus T bulk x. This definition is used for constant heat flux case this definition would be would indicate definition of h in case of constant wall temperature

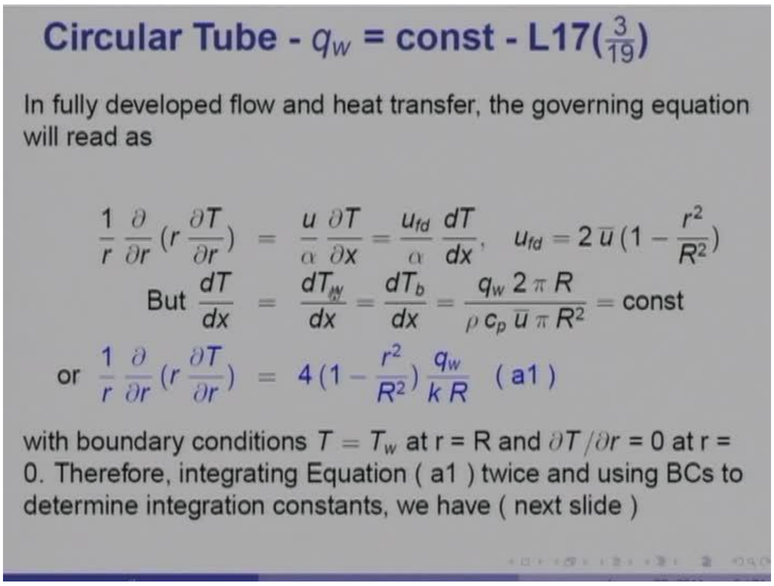
You will see that h becomes constant. So, fully developed heat transfer and constancy of h implies that d phi by d x must be equal to 0. It is the dimensionless temperature which must be 0.

Remember, we defined fully developed flow, d u by d x itself equal to 0. In heat transfer, we say fully developed dimensionless temperature gradient with x is 0 is the condition for fully developed heat transfer.

It implies h equal to constant. Although T wall and T bulk are indeed temperatures at every radius may vary with x and r as long as T wall T bulk. The heat flux that ratio is remains constant or h is constant we say the fully developed heat transfer has been reached.

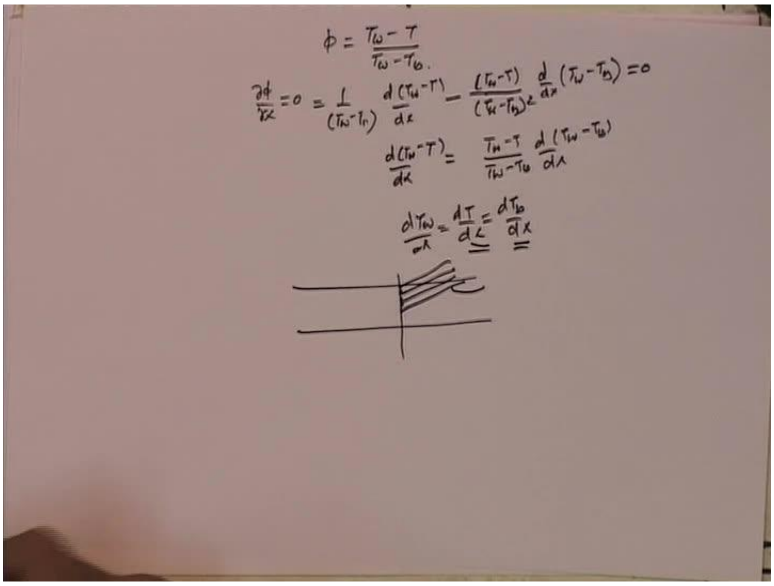
Let us take, the first case, the simplest case which you have studied in your undergraduate course and that of a circular tube with constant heat flux at the wall.

(Refer Slide Time: 09:44)



In that case, the governing equation as you will recall is simply one over r d by d r r d T by d r equal to u divided by alpha, alpha is a thermal diffusivity, k by rho c p into d T d x. Since, q wall is constant d T d x would be replace by d T bulk d x and it will actually also mean d T wall by d x.

(Refer Slide Time: 10:30)



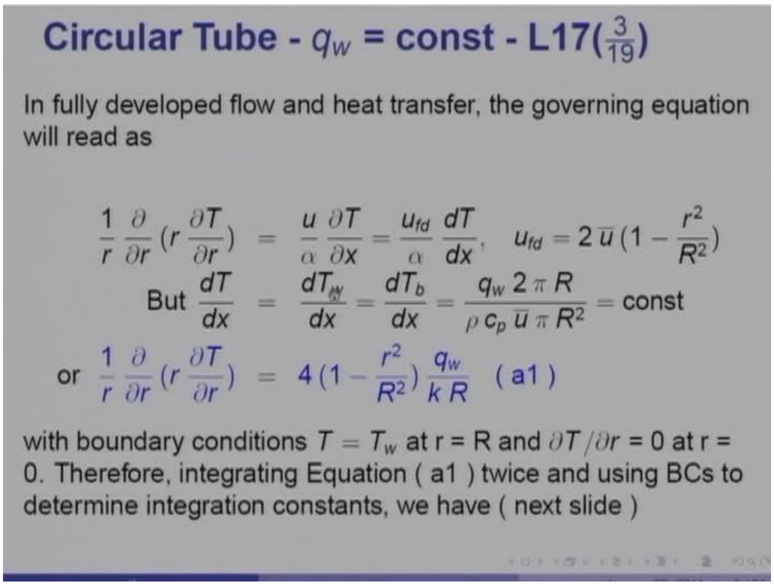
From our definition, you can see if d T bulk if I have to say d phi by d x equal to 0 and d T x equal to d b d T bulk x. Then, d T wall by d x will also be 0. To make it explicit let us say, d phi by d x equal to 0 would imply that phi is equal T wall minus T divided by T wall minus T bulk.

This would imply 1 over T wall minus T bulk into d T wall minus T by d x minus 1 over T wall minus T over T wall minus T bulk square d T by d x T wall minus T bulk equal to 0.

Which essentially gives me d T w minus T by d x equal to T wall minus T over T wall minus T bulk into d T wall minus T bulk by d x. This implies that all d T w by d x is equal to d T by d x and is also equal to d T bulk by d x.

The implication is that once the fully developed flow is reach, temperatures at all radii increase constant with x equal to an all radii d T x is equal to d T by d x.

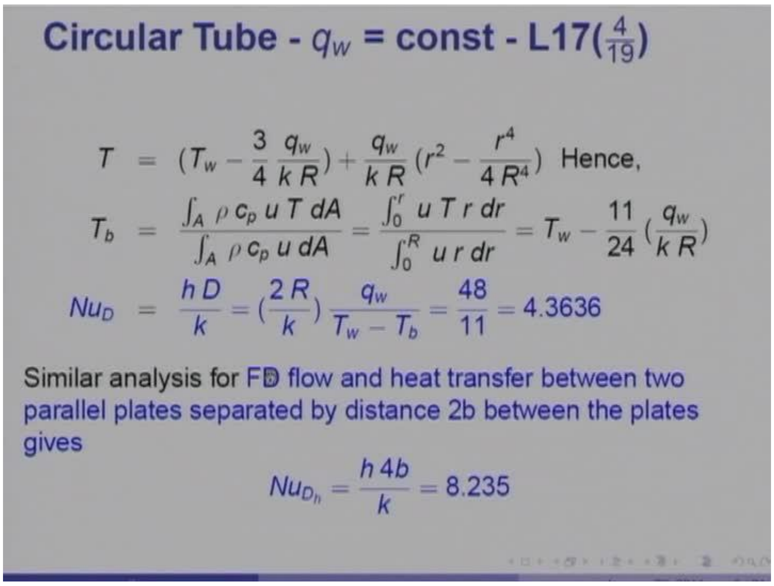
(Refer Slide Time: 09:44)



You will see that d T bulk by d x is simply; this is m dot into c p and this is q wall into perimeter, which is a constant. Therefore, the equation would simply become 1 over r d by d r r d T by d r.

Fully developed flow in a circular tube is given as 2 u bar 1 minus r square by R square and this will be that. If I integrate this equation twice with boundary conditions T equal to T wall at r equal to R and d T by d r equal to 0 at the axis of symmetry. Then, I can determine 2 constants of integration and the result is shown on the next slide here, T is equal to T wall minus 3 by 4 q wall over k R plus q wall over k R into r square minus r 4 R.

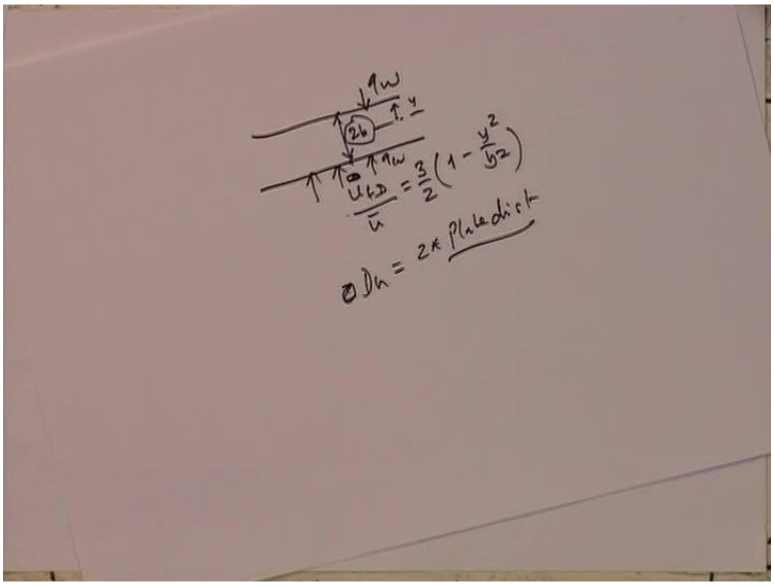
(Refer Slide Time: 12:51)



Now, I evaluate T bulk which are you will recall and we assume constant properties so rho c p just cancels. If I have to substitute this for temperature and the velocity profile from the previous slide then, I would get T w minus 11 by 24 q wall over k R.

In all these cases, integrations are very important and you have to take lot of care to make sure that you have made no errors in evaluating the temperature in evaluating the integrals. You can now see that Nusselt number, which is defined as h D by k will be equal to 2 R by k q wall over T wall minus T bulk T wall minus T bulk is 11 by 24 q wall R by a. So that gives us 48 by 11 equal to very well-known result 4.3636, which you derived in your undergraduate course.

(Refer Slide Time: 14:13)

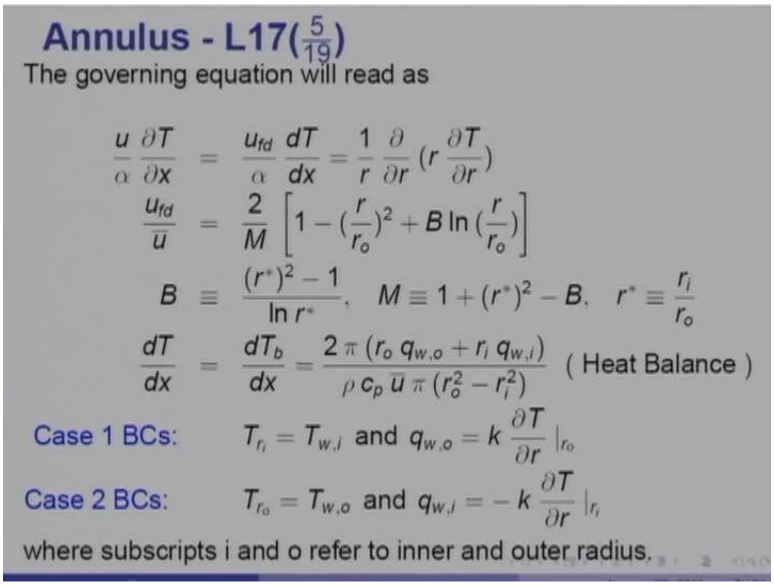


A similar analysis for flow between parallel plates, which are separated by a distance 2 b. If I have 2 parallel plates and the distance between the plates is 2 b and if I measure y from the axis symmetry. Then, you fully developed divided by u bar. In this particular case is 3 by 2 1 minus y square by b square.

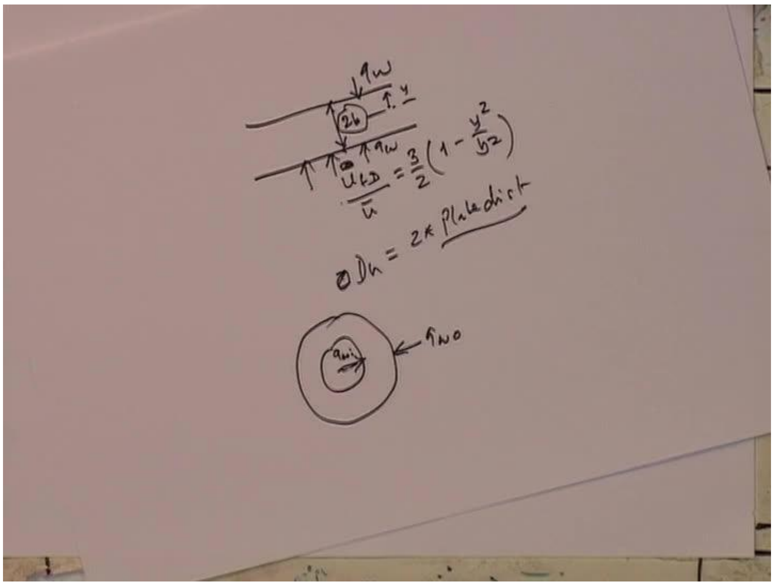
As you recall and if I carry out the similar analysis for constant heat flux at both the walls then the answer, I would get is based on hydraulic diameter. The hydraulic diameter for plate distance of 2 b and D h is 2 times the plate distance.

That is equal to 4 b so h 4 b by k would give you 8.235. Very simple cases that can be done by pencil and paper integration, no difficulty at all; you do not need numerical integration or anything like that.

(Refer Slide Time: 15:23)



(Refer Slide Time: 15:54)



Let us now turn our attention to little bit more complicated thing. Again, I am going to consider now, if flow in an annulus flow inside an annulus, you can get variety of it. It may be heated from inside or it may be heated from outside q wall o or q wall i.

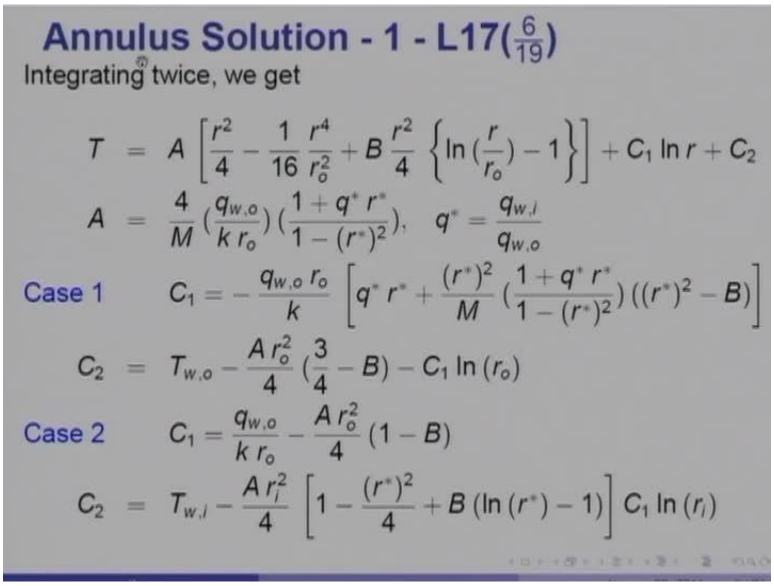
We are going to consider this particular case; in fact, both the cases in turn. Let us look at, you will recall from our earlier analysis that fully developed velocity profile is given as 2 by m 1 minus r by r o square plus B l n r by r o, which has a logarithmic term in it.

B itself is this, m is this and r star is r i by r o, the radius ratio of the annulus. The equation proper remains the same, it does not change at all only thing is d T d x here will be d T bulk by d x.

That would be equal to 2 pi r o q wall o plus r i q wall i divided by rho c p u bar pi r o square minus r i square. We can have 2 types of boundary conditions - in the first case outside wall is heated. So, the q wall o is given equal to k d T by d r at r o.

At the inner radius r I, we have T w i. On the other hand, we can also have a case in which the inner heat flux is given and the outer wall temperature is T w o.

(Refer Slide Time: 17:08)

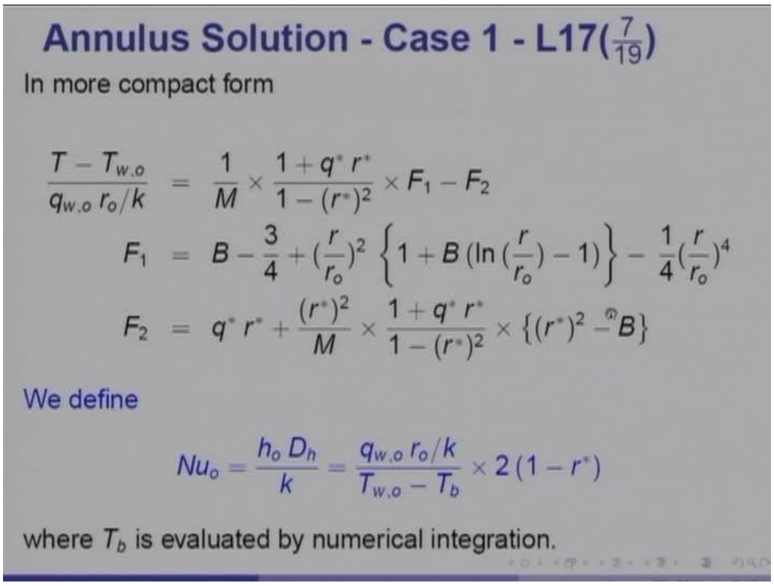


Annulus solution. If I integrate this particular equation with substituting for d T by d x d T bulk by d x and u fd equal to this and integrate this equation twice, which is mind you quite a elaborate integration because logarithms are involve.

I would get temperature itself as that, where A itself A the multiplier of this square bracket is this and q star is inner wall heat transfer divided by outer wall heat flux. I will consider 2 cases as I said.

In case one, where outer wall is heated c 1 turns out to be this and c 2 turns out to be this. In the other case, where inner wall is heated it will turn out to be this and c 2 is equal to that.

(Refer Slide Time: 18:20)

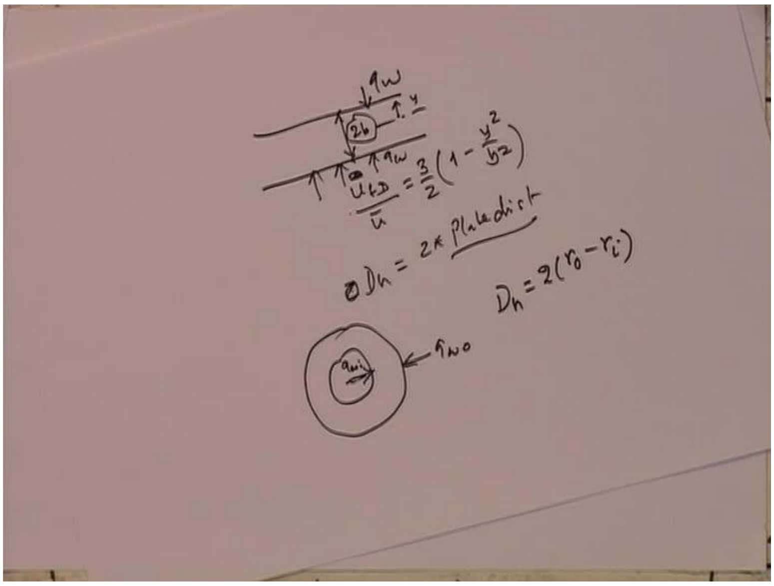


These things require very careful algebra, in order to avoid any errors. I can express the solution that of the previous slide in a more compact form. As T minus T w o q wall o r o by k, which has dimensions of temperature q wall r o by k has dimension equal to 1 over m into 1 plus q star r star plus mind you q star is q wall I minus q wall o.

1 minus r star r star is r i by r o whole square multiplied by a function F 1 minus function F 2 and I have given here the values of F 1 and F 2.

We now define; let us say, in case 1 where q wall o is heated. I will define N u o equal to at the outer wall h o D h by k, where the heat transfer is specified and that would be equal to q wall r o by k T wall minus T bulk into 2 into 1 minus r star square.

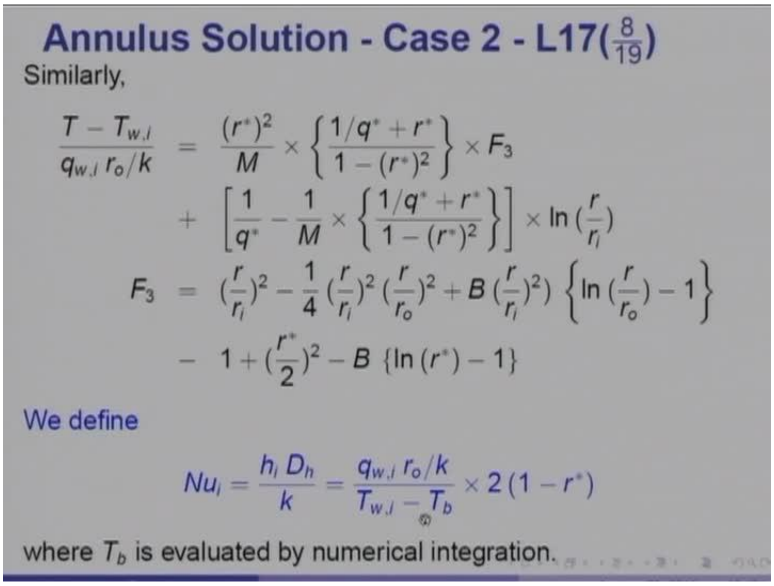
(Refer Slide Time: 19:29)



This is nothing but for the annulus D h is equal to 2 times r o minus r i and that is what is reflected here. So, if I take r i common you will get that.

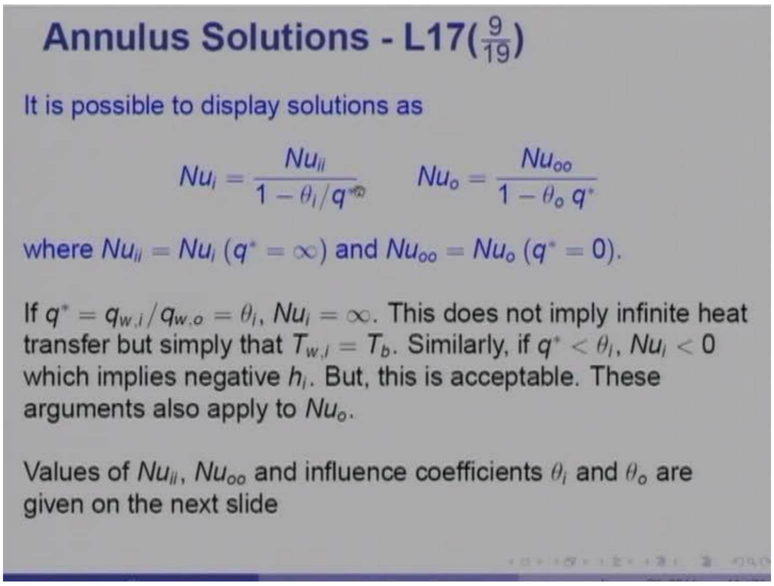
With this temperature, you must evaluate the bulk temperature. The bulk temperature evaluation becomes extremely difficult because there are logarithmic terms involved and numerical integration is the best way out. It does not require too much effort.

(Refer Slide Time: 19:57)



For case 2, where inner wall is heated that is q wall i is given. The solution can be expressed compactly in this fashion r star square by m into 1 over q star plus r star over that multiplied by a function F 3 plus all this and F 3 itself is given by a long expression like that. These expressions take quite a bit of algebra to arrive at and again in this particular case, I will define N u i equal to h i D h by k equal to q wall i r o by k divide by T wall i minus T bulk into 2 into 1 minus r.

(Refer Slide Time: 20:51)



Again using this temperature profile, I must evaluate T bulk which is required here by numerical integration. It is very fortuitous that the N ui N uo for variety of radius ratios can actually be expressed in the form I have indicated.

N ui would be equal to N uii into 1 minus theta i by q star and N uo can be expressed as equal to N uoo into 1 minus theta naught minus q star, where Nuoo and theta o and Nuii and theta i are simply functions of the radius ratio.

Now, q star was actually equal to q w i divided by q w o is equal to theta i. If suppose that was equal to theta i. Then, this gives a odd result that N ui would be equal to infinity but that should not worry us because all it implies is that it does not imply infinite heat transfer but simply that the inner wall temperature turns out to be equal to the bulk temperature. Therefore, N ui goes to infinity. Similarly, if q star is less than theta i then N ui will turn negative which implies negative h i.

Again, as we said repeatedly that this is not a particularly unacceptable situation. All it implies is that, since N ui is negative T wall must be greater than T bulk T wall i must be greater than T bulk.

About T wall i must be less than T bulk and as a result the N ui has turned out to be negative. Similar arguments would apply to N uo. So values of N uii N uoo theta i i and theta o are given on the next slide.

But to write the previous solutions in this manner, you again have to do little bit of algebraic manipulation to show that this theta i and N uii are functions of radius and only q wall minus q wall i over q wall o separates out as a function.

(Refer Slide Time: 23:18)

Annulus Solutions - L17($\frac{10}{19}$)

Annulus Solutions					
r^*	Nu_{ii}	θ_i	Nu_{oo}	θ_o	
0.0	∞	∞	4.364	0.0	circular tube
0.05	17.81	2.183	4.791	0.0293	
0.10	11.906	1.383	4.834	0.0561	
0.20	8.499	0.904	4.882	0.1038	
0.30	7.241	0.712	4.928	0.1454	
0.40	6.584	0.601	4.975	0.1822	
0.50	6.182	0.527	5.033	0.2153	
0.60	5.911	0.474	5.100	0.2455	
0.70	5.720	0.432	5.166	0.2733	
0.80	5.579	0.397	5.233	0.2991	
0.90	5.471	0.369	5.306	0.3233	
1.00	5.385	0.346	5.385	0.346	parallel plates

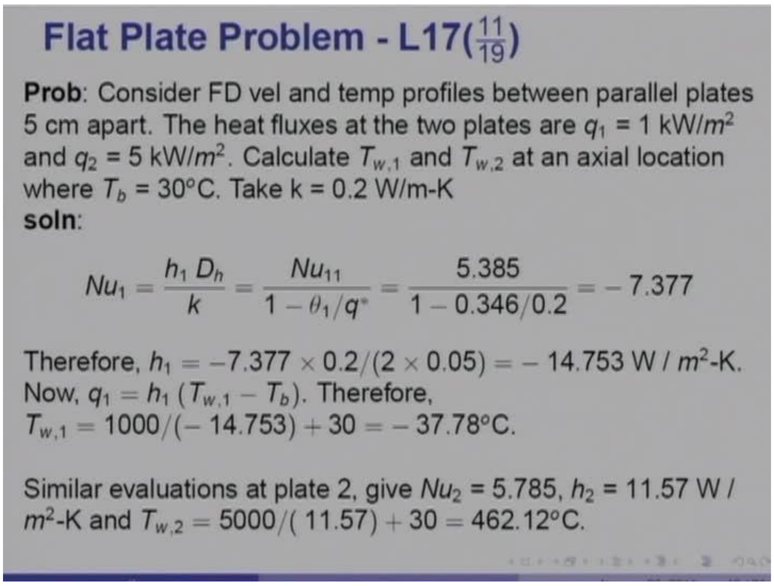
Here are the solutions that I have present from r star equal to 0, which is itself the circular tube and you can see N uii has no meaning but N uoo is 4.364 and its influence coefficient is 0 and N uo itself as you can see here.

Influence coefficient is 0 and N uo would be equal to N uoo. Therefore, that would be equal to 4.364. As I go on increasing the inner radius then you can see that the coefficients and the N uoo values go on changing.

When r star becomes 1, you have flow between parallel plates. In these case, both N uii and N uoo and theta i and theta o exactly identical. As they should be and our table confirms that they turn out to be the same.

Usually Nusselt numbers are not readily available for annulus but with this method you can see now you can evaluate them for any situation, for any q wall i and any q wall o that may be prescribed. You can readily recover the solutions for entire family of annulus solutions.

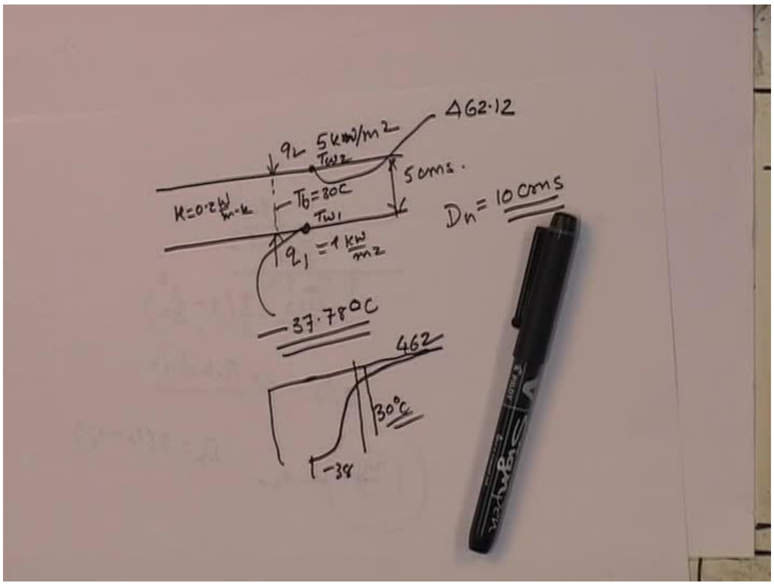
(Refer Slide Time: 24:50)



I will deliberately consider now a problem, in which I am considering flow between parallel plates that is the last entry in the table. Let us say, this is q 2 and this is q 1, the velocity is fully developed.

If you read the problem, it is like this. The flow between parallel plates, which are 5 centimeters apart. Therefore D h will be 10 centimeters and I have said q 1 is equal to 1 kilowatt per meter square and this is 5 kilowatts per meter square.

(Refer Slide Time: 24:58)



Since, the flow is fully developed the bulk temperature is rising with x and I am simply considering the k x axial position, where T bulk is 30 T bulk is 30 degree centigrade and the conductivity of the fluid is point 2 watts per meter kelvin.

The question is what will be T w 1 and what will be T w 2? That is the question that I asked. For example, the solution would run something like this N u1 will be h 1 D h by k equal to N u1 1 over 1 minus theta 1 q star.

If you see the influence coefficient for parallel plates is 0.346, N uii is 5.385 and N u1 will become 5.385 1 minus 0.346 divide by q star which is 0.2.

Therefore, the Nusselt number turns out to be negative 7.377 minus 7.377 and the heat transfer coefficient will also turn out to be negative minus 14.753. Since, q wall is 1 kilowatt h 1 is minus 14.753 and T bulk is 30, I can evaluate T w 1 as minus 37.8. So, T w 1 evaluates to minus 37.78 degree centigrade.

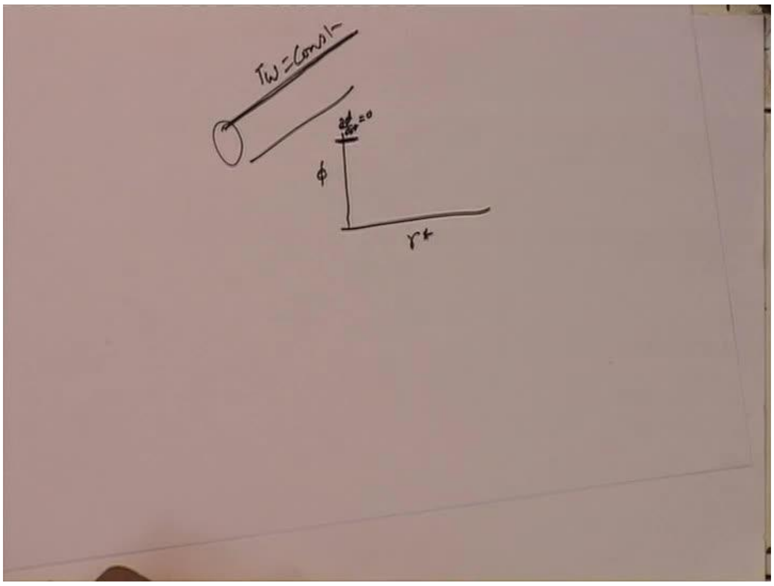
What about the outer wall? You can do the same thing N u2 equal to h 2 D h by k equal to N u2 2 minus 1 minus theta 2 into q star and again you will see N u2 will turn out to be 5.785. In this case, h 2 would be 11.57 and T w 2 turns out to be 462.12.

You can see what a great temperature difference there is. Basically, you have a situation, where very high temperature on this side and a very low temperature on this side. So minus 38 here, 462 here and the average temperature is 30 degree centigrade.

It is for this reason that we are interested in finding out the solutions because the wall no wall should become too hot or too cold because it might affect other processes on outside or inside of the annulus.

As much as the bulk temperature is only 30 degree centigrade, the wall temperatures can be enormously different for just very small heat fluxes of 1 kilowatt to 5 kilowatt meter square.

(Refer Slide Time: 29:28)



It is for this reason that we take particular care in evaluating our Nusselt number accurately in annulus flows. I have turned turn my attention to circular tube heat transfer, in which T w is constant and the flow is fully developed.

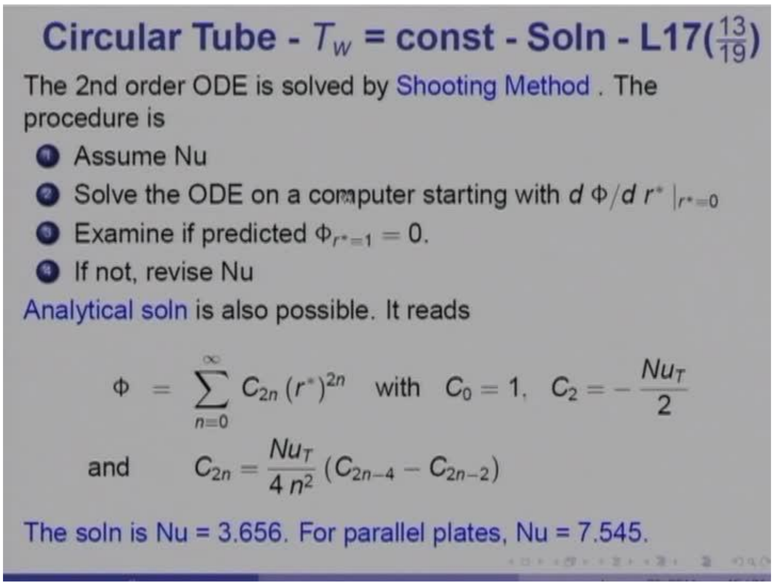
You can say that in this particular case, N u sub T means T wall is constant it will be h 2 R divided by k and that would be equal to d T d r at R into 2 R divided by T wall minus T bulk and that should be a constant.

Because h itself we say the fact that it is fully developed means h is constant and Nusselt number is a constant. Then, if you go back to slide 2 and recall that this is the condition for fully developed heat transfer for h equal to constant.

Then, you will see that d T by d x would be simply phi times d T bulk by d x and d T bulk by d x would be from heat balance 2 alpha divided by u bar R equal to q d T by d r. Remember, k d T d r at R equal to r is simply the heat flux that is coming in and d T bulk by d x would be related to d T by d r at the wall. If this was the original equation and if I now substitute for d T d x in terms of phi then the equation would transform to this form d 1 over r d by d r r star d phi by d r star equal to minus 2 N uT phi into 1 minus r star square. This is nothing but a second order ordinary differential equation with a boundary condition that phi r star at 1 which is the wall it is equal to 0 phi is 0 and d phi by d r star at the axis symmetry is 0 because it is a symmetry line and r star is equal to r divided by R. So, this kind of a second order ordinary differential equation is really solve by shooting method shooting method as i show you here.

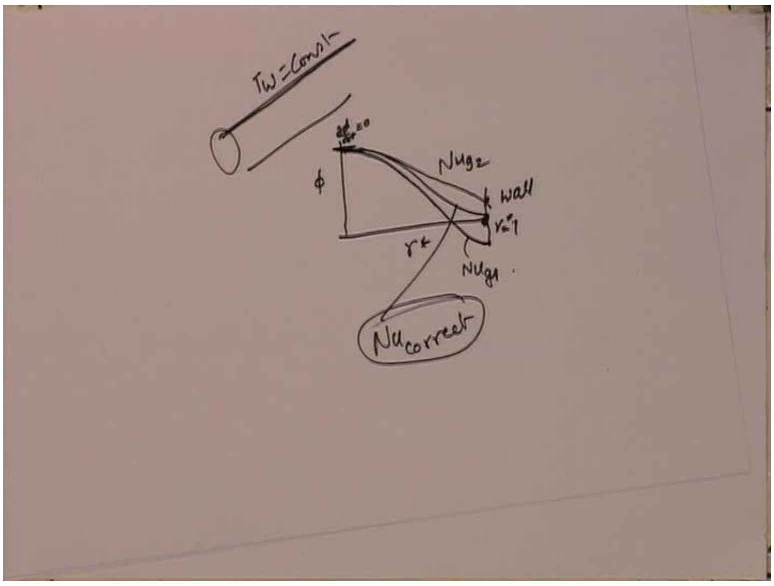
What one does is that, let us say this is r star and we want d phi by d r star equal to 0. The actual value of phi would not matter. All that we want is d phi by d r star equal to 0 it is a dimensionless quantity and we say d phi by d r star is equal to 0.

(Refer Slide Time: 32:15)



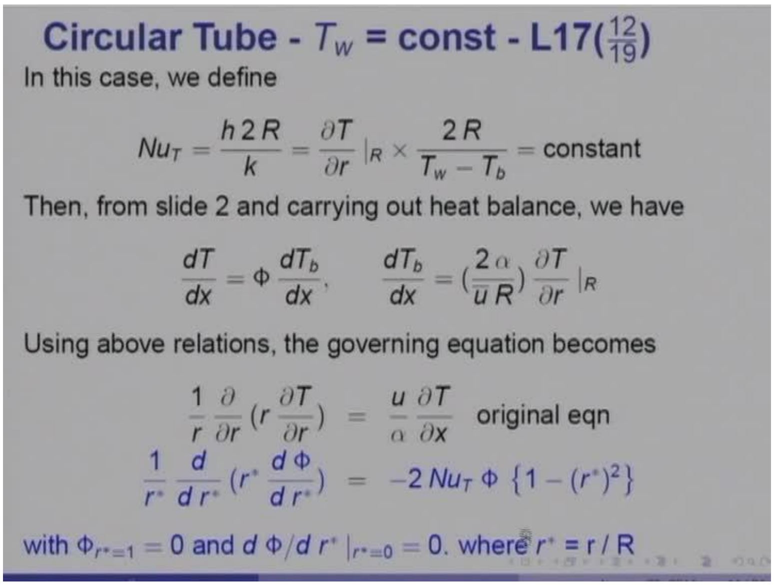
We assume a value of Nusselt number and solve the ordinary differential equation on a computer perhaps and you will come up with a value of phi at r star equal to 1 which is the wall, where we want phi star to be 0.

(Refer Slide Time: 32:31)



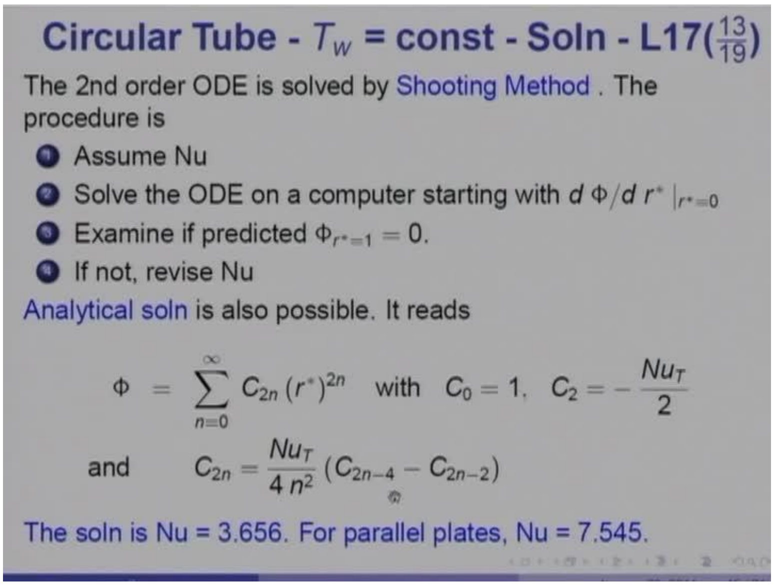
If our N u guess was incorrect then the chances are that you will come up with this. So, this is N u guess 1. Obviously, it is not equal to 1 as I want it. I take another guess and you will see you I may come up with that N ug2 and the error is on the positive side. The error here was on the negative side and I can use a bisection method to refine my error and generate a solution. Ultimately, I will come up with a value of N, which gives me phi at r star equal to 1 equal to 0 which is what I want.

(Refer Slide Time: 33:34)



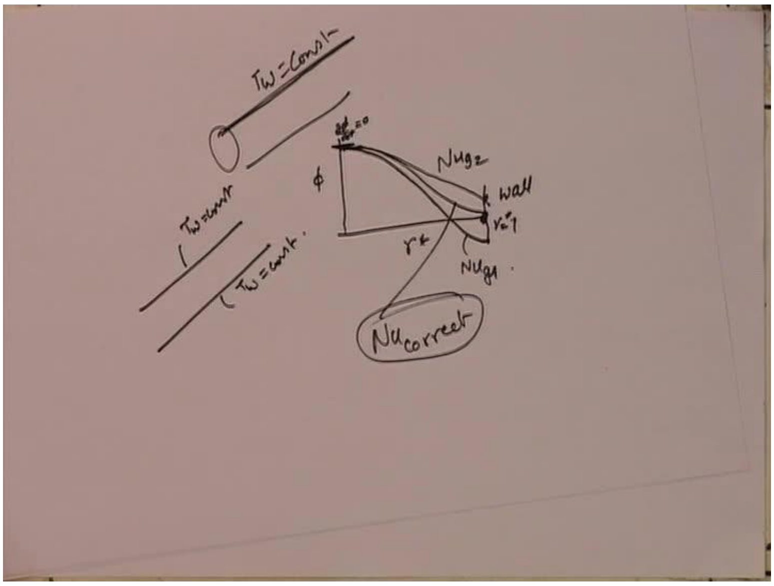
So, this is the N u correct. This would be the correct value of N u and if you do that in this equation you have to go on assuming a value of N uT and solve the equation.

(Refer Slide Time: 32:15)



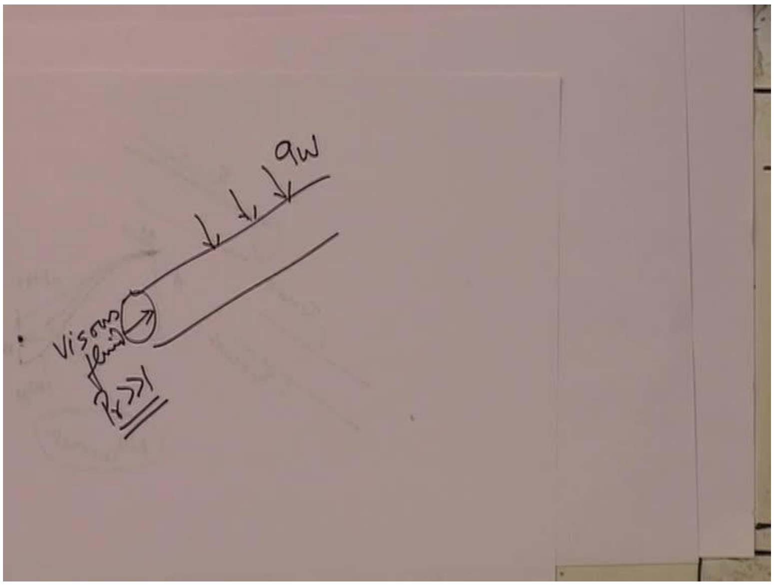
You will see N ut turns out to be 3.656 or sometimes taken as simply 3.66. It is also possible to solve this equation analytically in a series form and the solution has been given here.

(Refer Slide Time: 34:12)



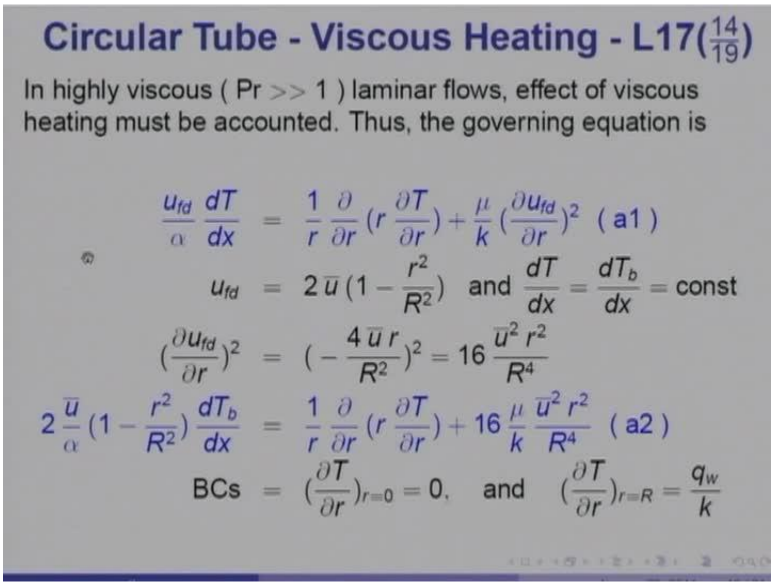
Again the value of N uT works out to be 3.656. If I have to do this same problem for flow between parallel plates with T wall equal to constant on both sides. Then, the Nusselt number that I would develop is a 7.545. So, that I leave you as an exercise to be done.

(Refer Slide Time: 34:41)



We now turn our attention to one more case and that is the case of highly viscous fluids, flowing in a tube which is highly viscous; its Prandtl number is much greater than 1.

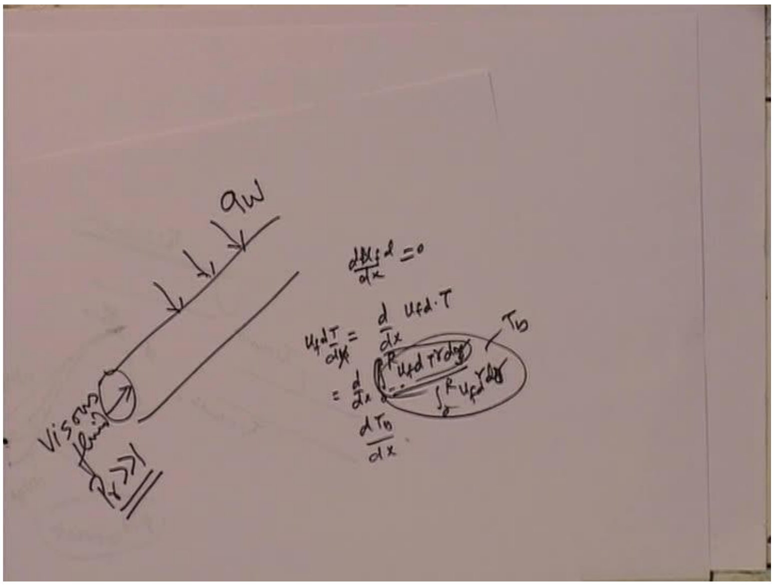
(Refer Slide Time: 35:05)



In this case, there is a constant wall heat flux is applied. In that case, you will recall from our energy equation that because viscosity is very high viscous dissipation term mu d u by d r whole square becomes as important as the conduction term. Therefore, it must be retained in the energy equation, u fd is 2 u bar 1 minus r square by R square and d T by d x equal to d T bulk by d x is equal to constant, d u fd by d r square from this thing will evaluate the 16 u bar square r square by R 4 and substituting these things in here, I get 2 u bar by alpha 1 minus r square by R square d T bulk by d x equal to 1 over r d by d r r d T by d r plus 16 times mu by k u bar square r square divided by R 4 this is equation a2.

The original equation is a1 and the boundary conditions are again at the axis of symmetry, this would be 0. At the wall, it is equal to q wall equal to k d T d r. The main thing is I said d T bulk by d x will be constant but what will be its value?

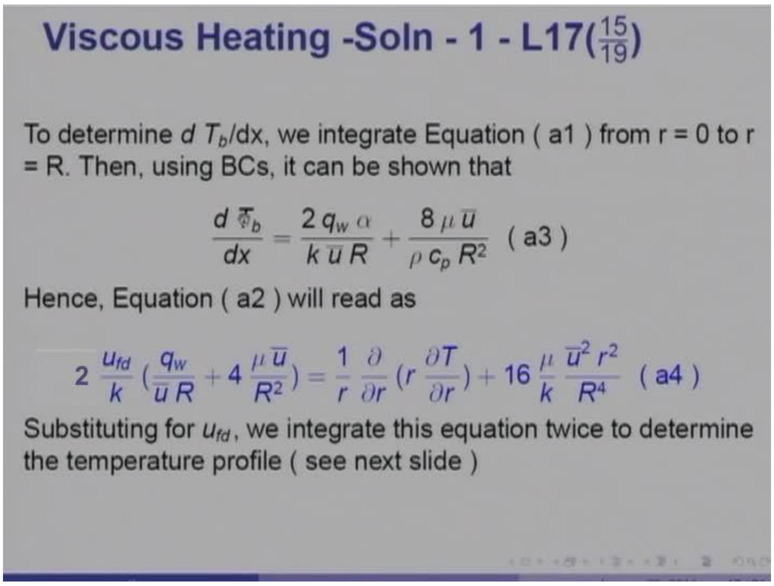
(Refer Slide Time: 36:40)



That is what we need to evaluate. To do that on the next slide, what I will do is, I will simply integrate this equation. You can see the left hand side, if I integrate over area u fd and d u fd by d x is equal to 0, because its fully developed u. fd d T by d x will actually become d x will actually become d by d x of u fd into temperature and I integrate this d by d x of 0 to R u fd T d y.

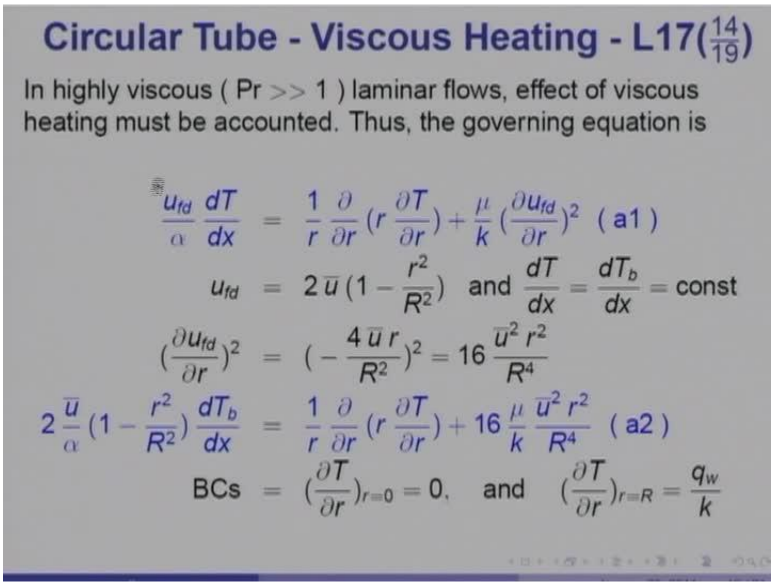
Then, you can see this will give me d T bulk by d x because when this is divided by in integral 0 to R u fd y d y, I mean d r r d r rather this is the definition of T bulk.

(Refer Slide Time: 37:40)



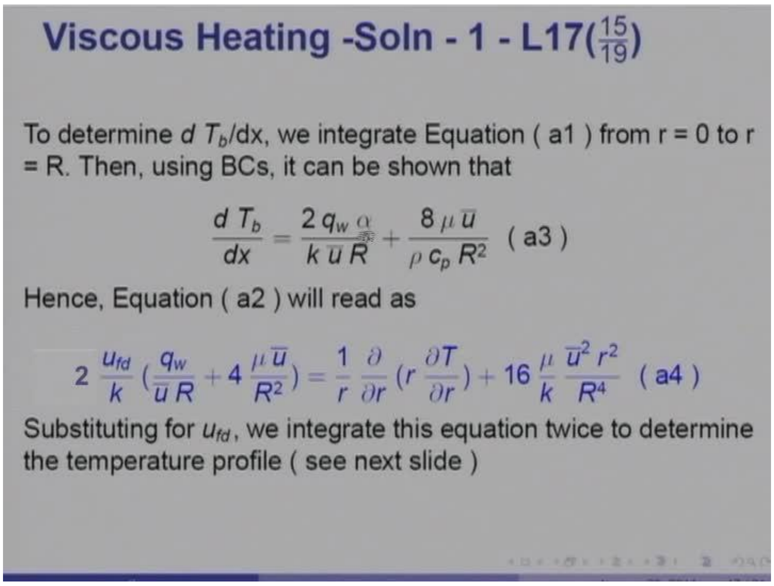
This is the constant u bar r d r is nothing but the mass flow rate through the channel or through the circular tube. You will see that to determine d T bulk by d x, I would simply integrate this equation from 0 to r of both sides of this equation.

(Refer Slide Time: 35:05)



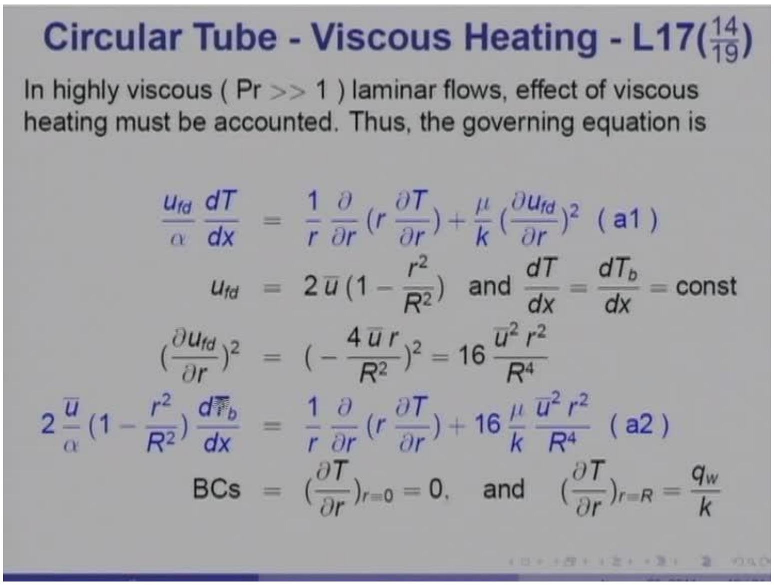
The left hand side will give me d T bulk by d x. This I will evaluate because I will get r d T d y d r at the wall and r d t by d r at the axis, which is 0. I have substitute q wall, which is known for the upper boundary condition.

(Refer Slide Time: 37:40)



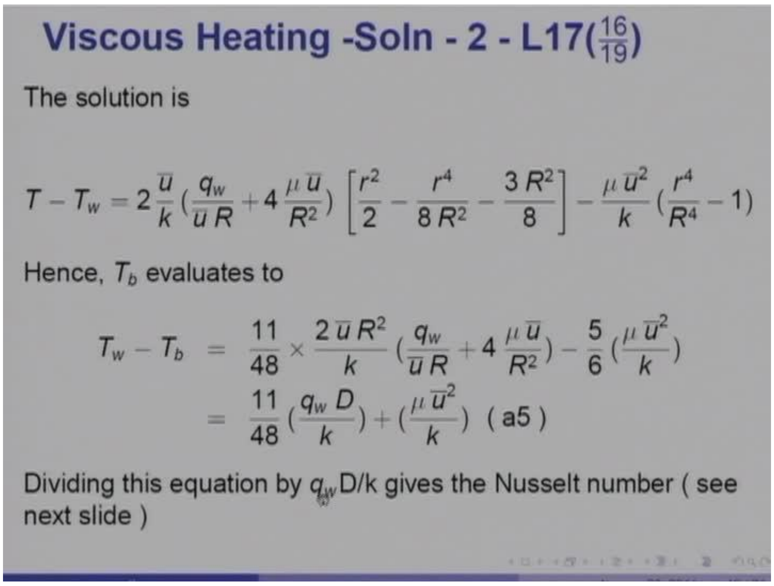
Likewise, you integrate this from 0 to r. The result is d T bulk by d x would turn out to be 2 q wall alpha k u bar R, which is the case when there is no viscous heating is included but now you see that the bulk temperature rise is also influenced by the amount of viscous heating that has taken place.

(Refer Slide Time: 35:05)



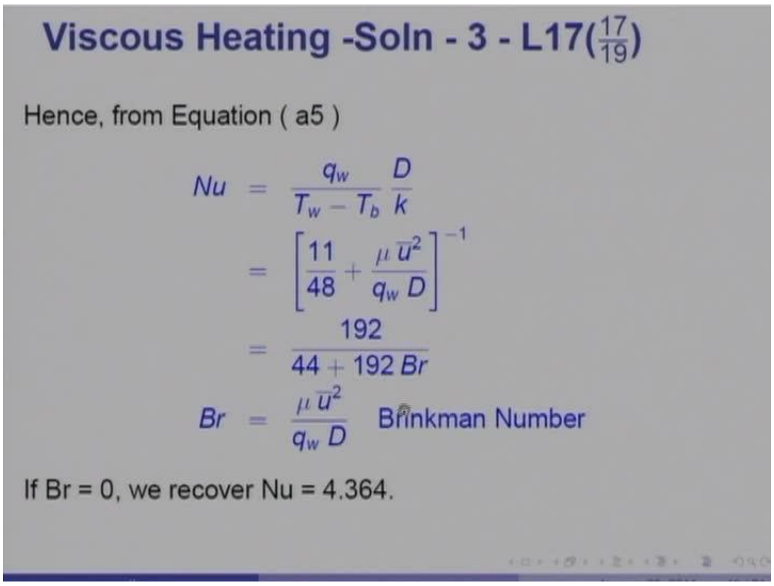
That is the equation a3. If I substitute d T bulk by d x in equation a2, which is this equation so if I substitute for d T bulk by d x, I will have an equation which looks like this. I must integrate this equation twice and determine 2 constants of integration to determine the temperature profile.

(Refer Slide Time: 37:40)



The temperature profile looks something like this. Remember, again if mu was 0 that is if viscous dissipation was neglected then that term would be 0 and here that term will. We will recover our original case of solution without viscous heating. Then, the T bulk evaluates this temperature must be integrated with u fd and T wall minus T bulk would evaluate to 11 by 48 q wall D by k plus mu u bar square by k.

(Refer Slide Time: 39:37)



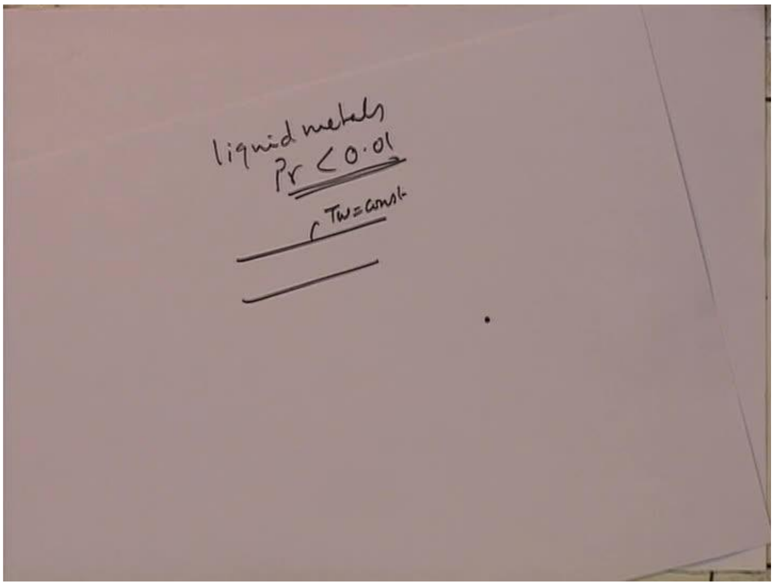
Now, it is very easy to define, if I divide both sides by q wall D by k then the Nusselt number would be obtained as here 11 by 48 plus mu u bar square divided by q wall d raised to minus 1.

This quantity is called the Brinkman number, after the scientist who first solves this kind of a problem. This can be written as 192 divided by 44 by 192 multiplied by Brinkman number.

A Brinkman number is 0; then you would readily recover 192 by 44 equal to 4.364. The effect of brinkman number for high Prandtl number fully developed heat transfer becomes (( )) higher the brinkman number depressed would be the heat transfer coefficient

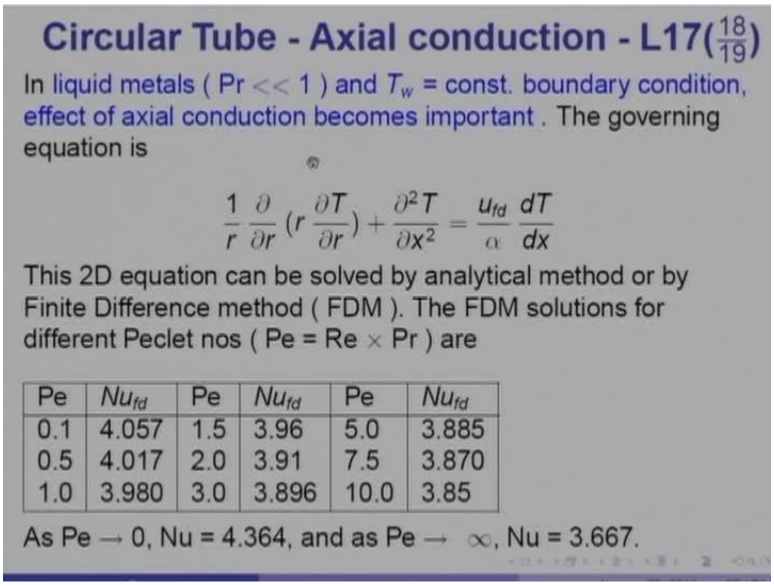
I consider the case of liquid metals, when Prandtl number is very small. As you will recall, for liquid metal the Prandtl number is less than 0.01.

(Refer Slide Time: 40:51)



The actual conduction term, which we have been neglecting so for becomes important, particularly when the wall temperature is constant; you have liquid metal heat transfer. Then, the governing equation would look as I have shown here.

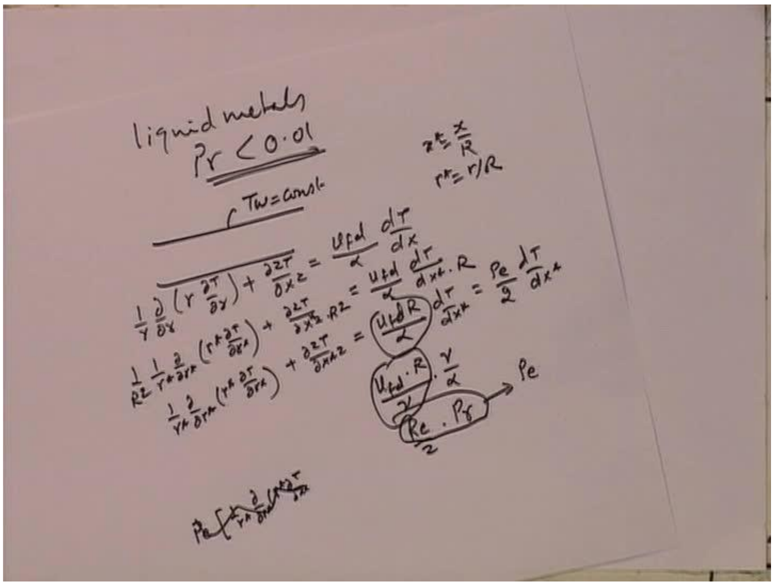
(Refer Slide Time: 41:11)



There will be the radial conduction term and the axial conduction term equal to u fd by alpha d T by d x. Now, this is a 2 dimensional equation it involves dependent variables r and x.

It can be solved analytically or now a day's much more simply by finite difference method. I give below the finite difference method solutions for different values of Peclet number. I wanted to appreciate how Peclet number comes about.

(Refer Slide Time: 41:51)



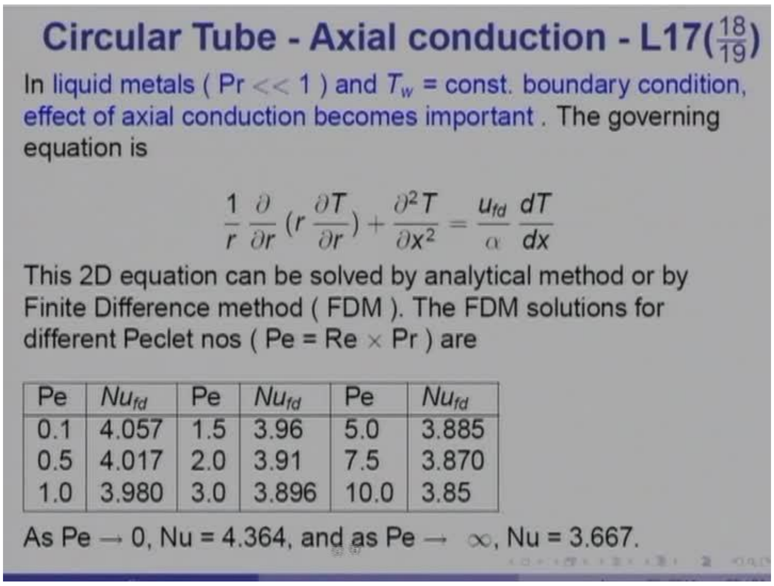
So, the governing equation is 1 over r d by d r r d T by d r plus d 2 T by d x square equal to u fd divided by alpha d T by d x. Now, if I define x star is equal to x by R and r star is equal to r by R. Then, you will notice that this will become simply 1 over r star d by d r star into r star d T by d r star plus d 2 T by d x square R star square and equal to u fd by alpha d T by d x star into R.

You will see that this this becomes sorry there would be 1 over R star here also. if I multiply through by r star r square. Then, you will see this becomes d by d r star into r star over d T by d r star plus d 2 T by d x star whole square is equal to u fd R divided by alpha d T by d x star.

What is that? u fd into R divided by nu into nu divided by alpha. If I do that, then you will see that, this is Reynolds number divided by 2 and this is Prandtl number nu divided by r the product of Reynolds and Prandtl is called the Peclet number.

This is essentially the Peclet number divided by 2 d T by d x star. Therefore, the equation can be written as Peclet number into 1 over r star d by d r star r star d T by d r star and so on and so forth.

(Refer Slide Time: 41:11)

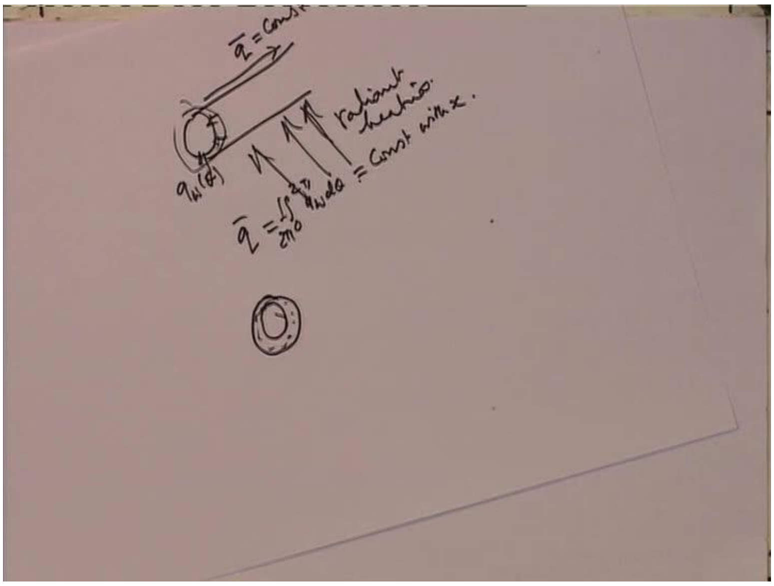


The Peclet number essentially determines the behavior of the solution. I have obtained this, remember because Prandtl number is so low the product of Reynolds and Prandtl can be very low in laminar flow.

So, I have taken values of 0.1, 0.5, 1.0, 1.5, 2.0, 3.0, 5.0, 7.5 and 10 and obtain solutions by finite difference method and here are the solutions of very low Peclet number. The Nusselt number is 4.057 but as I increase the Peclet number, which means allowing for more and more actual conduction, then you will see that this becomes even more the Nusselt number goes on reducing, when Peclet number is about 10 and the wall temperature is constant; so a large Peclet numbers the actual conduction effect goes on almost becoming negligible and you arrive at 3.85.

In effect as Peclet number tends to 0, you get only constant heat flux solution. So, N u equal to 0.364 and as Peclet number tends to infinity you get the constant wall temperature solution 3.667.

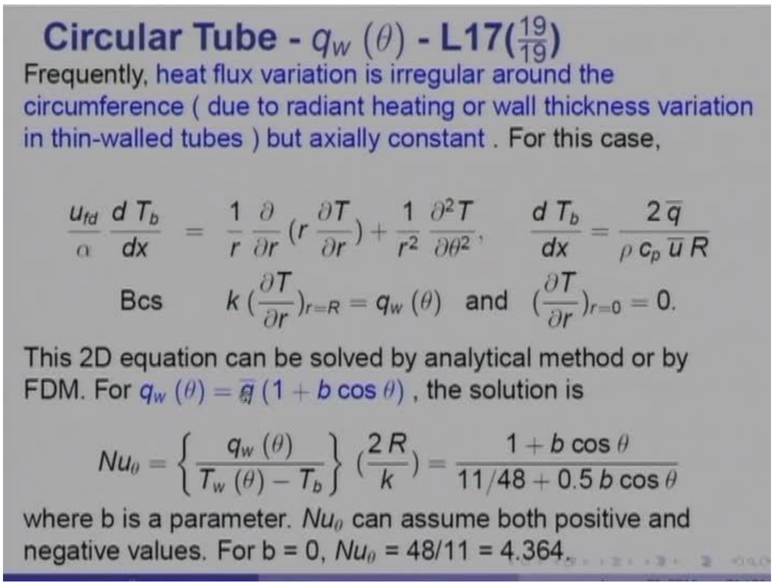
(Refer Slide Time: 45:40)



There is yet another case of considerable interest, which you might like to know and this is the case of a tube, which may be receiving let us say radiant heating. Then, clearly around the circumference it will have a variable heat flux q wall theta; although, axially it will be constant. At each cross section q bar will be simply 0 to 2 pi 1 over 2 pi q wall d theta would be a constant with x. So, this is the case which would occur for radiant heating.

Sometimes, it will also occur for a tube of uneven thickness and it is being electrically heated. So, there is internal heat generation within the tube and therefore it will receive circumferentially varying heat transfer.

(Refer Slide Time: 47:20)



In that case, because of the constant heat flux condition, the equation would become again d T bulk by d x would be constant given by that and but now you must allow for conduction both radially as well as in the circumferential direction. Again this 2 D equation can be solved either analytically or by finite difference method and for q wall theta as this simple circumferential variations of heat flux.

The solution turns out to be this 1 plus b cos theta 11 by 48 0.5 b cos theta, where b is simply a parameter. You can see, in this case N u theta along the periphery of the Duct can assume both positive as well as negative values but that should not disturb you because that is expected. All it tells you is whether the T wall bulk is greater than T bulk or less than T bulk but if b is equal to 0 which implies q wall theta is uniform along. Then, you readily recover 4.364 and that is what we had. So, this particular type of problem is very important because hot spots have to be avoided on walls, when you have uneven heating on all the side.

In the next lecture, we shall consider heat transfer in non-circular and arbitrary section Ducts. Thank you.