Matrix Theory
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Lecture 37
Unitary Matrices

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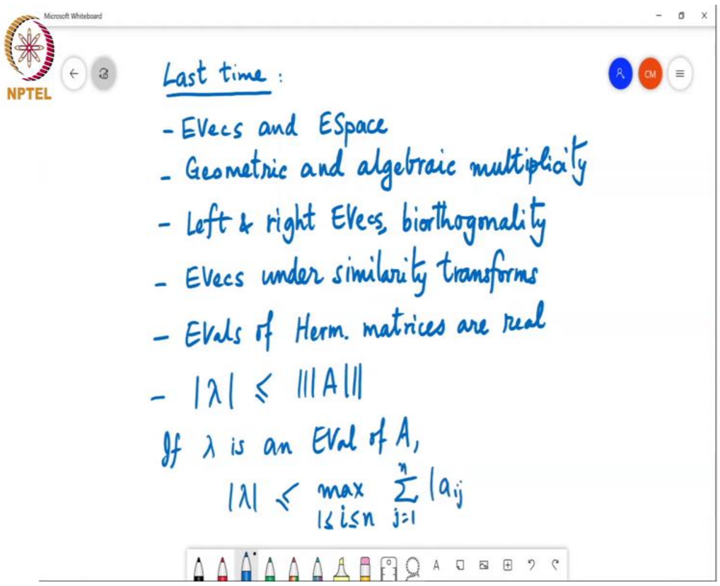

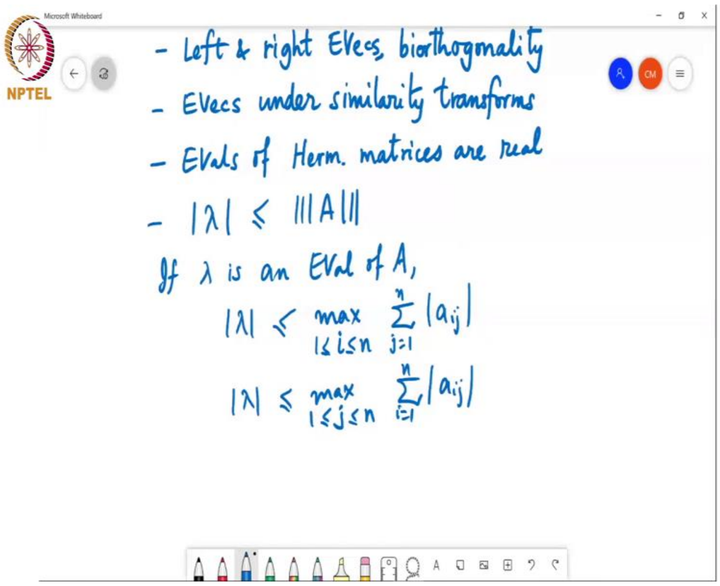

So, the last time we looked at several new things Eigen vectors and an Eigen space you can define with respect to particular Eigen values. We looked at geometry the concept of geometric and algebraic multiplicity of an eigenvalue. We also defined left and right Eigen vectors and the principle of y orthogonality namely that if you take two distinct eigenvalues of A matrix then any left eigenvector of the matrix corresponding to the first Eigen value is going to be orthogonal to any right eigenvector of the matrix corresponding to the other eigenvalue.

And we also looked at how Eigen vectors get changed under a similarity transform essentially the new Eigen vectors become s times the old Eigen vector. Also, we saw that if you take a real symmetric matrix or a complex Hermitian matrix its eigenvalues are always real. And if you have an operator norm then for any eigenvalue of the matrix A mod of lambda is less than or equal to the operator norm of the matrix A.

So, an immediate consequence of this is that if lambda is an eigenvalue of A then mod lambda is less than or equal to max 1 less than or equal to i less than or equal to n sigma j equal to 1 to n mod aij and mod lambda is less than or equal to max 1 less than or equal to j less than or equal to n sigma i equal to 1 to n mod aij. So, the mod rows terms or the mod columns sums. If you take the largest of them they will always be an upper bound on any the modulus of any Eigen value of the matrix. Why is this true?

Student: Was there l 1 L infinity norm of matrix?

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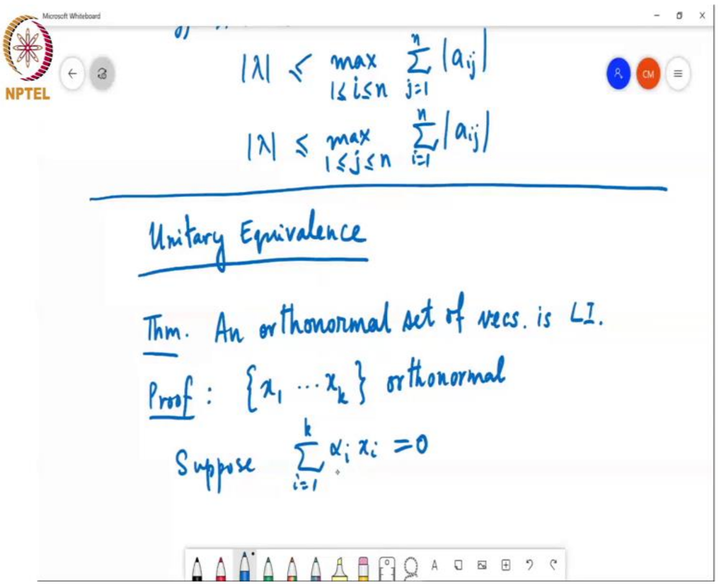

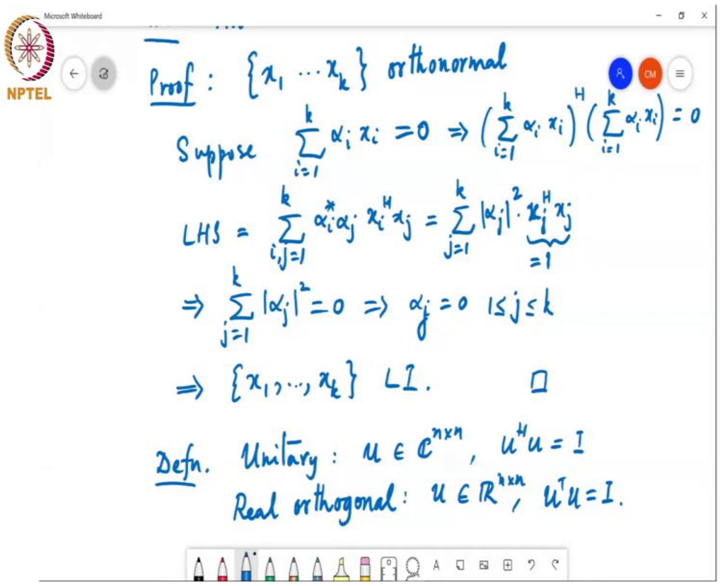

Professor: So, that is it. So, now we will continue on. So, the next thing I want to discuss is the idea of unitary equivalence. So, this is I mean just to kind of break the mystery here. Unitary equivalence or unitary similarity the first two similarity under unitary transformations. Remember that two matrices are similar if you A and B are similar if you can write B as S inverse A S for any invertible matrix S.

But if this S matrix S happens to be a unitary matrix. We say that A and B are unitarily similar or unitarily equivalent. And so basically that is the core idea here. And this unitary equivalence is very closely related to one very important theorem that I am going to discuss soon which is called the sure unitary triangular position theorem. So, it forms the basis for such as for that theorem. And So, this is like the prelude leading up to that theorem.

Also recall that we say a set of n vectors x1 to xn are orthogonal if the inner product between any pair of vectors is 0 as long as you are picking distinct vectors. In addition, if each of those vectors are have unit norm then we say that they are orthonormal. And when defining these things we typically only consider the usual inner product xi Hermitian xj and the usual Euclidean norm which is xi Hermitian xi.

And so then under this definition we say that these vectors form an orthonormal set. Also, if you are given a set of orthogonal vectors and the if these vectors are nonzero, you can obtain an orthonormal set of vectors from this set of orthogonal vectors by simply normalizing each of those vectors. So, for example if y1, y2 up to yk are orthogonal vectors and they are nonzero. Then if I define xi to be yi divided by square root of yi Hermitian yi.

Then these x1 x2 up to xk will form an orthonormal set of vectors. So, obviously orthonormal vectors are non-zero by definition. So, we have the following result an orthonormal set of vectors is linearly independent. This is very simple so I will just quickly write this out. So, if x1 through xk are an orthonormal set of vectors. Now, we need to show that they are linearly independent. And so, suppose sigma i equal to 1 to k alpha i xi equals 0 then we need to show that all these alphas must be equal to 0.

So, if this is the zero vector then we know that this implies sigma i equal to 1 to k alpha i xi Hermitian times sigma i equal to 1 to k alpha I xi is equal to 0. 0 vector inner product with itself will give you 0 and when I expand this inner product so the left-hand side is equal to sigma over i j equal to 1 to k alpha i alpha j xi Hermitian xj. But xi Hermitian xj is equal to 0 if i is not equal to j because they are a normal set of vectors.

So, this is equal to sigma say j equal to 1 to k alpha j there is a star missing here. Alpha j mod squared times xi xj Hermitian xj which is equal to 1 I will actually write it out. So, that it is clear xj Hermitian xj and this equals 1. So, this means that sigma j equal to 1 to k mod alpha j squared equals 0 which is only possible if alpha i equals 0 for alpha j equals 0 1 less than or equal to j less than or equal to k.

So, which means that x1 to xk are linearly independent. So, now we have seen this already but just for the sake of completeness. a unitary matrix is a matrix u in c to the n cross n such that u Hermitian u is the identity matrix. And at some point, we may need this. So, I will make a distinction between complex matrices and real valued matrices by calling the equivalent of this for real valued matrices as a real orthogonal matrix.

So, these are some abuse of terminology but just this is just for the sake of concreteness when we are using these matrices. So, when I say it is real orthogonal, I just mean u transpose u equal to identity and U is a real valued n cross n matrix.

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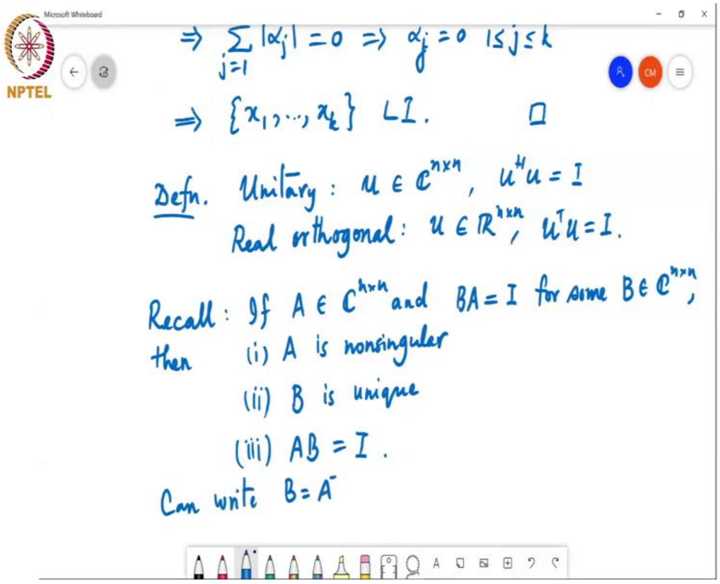

Now before I state the next result, I want to recall one little property that again we have seen earlier. So, if A is in C to the n cross n and BA equals the identity matrix for some B belonging to C to the n cross n. Then 1 A is non-singular, 2 B as unique, and 3 AB is also equal to a identity matrix. And so, as a consequence we can write B equals A inverse.