Convective Heat and Mass Transfer

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Module No. # 01

Lecture No. # 11

Integral Solutions to Laminar Vel BL

In this lecture, I will take up solution of the Integral momentum equations or the Integral equations of the velocity boundary layer.

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My task would be to introduce you to the general solution procedure and then using this procedure, study the effects of the pressure gradient, suction and blowing. I recall again that Integral solutions can be obtained for arbitrary variations of the free stream velocity U infinity at the edge of the boundary layer and the wall velocity at the solid surface; we call it as suction or blowing.

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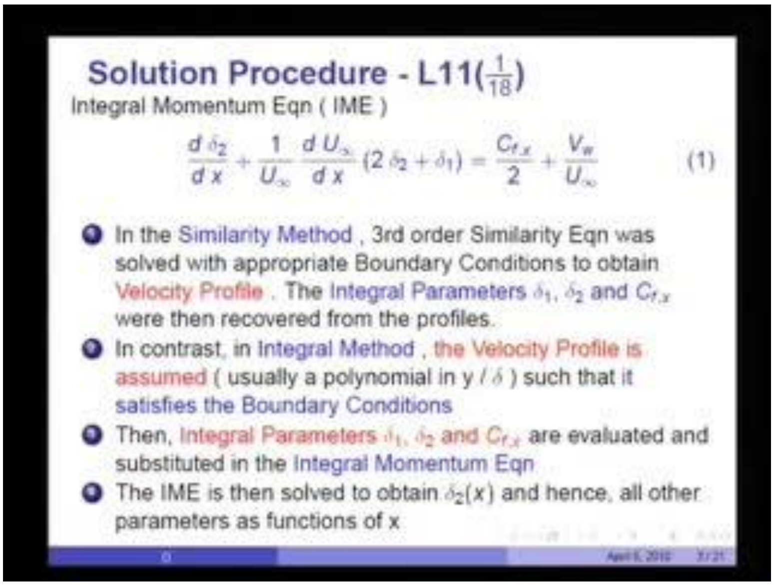

To refresh our memory, the Integral momentum equation reads as d delta 2 by d x plus 1 over U infinity d U infinity by d x plus 2 delta 2 plus delta 1 equal to C f x by 2 plus V w by 2.

How do we solve this equation? Just recall, the procedure here is quite the reverse of what we adopted for similarity method. In the similarity method, a third order similarity equation was solved with appropriate boundary conditions to obtain the velocity profile.

The velocity profile came out of the solution of the ordinary differential equation and from which the Integral parameters such as delta 1 delta 2 and C f x were recovered by integrating and differentiating the profiles. In contrast, in Integral method the velocity profile is assumed usually a polynomial in y by delta. Such that, it satisfies the boundary conditions having assumed this profile. We evaluate the Integral parameters delta 1 delta 2 and C f x and substitute them in the Integral momentum equation. The IME is then solved to obtain delta 2 as a function of x and hence all other parameters as functions of x. So, the procedure is quite the reverse of the similarity method and to the extent that we have assumed the velocity profile that which satisfies boundary conditions but none the less is an approximation to what could be the real velocity profile at a given x.

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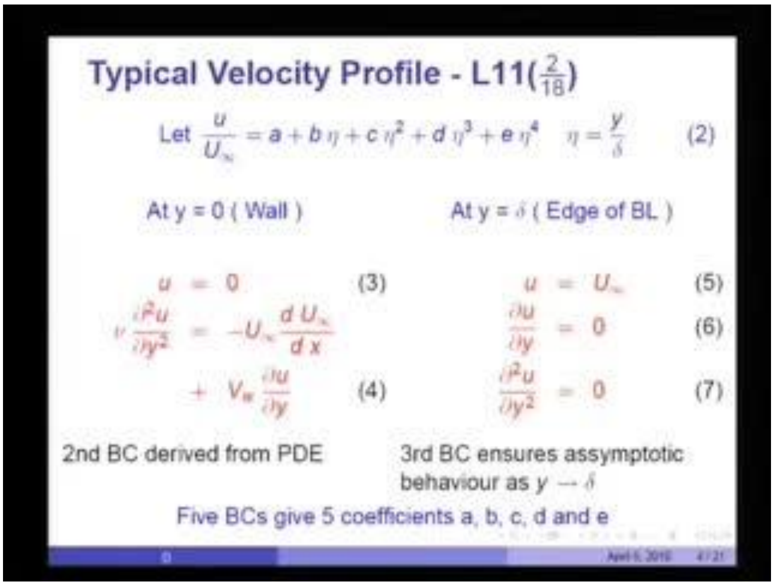

We say, let u over U infinity, which is the dimensionless velocity and it would be a function of x and y be a polynomial in eta variable, where eta is y by delta a plus b eta plus c eta square plus d eta cube and e eta 4. So, there are 5 constants to be determined a, b, c, d and e and this we do by invoking 5 boundary conditions. The boundary conditions are as follows - At the wall y equal to 0, u is equal to 0, which will render u equal to 0

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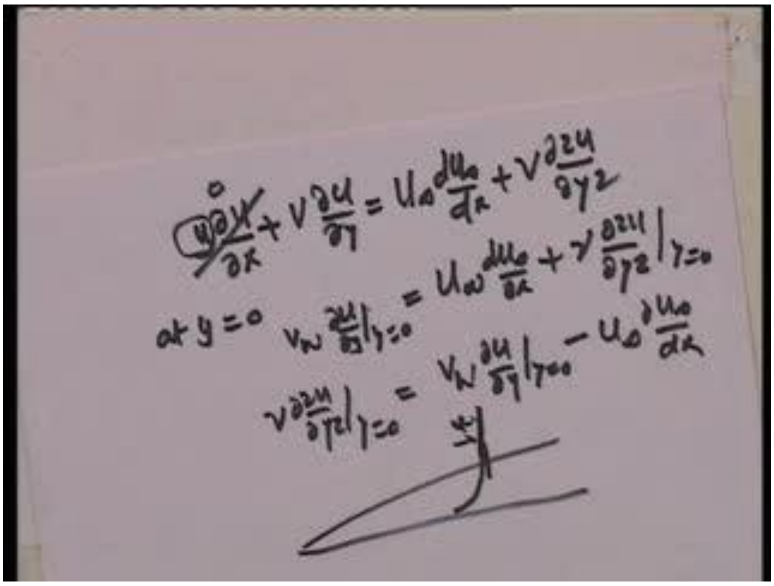

The second condition is that, if you look at the momentum equation, if I write this equation at y equal to 0, then clearly that term will be 0 because u itself is 0. However, v will be v w into d u by d y at y equal to 0. This term will survive, U infinity d U infinity by d x plus nu d 2 u by d y square at y equal to 0. I used this as a boundary condition, to say that nu d 2 u by d y square at y equal to 0 is equal to v w d u by d y at y equal to 0 minus U infinity d U infinity by d x.

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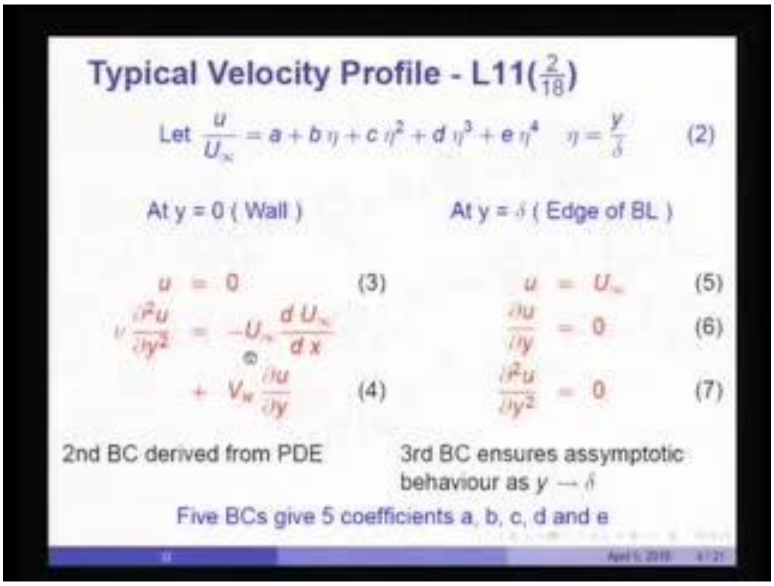

That is what I have written here as the equation 4, which is the second boundary condition at the wall. At the edge of the boundary layer, y equal to delta u will equal U infinity and therefore the left hand side will be equal to 1 and so will each of these etas will be 1. You will have a condition a plus b plus c plus d plus e equal to 1. Also d u by d y is equal to 0 at the edge of the boundary layer d u d y is 0. To the extent that the velocity approaches U infinity, asymptotically the continuity of this first derivative survives d u by d y equal to 0 survives as we increase y. Therefore, the last boundary condition is that d 2 u by d y square will also be equal to 0.

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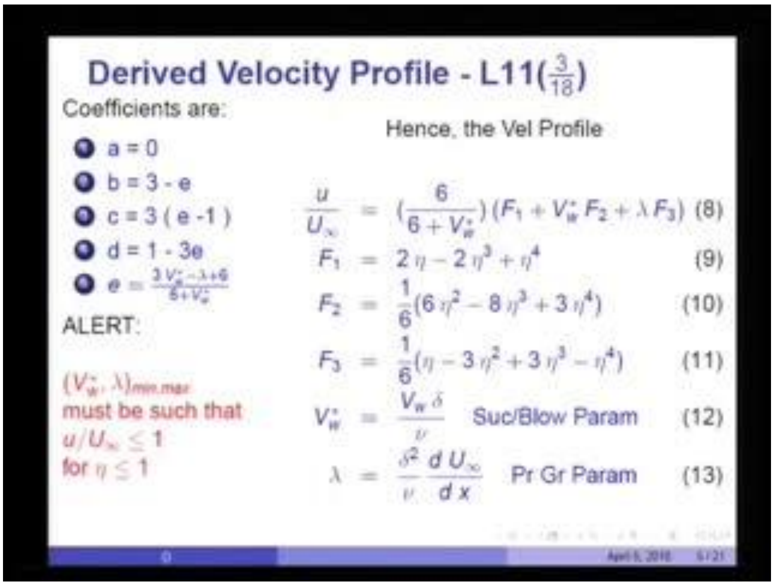

I have 1 2 3 4 and 5; five boundary conditions which will enable me to determine a, b, c, d and e. Let us see, what those coefficients look like? Each of these coefficients can be represented in terms of the coefficient e, a equal to 0 because of u is equal to 0 at y equal to 0, b will be equal to 3 minus e, c will be equal to 3 into e minus 1, d will be 1 minus 3 e and e will equal 3 times V w star minus lambda plus 6 divided by 6 plus V w star.

With these five boundary conditions, if I were to write out the equation, the velocity profile will look like u over U infinity 6 by 6 plus V w star a function F 1 a second function F 2 multiplied by V w star and a third function F 3 multiplied by lambda.

F 1 is simply 2 eta minus 2 eta cubes plus eta raised to 4. F 2 is this function F 3 is this function V w star is a dimensionless V w delta by nu. It is suction and blowing parameter and lambda is delta square by nu d U infinity by d x is a pressure gradient parameter associated with variation of free stream velocity U infinity with respect to x.

I did say that we have to choose the values of V w star and lambda very carefully because as much as we can allow for any variations of V w and U infinity, we must ensure that the assumed velocity profile is such that u over U infinity is always less than 1 inside the boundary layer for eta less than 1. So, values of V w star and lambda minimum or maximum must be such that this condition holds. Remember, V w star can be both positive or negative depends on suction or blowing and lambda depending on whether it is an adverse pressure gradient and a favorable pressure gradient can also be positive or negative. We have to choose these parameters within a certain restricted range only; so that the boundary layer approximations are well held.

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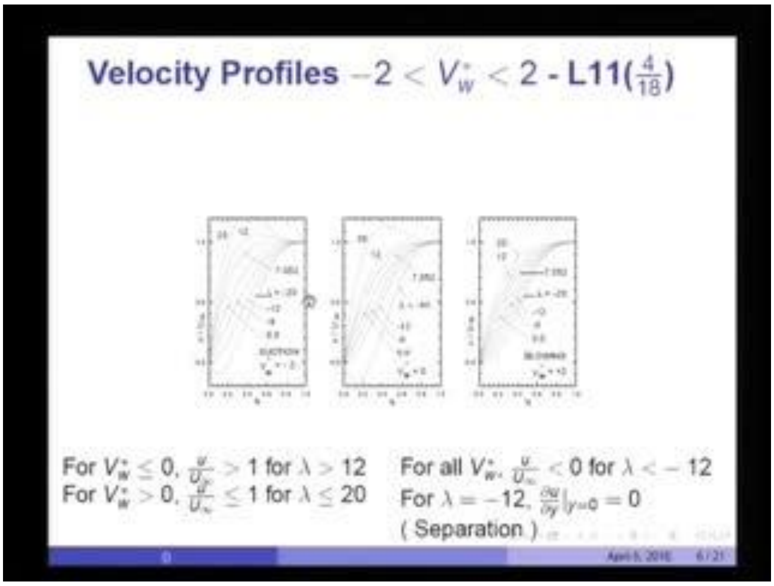

I have plotted 3 graphs. The middle graph is for no suction or blowing that is V w star equal to 0. When lambda is equal to 0, you will have the flat plate because d U infinity by d x is equal to 0. As you can see here, this will be the profile, lambda equal to positive values means accelerating boundary layer because d U infinity by d x then is positive, lambda negative would be d U infinity by d x because d U infinity by d x is negative and therefore, a decelerating boundary layer.

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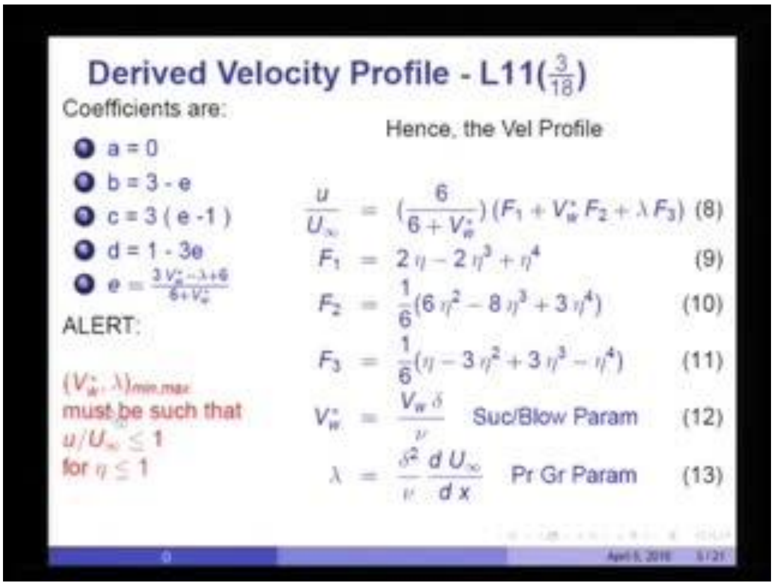

If I choose arbitrarily values of lambda, you will see up to lambda equal to 12 the u over U infinity values are well within 1 and up to minus 12 they are very much within 0 to 1. If I take lambda equal to 30, which is a very highly accelerating flow then u over U infinity exceeds 1. In fact that is what would happen for all values of lambda greater than 12. Therefore, such values of lambda are not admissible; likewise, values less than lambda equal to minus 12 are not admissible because velocity itself will turn negative this is the importance of this comment which I made here.

If u over U infinity exceeds 1 then delta 1 or delta 2 can become negative. Likewise, if u over U infinity is negative then delta 2 can be negative and that is not admissible.

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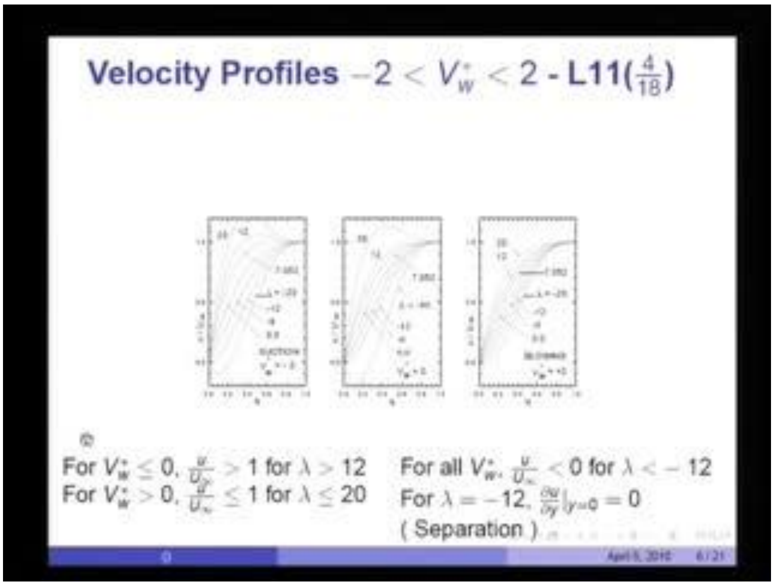

This is similar profiles for V w star equal to minus 2, which means the suction case and again I have plotted several values. Before I do, look at that let me go back again to V w star equal to 0 value, when lambda equal to 0, I said this is a flat plate solution but lambda equal to minus 12 gives us 0 gradient at the wall, which means separation must occur. It is a peculiar value, where lambda equal to 7.052 and the importance of that you will recognize a short while from now.

Let me go back again to the suction profile, here you will see for lambda equal to 12 plus 12 u over U infinity exceeds 1 and for 20 it exceeds very much. Likewise, here for lambda equal to minus 12 there is a velocity which goes less than 0.

On the blowing side, however up to lambda equal to 20 or little more, there is a u over U infinity well within 1. So, blowing permits much higher pressure gradients favorable pressure gradients. On the adverse pressure gradient side, again lambda equal to minus 2 is the limit.

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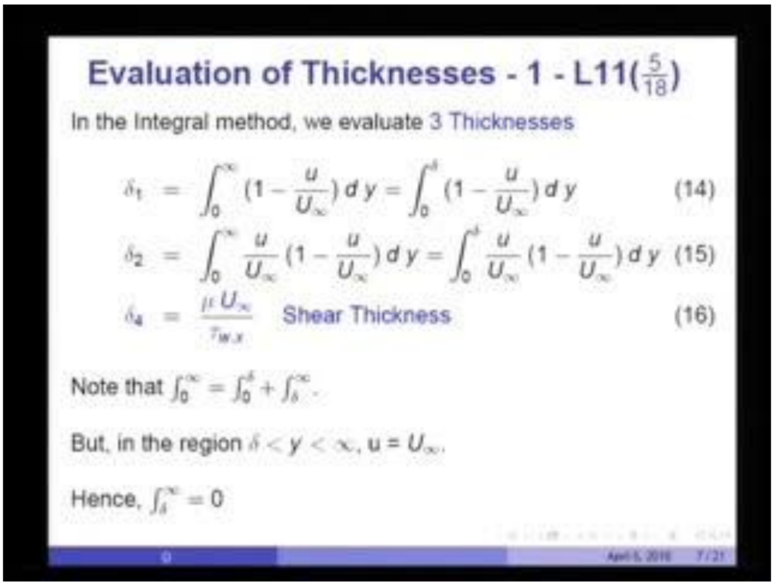

Integral method, we evaluate three thicknesses - the first one is very well known to you delta 1, which can be where the integration limit from 0 to infinity is replaced by 0 to delta because from delta to infinity u will remain equal to U infinity. Therefore, both these Integrals will be equal to 0 between delta and infinity and the limit infinity can be replaced by delta in both of them.

In addition, we define a shear thickness mu U infinity divided by shear stress. You will notice that it has a length dimension, we call delta 4 and it is called the shear thickness mu U infinity by tau wall x. The reason for this is, you will understand very shortly.

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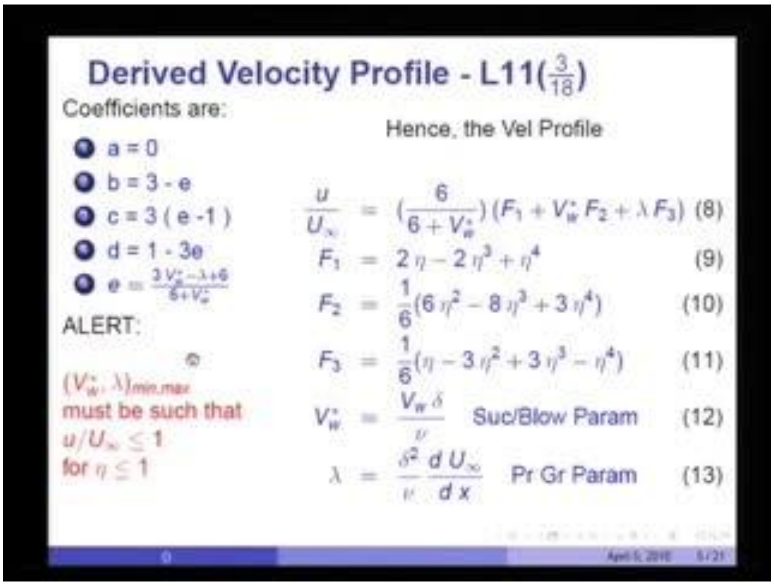

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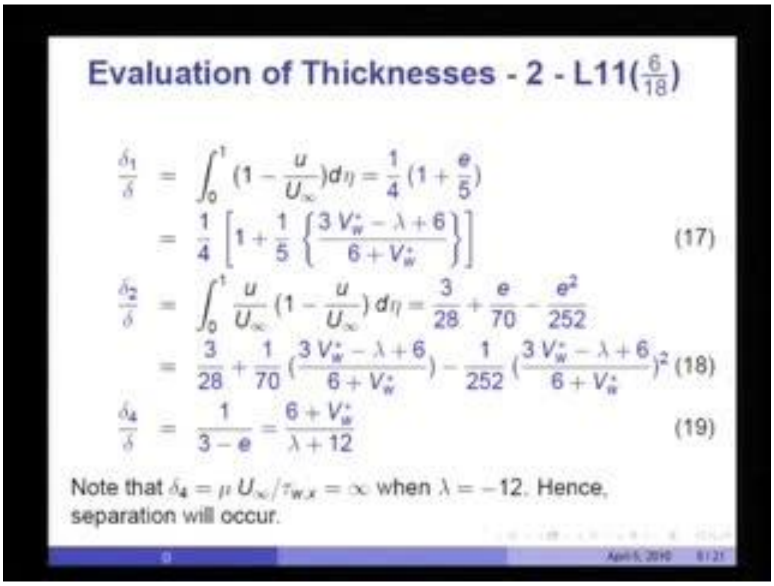

We have three thicknesses delta 1, delta 2 and delta 4. We could have recovered value of delta 4, even in the similarity method because we know how the tau wall x varies with x. If I substitute our velocity profile u over U infinity equal to all these as a function of eta into our definitions, then, you will see that delta 1 by delta would become 1 over 4 1 plus e by 5. If I represent e as 3 V w star minus lambda plus 6 over 6 plus V w star, then it will simply read like that delta 2 by delta. This integration involves product of u over U infinity square and that would result into 3 by 28 plus e by 70 minus e square by 252 or what I have given here by simply replacing e.

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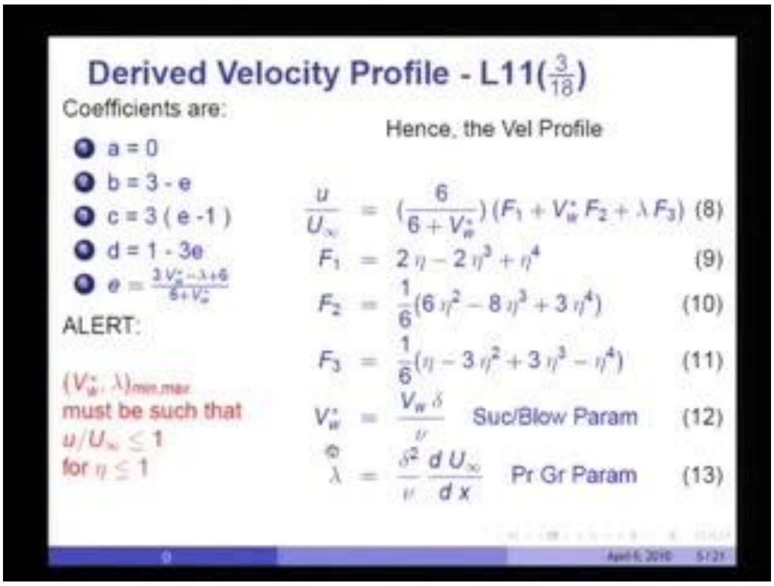

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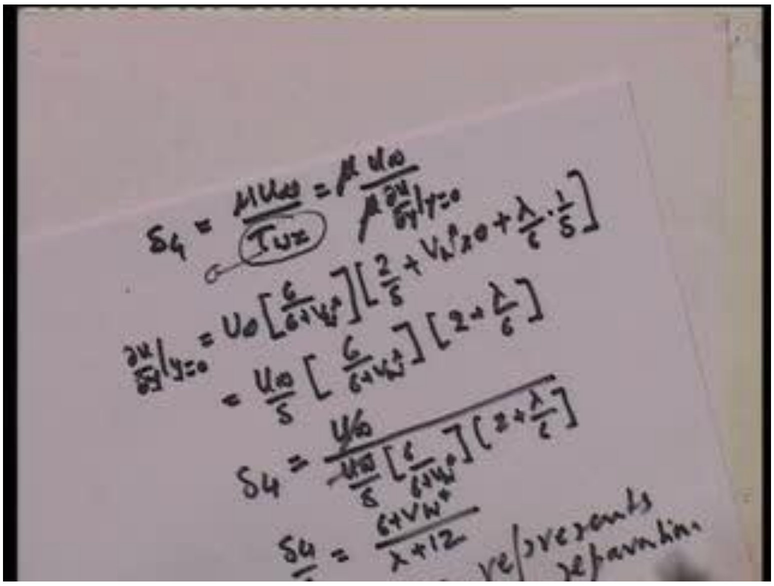

Most importantly, delta 4 by delta would simply result in 1 minus 3 e 6 plus V w star divided by lambda plus 12. This is quite easy to see, if we see our equation then remember delta 4 is mu over U infinity divided by tau wall x and this is equal to mu times U infinity divided by mu times d u by d y at y equal to 0. If we look at our velocity equation then you will see d u by d y at y equal to 0 will become equal to U infinity into

6 over 6 plus V w star into 2 by delta plus V w star into 0 plus lambda by 6 into 1 over delta.

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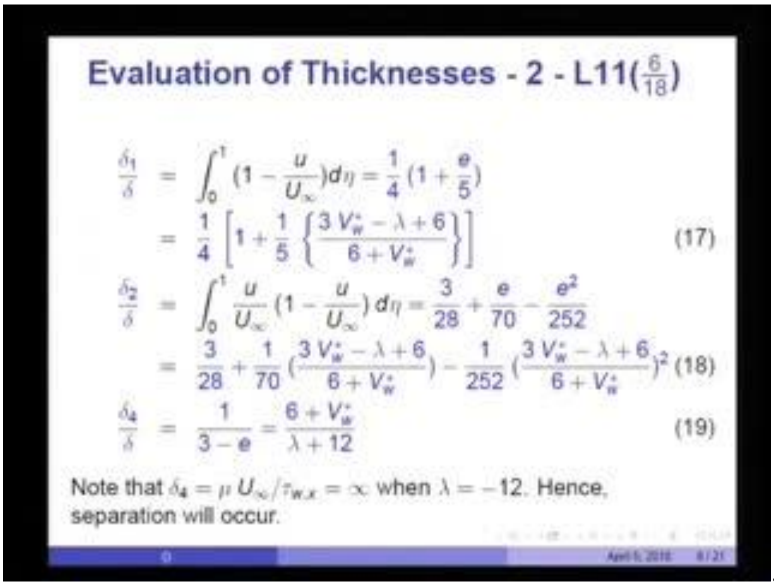

You will see, if I take delta common I will get this as U infinity by delta into 6 over 6 plus V w star into 2 plus lambda by 6 and delta 4 will become equal to mu mu gets cancelled here and U infinity divided by U infinity by delta 6 over 6 plus V w star into 2 plus lambda by 6. Therefore, this and this gets cancelled and delta 4 by delta will read as what I have shown there 6 plus V w star over lambda plus 12. This shows very clearly that if lambda is equal to minus 12, delta 4 will become infinity or if delta 4 becomes infinity then tau wall x must be 0. Therefore, lambda equal to minus 12 represents separation, values of u over infinity were not applicable in our profiles for lambda less than minus 12.

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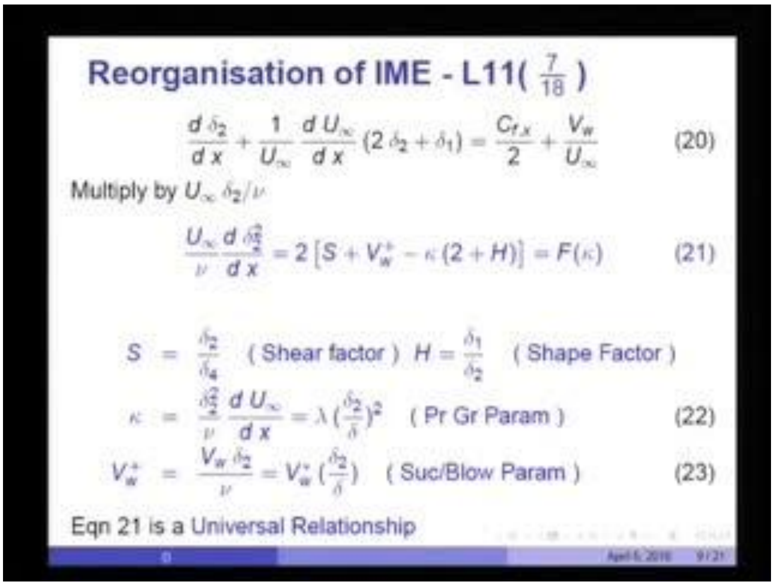

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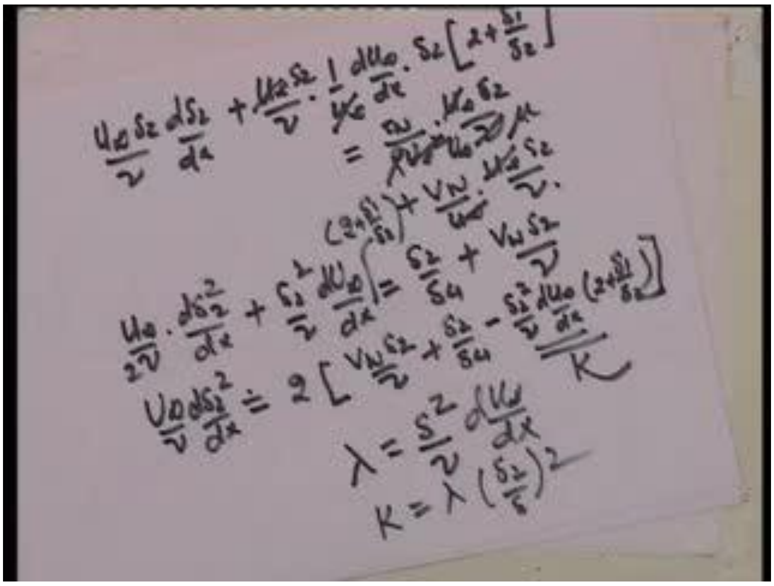

We reorganize the Integral momentum equation by multiplying each term by u delta 2 by nu. You will see that our equation would read as U infinity delta 2 by nu d delta 2 by d x plus U infinity delta 2 by nu divided by 1 over U infinity d U infinity by d x into delta 2 into 2 plus delta 1 by delta 2 equal to C f x which is by 2, which is tau wall over rho U infinity square into U infinity delta 2 by nu plus V w by U infinity into U infinity delta 2 by nu.

Then, you will see this term simply becomes U infinity by nu into d delta 2 square by d x divided by 2. In this case, U infinity gets cancelled with this and I get delta 2 square by nu d U infinity by d x equal to rho into nu becomes mu and nu infinity becomes U infinity. So, mu U infinity by tau wall is delta 2 by delta 4 plus U infinity gets cancelled with that I will be getting V w delta 2 by nu. Therefore, multiplying throughout by 2 I get U infinity by nu d delta 2 square by d x equal to 2 times V w delta 2 by nu plus delta 2 by delta 4 minus delta 2 square by nu oh sorry this should be multiplied by 2 plus delta 1 by delta 2 I forgot d U infinity by d x into 2 plus delta 1 by delta 2.

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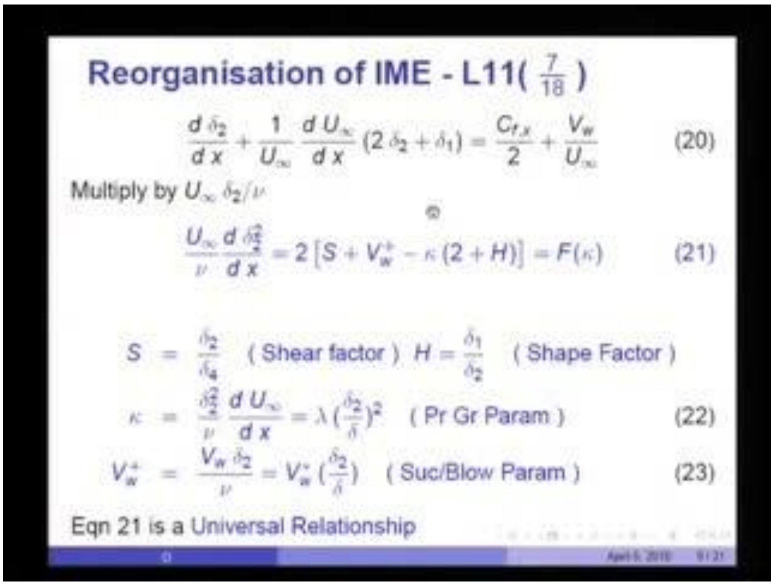

Again, the equation maintains its dimensionless form, Now, all I have done is in our derivation is delta 2 by delta 4 is replaced by s and we call it shear factor; delta 1 by delta 2 is replaced by h, which we call shape factor; delta 2 square divided by nu d u infinity by d x and that would simply be equal to lambda times delta 2 by delta whole square as a pressure gradient parameter. Remember, what was lambda? Lambda will be delta square by nu d U infinity by d x and therefore this parameter which is kappa. kappa will be simply lambda times delta 2 by delta whole square and that is the pressure gradient parameter and V w plus will be V w delta 2 by nu is equal to V w star delta 2 by delta because V w star was simply V w delta by nu. Each term is dimensionless, you can see this is velocity a length dimension because this is square and so this has a length dimension and this is nu.

So, it is a kind of a Reynolds number, which represents rate of growth of momentum thickness delta 2; kappa is a pressure gradient parameter. This is really a universal relationship what we mean that the relationship applies no matter what is the variation of U infinity or the wall velocity. It is a universal relationship between Integral parameters delta 2 delta 4 and delta 1.

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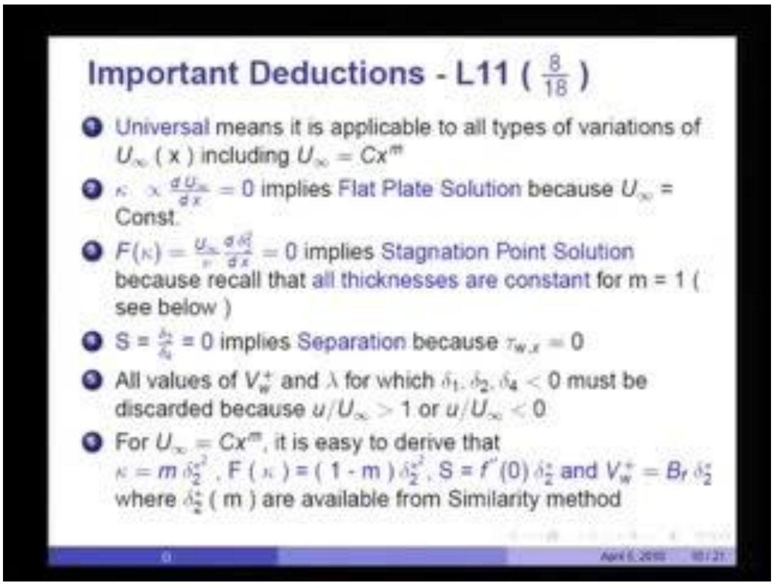

Let me go to the next slide. By universal, we mean that it is typically applicable to all types of variation on U infinity, including the ones we used in similarity method, U infinity equal to C x raise to m. kappa which is proportional to d U infinity by d x, when it is 0 implies a flat plate solution because U infinity equals constant.

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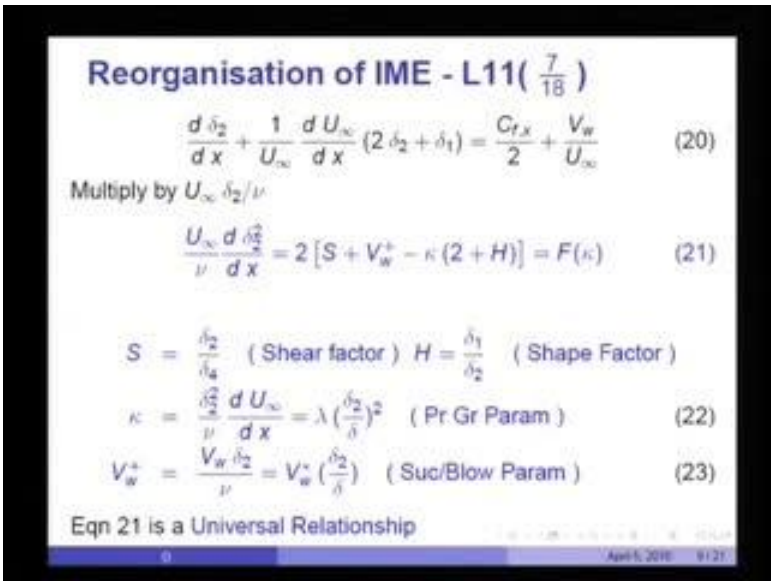

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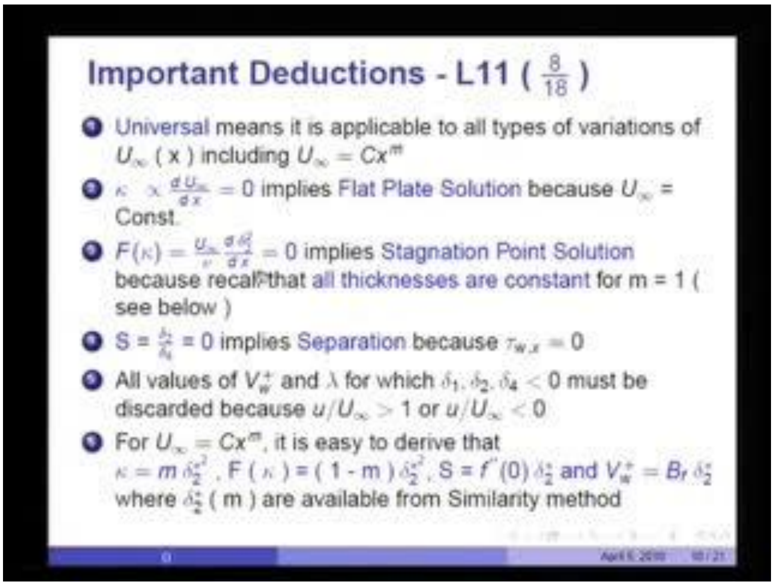

On the other hand, the right hand side F k or U infinity over nu d delta 2 square by d x equal to 0 implies that delta 2 is constant with x. As you will recall from our similarity method, it represents stagnation point solution because for m equal to 1 all thicknesses delta 1, delta 2, enthalpy thickness, heat transfer coefficient and all are constants with x that is the characteristic of a stagnation point solution.

Shear parameter equal to 0 delta 2 over delta 4 equal to 0 implies that delta 4 is infinite, which in turn implies that tau wall x is equal to 0 and it represent separation. All values of V w plus and lambda for which delta 1, delta 2 and delta 4 are less than 0 must be discarded because for these values, u over U infinity is either greater than 1 or u over U infinity is less than 0, which means those are inadmissible values of V w plus and delta lambda.

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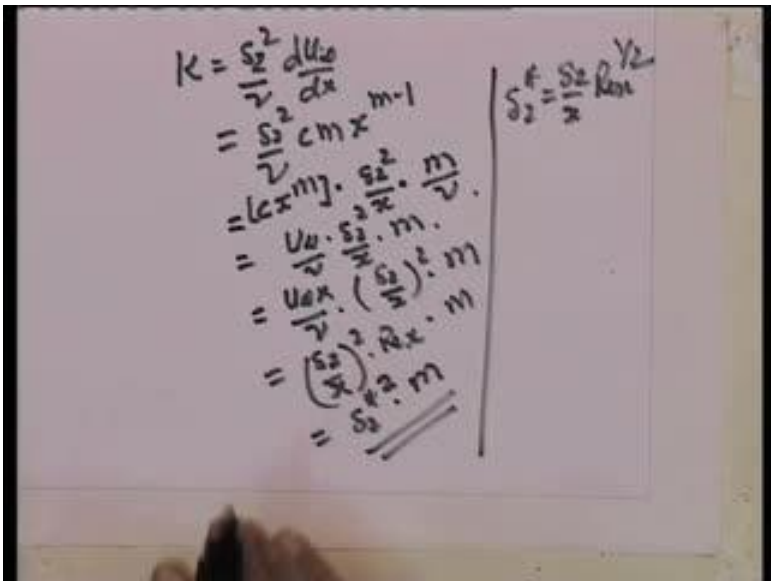

Now, for U infinity equal to C x m it is easy to derive that kappa will be equal to delta 2 star square divided by 2, I will show you how this is the case. If U infinity is equal C x raise to m and kappa is equal to delta 2 star by nu d U infinity by d x. Then, you will see that this becomes delta 2 square by nu C m x raise to m minus 1 or I can write this as C x raise to m into delta 2 square by x into m, I can write like that. This is nothing but C x to m is nothing but U infinity into delta 2 square by x into m, which I can write as U infinity oh sorry that should be a nu here. So, divide by nu I can write this as U infinity x by nu into delta 2 by x whole square into m that is equal to delta 2 by x square into R e x into m.

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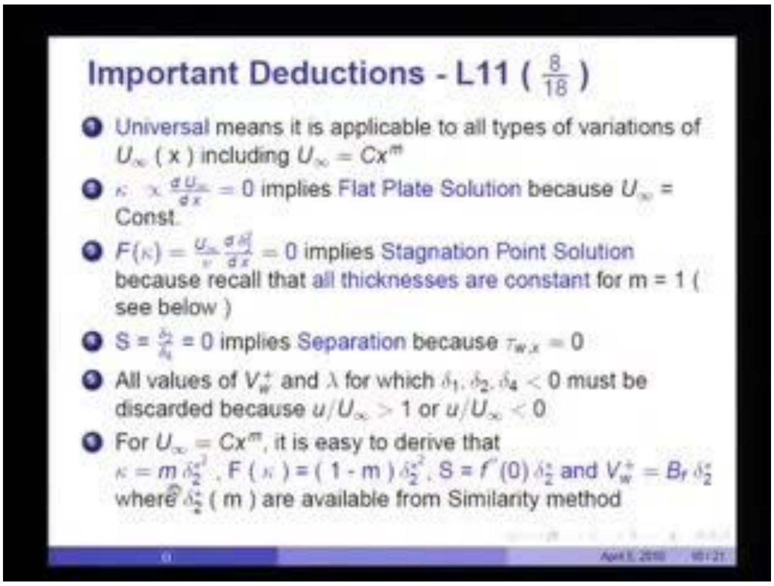

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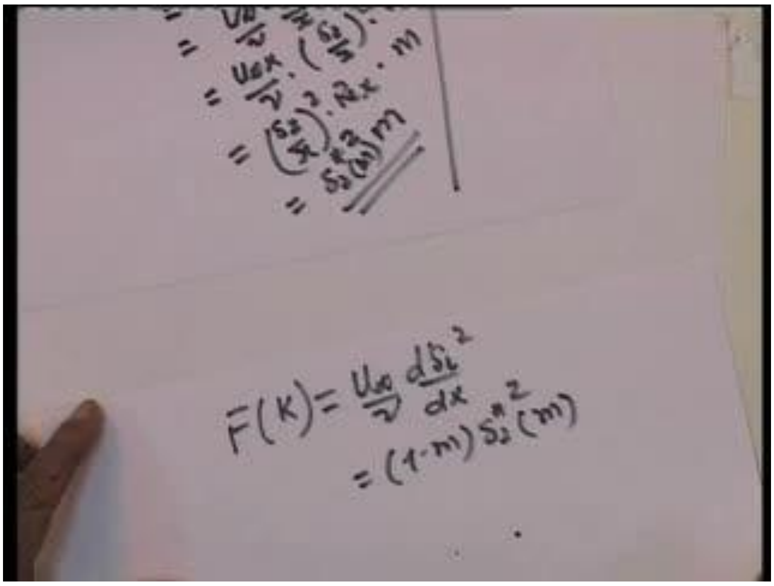

If you recall, in our similarity method, we had defined delta 2 star as delta 2 by x R e x to the half. So, essentially becomes delta 2 star square into m and that is what I shown here. Similarly, you can show that F kappa which is equal to U infinity by nu d delta 2 square by d x can be shown to be equal to 1 minus m delta 2 star square for that value of m.

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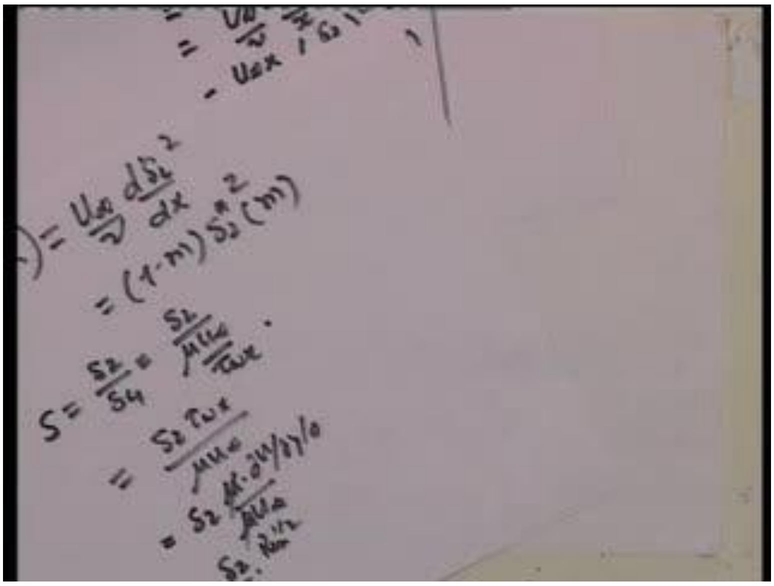

Similarly, here delta 2 square is corresponding to the value of m you are concerned with. You can also show S; for example, what is S? S is equal to delta 2 by delta 4 and that is equal to delta 2 divided by mu U infinity by tau wall x or that is equal to delta 2 tau wall x divided by mu U infinity. This term can be shown to be equal to f double prime 0 into delta 2 star, which simply a question of manipulation here and you will see the tau wall x.

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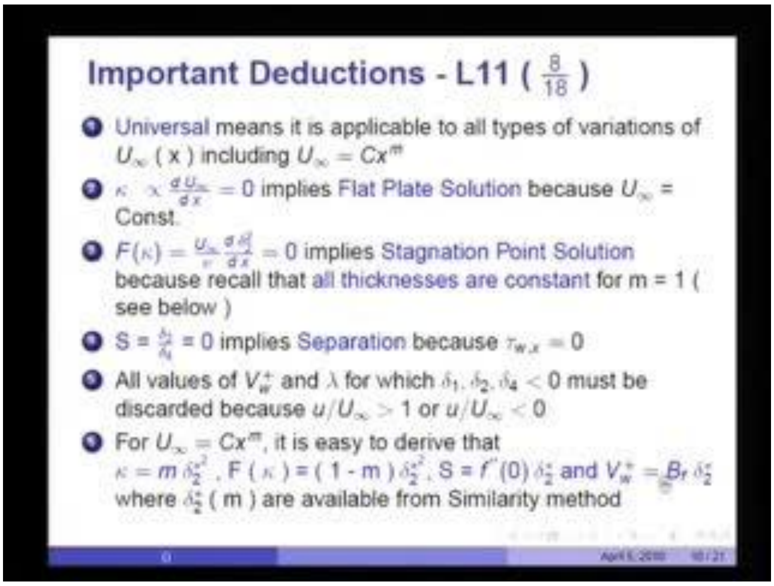

For example, this will be delta 2 into mu times d u by d y at 0 divided by U infinity into mu. So, mu and mu gets cancel and I can construct here delta 2 by x R e by x by half and you will get that f double prime. V w plus can also be shown to be B f; the similarity parameter B f delta 2 star and delta 2 star m are available from similarity methods. These deductions are very important as we shall see shortly.

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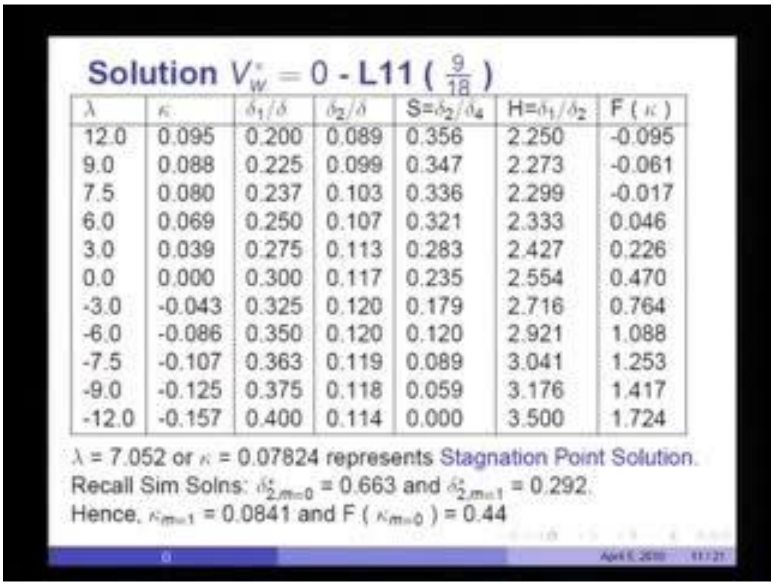

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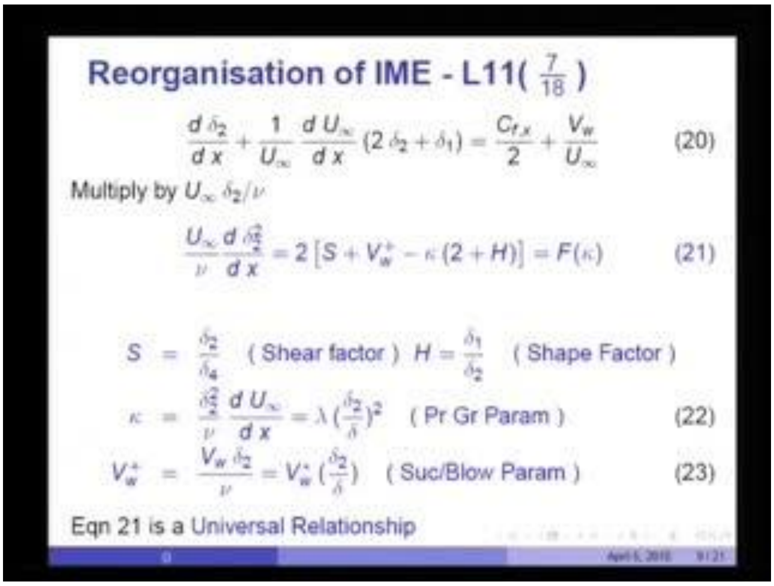

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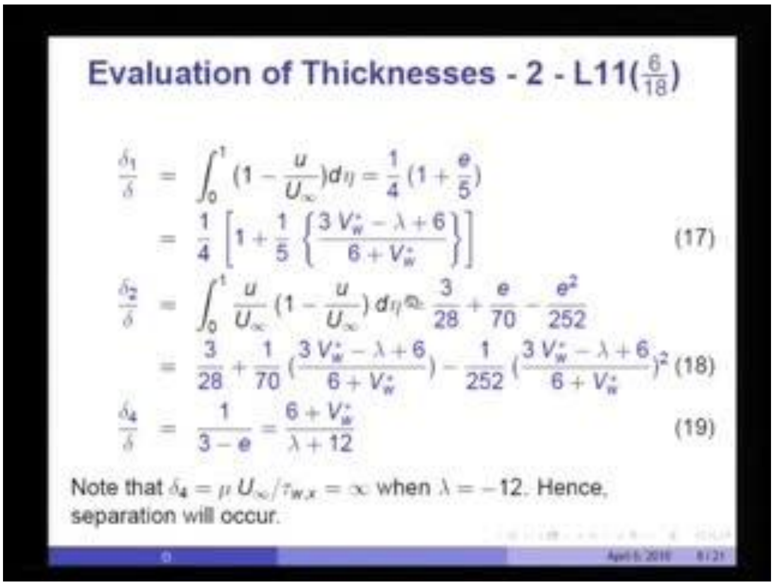

Let me plot the results. Remember, I have calculated S V w plus kappa, I have assumed values of kappa and calculated H also from the velocity profile that we assumed; that is delta 1 by delta, delta 2 by delta and delta 4 by delta.

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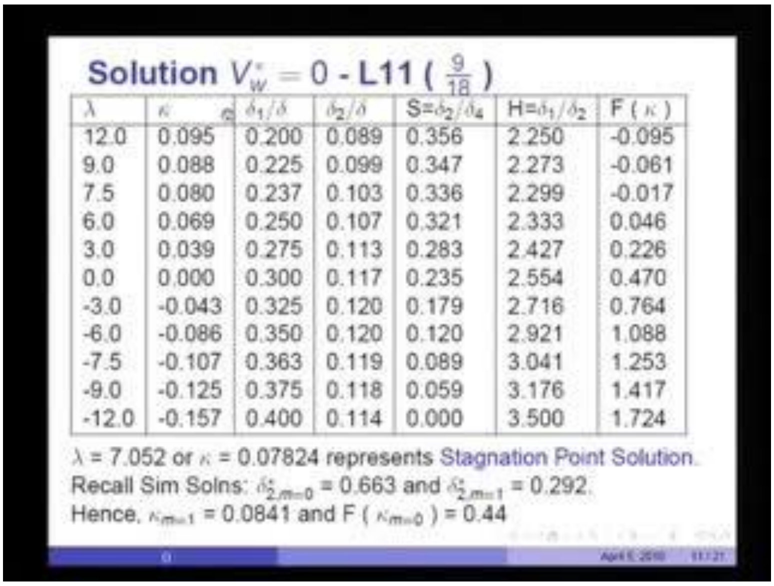

For a boundary layer without suction or blowing, the acceleration and deceleration parameters; acceleration parameter I have gone up to 12 and deceleration I have gone up to minus 12 because I cannot go less than minus 12. Then, the kappa values take these values, this is 0.095 and this goes on to minus 0.157 delta 1 by delta 2. As you can see compared to 0, where it was about 0.3 with acceleration the displacement thickness reduces and with deceleration it increases. Same thing holds for a momentum thickness divided by delta. It reduces and increases, inverse is true for shear thickness. The shear thickness goes on increasing whereas on the negative side, when there is suction it goes on reducing so much. So that at minus 12, it reduces exactly to 0.

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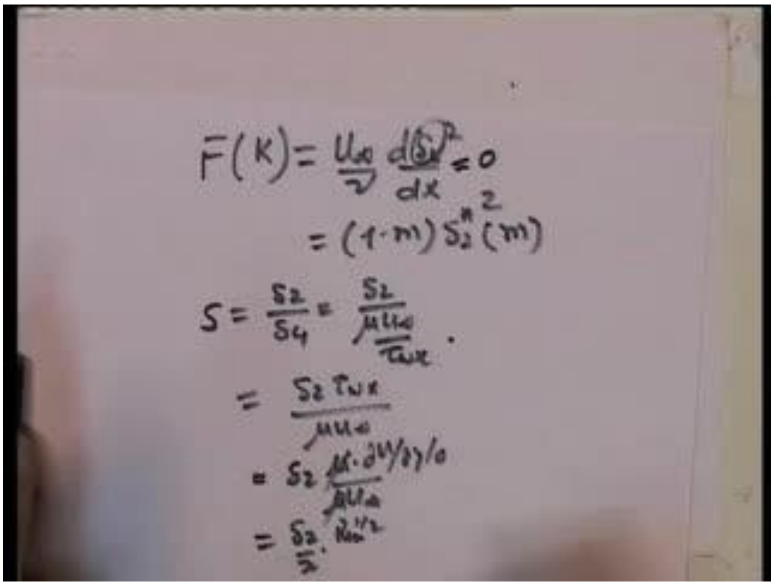

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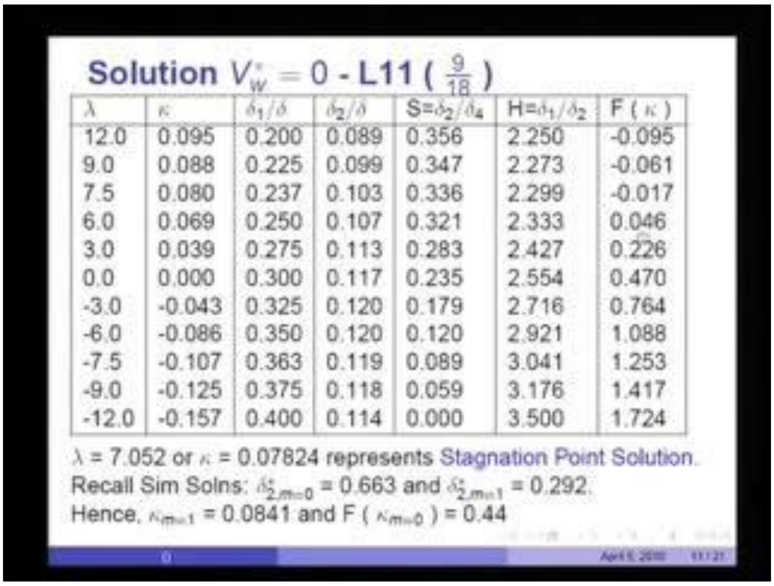

Solution $V_w^* = 0$ - L11 ($\frac{9}{18}$)

λ	κ	δ_1/δ	δ_2/δ	$S=\delta_2/\delta_4$	$H=\delta_1/\delta_2$	$F(\kappa)$
12.0	0.095	0.200	0.089	0.356	2.250	-0.095
9.0	0.088	0.225	0.099	0.347	2.273	-0.061
7.5	0.080	0.237	0.103	0.336	2.299	-0.017
6.0	0.069	0.250	0.107	0.321	2.333	0.046
3.0	0.039	0.275	0.113	0.283	2.427	0.226
0.0	0.000	0.300	0.117	0.235	2.554	0.470
-3.0	-0.043	0.325	0.120	0.179	2.716	0.764
-6.0	-0.086	0.350	0.120	0.120	2.921	1.088
-7.5	-0.107	0.363	0.119	0.089	3.041	1.253
-9.0	-0.125	0.375	0.118	0.059	3.176	1.417
-12.0	-0.157	0.400	0.114	0.000	3.500	1.724

λ = 7.052 or κ = 0.07824 represents Stagnation Point Solution.
Recall Sim Solns: $\delta^*_{2,m=0}$ = 0.663 and $\delta^*_{2,m=1}$ = 0.292.
Hence, $\kappa_{m=1}$ = 0.0841 and F ($\kappa_{m=0}$) = 0.44

The shape factor H delta 1 by delta 2 is remarkably a constant for highly on acceleration side but on decelerating side, it goes on increasing quite significantly. This is the right hand side with some values negative and then positive. Now, what is F k? To remember again F k is nothing but that value and when that is equal to 0, it means delta 2 is constant. Therefore, it represents stagnation point solution if you look at here the stagnation point solution will be somewhere here and its value will be lambda equal to 7.052 and kappa equal to minus 7824.

The similarity solution for delta 2 star m equal to 0 was 0.663 and delta 2 star m equal to 1 was 0.292. Therefore, kappa for m equal to 1 will be 0.0841 and F for k m equal to 0, kappa m equal to 0 would be 0.44 will make use of these numbers very shortly but remember this is the flat plate solution, 7.052 lambda is a stagnation point solution and minus 12 is a separation solution.

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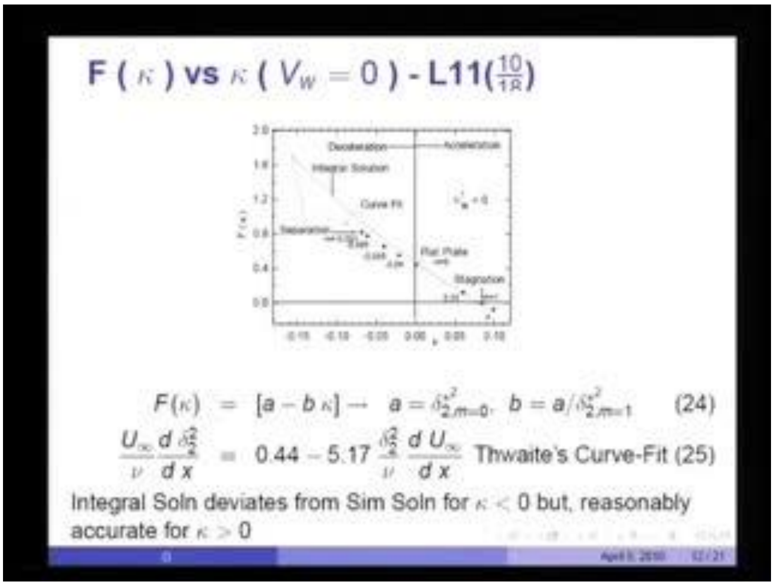

This is what I have plotted here. On the x axis, you have kappa the pressure gradient parameter and F kappa, which is the rate of growth of momentum thickness parameter on the y axis.

When kappa is positive, we have accelerating flow or favorable pressure gradient. When kappa is negative, we have the decelerating flow or adverse pressure gradient. The results are plotted, when there is no suction and blowing. Therefore, V w star is equal to 0.

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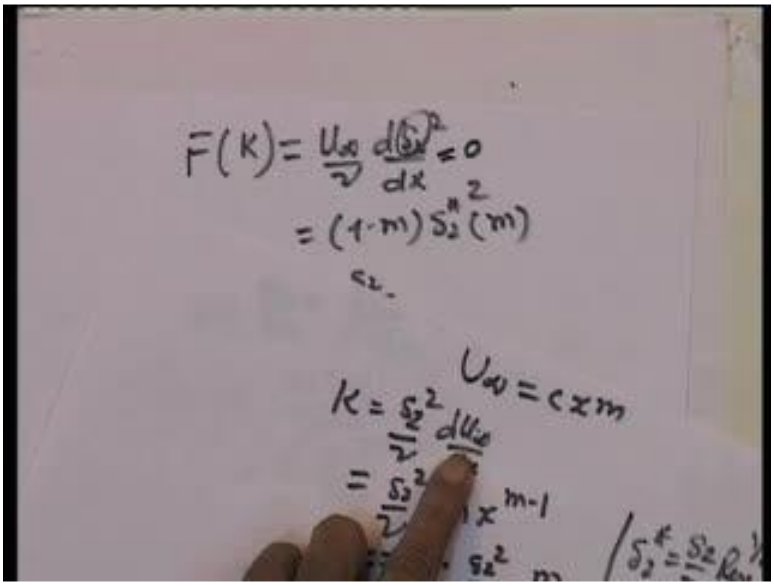

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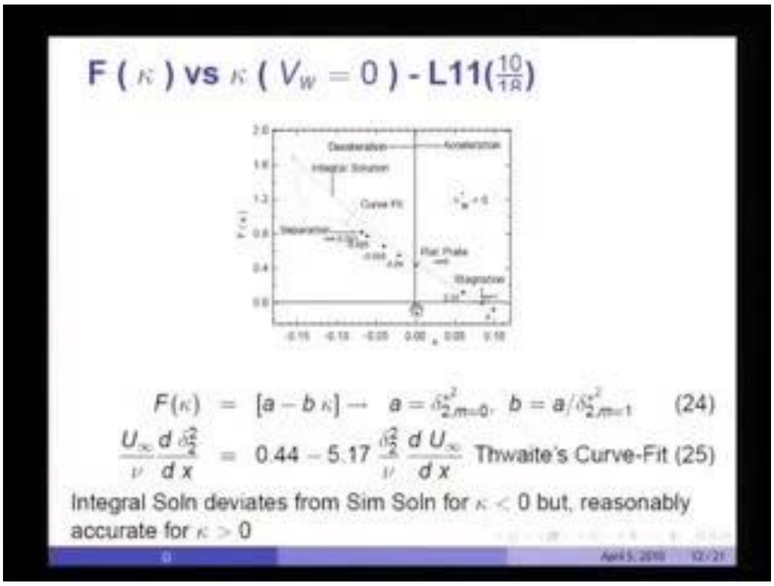

You will notice that when kappa is equal to 0, the value of F k must represent the flat plate solution. When F k is equal to 0, as we just said it must represent the stagnation point solution. So, the intercept on the y axis represents the flat plate solution whereas the intercept on the x axis represents the stagnation point solution and F k values turn negative, when you have very highly accelerating flow, whereas when the flow is decelerating F k values are positive.

Now, using the relationship between kappa and F k with Integral parameters, I have also plotted the parameters here the similarity solutions here. You will notice that this is m equal to 1 solution, this is m equal to 0.33 solution, minus 4 m equal to minus 4 minus 0.04 minus 0.065 minus 0.085 and minus 0.09 is the separation.

So, the separation is seen at about kappa equal to minus. Let us say about minus 0.07 in similarity solutions, whereas in Integral solution the separation occurs at minus 0.1567. On the Integral solution deviates from similarity solution for kappa less than 0 because we have allowed for arbitrary variation of U infinity but for specific variation U infinity equal to C x m the results go along there.

Now, in order to develop close form solution, we can see that at least for very moderate decelerating flows and accelerating flows, a near straight line approximation can be made. This was done by a scientist called Thwaite's. It is simply F kappa equal to a minus b kappa and the a will be the value of kappa equal to 0 and therefore represents flat plate solution a delta 2 star square m equal to 0, whereas b will be simply when F k is equal to 0 or the stagnation point solution then b will equal a divided by delta 2 star square of m equal to 1.

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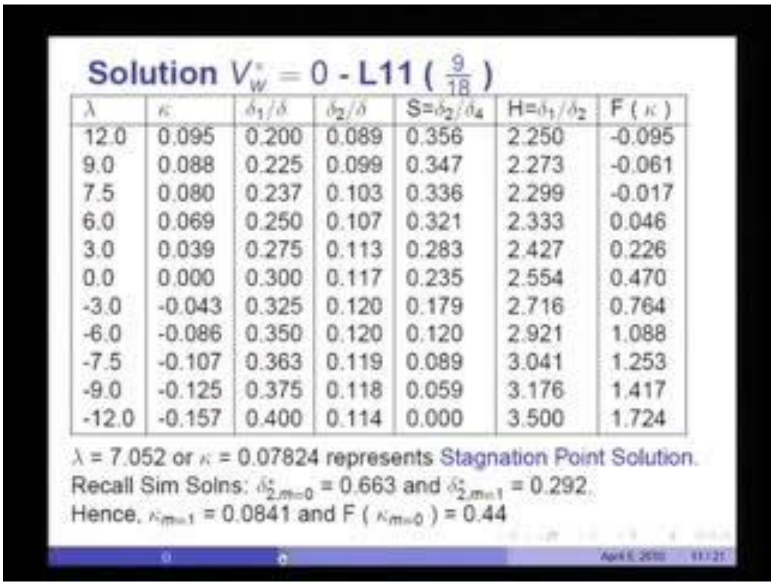

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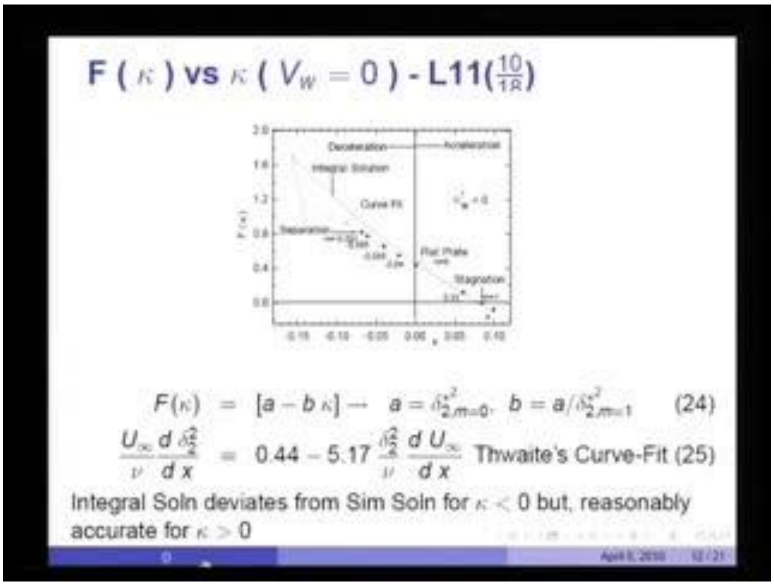

If you look at our previous slide, I have said delta 2 square star m equal to 0.663.So, delta 2 star square would be square of 0.663, which is 0.44 and delta 2 star m equal to 1 is 0.292. So, 0.44 divided by 0.292 will give me this value of 5.17. So, a will become equal to 0.44 whereas b will equal minus 0.4.

F k being equal to U infinity, rate of growth of momentum thickness is equal to a constant minus another constant times delta 2 square by nu d U infinity by d x. This is the Thwaite’s curve-fit, a universal curve fit for the case in which suction and blowing are absent. We will make use of this relationship as you will see on the next slide.

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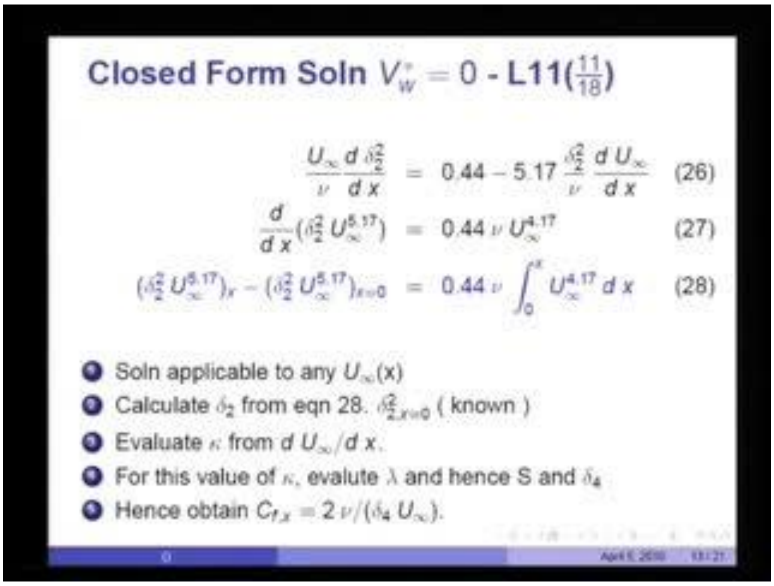

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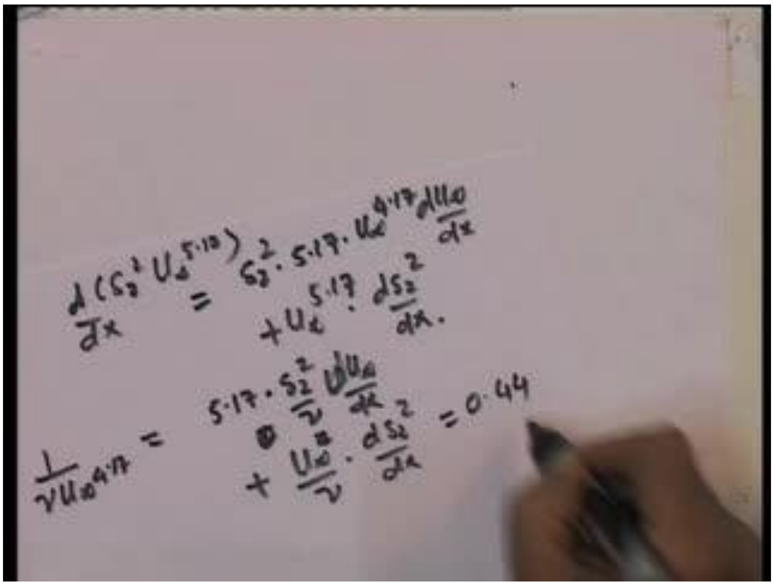

Just see, this was the relationship. I can manipulate these 2 terms; this term and this term as d by d x of d u delta 2 square U infinity 5.7 equal to 0.44 nu 4 u. To convince you, let me open up again the equation, then you will see d delta 2 square U infinity equal to 5.17 by d x will equal delta 2 square into 5.17 into U infinity raised to 4.17 d U infinity by d x plus U infinity raise to 5.17 into d delta 2 square by d x.

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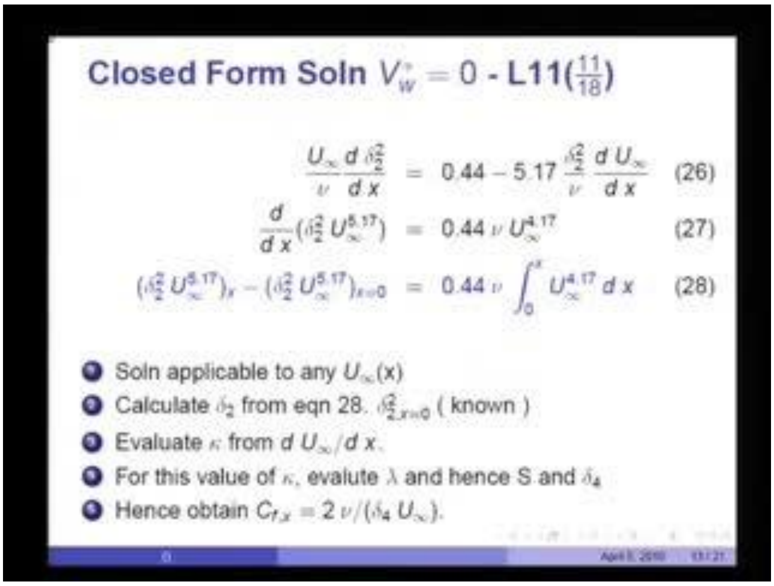

This is what it would mean and if I divide this 1 over nu, U infinity raise to 4.17, I will get 5.17 into delta 2 square by nu u d U infinity by d x plus U infinity by nu d delta 2 square by d x that is equal to 0.44. Therefore, you will see that I can write this equation in this form. If I were to integrate this equation from 0 to x then delta 2 square U infinity raise to 5.17 del x. We will equal delta 2 square U infinity raise 5 minus x equal to 0 equal to 0.44 nu 0 to x U infinity raise to 4.17 d x.

So, the solution is applicable to any arbitrary variation of U infinity and restriction imposed by similarity method is now removed. We use this relationship to calculate delta 2 at any x because U infinity at that x will be known. You must know delta 2 square at x equal to 0. If you start from x equal to 0 itself where delta 2 is 0 then of course that term will be 0. Evaluate kappa from d U infinity by d x. Now, that you know delta 2 square, you can evaluate our kappa which is.

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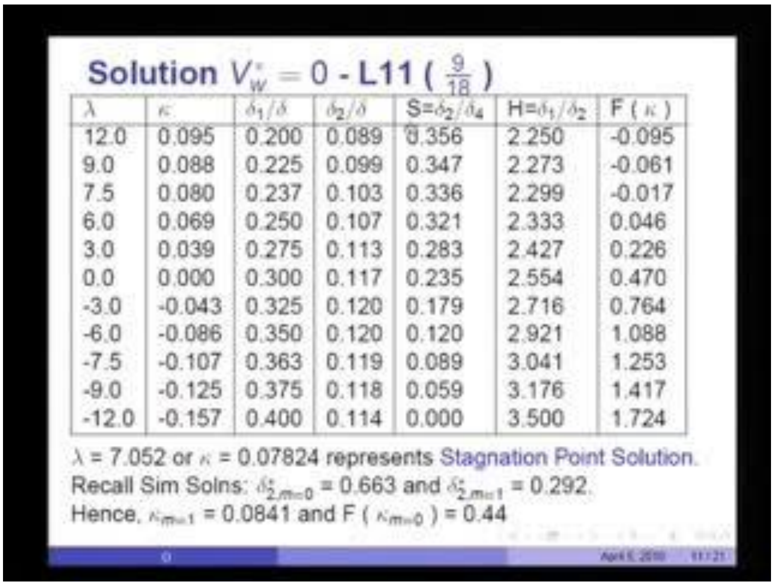

Solution $V_w^* = 0$ - L11 ($\frac{9}{18}$)

λ	κ	δ_1/δ	δ_2/δ	$S=\delta_2/\delta_4$	$H=\delta_1/\delta_2$	F (κ)
12.0	0.095	0.200	0.089	0.356	2.250	-0.095
9.0	0.088	0.225	0.099	0.347	2.273	-0.061
7.5	0.080	0.237	0.103	0.336	2.299	-0.017
6.0	0.069	0.250	0.107	0.321	2.333	0.046
3.0	0.039	0.275	0.113	0.283	2.427	0.226
0.0	0.000	0.300	0.117	0.235	2.554	0.470
-3.0	-0.043	0.325	0.120	0.179	2.716	0.764
-6.0	-0.086	0.350	0.120	0.120	2.921	1.088
-7.5	-0.107	0.363	0.119	0.089	3.041	1.253
-9.0	-0.125	0.375	0.118	0.059	3.176	1.417
-12.0	-0.157	0.400	0.114	0.000	3.500	1.724

λ = 7.052 or κ = 0.07824 represents Stagnation Point Solution.

Recall Sim Solns: $\delta^*_{2,m=0}$ = 0.663 and $\delta^*_{2,m=1}$ = 0.292.

Hence, $\kappa_{m=1}$ = 0.0841 and F ($\kappa_{m=0}$) = 0.44

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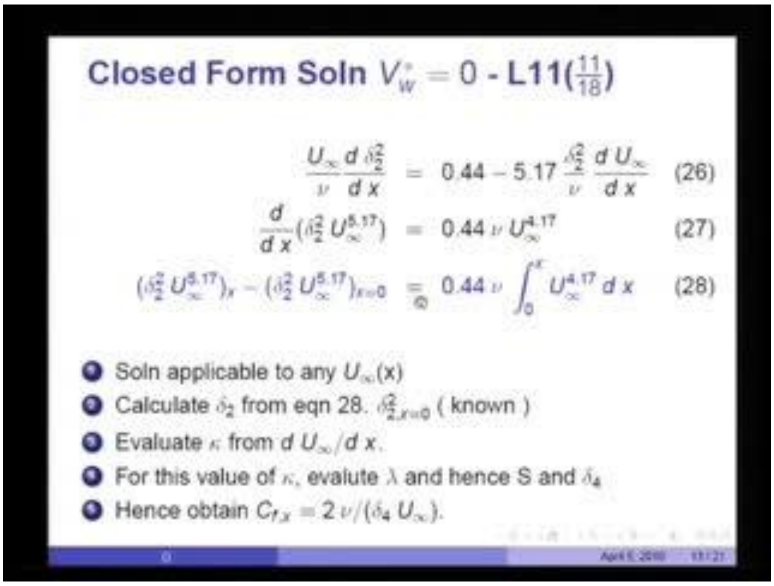

Once you have evaluated delta 2, you can now evaluate delta 2 square nu d U infinity by d x because you already know what U infinity x is. For this value of kappa, you can evaluate S value for knowing the kappa value; you can interpolate to get S value. You get a shear stress value from which you can evaluate S and delta 4 and from delta 4 you can evaluate the skin friction coefficient 2 nu over delta 4 U infinity, which is what we wish to evaluate anyway. That is the purpose.

In other words, knowing U infinity as a function of x, we get delta 2 as a function of x from which we get kappa from which we get S as a function of x. In fact, we get all other parameters delta 1 as a function of x and so on and so forth and because we know the shear factor variation with x. We can get C f x also as a function of x.

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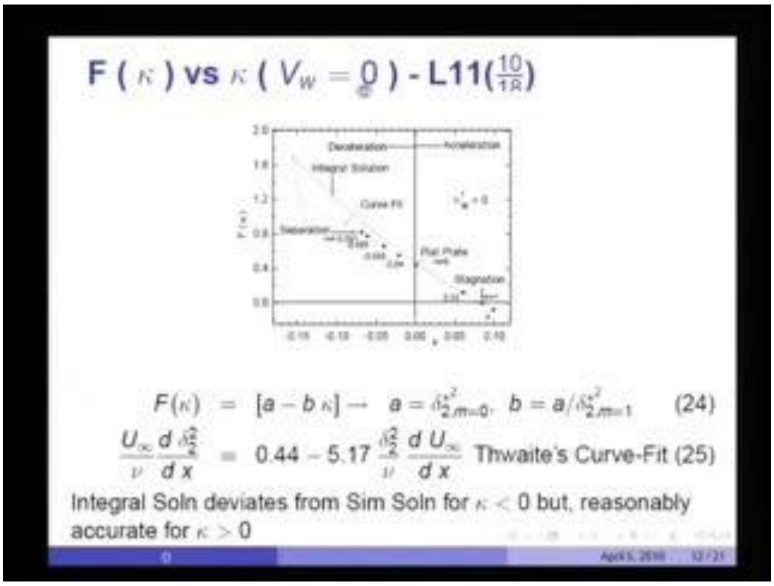

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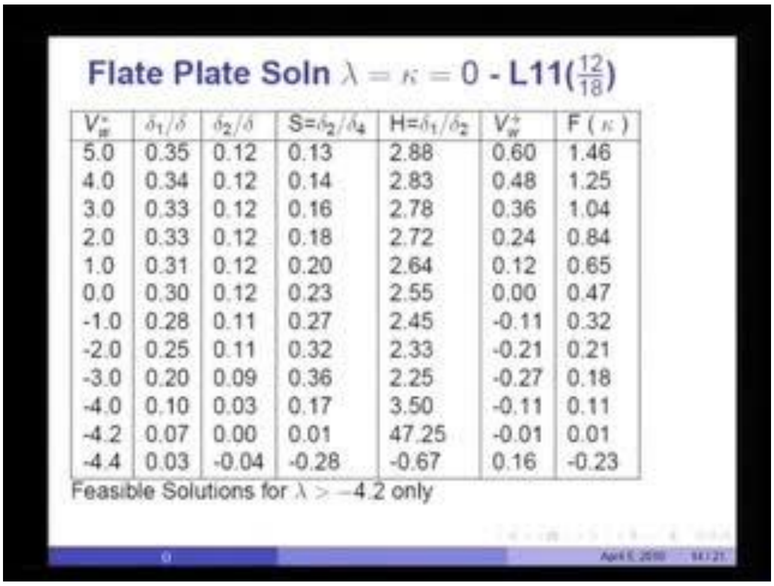

V_w^*	δ_1/δ	δ_2/δ	S=δ_2/δ_4	H=δ_1/δ_2	V_w^+	F (κ)
5.0	0.35	0.12	0.13	2.88	0.60	1.46
4.0	0.34	0.12	0.14	2.83	0.48	1.25
3.0	0.33	0.12	0.16	2.78	0.36	1.04
2.0	0.33	0.12	0.18	2.72	0.24	0.84
1.0	0.31	0.12	0.20	2.64	0.12	0.65
0.0	0.30	0.12	0.23	2.55	0.00	0.47
-1.0	0.28	0.11	0.27	2.45	-0.11	0.32
-2.0	0.25	0.11	0.32	2.33	-0.21	0.21
-3.0	0.20	0.09	0.36	2.25	-0.27	0.18
-4.0	0.10	0.03	0.17	3.50	-0.11	0.11
-4.2	0.07	0.00	0.01	47.25	-0.01	0.01
-4.4	0.03	-0.04	-0.28	-0.67	0.16	-0.23

In the previous table, I had held V w equal to the no suction and blowing. Now, I am saying, I am going to set lambda equal to 0, which is the case of a flat plate but allow for suction and blowing and that is what I have done here.

I allow for blowing parameter to go up to 5 on the suction side; I go up to minus 4.2 delta 1 by delta with blowing compared to V w star equal to 0 the delta 1 by delta increases as we expect delta 1 by delta decrease. As we increase suction, these also increase where it is not seen here because I have plotted results only up to second decimal place. On this side, it reduces but notice that at minus 4.4 delta 2 has already turned negative and that is not permitted.

I cannot go below lambda less than minus 4.2. So, feasible solutions are possible only for lambda greater than minus 4.2 as you can see here. The shear stress also has almost vanished here, which means this is where separation is about to take place.

The shape factor is increased enormously to 47.25 from the H average values around 2.7 on the positive side on the blowing side around this at moderate suction rates it is about 2.4 but it increases very rapidly to 47.5. This is almost the separation power profile and these are the values of F k.

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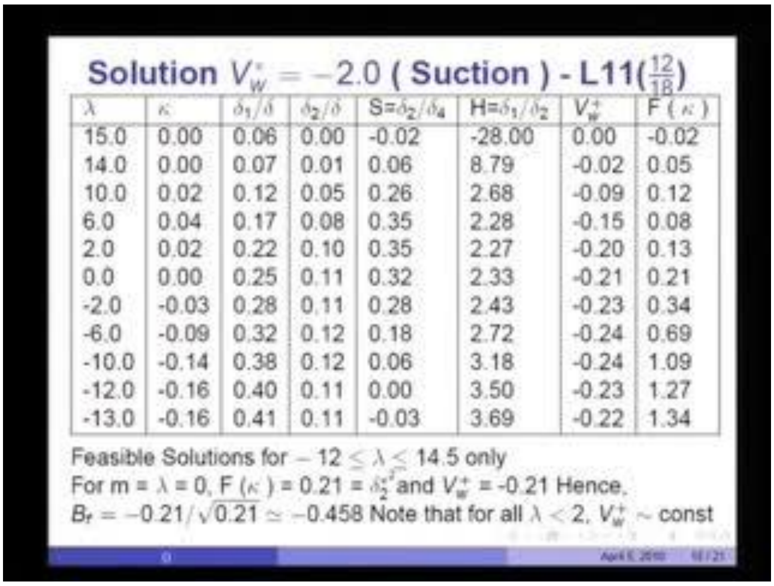

λ	κ	δ_1/δ	δ_2/δ	$S=\delta_2/\delta_4$	$H=\delta_1/\delta_2$	V_w^+	$F(\kappa)$
15.0	0.00	0.06	0.00	-0.02	-28.00	0.00	-0.02
14.0	0.00	0.07	0.01	0.06	8.79	-0.02	0.05
10.0	0.02	0.12	0.05	0.26	2.68	-0.09	0.12
6.0	0.04	0.17	0.08	0.35	2.28	-0.15	0.08
2.0	0.02	0.22	0.10	0.35	2.27	-0.20	0.13
0.0	0.00	0.25	0.11	0.32	2.33	-0.21	0.21
-2.0	-0.03	0.28	0.11	0.28	2.43	-0.23	0.34
-6.0	-0.09	0.32	0.12	0.18	2.72	-0.24	0.69
-10.0	-0.14	0.38	0.12	0.06	3.18	-0.24	1.09
-12.0	-0.16	0.40	0.11	0.00	3.50	-0.23	1.27
-13.0	-0.16	0.41	0.11	-0.03	3.69	-0.22	1.34

Likewise, I have now included effects x of both lambda n for a certain V w star, which is minus 0.2 that means it is a suction case with V w star equal to minus 0.2 and here I have gone up to 15 but notice that beyond 14.8 or S has become negative therefore this is not acceptable solution lambda is equal to 15 is not acceptable. On the adverse pressure gradient side, you will see I have gone up to minus 13 but at minus 12 it is 0 already. So, this is the separation occurs and minus 13 is minus 0.03 so this is not admissible. Effect of pressure gradient on at a certain suction state is valid between minus 12 and 14.5 only.

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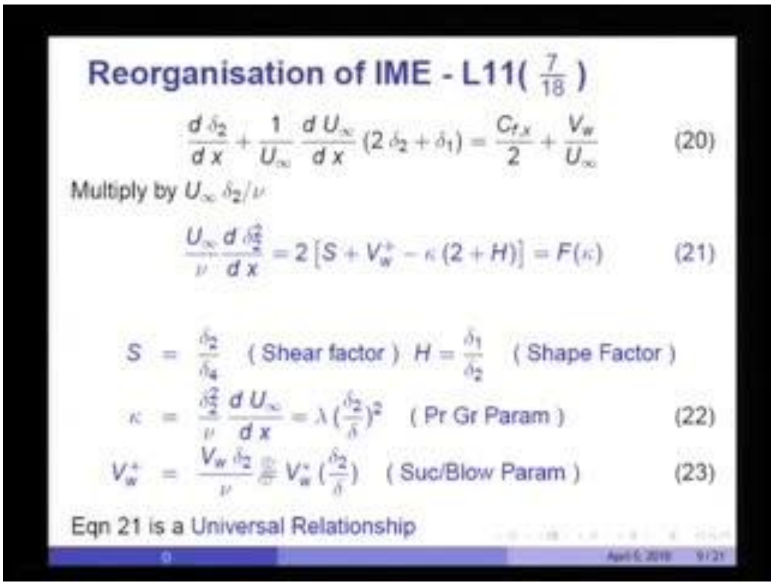

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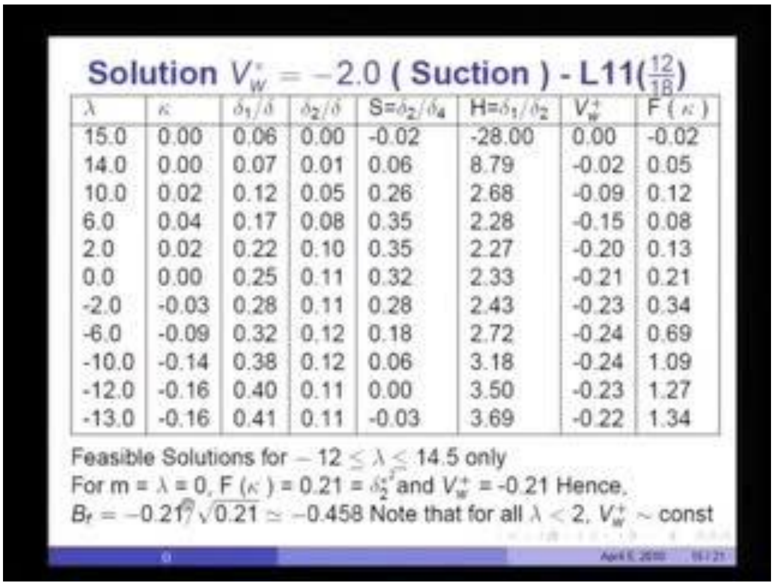

The remarkable feature of this solution is that for very mild acceleration to all the adverse pressure gradients, the value of V w plus is almost constant. In this, V w plus is almost constant on the suction side for lambda equal to 0 F k is about 21 you can see that 0.21 and that must equal delta 2 star square and V w plus equal to minus 0.21. This amounts to B f equal to minus 0.21 divided by root 0.21 0.458.

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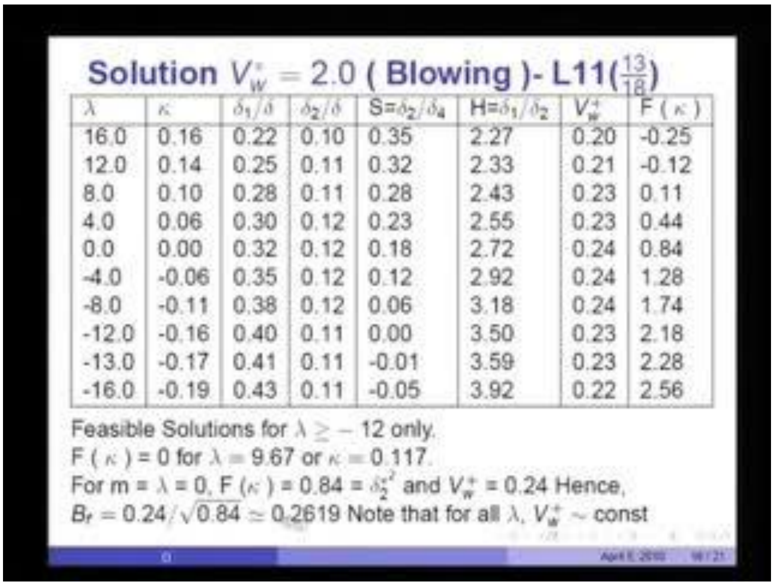

This is the blowing side and again you will see for adverse pressure gradients less than minus 12, you have negative. So, you cannot go below then again on V w plus is remarkably constant and this will correspond to B f equal to about 0.2619.

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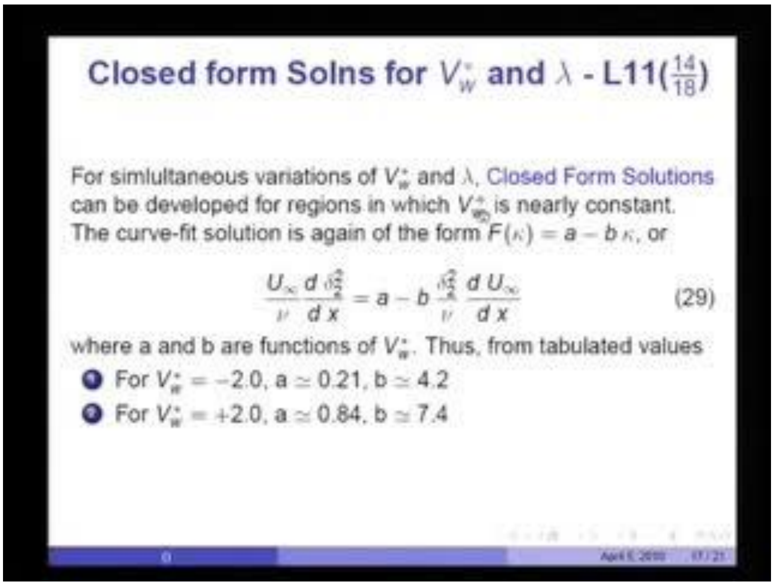

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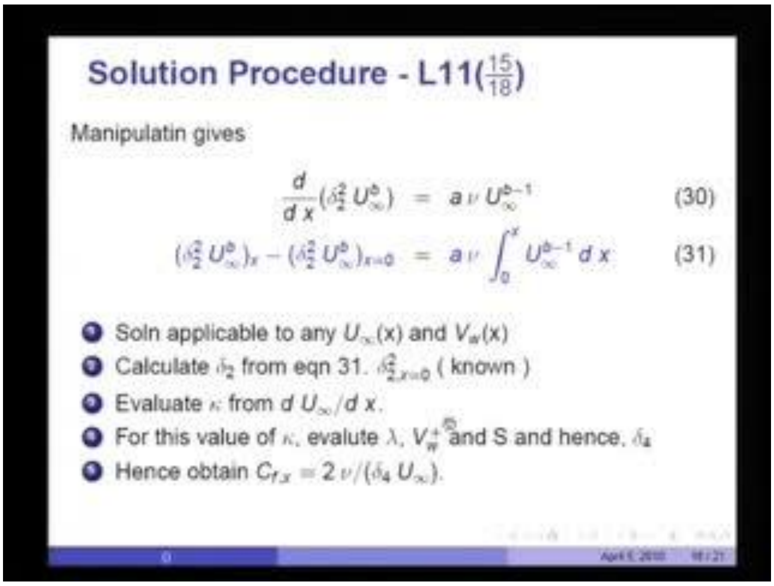

For simultaneous variations of V w star and lambda, close form solutions can again be developed in the regions in which V w plus is constant. You can curve fit F kappa equal to a minus b kappa or a relationship of this type can be established, where a and b are functions of V w plus. So, V w star equal to minus 0.2 you get a equal to 0.21, b equal to 4.2, V w star equal to plus 2 and you will get 0.84 and 7.4. Manipulation would give again d delta 2 d by d x of this equal to that and therefore you will get a solution.

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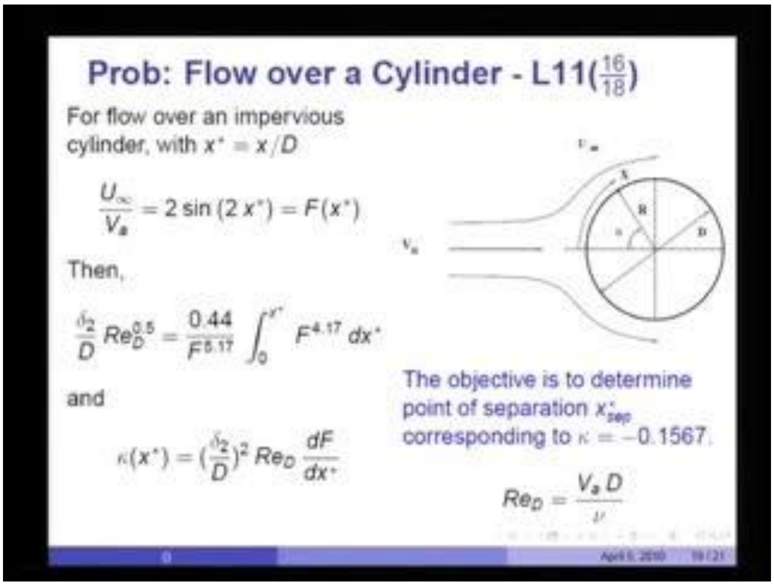

So, the procedure remains exactly the same as before only the values of a and b change with value of V w star. I am taking now, a case of flow over a cylinder; it is an impervious cylinder. There is no suction or blowing; there is an approach velocity V, a potential theory will show that the free stream velocity U infinity would vary as U infinity by V a equal to 2 times sine 2 x star where x star is x divided by diameter.

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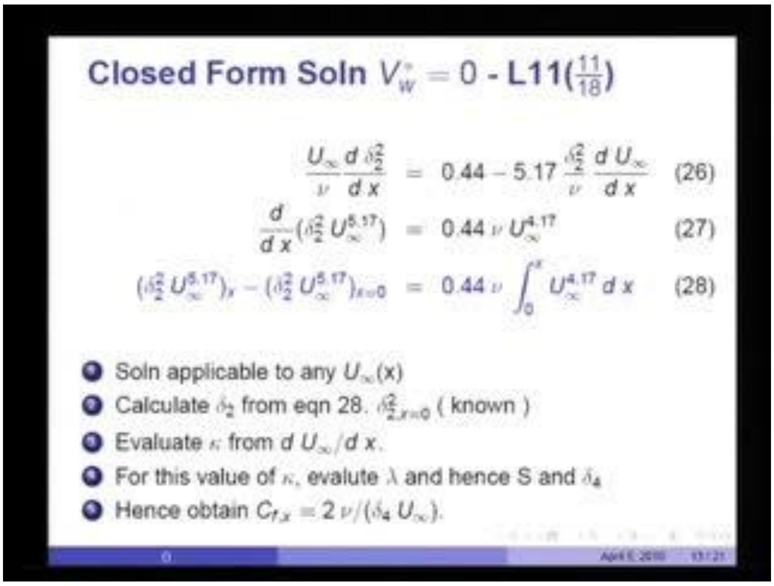

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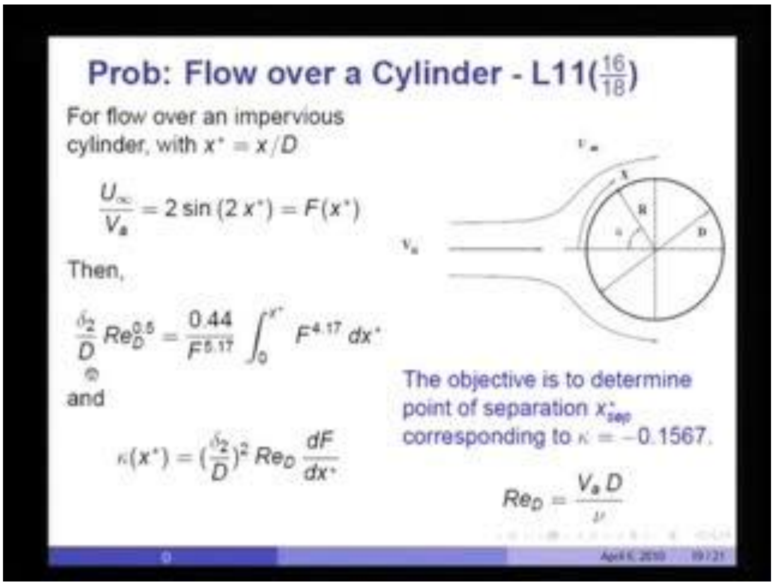

So, 2 x star is nothing but x divided by radius and I will call this F x star. Then, for this variation of free stream velocity, I can show that delta 2 by d R e D will be simply from equation here using this relationship, I can determine delta 2. Remember, delta 2 at x equal to 0 at the stagnation point will be 0 and from there I integrate. So, I can show that delta 2 by d R e d will be done and kappa will be this.

Our objective is to determine the location of the separation point corresponding to kappa equal to minus 1567 R e D is equal to V a D by nu, which is the Reynolds number defined for flow over a cylinder.

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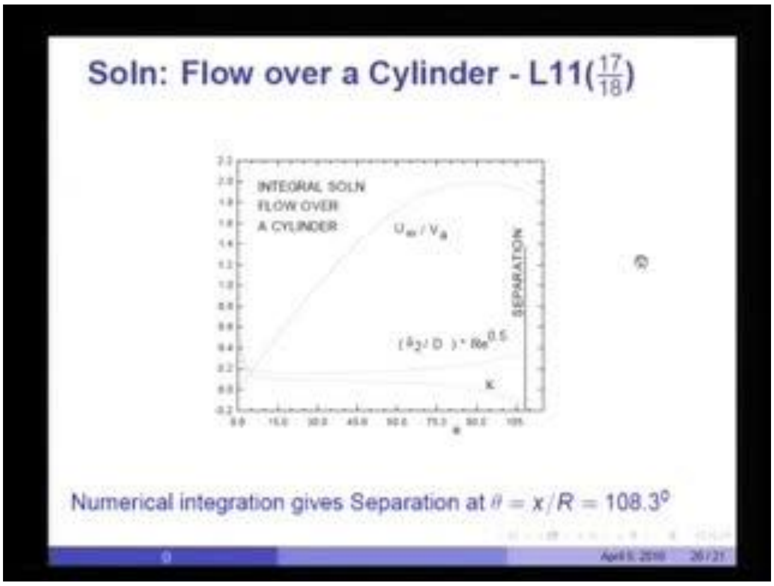

Here are the results - U infinity by V a is a sine function which goes and beyond 90 deceleration sets in whereas on the below 90 degrees there is a flow acceleration. This is the variation of kappa and you can see it has reached minus 0.1567 at about 108.3 degrees and therefore it is associated with separation. So, we have located the separation point from stagnation point from the unknown arbitrary velocity distribution.

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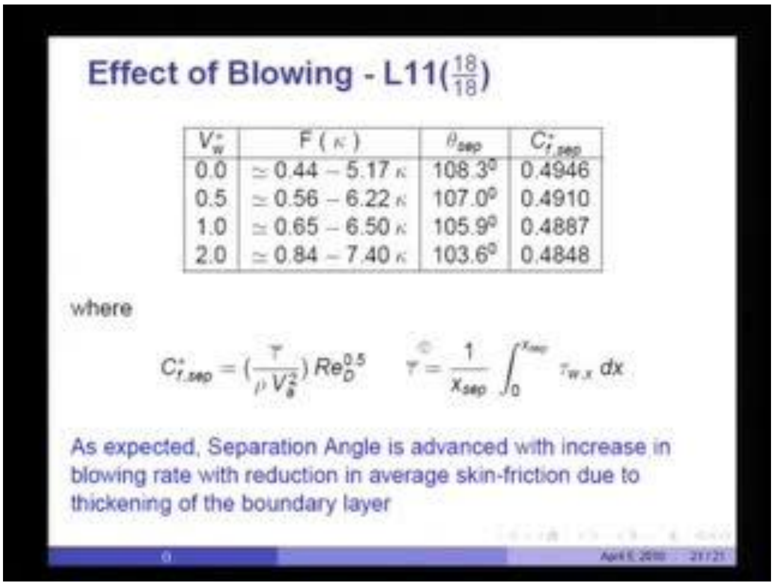

Similarly, now consider a cylinder. In which, there is blowing taking place from the cylinder surface. Then, I can curve fit as I said F kappa in this manner for different values of V w star and theta separation for V w star equal to 0 was 108 and that goes on reducing. You will expect where that a flow over cylinder with blowing would the separation would occur at an earlier location and that is what you see up to V w star at 0.2 the separation point has advanced. Average skin friction up to the separation point defined in this manner; it is highest at V w star equal to 0 but with blowing skin friction goes on reducing. As expected, separation angle is advance with increase in blowing rate with reduction in average skin friction due to thickening of the boundary layer.

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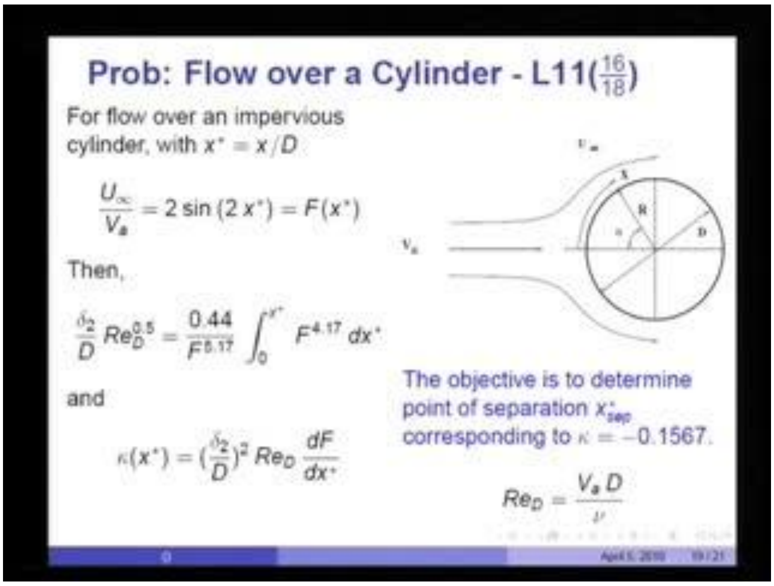

This shows you the power of the Integral method. What it cannot do is to go beyond the point of separation and complete the analysis of flow over a cylinder. It is a very useful tool to determine flow through conversion or diversion nozzles. For example, where the boundary layer, where the free stream velocity would either accelerate or decelerate with x and you want to determine the thickness of the boundary layers developing on the wall because it is this thickness, which determines the discharge coefficients of such nozzles.