Convective Heat and Mass Transfer

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Module No. # 01

Lecture No. # 12

Integral Solutions to Laminar Temp BL

In this lecture, we shall consider integral solutions to the temperature boundary layer equation.

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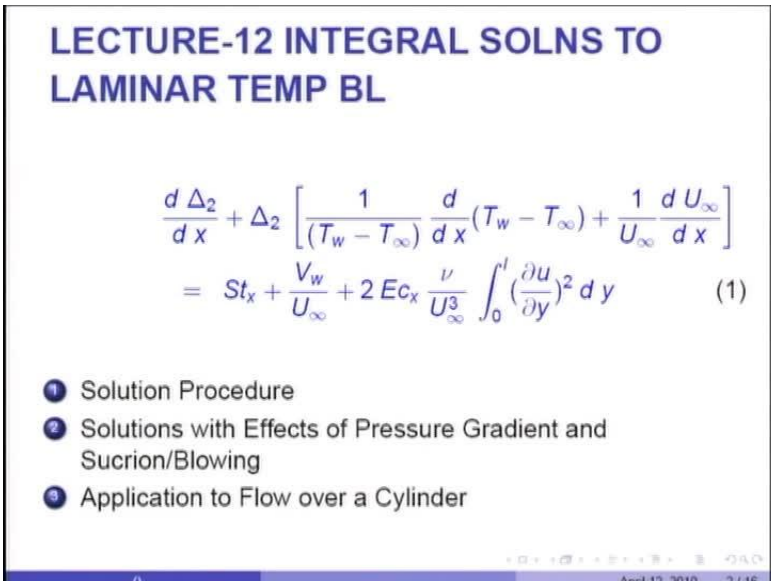

The equation you will recall is written here d delta 2 by d x, where delta 2 is the enthalpy thickness plus delta 2 times the wall temperature variation term, the pressure gradient term that is, the variation of U infinity. This equals the Stanton x, which is a dimensionless heat transfer coefficient at the surface plus the wall velocity term and the viscous dissipation term. Purpose here is to understand how the solution procedure is conducted. Then, we will look at solutions with effects of pressure gradient and suction and blowing with an application to flow over a cylinder.

Presently, the effect of wall temperature variation will not be taken nor will the effect of dissipation be considered.

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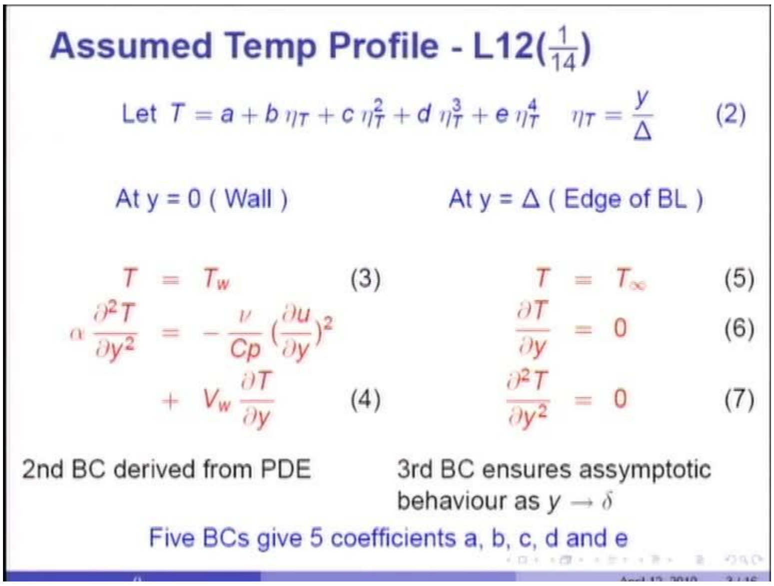

Like in velocity boundary layer solution, we first begin by saying - let the temperature be a function of distance from the wall, y divided by the thermal boundary layer thickness, delta. I have called eta T equal to y divided by delta. The assumed expression is as follows. It is a fourth order polynomial with five coefficients. These five coefficients are to be determined as we did in the case of velocity boundary layer equation with five boundary conditions.

The first of these is at y equal to 0; T is equal to T w and all eta Ts will be 0. Therefore, a will be equal to T w. The second condition at the wall derives from the differential equation of the boundary layer. It would be alpha times d 2 T by d y square, which is the diffusion term equal to the viscous dissipation term and the wall velocity term V w d T by d y. At the edge of the boundary layer, thermal boundary layer T will be equal to T infinity; the temperature derivative will be 0. The fact that temperature approaches T infinity asymptotically would entail that d 2 T by d y square should also be 0. So, we have five boundary conditions and five constants a, b, c, d and e to determine without going into the algebra.

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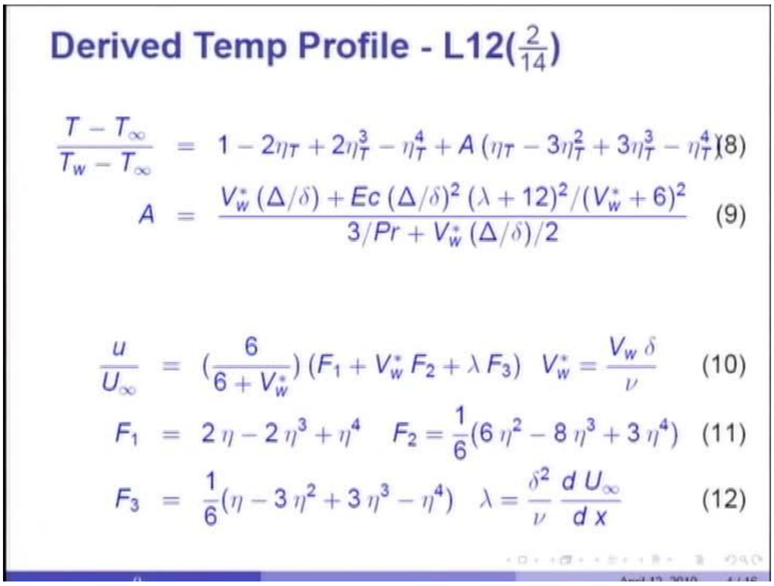

Here is the temperature profile as it looks. In dimensionless form the temperature profile reads as T minus T infinity divided by T w minus T infinity equals 1 minus 2 eta T plus 2 times eta cube T minus eta 4 T plus A times a function of eta T, where A captures the effects of wall velocity, viscous dissipation through the Eckert number, and the pressure gradient effect through lambda.

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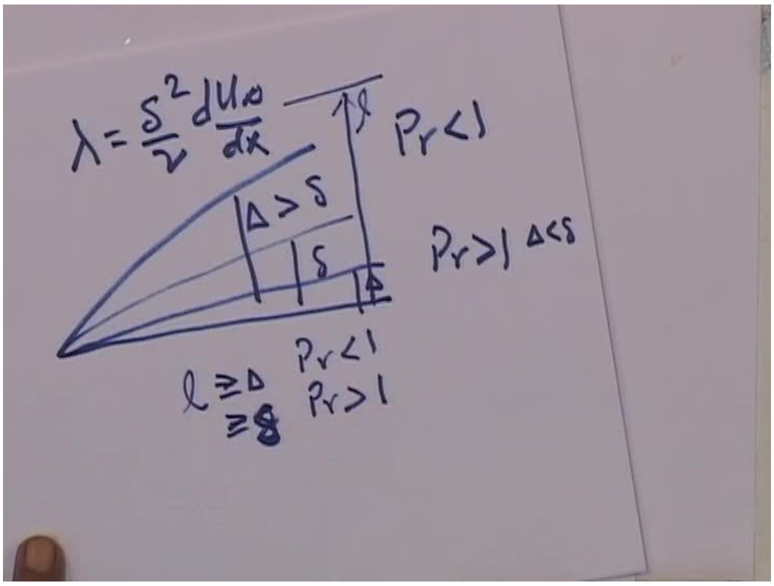

Lambda as you will recall is nothing but small delta square by nu d u infinity by dx. You will recall V w star; I have written it here V w star is simply V w small delta by nu

(Refer Slide Time: 03:52). The corresponding velocity profile is again reproduced to refresh your memory - F 1 is equal to that function F 2 multiplied by V w star is that function; F 3 multiplied by lambda is that function; that is, F 3 function. So, we now have a temperature profile and a velocity… As you can see, the expressions are quite complex, but nonetheless can be worked on a piece of paper.

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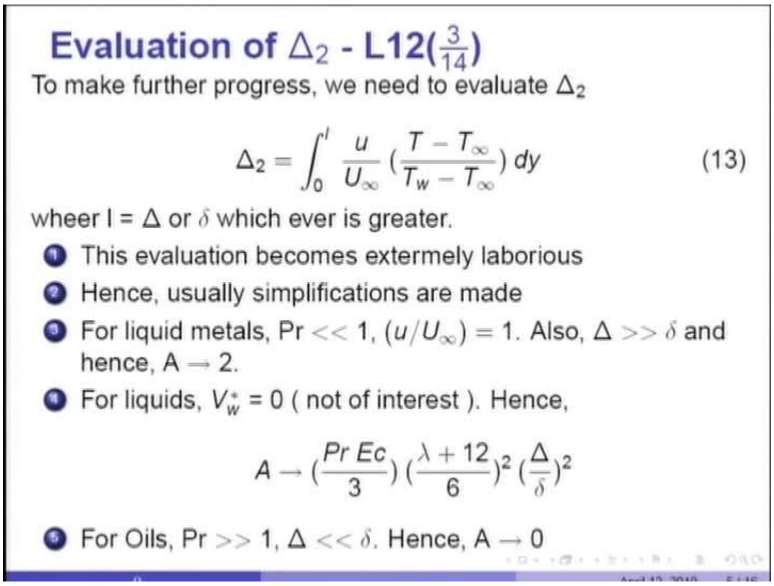

To make further progress in our equation here (Refer Slide Time: 04:24), we need to evaluate delta 2, the enthalpy thickness. The definition of enthalpy thickness is 0 to l U over U infinity into T minus T infinity divided by T w minus T infinity dy. Therefore, it is an integration of the two profiles: (Refer Slide Time: 04:45) the velocity profile and the temperature profile to 0 to l.

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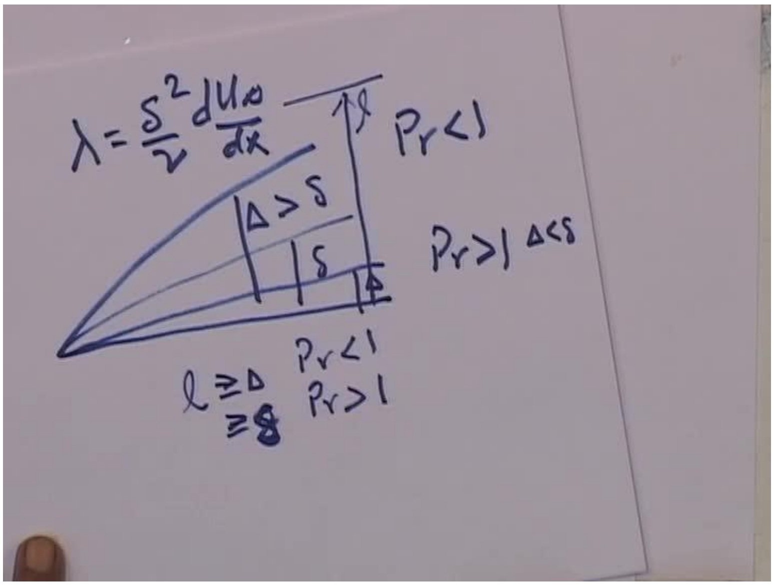

Remember: we recall that if this was the boundary layer development small delta; then if Prandtl was greater than 1, then thermal boundary layer thickness would be smaller than small delta. However, if Prandtl number was less than 1, then the thermal boundary layer thickness would be greater than small delta. We now choose l to be bigger than any of them; this is l. l therefore, is equal to delta or small delta. You can see that l will be equal to capital delta for Prandtl less than 1 and it would be small delta for Prandtl greater than 1. I mean, greater than or equal to would be appropriate way of writing it; this is small delta.

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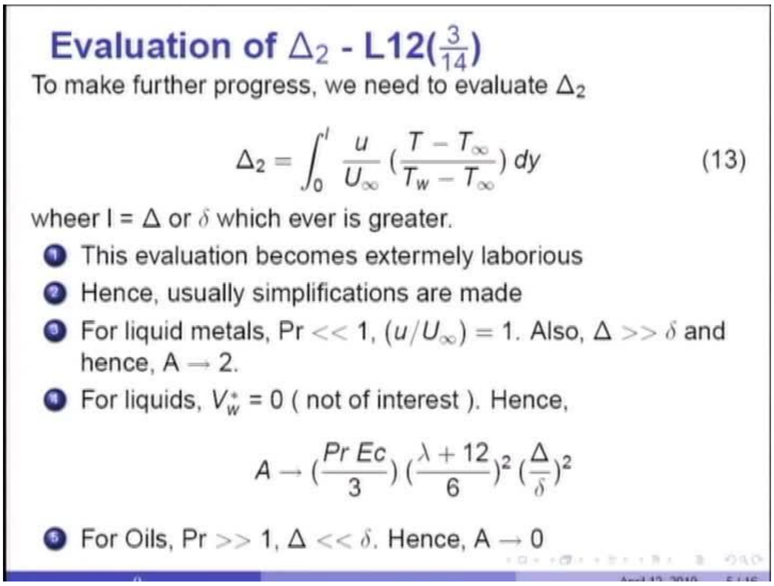

Now, you can imagine that because the two profiles are highly complex, the evaluation becomes extremely laborious of this quantity delta 2. Hence, usually simplifications are made. For example, for liquid metals, where Prandtl number is much less than 1; for all practical purposes, u over U infinity is equal to 1. That simplifies the integration to only the temperature profile. Also, delta is very much greater than small delta. Then you will see that delta is very much greater than delta and it appears in the numerator as well as denominator; 3 divided by Prandtl number; Prandtl number is of the order of 0.001. Therefore, this number is also very large. In other words, both the numerator and the denominator assume very high values and therefore, A would tend to 2.

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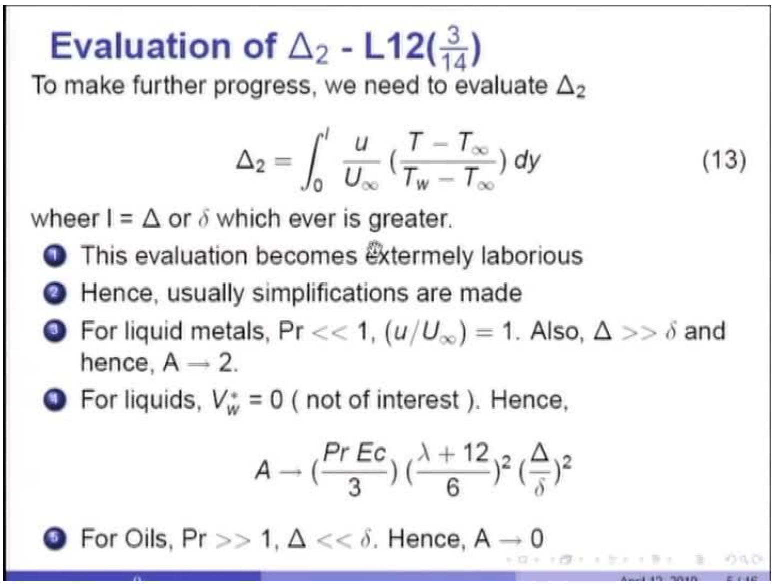

For liquids V w star equal to 0 is not of interest and in which case A. If I have dropped V w star terms, here (Refer Slide Time: 07:07) would simply be that term divided by this term. It would read as Prandtl Ec by 3 into lambda plus 12 by 6 whole square into delta by small delta whole square. If I were to consider oils in particular for which Prandlt number is very much greater than 1, then delta by small delta would be much smaller than 1. Therefore, A would practically tend to 0. So, one can make such assumptions to simplify this evaluation of the integral.

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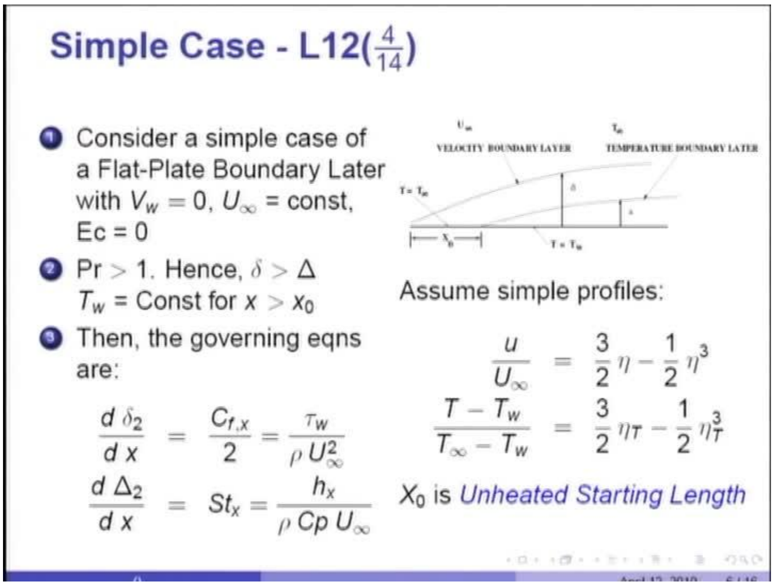

In order to explain how the procedure works, I am going to consider a very simple case of a flat plate boundary layer. I will ignore wall velocity V w equal to 0 because it is a flat plate boundary layer; U infinity will be a constant and Ec would be equal to 0.

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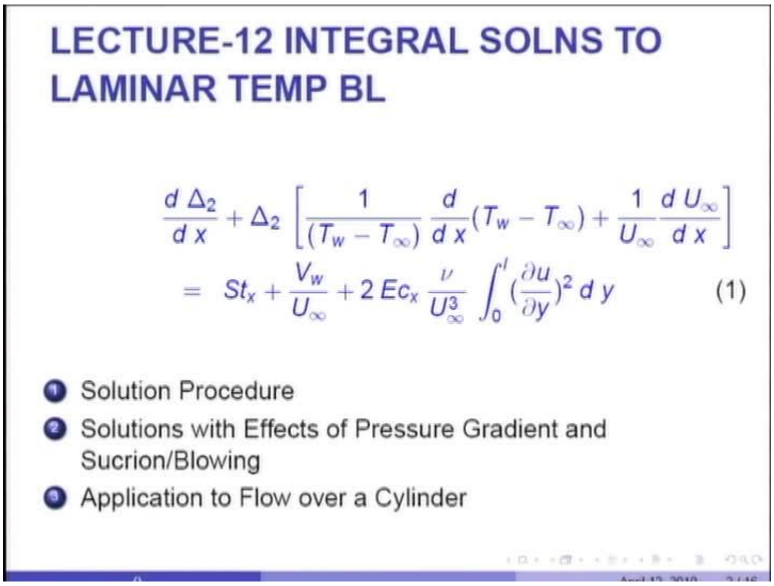

In other words, if you see this equation, there is no actual variation of temperature and there is no actual variation of velocity. So, that term goes to 0. There is no viscous dissipation considered nor is this considered. Therefore, d delta 2 by d x would equals Stanton x - is the most simple form of the energy equation.

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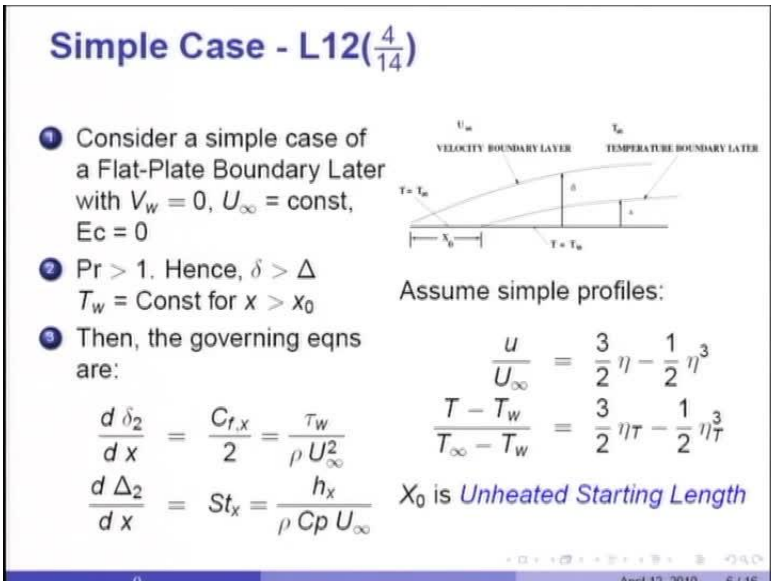

The corresponding velocity boundary layer equation will be d small delta 2 by dx equal to C f x by 2 equal to tau wall over rho U infinity square. I am also going to consider the case of Prandtl greater than 1. So, small delta will be bigger than the thermal boundary layer thickness, delta. I am going to postulate that the constant wall temperature boundary condition, T equal to T w starts at x greater than x naught.

Let us say T w is greater than T infinity. The temperature between 0 and x naught of the wall will be simply T infinity, but it suddenly raise to T w from x equal to x naught. x naught is called the Unheated Starting Length. There is a purpose for doing this. This analysis (Refer Slide Time: 09:21) would lead us to considering the effects of wall temperature variations. I shall consider this in the next lecture, but here simply appreciate the procedure of how these two equations are solved simultaneously.

To simplify matters instead of taking the longest fifth order profile, I am going to take very simple profiles u over U infinity equal to 3 by 2 eta minus 1 by 2 eta cube. You will see that this profile satisfies the boundary condition u equal to 0 at eta equal to 0, which is at the wall. It also satisfies the condition that u equals U infinity equal to 1 at eta equal to 1, which is correct. It also will satisfy du by dy equal to 0 at y equal to delta. You can see that condition is also satisfied. Therefore, the equation is quite ok. Likewise, the temperature profile is T minus T w over T infinity minus T w would be 3 by 2 eta T minus 1 over 2 eta T cubed.

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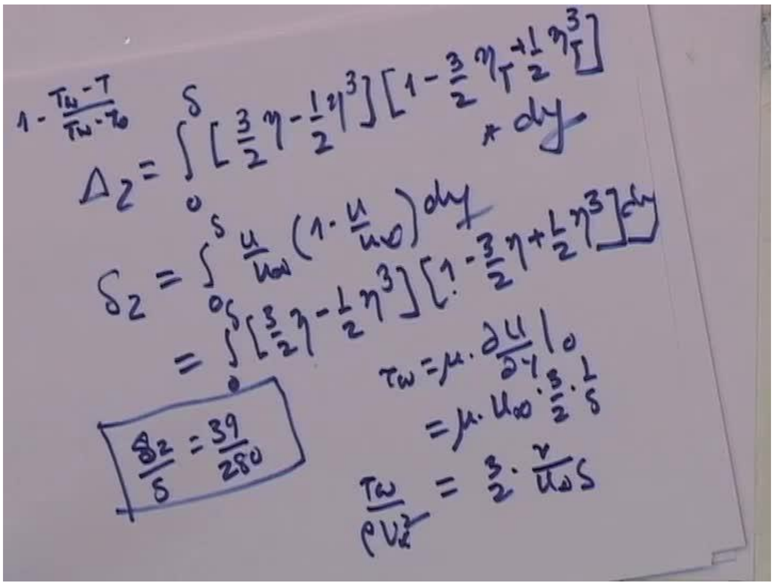

With these two very simple (Refer Slide Time: 10:34) temperature profile and velocity profile, delta 2 would now read as 0 to small delta 3 by 2 eta minus 1 by 2 eta cube into… as you can see here (Refer Slide Time: 11:00) we want T minus T infinity over T w minus T infinity. Therefore, this will be T minus T infinity over T w minus T infinity would be equal to 1 minus T w minus T over T w minus T infinity. Therefore, this will be (Refer Slide Time: 11:27) 1 minus 3 by 2 eta T plus 1 by 2 eta T cube into d y. This is how delta 2 will be evaluated.

The evaluation leads to first of all the momentum thickness, small delta 2; as you recall is 0 to small delta u over U infinity into 1 minus u over U infinity whole dy. Therefore, that will be equal to 0 to small delta 3 by 2 eta minus 1 by 2 eta cube into 1 minus 3 by 2 eta plus 1 by 2 eta cube into d y.

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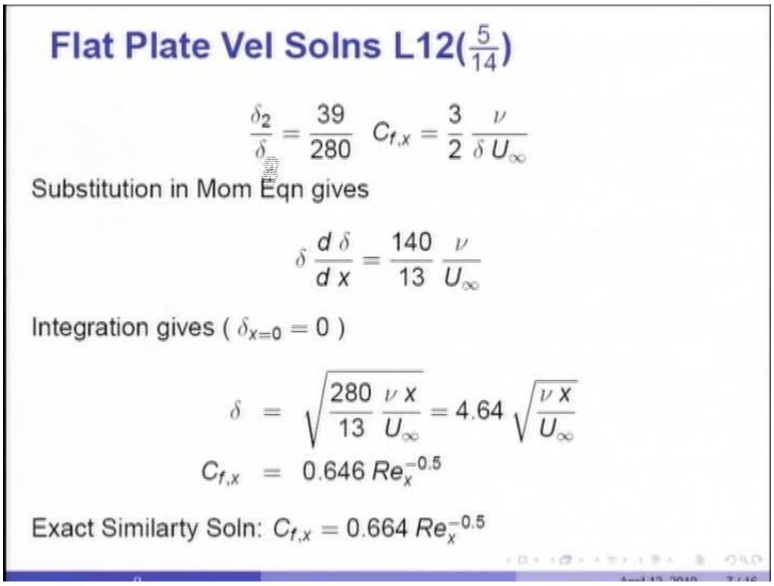

That evaluation gives you that small delta 2 by small delta will be 39 by 280. So, this is one result (Refer Slide Time: 12:36). From the velocity profile, tau wall, which is equal to mu times d u by d y at 0, you will see that this will become mu into U infinity into 3 by 2 into 1 over small delta. Tau wall over rho u infinity square would be equal to 3 by 2 into nu divided by U infinity small delta. That is what C f x would be and that is what I have shown here (Refer Slide Time: 13:14).

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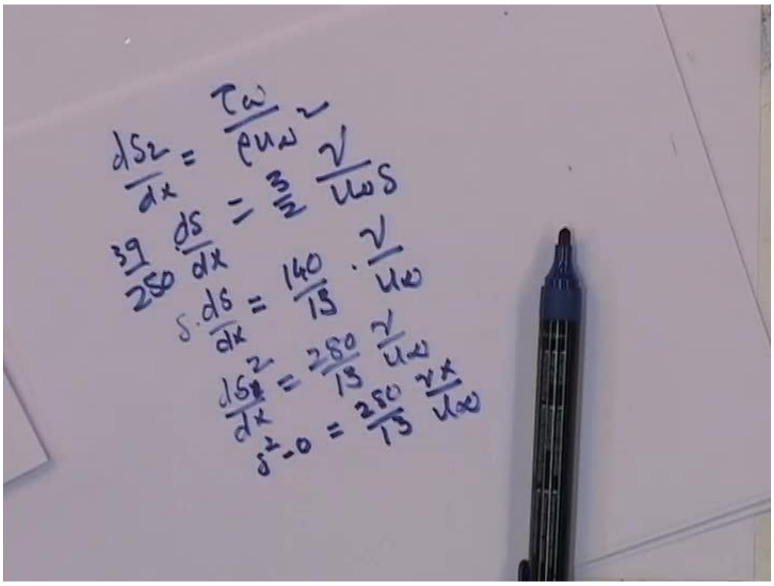

Now, if I substitute this solution small delta 2 by small delta equal to 39 by 280 into the momentum equation, which is d small delta 2 by d x equal to tau wall over rho U infinity square. Which we showed just now is equal to 3 by 2 into nu by U infinity small delta. Small delta 2 we said is 39 by 280 into d small delta by d x.

Then, you will see (Refer Slide Time: 13:49) that I get small delta into d small delta by d x equal to 140 by 13 into nu over U infinity, which is nothing but d small delta square by d x equal to 280 by 13 into nu by U infinity. If I integrate that, then I get small delta square minus 0 equal to 280 by 13 into nu x by U infinity.

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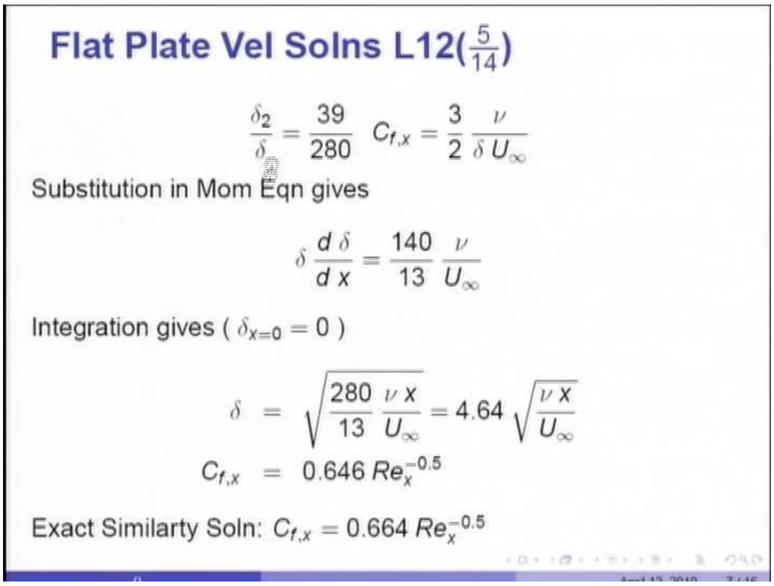

That is what I have shown here as the solution. Small delta is 0 at x equal to 0, which is the start of the boundary layer. So, you get small delta equal to under root 280 by 13 into nu x by U infinity, which is nothing but 4.64 under root nu x by u infinity.

If I now substitute this small delta (Refer Slide Time: 14:47) in the definition of C f x equal to 3 by 2 into nu over small delta U infinity, then it can be shown that C f x is 0.646 Reynolds x to the power minus 0.5. You will recall from our exact similarity solution that we had obtained C f x equal to 0.664 Reynolds x to the power minus 0.5.

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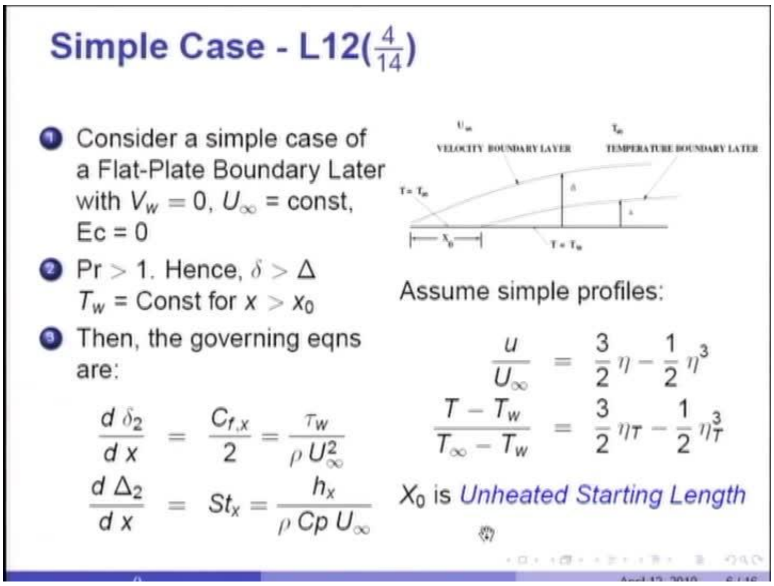

Even though we have chosen a very simple velocity profile here - 3 by 2 eta minus 1 over 2 eta cube, we have obtained a result, which is very close to the exact solution. It is for this reason that the integral method is called the approximate method because it depends on the approximation to the velocity profile that we have used. It is not that the equations are in exact, but it is the method of solution that is approximate.

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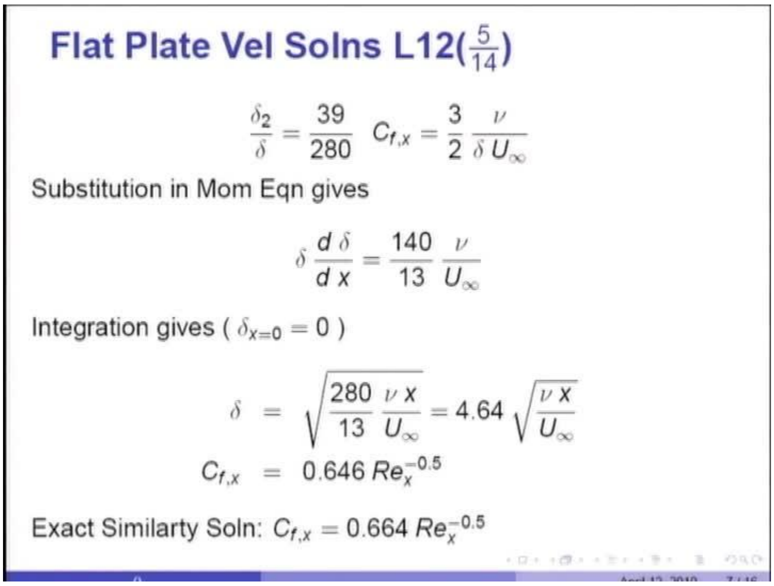

This is how one solves the momentum equation.

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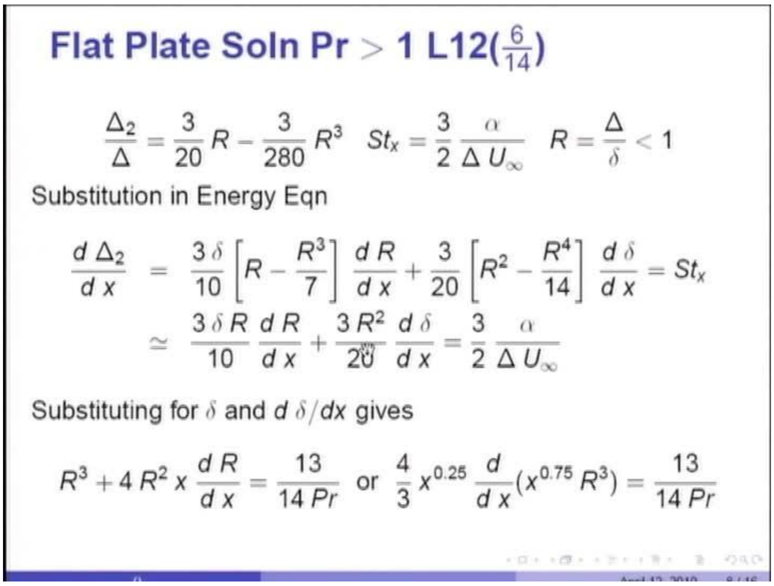

We now turn to the energy equation.

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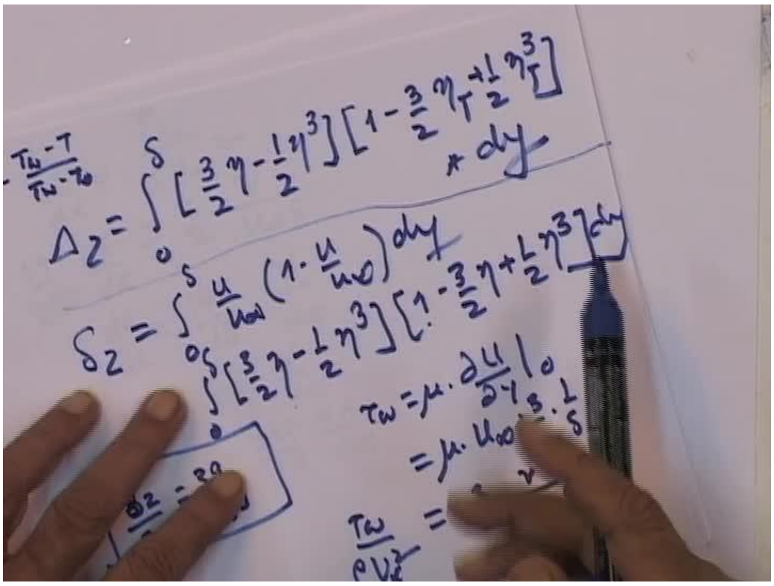

Then, as I said this is the definition of delta 2. If I integrate this equation, I would get (Refer Slide Time: 15:59) delta 2 by delta equal to 3 by 20 R minus 3 by 280 R cube.

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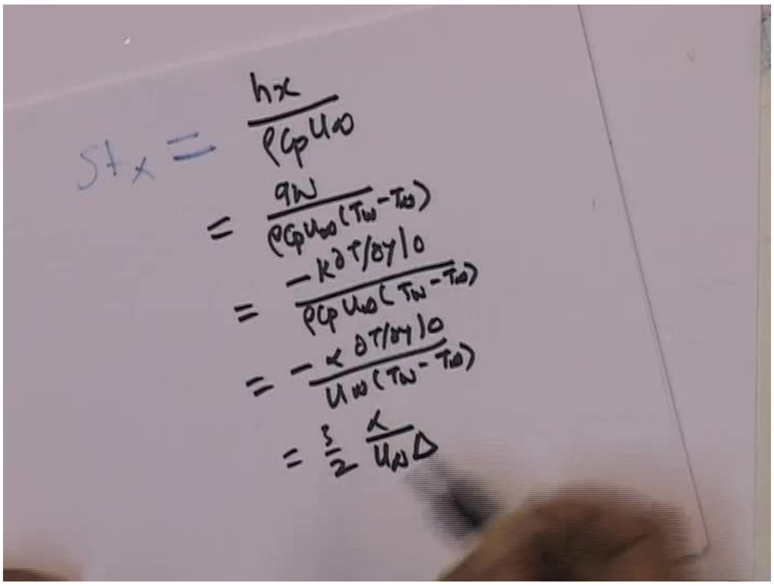

Stanton x is nothing but… You will recall Stanton x is h x by rho C p U infinity. That is equal to q wall over rho C p U infinity into T w minus T infinity. That is equal to minus k d T by d y at y equal to 0 divided by rho C p U infinity into T w minus T infinity. That will give me minus alpha times d T by d y at y equal to 0 over U infinity into T w minus T infinity. Therefore, you will see that this will reduce to very simply 3 by 2 because we have the temperature profile. Therefore, we can evaluate that as 3 by 2 into alpha by U infinity into delta.

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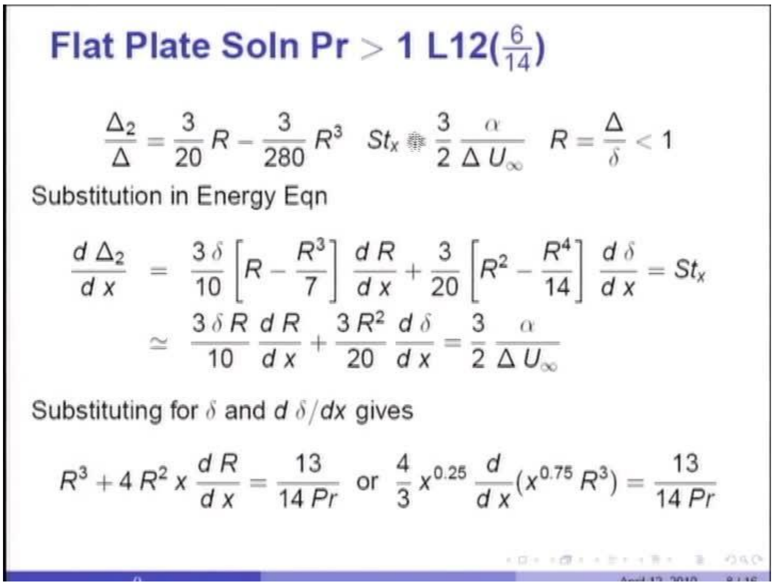

This is the definition. This is the evaluation of Stanton x; the right hand side of the energy equation. R here is delta by small delta. Since we are considering the case of Prandtl greater than 1, delta by small delta will always be less than 1.

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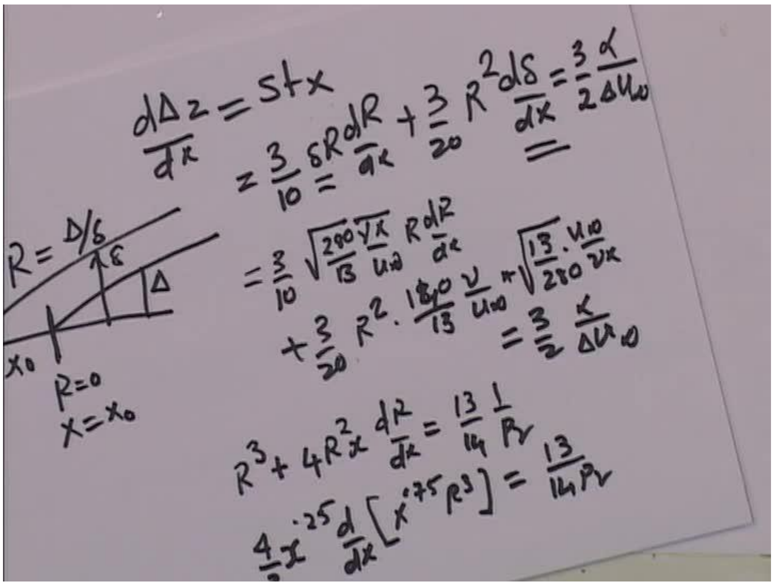

In the energy equation, if I substitute these results, d delta 2 by d x, what is the energy equation? equal to Stanton x. So, if I substitute these results, (Refer Slide Time: 17:36) I will get d delta 2 by d x equal to 3 small delta by 10 into R minus R cube by 7 into d R by d x plus 3 by 20 into R square minus R raised to 4 by 14 into d small delta by dx.

If I illustrate forward algebra, but notice each of these brackets - because R is less than 1, R cube by 7 will be much smaller than R. So, this can be ignored (Refer Slide Time: 18:01); likewise, R to the power 4 by 14 will be much smaller than R square. So, even that term can be ignored. Then, I will get d delta 2 by d x is very nearly equal to 3 small delta R by 10 into d R by d x plus 3 R square by 20 into d small delta by d x, which is equal to 3 by 2 U infinity.

I get (Refer Slide Time: 18:26) d delta 2 by d x is equal to 3 by 10 small delta R into d R by d x plus 3 by 20 R square into d small delta by d x equal to 3 by 2 into alpha over delta U infinity.

All right, where do we get small delta from? I need to get small delta and d small delta by d x. That is what we evaluated on the previous slide (Refer Slide Time: 18:58). We will see that small delta is just 3 by 10 into under root 280 by 13 into nu x by U infinity into R d R by d x plus 3 by 20 R square; d delta by d x will be simply 140 by 13 into nu by U infinity. In other words, this will be multiplied by 13 by 280 into U infinity by nu x. This is equal to 3 by 2 into alpha over delta U infinity.

This part of the equation (Refer Slide Time: 19:44) simplifies to this equation. You will see that all these can be simplified to R cubed plus 4 R squared x into d R by d x equal to 13 by 14 into 1 over Prandtl number. So, you will see that this left hand side (Refer Slide Time: 20:06) can be manipulated to read like that. This is nothing but 4 by 3 into x raised to 0.25 into d by d x of x raised to 0.75 into R cubed equal to 13 by 14 Prandtl number. So, I can now integrate this equation.

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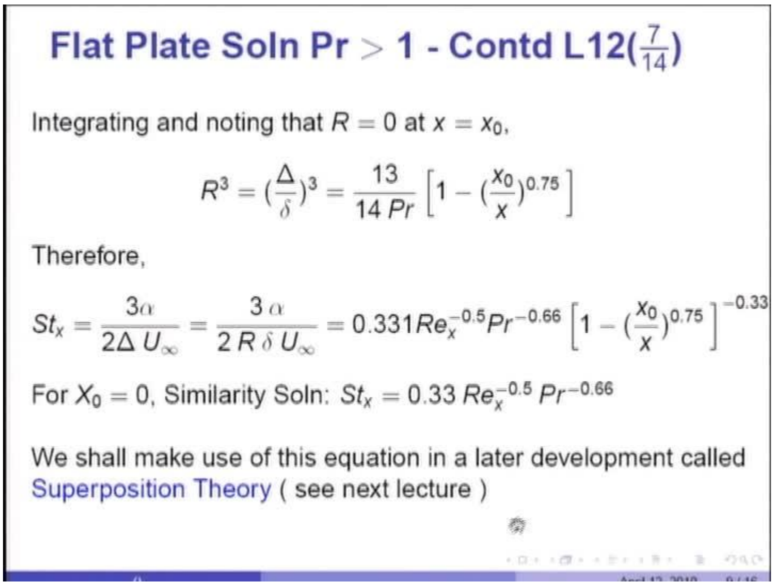

Integrating and noting that R; R is? Remember: (Refer Slide Time: 20:35) We are starting to heat from x equal to x naught and R is delta by small delta. So, this is delta, whereas this is delta. So, R is equal to 0 at x equal to x naught. If I make use of that, then I get the solution to that equation as R cubed equal to delta by small delta whole cube equal to 13 by 14 Prandtl number into 1 minus x naught by x raised to 0.75. Therefore, if I substitute this value, then you will see that Stanton x, which was already shown to be equal to (Refer Slide Time: 21:25) 3 by 2 into alpha over U infinity delta.

In this definition, (Refer Slide Time: 21:30) delta would be R times small delta and R is given by that expression. As you will see here, small delta is under root 280 by 13 into nu x by U infinity will give you this relationship.

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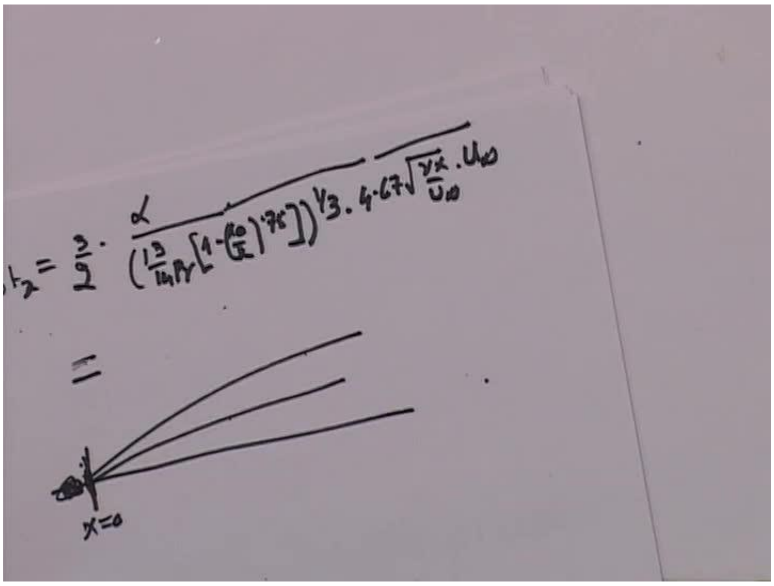

If you want to see how I have done this you will see that Stanton x therefore, will be 3 by 2 into alpha divided by R, which is 13 by 14 Prandtl number into 1 minus x naught by x raised to 0.75 raised to 1 by 3 into delta, which is 4.67 under root nu x by U infinity into U infinity, which is a constant. This is what Stanton x would read as.

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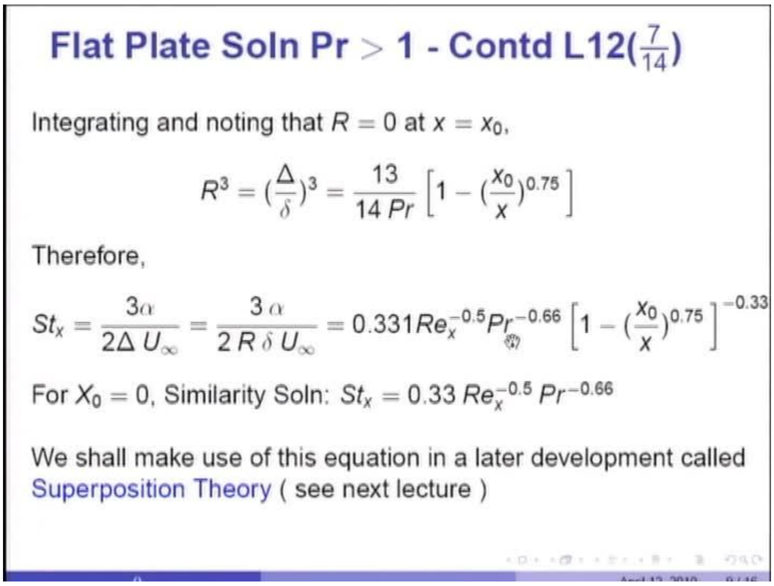

All these put together will result into Prandtl raised to minus 0.66 and 0.331 Re x to the power minus 0.5 into Prandtl to the power minus 0.66 into 1 minus x naught by x raised to 0.75 raised to minus 0.33. So, this is the solution to the case of unheated starting length x naught.

When obtaining similarity solutions we had thermal boundary layers and (Refer Slide Time: 23:12) velocity boundary layers growing at the same point, x equal to 0. Therefore, in effect, this was x naught equal to 0 and the similarity solution was 0.33 Reynolds x to the power minus 0.5 into Prandtl to the power minus 0.66. You can see that in spite of a very simple temperature profile and velocity profile, we have obtained an extremely accurate solution using integral method. However, you can see that similarity solution requires solution of 3 ODEs. The integral solution is simply pencil and paper method and many a times produces extremely accurate results.

This special solution (Refer Slide Time: 24:00) for heating started from x equal to x naught will be used later on to generate solutions when the wall temperature varies arbitrarily with x. I will be using what is called the superposition theory and which is something I will discuss in the next lecture. However, at the moment, let us turn our attention to the case of pressure gradient. Presently, in the flat plate case, the pressure gradient was 0. Therefore, the momentum equation was very simple and so was the energy equation. However, now you will see that our momentum and energy equations would be more complex.

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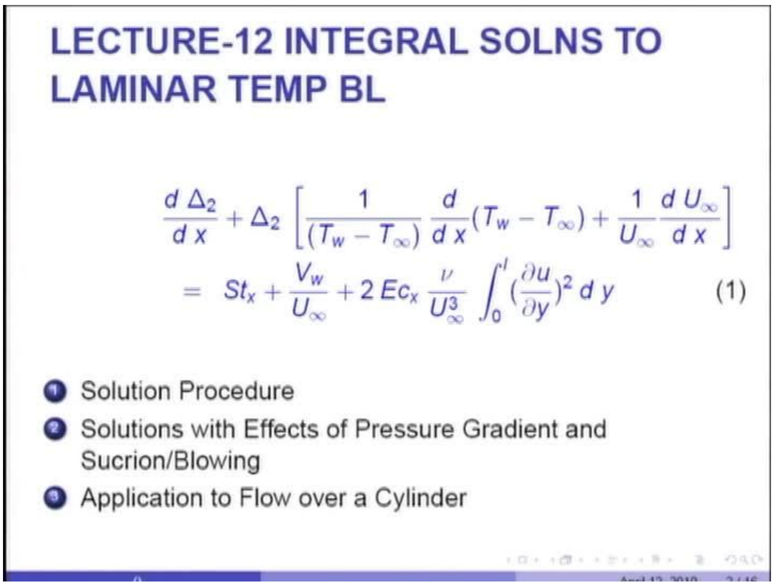

If I turn to the first slide, you will see that I am not considering wall temperature variation, I am not considering V w, but I am going to include this particular term, which contains the pressure gradient term.

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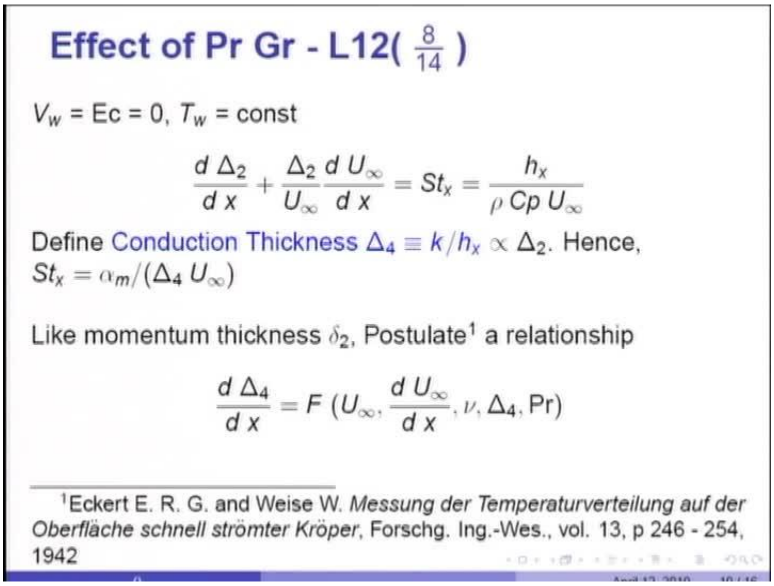

Then, you will see that the equation governing this situation is simply this - V w, Ec are all 0 and T w is constant. Therefore, the equation would read as d delta 2 by d x plus delta 2 by U infinity into d U infinity by dx is equal to this Stanton x.

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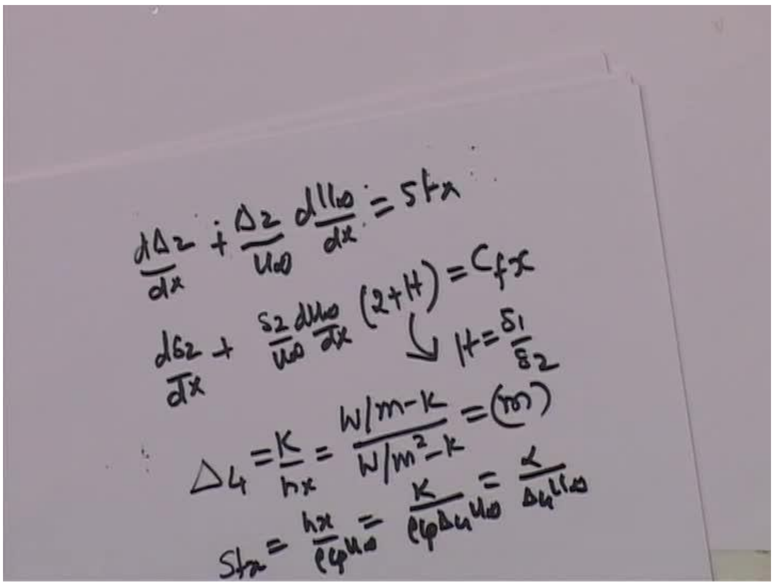

So, the equation is d delta 2 by d x plus delta 2 by U infinity into d U infinity by d x equal to Stanton x. The corresponding momentum equation was d small delta 2 by d x plus small delta 2 by U infinity into d U infinity by d x into 2 plus H equal to C f x, where H was equal to small delta 1 by small delta 2. You can see that the two equations are very similar. Here to make further progress, I am going to define what is called as conduction thickness - delta 4 equal to k by h x (Refer Slide Time: 26:03).

Delta 4 will be defined as k by h x. You recall that the units of k are watts per meter Kelvin, whereas units of h x will be watts per meter square Kelvin. This has the (Refer Slide Time: 26:22) units of meters. Therefore, it is called the conduction thickness. It will be proportional to all other thermal boundary layer thicknesses such as delta, delta 2 and so on and so forth (Refer Slide Time: 26:36). Incidentally, Stanton x would readily follow. It will be simply alpha m divided by delta 4 U infinity because you will know that Stanton x is h x over rho C p U infinity. However, h x is nothing but k by delta 4 U infinity. Therefore, this is equal to thermal diffusivity - delta 4 U infinity.

Because of the similarity of the momentum and thermal boundary layer thickness thermal energy equations (Refer Slide Time: 27:14), we shall postulate as done by Eckert. The reference is given here (Refer Slide Time: 27:20). It is a German paper, a very old paper of 1942.

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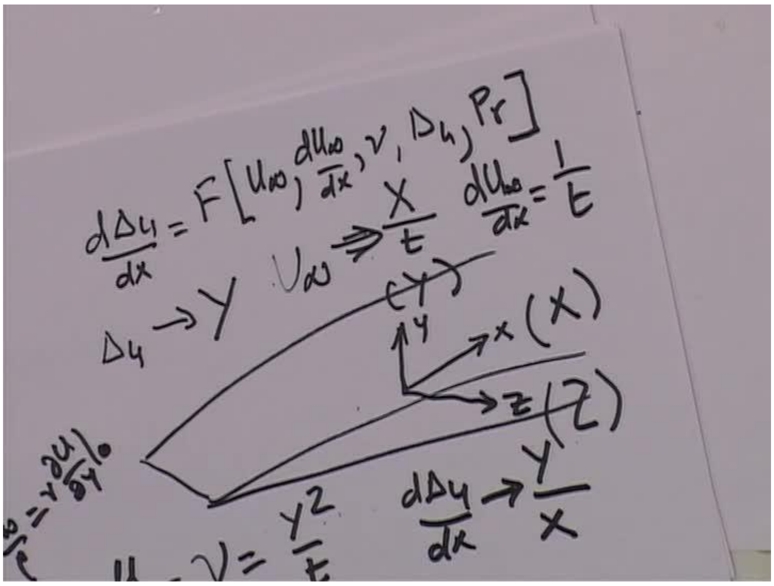

He said we will postulate that the rate of growth of conduction thickness d delta 4 by d x will be first of all governed by the rate of growth of momentum thickness delta 2, which in turn was governed by U infinity, d U infinity by d x and nu. So, that will be the first three parameters here (Refer Slide Time: 27:57). Then, it will also be governed by delta 4 and Prandtl number. So, the rate of growth of a conduction thickness would be done.

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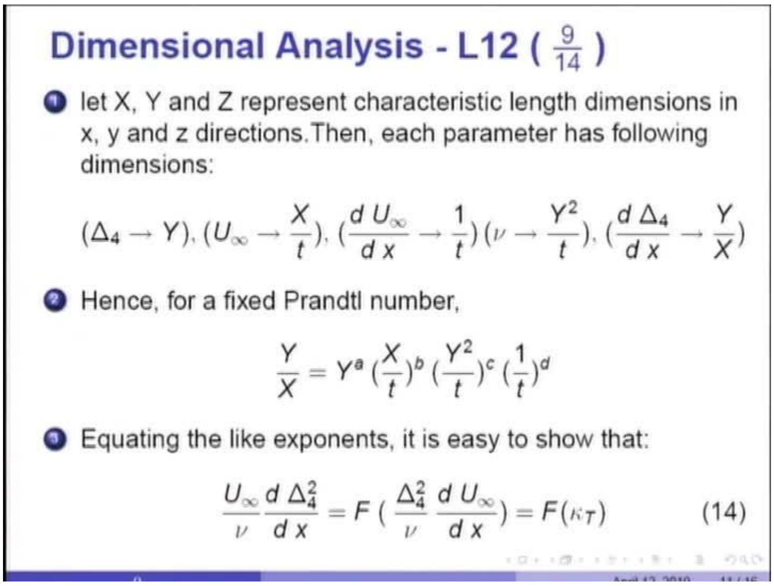

We are following a strategy very analogous to the results we had derived for integral momentum equation. Now, we shall say - we will carry out dimensional analysis. Let us say if this was the boundary layer (Refer Slide Time: 28:25) growing on a flat plate or any other plate with a pressure gradient, then in the stream-wise direction, this is x, this is the transverse direction y and this is the lateral direction z. If I say the characteristic dimension in direction x is capital X, the characteristic dimension in direction y is capital Y and characteristic dimension in direction z is Z.

In other words, this is Z, this is X (Refer Slide Time: 29:03) and here Y. Then, you will see that delta 4 being a transverse thickness will have dimension Y. U infinity being a stream-wise velocity would have dimension x by t. d U infinity by d x would be 1 over t; simply divide this by X. nu would be Y squared by t because this is kinematic viscosity mu by rho and the shear stress. This is nothing but tau wall by rho equal to nu times d u by d y at y equal to 0. Therefore, you can see that the shear stress, which acts along the surface on an area y z would have units. Therefore, the nu will have units of Y square by t and then finally, d delta 4 by d x would be simply (Refer Slide Time: 30:25) Y divided by X.

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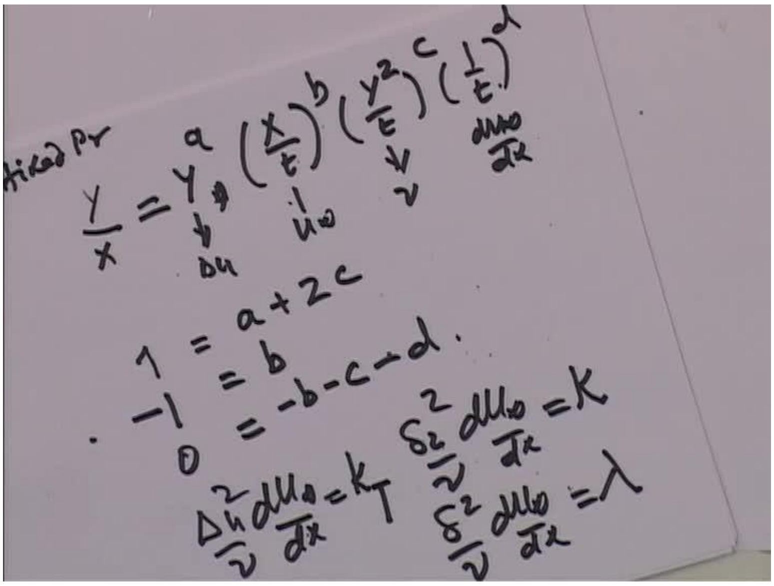

If I were to carry out the dimensional analysis, you will see I will get Y by X, which is d delta 4 by d x equal to first of all delta 4, which is Y raised to a. Then, U infinity, which is X by t raised to b; this is delta 4 and this is U infinity. Then, into Y squared by t raised to c, which is nothing but nu and 1 over t raised to d, which is d U infinity by d x. So, I have captured all for a fixed Prandtl number.

If I equate the like powers, power of Y on the left hand side is 1, which will be equal to a plus 2 c; power of X is minus 1, which will be equal to b; and the power of time is 0, which will be equal to minus b minus c minus d.

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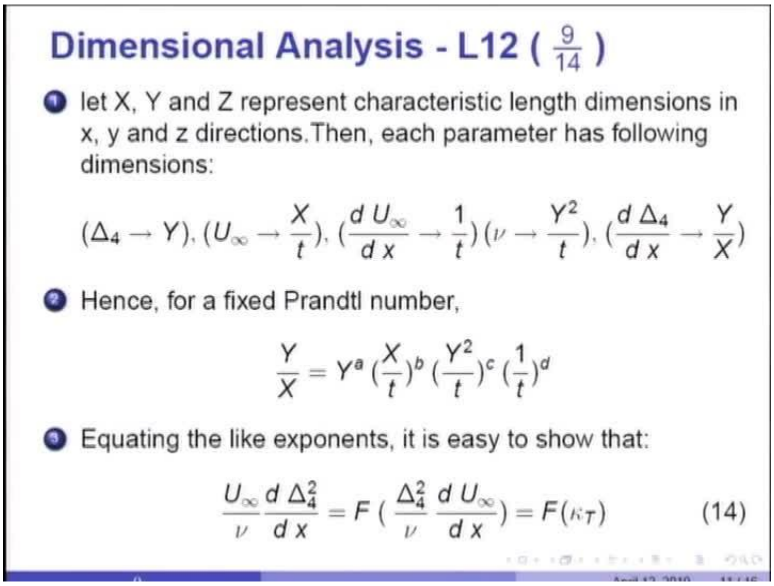

With this, I can determine a b c and d; let us say everything in terms of d. You will get U infinity by nu into d delta 4 by d x as a function of d delta 4 by nu into d U infinity by d x. Remember: This (Refer Slide Time: 31:57) delta 4 square by nu into d U infinity by d x is very similar to small delta 2 squared by nu into d U infinity by d x, which we had called kappa. Also, small delta square by nu into d U infinity by dx, which was called lambda. It is very analogous and therefore, I call it kappa T to indicate that this is a thermal dynamiter associated with conduction thickness delta 4.

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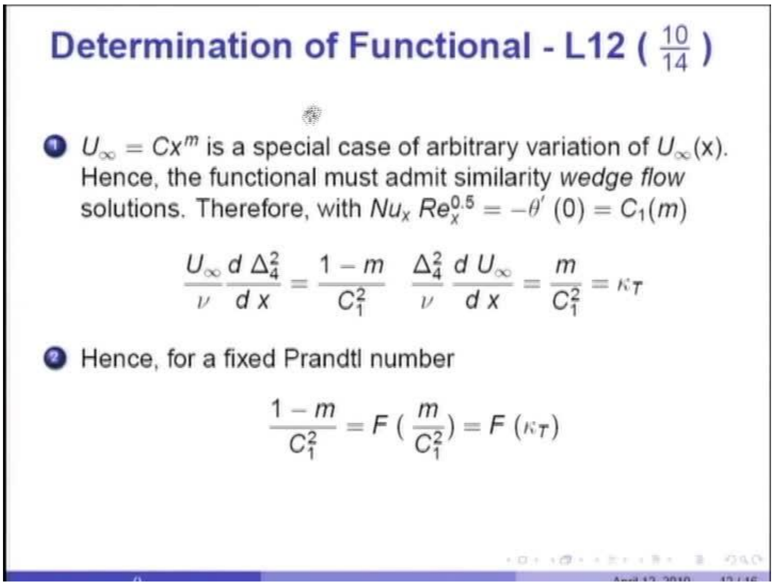

We have a relationship very similar to what we had in integral momentum equation. Therefore, I can evaluate both these quantities (Refer Slide Time: 32:35) from similarity solution U infinity equal to Cx to the power m, which is nothing but a special case of arbitrary variation of U infinity. The functional must admit similarity wedge flow solutions.

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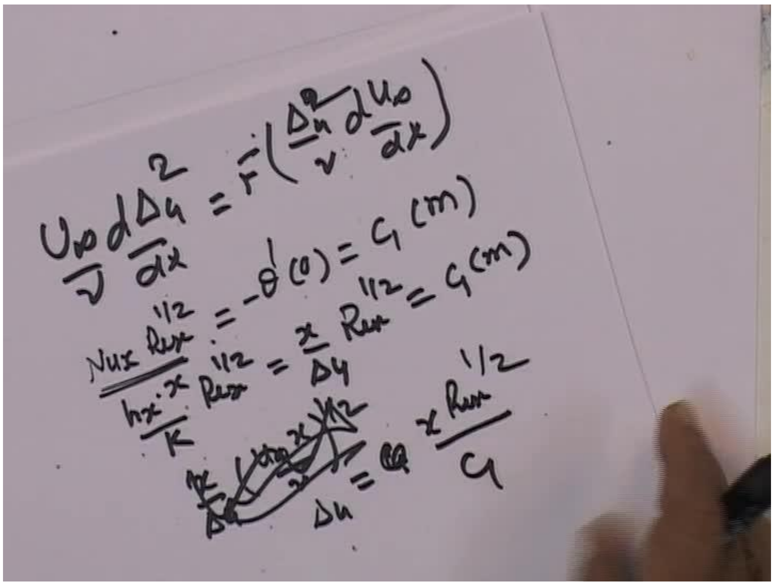

Both these functional (Refer Slide Time: 32:52) U infinity over nu into d delta 4 square by dx must equal F times delta 4 square by nu into d U infinity by d x. It must admit wedge flow solution.

Remember, we have (Refer Slide Time: 33:15) Nu x; Our wedge flow solutions are Nu x into Reynolds x to the power half equal to minus theta prime 0, which is c 1. This is because theta prime 0 is a function of m and it is a constant.

Now what is this? (Refer Slide Time: 33:39) Nu x Re x to the power half. It is hx x by k into Re x to the power half. Therefore, this is nothing but x by delta 4 into Re x to the power half. Therefore, it is equal to c 1 raised to m. Therefore, delta 4 is simply x Re x to the power half divided by c 1.

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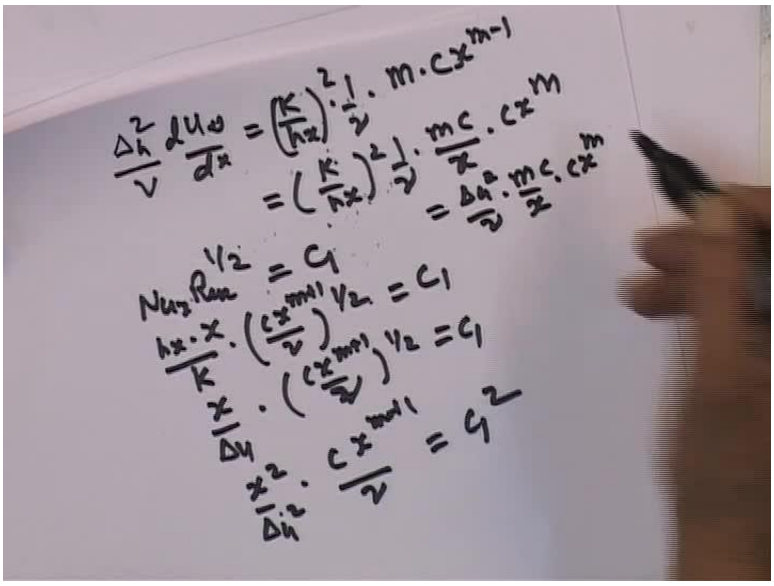

Delta 4 square by nu into d U infinity by d x will be simply k by h x whole squared into 1 over nu into m times c x raised to m minus 1, which is k by h x whole squared into 1 over nu into m c by x into c x raised to m. Nu x Re x raised to half is equal to c 1 or that is equal to hx x by k into cx raised to m plus 1 by nu raised to half equal to c 1. h x by k is nothing but x by delta 4 into c x raised to m plus 1 by nu raised to half equal to c 1. Therefore, c 1 square would be x square over delta 4 square into c x raised to m plus 1 by nu equal to c 1 square.

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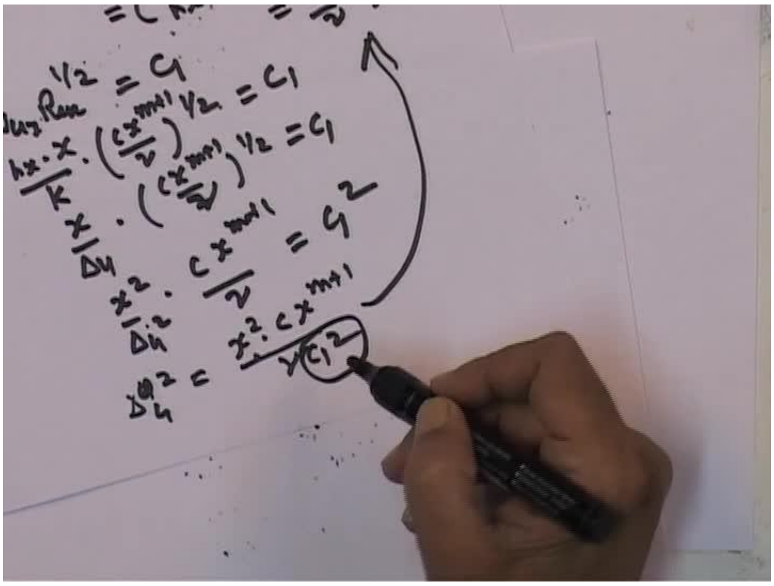
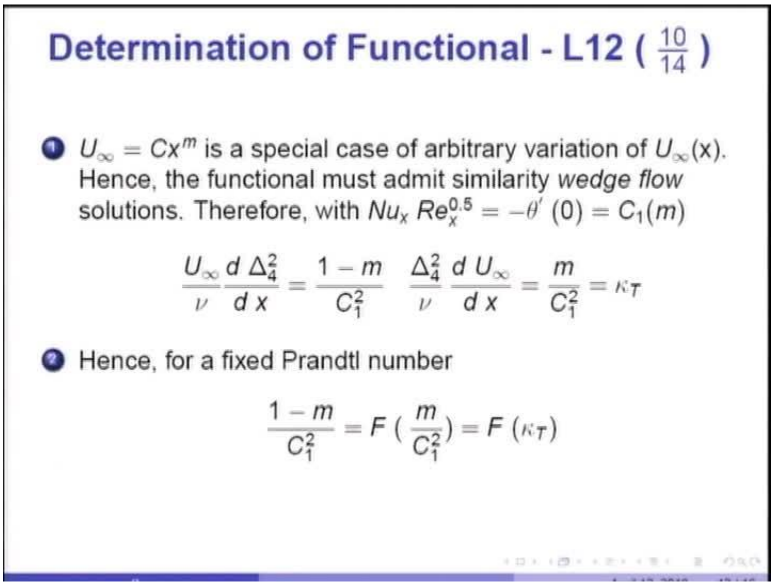


Therefore, delta 4 square is equal to x square into c x raised to m plus 1 divided by nu c 1 square. If I substitute that in here (Refer Slide Time: 36:20), you will see that I will get cancellation and it will become simply this m divided by c 1 square would survive.

(Refer Slide Time: 36:33)



That is equal to kappa T and likewise we can show that U infinity over nu into d delta 4 square by d x is 1 minus m by c 1 square. The relationship that we had here (Refer Slide Time: 36:46) is simply equal to 1 minus m by c 1 squared equal to F of m by c 1 squared equal to F kappa T, where c 1 is that belonging to the m chosen. Therefore, by selecting different values of m and correspondingly c 1, I can plot this variation.

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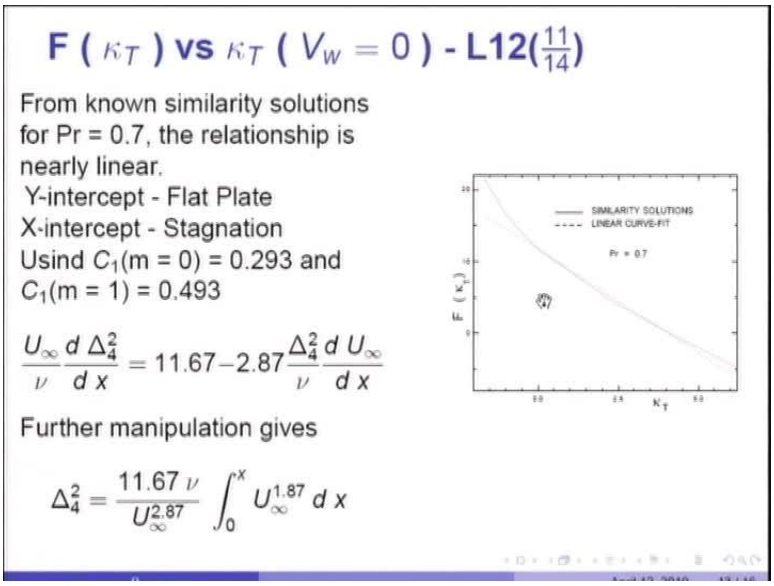

This is what I have done on this slide here. You can see - this is kappa T, this is function of kappa T, the right hand side; this is kappa T equal to 0 line and this is F kappa equal to 0 line. So, this is where the axis 0 is; at this point. What I have plotted here are the similarity solutions, but you can see that for considerable variation of kappa T, the variations are actually very close to a linear variation. The graph has been plotted while looking at results for Prandtl equal to 0.7. You can create similar graph for other values of Prandtl number for which we have the similarity solutions.

From known similarity solutions for Prandtl equal to 0.7, the relationship is linear. Therefore, you can see the x intercept here (Refer Slide Time: 38:13) will give you the stagnation point solution. The y intercept will give you the d U infinity by d x or equal to F kappa T equal to 0 solution and that will be flat plate. Then, using the value of Prandtl equal to 0.7, c 1 m equal to 0 is 0.293 and c 1 m equal to 1 is equal to 0.493.

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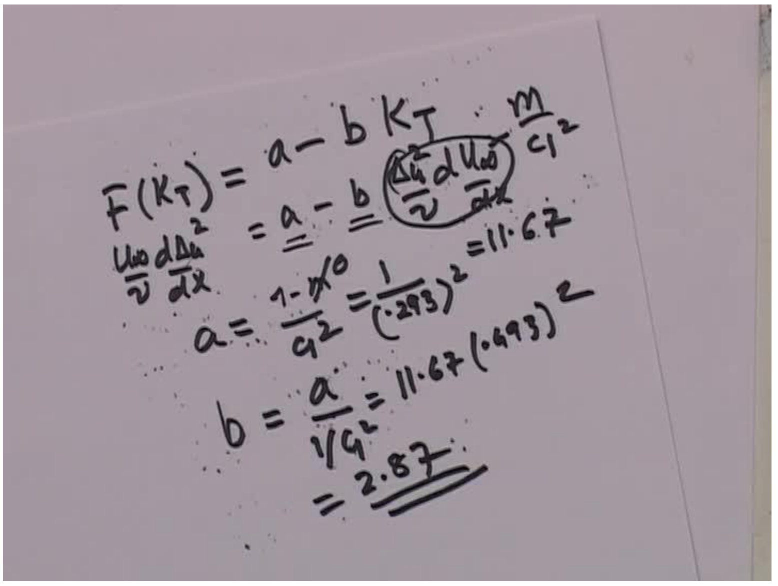

As we did in the wedge flow solutions of Thwaites, F kappa T is related to a minus b kappa T. This is a minus b delta 4 square by nu into d U infinity by d x and this is U infinity by nu into d delta 4 square by d x. So, I have to determine a and b.

The first thing I do is to look at the flat plate solution for d U infinity by d x. Therefore, as you can see, a will be simply equal to 1 minus m, which is equal to 0 divided by c 1 square. That is equal to 1 over 0.293 whole squared, which is 11.67.

Now, b would be determined, where this is 0. You will recall that for stagnation point solution, all thicknesses are constant. Therefore, this is equal to 0 (Refer Slide Time: 39:49). Then, b will be simply a times this quantity, which is m by c 1 square and m is equal to 1; which is 1 over c 1 square. That will be equal to 11.67 into 0.493 whole squared, which gives you 2.87.

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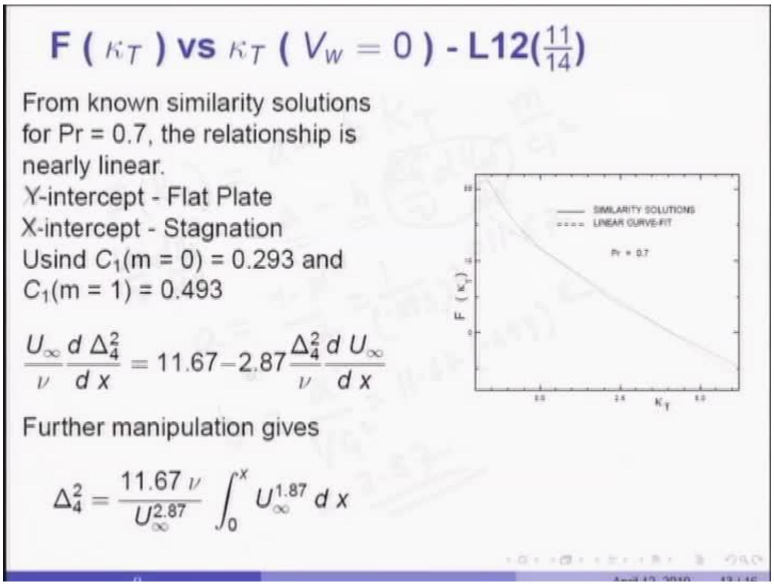

That is how I generate the linear curve fit to the integral energy equation. This can be manipulated to give delta 4 square equal to 11.67 nu by U infinity raised to 2.87 into 0 to x U infinity raised to 1.87 d x; exactly as we did in case of momentum.

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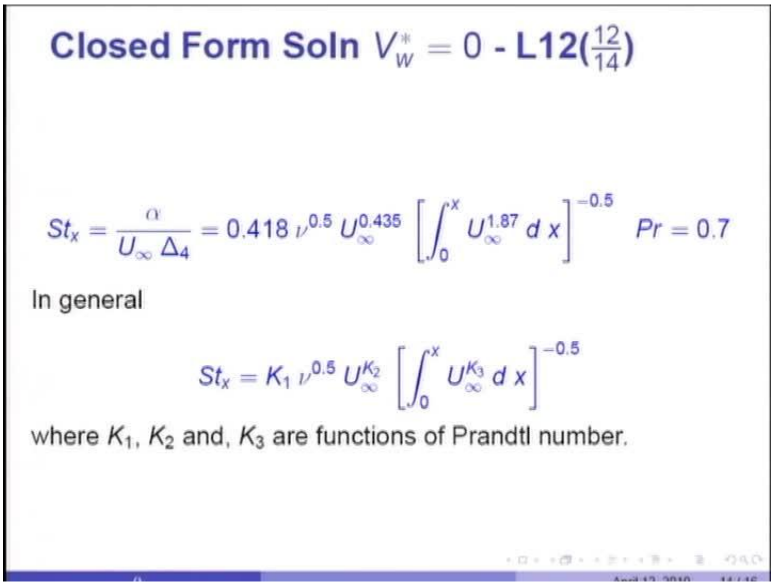

Then, the closed form solution having obtained delta 4 in this manner (Refer Slide Time: 40:49). So, delta 4 square will be square root of this. I get the solution for Stanton x (Refer Slide Time: 40:54) alpha by U infinity delta 4 equal to all these for Prandtl (( )).

All these constants - this one, this one and this one result from the curve fit, which we obtained for Prandtl equal to 0.7 (Refer Slide Time: 41:08).

If I had chosen any other Prandtl number I would get both these constants to be somewhat different. Therefore, I can generalize and say that (Refer Slide Time: 41:18) Stanton x is K 1 nu raised to 0.5 into U infinity k 2 into 0 to x U infinity k 3, where all these are functions of Prandtl number. This is something you can do by yourself because you already have the similarity solutions available to you for different Prandtl numbers for m equal to 0 and m equal to 1. Therefore, you can generate the solutions.

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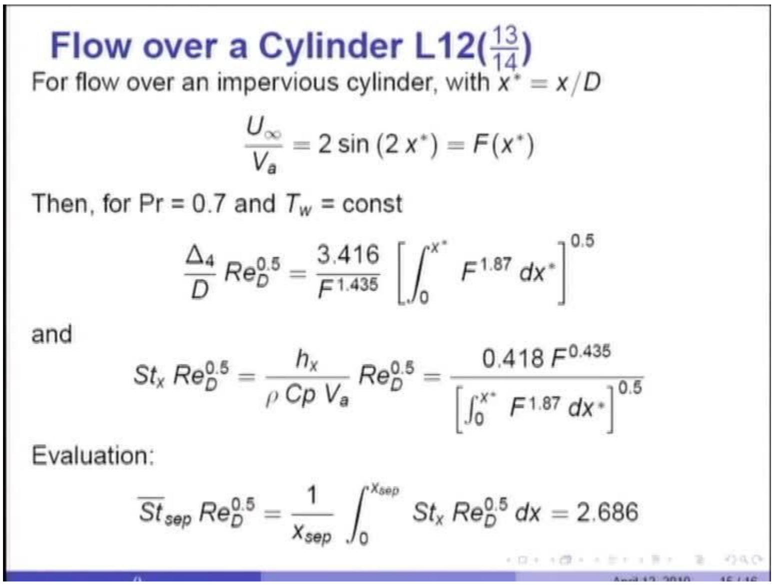

I will now consider the flow over a cylinder that we had considered earlier. For which, U infinity by V a is 2 sine 2 x star, where x star is x by d and I had called this as F x. Then, for Prandtl number 0.7 and T w equal to 0, it is possible to integrate by substituting for (Refer Slide Time: 42:04) U infinity our function, it is possible to show that delta 4 by D into R e D raised to 0.5 would be read like that. Stanton x would read in this fashion (Refer Slide Time: 42:16). If I were to integrate this function from 0 to separation 1 over x separation 0 to x separation, I get the result 2.686.

(Refer Slide Time: 42:35)

Angular Variations Pr = 0.7 L12($\frac{14}{14}$)

θ deg	$(\Delta_4/D)Re_D^{0.5}$	$St_x Re_D^{0.5}$	$C_{f,x} Re_D^{0.5}$
0.0573	0.242E+01	0.296E+03	0.151E-02
0.515	0.117E+01	0.679E+02	0.340E-01
2.00	0.105E+01	0.194E+02	0.159E+00
4.98	0.103E+01	0.801E+01	0.411E+00
10.0	0.102E+01	0.402E+01	0.832E+00
30.0	0.105E+01	0.136E+01	0.230E+01
50.0	0.113E+01	0.821E+00	0.322E+01
70.0	0.128E+01	0.592E+00	0.338E+01
80.0	0.139E+01	0.521E+00	0.317E+01
90.0	0.153E+01	0.465E+00	0.282E+01
100.0	0.173E+01	0.419E+00	0.234E+01
105.0	0.184E+01	0.401E+00	0.210E+01
108.3	0.194E+01	0.388E+00	0.192E+01

To see exactly the values of local values of Stanton x and how delta 4 varies, here are the angle along the cylinder starting with the stagnation point at 0. So, you have delta 4 by D, conduction thickness is 2.42. It reduces to 1.17, 1.05 and then again starts rising. As you recall, this is the point of separation.

Stanton x is very high. The Nusselt number is very high at this time or the heat transfer coefficient is high at the stagnation point. However, then gradually reduces as you move towards the vertical axis of the cylinder and becomes 0.388. C f x on the other hand starts with the very low value and rises and again when the flow begins to decelerate the C f x begins to fall again here. Whereas, during the acceleration part, the C f x goes on increasing with the angle. With this, I conclude the effects of pressure gradient on heat transfer in integral solutions.