Matrix Theory
Professor Chandra R. Murthy
Department of Electrical Communication Engineering
Indian Institute of Science, Bangalore
Lecture 57
Hermitian and symmetric matrix

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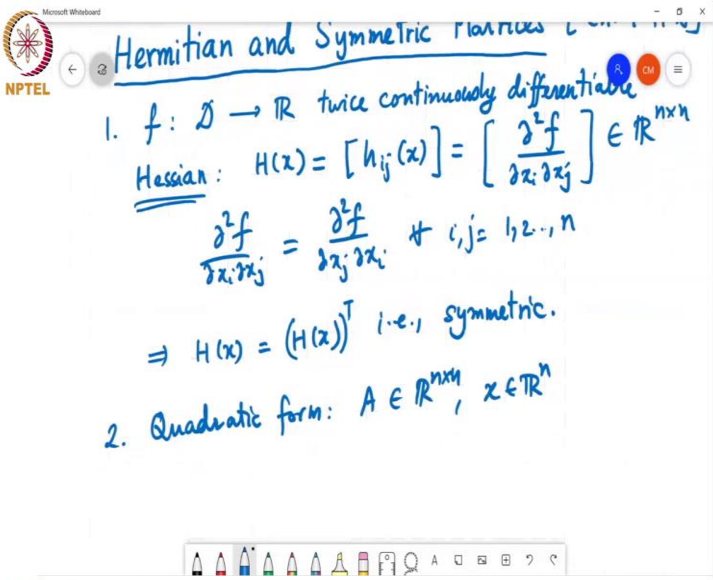

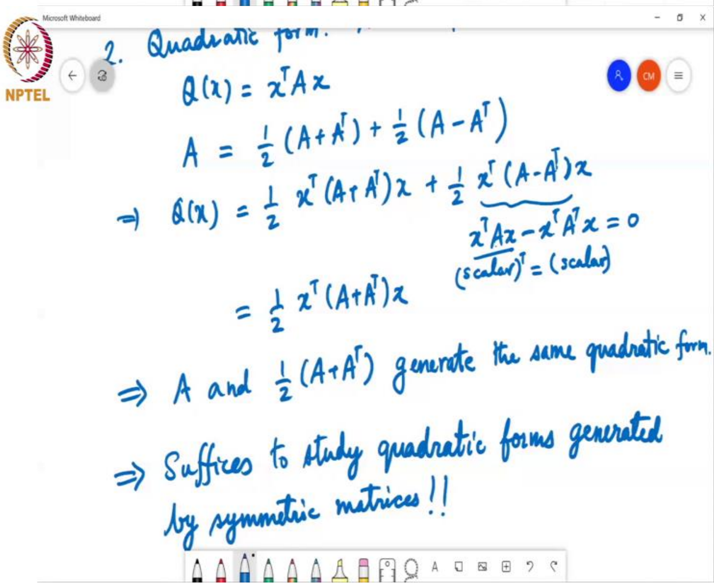

And now, we move on to chapter 4 in Horn and Johnson which is Hermitian and symmetric matrices. So, Hermitian and symmetric matrices are special matrices which are, I mean distinguishing them only in terms of complex versus real Hermitian matrix is one where A conjugate transpose equals A and a symmetric matrix is a real-valued matrix where A is equal to A transpose, but these arise naturally in a variety of applications.

So, for example, if you are looking at the, looking at a function f, which is defined from some n-dimensional real space, which I call D to the real line, then and suppose it is twice continuously differentiable. We define the Hessian of, Hessian matrix for this f as a matrix H of x which is an n cross n matrix whose entries are hij of x and each hij is the second partial derivative of this f with respect to xi and xj. So, this is a function of n variables, you take the second partial derivative with respect to xi and xj and this is a matrix that will be in R to the n cross n.

And a key property of this is very important in optimization and especially in optimization, differential equations, and so on. But, the key property of this partial derivatives is that dou squared f over dou xi do xj is equal to dou squared f over dou xj dou xi, is the fundamental property of these partial derivatives for every I, j equal to 1, 2, up to n. And as a consequence, H is equal to H transpose, that is it is symmetric. Similarly, if we look at what is known as the quadratic form.

So, suppose we are given A in R to the n cross n and x in r to the n, then consider Q of x which could be one example of this function f here, Q of x is equal to x transpose Ax. So, note that we can always write A as one half of A plus A transpose plus one half of A minus A transpose, this can always be written, basically half a transpose cancels with minus half a transpose, and a half A and half A add up to give A.

So, this is always true. So, if I substitute for this in here that means that Q of x is equal to half x transpose A plus A transpose x plus half x transpose A minus A transpose times X and if A is a symmetric matrix, then A may not even be symmetric here. So, if you just examine this x, x transpose A minus A transpose x this is equal to x transpose Ax minus x transpose A transpose x, but this is a scalar. So, its transpose is equal to itself. So, for any scalar, this is equal to scalar.

And so, the transpose of this is x transpose A transpose x and so these two are always going to be equal. So, this is equal to 0. And so, this Q of x is equal to one half x transpose A plus A transpose times x. Now, what that means is that A and one-half A plus A transpose generate the same quadratic form. So, basically that means, this is a very, very crucial point. So, that means that if you are interested in studying quadratic forms, it is sufficient to study quadratic forms generated by symmetric matrices. So, we will be studying quadratic forms generated by symmetric matrices.

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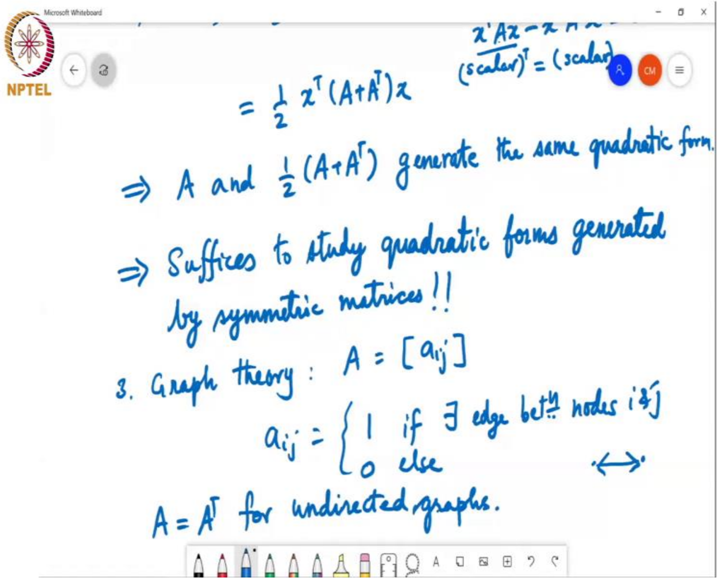

And finally, a third example is that I am out of time, so I will just very briefly mentioned it in graph theory. We define an adjacency matrix A graph is defined by a set of vertices and edges and an edge exits, so the set of vertices you can denoted by 1 to n. So, if there are n vertices in the graph, then you can label those vertices as 1 to n and edges are pairs of indices, pairs of vertices i, j, i1, j1, i2, j2, etcetera.

And an edge exists if the two nodes i and j are connected to each other. And we define an adjacency matrix to be a matrix A, which contains entries aij, where aij equals 1 if there exists an edge between nodes i and j and 0 otherwise. And if the graph is undirected, undirected means the edges do not have a direction associated with them.

So, you have two nodes, you define an edge like this, not with an arrow going like this. It is an undirected graph, undirected graphs. So, these are three examples where symmetric matrices arise. And so, with this background, we will start in the next class studying about Hermitian and symmetric matrices. They have lots of very intricate very beautiful properties and so we will cover them.