Convective Heat and Mass Transfer

Prof. A. W. Date
Department of Mechanical Engineering
Indian Institute of Technology, Bombay

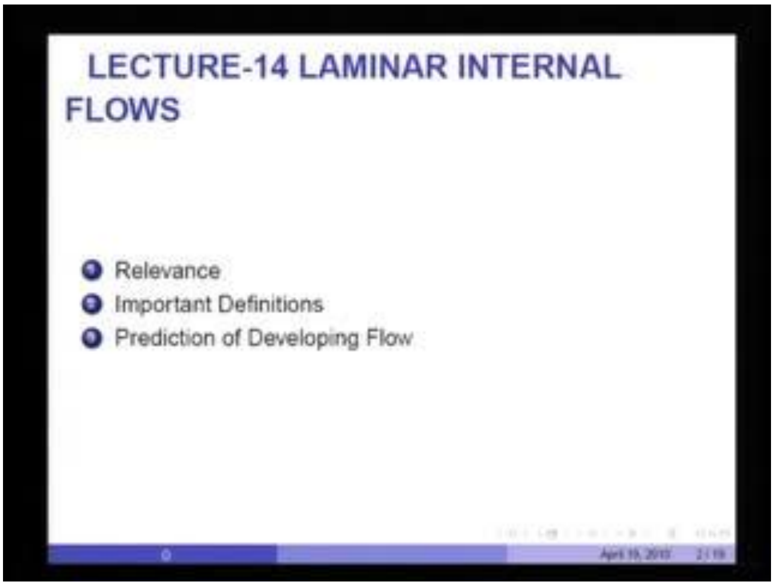
Module No. # 01

Lecture No. # 14

Laminar Internal Flows

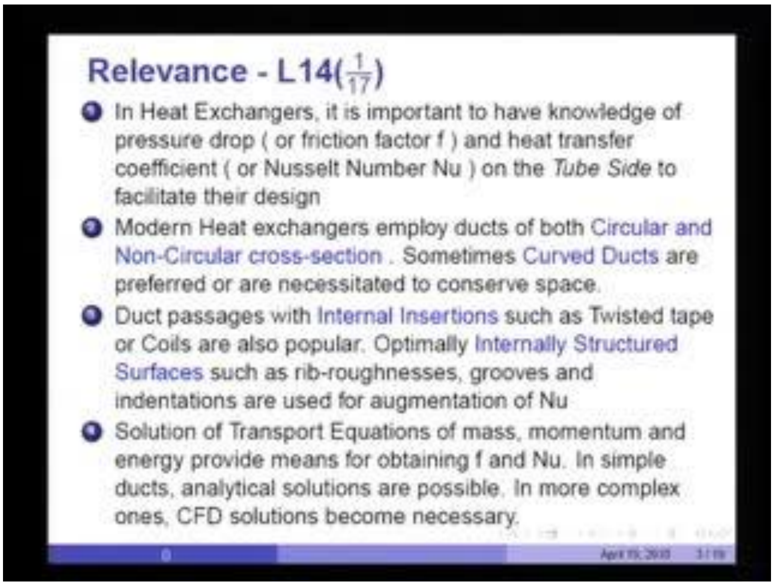
We have looked at both velocity and temperature boundary layers. We have obtained similarity solutions as well as integral solutions for a variety of conditions; the free stream velocity, the suction and blowing velocity, and the pressure gradient which of course, is accounted by free stream velocity variations as well as the wall temperature varies with and without effect of viscous dissipation.

(Refer Slide Time: 01:02)



We now turn to another important class of flows called internal flows that is, the topic of my discussion today is b laminar internal flows. I will explain first, the relevance of internal flows with important definitions and prediction of developing flow as an example of application.

(Refer Slide Time: 01:24)

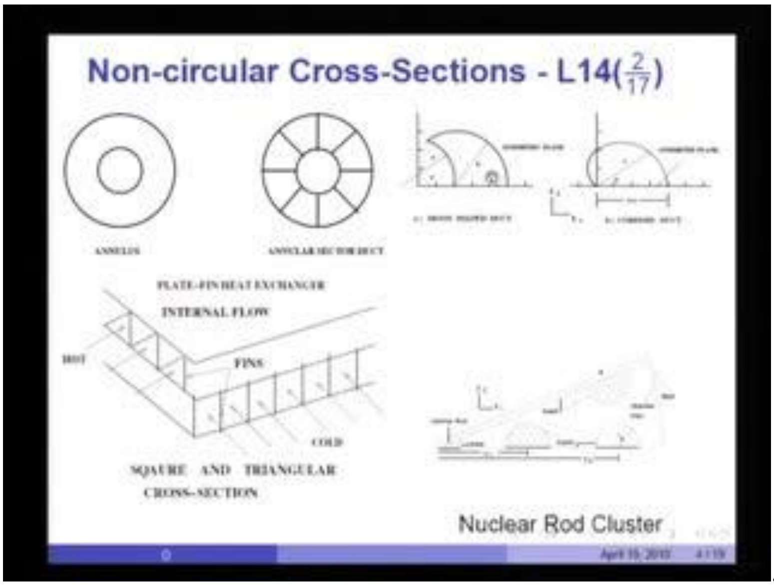


Internal duct flows are principally of interest in heat exchangers where it is important to have knowledge of the pressure drop - for a given length of the tube - or what we call the friction factor and also the heat transfer coefficient or the Nusselt number on the tube side of the heat exchangers to facilitate their design. Modern heat exchangers of course, employ ducts of both circular and non-circular cross sections. Sometimes even, curved ducts are preferred or are necessitated big in order to conserve space.

Duct passages with internal insertions such as the twisted tape or coils are also popular. Optimally internally structured surfaces such as rib-roughness, grooves and indentations are also used for augmentation of the heat transfer coefficient or the Nusselt number. Most of these modern applications represent examples of very complex ducts as oppose to the very simple case of a flow in a round tube of flow between two infinite parallel plates.

So, solution of transport equations of mass, momentum and energy provide means for obtaining f and Nu which is up interest in design. In simple ducts, analytical solutions are possible. In more complex ducts however, one needs CFD solution and unfortunately that would be a topic outside the scope of the present lectures but, we shall consider a few non-circular ducts and treat them analytically. In the slides to follow, I will show you few examples of ducts of non-cross sections and structured internal ducts and also curve ducts.

(Refer Slide Time: 03:36)



So, here are a set of examples. The first one on the left here is of course, the simple annulus with the inner tube and an outer tube. The flow is between the 2 tubes; sometimes the space between the inner and outer tubes is also connected by fins. What we have is an annular sector duct. The fluid flows through this sector and all sectors behave in the similar fashion and therefore, interest is in an annular sector.

The plate pin needs exchanger - is of course, well known to you; this is the top plate, this is the middle plate, this is the bottom plate and there would be several stacks of plates like this (Refer Slide Time: 04:25). The flow is cross flow in the sense, then between the top 2 plates the flow hot fluid may flow from left to right. Here, in the bottom 2 plates the flow may be directed as shown here the coal fluid.

Again, the 2 plates are connected by fins which form ducts of triangular cross section. Here, the coal fluid flows through duct of square or rectangular cross sections. So they are very commonly encountered ducts in plate fin geometry of more complexity in extremely small feed exchangers. What I have called mini heat exchangers or micro heat exchangers with very high surface area to volume ratios bordering on 1200 to 1500 meter square for cubic meter, ducts of extremely complex shape are employed.

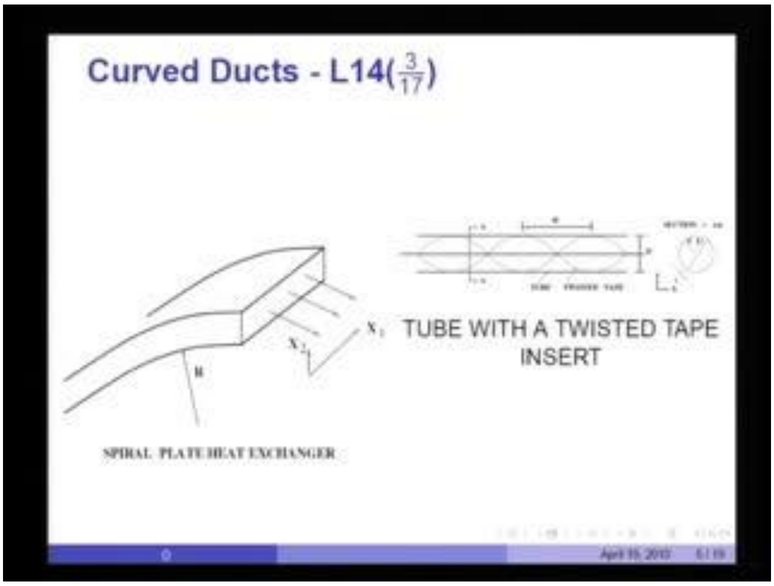
Here is an example of a duct which is moon shaped and what I have shown here is a half section of the moon, x axis being the symmetry axis. This is the inner edge of the moon;

this is the outer edge of the moon. Likewise the ducts which are of cardioid shape again x axis represents the symmetry axis and I mean the symmetry plane and this is the outer boundary of the cardioid duct.

In nuclear rod clusters, in a circular shell there are large number of rods kept there is a 1 rod in the center then, there are several rod in the first rod ring; then, another set of rods in the second rod ring and so on and so forth. So it may go like 1 6 and so, this is the 19 rod plus the geometry, so it would go like 1 6 7 and 12 in the outer ring making 19 rods.

The flow of course is, this is the symmetric plane again, this is also symmetry plane and the flow takes place in the inter space between the rods and you can see how complex the flow cross section; then it is a very good example of a complex shape duct, that is encountered in nuclear reactors. The plate fin heat exchanger that I show you here (Refer Slide Time: 07:07), if the plates here are flat in order to save space many times the plates are actually wound in a spiral.

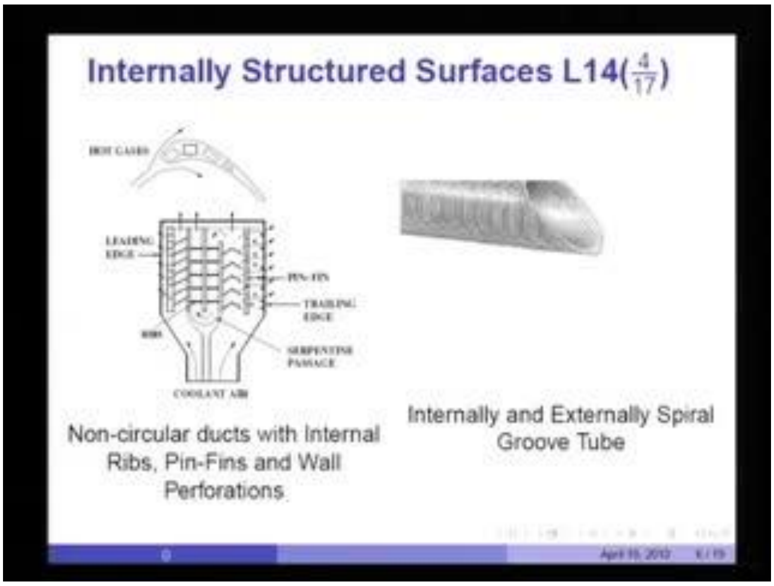
(Refer Slide Time: 07:18)



What you then have are several spirals starting from the center and becoming ever bigger in the radius R and what I have shown here is a typical section of a curve spiral plate heat exchanger duct.

So this is the outer wall, this is the inner wall and the duct itself is of has a radius of curvature R the fluid is flowing through this and coming up like that. Another example is that of a plane tube with a circular cross section but, in which a metal strip of width equal to the diameter the tube is twisted about the axis of the tube. What the tape does then is to divide the circular cross section into 2 semicircular ones and each semicircular cross section then twist along the axis of the tube as one goes down the tube in the flow direction. So, you have is 2 semicircular cross sections twisting about the axis of the tube.

(Refer Slide Time: 08:38)



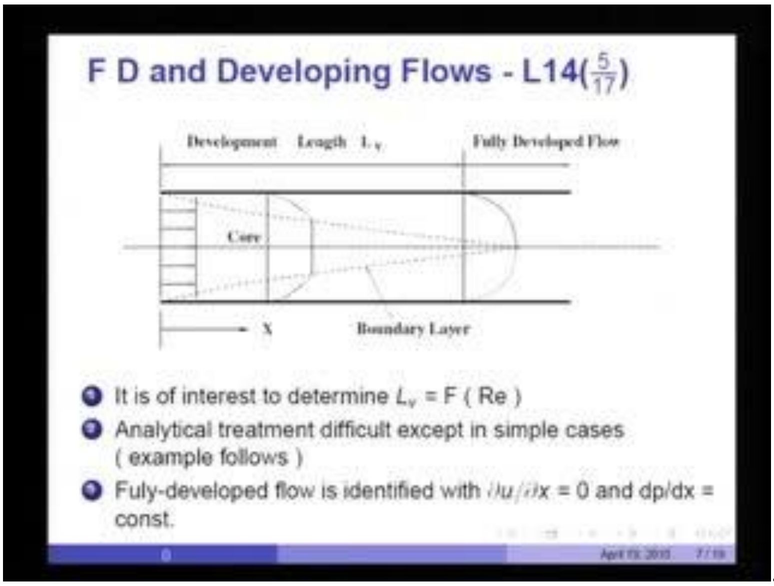
It forms a very complex curved duct with a non-circular geometry. Internally structured surfaces - the gas turbine blade is a classic example of how surfaces are internally structured. First of all, notice that this is the cross section of the blade and you will see ducts of non-circular cross section. Each duct has ribs in it; sometimes straight sometimes at an angle as you can see here, some ducts are straight whereas, others are having a bend. Then in some ducts some part of the blade you have a duct which has ribs and then one wall is perforated so that the flow goes this way and then also comes out with a impinging air on the leading edge of the blade and the remainder air goes out of the blade to the top.

Likewise at the rear end, the trailing edge of the blade you have flow coming in from here and which passes on through this perforation in this wall as well as some flow goes directly into this trailing edge passage. In this passage you have number of solid pin-fins their design to make the trailing edge strong and you have perhaps the most complex of internally structured passages or ducts that you can encounter in engineering practice in a gas turbine blade.

A tube of this type which is extensively used in refrigeration and air conditioning industry and has spiral grooves etched on both inside surface as well as the outside surface. These grooves are cutout, this is the copper tube and you can see how tiny the near surface structure is. In fact, the height of the grooves a is quite small and in fact if you have similarity with turbulent laminar sub layer in a turbulent flow, then the height of the tube just exceeds the laminar sub layer.

The purpose of the tube grooves is to make sure that the sluggish laminar sub layer is continuously disrupted and therefore, enabling enhance heat transfer; same is the purpose of internally rib roughness and so on and so forth.

(Refer Slide Time: 11:46)



Our interest here now is on laminar flows, all the practical applications are in turbulent regime. We shall take up the turbulent flow a little later but, let us concentrate first on the laminar flow in a duct. Let us say the flow enters with the uniform velocity u bar then, as soon as it encounters the top and the bottom wall there would be viscous action and therefore, a boundary layer development as shown here. Inside the boundary layer, there would be a reduction in velocity from its initial value and to compensate for that there would be increase in velocity inside what is call the core region.

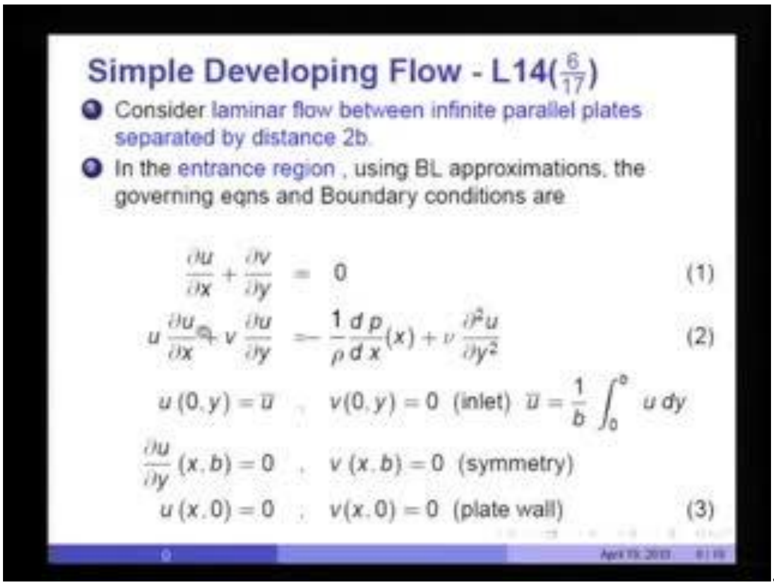
For all practical purposes, one could imagine the core to be almost of uniform velocity but, the velocity itself would be greater than what it was at the inlet. So we have a core region with the velocity continuously accelerates and is greater than that of the inlet velocity but, in the boundary layer region you have a velocity which is lower than the inlet value.

Ultimately the boundary layers from the 2 walls meet, when they meet we say the flow is no longer in state of development but, has reached a state of fully developedness; by that we mean that the velocity profile that is now generated would sustain itself without any change in the axial direction. Therefore, the fully developed flow is identified with the du dx equal to 0 where u is the velocity in the axial direction as well as the pressure gradient in the axial direction is constant.

What is of interest from practical stand point is the estimate of development length L v and L v as we can very well imagine would be a function of Reynolds number. The higher Reynolds number, the higher will be the length L v in this entrance region of the duct.

Analytical treatment is usually quite difficult except in very simple cases; for more complex cases of complex cross sectional ducts and so on so forth or curve ducts, one really has to adopt CFD base procedures computer procedures. None the less I would take up a very simple case as shown on the next slide. It is essentially a case in which these are two infinite parallel plates both in the z direction as well as in the x direction; the z direction being and the y direction is measured from the bottom of plate a going forward.

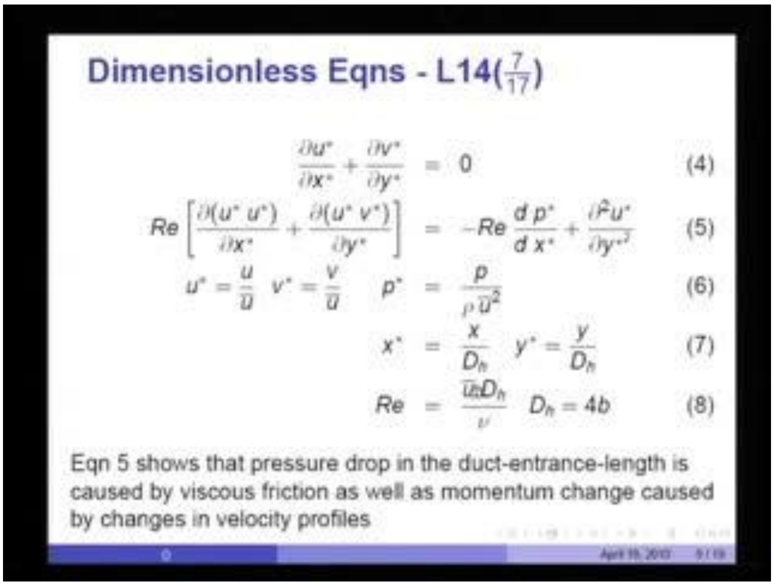
(Refer Slide Time: 15:03)



The inter plate distance is going to be 2b, so the distance between one wall and the axis symmetry is simply b as you will see on the next slide. So, we consider laminar flow between infinite parallel plates separated by distance 2b and in the entrance region using boundary layer approximations the governing equations and the boundary conditions are as given here. First of all, you will have the continuity equation then, the momentum equation in which you have the conductive terms and then the viscous term.

The velocity at x equal to 0 for all y is the average velocity u bar with which the fluid enters the duct. The v in the inlet plane again is 0 and u bar at any cross section would be 1 over b 0 to b u dy would be the average velocity. At the symmetry plane du by dy at all x will be 0 and so the v at the symmetry plane will be 0. At the bottom wall x 0, u will be 0 and so it would be b 0 that means, there is no suction or blowing into the duct all this is quite familiar and understandable by now.

(Refer Slide Time: 16:16)

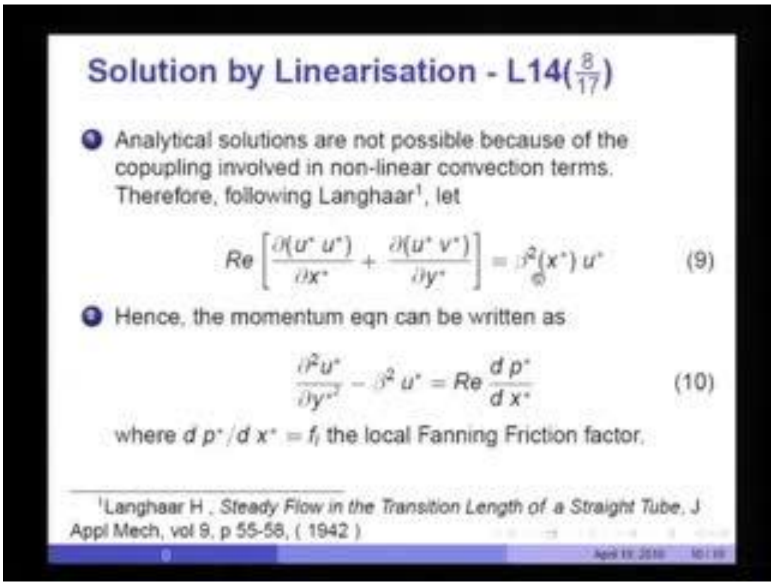


We non-dimensionalize this equation first in which velocity is made as u star as u over u bar, v star equal to v over u bar, p star equal to p over rho u bar square, x star the distance x over D h, y star y over D h; D h is the hydraulic diameter. Reynolds number also is defined based on the mean velocity u bar and the hydraulic diameter. D h for flow between parallel plates separated by distance 2b is twice the distance between the plate and therefore, equal to 4b is the hydraulic diameter.

Equation 5 here shows that, the pressure gradient is balanced by both the momentum change terms or the convection terms and also the viscous terms, the d 2 u d y square. So viscous friction as well as momentum change is caused by change in velocity profiles along the length predict balances the pressure gradient which itself varies with x. Now what makes it difficult to obtain solutions to this type of equations is the fact that remembers this first of all, these are 2 equations and we had 3 unknowns: u, v and pressure.

So, we really have problem here that unlike in the boundary layers where we specified the pressure gradient; we cannot do so in internal close and we have to do some tricks. So, the coupling that exists between the continuity equation and the momentum equation is due to convection terms here and that is what makes this equation said in non-linear equation said because of the coupling and we have to find ways to override this coupling.

(Refer Slide Time: 18:29)

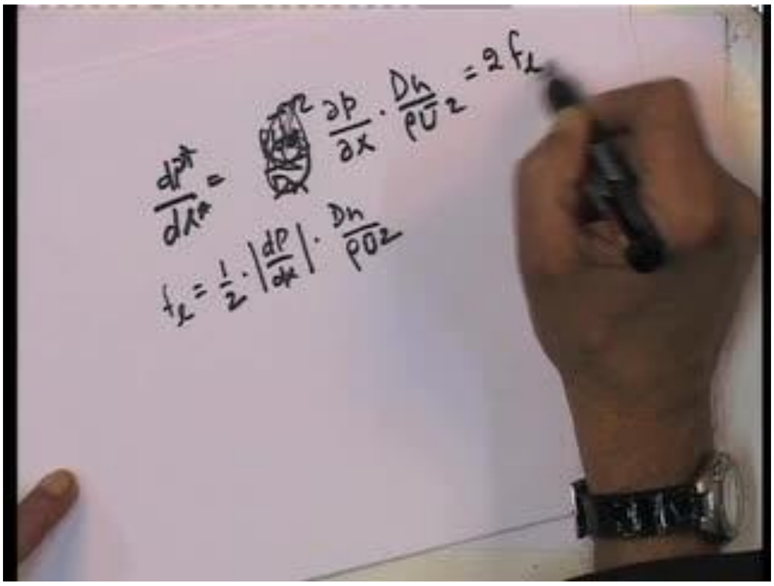


This is precisely what was done by a man call Langhaar in a paper published in 1942; title - steady flow in the transition length of a straight tube and was published in journal applied mechanics volume 9.

He said the left hand side or the momentum change terms or functions of x and y but, we shall write them as beta squared multiplied by velocity; beta square is a constant at 1 cross section, its value will change from cross section to cross section but, at a given cross section beta square is a constant and u star is the function of x and y of course, so that the left hand side and the right hand side I have the same dimensions.

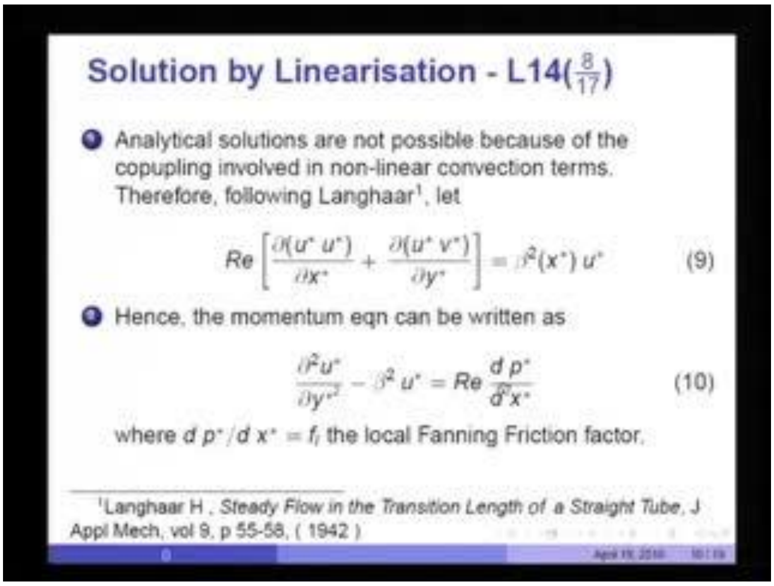
So, if I substitute for the left hand side beta square into u star then, the momentum equation can be written as d 2 u star by dy star square minus beta square u star equal to Re dp star by dx star.

(Refer Slide Time: 19:52)



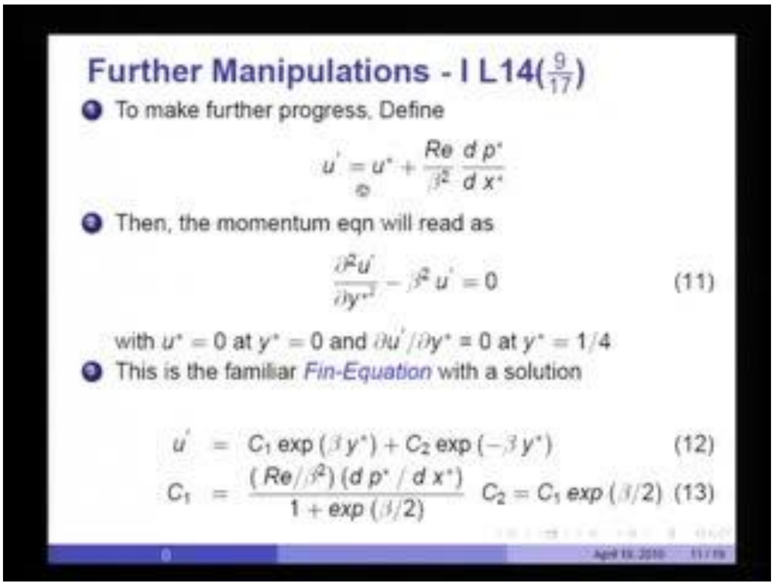
You will quite easily derive, that the pressure gradient dp star by dx star; I will show you that remember dp star by dx star is essentially rho u infinity rho u bar square into dp divided by D h dx. In other words, we define f l the local friction factor as 1 over 2 dp dx into D h by rho u bar square sorry this should be D h over rho u bar square then you will notice that this is nothing but, dp star by dx star is nothing but, 2 times f l.

(Refer Slide Time: 20:49)

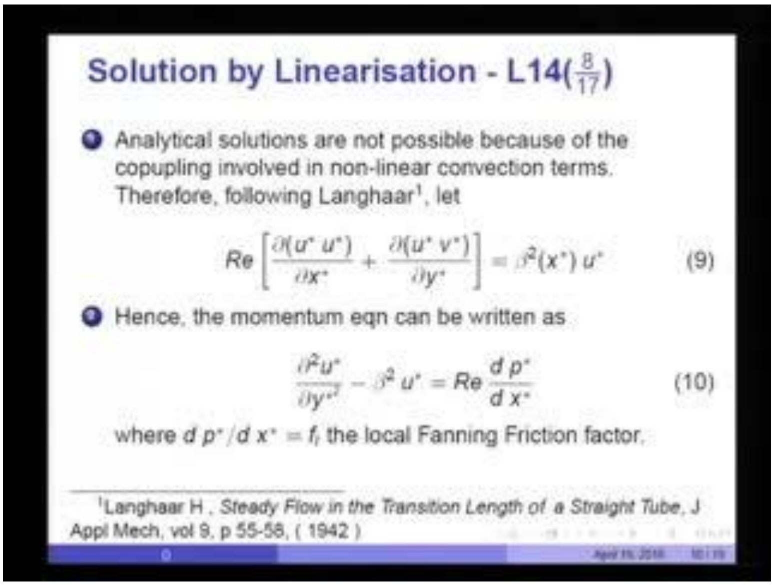


Essentially this is the frictional term, this is in a way represents now the convection term and this is the pressure gradient term (Refer Slide Time: 20:50).

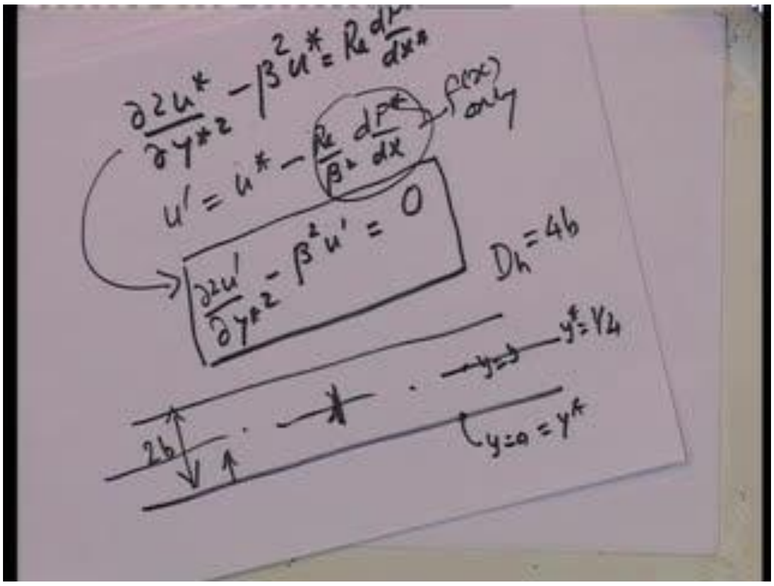
(Refer Slide Time: 21:03)



(Refer Slide Time: 21:08)



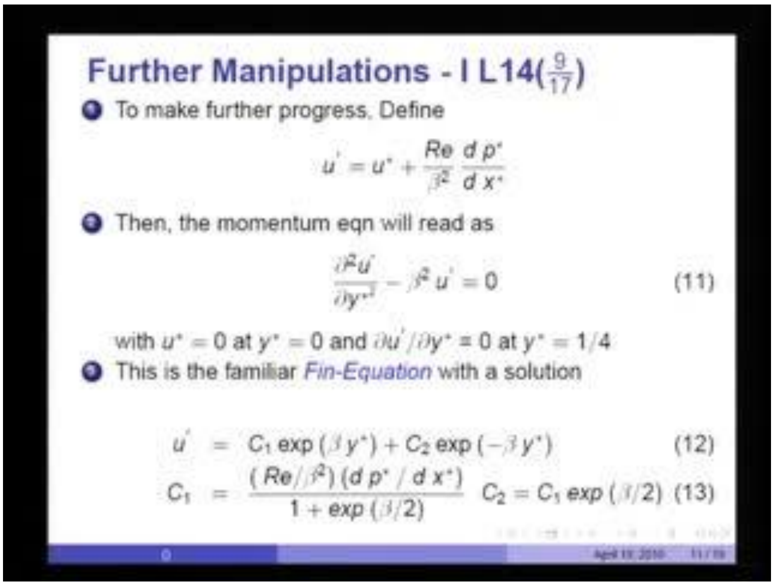
(Refer Slide Time: 21:17)



We will manipulate this a little further. So we define u dash, let me write down this equation, the equation that we wrote down is d 2 u star by dy star square minus beta square u star equal to Re dp star by dx star.

So I introduce here u dash equal to u star minus Re by beta square dp star by dx then, this equation if I substitute for u star and notice that this is the function of x only and therefore, with y this will simply become d 2 u dash by dy star square minus beta square u dash equal to 0.

(Refer Slide Time: 22:14)

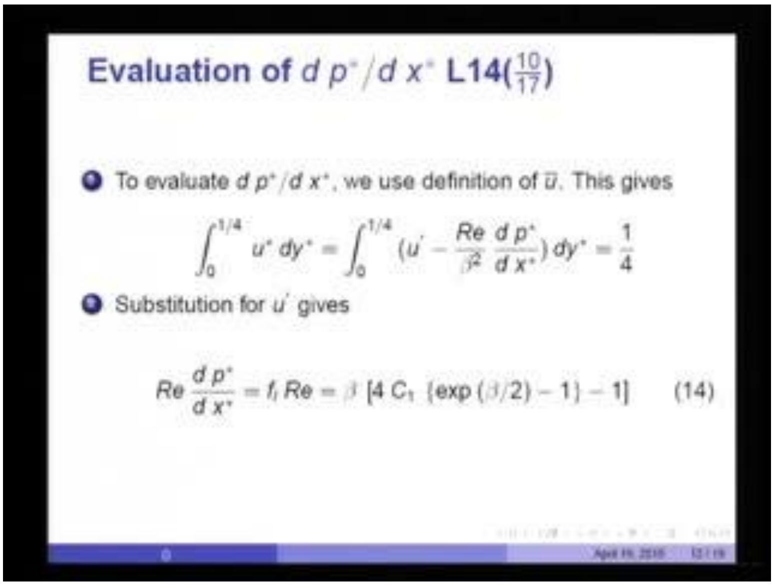


Now, this is with boundary condition u star equal to 0 at the wall y star equal to 0. At the axis of symmetry where y star is equal to 1 by 4 remember, this is the duct of length 2b this is the axis symmetry, we are measuring y this way; so this is y equal to 0 and this is y equal to b but, D h is equal to 4b so this becomes y star equal to 1 by 4, b divided by 4b and this is also equal to y star (Refer Slide Time: 22:10).

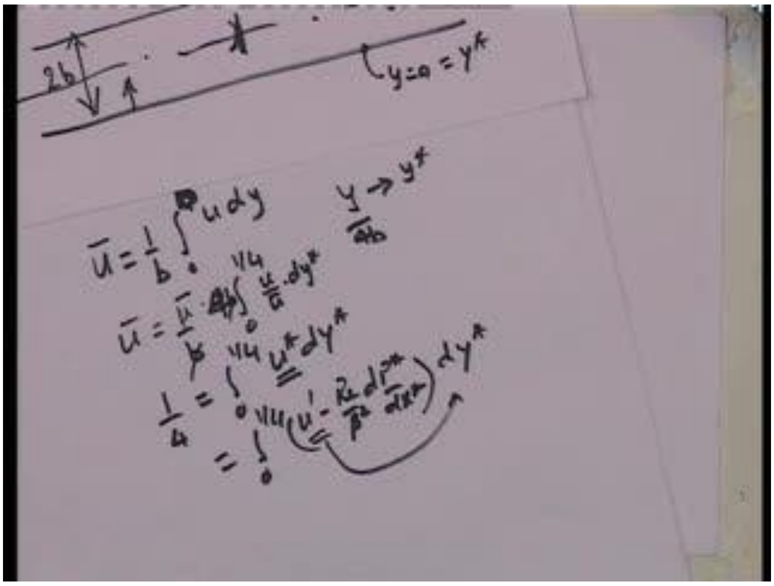
So, at y star equal to 1 by 4 the velocity gradient will be 0, the velocity gradient in y direction would be 0 and that is what this equation has 2 boundary conditions: u star equal to 0 at y star equal to 0 and du dash by d y star equal to 0 at y star equal to 1 by 4.

Now, this varies familiar fin equation in heat conduction that all of you are familiar with and therefore, the solution is very simple. It is a C 1 exponential beta y star plus C 2 exponential of minus beta y star and if I make use of these 2 conditions, I can determine C 1 and C 2 they evaluate in the following way. C 1 is Reynolds by beta square dp star by dx star dived by 1 plus exponential of beta by 2 and C 2 equal to C 1 exponential of beta by 2. So, this quite straightforward algebra to really evaluate C 1 and C 2 which incidentally is the function of C 1 into exponential of beta by 2.

(Refer Slide Time: 24:14)

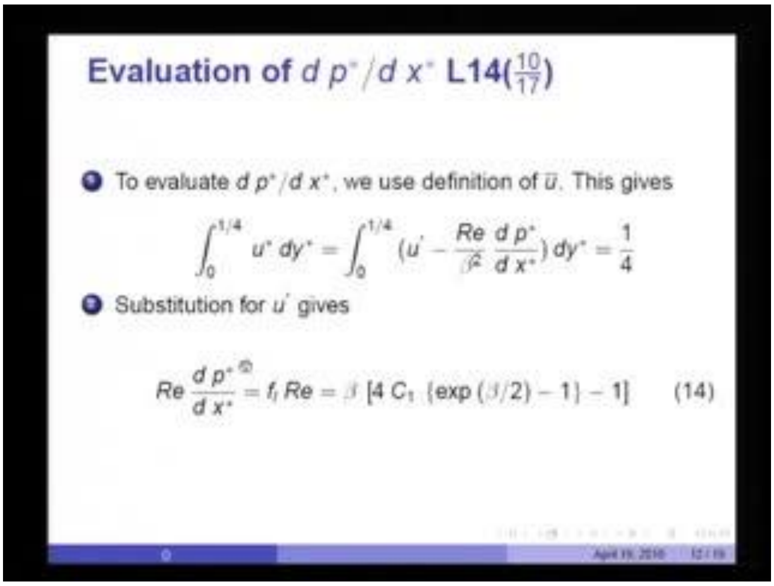


(Refer Slide Time: 24:22)



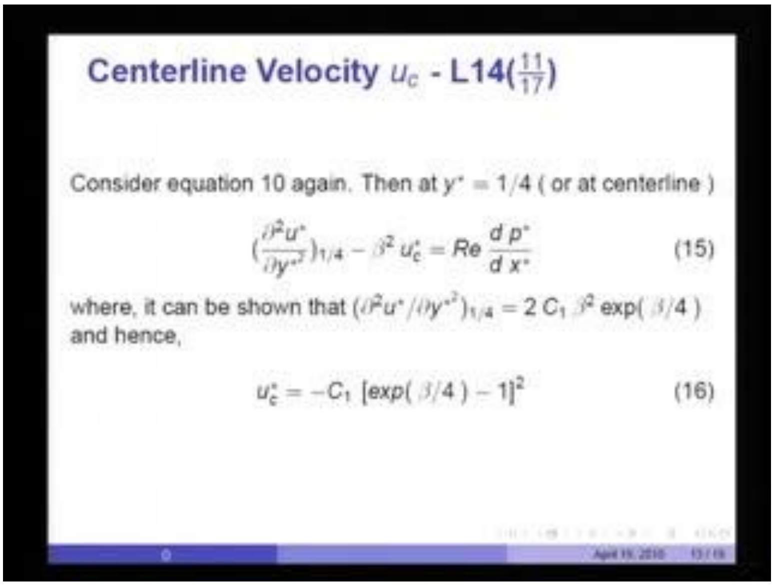
Now to evaluate pressure gradient, we make use of the definition of mean velocity. We said u bar is equal to 1 over b integral 0 to y u dy sorry 0 to b if I change y to y star then essentially you get, 1 over b into u bar into 4, 0 to 1 by 4 u over u bar dy star, dy would be equal to 4b, so this is 4 times b and this would be u bar and therefore, b b gets canceled and you will get 1 by 4 will be equal to 0 to 1 by 4 u star dy star.

(Refer Slide Time: 25:38)

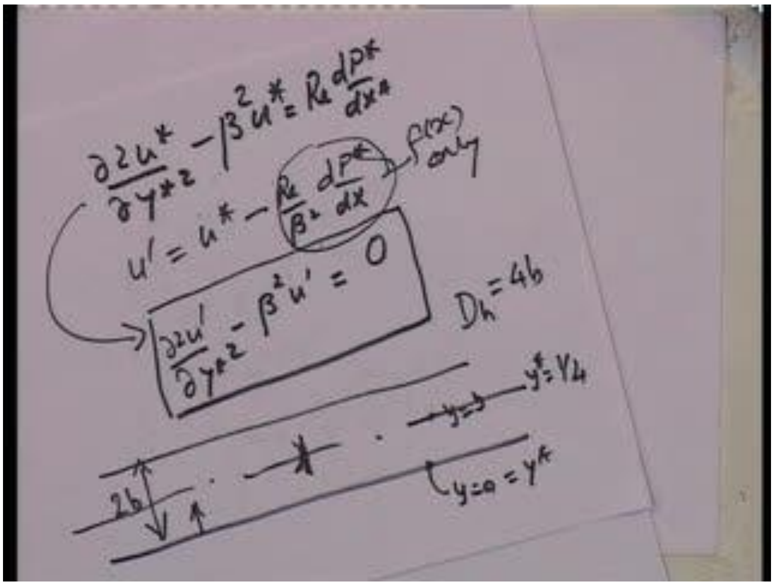


If I now substitute for u star 0 to 1 by 4, u dash minus Reynolds by beta squared dp star by dx star dy star. I know u dash as a function of y and therefore, integrate then you will see that you can get Re dp star by dx star equal to f l Reynolds equal to beta times 4 C 1 exponential of beta minus 2. So knowing the value or assuming a value of beta, you can always calculate beta C 1 and therefore, evaluate expression d Re dp star by dx star, the local pressure gradient.

(Refer Slide Time: 26:01)

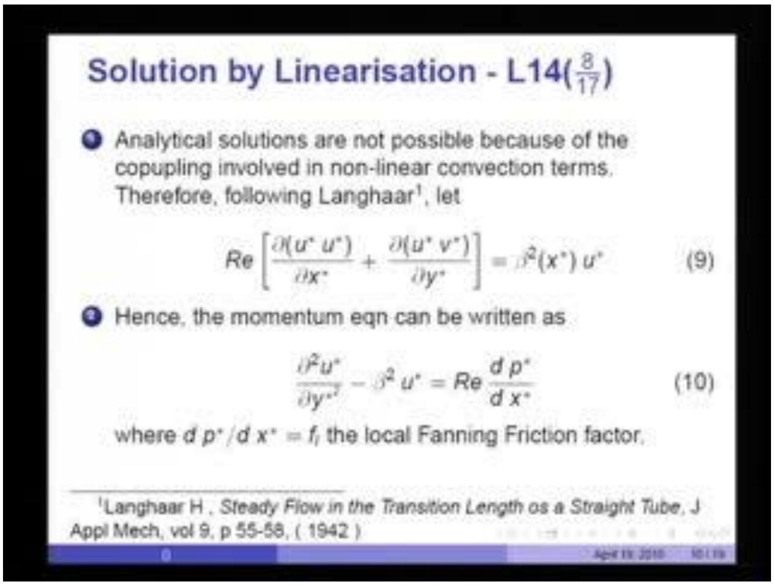


(Refer Slide Time: 26:10)

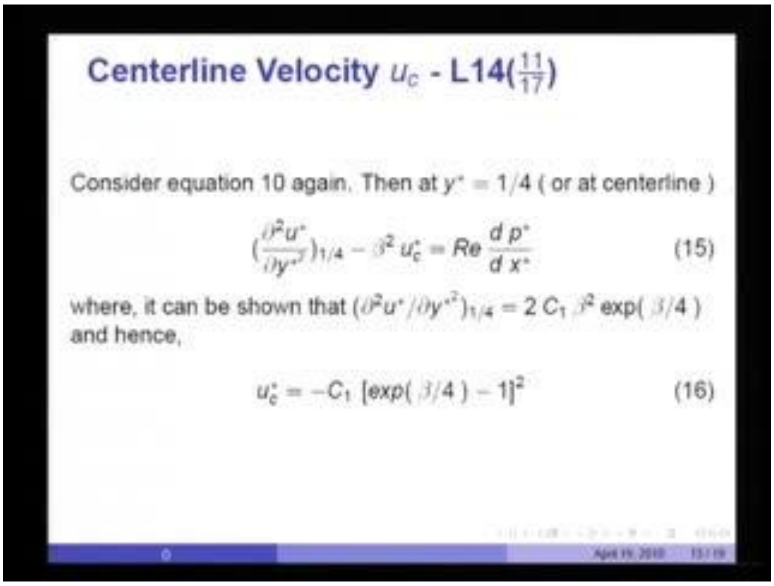


Another quantity of interest is the centerline velocity how does the velocity at the centerline change of course, the velocity begins at x equal to 0, u c will be equal to u bar at x equal to 0 but, it will go on increasing till the flow become fully developed.

(Refer Slide Time: 26:23)



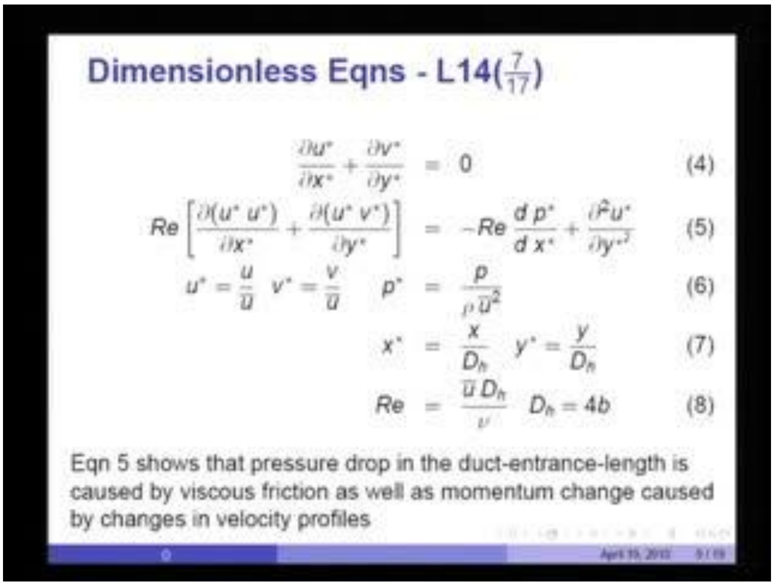
(Refer Slide Time: 26:33)



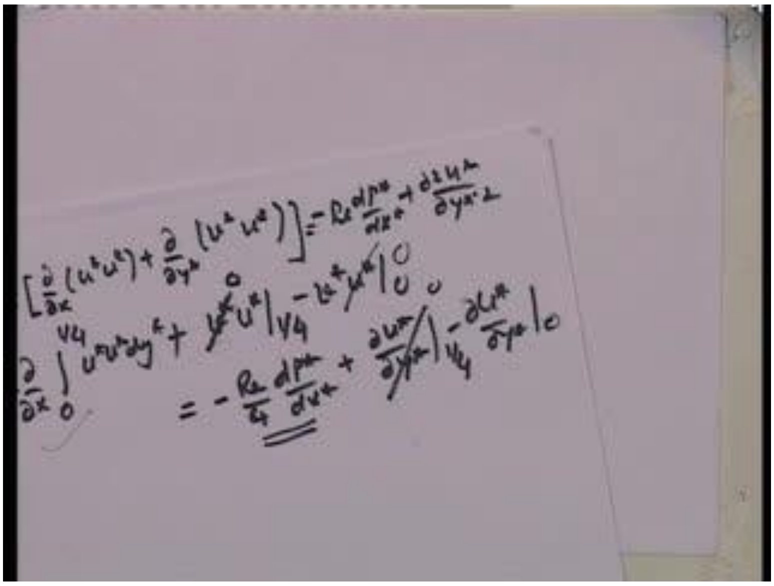
So if we consider this equation again, equation 10 then write it at the axis of symmetry y star equal to 1 by 4; it will simply become d 2 u dy star square at 1 by 4 minus beta squared u star c equal to Re dp star by dx star where it can be shown that the evaluation of d 2 u star dy star 1 by 4 is nothing but, 2 C 1 beta square over exponential of beta by 4 and therefore, a little algebra of about 2 3 lines will give u star c is a function of again beta and C 1.

So, we have evaluated the local pressure gradient, we have evaluated the local value of the centerline velocity for a given beta which incidentally means for a given value of x along the duct. The next question is of course, which value of beta corresponds to which value of x? Because beta has been arbitrarily chosen numerical value and we still have to work out, what is it the connection between beta and x.

(Refer Slide Time: 27:48)



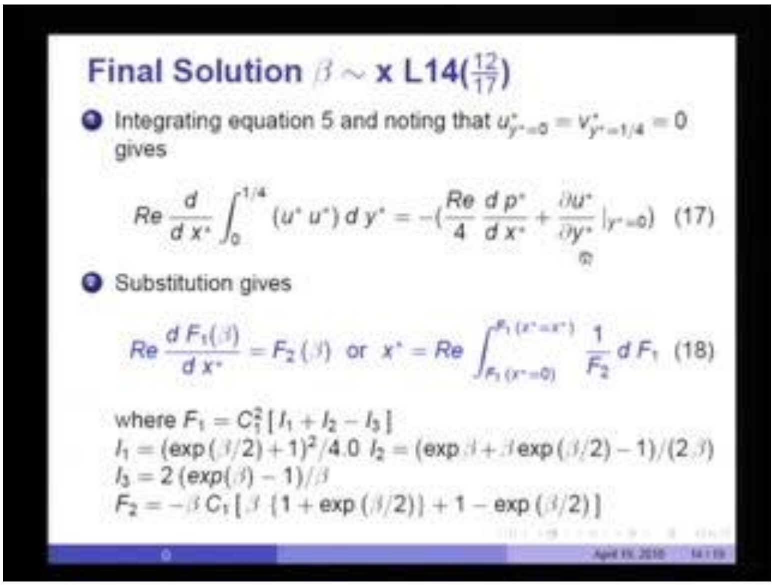
(Refer Slide Time: 28:01)



So, let us go back to equation numbers 5, this was our original equation (Refer Slide Time: 27:50). If I integrate each term of this equation from the lower wall to the axis of symmetry, the equation reads as Re times d by dx of u star u star plus d by dy star of v star u star equal to minus Re dp star by dx star plus d 2 u star by dy star square.

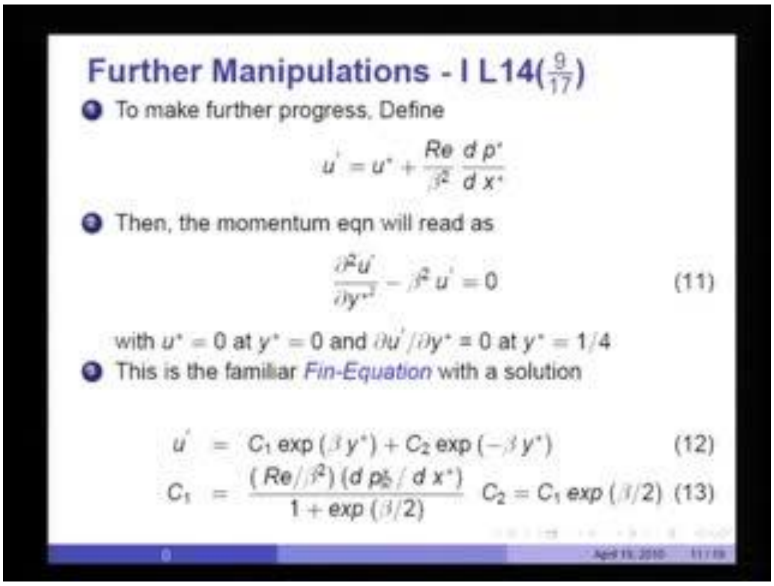
If I integrate this equation from 0 to y star, then I will get d by dx of 0 to 1 by 4 u star u star dy star plus v star u star at 1 by 4 minus v star u star at 0 equal to minus Reynolds by 4 dp star by dx star because this is not a function of y. So simply I get at 1 by 4 plus d 2 d u star by dy star at 1 by 4 minus du star dy star at y star equal to 0. So this term remains intact but, at the axis of symmetry v star is 0 likewise u star is 0 at the wall, so that term is also 0.

(Refer Slide Time: 29:56)



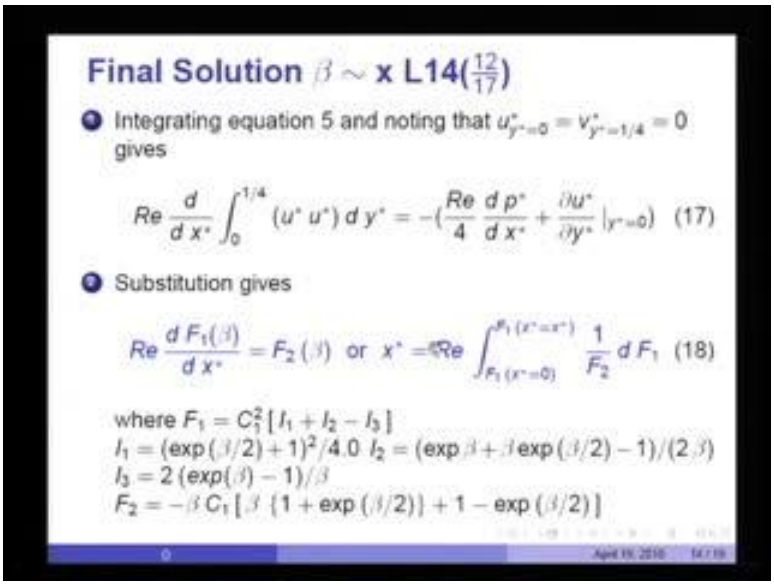
This term remains intact du star by dy star at the axis symmetry 0 and therefore, that term goes to 0 and I get this term of course, survives because there is a velocity gradient right at the wall. Therefore, the equation will now read as I have shown here (Refer Slide Time: 30:00) Re d by dx star 0 to 1 by 4 u star u star dy star equal to minus Re by 4 dp star by dx star plus du star by dy star y star equal to 0.

(Refer Slide Time: 30:24)



So, if I now substitute u for u star, u in terms of u dash here and evaluate du star by dy star from velocity profile where u dash is equal to u star. So, I substitute for u star in terms of u dash and then carry out the integration and differentiation.

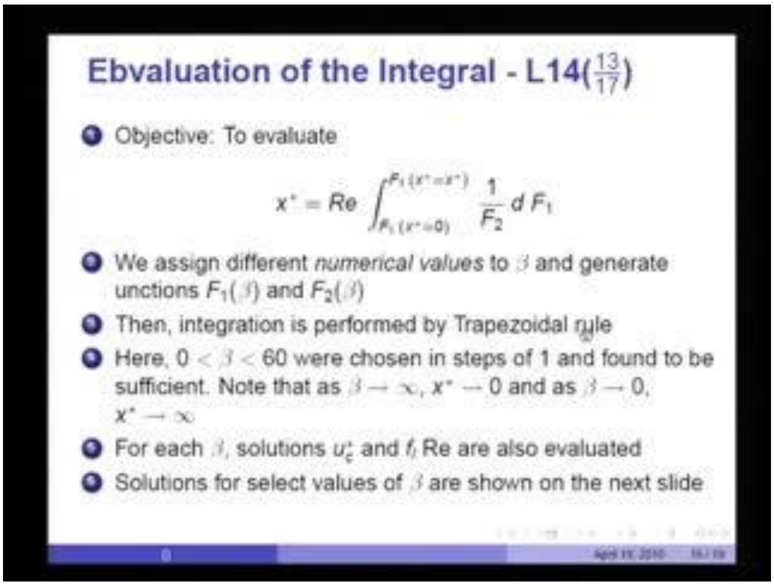
(Refer Slide Time: 30:31)



Then after some algebra about 1 page, you derive an equation then Re equal to d F 1 by dx star equal to F 2 where F 1 and F 2 are functions of beta as shown here. F 1 would be C 1 star plus I 1 plus I 2 minus I 3, I 1 is equal to that term I 2 is equal to that term and I 3 is equal to that term and F 2 itself would again be a function of beta and C 1.

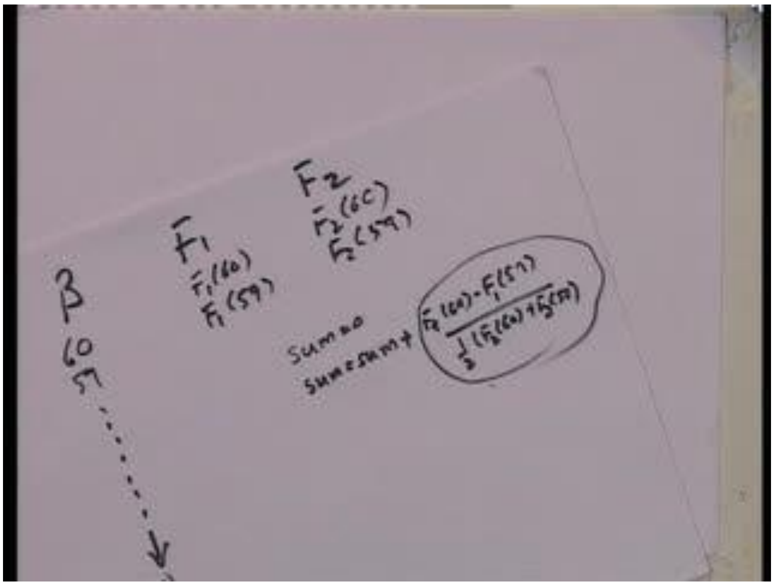
Another way of writing this equation is to say x star equal to Re times integral value of F 1 at x star equal to 0 to value of F 1 at x star equal to x star 1 over F to d F 1, it is this that establishes the relationship between x star and beta that we wish to find out because F 1 and F 2 are functions of beta.

(Refer Slide Time: 31:32)



How is this done? It is done as I shown here, we want to evaluate this integral; so we first assign different numerical values to beta and generates functions F 1 beta and F 2 beta and tabulate them.

(Refer Slide Time: 31:58)

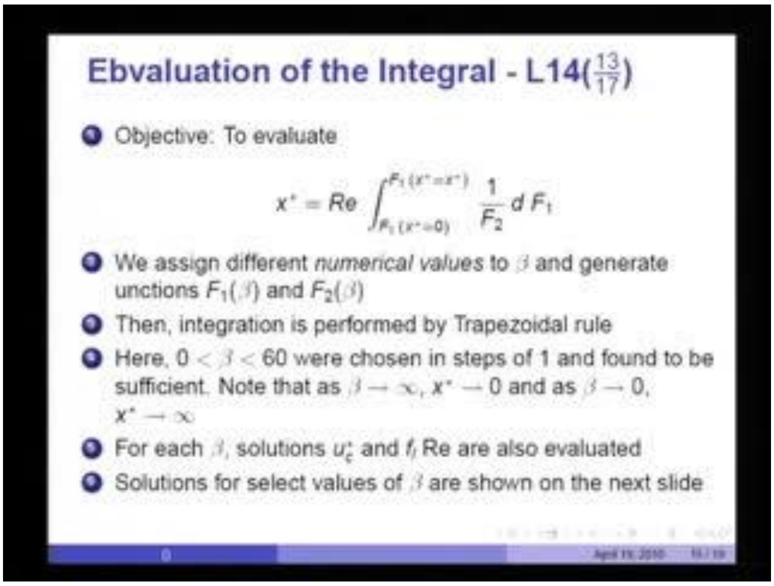


Then we simply carry out integration by trapezoidal rule, so essentially you assume certain values of beta, F 1 and F 2 a large value let us say I have used here 60, so you have a value of F 1 60 F 2 60 and so on so forth; then 59 F 1 59 F 2 59 and so on so forth and you go on to towards tending to 0, so we have for each beta value is function.

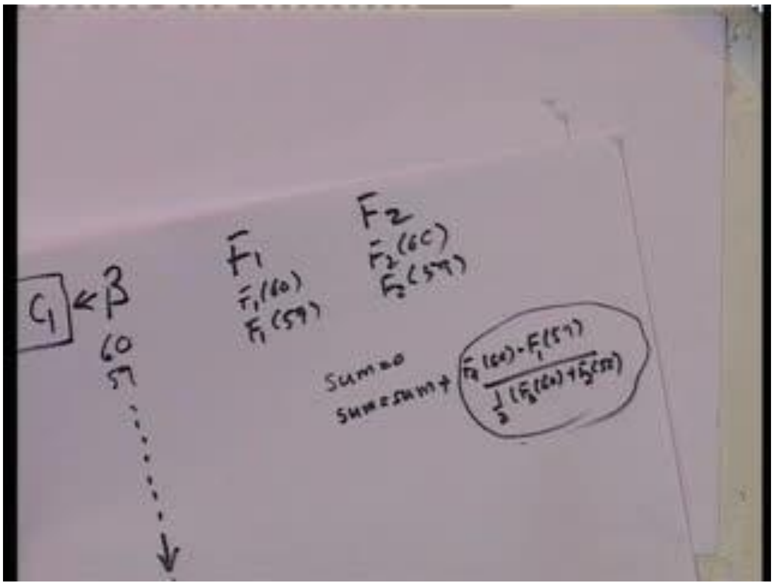
So then what you do is simply in order to carry out you say sum equal to 0 and then you say sum is equal to sum plus F 1 60 minus F 1 59 divided by half of F 2 60 plus F 2 59 as the integral this is the integrant 1 over F 2 d F 1 and simply go on adding these terms and you carried out the integration, this is the trapezoidal rule which I just draw.

So now of course, you can make beta arbitrarily very large it can go up to infinity and on other side will go to 0. I took several values and I found going up to 60 was quite good enough because as beta tends to infinity x star tends to 0 and has beta tends to 0 x star tends to infinity; so of course, in u of infinity one can chose any value.

(Refer Slide Time: 33:43)

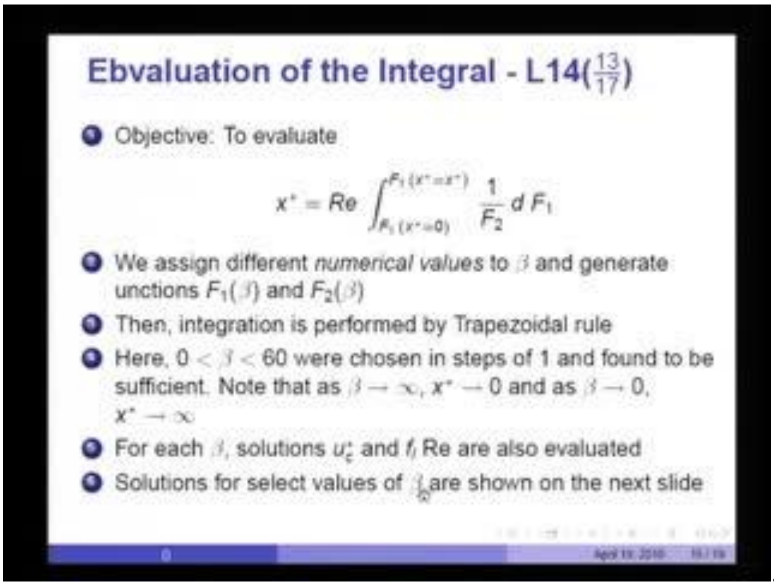


(Refer Slide Time: 34:06)



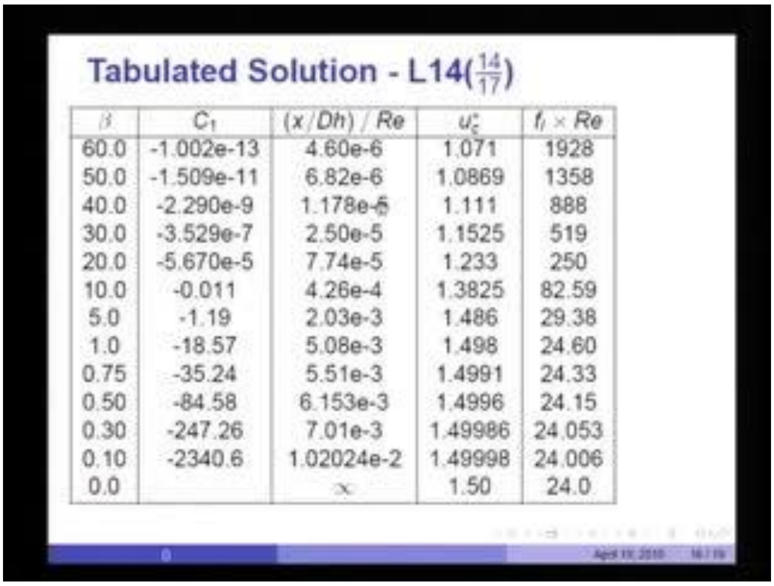
Now of course, once you have chosen beta values, for these beta values you can calculate 3 mode quantities one is C 1, second one is u c and the third one is f l.

(Refer Slide Time: 34:21)



So, these are the 3 quantities you can calculate the all functions of beta and once integration relates the value of x star to beta, we know the these are the values that correspond to x star. Although large numbers of solutions were generated for very tiny steps of beta I am showing you solution for select values of beta.

(Refer Slide Time: 34:37)



Tabulated Solution - L14($\frac{14}{17}$)

β	C_1	$(x/Dh)/Re$	u_c^*	$f_l \times Re$
60.0	-1.002e-13	4.60e-6	1.071	1928
50.0	-1.509e-11	6.82e-6	1.0869	1358
40.0	-2.290e-9	1.178e-5	1.111	888
30.0	-3.529e-7	2.50e-5	1.1525	519
20.0	-5.670e-5	7.74e-5	1.233	250
10.0	-0.011	4.26e-4	1.3825	82.59
5.0	-1.19	2.03e-3	1.486	29.38
1.0	-18.57	5.08e-3	1.498	24.60
0.75	-35.24	5.51e-3	1.4991	24.33
0.50	-84.58	6.153e-3	1.4996	24.15
0.30	-247.26	7.01e-3	1.49986	24.053
0.10	-2340.6	1.02024e-2	1.49998	24.006
0.0		∞	1.50	24.0

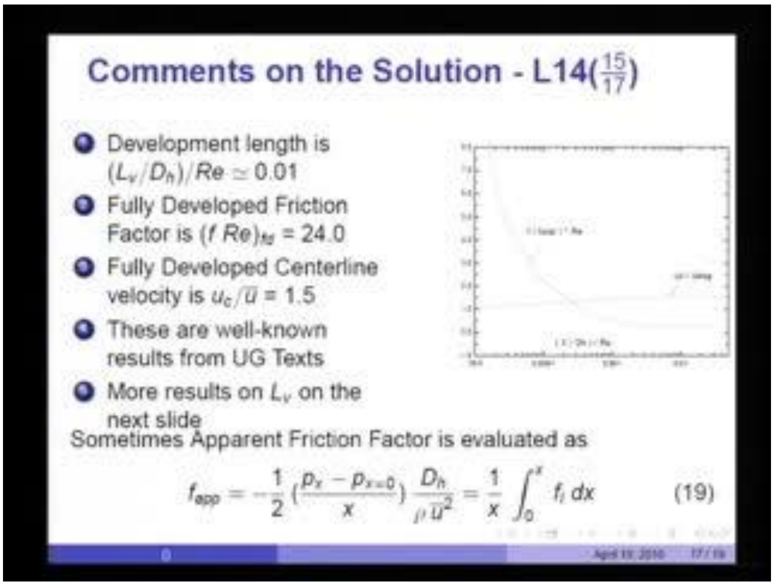
So, here are the values at beta equal to 60 C 1 turns out to be extremely small and negative it corresponds to x equals to 4.60 into 10 is to minus 6. Here x is divided by D h the hydraulic diameter and then divided by Reynolds number; u c star is simply the centerline velocity divided by u bar and this is f l multiplied by Reynolds number which is the local friction factor.

You can see for each value of beta a value of x has been discovered; as beta goes on reducing x goes on increasing and finally, for beta equal to 0.1, x is equal to 0.01, u c star becomes equal to 1.49998 almost 1.50. So asymptotically, you can see from about 0.005 onwards a change in friction factor or essentially the pressure gradient is very small; the change in centerline velocity is also very small.

Essentially a state of fully developness has been reached asymptotically at infinity of course, u c star becomes 1.5 and 24. These incidentally can be shown even from the analytical solution that we already have. So you can see, as the duct, as the flow develops, the centerline velocity increases, and friction factor itself multiplied by Reynolds number of course, decreases with x.

Solutions of this type can be generated; here, I have generated them for flow between parallel plates where the result in equation is the simple fin equation but, you can also do this for flow in a circular tube where the resulting equation is a Bessel’s equation. Also in case of annuals, entrance region of an annuals the resulting equation is a Bessel's equation. The algebra there is much longer but, here it is a simple algebra therefore, I chose to do the flow between parallel plates which in itself is upgrade interest.

(Refer Slide Time: 36:50)



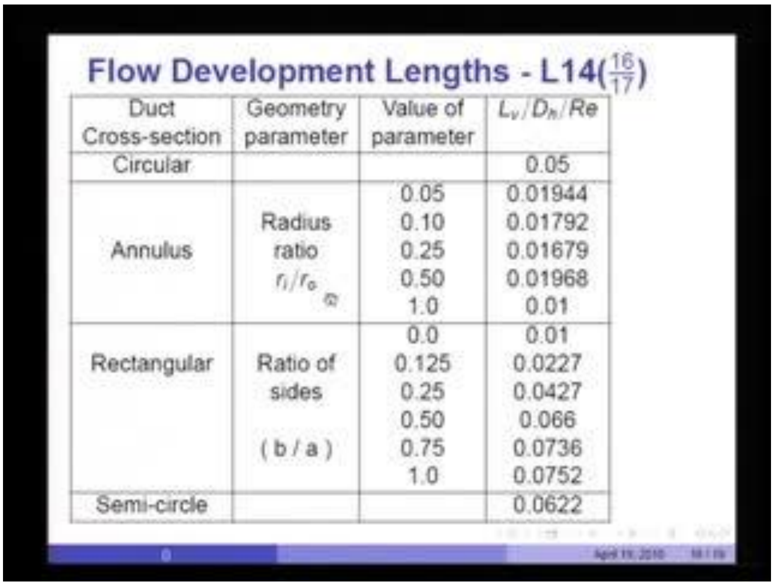
This is the variation of friction factor versus Reynolds numbers starts from high value and goes down to a constant 24 at about 0.01. Likewise centerline velocity goes from 1 and ends up with 1.5, so fully developed friction factor is 24, fully developed centerline velocity is u c by u bar 1.5. We can now treat this distance L v by D h Re approximately equal to 0.01, the behavior near the fully developness is very asymptotic. Therefore, one could not really fix precisely the value of development length with purely by observation; we can say L v by D h Reynolds of 0.01 is a good estimate of the flow development length which we said was about objective.

Now, all these results that I mentioned and of the fully developed flow are well known from the UG texts either from analytical solutions for annulus and circular tube or in case of ducts of non-circular cross section where CFD analysis is used you can get values of development lengths and I will show them on the next slide.

But here note that, instead of the local friction factor that I have plotted here sometimes people preferred because the local friction factor gives you variation of the local pressure gradient but, people are interested sometime to measure actually the local value of pressure itself and that means what they define, what is called does an apparent friction factor? Because this is what you will measure in an experiment.

Apparent friction factor is defined as minus 1 over 2 P x minus P x 0 divided by x D h over rho u bar square and it is simply 1 over x 0 to x f l dx. So our solution for f l can be integrated, you get apparent friction factor and it would go something like just likely above the local friction factor and I can predict at long length you will predict that apparent friction factor is also into Reynolds number again 24.

(Refer Slide Time: 39:20)

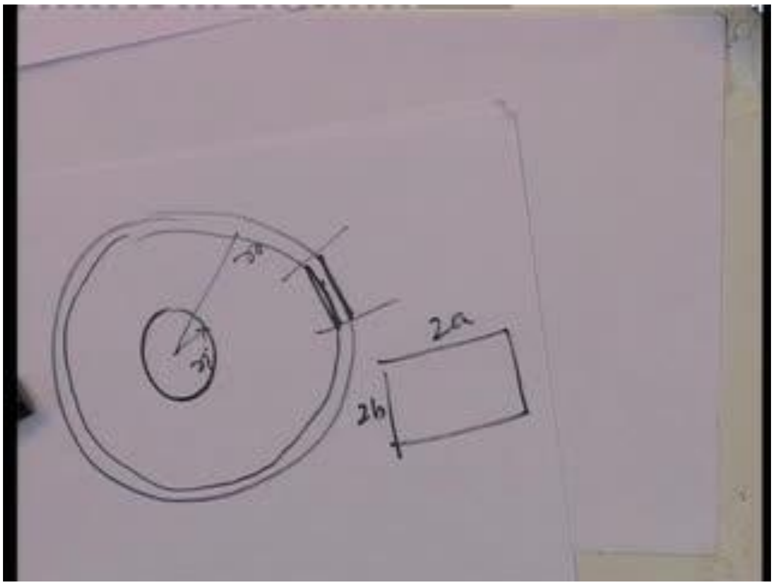

Flow Development Lengths - L14($\frac{16}{17}$)

Duct Cross-section	Geometry parameter	Value of parameter	$L_v/D_h/Re$
Circular			0.05
Annulus	Radius ratio r_i/r_o	0.05	0.01944
		0.10	0.01792
		0.25	0.01679
		0.50	0.01968
		1.0	0.01
Rectangular	Ratio of sides (b / a)	0.0	0.01
		0.125	0.0227
		0.25	0.0427
		0.50	0.066
		0.75	0.0736
		1.0	0.0752
Semi-circle			0.0622

Let us look at the values of development lengths predicted for different ducts and here I have chosen a few ducts of cross section circular tube its development length is 0.05 L v divided by D h divided by Reynolds for an annulus of different radius ratios r i by r o; so if the inner radius is very small it is 0.01944, if it is 0.1 0.17 and so on and so forth.

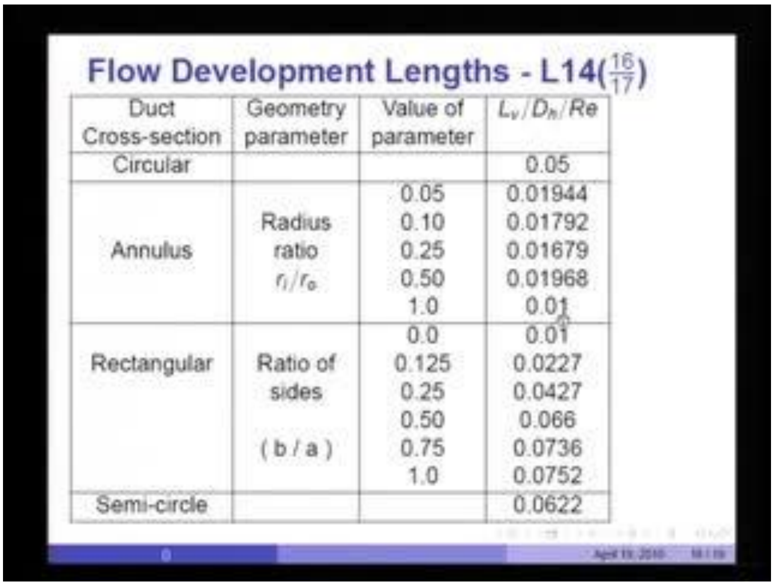
Note that these values need not be monotonous because the value of D h itself goes on changing for different radius ratios and so does the values of Reynolds number which is the function of hydraulic diameter. So this need not be monotonously increasing or decreasing the values are simply numbers to be used directly in an analysis.

(Refer Slide Time: 40:09)



But notice that, when r i by r o tends to 1; this is r i and this is r o when r i by r o tends to 1 between is r i is very close to r o then for all practical purpose the flow in the annuals is much like the flow in a between parallel plates and predictably you are seeing that the development length is 0.01 which we actually calculated through our analysis.

(Refer Slide Time: 40:38)

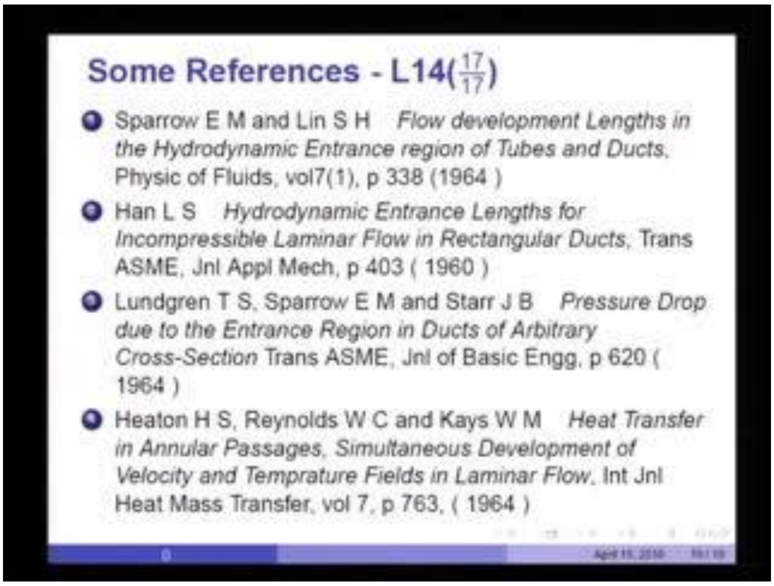


Duct Cross-section	Geometry parameter	Value of parameter	$L_v/D_h/Re$
Circular			0.05
Annulus	Radius ratio r_i/r_o	0.05	0.01944
		0.10	0.01792
		0.25	0.01679
		0.50	0.01968
		1.0	0.01
Rectangular	Ratio of sides (b / a)	0.0	0.01
		0.125	0.0227
		0.25	0.0427
		0.50	0.066
		0.75	0.0736
		1.0	0.0752
Semi-circle			0.0622

Similarly, in rectangular ducts of side ratio b by a or essentially it is 2b by 2a then the ratio b by a gives you the aspect ratio. When b by a is 0 again when b is very tiny you essentially get flow between parallel plates because a then is in finite and the L v by D h ratio is 0.01 as you predicted for 0.125 0.27 0.25427 0.5 and so on so forth. When you tend to 1 that is the perfect square the development length is 0.0752 here is a result for semi-circle, this has been calculated by numerical analysis and L v by D h Reynolds number is 0.0622.

So, the development length normalize with hydraulic diameter and Reynolds number has a notionally fixed value for a given duct and is always as you will see when we go on to heat transfer calculation it is very essential that we know what this development lengths are, so that we can account for their presence in the heat transfer analysis. Analytical work of this type is almost now abundant because most people use CFD base solution procedures.

(Refer Slide Time: 42:44)



But, it would be useful for you to know who whether early contributed to this very interesting aspect of internal flows and therefore, I give you a few references the first one is by sparrow and Lin published in 1964, it is called flow development lengths in hydrodynamic entrance region of tubes and ducts, physic of fluids volume 17. Another one by Han hydrodynamic entrance lengths for incompressible laminar flow in rectangular ducts published in 1960.

Lundgren sparrow and star pressure drop due to the entrance region in ducts of arbitrary cross-section, this is in journal of basic engineering 64 and then for annular passages there is the paper by Heaton Reynolds and Kays international journal of heat mass transfer.

So as I said, the flow in the entrance region is quite complex situation and one needs exact analytical solutions for class room work can only be worked out for simple cases and I gave you example of one case and then concluded with magnitudes of the developedness length as encountered in deferent ducts.

In the next lecture, we will take up the case of fully developed flow inside ducts of circular and non-circular cross section.