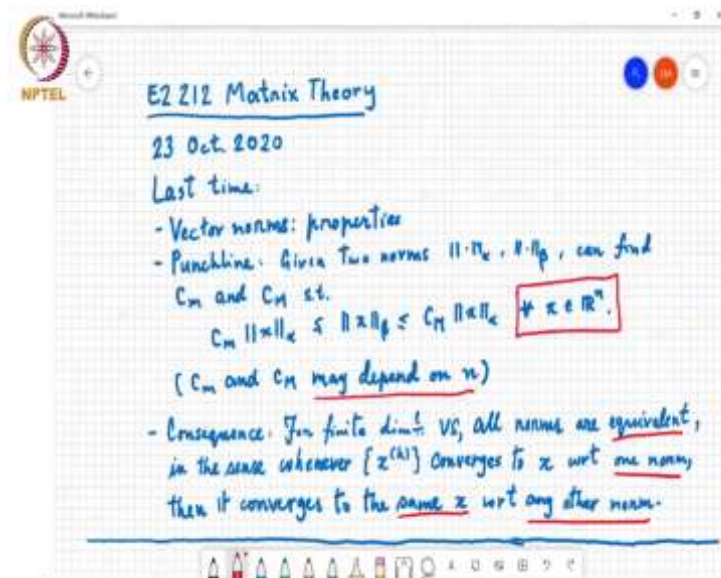


Matrix Theory
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Summary of equivalence of norms

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So, last time we were looking at properties of vector norms and the punchline was that given two norms α and β , we can find constant C small m and C capital M , such that the β norm of x is sandwiched between C small m times the α norm of x and C capital M times the α norm of x for every x in \mathbb{R}^n .

So, that is the key part is that this is a bound as valid regardless of which x you choose in \mathbb{R}^n and these constant C small m and C capital M may depend on n the dimension of the space over which you want to find these bounds. The consequence of this is that over finite dimensional vector spaces, all norms are equivalent in the sense that whenever x_k converges to x with respect to say the α norm, then it converges to the same x with respect to any other norm.

So, in particular, since all norms are equivalent, all norms are equivalent to the infinity norm, which is the max entry, which intern means that if the limit of x_k as k goes to infinity is x with respect to any norm that is true, if and only if the limit as k goes to infinity, the i th entry of x_k being equal to x_i for i equal to 1, 2 up to n that means, component wise convergence is equivalent to convergence with respect to any norm.

So, the other thing here is the same x if it converges to this one norm it converges to respect to any other norm. So, this is what we saw in the previous class. So, today we will proceed with a little bit more discussion about norms.